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Children as Family Public Goods: Some Implications for Fertility

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A two-stage bargaining model is developed to describe how fertility decisions are made in a strategic family setting. Given the assumption that family contracts are incomplete and cannot be used to enforce optimal behavior, it is shown that investments in children (i.e. the fertility rate) may be sub-optimal. This is because the woman may find it in her interest to invest too little in children in stage 1 of the model in order to protect her bargaining status in stage 2. I then consider in the context of this model the impact on fertility rates of changes in child custody rules (in the case of divorce), the wage rate, and the male-female wage differential. I conclude by exploring how the introduction of child subsidies can change the results. (*JEL* D13, H55, J13, J14, J22)

Keywords: Family bargaining; Fertility; Child subsidies; Labor Participation Rate

*This article is based on chapter 5 of my PhD dissertation.

1 Introduction

The extent to which children are 'social public goods' will differ from one country to another. Children have public good characteristics in countries like Italy where the fertility rate is significantly below reproduction-rate and the pension system is of the pay-as-you-go type. Children are public goods to a far lesser extent in developing countries where fertility rates are high or in countries that do not have pay-as-you-go pension systems. The extent to which children are public goods therefore depends not only on the fertility rate of a country but also on the institutional setting. Large budget deficits and pay-as-you-go pension systems increase the public good value of children.

Whether or not children should be viewed as public goods from a societal perspective, they certainly exhibit some of the characteristics of public goods inside the family. In a family partnership consisting of both a woman and a man it is typically the woman who incurs most of the costs of having children – both physically through pregnancy and in terms of opportunity costs on the job market – while both parents can equally enjoy the benefits from having children.

I attempt here to model individuals' fertility behavior in a family setting. I look at two different ways in which a family might be organized: first, a traditional way where husband and wife attempt to maximize a joint family utility function and second, a more strategic bargaining approach where individuals' still work together but attempt to secure their bargaining position while doing so. I consider how changes in outside options like wage-rates and divorce laws may alter fertility behavior for both of these alternative family models.

In the Family Economics literature there are two main ways of modelling family decisions. The first class consists of joint family utility models in the tradition of Samuelson (1956) and Becker (1965, 1981). Becker's first models on the family in the 1960's were the beginning of a school of thought called 'New Home Economics'. The starting point was Becker's belief that economic reasoning is not limited to market transactions but provides a useful framework for understanding all human behavior. Models in the tradition of new home economics treat families as 'little firms' that try to maximize 'family production'. These models assume that there exists a joint family utility function that reflects the interests of all family members. The complementarity between the family members (husband and wife) determines the size of the 'marriage gains'. Since family gains arise from specialization, family utility is maximized when family members allocate their investments and their working time to that sector where they have the higher comparative advantage (the sectors being market work or household activities). Each person maximizes personal as well as total family utility by choosing the optimal investment level of human capital in either the household or the market sector. Given that all family members are identical in the beginning, these models predict equal distribution of 'family gains' among the family members. Models in this tradition (e.g. Becker, Murphy and Tamura 1990) acknowledge the impact of outside options on the fertility rate of households and predict that the demand for children falls when they become more expensive. But while these models make explicit the decision process inside families with respect to having children, they do so in an environment that has little resemblance with the 'real world'. Maximizing a joint family utility function in a 'first best world' necessarily leads to a Pareto optimal outcome but – as Pollak (1985) stresses – "the New Home Economics literature ignores the internal organization and structure of families and households". In my view, this approach can serve as an important benchmark, but does not capture the decision process of self-interested individuals.

The second class consists of bargaining models, most of the time using cooperative game theory (see for example Pollak 1985, Eswaran 2002, Murphy 2002, Stevenson 2007, and Iyigun and Walsh 2007). Bargaining models employ game theory to model family behavior. These models use individual utility functions and do not assume the existence of a joint family utility function. "Spouses are assumed to have conflicting preferences and to resolve their differences in the manner prescribed by some explicit bargaining model" (Pollak, 1985). These models stress the importance of outside options (threat points) in determining the distribution of resources inside families.

In this study I construct a bargaining model to explore how intra-family bargaining impacts on fertility choices. The model allows me not only to analyze the intra-family bargaining in stage 2 but also the individual family members' strategic behavior in stage 1 which determines their threat-points. In this sense my model differs from most of the bargaining literature which typically takes the individuals' threat-points as exogenously given (for a recent exception see Iyigun and Walsh 2007).[1]

My model has two stages. In the first stage the woman has to decide on her relative investment in having children and human capital accumulation. The man spends all his time accumulating human capital. At the end of stage 1, the woman and man can divorce. In stage 2, the woman and man must each decide how to allocate their time between caring for the children and working in the labor market. Divorce affects each spouse's utility since children are treated as intra-family public goods. They cease to be public goods after divorce. Hence there is a marriage surplus. It is assumed this surplus is split according to the Nash (1950) bargaining rule. I further assume that the marriage contract the spouses enter into is incomplete. By that I mean that it is either not possible to write a contract that specifies the optimal future behavior of the spouses or one that regulates future pay-offs depending on optimal behavior. This concept of 'incomplete contracts' is borrowed from the Industrial Organization literature where it is often discussed in connection with the so-called 'hold-up' problem (see e.g. Hart 1995). I show how the woman may invest more time in human capital accumulation in stage 1 and less in children than is optimal (in terms of maximizing family utility) in order to improve her bargaining position in stage 2. This is because the bargaining power of the woman in the model is determined during stage 1 when investments in human capital are made. Investments in market-related human capital generally increase a woman's bargaining power as she can always recoup the return on this investment (in terms of wages) during stage 2. Stage 1 investments in children

[1] My model is related to those of Eswaran (2002) and Iyigun and Walsh (2007). These authors, however, approach the problem from the perspective of developing countries undergoing demographic transitions. My model by contrast is not couched in a development context.

generally do not increase a woman's bargaining power as she typically will not recoup all the returns on her investments. This situation occurs in part because of divorce custody laws (the man may receive partial or full custody over the children in stage 2 even though the woman incurred the investment cost of children) and also because there is no outside market for her investment in period 2 – that is other people do not derive the same level of utility from the couple's children as they do themselves.

Having developed the model I then explore the impact of changes in child custody rules in the case of divorce, changes in the wage rate, and changes in the male-female wage differential on fertility rates. I conclude by considering how the introduction of child subsidies can change the results. In particular, I show that child subsidies can help ameliorate the problem of an inefficiently low fertility rate. This result ties in with the finding of Cigno, Luporini and Pettini (2003) and Cigno and Werding (2007) that unfunded public pension systems can also cause the fertility rate to be inefficiently low. These authors likewise recommend child subsidies as one way of dealing with this problem. My findings here therefore complement theirs.

2 The Model

A two stage model is examined. To facilitate the analysis it is assumed that couples are already matched up before the commencement of the "game" and that husband and wife have identical human capital at that point.[2] It is assumed that the spouses are rational individuals with stable preferences. Marriage occurs in order to maximize utility, and there is no altruism. The utility functions are increasing in income and children. Children enter into the utility function through some combination of quantity and quality, where quality, in turn, is an increasing function of the time parents spend at home with the children.

I consider two periods: the investment period (stage 1) and period 2 which represents 'the rest of time' (stage 2). All future consumption is collapsed into period

[2]Assortive mating (see for example Becker 1981) is therefore not considered here.

2.

Stage 1:

Stage 1 is the investment period. A woman has one unit of time which can be allocated either to the accumulation of human capital or to having children. Let t_1^F denote the amount of time allocated to human capital accumulation. It follows that $1 - t_1^F$ is allocated to having children. The total amount of human capital accumulated is given by the function $H(t_1^F)$, where H is increasing in t_1^F. The number of children a woman has is determined by the function $G(t_1^F)$, where $G(1) = 0$, $G'(t_1^F) < 0$ and $G''(t_1^F) < 0$.

It follows, therefore, that the opportunity cost of having children is the human capital that the woman could have accumulated during this time in the market. The woman's decision problem in period 1 is therefore to choose the optimal level of t_1^F.

A man can also engage in human capital accumulation, with the same return per unit of time as a woman. The essential asymmetry of the model is that only a woman can have children. This means that a man does not have any decision to make in stage 1. His complete allocation of time will be devoted to human capital accumulation. For the remainder of the model, it will mainly be this difference in human capital accumulation in stage 1 that will make men distinguishable from women.[3]

Stage 2:

The 'quality' of the children is determined in stage 2. In stage two the woman again has an allocation of one unit of time. This can be devoted to either working in the labor market (t_2^F) or to staying at home looking after the children ($1 - t_2^F$). If the woman has no children in stage 1, she will necessarily spend all her time in stage 2 in the labor market. I will assume that a woman's labor income (y^F) is equal to the product of the time she spends working and her level of human capital:

$$y^F = t_2^F H(t_1^F).$$

Time spend at home improves the quality of the children. However, the extent of the

[3] An additional potential difference between men and women will be that we allow for differences in male and female return on identical levels of market related human capital (see discussion below).

quality improvement depends both on the woman's and man's time spent at home. It is in this sense that children are family public goods in this model.

Before discussing further this issue of child quality, I consider first the man's allocation of time in period 2. The situation here is entirely symmetric with that of the woman. The man devotes time t_2^M to the labor market, and time $1 - t_2^M$ to staying at home looking after the children. The man's labor income is determined as follows:

$$y^M = \alpha t_2^M H(1).$$

The parameter α captures wage differentials between men and woman. When $\alpha > 1$, this implies a man is paid more than s woman with the same level of human capital. The reverse is true when $\alpha < 1$. The empirical evidence suggests that $\alpha > 1$ (see for example Oaxaca 1973, Gunderson 1989, Card and DiNardo 2002, and Blau and Kahn 2003).

The utility function of each player in this model has two arguments. The first argument is income. It should be noted that income here is not pooled. The man and woman have separate incomes. The second argument is a quality adjusted child quantity index, denoted here by D. The quality adjusted child quantity index is defined as follows:

$$D = D[t_2^F, t_2^M, G(t_1^F)],$$

where D is a decreasing function of t_2^F, t_2^M and t_1^F. That is, the quality adjusted child quantity index is an increasing function of the number of children a woman has at stage 1, and of the time spent at home by both the woman and man looking after the children in stage 2. The man and woman are assumed to be equally well adapted to looking after the children and doing household tasks in stage 2. That is, the woman's stage 1 investment in having children does not make her any better equipped than her husband in educating and caring for them during stage 2. To simplify matters, I will assume that D has the following functional form:

$$D = (2 - t_2^F - t_2^M)G(t_1^F).$$

An important implication of this functional form is that when both parents spend all their time working in stage 2, neither derives any utility from their children. To put it another way, when both parents spend all their time working in stage 2, the quality adjusted child quantity index is zero in both parents' utility function irrespective of the total number of children. However, as long as one parent spends some time with the children during stage 2 both parents reap the rewards and derive positive utility from their children.

I make three further simplifications. First, I assume that human capital accumulation is a linear function of time invested.[4] That is,

$$H(t) = h \times t.$$

Second, I assume that stage 2 income is a linear function of accumulated human capital. Third, I assume that utility is equal to the sum of income and the quality adjusted child quantity index. Hence, a married woman's utility function can be written as follows:

$$U_m^F = ht_2^F t_1^F + (2 - t_2^F - t_2^M)G(t_1^F). \tag{1}$$

Similarly, a married man's utility function is as follows:

$$U_m^M = \alpha h t_2^M + (2 - t_2^F - t_2^M)G(t_1^F). \tag{2}$$

From a comparison of (1) and (2) it can be seen that the quality adjusted child quantity index enters into both parents utility functions, and hence children are family public goods.

[4]Assuming constant returns to scale in human capital accumulation as a function of time invested is a common assumption in the literature. However, Trostel (2004) presents empirical evidence that suggests the existence of significant increasing returns at low allocations of time and significant decreasing returns at high allocations of time to human capital accumulation. But even taking these findings into account assuming a linear human capital production function is still a good first approximation.

3 Becker Type Household Decisions

As a reference case, let us first assume with Becker (1981) that the joint family utility function gets maximized. Family behavior is then the result of maximizing a single household utility function that describes the joint interests of all household members.

The maximization of the Joint Family Utility function is determined by backward induction. First the optimal stage 2 time allocations (the values of t_2^F and t_2^M) are derived as functions of t_1^F. Then the optimal stage 1 investment in children (the value of t_1^F) is determined. Joint family utility is given by adding the previously considered individual utility functions of husband and wife when married. That is, the joint family utility function is determined by adding (1) and (2):

$$\begin{aligned} U_m^F + U_m^M &= ht_1^F t_2^F + (2 - t_2^F - t_2^M)G(t_1^F) + \alpha h t_2^M + (2 - t_2^F - t_2^M)G(t_1^F) \\ &= ht_1^F t_2^F + \alpha h t_2^M + 2(2 - t_2^F - t_2^M)G(t_1^F). \qquad (3) \end{aligned}$$

Treating t_1^F as given in stage 2, the following stage 2 first order conditions are obtained:

$$\frac{\partial(U^F + U^M)}{\partial t_2^F} = ht_1^F - 2G(t_1^F),$$

$$\frac{\partial(U^F + U^M)}{\partial t_2^M} = \alpha h - 2G(t_1^F).$$

Given that t_2^F and t_2^M do not feature in either of these first order conditions it follows that we will only observe corner solutions in stage 2. There are three possible cases:
(i) $t_2^F = t_2^M = 0$: both husband and wife spend stage 2 looking after children only.
(ii) $t_2^F = 0, t_2^M = 1$: the wife spends all of her stage 2 time at home with the children while the husband works full time.
(iii) $t_2^F = t_2^M = 1$: both spouses spend stage 2 working in the market only.
The case where $t_2^F = 1$ and $t_2^M = 0$ cannot be observed when $\alpha \geq 1$, since the fact that $t_1^F \leq 1$ implies that it is not possible for $\partial(U^F + U^M)/\partial t_2^F > 0$ and $\partial(U^F + U^M)/\partial t_2^M < 0$ to hold simultaneously. To determine the optimal solution, the joint utilities of these three cases must be considered in turn.

Case (i):

$$U^F + U^M = 4G(t_1^F)$$

It follows that for the case in which both partners find it optimal to spend stage 2 looking after their children only, the joint family utility function (3) is maximized at $t_1^F = 0$. To check that the solutions for stage 2 assumed here are consistent with the stage 1 solution, I substitute $t_1^F = 0$ back into the first order conditions for stage 2, as follows:

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^F}\right|_{t_1^F=0} = -2G(0) < 0,$$
$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^M}\right|_{t_1^F=0} = \alpha h - 2G(0).$$

Given that $\partial(U^F + U^M)/\partial t_2^F|_{t_1^F=0} < 0$, it follows that $t_1^F = 0$ implies that $t_2^F = 0$. However, $\partial(U^F + U^M)/\partial t_2^M|_{t_1^F=0} < 0$ only holds when $\alpha h < 2G(0)$. That is, the man's wage income is smaller than the maximum possible family gain he could get. Hence this solution is only internally consistent when this inequality is satisfied. In this case, $U_m^F + U_m^M = 4G(0)$.

Case (ii):

$$U^F + U^M = \alpha h + 2G(t_1^F)$$

For cases where the wife spends all of her stage 2 time looking after children while the husband spends all of his time in the market, $U^F + U^M$ is again maximized at $t_1^F = 0$. It follows from the results obtained for case (i) that this solution is only internally consistent when $\alpha h > 2G(0)$. In this case, $U_m^F + U_m^M = \alpha h + 2G(0)$.

Case (iii):

$$U^F + U^M = ht_1^F + \alpha h$$

It follows that when both partners spend all of stage 2 in the market, (3) is maximized at $t_1^F = 1$. Substituting $t_1^F = 1$ back into the first order conditions for stage 2, yields the following:

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^F}\right|_{t_1^F=1} = h > 0,$$

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^M}\right|_{t_1^F=1} = \alpha h > 0.$$

Hence when $t_1^F = 1$, it follows that $t_2^F = t_2^M = 1$. The solution therefore is internally consistent. In this case, $U_m^F + U_m^M = (1+\alpha)h$.

Therefore, in cases (i) and (ii), $t_1^F = 0$ (the woman specializes in children only), while in case (iii), $t_1^F = 1$ (the woman specializes in market activities only). From a comparison of these three cases, it can be seen that case (i) is optimal when $2G(0) > \alpha h$, case (ii) is optimal when $h < 2G(0) < \alpha h$, and case (iii) is optimal when $2G(0) < h$. When $2G(0) > h$, the maximum family gain a woman can get exceeds her maximum possible wage income. Case (ii) is never optimal when $\alpha = 1$ (except in the special case where $2G(0) = h$, where all three cases generate the same family utility).

No interior solutions are observed in the specification of the model considered here. It should be noted, however, that interior solutions in the stage 2 choices of the woman and man may arise in more general specifications of the model, such as a utility function that has interaction terms involving money income and the quality adjusted child quantity index, or a more flexible D function.

In this section it is assumed that both spouses behave in a manner consistent with maximization of the sum of their individual utilities. That is, we assume cooperation between husband and wife with respect to stage 1 as well as stage 2 behavior. However, it is important to note that when the cooperative solution is case (ii), where the man specializes in working and the woman in having and caring for children, the cooperative solution will not necessarily coincide with the noncooperative solution. In the absence of a transfer from the man to compensate her for the income she has lost from having and caring for children, the woman may switch from specializing in children under joint maximization to specializing in working in the noncooperative case. This is because the woman's utility level in this case is $G(0)$. If instead, she specializes in accumulating human capital in stage 1, and working in stage 2, her utility level is h. Assuming $\alpha < 2$, in this case it will follow that $G(0) < h$, and hence the woman's utility is higher when $t_1^F = t_2^F = t_2^M = 1$ than when $t_1^F = t_2^F = 0$, $t_t^M = 1$, even though the latter maximizes family utility.[5]

The Becker model focuses on maximizing family utility on the grounds that it must then be possible to compensate all family members in such a way to make this alternative a Pareto improvement on all other alternatives. However, this ignores the fact that the woman may have to commit to her human capital level prior to the payment of any compensation, and that it may not be possible to commit to this payment in advance. In such a situation, the family may not be able to achieve the family utility maximizing outcome. I investigate this issue in the next section.

4 Bargaining Approach

In this section it is assumed that each family member tries to maximize his or her own utility function rather than a joint family utility function. If the family members could agree before the investment period on how to distribute the gains from their 'cooperation' and write a binding contract they could collectively receive the same outcome as if they maximized a joint household utility function. The situation considered here is one in which no such binding agreements can be made. In the industrial organization literature such a situation is referred to as the "hold-up" problem (see Tirole 1988 and Hart 1995).[6] In addition, I assume in stage 2 the family can either remain married or get divorced. If married, gains from marriage are split via a bargaining process. If the partners get divorced each realizes his/her outside options.

4.1 The Possibility of Divorce

It is assumed here that no-fault divorce can occur in period 2. That is, the divorce can be initiated by either spouse without consent of his or her partner. Zelder (1993)

[5]It is always optimal for the man to choose $t_2^M = 1$ when $t_1^F = 1$, since there are then no children and hence no reason to stay at home in stage 2.

[6]The two most common reasons for the existence of the hold-up problem are transaction costs in writing a complete contract and the impossibility of specifying all possible future states that a complete contract would have to cover.

investigates the effect of the public good, children, on the occurrence and efficiency of divorce when the legal regime switches from fault to no-fault divorce.[7] He concludes that more inefficient divorces will occur under a no fault divorce regime because of the non-transferability of the family public good children. He does not, however, investigate how different divorce regulations influence the provision of the public good 'children'. This is exactly the point I want to address.

I borrow from Zelder (1993) the concept of a visitation rate, that regulates the division of the public good between the parents in case of a divorce. Let the husband's visitation rate be v and the wife's visitation rate (1-v), with v lying in the interval [0,1]. If the family remains married during stage 2 both have unlimited 'access' to their children so the effective visitation rate is 1 for both of them. If divorce occurs, children cannot be enjoyed by both parents simultaneously and therefore children cease to be family public goods.

The different custody rules in case of a divorce can be expressed in terms of the visitation rule:

1) $v = 0$: the woman gets sole custody over the children in case of divorce.

2) $v = 1$: the man gets sole custody over the children in case of divorce.

3) $v = 1/2$: joint custody.

The impact of a divorce therefore is to set an upper bound on the amount of time each parent can spend with the children. The bounds are as follows:

$$t_2^F \geq v, \quad t_2^M \geq 1 - v.$$

That is, when divorced, the man can spend a maximum proportion v of his time allocation in stage 2 with the children, while the woman can spend a maximum of $1 - v$ of her time in stage 2 with the children.

Divorce changes the utility maximization problem of the woman and the man. If divorce occurs, children cease to be a family public good and gains from marriage will

[7]In this setting divorce is efficient if total joint utility in the married state is no larger than the sum of individual utilities in the divorced state.

be zero. That is, the children respond positively to the time their parents spend with them, which in turn increases the utility the parents derive from the children. Both parents therefore benefit from the time either parent spends with the children if they all live in the same household. If, by contrast, the parents have divorced, the positive effect on the children is not shared by the absent parent.

4.2 The Dynamics of the Bargaining Game

I consider a bargaining game with two parts. The first corresponds to stage 1 discussed above. That is, the woman decides on how to allocate her stage 1 time between accumulating human capital and having children and chooses t_1^F. In the second part, the woman and man must decide whether to stay married or to divorce. They stay married if they are able to agree on how to split the marriage surplus. If they are unable to reach agreement they divorce. They also choose how to allocate their period two time between work and children by choosing t_2^F and t_2^M, respectively. In the case of divorce, these choices are constrained by the visitation rule.

4.2.1 Nash Bargaining

I assume that the division of the marriage surplus is determined by Nash (1950) bargaining. That is, during stage 2 the couple will decide on how to split the marriage surplus in a cooperative way. Nash bargaining maximizes the product gain from cooperation. It leads to a situation where each partner receives his/her divorce utility (which corresponds with his/her threat-point) plus half of the marriage gains. The model is again solved by backward induction. That is, both husband and wife first calculate their optimal stage 2 behavior and corresponding pay-offs for all possible stage 2 starting positions. Then, given this information, they calculate their optimal stage 1 behavior. Note again, that I assume that no binding contract can be written specifying either party's stage 1 or stage 2 behavior in advance. That is, neither the woman's optimal stage 1 investment in children nor an optimal stage 2 transfer payment can be made part of a binding contract.

Stage 2

The total marriage gain is given by the difference between the sum of their married and divorce utilities. It is calculated as follows:

$$\Delta U^{F+M} = \max(U_m^F + U_m^M) - U_d^F - U_d^M,$$

where U_m^F and U_m^M denote the utility of the woman and man when they are married, and U_d^F and U_d^M denote the utility of the woman and man when they are divorced.

$$U_m^F = ht_1^F t_2^F + (2 - t_2^F - t_2^M)G(t_1^F)$$

$$U_m^M = \alpha h t_2^M + (2 - t_2^F - t_2^M)G(t_1^F)$$

$$U_d^F = ht_1^F t_2^F + (1 - t_2^F)G(t_1^F),$$

$$\text{subject to} \quad t_2^F \geq v.$$

$$U_d^M = \alpha h t_2^M + (1 - t_2^M)G(t_1^F),$$

$$\text{subject to} \quad t_2^M \geq 1 - v.$$

Substituting in from the individual utility functions we obtain the following:

$$\Delta U^{F+M} = ht_1^F(\hat{t}_2^F - \tilde{t}_2^F) + \alpha h(\hat{t}_2^M - \tilde{t}_2^M) + [2(2 - \hat{t}_2^F - \hat{t}_2^M) - (2 - \tilde{t}_2^F - \tilde{t}_2^M)]G(t_1^F),$$

where $\hat{t}_2$ denotes a stage two married time allocation, and $\tilde{t}_2$ a stage 2 divorced time allocation.

Given this information about the marriage surplus and how it will be divided in stage 2 if they remain married, the woman now faces the following utility maximization problem: Her utility function V^F is given by the sum of her divorce utility plus her share of the marriage gains. It is assumed that the marriage gains are split via Nash bargaining. Given that t_1^F is already fixed by the beginning of the bargaining process, then, assuming the woman and man have equal bargaining abilities, it follows that they split the marriage surplus equally. The woman's overall utility function therefore becomes:

$$V^F = ht_1^F \tilde{t}_2^F + (1 - \tilde{t}_2^F)G(t_1^F) + \Delta U^{F+M}/2$$

$$= \frac{ht_1^F(\tilde{t}_2^F + \hat{t}_2^F)}{2} + \frac{\alpha h(\hat{t}_2^M - \tilde{t}_2^M)}{2} + \left[(2 - \hat{t}_2^F - \hat{t}_2^M) + \frac{(\tilde{t}_2^M - \tilde{t}_2^F)}{2}\right] G(t_1^F).$$

In stage 2 she will maximize this utility function with respect to $\tilde{t}_2^F$ as well as $\hat{t}_2^F$ to find her optimal behavior when divorced as well as married. This yields the following stage 2 first order conditions:

$$\frac{\partial V^F}{\partial \tilde{t}_2^F} = \frac{ht_1^F}{2} - \frac{G(t_1^F)}{2},$$

$$\frac{\partial V^F}{\partial \hat{t}_2^F} = \frac{ht_1^F}{2} - G(t_1^F).$$

Similarly, the man's overall utility function is now as follows:

$$V^M = \alpha h \tilde{t}_2^M + (1 - \tilde{t}_2^M)G(t_1^F) + \Delta U^{F+M}/2$$

$$= \frac{\alpha h(\tilde{t}_2^M + \hat{t}_2^M)}{2} + \frac{ht_1^F(\hat{t}_2^F - \tilde{t}_2^F)}{2} + \left[(2 - \hat{t}_2^F - \hat{t}_2^M) + \frac{(\tilde{t}_2^F - \tilde{t}_2^M)}{2}\right] G(t_1^F).$$

Maximizing this utility function with respect to $\tilde{t}_2^M$ as well as $\hat{t}_2^M$ yields the following stage 2 first order conditions:

$$\frac{\partial V^M}{\partial \tilde{t}_2^M} = \frac{\alpha h}{2} - \frac{G(t_1^F)}{2},$$

$$\frac{\partial V^M}{\partial \hat{t}_2^M} = \frac{\alpha h}{2} - G(t_1^F).$$

Given that $\tilde{t}_2^F$, $\hat{t}_2^F$, $\tilde{t}_2^M$ and $\hat{t}_2^M$ do not feature in any of these first order conditions, it follows again that we will only observe corner solutions in stage 2. Both parties' optimal period two behavior is determined by the values of h and α as well as t_1^F.

In stage 2 the spouses must decide whether to stay married or get divorced. The fact that the gains from marriage are always positive when children are present means that rational individuals will never divorce in stage 2 if there was investment in children during state 1 ($t_1^F < 1$). If no children are present, divorce may occur during stage 2. As marriage gains are equal to zero in that case anyway, there is no difference between the woman's and man's married and divorce payoffs.

Stage 1

During stage 2 it is assumed that the couple behaves in a cooperative way and splits gains from marriage according to the Nash bargaining rule. However, investment choices in stage 1 are made non-cooperatively. That is, rather than maximizing the joint family utility function, each family member chooses their behavior so as to maximize their own private utility function. Considering first the stage 2 first order conditions for the woman, it can be seen that there are three possible cases.

(i-F) $ht_1^F/2 < ht_1^F < G(t_1^F) \Rightarrow \tilde{t}_2^F = v, \hat{t}_2^F = 0$

She spends as much time as possible with children in stage 2

(ii-F) $ht_1^F/2 < G(t_1^F) < ht_1^F \Rightarrow \tilde{t}_2^F = 1, \hat{t}_2^F = 0$

She stays with children when married but works full time when divorced in stage 2.

(iii-F) $G(t_1^F) < ht_1^F/2 < ht_1^F \Rightarrow \tilde{t}_2^F = 1, \hat{t}_2^F = 1$

She works all the time during stage 2 whether or not she is married.

Similarly for the man, there are also three possible cases for stage 2:

(i-M) $\alpha h/2 < \alpha h < G(t_1^F) \Rightarrow \tilde{t}_2^M = 1 - v, \hat{t}_2^M = 0$

He spends as much time as possible with children in stage 2.

(ii-M) $\alpha h/2 < G(t_1^F) < \alpha h \Rightarrow \tilde{t}_2^M = 1, \hat{t}_2^M = 0$

He stays with children when married but works full time when divorced in stage 2.

(iii-M) $G(t_1^F) < \alpha h/2 < \alpha h \Rightarrow \tilde{t}_2^M = 1, \hat{t}_2^M = 1$

He works all the time during stage 2 whether or not he is married.

Using the fact that $t_1^F \leq 1$ as well as $\alpha \geq 1$, the female and male cases can be linked as follows:

(i-F) is consistent with any of (i-M), (ii-M) and (iii-M),

(ii-F) is consistent with (ii-M) and (iii-M),

(iii-F) is consistent only with (iii-M),

(i-M) is consistent only with (i-F),

(ii-M) is consistent with (i-F) and (ii-F),

(iii-M) is consistent with any of (i-F), (ii-F) and (iii-F).

In total, therefore, there are six cases that must be considered. I will solve for the optimal value of t_1^F, denoted here by $\bar{t}_1^F$, for each in turn. This is done for each of the 3 possible divorce visitation rules (i.e. $v = 0$, $v = 0.5$, and $v = 1$).

4.3 Discussion of the Six Possible Cases

Case 1 ((i-F) and (i-M)): Both spend as much time as possible with children in stage 2.

$$V^F = \frac{ht_1^F v}{2} + \left(\frac{5}{2} - v\right) G(t_1^F) - \frac{\alpha h(1-v)}{2}.$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{hv}{2} + \left(\frac{5}{2} - v\right) G'(t_1^F)$$

$$\Rightarrow G'(\bar{t}_1^F) = -\frac{hv}{5-2v}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{h}{8}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{h}{3}$$

It was assumed earlier that $G(t_1^F)$ had the following properties: $G(1) = 0$, $G'(t_1^F) < 0$ and $G''(t_1^F) < 0$. As $0 < h/8 < h/3$, we know that in this case the woman's optimal investment in children during stage 1 decreases with increases in the husband's stage 2 visitation rights in case of divorce. For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$, $\partial V^F/\partial \hat{t}_2^F$, $\partial V^M/\partial \tilde{t}_2^M$ and $\partial V^M/\partial \hat{t}_2^M$ are all negative when evaluated at $\bar{t}_1^F$. These inequality conditions yield the following four constraints:

$$\frac{\partial V^F}{\partial \tilde{t}_2^F} < 0 \Rightarrow h\bar{t}_1^F < G(\bar{t}_1^F),$$

$$\frac{\partial V^F}{\partial \hat{t}_2^F} < 0 \Rightarrow h\bar{t}_1^F < 2G(\bar{t}_1^F),$$

$$\frac{\partial V^M}{\partial \tilde{t}_2^M} < 0 \Rightarrow \alpha h < G(\bar{t}_1^F),$$

$$\frac{\partial V^M}{\partial \hat{t}_2^M} < 0 \Rightarrow \alpha h < 2G(\bar{t}_1^F).$$

If the third condition is satisfied, all the others must be as well.

Case 2 ((i-F) and (ii-M)): Both spend all their stage 2 time with children when married but husband does work full-time when divorced in stage 2.

$$V^F = \frac{ht_1^F v}{2} - \frac{\alpha h}{2} + \left[2 + \frac{(1-v)}{2}\right] G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{hv}{2} + \left[2 + \frac{(1-v)}{2}\right] G'(t_1^F).$$

$$\Rightarrow G'(\bar{t}_1^F) = -\frac{hv}{5-v}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{h}{9}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{h}{4}$$

As $0 < h/9 < h/4$, the woman's optimal stage 1 investment in children is again decreasing as the husband's visitation right in case of divorce (v) increases.

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$, $\partial V^F/\partial \hat{t}_2^F$, and $\partial V^M/\partial \hat{t}_2^M$ are all negative while $\partial V^M/\partial \tilde{t}_2^M$ is positive, when evaluated at $\bar{t}_1^F$. These inequality conditions again yield four constraints. The only difference with case (i) is that now the direction of the inequality sign in the third condition is reversed. It is now necessary to check the third condition and whichever is most binding out of the first and fourth conditions.

Case 3 ((i-F) and (iii-M)): Wife spends all of stage 2 with children while husband works full-time.

$$V^F = \frac{ht_1^F v}{2} + \left[1 + \frac{(1-v)}{2}\right] G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{hv}{2} + \left[1 + \frac{(1-v)}{2}\right] G'(t_1^F)$$

$$\Rightarrow G'(\bar{t}_1^F) = -\frac{hv}{3-v}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{h}{5}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{h}{2}$$

As $0 < h/5 < h/2$, the woman's optimal stage 1 investment in children is again decreasing as the husband's visitation right in case of divorce (v) increases.

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$ and $\partial V^F/\partial \hat{t}_2^F$ are negative, and $\partial V^M/\partial \hat{t}_2^M$ and $\partial V^M/\partial \tilde{t}_2^M$ are positive, when evaluated at $\bar{t}_1^F$. Now it is the first and third conditions that are most binding.

Case 4 ((ii-F) and (ii-M)): Both spend all time with children when married and work full-time when divorced.

$$V^F = \frac{ht_1^F}{2} - \frac{\alpha h}{2} + 2G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{h}{2} + 2G'(t_1^F)$$

$$\Rightarrow G'(\bar{t}_1^F) = -\frac{h}{4}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$ and $\partial V^F/\partial \tilde{t}_2^M$ are positive, and $\partial V^M/\partial \hat{t}_2^F$ and $\partial V^M/\partial \hat{t}_2^M$ are negative, when evaluated at $\bar{t}_1^F$.

Case 5 ((ii-F) and (iii-M)): Wife spends all time with children when married but works full-time when divorced. Husband works full-time regardless.

$$V^F = \frac{ht_1^F}{2} + G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{h}{2} + G'(t_1^F)$$

$$\Rightarrow G'(\bar{t}_1^F) = -\frac{h}{2}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \hat{t}_2^F$ is negative, and $\partial V^M/\partial \tilde{t}_2^F$, $\partial V^F/\partial \tilde{t}_2^M$, and $\partial V^M/\partial \hat{t}_2^M$ are positive, when evaluated at $\bar{t}_1^F$.

Case 6 ((iii-F) and (iii-M)): Both spouses spend all their time in market in stage 2.

$$V^F = ht_1^F$$

$$\frac{\partial V^F}{\partial t_1^F} = h$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$, $\partial V^M/\partial \hat{t}_2^F$, $\partial V^F/\partial \tilde{t}_2^M$, and $\partial V^M/\partial \hat{t}_2^M$ are all positive, when evaluated at $\bar{t}_1^F$. If the second condition is satisfied, all the others must be as well.

For cases 4 and 5, the woman's optimal stage 1 investment in children does not depend on the husband's visitation right in the event of divorce. This is because, in these cases, the woman does not spend any time with her children in stage 2 in the event of divorce and hence the visitation rules are non binding. In case 6, there are no children.

The first five cases can generate interior solutions for t_1^F (the woman's time spent accumulating market related human capital during stage 1). Thus there may be positive investments in children in these cases. For the sixth case $t_1^F = 1$ and there will be no investment in children in stage 1. Given that by assumption $G''(t_1^F) < 0$, we know that the interior solution must be a maximum. A sufficient condition to ensure that the interior solutions for t_1^F, when $v > 0$, all lie in the range (0,1) is that $G'(1) = 0$, $|G'(0)| > h/2$ and $h < 2$. For $v = 0.5$ or 1, the bound on $G'(0)$ can be tightened to $|G'(0)| < h/9$. Given these constraints on $G'(0)$ and $G'(1)$, the following ranking of solutions for t_1^F is obtained:

A low value of $\bar{t}_1^F$ implies a high fertility rate. I rank these cases (and subcases) according to the optimal number of children that will be born in each of them:

- fertility will be greatest and equal to the maximum possible number of children in the following cases: case 1 (when $v = 0$), case 2 (when $v = 0$) and case 3 (when $v = 0$)
- the second largest fertility level will be obtained in case 2 when $v = 0.5$
- the third largest fertility level will be obtained in case 1 when $v = 0.5$
- the fourth largest fertility level will be obtained in case 3 when $v = 0.5$
- the fifth largest fertility level will be obtained in case 2 when $v = 1$ or case 4 (independent of v)
- the sixth largest fertility level will be obtained in case 1 when $v = 1$
- the seventh largest fertility level will be obtained in case 3 when $v = 1$ or case 5 (independent of v)
- the lowest fertility level (with fertility equal to zero) will be obtained in case 6 (independent of v)

If there are no children, as is the situation in case 6, whether the couple remain married or divorce is irrelevant in this model.

The case actually observed is the one that maximizes the woman's utility V^F. In general, this will depend both on the parameters h, α and v, and the functional form of $G(t_1^F)$. Here I consider a few examples and contrast the results obtained in the bargaining model with those obtained under joint family maximization.

4.4 Illustrative Examples

In the following examples, I set $v = 0$ and $\alpha = 1.4$. That is, I assume that, in the event of divorce, the woman gets custody of the children and that men get a 40 percent wage premium over women.[8] In addition, $G(t_1^F)$ is assumed to have the following functional form: $G(t_1^F) = 5(1 - t_1^F)^{1/2}$. This means that the maximum number of children a woman can have is 5. I now consider how varying h (the rate at which human capital is accumulated) in the range from 5 to 12 affects the outcome. The solutions for $U^F + U^M$ for each of the three joint utility maximization cases, as functions of h, are as follows:

Case (i): $U^F + U^M = 20$

Case (ii): $U^F + U^M = 1.4h + 10$

Case (iii): $U^F + U^M = 2.4h$

The solutions for V^F for each of the six bargaining cases, as functions of h, are as follows:

Case 1: $V^F = 12.5 - 0.7h$

Case 2: $V^F = 12.5 - 0.7h$

Case 3: $V^F = 7.5$

Case 4: $V^F = 10 - 0.7h$ for $h < 10$

$V^F = 50/h - h/5$ for $h \geq 10$

Case 5: $V^F = 5$ for $h < 5$

$V^F = h/2 - 25/(2h) + 5$ for $h \geq 5$

Case 6: $V^F = h$

- Example 1: $h = 5$

Joint maximization:

[8] For a discussion of how changes in α influence the results see section 5.

Case (i): $U^F + U^M = 20$

Case (ii): $U^F + U^M = 17$

Case (iii): $U^F + U^M = 12$

Hence case (i) is observed: $t_1^F = t_2^F = t_2^M = 0$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \;\; V^F = 9$

Case 2: $\bar{t}_1^F = 0, \;\; V^F = 9$

Case 3: $\bar{t}_1^F = 0, \;\; V^F = 7.5$

Case 4: $\bar{t}_1^F = 0, \;\; V^F = 6.5$

Case 5: $\bar{t}_1^F = 0, \;\; V^F = 5$

Case 6: $\bar{t}_1^F = 1, \;\; V^F = 5$

Hence either case 1 or 2 is observed: $t_1^F = t_2^F = t_2^M = 0$.[9]

- Example 2: $h = 8$

Joint maximization:

Case (i): $U^F + U^M = 20$

Case (ii): $U^F + U^M = 21.2$

Case (iii): $U^F + U^M = 19.2$

Hence case (ii) is observed: $t_1^F = t_2^F = 0$ and $t_2^M = 1$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \;\; V^F = 6.9$

Case 2: $\bar{t}_1^F = 0, \;\; V^F = 6.9$

Case 3: $\bar{t}_1^F = 0, \;\; V^F = 7.5$

Case 4: $\bar{t}_1^F = 0, \;\; V^F = 4.4$

Case 5: $\bar{t}_1^F = 0.61, \;\; V^F = 7.44$

Case 6: $\bar{t}_1^F = 1, \;\; V^F = 8$

Hence case 6 is observed: $t_1^F = t_2^F = t_2^M = 1$.

- Example 3: $h = 10$

[9] Divorce is never observed as an equilibrium outcome and hence there is no need here to distinguish here between 1 and 2. Both cases generate the same $\hat{t}_2^F$ and $\hat{t}_2^M$.

Joint maximization:

Case (i): $U^F + U^M = 20$

Case (ii): $U^F + U^M = 24$

Case (iii): $U^F + U^M = 24$

Hence the family is indifferent between cases (ii) and (iii): that is between $t_1^F = t_2^F = 0, t_2^M = 1$, and $t_1^F = t_2^F = t_2^M = 1$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \ \ V^F = 5.5$

Case 2: $\bar{t}_1^F = 0, \ \ V^F = 5.5$

Case 3: $\bar{t}_1^F = 0, \ \ V^F = 7.5$

Case 4: $\bar{t}_1^F = 0, \ \ V^F = 3$

Case 5: $\bar{t}_1^F = 0.75, \ \ V^F = 8.75$

Case 6: $\bar{t}_1^F = 1, \ \ V^F = 10$

Hence case 6 is observed: $t_1^F = t_2^F = t_2^M = 1$.

- Example 4: $h = 12$

Joint maximization:

Case (i): $U^F + U^M = 20$

Case (ii): $U^F + U^M = 26.8$

Case (iii): $U^F + U^M = 28.8$

Hence case (iii) is observed: $t_1^F = t_2^F = t_2^M = 1$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \ \ V^F = 4.1$

Case 2: $\bar{t}_1^F = 0, \ \ V^F = 4.1$

Case 3: $\bar{t}_1^F = 0, \ \ V^F = 7.5$

Case 4: $\bar{t}_1^F = 0.31, \ \ V^F = 2.97$

Case 5: $\bar{t}_1^F = 0.83, \ \ V^F = 9.96$

Case 6: $\bar{t}_1^F = 1, \ \ V^F = 12$

Hence case 6 is observed: $t_1^F = t_2^F = t_2^M = 1$.

A comparison of these results reveals that the level of fertility falls both in the joint utility maximization and bargaining models as h rises. This result follows from the fact that working in the labor market becomes more and more attractive relative to having children as h rises. Furthermore, the woman tends to choose a lower level of fertility in stage 1 in the bargaining model than would be consistent with joint utility maximization. This can be seen in the examples where $h = 8$ and 10. The woman does this to protect her bargaining position in stage 2.

5 Some Implications for Fertility Levels

Until the 1960s, child custody was almost always assigned to the mother. Rising claims of sex discrimination in custody decisions by fathers, the rise of the feminist movement, and the entry of large numbers of women into the workforce weakened the concept of a primary maternal caretaker (see Kelly 1994, and Cancian and Meyer 1998). As a result, male visitation rates have since increased significantly.

5.0.1 The Impact of v

The impact on fertility of a rise in the male visitation right, v, can be explored in the context of the model. I do not explore the possibility of interdependence between female labor participation and child custody rules. Rather, I assume that visitation rates are set exogenously. An increase in v from 0 to 0.5 reduces fertility for cases 1-3, but has no impact in cases 4-5 since the woman spends all her time working in the event of divorce as discussed above. In case 6 there are no children. Assuming that the fertility function $G(t_1^F)$ differs across women, we would expect to observe all six cases across the whole population. Overall, therefore, we would expect a rise in v to reduce fertility. The possibility of such a causal relationship has received little if any attention in the literature.

5.0.2 The Impact of α

Changes in the male-female wage differential as captured here by the parameter α may also impact on fertility. Card and DiNardo (2002) find that male-female wage differential in the US fell significantly between 1967 and 1994 and then remained approximately stable between 1994 and 2000. Hence although α remains greater than 1, its value has fallen in recent decades. Changes in α do not affect fertility levels in any of the six cases of section 5. However, changes in α can switch the outcome from one case to another. More precisely, a fall in α acts to reduce the likelihood that case (iii-M) is observed and increase the likelihood that case (i-M) is observed. This reduces the likelihood that cases 3, 5 and 6 are observed, and increases the likelihood of observing case 1. Overall, therefore, a reduction in α should raise fertility since it reduces the relative bargaining power of the man.

5.0.3 The Impact of h

Changes in wage rates can also impact on fertility. Here I will equate a rise in wages with an increased value of the parameter h. Strictly speaking, h here measures the rate at which human capital accumulates rather than the wage rate. However, given that the human capital accumulation function is assumed to be linear, a rise in h is equivalent to a rise in the wage rate in the model. The empirical evidence on trends in wage rates in the US in recent decades is mixed. Sullivan (1997) discusses how real hourly earnings rose from 1964 to 1972, but then fell from 1972 to 1996. By contrast, real hourly compensation rose throughout this whole period. Sullivan (1997) argues that the difference is due to measurement problems with the real hourly earnings series. Focusing on real hourly compensation, it follows that h has risen in recent decades. A rise in h acts to reduce fertility in cases 1-5, except when $v = 0$ in cases 1-3. In case 6, fertility is already zero to begin with. A rise in h also reduces the likelihood that cases (i-F) and (i-M) are observed (parents staying at home with the children in stage 2) and increases the likelihood of observing cases (iii-F) and (iii-M) (parents working in the market in period 2), thus further acting to reduce fertility. This finding is as expected, since a rise in the wage rate increases the attractiveness of working as compared with having children.

Overall, the cumulative impact of recent changes in v, α and h in the US are broadly consistent with a decline in fertility. The fertility rate did decline until the early 1970s, since when it has been more or less stable (see d'Addio and d'Ercole 2005). One possible way of reconciling this finding with the predictions of the model is that starting in the 1970s, real wages rose faster for workers on higher incomes. The wages of lower income workers – who also tend to have higher fertility rates – have stagnated. Hence for this group h has not risen.

6 Extension: The Case of Child Subsidies

Suppose now that the government pays a subsidy a in stage 2 to the parents of each child born. This payment is financed through a proportional tax rate s on income in stage 2. I will treat both a and s as exogenous, and hence I do not impose a balanced budget on the government. When the parents are married, I assume that the child subsidy is paid to the woman. In the event of divorce, the child subsidy is split in accordance with the visitation rights. That is, the man receives av and the woman $a(1-v)$.

6.1 Introducing Child Subsidy a in Becker-type Model

Considering first the cooperative Becker type scenario, the utility of a married woman can be written as follows:

$$U_m^F = (1-s)ht_1^F t_2^F + (2 + a - t_2^F - t_2^M)G(t_1^F).$$

Similarly, a married man's utility function is as follows:

$$U_m^M = (1-s)\alpha h t_2^M + (2 - t_2^F - t_2^M)G(t_1^F).$$

The joint family utility function therefore is given by

$$U^F + U^M = (1-s)ht_1^F t_2^F + (4 + a - 2t_2^F - 2t_2^M)G(t_1^F) + (1-s)\alpha h t_2^M.$$

Treating t_1^F as given in stage 2, the following stage 2 first order conditions are obtained:

$$\frac{\partial(U^F + U^M)}{\partial t_2^F} = (1-s)ht_1^F - 2G(t_1^F), \tag{4}$$

$$\frac{\partial(U^F + U^M)}{\partial t_2^M} = (1-s)\alpha h - 2G(t_1^F). \tag{5}$$

Again, given that t_2^F and t_2^M do not feature in either of these first order conditions it follows that we will only observe corner solutions in stage 2. The three possible cases are the same as before:

(i) $t_2^F = t_2^M = 0$: both husband and wife spend stage 2 looking after children only.

(ii) $t_2^F = 0, t_2^M = 1$: the wife spends all of her stage 2 time at home with the children while the husband works full time.

(iii) $t_2^F = t_2^M = 1$: both spouses spend stage 2 working in the market only.

Case (i):

$$U^F + U^M = (4+a)G(t_1^F)$$

It follows that $U^F + U^M$ is maximized at $t_1^F = 0$. The consistency of this solution can be checked by substituting $t_1^F = 0$ back into the first order conditions for stage 2 as follows:

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^F}\right|_{t_1^F=0} = -2G(0) < 0,$$

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^M}\right|_{t_1^F=0} = (1-s)\alpha h - 2G(0).$$

Given that $\partial(U^F + U^M)/\partial t_2^F|_{t_1^F=0} < 0$, it follows that $t_1^F = 0$ implies that $t_2^F = 0$. Given that $\partial(U^F + U^M)/\partial t_2^M|_{t_1^F=0} < 0$ only holds when $(1-s)\alpha h < 2G(0)$, the internal consistency of this solution requires that this inequality is satisfied. In this case, $U_m^F + U_m^M = (4+a)G(0)$.

Compared with the no-subsidy situation in section 4, total family utility has increased by $aG(0)$. For a family in case (i), the child subsidy scheme is therefore strictly welfare enhancing.

Case (ii):

$$U^F + U^M = (1-s)\alpha h + (2+a)G(t_1^F)$$

It follows that $U^F + U^M$ is again maximized at $t_1^F = 0$. From the results obtained for case (i), it can be seen that this solution is only internally consistent when $(1-s)\alpha h > 2G(0)$. In this case, $U_m^F + U_m^M = (1-s)\alpha h + (2+a)G(0)$.

Compared with section 4, total family utility is decreased by $s\alpha h$, but increased by $aG(0)$. Whether total family wellbeing is improved by the child subsidy scheme depends therefore on whether $s\alpha h < aG(0)$.

Case (iii):

$$U^F + U^M = (1-s)ht_1^F + (1-s)\alpha h + aG(t_1^F)$$

The stage 2 first order conditions are given by (4) and (5). Differentiating with respect to t_1^F yields the following for stage 1:

$$\frac{\partial(U^F + U^M)}{\partial t_1^F} = (1-s)h + aG'(t_1^F).$$

A sufficient condition for this case to generate an interior solution for t_1^F, denoted here by $\bar{t}_1^F$, is that

$$|G'(0)| < \frac{(1-s)h}{a} < |G'(1)|. \tag{6}$$

For this solution to be compatible with $t_2^F = t_2^M = 1$, it must be the case that

$$\left.\frac{\partial(U^F + U^M)}{\partial t_2^F}\right|_{t_1^F = \bar{t}_1^F} > 0, \quad \text{and} \quad \left.\frac{\partial(U^F + U^M)}{\partial t_2^M}\right|_{t_1^F = \bar{t}_1^F} > 0.$$

These conditions translate to the following:

$$(1-s)h\bar{t}_1^F > 2G(\bar{t}_1^F),$$

$$(1-s)\alpha h > 2G(\bar{t}_1^F).$$

As $\alpha \geq 1$ and $\bar{t}_1^F < 1$, the first of these inequalities is more binding. It follows therefore that for this solution to be valid, it must be the case that $\bar{t}_1^F > \breve{t}_1^F$ where $(1-s)h\breve{t}_1^F = 2G(\breve{t}_1^F)$. In this case, we obtain that $U_m^F + U_m^M = (1-s)h\bar{t}_1^F + (1-s)\alpha h + aG(\bar{t}_1^F)$. In section 4, case (iii) always implied a corner solution in both stages 1 and 2. This is no longer the case here when condition (6) is satisfied. If instead $(1-s)h > a|G'(1)|$, the same corner solution with $\bar{t}_1^F = 1$ will be observed as in section 4. In this case, total family utility is given by $U^F + U^M = (1-s)(1+\alpha)h$. This indicates that a family that has no children will be strictly worse off as a result of the child subsidy scheme as they will have to pay a child subsidy tax without receiving the child subsidy.

From a comparison of the three cases, it can be seen that case (i) is optimal when $2G(0) > (1-s)\alpha h$. When $2G(0) < (1-s)\alpha h$, determining which out of cases (ii) and (iii) is more problematic. A sufficient condition for (ii) to be optimal is that $(1-s)h < 2G(0) < (1-s)\alpha h$. A sufficient condition for (iii) to be optimal is that $(2+a)G(0) < (1-s)h$. Without specifying a functional form for $G(t_1^F)$ it is difficult to say more than this. In comparison with section 4, the introduction of the child subsidy scheme increases the likelihood that the case in which both parents spend stage 2 with their children – that is case (i) – is optimal.

6.2 Introducing a Child Subsidy a into the Bargaining Model

In the bargaining model, the married and divorced utilities of the woman and man are as follows:

$$U_m^F = (1-s)ht_1^F t_2^F + (2 - t_2^F - t_2^M)G(t_1^F) + aG(t_1^F)$$

$$U_m^M = (1-s)\alpha h t_2^M + (2 - t_2^F - t_2^M)G(t_1^F)$$

$$U_d^F = (1-s)ht_1^F t_2^F + (1 - t_2^F)G(t_1^F) + (1-v)aG(t_1^F),$$

$$\text{subject to} \quad t_2^F \geq v.$$

$$U_d^M = (1-s)\alpha h t_2^M + (1 - t_2^M)G(t_1^F) + va,$$

$$\text{subject to} \quad t_2^M \geq 1 - v.$$

Substituting in from the individual utility functions we obtain the following:

$$\Delta U^{F+M} = (1-s)ht_1^F(\hat{t}_2^F - \tilde{t}_2^F) + (1-s)\alpha h(\hat{t}_2^M - \tilde{t}_2^M) + [2(2-\hat{t}_2^F - \hat{t}_2^M) - (2-\tilde{t}_2^F - \tilde{t}_2^M)]G(t_1^F),$$

where $\hat{t}_2$ again denotes a stage two married time allocation, and $\tilde{t}_2$ a stage 2 divorced time allocation.

Assuming again that the marriage surplus is split via Nash bargaining, the woman's overall utility function becomes:

$$V^F = (1-s)ht_1^F\tilde{t}_2^F + [1 + (1-v)a - \tilde{t}_2^F]G(t_1^F) + \Delta U^{F+M}/2$$

$$= \frac{(1-s)ht_1^F(\tilde{t}_2^F + \hat{t}_2^F)}{2} + \frac{(1-s)\alpha h(\hat{t}_2^M - \tilde{t}_2^M)}{2} + \left[(2 - \hat{t}_2^F - \hat{t}_2^M) + (1-v)a + \frac{(\tilde{t}_2^M - \tilde{t}_2^F)}{2}\right] G(t_1^F).$$

Differentiating V^F with respect to $\tilde{t}_2^F$ and $\hat{t}_2^F$ yields the following stage 2 first order conditions:

$$\frac{\partial V^F}{\partial \tilde{t}_2^F} = \frac{(1-s)ht_1^F}{2} - \frac{G(t_1^F)}{2},$$

$$\frac{\partial V^F}{\partial \hat{t}_2^F} = \frac{(1-s)ht_1^F}{2} - G(t_1^F).$$

Similarly, the man's overall utility function is now as follows:

$$V^M = (1-s)\alpha h\tilde{t}_2^M + (1 + va - \tilde{t}_2^M)G(t_1^F) + \Delta U^{F+M}/2$$

$$= \frac{(1-s)\alpha h(\tilde{t}_2^M + \hat{t}_2^M)}{2} + \frac{(1-s)ht_1^F(\hat{t}_2^F - \tilde{t}_2^F)}{2} + \left[(2 - \hat{t}_2^F - \hat{t}_2^M) + va + \frac{(\tilde{t}_2^F - \tilde{t}_2^M)}{2}\right] G(t_1^F).$$

Maximizing this utility function with respect to $\tilde{t}_2^M$ as well as $\hat{t}_2^M$ yields the following stage 2 first order conditions:

$$\frac{\partial V^M}{\partial \tilde{t}_2^M} = \frac{(1-s)\alpha h}{2} - \frac{G(t_1^F)}{2},$$

$$\frac{\partial V^M}{\partial \hat{t}_2^M} = \frac{(1-s)\alpha h}{2} - G(t_1^F).$$

Given that $\tilde{t}_2^F$, $\hat{t}_2^F$, $\tilde{t}_2^M$ and $\hat{t}_2^M$ again do not feature in any of these first order conditions, it follows that we will only observe corner solutions in stage 2. Both parties' optimal period two behavior is determined by the values of s, h and α and t_1^F. Considering first the stage 2 first order conditions for the woman, there are three possible cases.

(i-F) $(1-s)ht_1^F/2 < (1-s)ht_1^F < G(t_1^F) \Rightarrow \tilde{t}_2^F = v, \hat{t}_2^F = 0$

She spends as much time as possible with children in stage 2. Any positive tax rate s will increase the likelihood that case (i-F) applies.

(ii-F) $(1-s)ht_1^F/2 < G(t_1^F) < (1-s)ht_1^F \Rightarrow \tilde{t}_2^F = 1, \hat{t}_2^F = 0$

She stays with children when married but works full time when divorced in stage 2.

(iii-F) $G(t_1^F) < (1-s)ht_1^F/2 < (1-s)ht_1^F \Rightarrow \tilde{t}_2^F = 1, \hat{t}_2^F = 1$

She works all the time during stage 2 whether or not she is married. Any positive tax rate s will reduce the likelihood that case (iii-F) applies.

Similarly for the man, the three cases for stage 2 are:

(i-M) $(1-s)\alpha h/2 < (1-s)\alpha h < G(t_1^F) \Rightarrow \tilde{t}_2^M = 1-v, \hat{t}_2^M = 0$

He spends as much time as possible with children in stage 2.

(ii-M) $(1-s)\alpha h/2 < G(t_1^F) < (1-s)\alpha h \Rightarrow \tilde{t}_2^M = 1, \hat{t}_2^M = 0$

He stays with children when married but works full time when divorced in stage 2.

(iii-M) $G(t_1^F) < (1-s)\alpha h/2 < (1-s)\alpha h \Rightarrow \tilde{t}_2^M = 1, \hat{t}_2^M = 1$

He works all the time during stage 2 whether or not he is married. Again, a positive tax rate s increases the likelihood of case (i-M) and decreases the likelihood of case (iii-M) applying.

This again leads to the same six cases as in section 5.

Case 1 ((i-F) and (i-M)):

$$V^F = \frac{(1-s)hvt_1^F}{2} - \frac{(1-s)\alpha h(1-v)}{2} + \left[\frac{5+2(1-v)a-2v}{2}\right] G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{(1-s)hv}{2} + \left[\frac{5+2(1-v)a-2v}{2}\right] G'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)hv}{5+2(1-v)a-2v}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{(1-s)h}{2(4+a)}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{(1-s)h}{3}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial\tilde{t}_2^F$, $\partial V^F/\partial\hat{t}_2^F$, $\partial V^M/\partial\tilde{t}_2^M$ and $\partial V^M/\partial\hat{t}_2^M$ are all negative when evaluated at $\bar{t}_1^F$. These inequality conditions yield the following four constraints:

$$\frac{\partial V^F}{\partial \tilde{t}_2^F} < 0 \Rightarrow (1-s)h\bar{t}_1^F < G(\bar{t}_1^F),$$

$$\frac{\partial V^F}{\partial \hat{t}_2^F} < 0 \Rightarrow (1-s)h\bar{t}_1^F < 2G(\bar{t}_1^F),$$

$$\frac{\partial V^M}{\partial \tilde{t}_2^M} < 0 \Rightarrow (1-s)h\alpha h < G(\bar{t}_1^F),$$

$$\frac{\partial V^M}{\partial \hat{t}_2^M} < 0 \Rightarrow (1-s)\alpha h < 2G(\bar{t}_1^F).$$

The righthand side of these inequality constraints is the same as in section 5. The lefthand side has been multiplied by $(1-s)$ for each inequality. Since by assumption $s > 0$ for any positive child subsidy a, this will make it more likely that all constraints hold in case 1.

Case 2 ((i-F) and (ii-M)):

$$V^F = \frac{(1-s)hvt_1^F}{2} - \frac{(1-s)\alpha h}{2} + \left[\frac{4+(2a+1)(1-v)}{2}\right] G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{(1-s)hv}{2} + \left[\frac{4+(2a+1)(1-v)}{2}\right] G'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)hv}{4+(2a+1)(1-v)}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{(1-s)h}{9+2a}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{(1-s)h}{4}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$, $\partial V^F/\partial \hat{t}_2^F$, and $\partial V^M/\partial \hat{t}_2^M$ are all negative while $\partial V^M/\partial \tilde{t}_2^M$ is positive, when evaluated at $\bar{t}_1^F$.

Case 3 ((i-F) and (iii-M)):

$$V^F = \frac{(1-s)hvt_1^F}{2} + \left[\frac{2+(2a+1)(1-v)}{2}\right] G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{(1-s)hv}{2} + \left[\frac{2+(2a+1)(1-v)}{2}\right] G'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)hv}{2+(2a+1)(1-v)}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = 0, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{(1-s)h}{5+2a}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{(1-s)h}{2}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$ and $\partial V^F/\partial \hat{t}_2^F$ are negative, and $\partial V^M/\partial \hat{t}_2^M$ and $\partial V^M/\partial \tilde{t}_2^M$ are positive, when evaluated at $\bar{t}_1^F$.

Case 4 ((ii-F) and (ii-M)):

$$V^F = \frac{(1-s)ht_1^F}{2} - \frac{(1-s)\alpha h}{2} + [2+(1-v)a]G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{(1-s)h}{2} + [2+(1-v)a]G'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)h}{4+2(1-v)a}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = -\frac{(1-s)h}{4+2a}, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{(1-s)h}{4+a}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{(1-s)h}{4}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$ and $\partial V^F/\partial \tilde{t}_2^M$ are positive, and $\partial V^M/\partial \hat{t}_2^F$ and $\partial V^M/\partial \hat{t}_2^M$ are negative, when evaluated at $\bar{t}_1^F$.

Case 5 ((ii-F) and (iii-M)):

$$V^F = \frac{(1-s)ht_1^F}{2} + [1+(1-v)a]G(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = \frac{(1-s)h}{2} + [1+(1-v)a]G'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)h}{2+2(1-v)a}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = -\frac{(1-s)h}{2+2a}, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{(1-s)h}{2+a}, \quad G'(\bar{t}_1^F)\Big|_{v=1} = -\frac{(1-s)h}{2}$$

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \hat{t}_2^F$ is negative, and $\partial V^M/\partial \tilde{t}_2^F$, $\partial V^F/\partial \tilde{t}_2^M$, and $\partial V^M/\partial \hat{t}_2^M$ are positive, when evaluated at $\bar{t}_1^F$.

Case 6 ((iii-F) and (iii-M)):

$$V^F = (1-s)ht_1^F + (1-v)aG(t_1^F)$$

$$\frac{\partial V^F}{\partial t_1^F} = (1-s)h + (1-v)aG'(t_1^F)$$

$$\Rightarrow G'(t_1^F) = -\frac{(1-s)h}{(1-v)a}$$

$$G'(\bar{t}_1^F)\Big|_{v=0} = \frac{(1-s)h}{a}, \quad G'(\bar{t}_1^F)\Big|_{v=0.5} = -\frac{2(1-s)h}{a}$$

A corner solution is obtained when $v = 1$. In this case, $\partial V^F/\partial t_1^F = (1-s)h > 0$. In section 5, case 6 always generates a corner solution in stage 1, irrespective of the value of v. This is no longer the case here, except when $v = 1$. Only in this case does the woman get no benefit from having children if she divorces.

For this solution to be internally consistent, it must be the case that $\partial V^F/\partial \tilde{t}_2^F$, $\partial V^M/\partial \hat{t}_2^F$, $\partial V^F/\partial \tilde{t}_2^M$, and $\partial V^M/\partial \hat{t}_2^M$ are all positive, when evaluated at $\bar{t}_1^F$. If the second condition is satisfied, all the others must be as well. With a positive s, these conditions are less likely to be satisfied than before.

6.3 The Impact of Child Subsidy a - An Illustration

In the following examples, I illustrate the impact of the child subsidy for a particular parameterization of the model. I again set $v = 0$, $\alpha = 1.4$ and $G(t_1^F) = 5(1-t_1^F)^{1/2}$. In addition, I assume that the government budget with regard to the child subsidy is balanced for a family where $t_1^F = 0.5$ and $t_2^F = t_2^M = 1$. This restriction yields the following government budget equation linking a and s:

$$\frac{sh}{2} + s\alpha h = \frac{5a}{4}.$$

Setting $a = 1$, it therefore follows that $s = 0.658/h$.

The solutions for $U^F + U^M$ are as follows:

Case (i): $U^F + U^M = 25$

Case (ii): $U^F + U^M = 1.4(h - 0.658) + 15$

Case (iii): $U^F + U^M = 2.4(h - 0.658) + 5$

The solutions for V^F for each of the six bargaining cases, as functions of h, are as follows:

Case 1: $V^F = 17.5 - 0.7(h - 0.658)$

Case 2: $V^F = 17.5 - 0.7(h - 0.658)$

Case 3: $V^F = 12.5$

Case 4: $V^F = 15 - 0.7(h - 0.658) \quad \text{for } h < 15.658,$[10]

Case 5: $V^F = 10$ for $h < 10.658$

$$V^F = 10 + [(h - 0.658)/2][1 - 100/(h - 0.658)^2] \text{ for } h \geq 10.658$$

Case 6: $V^F = h + 4.342$ for $h < 3.158$

$$V^F = 5 + (h - 0.658)[1 - 6.25/(h - 0.658)^2] \text{ for } h \geq 3.158$$

- Example 1: $h = 5$

Joint maximization:

Case (i): $U^F + U^M = 25$

Case (ii): $U^F + U^M = 21.08$

Case (iii): $U^F + U^M = 15.42$

Hence case (i) is observed: $t_1^F = t_2^F = t_2^M = 0$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \quad V^F = 14.46$

Case 2: $\bar{t}_1^F = 0, \quad V^F = 14.46$

Case 3: $\bar{t}_1^F = 0, \quad V^F = 12.5$

Case 4: $\bar{t}_1^F = 0, \quad V^F = 11.96$

Case 5: $\bar{t}_1^F = 0, \quad V^F = 10$

Case 6: $\bar{t}_1^F = 0.67, \quad V^F = 7.90$

Hence either case 1 or 2 is observed: $t_1^F = t_2^F = t_2^M = 0$.

- Example 2: $h = 8$

Joint maximization:

Case (i): $U^F + U^M = 25$

Case (ii): $U^F + U^M = 25.28$

Case (iii): $U^F + U^M = 22.62$

Hence case (ii) is observed: $t_1^F = t_2^F = 0$ and $t_2^M = 1$.

[10] This inequality constraint is obtained by setting $G'(\bar{t}_1^F)|_{v=0} = (1 - s)h/(4 + 2a)$, and then finding what restriction must be imposed on h to ensure that $\bar{t}_1^F > 0$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \;\; V^F = 12.36$

Case 2: $\bar{t}_1^F = 0, \;\; V^F = 12.36$

Case 3: $\bar{t}_1^F = 0, \;\; V^F = 12.5$

Case 4: $\bar{t}_1^F = 0, \;\; V^F = 9.86$

Case 5: $\bar{t}_1^F = 0, \;\; V^F = 10$

Case 6: $\bar{t}_1^F = 0.88, \;\; V^F = 11.49$

Hence case 3 is observed: $t_1^F = t_2^F = 0$ and $t_2^M = 1$.

- Example 3: $h = 10$

Joint maximization:

Case (i): $U^F + U^M = 25$

Case (ii): $U^F + U^M = 28.08$

Case (iii): $U^F + U^M = 27.42$

Hence case (ii) is observed: $t_1^F = t_2^F = 0$ and $t_2^M = 1$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \;\; V^F = 10.96$

Case 2: $\bar{t}_1^F = 0, \;\; V^F = 10.96$

Case 3: $\bar{t}_1^F = 0, \;\; V^F = 12.5$

Case 4: $\bar{t}_1^F = 0, \;\; V^F = 8.46$

Case 5: $\bar{t}_1^F = 0, \;\; V^F = 10$

Case 6: $\bar{t}_1^F = 0.93, \;\; V^F = 13.67$

Hence case 6 is observed: $t_1^F = 0.93$, and $t_2^F = t_2^M = 1$.

- Example 4: $h = 12$

Joint maximization:

Case (i): $U^F + U^M = 25$

Case (ii): $U^F + U^M = 30.88$

Case (iii): $U^F + U^M = 32.22$

Hence case (iii) is observed: $t_1^F = t_2^F = t_2^M = 1$.

Bargaining

Case 1: $\bar{t}_1^F = 0, \ \ V^F = 9.56$

Case 2: $\bar{t}_1^F = 0, \ \ V^F = 9.56$

Case 3: $\bar{t}_1^F = 0, \ \ V^F = 12.5$

Case 4: $\bar{t}_1^F = 0, \ \ V^F = 7.06$

Case 5: $\bar{t}_1^F = 0.22, \ \ V^F = 11.26$

Case 6: $\bar{t}_1^F = 0.95, \ \ V^F = 15.79$

Hence case 6 is observed: $t_1^F = 0.95$ and $t_2^F = t_2^M = 1$.

A comparison of these results with those obtained in section 4 reveals that the child subsidy does indeed increase the fertility rate. This is particularly true for the bargaining model where, in the absence of the child subsidy fertility is zero ($t_1^F = 1$) when $h = 8$, 10 or 12, while in the presence of the child subsidy fertility is positive for all three values ($t_1^F < 1$).

7 Conclusion

Perhaps the main insight to emerge from this study is how the fundamental asymmetry between women and men, in terms of child bearing costs, can cause women to overinvest – from the perspective of joint utility maximization – in human capital accumulation, and hence underinvest in children. Women do this to protect their intra-family bargaining position. This phenomenon, by construction, is completely missed by joint utility maximization models. By combining a bargaining model with the assumption of an incomplete contracts framework one perhaps gets closer to understanding how families reach their actual fertility decisions. Bargaining models, such as the one developed here, therefore are a useful additional tool for understanding fertility patterns and how they respond to exogenous shocks. In particular, they can shed light on the likely impact on fertility rates of government initiatives such as child subsidies, as well as changes in the male-female wage differential, wages rates and custody rules.

References

Becker, G. (1965) A Theory of the Allocation of Time, *Economic Journal* 75, 493-517.

Becker, G. S. (1981), *A Treatise on the Family.* Harvard University Press: Cambridge MA.

Becker, G. S, Murphy K. M. and Tamura R. (1990), Human Capital, Fertility, and Economic Growth, *The Journal of Political Economy* 98(5), Part 2, S12-S37.

Blau, F. and Kahn, L. (2000), Understanding International Differences in the Gender Pay Gap, *Journal of Labor Economics* 21(1), 106-144.

Cancian, M. and Meyer, D. R. (1998), Who Gets Custody, *Demography* 35(2), 147-157.

Card, D. and DiNardo, J. E. (2002), Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles, *Journal of Labor Economics* 20(4), 733-783.

Cigno, A., Luporini, A., and Pettini, A. (2003): Transfers to families with children as a principal-agent problem, *Journal of Public Economics* 87, 1165-1177.

Cigno, A. and Werding M. (2007), *Children and Pensions*, CESifo Book Series. MIT Press: Cambridge MA.

d'Addio, A. C. and d'Ercole M. M. (2005), *Trends and Determinants of Fertility Rates in OECD Countries: The Role of Policies*, OECD Social, Employment and Migration Working Papers 27.

Eswaran, M. (2002), The Empowerment of Women, Fertility, and Child Mortality: Towards a Theoretical Analysis, *Journal of Population Economics* 15(3), 433-454.

Gunderson, M. (1989), Male-Female Wage Differentials and Policy Responses, *Journal of Economic Literature* 27, 46-72.

Hart, O. (1995), *Firms, Contracts, and Financial Structure.* Oxford: Clarendon Press.

Iyigun, M. and Walsh, R. P. (2007), Endogenous Gender Power, Household Labor Supply and the Demographic Transition, *Journal of Development Economics* 82(1), 138-155.

Kelly, J. B. (1994), The Determination of Child Custody, *Children and Divorce* 4(1), 121-142.

Murphy, R. D. Jr. (2002), A Good Man Is Hard to Find: Marriage as an Institution, *Journal of Economic Behavior and Organization* 47(1), 27-53.

Nash, J. F. (1950), "The Bargaining Problem," *Econometrica* 18(1), 155-162.

Oaxaca, R. (1973), Male-Female Wage Differentials in Urban Labor Markets, *International Economic Review* 14(3), 693-709.

Pollak, R. (1985), A Transaction Cost Approach to Families and Households, *Journal of Economic Literature* 23(2), 581-608.

Samuelson, P. A. (1956), "Social Indifference Curves," *Quarterly Journal of Economics* 70(1), 1-22.

Stevenson, B. (2007), The Impact of Divorce Laws on Marriage-Specific Capital, *Journal of Labor Economics* 25(1), 75-94.

Sullivan, D. (1997), Trends in Real Wage Growth, *Chicago Fed Letter* 115, The Federal Reserve Bank of Chicago.

Tirole, J. (1988), *The Theory of Industrial Organization.* Cambridge MA; MIT Press.

Trostel, P. A. (2004), Returns to Scale in Producing Human Capital from Schooling, *Oxford Economic Papers* 56(3), 461-84.

Zelder, M. (1993), Inefficient Dissolutions as a Consequence of Public Goods: The Case of No-fault Divorce, *Journal of Legal Studies* 22(2), 503-520.