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Fertility Decisions and the Sustainability of Defined Benefit Pay-As-You-Go Pension Systems

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The sustainability of a defined benefit pay-as-you-go (DBPAYG) pension system is investigated in the context of an overlapping-generations model of endogenous fertility with heterogeneous agents. The model places particular emphasis on the time costs of child rearing. It illustrates the mechanism by which such a pension system can increase the opportunity cost of having children and hence sow the seeds of its own destruction. The model is then extended to allow for fertility-based payments. Such a system is more likely to be sustainable. The model highlights a number of issues that are of relevance to a number of OECD countries that have generous DBPAYG pension systems and falling fertility rates. (*JEL* H55, J13, J14)

Keywords: Pay-as-you-go pension; Defined benefit; Overlapping generations; Endogenous fertility; Labor participation Rate; Heterogeneous agents

*This article is based on chapter 4 of my PhD dissertation. See Steurer (2008).

1 Introduction

Most OECD countries have pay-as-you-go (PAYG) public pension systems in which current pension payments are financed by contributions of the current working population. Hence the size of the retired population relative to the working population (the old age dependency ratio) is an important determinant of the sustainability of pension systems. Falling fertility rates in most OECD countries together with rising life expectancy and earlier retirement are increasing the old-age-dependency ratio and thereby threatening the sustainability of public pension systems (see for example Disney (2000), Rother, Catenaro and Schwab (2004) and Cournede and Gonand (2006)).[1] For this reason there exists pressure in most OECD countries to reform the pension system. One possible avenue is to switch from a PAYG to a funded system. Such a switch is politically hard to achieve in a democratic society since it will make at least one generation strictly worse off (see Breyer (1989)). Rather than investigating whether and how such a switch from a PAYG to a funded pension system should occur, I take the existence of a PAYG pension system as given and investigate the sustainability of such a system.

In general, a PAYG pension system can take either the 'defined contribution' or 'defined benefit' form. Defined contribution (DCPAYG) systems levy a fixed contribution or wage tax per period and divide the resulting amount among the current pensioners. In defined benefit pay-as-you-go (DBPAYG) systems pensioners have the right to claim a certain pension and the government levies the tax that is necessary to cover these obligations. With stable population structures and per capita incomes contributions paid and benefits received are the same under both systems. But the pension received by the old and contributions paid by the young differ when the population structure is changing. If the population is decreasing pensioners in a DCPAYG system receive smaller and smaller pensions as the old age dependency ratio increases. In a DBPAYG

[1] By sustainability I mean that the pension system is able to deliver on the payments promised to each generation while maintaining a balanced budget.

system the pension payments to the old remain unaltered and the current working population have to bear the burden of the aging society. Their contribution rates rise in order to finance the pensions of the older generation.

PAYG pension systems also differ in the amount of intragenerational redistribution present in the system. The pension could be uniform (i.e., highly redistributive) or an increasing function of an individual's contributions. For example, public pensions are fairly uniform in the U.K., the Netherlands and Australia. Such a system is often referred to as a flat-rate pension. In other countries (e.g., Germany, Austria, or Italy) public pension benefits vary significantly depending on an individual's contributions during their working life. These are so-called earnings-related pensions. Describing the German pension system, Börsch-Supan (2000) writes:

> "[P]ublic pensions are roughly proportional to labor income averaged over the entire life course and feature only few redistributive properties."

In practice, most OECD countries have earnings related DBPAYG systems. I therefore focus primarily on the implications of these systems.

In recent years the focus in the pension literature has shifted towards the implications of falling fertility rates on the sustainability of PAYG pension systems, with fertility modeled endogenously. In a model of endogenous fertility the relationship between fertility rates and PAYG pension systems works in two directions. Under the assumption that the demand for children is at least partially an investment decision (parents expecting children to provide for them during old age) it is possible to show that the introduction of a public pension system is very likely to reduce the demand for children. Not only is it the case that low rates of fertility put pressure on the public pension system; the public pension system itself contributes to the 'old age crisis' by reducing the demand for children (see for example Cigno (1993), Nishimura and Zhang (1995), Kolmar (1997), Wigger (1999), Cigno, Luporini and Pettini (2003), and Cigno and Werding (2007)).

As in Cigno (1993), Rosati (1996) I assume that the sole reason for having children is that they provide parents with support during old age. This support is provided by the child in the form of a fixed amount money transfer to the aging parent. Given this assumption, an earnings-related pension system will reduce the incentive to have children more than a flat-rate system. This is the case as under an earnings-related system, people who substitute child raising with increased investments in market related human capital not only earn higher incomes (leading to higher pension claims), they also do not miss contribution times due to child rearing (also leading to higher pension claims).

It also makes a difference whether this interdependency of fertility levels and pension sustainability is analyzed in a DBPAYG or defined contribution PAYG (DCPAYG) setting. While it can be shown that the introduction of a DCPAYG public pension system (potentially) reduces fertility levels - the question of sustainability cannot be addressed in such system. The contribution rate (tax rate) levied on working income remains stable in DC systems and once introduced a DC system never goes bust. The increase in the tax burden due to decreased fertility rates and population aging, which is presently experienced in many OECD countries, cannot be modeled in an DC pension system. Some authors address the issue of rising contribution rates in DC PAYG systems by exogenously introducing such increases in the contribution rates (see for example Kolmar (1997), Nishimura and Zhang (1992, 1995), and Zhang and Zhang (1998)). However, the DBPAYG models developed here allow me to endogenously model the development of the contribution rate needed to keep the system running.

Since too low rates of population growth are bad for PAYG pension systems, the decision of an individual to have a child creates a positive externality. Under a DC-PAYG system, the beneficiaries of this externality are the rest of the population of the same generation as the parent. The child in period t will subsequently pay pension contributions in period $t+1$, which will be paid out to the then retired parent generation.[2]

[2]It is assumed throughout that generation t is born in period $t-1$, is of young (working) age during period t and old during period $t+1$.

Under a DBPAYG system, the beneficiaries are the rest of the population of the same generation as the child. This is because one extra child in period t reduces the pension burden that must be borne by working age adults in period $t+1$.

Endogenous fertility pension models often assume that individuals supply labor *inelastically* into the labor market. In return they receive money income which can be used to rear children, buy consumer goods, and save for old age (if a capital market exists). This representation omits an important factor. While rearing children can involve quite large monetary costs, a significant part of the costs is the time spent raising them. This is especially the case in industrialized countries where the opportunity costs of child rearing are high. I assume that raising children is time intensive. The consequence of this assumption is that labor supply also becomes an endogenous variable.

This article extends the literature on public pensions in a number of ways. First, it uses a model with time intensive child rearing and heterogeneous agents to analyze fertility decisions and their implications for the sustainability of public pension systems. Second, by explicitly modeling an earnings-related DBPAYG pension system it discusses the specific structure of pension system found in many countries around the world (especially in Western Europe). Given this setup, the tax rate needed to keep the pension system running can be endogenously determined. And this in turn allows us to address the question of sustainability of the pension system in an endogenous manner. Third, a variant on the earnings-related DBPAYG model is developed which includes a fertility based component and is designed to enhance the sustainability of the system. Using simulations I show that having a fertility based component in the DBPAYG pension system increases the likelihood that the system will be sustainable. This is consistent with the findings of Cigno, Luporini and Pettini (2003) and Cigno and Werding (2007), who also consider the impact of fertility payments on PAYG pension systems. I conclude by discussing some of the implications of my findings.

2 An Overlapping Generations Model of an Earnings-Related DB-PAYG Pension System

I use an overlapping generations model in which the actual lifetime of a person is three periods. Only two of these periods – active life (young) and retirement (old) – will be modelled explicitly since it is assumed that an individual cannot influence their utility during childhood. These two time periods (young age and old age) are assumed to be of equal length.[3] Each generation consists of two groups of agents: $(1-\theta)$ percent of the agents in any cohort are high skilled while θ percent of each cohort have low skill levels. The distribution of skill levels in any generation t are assumed to be constant over time. Thus, generation $t-1$ consists of N_{t-1} agents, $(1-\theta)$ percent of them with high and θ percent of them with low skill levels, generation t consists of N_t agents, again $(1-\theta)$ percent with high and θ percent with low skill levels etc. Generation t is young during period t and old during period $t+1$. High skilled individuals are more productive than low skilled individuals. This difference in productivity will be reflected in their relative wage rates, which are given by w_t^h and w_t^l respectively.

To keep the analysis as simple as possible it is assumed that all young agents can bear children and that child raising is equally time intensive for high as well as low skilled individuals. When agents are young they can decide how to spend their time – one unit – between market work and having children. To highlight the importance of time in raising children it is assumed that children grow up on parental time alone. Each potential parent has one unit of time that they can either allocate to the activities of child rearing or working in the market. t denotes the amount of time that an individual

[3] The relative period length is important in determining the costs of the public pension system. The longer the old live and the sooner they retire (the longer the second period) the higher the dependency ratio and the more expensive the system is for the young. In some European countries early retirement has become the norm not the exception. For example, 85 percent of Austrian men and 70 percent of Austrian women who retire do so before the official retirement age of 65/60 years (Koch and Thimann (1999)).

spends on child rearing. Individuals 'produce' children via a linear 'child production' function $G(t)$ with $G(t) = t/b$ which is equivalent to $t(G) = bG_t$. Since $t \in [0, 1]$, $1/b$ is the maximum number of children an agent can raise and is determined by human physiology and medical standards (which are assumed to be fixed). For reasons of convenience it is assumed that the number of children born to an individual can take any value in this range and that the skill level of children is independent of their parents' skill level and determined by chance only.

The economy considered is a small open economy with prices and wage rates given exogenously. But labor is assumed to be immobile and no immigration occurs.[4]

In the absence of a government pension individuals have to provide themselves for their old age consumption. They have two ways of doing so. One possibility is to save part of their income during young age and put it into the capital market where it will earn interest. The second possibility is to have children which will provide them with support during old age.

In the model children are viewed purely as investment goods. The support children provide for their ageing parents is illustrated by the parameter k. Here k is modeled as a transfer from adult child to elderly parent.

Children are willing to support their elderly parents in this way because 'it is tradition to do so'. This family tradition is enforced by the threat of social punishment for individuals that reneg on their obligations. The value of the social punishment needs to be higher than the value of k that individuals would have to spend on their parent.[5] Cigno (1993) and Rosati (1996) show how such a family constitution can be self-enforcing (without relying on social codes and punishments). An alternative way

[4]Börsch-Supan (1998, 2000) states that Germany would need an immigration stream of about 800,000 persons a year (about 2.5 times the current net immigration rate) to compensate fully for population aging.

[5]For individuals who themselves rely on the support of children when old, the family tradition becomes self-enforcing if it is assumed that individuals who do not care for their parents will not be cared for by their children.

to explain positive transfers from children to parents would be to assume that children have altruistic feelings towards their parents (as done in Nishimura and Zhang (1995), and Wigger (1999)).

If no government pension system exists, a person i of generation t with high skill level will solve the following utility maximization problem at the beginning of young age.

$$Max \quad U_t^i = (c_{i,t}^y)^{\alpha}(c_{i,t}^o)^{(1-\alpha)}$$

$$subject \;\; to: \quad c_{i,t}^y = w_t^h(1 - bG_t^i)(1 - s_t^i) - k, \tag{1}$$

$$c_{i,t}^o = k(1 + \gamma_t)G_t^i + w_t^h(1 - bG_t^i)s_t^i R_{t+1},$$

$$0 \leq bG_t^i \leq 1,$$

where $c_{i,t}^y$ denotes person i's consumption when young (in period t) and $c_{i,t}^o$ her consumption when old (in period $t+1$). $(1 + \gamma_t)$ represents the growth rate of wages from period t to $t+1$.[6] It is assumed here that the support from child to parent also grows at this rate, such that a person of generation t can expect transfers of $k(1+\gamma_t)G_t^i$ when retiring in period $t+1$. The subscript for the individual, i, will be left out in the following analysis if the meaning remains clear from the context. Since bG_t^i denotes the time individual i spends on rearing children, $(1 - bG_t^i)$ is the time individual i spends on the labor market.

For now it is assumed that for high skilled individuals the return from saving beats the return available via the family tradition. That is: $bw_t^h R_{t+1} > k(1+\gamma_t)$. This implies that high skilled individuals will have positive savings but no children, thus the average number of children born to high skill individuals is zero ($G_t^h = 0$). Solving the above utility maximization problem we find that a high skill person's saving rate in period t will be

$$s_t^h = (1 - \alpha) - \frac{k(1 - \alpha)}{w_t^h}.$$

[6] It is assumed here that high and low skilled wage rates both grow at the same rate.

Thus, i's saving rate depends positively on the relative importance i puts on old age consumption and the wage rate when young. It depends negatively on the size of k. It does however not depend on the interest rate R_{t+1}.[7]

For a low skilled individual j of generation t the budget constraints for young and old age consumption will look as follows:

$$c^y_{j,t} = w^l_t(1 - bG^j_t)(1 - s^j_t) - k,$$

$$c^o_{j,t} = k(1 + \gamma_t)G^j_t + w^l_t(1 - bG^j_t)s^j_t R_{t+1},$$

For now it is assumed that for low skilled individuals the return available by investing in children is higher than the return via the capital market: $bw^l_t R_{t+1} < k(1+\gamma_t)$. Thus, a low skill person will invest in children only. This means that the average savings rate for low skill individuals will be equal to zero $s^l_t = 0$. This then implies that the first order condition of the Cobb-Doublas utility function with respect to the budget constraint of a low skilled individual can be stated as follows:

$$\frac{\partial \ln U}{\partial G^i_t} = \frac{-\alpha b w^l_t}{w^l_t(1 - bG^i_t) - k} + \frac{(1-\alpha)}{G^i_t}$$

From this follows that the optimal number of children for a low skill individual will raise is given by

$$G^l_t = \frac{1}{b}[(1-\alpha) - \frac{k(1-\alpha)}{w^h_t}]. \tag{2}$$

3 ER-DB-PAYG pension system

For the reasons mentioned in section 1 above, the main model presented here will be an earnings related defined benefit pay-as-you-go (ER-DB-PAYG) model.

Public pension benefits are determined by a percentage formula over lifetime income. The structure of the pension system is given exogeneously (e.g. by law or constitution). The parameter $x \in [0, 1]$ is used to calculate each individual's pension claim. It denotes

[7] This is a consequence of the Cobb-Douglas utility function used here.

the replacement rate of the PAYG pension system. Multiplying i's period t income by x will give her public pension benefit in period $t+1$. Individual pensions therefore depend on x, the wage rate, and on the amount of time a person has spent on the labor market, $(1-bG_t^i)$. The individual treats her promised pension payment as a parameter when maximizing utility. Even though x can in principle be changed from one period to the next it is assumed that such a change cannot be applied to people who worked under the old system. That is, existing pension contracts have to be honored.[8]

To finance the PAYG pension system, the government levies an income tax, v_t, on income. The level of v_t is determined endogenously to balance the government's budget in each period.

3.1 High Skilled Individuals

In this system the constraints of high skilled individuals of generation t will change to:

$$c_{i,t}^y = w_t^h(1-bG_t^i)(1-v_t)(1-s_t^i)-k, \tag{3}$$

$$c_{i,t}^o = k(1+\gamma_t)G_t^i + xw_t^h(1-bG_t^i) + w_t^h(1-bG_t^i)(1-v)s_t^i R_{t+1},$$

$$0 \leq bG_t^i \leq 1,$$

We assumed that high skill i finds it optimal to save and invest in the capital market rather than having children if no pension system exists. For this to still be her optimal strategy it must be the case that:

$$w_t^h[(1-v_t)R_{t+1}+x] - \frac{k(1+\gamma_t)}{b} > 0. \tag{4}$$

The pension benefits x increase, while the tax payments v_t decrease the opportunity cost of having children. Thus, high pension contribution rates v_t reduce the desirability of market participation (and with it saving).

[8] It is assumed here that individuals view promised pension benefits like private property and that this view is justified by the country's political and legal system. For a further discussion on this and similar issues see Diamond (1997).

For the following analysis it is assumed that the [4] is satisfied which implies that i finds the return of the market higher than the return via the family tradition. Her optimal saving rate becomes:

$$s_t^i = (1-\alpha) - \frac{\alpha x}{R_{t+1}(1-v_t)} - \frac{k(1-\alpha)}{w_t^h(1-v_t)}$$

Thus, compared with the case without government pension, the per capita saving rate of a high skilled individual has been reduced for all positive values of x and/or v_t. This is the crowding out effect of the ER-DB-PAYG pension system that is widely discussed in parts of the pension literature.[9] Note that if the pension benefits are very generous, such that

$$x \geq \frac{(1-\alpha)R_{t+1}}{\alpha}[(1-v_t) - \frac{k}{w_t^h}]$$

all private saving will be crowded out.

3.2 Low Skilled Individuals

If an ER-DB-PAYG pension system exists, the budget constraints for young and old age consumption for a low skilled individual j of generation t will look as follows:

$$c_{j,t}^y = w_t^l(1-bG_t^j)(1-v_t)(1-s_t^j) - k,$$

$$c_{j,t}^o = k(1+\gamma_t)G_t^j + xw_t^l(1-bG_t^j) + w_t^l(1-bG_t^j)s_t^j R_{t+1},$$

For j to find it optimal to still have children once the public pension system is introduced it must be the case that the return from having a child is larger than the income and pension payments forgone by having a child. The participation constraint for having a child for a low skill individual is as follows:

$$w_t^l[(1-v_t)R_{t+1} + x] - \frac{k(1+\gamma_t)}{b} < 0. \tag{5}$$

The ER-DB PAYG pension system thus has a twofold influence on the opportunity cost of having children. The higher the level of benefits x, the higher the opportunity

[9] For an empirical discussion of the crowding out of private savings due to PAYG pension systems see for example Feldstein (1985).

cost of having a child. On the other hand, the higher the tax rate, v, the lower the opportunity cost of a child. The higher v_t the more likely that (5) is satisfied and having children will be the preferred vehicle to transport consumption into old age. This is the case as increases in v_t hurt the return on private savings more than the return on children.

As long as (5) is satisfied the return from having children outweighs that of market participation and the optimal number of children born to a low skilled person becomes:

$$G_t^l = \frac{1}{b}[1 - \frac{\alpha k(1+\gamma_t)}{k(1+\gamma_t) - xw_t^l b} - \frac{(1-\alpha)k}{(1-v_t)w_t^l}]. \tag{6}$$

It is interesting to note that while a high v_t reduced the relative cost of children in (5), the number of children a person will rear depends negatively on v. The first derivative of (6) with respect to v_t is equal to:

$$\frac{\partial G_t^i}{\partial v_t} = -\frac{(1-\alpha)k}{(1-v_t)^2 w_t^l b}.$$

This term is always negative.

Thus, while increases in v_t make it more likely that (5) is satisfied and hence an individual has children, once (5) is satisfied a further increase in the tax rate v_t will act to reduce fertility. This is because an increase in v_t will reduce young age consumption $c_{j,t}^y$. An agent will respond by trying to smooth consumption (i.e. increasing $c_{j,t}^y$ and reducing $c_{j,t}^o$) by reducing fertility. The overall influence of v_t on fertility is thus not clear.

The influence of x on individual fertility is straightforward. Increases in x will increase the opportunity cost of having a child in (5), as well as the optimal number of children in (6). The ER-DB-PAYG system reduces j's fertility for all levels of $x \in (0, 1]$. Thus, the ER-DB-PAYG system crowds out fertility. A similar crowding out effect of individual fertility due to PAYG pensions has been mentioned in Cigno (1993), Rosati (1996), and Wigger (1999).

It is interesting to note that the introduction of the PAYG system will definitely decrease saving by high skilled individuals. But if the introduction of the PAYG system alters (5) such that low skill individuals find it optimal to switch from having children as old age support to relying on a combination of public pension and private saving, then their per capita savings rate increases. As long as x is not too high (such that all private saving is crowded out) it could be that the increase in saving by the low skilled would off-set the reduction in saving by the high skilled. Thus, the introduction (or expansion) of an ER-DB-PAYG system could in principle decrease, increase or keep total per capita saving unchanged. This ambigous effect of PAYG pension systems on per capita saving occurs when individuals can use children as well as the capital system to transport consumption into old age (this implies that children are - at least partially - seen as investment goods). This ambiguous effect of PAYG pension systems on per capita saving (and growth) have also been analyzed by Cigno and Rosati (1996), Zhang and Zhang (1998), and Wigger (1999).

3.3 The derivation of the budget balancing tax rate

In the following it is assumed that the separation is perfect. That is, low skilled individuals will find it optimal to transport consumption possibilities to old age via having children, while high skilled individuals will find it optimal to use the capital market.

In such a separating case, an old person at time t receives $xw_{t-1}^l(1 - bG_{t-1}^l)$ in the form of a public pension if she is of the low skilled type and xw_{t-1}^h if she is of the high skilled type. To provide all old agents at time t with their pensions, the government therefore has to pay out:

$$[xw_{t-1}^l(1 - bG_{t-1}^l)\phi + xw_{t-1}^h(1 - \phi)]N_{t-1}. \tag{7}$$

Where G_{t-1}^l denotes the average number of children born to low skill individuals in period $t - 1$. Note that the sum of generation $t - 1$'s children makes up generation t, i.e., $\phi N_{t-1}G_{t-1}^l = N_t$. When the government levies a tax rate v_t, the collective tax payments of the young in period t amount to:

$$\phi N_t w_t^l(1 - bG_t^l)v_t + (1 - \phi)N_t w_t^h v_t. \tag{8}$$

Since the current young pay the pensions of the current old, the collective tax payments of the young must be sufficient to finance the public pensions of the old. That is, it must be the case that [7] must be equal to [8].

The tax rate on income that the government has to levy in order to balance the budget in period t, as a function of fertility, is therefore:

$$v_t(G_t^l) = \frac{x[\phi(1-bG_{t-1}^l)w_{t-1}^l + (1-\phi)w_{t-1}^h]}{(1+n_t)[\phi(1-bG_t^l)w_t^l + (1-\phi)w_t^h]} \quad for \quad G_t^l < \bar{G}_t \tag{9}$$

with

$$\bar{G}_t = \frac{w_t^l\phi + w_t^h(1-\phi)}{w_t^l\phi b} - \frac{x[w_{t-1}^h(1-\phi) + w_{t-1}^l\phi(1-bG_{t-1})]}{w_t^l\phi bG_{t-1}(1+\gamma)}.$$

and $N_t/N_{t-1} = (1+n_t)$.

Let $L_t = N_t[\phi(1-bG_t^l) + (1-\phi)w_t^h/w_t^l]$ denote the quality-adjusted labor force in period t and let $(1+l_t) = L_t/L_{t-1}$ denote the growth rate of this quality adjusted labor force. The growth rate of this quality adjusted labor force can be further broken down into the growth rate of population and the growth rate of quality adjusted labor participation:

$$(1+l_t) = (1+n_t)(1+g_t).$$

The growth rate of the quality adjusted labor participation rate is given by:

$$(1+g_t) = \frac{\phi(1-bG_t) + (1-\phi)w_t^h/w_t^l}{\phi(1-bG_{t-1})(1-\phi)w_{t-1}^h/w_{t-1}^l}.$$

Then (9) can be re-written as

$$v_t(G_t^l) = \frac{x}{(1+g_t)(1+\gamma_t)(1+n_t)} \quad for \quad G_t^l < \bar{G}_t \tag{10}$$

with $\bar{G}_t$ as above.

The above equation represents the government's budget balancing equation for any level of aggregate fertility in the range $[0, \bar{G}_t]$. If aggregate fertility lies outside this range even a tax rate of 100 percent could not balance the budget. $v_t(G_t)$ is strictly increasing in G_t for $G_t < \bar{G}_t$. $G_t(v_t)$ on the other hand is strictly decreasing in v_t.

The budget balancing tax rate for period t is found by setting [6] equal to [10], as shown in Figure 1 for the case of the parameter values used in the simulations in section 5. This equilibrium tax rate generates tax revenue exactly equal to the amount needed to finance generation $t-1$'s pension payments.

Insert Figure 1

4 Including Fertility Based Claims in a DBPAYG Pension System

A fertility based component is now added to the pension system analyzed above. Under this fertility based ERDB system the public pension received by an old person of generation t now consists of two parts: a part based on the contribution in the labor market and a part based on individual fertility. The total payment to a high skill individual of generation t combines these two parts:

$$x[(1-f)w_t^h(1-bG_t^i)] + x[fG_t^i],$$

where xfG_t^i is the fertility based part of the public pension system with the parameter $f \in [0,1]$ set by the Government. The pension formula for low skill individuals is identical except that it features the wage rate w_t^l instead of w_t^h. For a similar fertility based component see Kolmar (1997).

When $f = 0$, there are no fertility based payments and hence the system is identical to the one described in the previous section. In the other extreme case with $f = 1$, public pension payments depend only on individual fertility.

The parameter f can in principle be adjusted over time by the government. As before it is assumed however that such changes cannot be applied in retrospect. That means, existing pension claims must be valued. The government therefore takes f as well as x as given when calculating the budget balancing tax rate for a particular period t.

For any given tax rate $v_t \in [0,1]$, an individual i of generation t now maximizes the following intertemporal Cobb-Douglas utility function:

$$Max \quad U_t^i = (c_{i,t}^y)^{\alpha}(c_{i,t}^o)^{(1-\alpha)}$$

$$subject\ \ to: \ \ c_{i,t}^y = w_t^h(1-bG_t^i)(1-v_t)(1-s_t^i) - k, \tag{11}$$

$$c_{i,t}^o = k(1+\gamma_t)G_t^i + x[(1-f)w_t^h(1-bG_t^i) + fG_t^i] + w_t^h(1-bG_t^i)(1-v_t)s_t^iR_{t+1},$$

$$0 \le bG_t^i \le 1.$$

Apart from the extra term in the constraint determining old age consumption, this maximization problem is identical to the one considered in (3).

Again, high skill individuals invest in the market only as long as the return from doing so outweighs the benefits available by having children:

$$w_t^h[x(1-f) + (1-v_t)R_{t+1}] - \frac{k(1+\gamma_t)+xf}{b} > 0. \tag{12}$$

This inequality can be rewritten as:

$$v_t < 1 - \frac{k(1+\gamma_t)+xf}{R_{t+1}bw_t^h} + \frac{x(1-f)}{R_{t+1}}$$

For now it is assumed that v_t satisfies this inequality. If this is the case, $G_t^i = 0$ and individual saving of the high skilled individuals will become:

$$s_t^h = (1-\alpha) - \frac{\alpha(1-f)x}{(1-v_t)R_{t+1}} - \frac{k(1-\alpha)}{w_t^h(1-v_t)}.$$

Ceteris paribus, this savings rate of high skill individuals depends positively on f as can be seen by

$$\frac{\partial s_t^h}{\partial f} = \frac{\alpha x}{R_{t+1}(1-v_t)}.$$

Thus, ignoring the impact of v_t, saving by high skill individuals would increase with an introduction of the fertility part of the pension system. The reason is that the introduction of the fertility based part of the PAYG system reduces the pension benefit of individuals without children. Thus for individuals without children, consumption smoothing requires a transfer of young age resources to old age resources via additional saving. Of course, v_t will be impacted by a change in f (v_t can either rise or fall in response to an increase in f) and we will discuss this relationship further in the simulation section below.

Low skilled individual j will maximize her utility function subject to the following constraints.

$$c^y_{j,t} = w^l_t(1 - bG^j_t)(1 - v_t)(1 - s^i_t) - k,$$

$$c^o_{j,t} = k(1 + \gamma_t)G^j_t + x[(1 - f)w^l_t(1 - bG^j_t) + fG^j_t] + w^l_t(1 - bG^j_t)(1 - v_t)s^j_t R_{t+1},$$

$$0 \leq bG^j_t \leq 1.$$

The opportunity cost of children has been reduced by the fertility based PAYG system relative to the ER-DB-PAYG system of the previous section. For low skilled individuals the return on saving will therefor still be lower than the return available from having children. Thus, $s^l_t = 0$ and $G^l_t > 0$.

Solving for the average number of children born to a low skill individual G^l_t gives after some restructuring:

$$G^l_t = \frac{1}{b}[1 - \frac{\alpha[k(1 + \gamma_t) + xf]}{k(1 + \gamma_t) + xf - x(1 - f)bw^l_t} - \frac{(1 - \alpha)k}{(1 - v_t)w^l_t}] \tag{13}$$

for $v_t \leq \bar{v}_t$.

4.1 The derivation of the budget balancing tax rate

As before the government uses the tax revenue of generation t to pay for generation $t-1$'s pensions while balancing the budget. Since an individual old person of generation $t - 1$ will receive $x[(1 - f)w^l_{t-1}(1 - bG^l_{t-1}) + fG^l_{t-1}]$ in the form of a public pension if she is of the low skilled type and $x(1 - f)w^h_{t-1}$ if of the high skilled type.

In a separating case a young person at time t will pay $v_t(1 - bG^l_t)w^l_t$ in the form of payroll taxes if unskilled and $v_t w^h_t$ if skilled.

Balancing the budget while honoring the pension benefits of the older generation implies therefore that the budget balancing payroll tax v_t is equal to:

$$v_t(G_t^l) = \frac{x(1-f)[\phi w_{t-1}^l(1-bG_{t-1}^l) + (1-\phi)w_{t-1}^h] + x\phi f G_{t-1}^l}{(1+n_t)[\phi w_t^l(1-bG_t^l) + (1-\phi)w_t^h]}$$

for $G_t^l < \bar{G}_t$ with

$$\bar{G}_t = \frac{[-fxN_{t-1}\phi G_{t-1} + N_t\phi(w_t^l - w_t^h) + N_t w_t^h]}{N_t\phi w_t^l b} - \frac{(1-bG_{t-1}\phi)x(1-f)w_t^l}{(1+n_t)\phi w_t^l b(1+\gamma)}$$

As in the last section this budget balancing tax rate can be rewritten in terms of the growth rates of population, labor participation, and wages:

$$v_t(G_t^l) = x\left[\frac{(1-f)}{(1+n_t)(1+\gamma_t)(1+g_t)} + \frac{f\phi G_{t-1}^l}{(1+n_t)w_t^l L_t}\right] \tag{14}$$

for $G_t^l < \bar{G}_t$, with $\bar{G}_t$ as above.

The equilibrium tax rate for period t is found by setting [6] equal to [14], as shown in Figure 2 for the case of the parameter values used in the simulations in section 5 and a fertility payment rate f of 0.2.

Insert Figure 2 Here

The budget balancing tax rate v_t tends to increase in the first period after the introduction (or increase) of the fertility based component of the PAYG pension system. This is because the incentive to have more children reduces the tax base to finance existing pension obligations. However, by the second period after the introduction of the fertility based component the budget balancing tax rate v_t tends to fall below its original level due to the increase in the population growth rate. The effect of a change in f on other parameter values is discussed further in the next section.

5 Simulations

The dynamics of the model are analyzed here using simulations. There are eight parameters in the model. α, b, k, N_t/N_{t-1}, w_t^l, w_t^h, $(1+\gamma_t)$, and x. The first simulation evaluates the impact of wage growth and PAYG replacement rates on population growth and sustainability of the pension system over time. The growth rate of wages, $(1+\gamma_t)$, and the PAYG replacement rate, x, are allowed to vary while the remaining parameters are set to the following values: $\alpha = 0.5$, $b = 0.1$, $w_t^h = 3$, $w_t^l = 0.9$, $G_{t-1}^l = 2$, $k = 0.2$, $\theta = 0.5$.

In the following simulations α is set equal to 0.5. This means that for each individual consumption is equal in both the young and old period. Reflecting their higher productivity, high skilled individuals earn more than low skilled individuals, with $w_t^h = 3$ and $w_t^l = 0.9$. It is assumed that both types of wages grow over time at the rate $(1 + \gamma_t)$. The child contribution rate k is set equal to 0.2, which means that young people have to transfer 1/15th of their gross income to their elderly parents if they are of the high skill type and 2/9th if they are of the low skill type. The maximum number of children per person is set to ten, so that $b = 0.1$. The number of children born in the starting period is given by $G_{t-1}^l = 2$. The percentage of low skilled skilled individuals in the population is given by θ which is set to 0.5 in the simulations.

5.1 The influence of x and γ on sustainability

Figures 3 and 4 show the population dynamics for all possible combinations of x and γ_t. The figures were constructed using the following iterative process. In every round the equilibrium allocation of G_t and v_t is determined endogenously. Unless the system collapses right away, these values then determine the starting values for the next period.

Insert Figure 3 Here

In Figure 3, γ, x combinations that lead to a stable population are represented by the dark blue upward sloping line in double thickness. When only low skill individuals have children and with $\theta = 0.5$ this occurs at $G_t = G_t^l = 2$.

Combinations of high wage growth and/or low PAYG replacement rates lead to increasing population over time. On the other hand, combinations of low wage growth and/or generous replacement rates lead to decreasing population over time which make the PAYG system unsustainable.

The higher γ the more attractive children are as old age security as parents can participate in the growing income of their children via their children's old age support k. If γ_t is high enough even high skilled individuals will find the returns offered by the family tradition more attractive than returns from the market and switch to having children themselves. On the other hand, if γ_t is too low even low skilled individuals opt out of having children and rely on the market instead. For a gross interest rate of $R_{t+1} = 2$ these two "participation" constraints for low and high skilled individuals are represented by the two thin green lines in Figure 3.

If a $\gamma_t - x$ combination falls below the lower of these participation lines, even low skilled individuals opt out of having any offspring and instead use the capital market together with PAYG pension system to secure their retirement consumption. Fertility levels in this case fall to zero.[10] $\gamma - x$ combinations that fall into the area above the upper "participation constraint" indicate that high and low skilled individuals want to have children and rely on the family tradition when old rather than the capital market. Fertility rates are very high in this case but average per capita saving is zero.[11]

The two "participation" constraints depend on the return on the capital market as indicated by R_{t+1}. Figure 3 assumes that $R_{t+1} = 2$. For interest rates higher than this both participation constraints move up, while they would move down if interest rates were lower.

Area A in Figure 3 represents x and γ_t combinations that lead to increasing population over time with complete separation between high and low skilled individuals. In this region the fertility rate of low skilled individuals is high enough to make the entire population increase over time. High skilled individuals do not have any children and instead save for their retirement. Assuming no restriction in natural resources, the

[10] There is a certain amount of myopia assumed in this model. Individuals – even though otherwise rational – do not believe that the PAYG pension system can collapse due to a lack of young people. If they were completely rational, they would predict the collapse and hence not rely on it to provide old age consumption.

[11] For a given γ_t, fertility levels decline as the replacement rate x increases. The reason for this is that a higher x makes market participation more attractive for low skilled individuals.

population keeps growing, making the PAYG system cheap to run. The tax rate needed to finance the PAYG system stays low (or decreases) over time, making the internal rate of return of the PAYG system, which is given by $x/v_t = (1+\gamma_t)(1+l_t)(1+n_t)$, an attractive alternative to investment in the capital market at an interest rate of R_{t+1}.[12]

Combinations of x and γ_t that lie in Area B lead to a fertility decline over time. Systems characterized by these parameter combinations are not sustainable in the long run and eventually collapse due to a lack of young people. The further the $\gamma_t - x$ combination lies away from the separation line between areas A and B, the less sustainable it becomes. Just like region A, B is also characterized by complete separation in the retirement behavior of low and high skilled individuals: low skilled individuals invest in children while high skilled individuals save in the capital market.

Only parameter combinations that lie on the thick blue line along the boundary between areas A and B lead to fertility rates that can guarantee stable population growth.

C - the region above the "participation constraint" of high skilled individuals is characterized by rapidly increasing population and no private saving by either group.

In contrast, parameter levels of γ_t and x that lie below low skilled individuals' "participation constraint" in region D, lead to a collapse of the PAYG pension system due to a lack of a next generation. Since both low and high skilled individuals use the capital market and the public pension system instead of children to transport consumption

[12] This is a variation on the Aaron (1966) condition for a system with endogenous fertility. If $(1+\gamma_t)(1+n_t)(1+l_t) > R_{t+1}$, welfare of high skilled individuals is higher under the PAYG system compared with an unfunded system. On the other hand, high skilled individuals would be better off under an unfunded system (or no system at all) if the inequality is reversed. Thus, this inequality tells us only about the welfare of individuals that participate in the market. Individuals that rely on children care about whether the return from the pension system is bigger or smaller than the return from children, given by $k/(b(1+\gamma_t))$. Thus their "Aaron condition" is given by $(1+\gamma_t)(1+n_t)(1+l_t) > R_{t+1} > k/(b(1+\gamma_t))$. If it is then the PAYG system increases their individual welfare, and otherwise not.

possibilities into their retirement period, fertility falls to zero.

Figure 4 shows how these regions of sustainability change when a fertility based component is introduced into the ER-DB-PAYG pension system. The most important change is that the growth rate of wages (γ_t) needed to sustain a pension system with a particular replacement rate is reduced. This improves the sustainability of the PAYG system. As a consequence the thick black line that symbolizes the $x - \gamma$ combinations that lead to stable population growth moves downwards as f increases. Also, the participation constraints for high and low skill types move downwards as f increases which is illustrated by a downwards shift from the light green to the light blue lines in Figure 4. This indicates that both types of individuals start to have children at a lower growth rate of wages than previously, improving the sustainability of the PAYG system.

Insert Figure 4 Here

The improvement in sustainablity can be attributed to the inclusion of the fertility based part in the ER-DB-PAYG pension system which gives individuals a greater financial incentive to have children. In other words the public good character of children is reduced by the introduction of a fertility based pension component.

One potential shortcoming of the above analysis is that x and γ are not strictly independent in reality. As discussed in section 3 above, high levels of x crowd out private saving of the high skilled individuals as well as fertility levels of the low skilled. Both of these crowding out effects have an influence on the wage rate and thus γ. In a neoclassical growth model the crowding out of private saving puts downward pressure on wage rates due to a reduction in the capital-labor ratio. However, one can also argue that the decrease in fertility increases the capital-labor ratio of the next generation and therefore puts upward pressure on future wages.[13] Thus, while it is clear that there are interdependencies between x and γ it is not always clear how a change in x influences

[13]In the present model a reduction in fertility would also increase the current participation rate which then would decrease this period's capital-labor ratio and therefore reduce current wages.

γ.

An analysis of the exact interaction of x and γ_t in the present framework goes beyond the scope of this article. Traditionally OLG models without endogenous fertility find that PAYG pension systems crowd out economic growth because of the first of these effects (the crowding out of private saving). Models with endogenous fertility and children as investment goods (e.g., Cigno and Rosati (1996)) however find that PAYG systems could increase economic growth (and therefore wage growth) because of the second effect (the reduction in fertility).

The effect of x on per capita saving rates is also not clear cut. In the present model an increase in the level of x leads to a decrease in per capita saving by high skilled individuals but to an increase in per capita saving by low skilled individuals. Again, the aggregate effect could be positive or negative.

5.2 The influence of x and k on sustainability

Figure 5 shows how the sustainability of the PAYG system is affected by different levels of x and k. It is assumed here that wages remain constant over time with $(1 + \gamma_t) = 1$. The straight downward sloping lines illustrate the "participation constraints" for high and low skilled individuals respectively. They divide the $k - x$ space into three main areas. In the lowest part of the graph (below the low skill participation constraint) the return from children is not high enough to induce either type of individual to invest in children. If this is the case the fertility rate drops to zero immediately while the per capita saving rate is quite high. This is of course followed by a collapse of the PAYG system in the next period since there is no one around to pay the existing pension liabilities.

Insert Figure 5 Here

The top part of the graph above the high skill participation constraint (Area C') illustrates the situation in which family tradition forces children to provide so generously for their ageing parents that both low and high skill individuals want to rely on children rather than the market for old age protection. Fertility rates are high in area C' and even high PAYG replacement rates are sustainable over time but per capita private savings are zero in this region. However, for a given level of k fertility declines as one moves from the left to the right side of the graph. As every individual (whether or not they find it optimal to use the market to transfer consumption into old age) spends at least part of her young age working in the market to finance young age consumption, a high replacement rate x automatically covers some of their old age consumption even if they were planning on having children. Thus – other things equal – the higher x the greater the share of old age consumption that gets automatically covered by the public pension and the lower the number of children born.

For x-k combinations that lie along the separating line between areas C' and D' population growth would be equal to replacement levels: $G_t = G_t^h + G_t^l = 2$. For combinations that lie in region D' fertility by high and low skilled individuals would lie below the replacement level of $G_t = G_t^h + G_t^l = 2$.

If the $k - x$ combinations lie between the two downward sloping participation constraints (pictured in red), low skilled individuals find it optimal to have children while high skilled individuals save. Combinations of k and x that fall into area A' lead to population increase over time, while combinations of k and x that fall into area B' are characterized by decreasing populations. x and k combinations that lie along the separation line between areas A' and B' lead to stable populations over time with $G_t = G_t^l = 2$.

For systems that are characterized by the parameter combinations of region B', fertility rates are below replacement level. In such cases, a DBPAYG system is not sustainable in the long run. A reduction in the pension benefit rate x is one way a government could try to prevent a collapse of the public pension system. Another possibility to increase sustainability is to implement policies that will induce an increase in the aggregate fertility level. One potential policy to achieve this is to include a fertility based component into the ER-DB-PAYG pension system.

Figure 6 illustrates what happens to sustainability if a fertility based component f is introduced into the pension system. The main difference to Figure 5 is that now with $f > 0$ the region that represents growing population over time (A'') occupies a much larger part of the graph. A'' now contains many of the combinations of k and x that fell into the region of permanently shrinking populations B' before. Figure 6 assumes a fertility component of the pension system of $f = 0.2$. This trend becomes even more pronounced if higher fertility based components of the PAYG system, f, are considered. The arrows in Figure 7 illustrate how the lines move as f increases from 0 to 0.2.

Insert Figure 6 Here

Insert Figure 7 Here

Hence the inclusion of a fertility based component in the DBPAYG pension system increases the likelihood that the system will be sustainable. This finding is consistent with those of Cigno, Luporini and Pettini (2003) and Cigno and Werding (2007).

6 Conclusion

The overlapping-generations model of endogenous fertility developed here illustrates how an earnings related defined benefit pay-as-you-go (ER-DB-PAYG) pension system can reduce the number of children born and hence sow the seeds of its own destruction. It is also shown that the sustainability of a PAYG pension system can be increased by including a fertility based component. These issues are of direct relevance to a number of OECD countries that have generous ER-DB-PAYG pension systems and falling fertility rates.

My simulations show that having a fertility based component in the DBPAYG pension system increases the likelihood that the system will be sustainable. In particular, for parameter values that are plausible for many European countries, introducing a modest fertility payment may be sufficient to transform an unsustainable pension system into one that is sustainable. It should be politically easier to find a majority to agree with a change in f than with a switch from an unfunded to a funded pension system or a reduction in x_t. This is because changing the size of the pension parameter f does not place as big a burden on the 'switching generation'. Assuming f can be altered if fertility levels are too high or too low, this system also provides governments with the necessary flexibility to cope with changing population structures without increasing uncertainty for pensioners.

The benchmark for this article is OECD countries with aging populations and generous PAYG pension systems. However, the analysis may also be relevant to developing countries. Most developing countries have little or no public pension provision, but high reliance on family ties. A parent's only source of income during old age comes from their children's contributions. Those countries are characterized by low or zero x values but relatively high k values. As can be seen from Figure 3 this combination lies in region A, the area with exploding population. A country facing too high fertility rates could therefore implement a defined-benefit pension system in order to reduce fertility rates. A moderate defined benefit pension system would give individuals the possibility to obtain old age consumption without relying completely on their children. This will increase their incentives to work in the market and have less children. Also, the introduction of such a pension system would reduce the incentives to work outside the 'official' market (in 'black market activities') since pension payments are at stake.

It is of course the case that parents do not raise their children only as old age security. They also derive emotional satisfaction from them. In this sense children could be seen as durable consumer goods and one could argue that the utility derived from having children should feature in a parent's utility function. This approach has been used by Kolmar (1997) and Wigger (1999) among others. The present model could easily be adapted to include children in the utility function by adding a 'child-consumption term'. This kind of utility function would induce people to invest more time in children and make the declining fertility result less dramatic since people would not stop having children completely even if k is zero. However, putting children into the utility function would not reverse the relationship between rising opportunity costs of child-bearing and falling fertility rates and would therefore not eliminate the externality effects of fertility decisions. The basic conclusions of the model would still remain the same.

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Figure 1:
Finding the budget balancing tax rate in period t for f=0

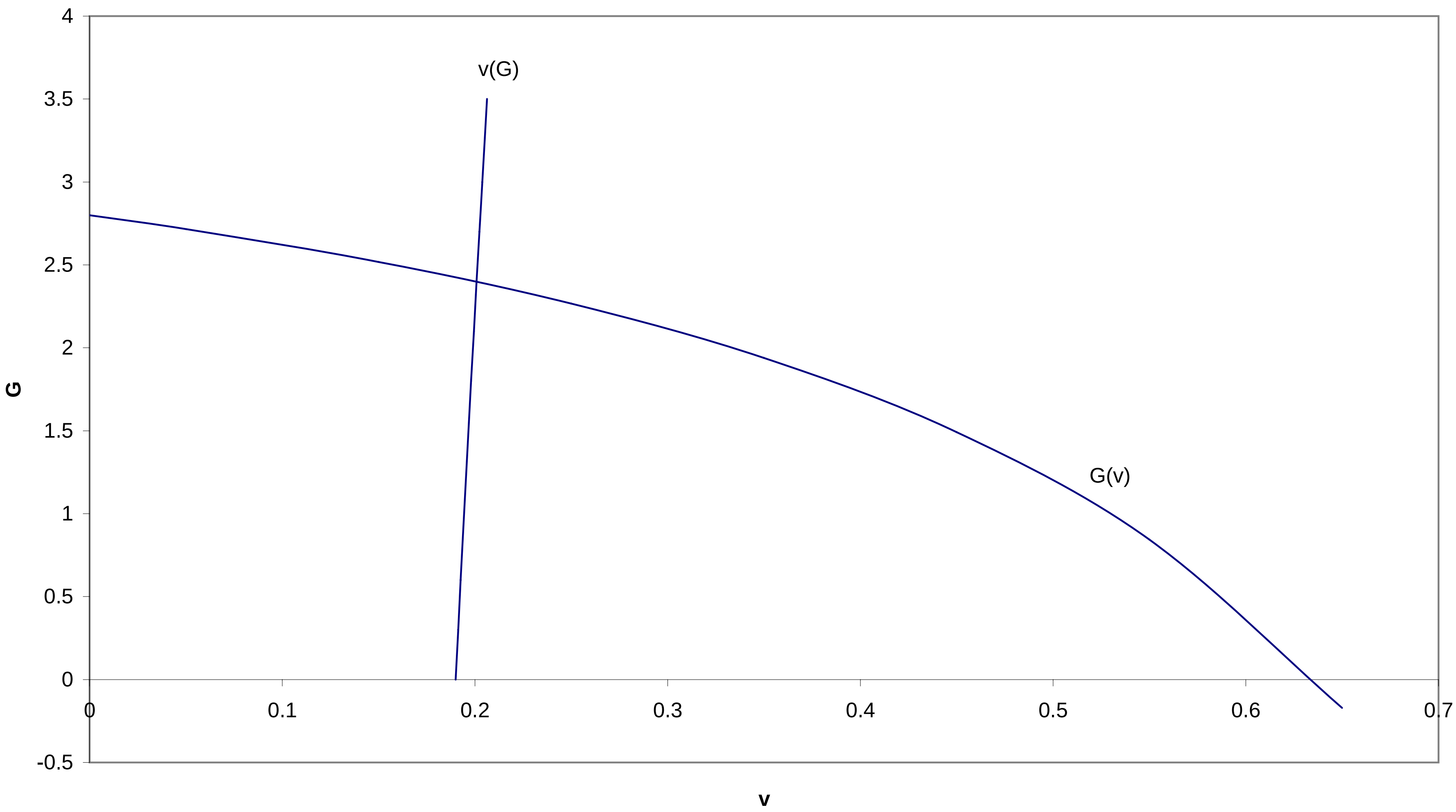

Figure 2:
Finding the budget balancing tax rate for period t when f=0.2

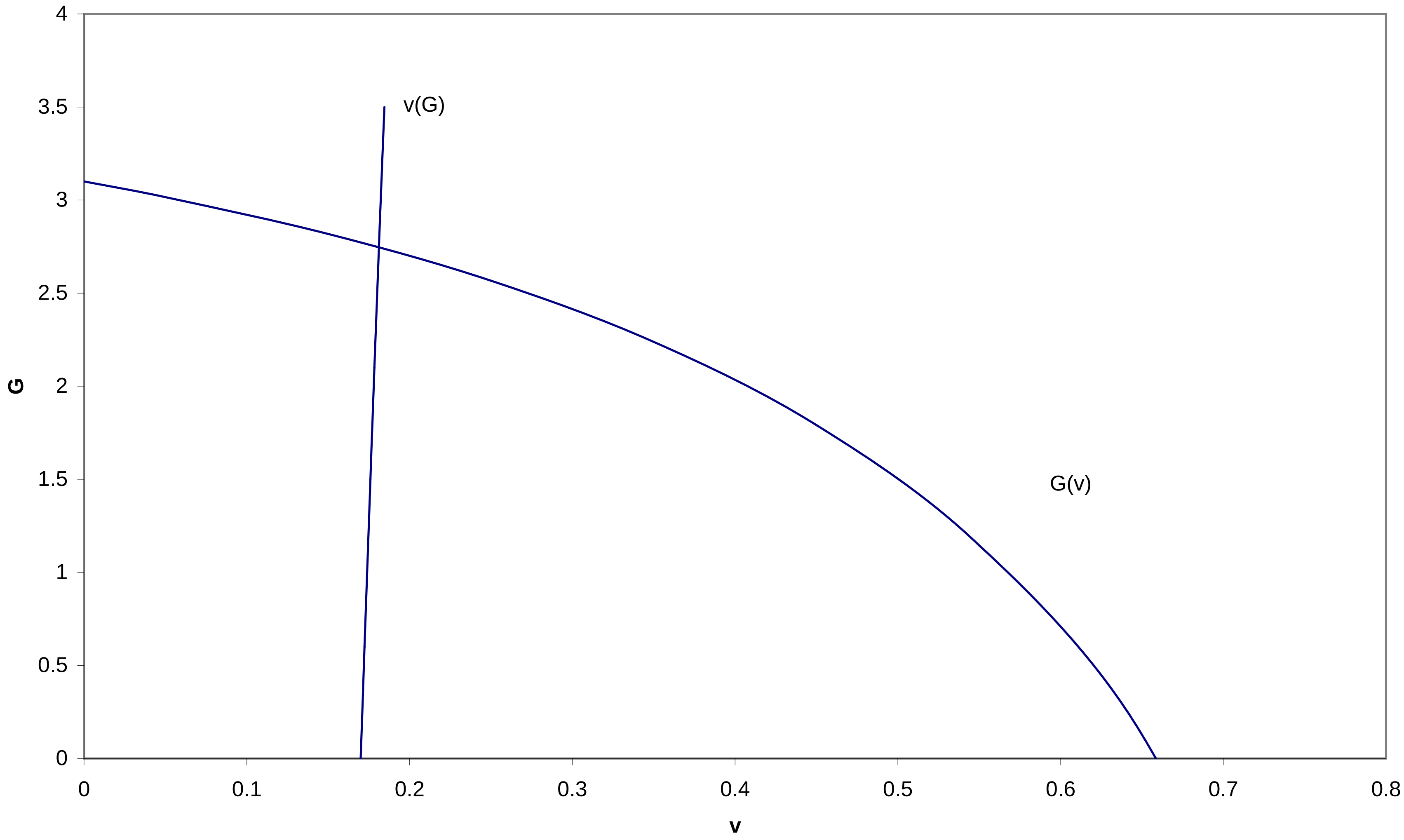

Figure 3:
x-gamma combinations for f=0

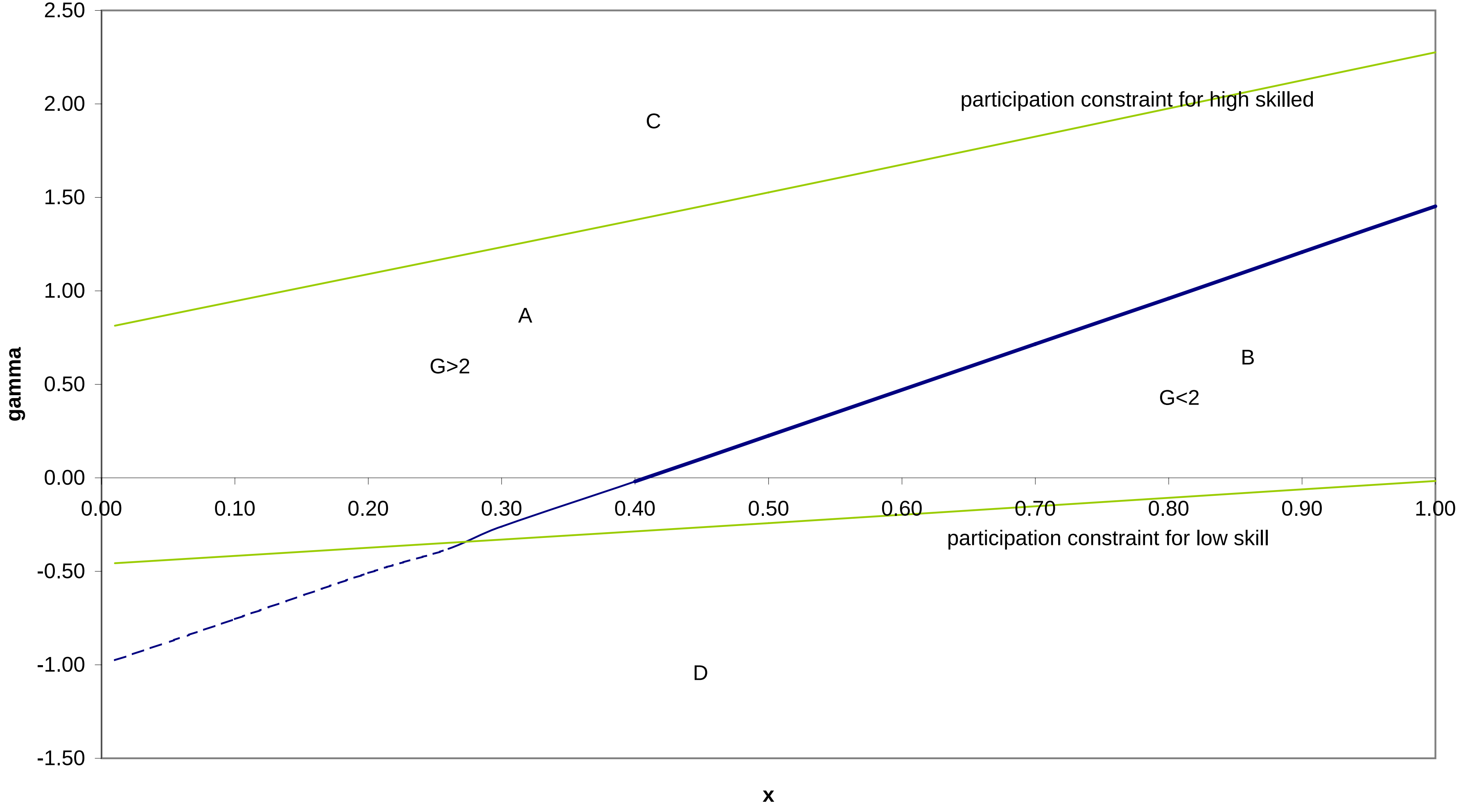

Figure 4:
gamma-x combinations

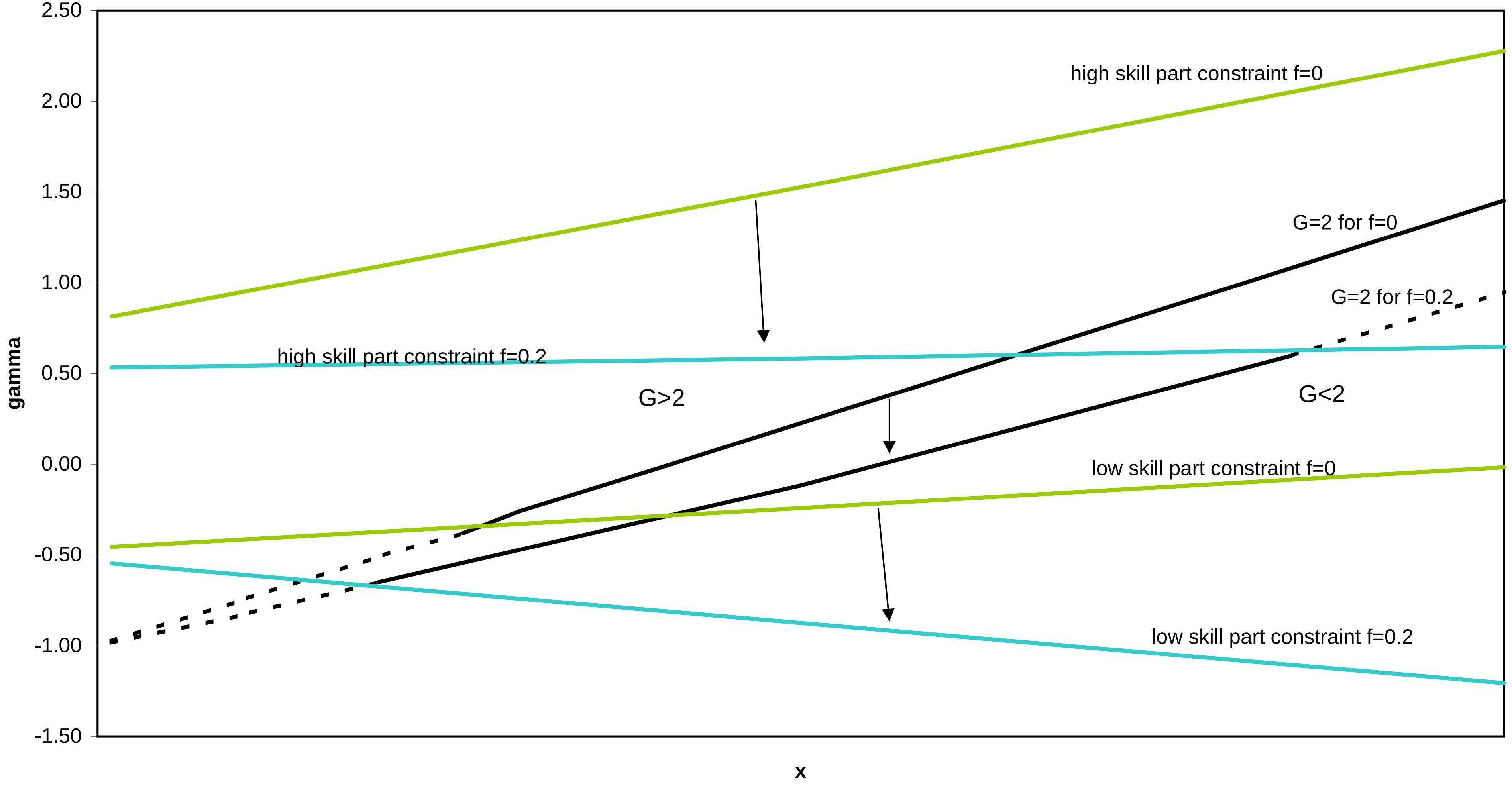

Figure 5:
sustainable regions for x-k combinations when f=0

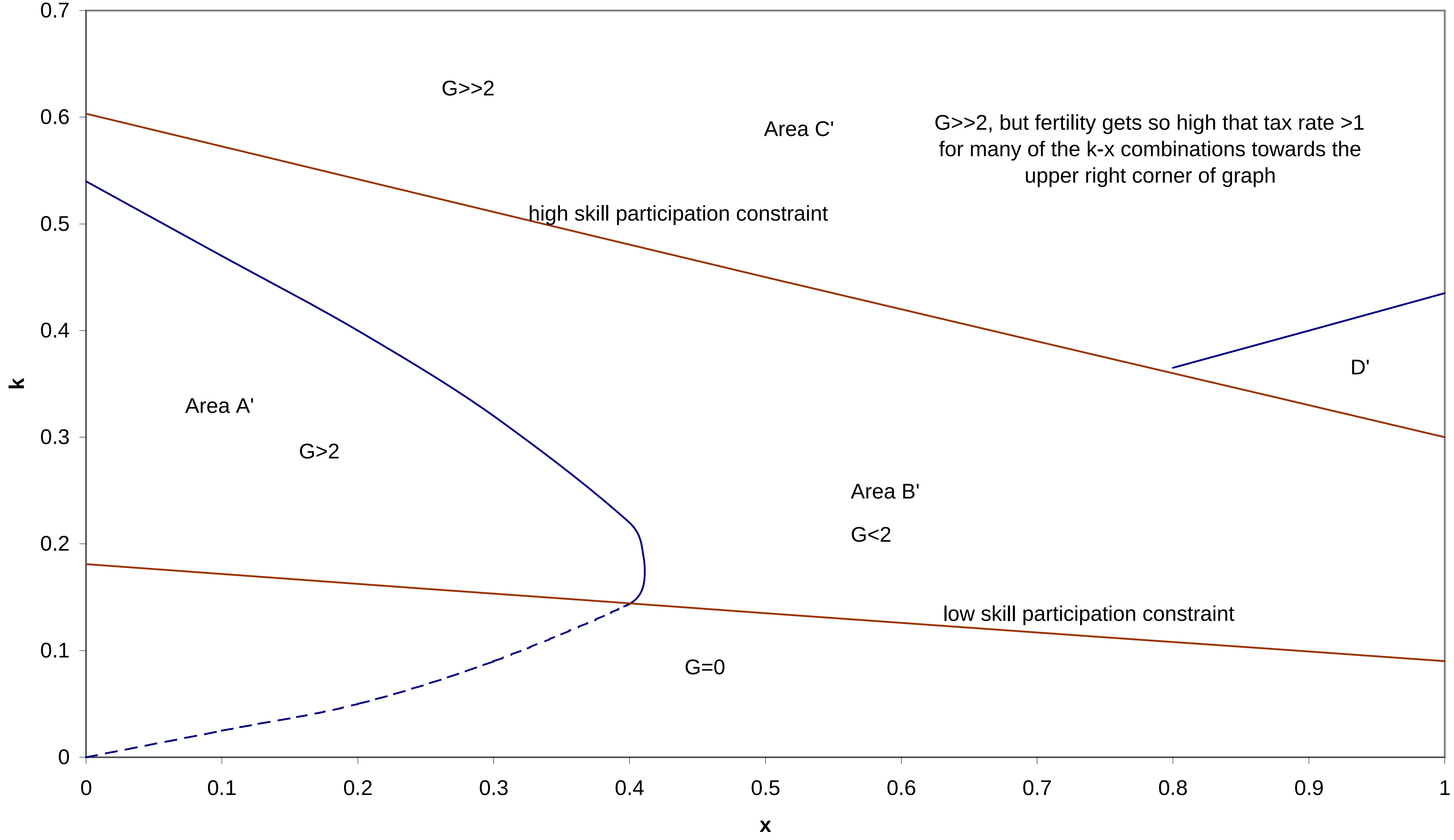

Figure 6:
sustainable combinations of x and k for f=0.2

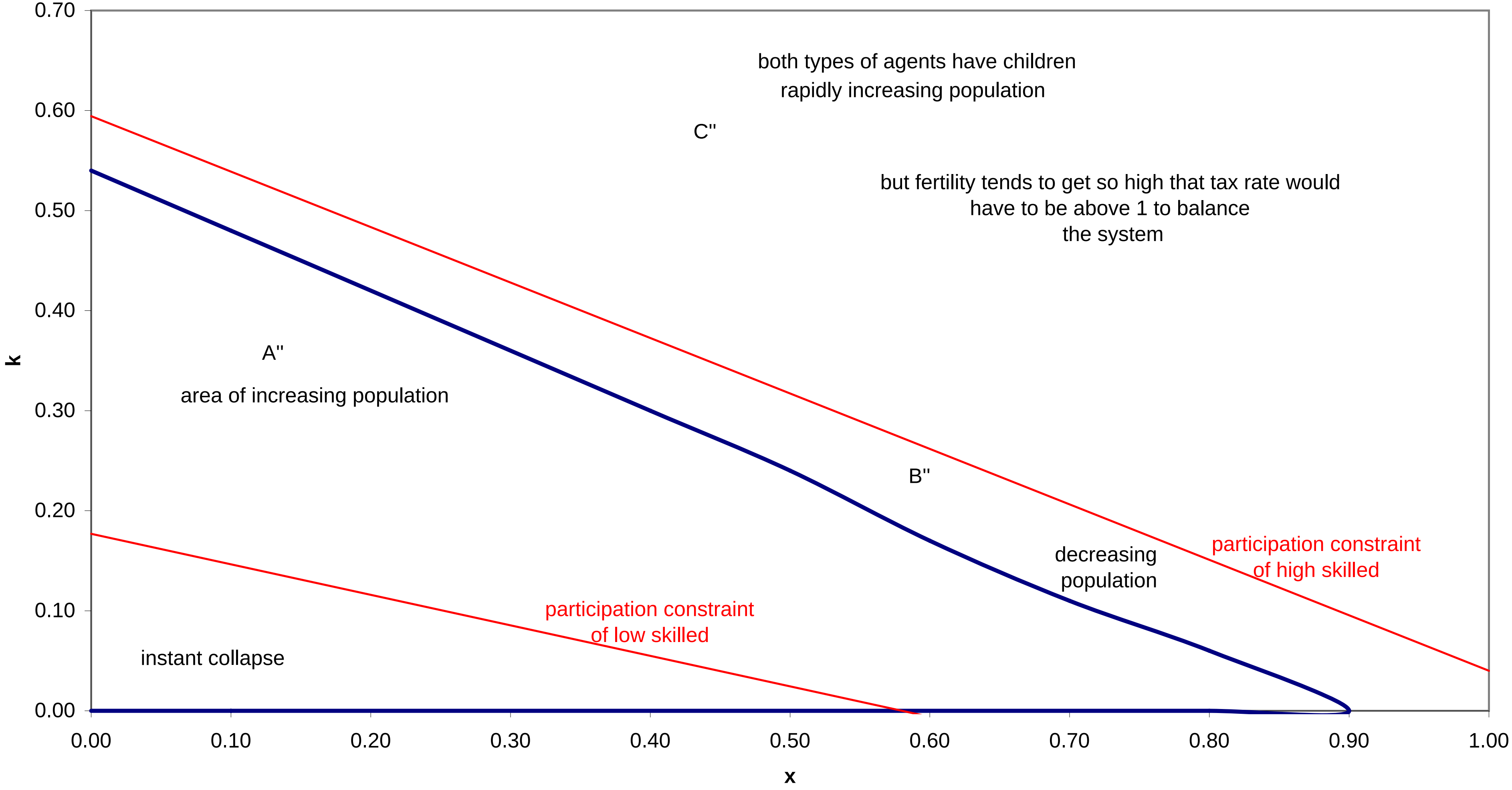

Figure 7:
sustainable regions of x-k combinations f=0 and f=0.2