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Temptation and Social Security in a Dynastic Framework*

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Abstract

We investigate welfare and aggregate implications of a pay as you go (PAYG) social security system in a dynastic framework in which agents have self-control problems. The presence of these two additional factors at the same time affects individuals' intertemporal decision problems in two opposite directions. That is, on the one hand individuals prefer to save more because of their altruistic concerns, on the other hand, they prefer to save less because of their urge for temptation towards current consumption. Individuals' efforts to balance between the long-term commitment (consumption smoothing and altruism) and the short-term urge for temptation result in self-control costs. In this environment the existence of social security system provides not only consumption smoothing and risk-sharing mechanisms but also a channel that reduces the severity of temptation. We find that the adverse welfare effects of a PAYG system are further mitigated relative to the environments that incorporates altruism and self control issues separately.

JEL Classification: E21, E62, H55

Keywords: Temptation; Self-control preferences; Altruism; Social security; Dynamic general equilibrium; Overlapping generations; Welfare.

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1 Introduction

The U.S. and many other countries run a pay as you go (PAYG) social security system in which tax revenues collected from current workers pay benefits of current retirees. Because the PAYG system affects individuals' fundamental economic decisions substantially and social security expenses is one of the large expenditure items in a government's budget, economists have analyzed the effects of the PAYG social security system extensively.

In the standard models of social security it is assumed that individuals save to smooth their consumption and to insure against idiosyncratic productivity shocks and longevity risk. Yet, individuals' savings decisions might also be affected by their altruistic concerns and self-control problems. Although there are studies that conduct welfare analyses of a PAYG system by using model economies that incorporate either individuals' altruistic concerns or self-control problems, the welfare implications of a PAYG system are not known if the two factors, altruism and temptation, coexist. We develop a model economy that incorporates these two factors and investigate the welfare implications of a PAYG system accordingly. In particular, we address the following two questions. First, what are the effects of temptation and altruism together on individuals' inter-temporal allocations and welfare? Second, what are the role and effects of a PAYG system in that environment?

The benefits and costs of the PAYG system are well documented in the literature: the PAYG system provides an insurance against longevity and income risks but, at the same time, it distorts an individual's saving and labour supply decisions. Using a deterministic standard overlapping generations (OLG) model, Diamond [1965] shows that social security reduces the steady state capital stock because it taxes workers with high propensities to save to pay benefits of retirees with low propensities to save. A decrease in a steady state capital stock, in turn, reduces welfare when the economy is dynamically efficient. Auerbach and Kotlikoff [1987] assess the magnitude of welfare losses using a deterministic large scale OLG model and find that social security always results in a welfare loss. Because private annuity markets that provide insurance against longevity risk are thin, social security has an important insurance role.[1] This, in turn, creates a potential for welfare improvement. Hubbard and Judd [1987] analyze the extent of insurance benefit of social security in an incomplete market environment where individuals have random lives. They show that even in the environment that social security provides insurance against longevity risk, it still decreases welfare. Imrohoroglu et al. [1995] extend Hubbard and Jude's work by adding an individual earning uncertainty to the model through the channel of unemployment risk. They show that social security might enhance welfare, but their results are driven from the dynamic inefficiency in the model economy. To eliminate the dynamic inefficiency, Imrohoroglu et al. [1999] incorporate a fixed production factor, land, to a model of social security. They show that having social security in the economy reduces welfare because the adverse effects are dominant.

Inter-generational transfers have important implications on individuals' choices. This in

[1] See Diamond et al. [2005] for more information regarding with annuity markets.

turn affects the extend of the PAYG system's costs and benefits. There is a strand of the literature that analyzes the effects of social security by using dynastic models. In his seminal work, Barro [1974] shows that if a bequest motive is operative then private transfers can neutralize the effects of public transfers and public debt i.e. Ricardian equivalence holds. Fuster [1999] demonstrate that when individuals have bequest and inter-vivos transfer motives, social security is less detrimental to the capital stock in an economy. Fuster et al. [2003] develop a dynastic OLG model with inelastic labor supply. They show that the insurance role played by social security dominates its crowding-out effects on capital stock and hence, the steady state welfare increases with social security for most households. Fuster et al. [2007] extend their previous work by adding labor/leisure choice and find that individuals prefer to be born into an economy without social security mainly because of efficiency gains from removing distortions on labor supply.

Potential idiosyncrasies in individuals' preferences are another important source of uncertainty regarding with the welfare implications of social security. A number of studies show that social security can generate welfare gains when households lack the foresight to save adequately for their retirement.[2] Two types of preference structures, time-inconsistent and self-control preferences, have been employed to analyze various macroeconomic problems including social security.[3] Imrohoroglu et al. [2003] investigate the welfare effects of a PAYG system in an economy in which individuals have time-inconsistent preferences. Social security in that environment works as a commitment device. In other words, social security saves on behalf of individuals who, otherwise, would not save enough for their retirement because of their time-inconsistent preferences. Even in that environment, the existence of a PAYG program could not improve the welfare. In a similar vein, Fehr et al. [2008] calculate the welfare effects of removing social security by calibrating a model economy to the German economy. They calculate not only the steady state equilibrium but also the transition path. In contrast to the results of Imrohoroglu et al., they show that social security enhances welfare and the welfare gain is the highest if individuals have time-inconsistent preferences. Kumru and Thanopoulos [2008] incorporate self-control preferences into a model of social security. In their environment, social security does not play a role of a commitment device but it plays a role of a temptation reducing device. Interestingly, Kumru and Thanopoulos show that social security might enhance welfare by reducing individuals' self-control costs through the channel of reducing their available wealth each period.

[2] See Imrohoroglu et al. [2003] and Kumru and Thanopoulos [2008] for a detailed discussion in this issue.

[3] The experimental economics literature documents that subjects who face intertemporal choice problems often show preference reversals. The first formal analysis of preference reversals was conducted by Strotz [1956]. Later, Laibson [1997] adopt the structure that was created by Phelps and Pollak [1968] to analyze intergenerational altruism to model preference reversals. Laibson modify the standard exponential discounting model by incorporating an additional discounting factor that captures the present-bias. The new discounting factor distorts the time-consistent feature of the standard exponential model. Laibson's preference structure is often called as time-inconsistent preferences. Gul and Pesendorfer [2004] attempt to explain the same phenomenon by creating self-control preferences that depend not only on an agent's actual consumption but also on the agent's hypothetical temptation consumption. **?** provide empirical estimates that provide statistical evidence supporting the presence of temptation.

The effects of a PAYG social security system on an economy in which individuals are altruistic towards their children but have self-control problems have not yet been analyzed. In dynastic models individuals save not only for consumption smoothing and insurance purposes but also for altruistic motives i.e. to leave bequests and make inter-vivos transfers. Intergenerational private transfers mitigate the adverse effects of the PAYG system on savings. In contrast, when individuals face temptation they increase their current consumptions to be rid of the urge for temptation. Individuals' efforts to resist temptation create a self-control cost. In such environment the PAYG system provides an additional benefit: reducing self-control cost through the channel of reducing available wealth each period. Hence, its existence might not be as detrimental as compared to an environment in which there are no self-control problems.

In this paper we analyze the welfare implications of a PAYG social security system in a dynastic framework in which individuals have self-control preferences. To conduct our analysis we first develop a simple two period model. Our simple model helps us to understand how the interaction between altruistic concerns and urge for temptation influences the effects of the PAYG system on individuals' saving decisions. We show that altruism and temptation factors affect individuals' saving decisions in the opposite directions. That is on the one hand individuals prefer to save more because of their altruistic concerns, on the other hand they prefer to save less because of their urge for temptation towards current consumption. Furthermore, we show that altruistic individuals with self-control problems face larger self-control costs when they are young because they save not only for their old age consumptions but also for to leave bequests. In such environment, we show that the PAYG system offer an additional benefit: reducing self-control costs. Note that the same additional benefit is also available for non-altruistic individuals when they are young. In contrast to non-altruistic individuals, altruistic individuals face self-control costs when they are old too. Yet, the PAYG system does not offer any relief for old age self-control costs. Our simple model shows that the complex interaction between altruistic concerns, urge for temptation, and the PAYG system has substantial impacts on savings and self-control costs. This, in turn, raises the possibility that welfare implications of social security might differ from those already established by the previous studies, which incorporate altruistic concerns and self control issues separately to large scale general equilibrium OLG models.

Next, we develop a large scale model economy that comprises altruistic individuals with self-control preferences, competitive firms and a fully committed government. In that model, parents and children form a decision unit called a household in which resources are pooled and decisions are made jointly. A sequence of households in a family line, which is linked together through skill transmission and a bequest motive, creates a household dynasty. Households face demographic and skill shocks which are uninsurable. Our set up of household sector is quite similar to that of Fuster et al. [2003]. However, we deviate from them by incorporating the self-control preference structure created by Gul and Pesendorfer [2004]. We calibrate our model to the US data. In our simulation studies we confirm that the presence of two additional factors, altruism and temptation, mitigates the adverse welfare effects of a PAYG system.

The paper is organized as follows. Section 2 provides a concise introduction to self-control preferences and time inconsistency. We briefly present and compare the two theories and attempt to shed light on the different implications they have for the question at hand. Section 3 presents our two-period partial equilibrium model and analytical results. In section 4 we present our large scale model economy, describe the parameter values of it, present results of our policy experiments, and conduct sensitivity analysis. Section 5 concludes. The mathematical details of the solution to the simple model are delegated to Appendix A. Appendix B explains the computational techniques used in the paper and appendix C presents the remaining tables.

2 Temptation and self-control preferences

In this section we briefly highlight the similarities and differences between the time-inconsistent and the self-control preferences as well as motivate the use of the former in this paper.

2.1 Time inconsistent preferences

In the standard OLG models, preferences are defined over sequences of lifetime consumption, $\{c_1, c_2, ..., c_j, ..., c_J\}$. If individuals in the economy have time consistent preferences, and a deterministic life-span is equal to J, the utility ranking of the lifetime consumption sequences will not depend on their standpoint j : ranking of these sequences will be invariant with respect to the time the ranking took place.

The essence of the time inconsistent preferences is that the aforementioned invariance result no longer holds: The discounting structure sets up a conflict between today's preferences and the preferences that will be held in the future, commonly labeled as a "preference reversal." For example, from today's perspective, the discount rate between two far-off periods, j and $j+1$, is the long-term low discount rate, while from the time j perspective, the discount rate between j and $j+1$ is the short-term high discount rate. This can be modeled by the following preference structure adapted to the purposes of our model from Laibson [1997]:

$$U_j = E_j \left[u(c_j) + \delta \sum_{i=1}^{J-j} \beta^i u(c_{j+i}) \right].$$

When $0 < \delta < 1$, the above discounting structure can mimic the qualitative property of the generalized hyperbolic discounting function (namely a function implying discount rates that decline as the discounted event moves further away in time) but at the same time maintain most of the analytical tractability of the exponential discounting function. The preferences given in the above equation are dynamically inconsistent, in the sense that preferences at date j are inconsistent with preferences at date $j+1$.[4]

[4] To check this note that the MRS between periods $j+1$ and $j+2$ consumptions from the standpoint of the decision maker at time j is given by $\frac{u'(c_{j+1})}{\delta u'(c_{j+2})}$, which is not equal to the MRS between those same periods from the standpoint of the decision maker at $j+1$: $\frac{u'(c_{j+1})}{\beta\delta u'(c_{j+2})}$

Note that a major consequence of the time inconsistent preferences is that the optimal policy functions derived at age j for ages $j' > j$ will no longer be optimal when the individual arrives at age j'; and in the absence of any commitment technology, the individual's future behavior will deviate from that prescribed by the earlier policy functions.

2.2 Self-control preferences

An alternative way of modelling self-control issues is to use a class of utility functions identified by Gul and Pesendorfer [2004]. They provide a time-consistent model that addresses the preference reversals that motivate the time inconsistency literature.

Consider a set B of consumption lotteries, and a two-period setting. Gul and Pesendorfer [2004] have shown that under a specific assumption on choice sets (set betweenness) combined with other standard axioms that yield the expected utility function $U(.)$ defined as

$$U(B) := \max_{p \in B} \int (u(c) + v(c))\, dp - \max_{p \in B} \int v(c) dp$$

that represents the preference relation implied by the above axioms. The function $u(.)$ represents the individual's ranking over alternatives when she is committed to a single choice while when she is not committed to a single choice, her welfare is affected by the temptation utility represented by $v(.)$. Note that when B is a singleton, the terms involving $v(.)$ will vanish leaving only the $u(.)$ terms to represent preferences. However, if it is e.g. $B = \{c, c'\}$ with $u(c) > u(c')$ an individual will succumb to the temptation (that is, she will pick the commitment utility-reducing alternative, c') only if the latter provides a sufficiently high temptation utility $v(.)$ and offsets the fact that $u(c) > u(c')$, i.e., when

$$u(c') + v(c') > u(c) + v(c).$$

In this case the individual wishes she had only c as the available alternative, since under the presence of c', she cannot resist the temptation of choosing the latter.

When the above inequality is reversed, however, the individual will pick c in the second period, albeit at a cost of $v(c') - v(c)$.[5] We call the latter difference the "*cost of self-control.*"

In terms of the setting in the present paper, in every period an individual faces a consumption-savings problem. Each period, our individuals make a decision that yields a consumption for

[5] To see that, note that for $B = \{c, c'\}$ and $u(c) > u(c')$ we would have that

$$U(\{c, c'\}) = \max_{\tilde{c} \in \{c, c'\}} (u(\tilde{c}) + v(\tilde{c})) - \max_{\tilde{c} \in \{c, c'\}} v(\tilde{c}) =$$

$$= u(c) + v(c) - v(c')$$

and since by assumption $v(c') > v(c)$ this means that

$$U(\{c, c'\}) = u(c) - [v(c') - v(c)]$$

i.e. the utility of the choice c gets penalized by a positive number, the "cost of self-control". Note that in the case $v(c') < v(c)$ i.e. when there is congruence of the utility functions as to which alternative is the best, there is no temptation issue anymore; c is chosen at no penalty since the $v(.)$ terms in $U(\{c, c'\})$ cancel out.

that period and wealth for the next. However, each period these individuals face the temptation to consume all of their wealth, and hence, resisting to this temptation results in a self-control-related cost.

Under standard assumptions combined with the multi-period version of "set betweenness," we can represent self-control preferences in a recursive form for the purposes of our J period model which is delegated to sections 3 and 4.

The main difference between the above models is that the self control preferences do not imply dynamic inconsistency. Preferences are perfectly consistent. Moreover, it allows agents to exercise self-control, an option not existing in the time-inconsistent preferences. The difference in discounting was the source of preference reversals in the time-inconsistent preferences and the explanation of why individuals find immediate rewards tempting. Instead, Gul and Pesendorfer [2004]'s explanation assumes that agents maximize a utility function that is a "compromise" between the standard utility (or "commitment" utility) and a "temptation" utility.

Imrohoroglu et al. [2003] consider a setting similar to ours and analyze the consequences of time inconsistent preferences, while we follow the self-control paradigm in a similar finite-horizon setting. Gul and Pesendorfer [2004] show that for finite decision problems a time inconsistency model can be re-interpreted as a temptation model.

3 A simple two-period model

In this section we analyze the effects of altruism and self-control problems on individuals' allocation of resources by using a two period OLG model.

3.1 Individual's problem

An individual lives for two periods. At the beginning of her life she receives a bequest from her parents. While she supplies her 1 unit of labor endowment inelastically when she is young, she retires and receives a pension benefit when she is old. Because of her altruistic concerns, she saves a fraction of her wealth for her children in the second period. We also assume that she has self-control problems. The dynamic programing problem in the first period can be written as follows:

$$V_1(b_1) = \max_{c_1, s_1} \{u(c_1, s_1) + v(c_1, s_1)\} - \max_{\widetilde{c_1}, \widetilde{s_1}} \{v(\widetilde{c_1}, \widetilde{s_1})\}$$

subject to

$$\begin{aligned} c_1 + s_1 &= (1-\tau) w + b_1; \\ \widetilde{c_1} + \widetilde{s_1} &= (1-\tau) w + b_1, \end{aligned}$$

where V_1 is the first period's value function; c_1 is the first period's commitment consumption; $\widetilde{c_1}$ is the first period's hypothetical temptation consumption; s_1 is the first period's commitment saving; $\widetilde{s_1}$ is the first period's hypothetical saving; b_1 is the bequest received from the parents;

w is the wage income; τ is the social security tax rate; $u(.)$ is the momentary utility function; $v(.)$ is the temptation utility function. We assume that momentary and temptation utility functions take the following forms respectively: $u\left(c_1, s_1\right) = \left(\frac{c_1^{1-\sigma}}{1-\sigma} + \beta V_2\left(s_1\right)\right)$ and $v\left(c_1, s_1\right) = \lambda\left(\frac{c_1^{1-\sigma}}{1-\sigma} + \kappa_1 \beta V_2\left(s_1\right)\right)$, where V_2 is the second period's value function; λ is the parameter that captures the strength of temptation; and κ is the parameter that governs the nature of temptation. When $\kappa_1 < 1$, individuals are tempted towards current consumption; when $\kappa_1 > 1$, individuals are tempted towards future consumption; and when $\kappa_1 = 0$, individuals are tempted to consume all available wealth in a given period. Note that $\{v\left(c_1, s_1\right) - v\left(\widetilde{c_1}, \widetilde{s_1}\right)\}$ represents the cost of resisting to temptation i.e. the self-control cost.

In the second period she allocates her wealth between the second period consumption and bequest levels. The dynamic programming problem is given by

$$V_2\left(s_1\right) = \max_{c_2, b_2}\left\{u\left(c_2, b_2\right) + v\left(c_2, b_2\right)\right\} - \max_{\widetilde{c_2}, \widetilde{b_2}}\left\{v\left(\widetilde{c_2}, \widetilde{b_2}\right)\right\}$$

subject to

$$\begin{aligned} c_2 + b_2 &= Rs_1 + T; \\ \widetilde{c_2} + \widetilde{b_2} &= Rs_1 + T, \end{aligned}$$

where b_2 is the amount of bequest left if the commitment consumption level is chosen; $\widetilde{b_2}$ is the amount of bequest left when the temptation consumption level is chosen; R is the gross return from the savings, and T is the amount of pension benefit. Specifically, we assume that the second period utility function has a form of $u\left(c_2, b_2\right) = \frac{c_2^{1-\sigma}}{1-\sigma} + \gamma\frac{b_2^{1-\sigma}}{1-\sigma}$ and the temptation utility function has a form of $v\left(c_2, b_2\right) = \lambda\left(\frac{c_2^{1-\sigma}}{1-\sigma} + \kappa_2\gamma\frac{b_2^{1-\sigma}}{1-\sigma}\right)$. Similarly, $\left\{v\left(c_2, b_2\right) - v\left(\widetilde{c_2}, \widetilde{b_2}\right)\right\}$ is the self-control cost in the second period. The motive for bequest arises because individuals' preferences display "joy-of-giving" i.e. an individual's utility is an increasing function of the amount of bequest left.

3.2 Optimal allocation

We use the backward induction method to solve the individual's problem. In particular we first solve the individual's second period problem then we use that solution to solve the individual's first period problem. Each period's problem is solved in two steps. To solve the individual's second period problem we first solve the sub-problem of $\max_{\widetilde{c_2}, \widetilde{b_2}} v\left(\widetilde{c_2}, \widetilde{b_2}\right)$. Then, we solve the remaining sub-problem of $\max_{c_2, b_2} u\left(c_2, b_2\right) + v\left(c_2, b_2\right)$ by using the maximized values of hypothetical temptation consumption and bequest obtained in the previous step. The individual's first period problem is solved similarly. The details of the solution are delegated to Appendix A. The solution to the individual's optimization problem returns the following maximized values of the first and second period consumptions, saving and bequest.

$$c_1^* = \frac{1}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)\,w + b_1 + \frac{T}{R}\right), \tag{1}$$

$$s_1^* = \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)\,w + b_1\right) + \left(\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}} - 1\right)\frac{T}{R}, \tag{2}$$

$$c_2^* = \frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\left(\frac{R\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)\,w + b_1\right) + \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}T\right), \tag{3}$$

$$b_2^* = \frac{\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\left(\frac{R\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)\,w + b_1\right) + \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}T\right), \tag{4}$$

where, $\psi = \left(\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} - \lambda\right)$.

3.3 Analytical results

Our simple model is general enough to encompass several cases: when we set the temptation parameter $\lambda = 0$ and the altruism parameter $\gamma = 0$, we have a standard OLG model without temptation and altruism; when we set $\lambda > 0$ and $\gamma = 0$, we have a OLG model with temptation; and when we set $\lambda > 0$ and $\gamma > 0$, we have a model with altruism and temptation. Similarly, we can turn off (on) the public pension program by setting $\tau = 0(> 0)$ [and hence, $T = 0(> 0)$].

3.3.1 Temptation, altruism and savings

In this section we focus on analyzing the effects of temptation and altruism on savings. We start our analysis by using a standard OLG model then we extend it by incorporating self-control problems and altruistic concerns to the model to understand how the existence of temptation and altruism would affect individuals' inter-temporal allocations and welfare. To isolate the effects of temptation and altruism we close down the public pension program.

In the standard OLG model, individuals allocate consumptions over life-cycle to maximize their life-time utility i.e. they keep a fraction of their initial endowment for old age consumption. The unit cost of consumption in the fist period is the inverse market interest rate, $\frac{1}{R}$. The optimal saving rule is given by $s_1^{*(\lambda=0 \text{ and } \gamma=0)} = \frac{(\beta R)^{\frac{1}{\sigma}}}{1+(\beta R)^{\frac{1}{\sigma}}}\left(w + b_1\right)$.

Yet, when a non-altruistic individual has a self control problem i.e. $\lambda > 0$ and $\gamma = 0$, the optimal saving rule is given by

$$s_1^{*(\lambda>0 \text{ and } \gamma=0)} = \frac{\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left(w+b_1\right).$$

Since $s_1^{*(\lambda>0 \text{ and } \gamma=0)} < s_1^{*(\lambda=0 \text{ and } \gamma=0)}$, we conclude that individuals save less when they have self-control problems.

Proposition 1 *Individuals with urge for temptation towards current consumption save less when they are young.*

The intuition is as follows. Individuals try to balance their urge for high level consumption in a given period with the long term interest of resisting to it. Because the resistance to temptation comes with a cost (self-control cost), the optimal level of saving is lower than the case that there are no self-control issues.

We now introduce altruistic concerns to the model by assuming that individuals have "joy of giving" i.e. $\gamma > 0$. It is well established in the literature that altruistic individuals save more to leave bequests and make inter-vivos transfers. This result still holds even when individuals have self-control problems.

Proposition 2 *Altruistic individuals save more even if they have self-control problems.*

When individuals are altruistic and have self-control problems, the optimal saving rule is:

$$s_1^{*(\lambda>0 \text{ and } \gamma>0)} = \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left(w+b_1\right),$$

where $\psi = \left(\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} - \lambda\right)$. The variable ψ captures the effect of the interaction between altruism and temptation parameters on savings. Let consider the special case in which $\sigma = 1$ that results in $\psi = (1+\gamma) > 1$. By plugging this value of ψ into the above saving rule we can show that that $s_1^{*(\lambda>0 \text{ and } \gamma>0)} > s_1^{*(\lambda>0 \text{ and } \gamma=0)}$. It implies that altruistic individuals save more because they save not only for old age consumption but also save for to leave bequests. The similar result holds when $\sigma > 1$. In other words, the presence of altruism mitigates the negative effect of temptation on savings.

This result also reveals that temptation and altruism factors affect individuals' saving decisions in the opposite directions: while the existence of temptation negatively affects individuals' savings, the existence of altruism positively affect individuals' savings.

3.3.2 Temptation, altruism and self-control cost

Proposition 3 *Non-altruistic individuals with self-control problems face self-control costs only when they are young.*

A non-altruistic individual's temptation and commitment consumption levels in the first period are given by $\widetilde{c}_1 = w + b_1$ and $c_1 = \frac{(w+b_1)}{1+\left(\frac{\beta R}{1+\lambda}\right)^{\frac{1}{\sigma}}}$ respectively. Hence, the self-control cost of the non-altruistic individual can be written as

$$SCC_1^{(\lambda>0 \text{ and } \gamma=0)} = \lambda \left[\frac{\left(\frac{w+b_1}{1+\left(\frac{\beta R}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}}{1-\sigma} - \frac{(w+b_1)^{1-\sigma}}{1-\sigma} \right] < 0.$$

Non-altruistic individuals do not face self-control costs when they are old i.e. $SCC_2^{(\lambda>0 \text{ and } \gamma=0)} = 0$. The reason is simple. They have only one choice when they are old: consuming everything available. In other words, their temptation and commitment consumption levels are identical in the second period.

Proposition 4 *Altruistic individuals face self-control costs not only when they are young but also when they are old. In addition, self-control costs they face when they are young are larger.*

In the second period, altruistic individuals need to split their wealth between consumption and bequest. To leave a bequest they should resist to the temptation that creates a self-control cost as in the first period. Although the presence of altruism mitigates the negative effects of temptation on savings in the first period, it generates additional self-control cost in the second period:

$$SCC_2^{(\lambda>0 \text{ and } \gamma>0)} = \lambda \left[\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} \right)^{1-\sigma} - 1 \right] \frac{(Rs_1)^{1-\sigma}}{1-\sigma} < 0.$$

Moreover altruistic concerns result in higher self-control cost in the first period. When $\lambda > 0$ and $\gamma > 0$, the self-control cost $SCC_1^{(\lambda>0 \text{ and } \gamma>0)}$ is given by

$$SCC_1^{(\lambda>0 \text{ and } \gamma>0)} = \lambda \left[\frac{\left(\frac{(w_1+b_1)}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}}{1-\sigma} - \frac{(w_1+b_1)^{1-\sigma}}{1-\sigma} \right].$$

Since $SCC_1^{(\lambda>0 \text{ and } \gamma>0)} < SCC_1^{(\lambda>0 \text{ and } \gamma=0)} < 0$, we conclude that the self-control cost is larger for young altruistic individuals. The reason is as follows. In the altruistic framework individuals tend to save more when they are young to leave bequests. This means that the gap between commitment and temptation consumption levels widens. As a result, altruistic individuals face larger self-control cost in the first period. In other words, the more "joy of giving" individuals have the more self-control cost they face.

Corollary 1 *Leaving bequest is costly for altruistic individuals who have self-control problems.*

3.3.3 Social security, savings and self-control costs

An introduction of a PAYG social security system creates distortions on individuals' intertemporal allocations. The optimal saving rule in the presence of a social security system is given by

$$s_1^* = \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)w+b_1\right)-\left(1-\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)\frac{T}{R}.$$

The term $\left(1-\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)$ captures the direct effect of social security on savings. In other words, an increase in social security payments by one dollar would decrease savings by $\left(1-\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)$. The classic result that social security crowds out savings still holds. The term $\left(\tau\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)$ captures the distortion of the tax-financing instrument on income.

Proposition 5 *The presence of altruism mitigates the adverse effect of social security on saving.*

The strength of the crowding-out effect of social security on savings is diminished in the dynastic framework. When individuals are not altruistic i.e. $\gamma=0$, the adverse effect of social security on saving is equal to $\left(1-\frac{\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)$. Since $\left(1-\frac{\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right) > \left(1-\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)$, we conclude that the adverse effect of social security on saving is much smaller when individuals are altruistic.

Next, we analyze the effects of social security on the level of consumption and the self-control cost. To isolate the effects of the PAYG on the self-control cost, we assume that social security benefit payments are fair: individuals receive back their contributions plus interest income i.e. $T=R\tau w$. The consumption level and the self-control cost are given by $c_1^*=\frac{1}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}(w+b_1)$ and $SCC_1=\lambda\left[\frac{\left(\frac{(w+b_1)}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}}{1-\sigma}-\frac{((1-\tau)w+b_1)^{1-\sigma}}{1-\sigma}\right]$ respectively.

Proposition 6 *The introduction of social security program reduces the self-control cost in the first period in both altruistic and non-altruistic frameworks.*

Social security works as a forced saving mechanism in our model by assumption. It implies that while social security affects savings, it does not distort individuals' inter-temporal consumption allocations. In other words, consumption allocations remain identical before and after the introduction of the program. However, if individuals have self-control problems, social security can still create an additional benefit. It restrains young individuals' choice sets by reducing their available wealth through taxation. As a result, the young individuals' urges for temptation become less severe. This, in turn implies that the individuals face smaller self-control costs. Note that self-control costs become smaller as the tax rate τ increases.

Proposition 7 *Social security does not reduce old individuals' self-control costs in the altruistic framework.*

An altruistic old individual's self-control cost in an economy with PAYG system is the following:

$$SCC_2 = \lambda \left[\left(\frac{1}{1 + \left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} \right)^{1-\sigma} - 1 \right] \frac{(Rs_1 + T)^{1-\sigma}}{1-\sigma}.$$

An introduction of social security creates two opposite effects on available wealth in the second period. On the one hand it increases the transfer income received, on the other hand it reduces the investment income as a result of a decrease in savings in the first period. Since the former effect is larger than the latter effect, the introduction of social security creates a larger self-control cost in the second period as shown by the following:

$$\frac{\partial SCC_2}{\partial T} = \lambda \left[\left(\frac{1}{1 + \left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} \right)^{1-\sigma} - 1 \right] (Rs_1 + T)^{-\sigma} \left(-\frac{1}{1 + \left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}} + 1 \right) > 0.$$

While social security reduces the size of the choice set in the first period, it extend the size of the choice set in the second period. Therefore, it reduces the self-control cost when individuals are young and increases it when individuals are old.

Corollary 2 *The presence of altruism reduces the strength of the additional benefit offered by the PAYG system to individuals with self-control problems.*

4 Quantitative analysis

In the previous section our partial equilibrium model was simple enough to obtain a close form solution. Hence, we were able to draw intuitive conclusions regarding the effects of altruism, temptation, and social security on individuals' choices. However, our analytical results do not encompass the general equilibrium effects. We know that a change in inter-temporal allocations ultimately affects capital accumulation and market prices. These in turn create feedback effects on individuals' resource allocations. These feedback effects can have very important implications for the policy analysis at hand as shown in the previous literature. In addition, since the effects of social security might vary across different income groups, it is important to model the heterogeneity among households. In this section, we develop a large scale OLG model with heterogeneous individuals to fully analyze the effects of social security.

4.1 Model

4.1.1 Demographics

We consider an economy populated by overlapping generations. Every period t a generation of economically active individuals is born. All newly-born individuals are endowed with a skill level s that might be high (H) or low (L). Individuals face stochastic lives and live maximum $2J$ periods. Following Fuster et al. [2003], we assume that individuals' skill types affect their survival probabilities. A type s individual's probability of surviving up to age j conditional on surviving up to age $j-1$ is given by $v_j(s)$. The constant cohort share of generation j individual can be written as follows $\mu_1(s) = \lambda(s)(1+n)^J$, and $\mu_j(s) = \dfrac{v_j(s)}{1+n}\mu_{j-1}(s)$, for $j = 2, 3, ..., 2J$,where $\lambda(s)$ is the measure of period 1 individuals and n is the constant population growth rate. Similarly, the cohort size of individuals dying each period (conditional on survival up to the previous period) can be defined recursively as $\eta_j(s) = \frac{1-v_j(s)}{1+n}\mu_{j-1}(s)$.

4.1.2 Altruism and household dynasty

Individuals are assumed to be altruistic towards both their children and parents i.e. they value their children and parents' consumption stream. In other words, the two-sided altruism exists in our model economy. There are three possible transfer schemes from parents to children: inter-vivos transfers (parents are alive when they make transfers), bequests (transfer realizes when parents die at the maximum age), and unintended bequests (transfer realizes when parents die early). Similarly, children can transfer wealth to their parents to promote well-being of them.

An individual's life consists of two stages. The first stage of life, childhood, starts at age 1 and ends at at age J. The second stage of life, adulthood, starts at age $J+1$ and ends at $2J$. At age $J+1$, individuals' parents die and their children are born. Variable J denotes the number of periods that children and parents' lives overlap i.e. individuals' lives overlap with those of their parents in the first J periods and overlap with those of their own children in the last J periods. We call individuals as children in the first stage of life and as parents in the second stage of life.

In each period the surviving members of a family form a decision unit called *"household."* Depending on their demographic structures, households are classified into one of three groups. Group 1 households are made up of parents and children, group 2 households are made up of parents only, and group 3 households are made up of children only. If parents and children survive together, they pool resources and solve a joint utility maximization problem. This is the simplest way to incorporate two-sided altruism.[6] If children do not survive, parents run households on their own and the family line breaks after parents die. If parents die early, children take over and set up children only household. At age $J+1$, children themselves become new parents and start a new household with their own children. They again pool their resources and jointly solve a new household optimization problem.

Group 1 households last for J periods i.e. they last until parents die at age $2J$. A group 1 household can become a group 2 household if children die and become a group 3 household if parents die during J periods. However, group 2 and 3 households' types do not change within J periods. The transition probability matrix that describes the movements between groups is given by

$$\Omega\left(g_j, g_{j+1}\right) = \begin{bmatrix} v(s)^p_{J+j} v(s)^k_j & v(s)^p_{J+j}\left(1 - v(s)^k_j\right) & \left(1 - v(s)^p_{J+j}\right)\left(1 - v(s)^k_j\right) \\ 0 & v(s)^p_{J+j} & 0 \\ 0 & 0 & v(s)^k_j \end{bmatrix},$$

where $g_j = 1, 2$, or 3 and $v(s)^p_{J+j}$ and $v(s)^k_j$ are survival probabilities of a parent and children, respectively.

The sequence of households of parents, children, grandchildren etc. in a family line defines a household dynasty. Each individual is a member of two consecutive households (or decision making units). In other words, an individual first sets up a household with her parents and then she sets up a household with her off-springs. Our model shows the features of both the infinite horizon and overlapping generation frameworks. While skill transmission and two-sided altruism introduce an infinite horizon framework by generating a household dynasty that continues forever, our assumptions that each individual faces a random finite lifetime that overlaps with her parent and her children and a demographic shock that breaks a family line with a certain probability introduce a life-cycle framework.

4.1.3 Skill endowment

Individuals cannot change their skill types during their life-times, but it is possible that their children are born with different skill levels. We suppose that skill transmission across generations follows a two-state Markov process. The transition probability matrix is as follows:

$$\Pi\left(s^p, s^k\right) = \begin{bmatrix} \pi_{L,L} & \pi_{L,H} \\ \pi_{H,L} & \pi_{H,H} \end{bmatrix},$$

[6] If we assume that parents and children maximize different objective functions, a strategic game between parents and children arises. Solving models that incorporate such games requires a more complicated solution technique. Nishiyama [2002] provides more details on this.

where s^p and s^k denote a parent's and a child's skill levels respectively and π_{s^p,s^k} is the probability that the child is endowed with the skill level s^k conditional on his parent's skill level s^p. Individuals' labor efficiencies depend on both their skill levels and ages. We denote age and skill dependent efficiency by $e_j(s)$. There is no labor-leisure choice in our model economy i.e. individuals inelastically supply one unit of labor each period before the compulsory retirement age at $\hat{J}$.

4.1.4 Government and social security

The government runs a pay as you go (PAYG) social security system. The system is self-financing i.e. it is financed through payroll taxes collected from working age generations. In our calculation of social security benefits we follow Fuster et al. [2003], who use a benefit formula that mimics the current benefit formula used by Social Security Administration. The social security benefits for each skill group are calculated as follows:

$$P_L(M_L) = \Psi[\frac{0.9}{0.44}(0.2M) + \frac{0.33}{0.44}(M_L - 0.2M)],$$

$$P_H(M_H) = \Psi[\frac{0.9}{0.44}(0.2M) + \frac{0.33}{0.44}(1.25M - 0.2M) + \frac{0.15}{0.44}(M_H - 1.25M)],$$

where M denotes the economy's average earnings, M_H and M_L denote the average lifetime earning of high skill and low skill individuals respectively, and Ψ denotes the average replacement rate. The benefit formula reflects the progressive structure of social security benefits. Marginal replacement rates are lower for individuals who have higher average life-time earnings indexed to the productivity growth.

Total government expenditure, G is financed by labor and capital tax revenues and confiscated unintended bequests, b. The government runs a balanced budget. Payroll, capital, and labor tax rates are denoted by τ_{SS}, τ_L, and τ_K respectively.

4.1.5 Technology

Output Y is produced by an aggregate technology that uses labor N and capital K. The technology is represented by a Cobb-Douglas constant returns production function

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{5}$$

Output shares of capital and labor are given by α and $1-\alpha$, respectively. The exogenously given technology level A grows at a constant rate g and capital depreciates at a constant rate $\delta \in (0,1)$. Firms maximize their profits by setting wage and rental rates equal to marginal products of labor and capital respectively:

$$w_t = (1-\alpha)A_t \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{6}$$

$$r_t = \alpha A_t \left(\frac{K_t}{N_t}\right)^{\alpha-1} - \delta. \tag{7}$$

4.1.6 Household's problem

A household consists of a parent and children who pool incomes and solve a joint utility maximization problem.[7] The net wage income of a child at age j is defined as

$$y_j^k(s) = (1 - \tau_L - \tau_{SS})\, e_j(s) w.$$

While a working age parent earns wage income, a retired parent gets pension benefit. The net wage income of a parent at age j is given as

$$y_{J+j}^p = \begin{cases} (1 - \tau_L - \tau_{SS})\, e_{J+j}(s) w & \text{if } J + j \leq \hat{J}, \\ P_s(M_s) & \text{if } J + j > \hat{J}. \end{cases}$$

Hence, the growth-adjusted household budget constraint can be written as

$$(1 + \tau_C)\left(\xi_j^k c_j^k + \xi_j^p c_{J+j}^p\right) + (1 + g)\, a_{j+1} = (1 + (1 - \tau_K) r)\, a_j + \xi_j^p y_j^p + \xi_j^k y_{J+j}^k \text{ for } j = 1, ...J, \tag{8}$$

where ξ_j^k is an index function that is equal to $m = (1+n)^J$ if children are alive and 0 otherwise, while ξ_j^p is an index function equal to 1 if parents are alive and 0 otherwise. We assume that children either survive all together or die all together. Variable a_j denotes the household's asset holding at the beginning of age j and a_{j+1} is the asset holding in the next period. It is assumed that individuals face borrowing constraints, i.e. $a_j \geq 0$.

Individuals in this economy have self-control preferences. We follow Gul and Pesendorfer [2004] and DeJong and Ripoll [2007] to model those preferences. Individuals face temptation each period and resisting to that creates a self-control cost. They try to balance their current consumption urge with their long-term benefit from not succumbing into temptation. We represent momentary utility and temptation utility functions, and temptation consumption by $u(.)$, $v(.)$, and $\check{c}$ respectively.

A household in a dynasty starts with some initial assets in the form of bequests received from the previous household and then chooses sequences of consumption and savings to maximize its dynasty's expected utility. Let $V_j(\Phi_j)$ be the value function of a household at j given a set of state variable Φ_j. The set of state variable comprises the beginning of period asset holding, skill endowments and demographic structure of the household i.e. $\Phi_j = \left\{a_j, s^p, s^k, \xi_j^p, \xi_j^k\right\}$.

[7] During the rest of the paper, we will drop time subscripts from the equations in order to minimize the notational burden.

The household problem can be defined recursively in terms of a Bellman equation as

$$V_j(\Phi_j) = \max_{\left\{c_j^k, c_{J\ +j}^p, a_{j+1}\right\}} \left\{u\left(c_j^k, c_{J\ +j}^p\right) + v\left(c_j^k, c_{J\ +j}^p\right) + \beta EV_{j+1}(\Phi_{j+1})\right\} - \max_{\left\{\check{c}_j^k, \check{c}_{J+j}^p\right\}} \{v(\check{c}_j^k, \check{c}_{J+j}^p)\} \tag{9}$$

subject to (8) and the borrowing constraint $a_j \geq 0$. The variable EV_{j+1} is the expected value function, defined as follows:

$$EV_{j+1}(\Phi_{j+1}) = \begin{cases} \sum_{g=1}^{3} \Omega(g_j, g_{j+1}) V_{j+1}(\Phi_{j+1}) & \text{for } j = 1, .., J-1, \\ \Pi\left(s^{p'}, s^{k\prime}\right) \sum_{g=1}^{3} \Omega(g_J, g_1) \theta m V_1(\Phi_1) & \text{for } j = J. \end{cases}$$

The term $\left\{v\left(c_j^k, c_{J\ +j}^p\right) - \max_{\check{c}_j^k, \check{c}_{J+j}^p} v(\check{c}_j^k, \check{c}_{J+j}^p)\right\}$ denotes the the disutility of choosing consumption $\left(c_j^k, c_{J\ +j}^p\right)$ instead of $(\check{c}_j^k, \check{c}_{J+j}^p)$. We call this disutility as a household's self-control cost.

We assume that the momentary function has a standard form of $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The temptation function might take the following functional form: $v(c) = \lambda u(c)$. In this functional form higher values of λ imply increases in the share of the temptation utility. We assume that temptation functions are strictly increasing i.e. individuals are tempted to consume their entire wealth each period.

A household's momentary utility function (temptation utility function) is the weighted sum of the parent's and children' momentary utility functions (temptation utility functions). More precisely, $u\left(c_j^k, c_{J\ +j}^p\right) = \xi^k u(c_j^k) + \xi^p u\left(c_{J+j}^p\right)$ and $v\left(c_j^k, c_{J\ +j}^p\right) = \xi^k v(c_j^k) + \xi^p v\left(c_{J+j}^p\right)$ denote the household's momentary utility and temptation utility functions respectively.

The household problem includes several cases. If agents do not have self-control problems then we end up in an economy in which agents have standard in period constant relative risk aversion (CRRA) preferences. More specifically, the self-control cost drops out and the household problem becomes the following standard household problem:

$$V_j(\Phi_j) = \max_{\left\{c_j^k, c_{J\ +j}^p, a_{j+1}\right\}} \left\{u\left(c_j^k, c_{J\ +j}^p\right) + \beta EV_{j+1}(\Phi_{j+1})\right\}$$

subject to the budget constraint given by the equation 8 and $a_j \geq 0$.Similarly, if agents are not altruistic ($\theta = 0$) then our model turns into a pure life-cycle model.

Households face shocks to their demographic structure each period as expressed by the Markov switching matrix $\Omega(g_j, g_{j+1})$. Every J period when the new household is formed a shock to the occupational composition is realized via the Markov switching matrix $\Pi\left(s^{p'}, s^{k\prime}\right)$. This shock only affects the newborn generation and determines the type of household that this generation will form with their parents. The current household saving in the last period is the intended bequest, which is divided equally among m children and becomes the initial asset of the next households in the family line $a_1' = \frac{a_{J+1}}{m}$.

4.1.7 Recursive competitive equilibrium

Definition 1 *Given realizations of initial assets, skill levels* $\left\{s^p, s^k\right\}$*, exogenous skill transition probabilities* Π*, survival probabilities, and government policies* $\{\tau_C, \tau_L, \tau_{ss}, \tau_K, \Delta_G, \theta\}$*, a stationary recursive competitive equilibrium is a collection of value functions* $\{V_j(a_j, \Phi_j)\}_{j=1}^J$ *with* $\Phi_j = \left\{s^p, s^k, \xi^p, \xi^k\right\}$*, household decision rules* $\left\{c_{J+j}^p, c_j^k, a_{j+1}\right\}_{j=1}^J$*, a collection of sequences of time invariant distributions* $\left\{\mu_j(a_j, \Phi_j)\right\}_{j=1}^J$*, sequences of aggregate stocks of physical capital and labor* $\{K,\ L\}$*, and sequences of prices* $\{w, r\}$ *such that*

(i) *household decision rules* $\left\{c_{J+j}^p, c_j^k, a_{j+1}\right\}_{j=1}^J$ *solve the household maximization problem,* (9)*.*

(ii) *factor prices are determined by equations* (2) *and* (3)*,*

(iii) *aggregate stocks are given by*

$$
\begin{aligned}
K &= \sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j, \Phi_j)\, a_j(a_j, \Phi_j) + \sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \eta_j(a_j, \Phi_j)\, a_j(a_j, \Phi_j) \\
S &= \sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j, \Phi_j)\, a_{j+1}(a_j, \Phi_j) \\
C &= \overbrace{\sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j, \Phi_j)\, c_j(a_j, \Phi_j)}^{\text{aggregate consumption}}, \\
L &= \overbrace{\sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j, \Phi_j)\, e_j}^{\text{effective labor}},
\end{aligned}
$$

(iv) *commodity markets clear*

$$
C + (1+g)\, S + G = Y + (1-\delta)\, K,
$$

(v)

$$\overbrace{(1+r)B}^{debt\ payment} + \overbrace{\Delta_G Y}^{government\ consumption} = \overbrace{\sum_{j,s^p,s^k\xi^p,\xi^k} \int_a \mu_j(a_j,\Phi_j)\, we_j\tau_L}^{labor\ income\ tax\ revenue}$$

$$+\overbrace{\sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j,\Phi_j)\, a_j(a_j,\Phi_j)\,\tau_K}^{capital\ income\ tax\ revenue}$$

$$+\overbrace{\sum_{j,s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j,\Phi_j)\, c_j(a_j,\Phi_j)\,\tau_C}^{consumption\ tax\ revenue}$$

$$+\overbrace{\sum_{j,s^p,s^k,\xi^p=\xi^k=0} \int_a \eta_j(a_j,\Phi_j)\, a_j(a_j,\Phi_j)}^{aggregate\ accidental\ bequests}$$

$$+\overbrace{(1+n)(1+g)B}^{borrowing},$$

(vi) *social security system is self financing*

$$\overbrace{\sum_{j,s^p,s^k,a,\xi^p,\xi^k} \int_a \mu_j^i(a_j,\Phi_j)\, we_j\tau_{SS}}^{social\ security\ tax\ revenue} = \overbrace{\sum_{j=J_w+1}^{2J} \sum_{s^p,s^k,\xi^p,\xi^k} \int_a \mu_j(a_j,\Phi_j)\, P}^{pension\ payment},$$

(vii) *and the time invariant distribution satisfies*

$$\mu_1(a_1,\Phi_1) = \sum_{s^p} \sum_{\xi^{p'},\xi^{k'}} \int_a \Pi\left(s^{p'},s^{k'}\right)\Omega(g_1,g_J)\,\mu_J(a_J,\Phi_J),$$

$$\mu_{j+1}(a_{j+1},\Phi_{j+1}) = \sum_{\xi^p,\xi^k} \int_a \Omega(g_j,g_{j+1})\,\mu_j(a_j,\Phi_j), \quad \text{for } j=1,...,J-1.$$

4.2 Calibration

In this section, we briefly describe the parameter values of our model economy.

4.2.1 Demographic and labor market parameters

Economically active individuals are born at the age of 20 (model age of 1). They can live up to the the age of 90 (model age of 14) and retire at the age of 65 (model age of 9). Each period in our model corresponds to five years. The population growth rate n is assumed to be equal to the average of the US population growth rate between 1931 and 2006 which corresponds, on average, to 1.2% per year (U.S. Census Bureau [2006]). This implies that the number of children of an individual, m is equal to 1.52.

Labor-efficiency profiles of high- and low-skill individuals, $e_j(s)$ are calculated from the data on college and non-college graduates' earnings from the U.S.Census Bureau [2008]. The values for the transition probabilities are chosen following Fuster et al. [2003] as $\pi_{H,H} = 0.57$ and $\pi_{L,L} = 0.83$. In our model, conditional survival probabilities depend on individuals' skill levels. We take conditional survival probabilities for low- and high-skill individuals from Elo and Preston [1996], who present data for the conditional survival probabilities of college and non-college graduate males in the US.

4.2.2 Technology

The parameters describing the production side of the economy are chosen to match the long-run features of the US economy. We set the capital share of output to the standard value, which is $\alpha = 0.33$. The total factor productivity is chosen as $A = 1$. Nadiri and Prucha [1996] report estimates of physical capital and R&D capital depreciation rates are 5.9% and 12% respectively. Following Kydland and Prescott [1982] we set the capital depreciation rate δ equal to 6%. This implies that depreciation per period, $\delta^*$is equal to 0.2193[8]. Annual growth rate of technology g is taken as 2.1%, which is the actual average growth rate of GDP per capita taken over the time interval from 1959 to 1994 (Hugget and Ventura [1999]).

4.2.3 Government and social security

The US Social Security Administration calculates retirement benefits by using a concave, piece-wise linear benefit function. There is a negative correlation between average life time earnings and marginal replacement rates i.e. higher average lifetime earnings yield lower replacement rates. We choose the average replacement rate, Ψ equal to 0.40 that yields a realistic approximation of benefit formulas of high and low skill individuals in our economy. Government purchases to GDP ratio ($\frac{G}{Y}$) set equal to 0.18. The capital income tax rate τ_K, the consumption tax rate, and labor income tax rate τ_L are set equal to 40%, 5.5%, and 20% respectively.

4.2.4 Preferences

Throughout our analysis we use a concave temptation function. Empirical studies estimate the values of the relative risk aversion parameter γ between 1 and 10 (Auerbach and Kotlikoff [1987]). We conduct our benchmark analysis by setting $\gamma = 2$ and $\lambda = 0.0786$ following DeJong and Ripoll [2007]. The annual discount factor β is calibrated in such a way that the capital-output ratio of the model approximates that of the US economy.

The parameters of the benchmark calibration is given in Table 1.

[8] $\delta^* = 1 - (1-\delta)^{(\frac{years}{J-1})} = 1 - 0.94^4 = 0.2193$

Demographics	
Maximum possible life span $2J$	14
Obligatory retirement age J_w	9
Growth rate of population n	1.20%
Measure of individuals with high ability $\lambda(H)$	0.28
Labor efficiency profile $\{e_j(s)\}_{j=1}^{J_w-1}$	U.S. Census Bureau (2008)
Conditional survival probabilities $\{v_j(s)\}_{j=1}^{2J}$	Elo and Preston (1996)
Production	
Capital share of GDP α	0.33
Annual depreciation of capital stock δ	0.06
Annual per capita output growth rate g	2.1%
Markov Process for skills Π	$\begin{bmatrix} 0.83 & 0.17 \\ 0.43 & 0.57 \end{bmatrix}$
Preferences	
Annual discount factor of utility β	0.998
Scale factor of the temptation utility λ	0.0786
Risk aversion parameter γ	2.0
Government	
Social security replacement ratio θ	0.44
Labor income tax rate τ_L	0.2
Capital income tax rate τ_K	0.4
Consumption tax rate τ_C	0.055
Government purchases as a percentage of GDP	18%

Table 1: Parameter values of the benchmark calibration

4.3 Results

Before starting the welfare analysis of a PAYG system in an economy with altruism and temptation we want to demonstrate the effects of temptation on economic aggregates. The following two examples are created for this purpose.[9]

In our first example we want to show the effects of temptation on economic aggregates in a pure life cycle model ($\theta = 0$).[10] In order to do so, we first calibrate a pure life cycle model economy to the US data assuming that the time-discounting factor $\beta = 1.0126$. In this calibration exercise, we assume that individuals have time-consistent preferences ($\lambda = 0$) and the social security replacement rate is 40% ($\Psi = 0.40$). This rate is approximately equal to the average replacement rate in the US. We hit the long-term average of the US capital output ratio of 2.5 (see Table 2). Then we shut down the social security system ($\Psi = 0$) to isolate the effects of temptation better and vary values of the temptation parameter λ. As it can be seen in Table 3, the existence of temptation and the associated self-control cost cause a substantial reduction in the aggregate stock of capital. A comparison of of the first two rows of Table 3 reveals that the capital stocks decreases by 3.46%. It is interesting to see that this sharp

[9] Unless stated otherwise the risk aversion parameter $\gamma = 2$ in all of the following computational exercises.

[10] In other words there is no altruism in the model economy.

decrease is actually generated by very small value of the temptation parameter. We increase the value of the temptation parameter by 50% starting with the parameter value of 0.025. The initial 50% increase reduces the level of capital stock about 1.74%. For the higher values of the temptation parameter, a 50% increase reduces the capital stock at higher rates . In other words, when the severity of temptation increases the marginal cost of temptation in terms of a decrease in the capital stock increases.

In our second example we use a two sided altruistic framework ($\theta = 1$). First we calibrate the model to the US data assuming that the time-discount factor $\beta = 0.94937$. As in the above we assume that individuals have time-consistent preferences and the social security replacement rate is 40%. We hit the long-term average of the US capital-output ratio (see Table 4) then we shut down the social security system and vary the values of the temptation parameter. These two examples allow us to compare the distortions on the aggregate capital stocks in both altruistic and non-altruistic frameworks as a result of the existence of temptation. A quick comparison of the first two rows of Table 3 and Table 5 reveals that the existence of a self-control problem at the same strength reduces the capital stock more in the altruistic framework (while the capital stock decreases 3.46% in the pure life cycle framework, it decreases 4.70% in the altruistic framework). Starting from the parameter value of 0.025 we increase the value of the temptation parameter by 50%. The initial 50% increase creates a 2.22% decrease in the aggregate capital stock. As in the first example the marginal cost of temptation in terms of a decrease in the capital stock increases as the severity of temptation increases. It is clear that each 50% increase in the value of temptation parameter creates more distortions in the altruistic framework.

Kumru and Thanopoulos [2008], in a pure life cycle framework, show that temptation negatively affects individuals' savings decisions and hence, it reduces the overall capital stock. In a dynastic framework in which individuals have time-consistent preferences, Fuster et al. [2003] show that individuals save more than those in a pure life cycle framework because of their altruistic concerns. These two studies show that temptation and altruism are two factors that have opposite effects on the aggregate capital stock. Our simple two period model confirms results of Kumru and Thanopoulos [2008] and Fuster et al. [2003]. However, it has been unknown that how temptation and altruism factors interact and what are the effects of the interaction on the capital stock. A comparison of above two examples provide an answer to this question: the existence of temptation severely reduces the capital stock in both altruistic and pure life-cycle frameworks but the rate of decrease in the capital stock is larger when individuals are altruistic. In other words, same degree of temptation reduces the capital stock more in the dynastic framework. The mechanism that derives this result is as follows. The existence of altruism causes a higher level of savings that results in a higher level of wealth in each period. Yet, the higher level of wealth in each period translates into a higher level of self-control cost for a given level of temptation. Therefore, individuals decrease their savings more to escape from higher self control costs in the dynastic framework due to an increase in the strength of temptation.

Next we analyze both aggregate and welfare effects of a PAYG system in economies populated with individuals who have time-consistent and self-control preferences respectively. We first use a pure life cycle framework then we employ a two-sided altruistic framework.[11] In all our analysis we chose β in such a way that the capital-output ratio is about 2.5 when the social security replacement rate $\Psi = 0.40$. As in Fuster et al. [2003], households differ in terms of their demographic composition and skills. According to their demographic compositions households are divided into three groups: a household is made up of both a parent and children is called as Group 1 household and denoted by $G1$; a household is made up of the parent only is called as Group 2 household and denoted by $G2$; a household is made up of only of children are called as Group 3 household and denoted by $G3$. Since individuals differ in terms of their skill levels, a $G1$ type of household is divided further into four categories: LL, LH, HL, and HH. The first capital letter denotes the parent's skill level (H for high skill and L for low skill) and the second capital letter denotes children' skill levels. $G2$ and $G3$ types of households are divided into two more categories: H and L. For each case we create two tables. The first table shows the levels of capital stock, consumption, and output relative to the corresponding levels at 0% replacement rate economy. The second table shows the welfare effects of various replacement rates on different types of households. In particular, the first column of the second table reports the average replacement rate which varies between 0% and 100%. Each remaining column except the last column reports the steady state expected life-time utility of households relative to the corresponding levels at 0% replacement rate economy. The last column reports the average of the households' expected lifetime-utilities.

In the first case we use a pure life cycle framework in which individuals have time consistent preferences. This case corresponds to the standard social security models that are extensively analyzed (see for example Imrohoroglu et al. [1995]). Our results here are consistent with those of the previous studies. In particular, a higher social security replacement rate causes a lower level of capital stock and a lower level of expected utility for each type of $G1$ households because the PAYG system transfers resources from individuals who have high propensity to save to individuals who have high propensity to consume (see Table 6 and Table 7). Note that the only parent household $G2$ always prefers the highest social security replacement rate because its member receives much higher social security benefits than her contributions to the system through payroll taxes. Similarly the only children household $G3$ always prefers the lowest social security replacement rate because its members do not get any social security benefits although they contribute to the system through their payroll taxes. Therefore we generally do not pay much attention to $G2$ and $G3$ types of households' welfare.

In the second case we use a pure life cycle framework in which individuals have self-control preferences. This case corresponds to the model analyzed by Kumru and Thanopoulos [2008] and our results are consistent with their results. Each social security replacement rate reduces the capital stock more because of the existence of temptation (compare Table 6 and Table 8).

[11] Kumru and Thanopoulos [2008] analyzed the welfare effects of a PAYG social security system in a pure life cycle economy populated with individuals who have self-control preferences. We repeat a similar analysis here in to make the paper self-contained.

Yet the welfare cost of having a higher social security replacement rate is not as high as in Case 1 (see Table 9). In other words, each replacement rate reduces each $G1$ household's welfare less. In Case 2, the PAYG system offers an additional benefit: It reduces individuals' self-control cost by reducing their wealth each period. As a result, the PAYG system is less detrimental to the overall welfare.

In the third case we use a two-sided altruistic framework in which individuals have time-consistent preferences. The model we analyze corresponds to the model analyzed by Fuster et al. [2003]. A comparison of Table 6 and Table 10 reveals that the presence of altruism mitigates the crowding out effect of a PAYG system. Hence the steady state expected utility of a new born household is either reduced less or increased by a higher replacement rate in comparison to Case 1 (see Table 11).

Although in our initial examples we demonstrated that the existence of temptation in an altruistic framework reduces the capital stock more, it has not yet been known how these two factors together alter the welfare implications of a PAYG system. The three cases above demonstrate that altruism and temptation are quite effective to mitigate adverse welfare consequences of higher social security replacement rates.

The fourth case is created to analyze the welfare implications of a PAYG system in a two sided altruistic framework in which individuals have self-control preferences. This case allows us to explore how altruism, temptation and a PAYG system interact with each other and the effects of this interaction on economic aggregates and welfare of each type of household. To demonstrate the effects of temptation better in an altruistic framework we often compare the results in this case with those of Case 3.

Introducing social security with 40% replacement rate causes a crowding-out effect in both Case 3 and Case 4. In particular, while an economy with 0% replacement rate creates 9.29% more capital stock and 3.45% more output in Case 3 it creates only 8.09% more capital stock and 2.99% more output in Case 4 (see Table 10 and Table 14) We have already showed that the existence of temptation in an altruistic framework reduces the capital stock. Because the level of capital stock is lower in Case 4 than that of Case 3, the same replacement rate creates less severe crowding-out effect in Case 4. In other words, the marginal crowding-out effect of a replacement rate is smaller if the economy populated with individuals who have self-control problems. As in the previous cases 0% replacement rate economy of Case 4 creates the highest level of capital stock.

Table 11 shows that while LL, LH, and HH types Group 1 households prefer a 0% social security replacement rate, a HL type Group 1 household prefers a 100% social security replacement rate in Case 3.[12] Imrohoroglu et al. [2003] give a number of reasons regarding with the observed differences in preferences towards various social security replacement rates: First, because households' life spans differ the value they assign to the annuity role of social

[12] Although the model we used in Case 3 is very similar to that of Fuster et al. [2003], our welfare results slightly differ. In Fuster et al. [2003] HH types prefers 80% replacement rate and LL type prefers 44% replacement rates. However, both our results and their results imply that the average utility measure indicates 0% replacement rate as a welfare maximizing rate.

security differs. Second, social security redistributes income from high-skill individuals to low skill individuals through the progressive benefit formula and hence, low skill individuals might prefer higher replacement rates. Third, more borrowing constrained households might value social security less. Not surprisingly while a Group 2 household where only a parent is alive prefers the highest replacement rate a Group 3 household in which only children are alive prefers the lowest replacement rate. The reason is as follows: In the former case, a household receives much higher social security benefits compared to its life-time social security payroll tax payments and hence, the highest replacement rate is optimal for this group. In the latter case, a household does not receive any social security benefits and hence only a 0% replacement rate maximizes its welfare.

Welfare results in Case 4 are quite interesting. On average, the negative welfare effects of social security are mitigated when individuals have self-control preferences in a dynastic framework. However, welfare effects of social security vary significantly across different types of households. Households' wealth seems to be the key factor governing those differences. In this environment social security provides an additional benefit through the channel of reducing available wealth in each period. In other words, it plays a role of a temptation reducing device. Demand for such a device varies across different types of households. Richer households seems to demand more social security to reduce their large self-control costs. HL and HH types $G1$ households have relatively larger household wealth than those of LL and LH types $G1$ households. This, in turn implies that HL and HH types $G1$ households face larger self-control costs for a given strength of temptation and hence, they have a higher demand for a self-control reducing device. The existence of the PAYG system certainly help to meet their demands. In particular, an increase in the generosity of social security (an increase in the social security replacement rate) increases HL type's expected utility more when individuals have self-control preferences (compare tables 11 and 13). Note that both in Case 3 and Case 4, HL types prefer 100% replacement rate as in Fuster et al. [2003] but the presence of temptation makes the welfare gain more pronounced. Similarly, an increase in the generosity of social security decreases HH type household's expected utility less when individuals have self-control preferences (see tables 11 and 13).

LL and LH types $G1$ households have relatively low wealth. This implies that for a given strength of temptation these two types have lower self-control costs and hence, they have lower demand for a self-control reducing device. Hence, the additional benefit of social security as a self-control reducing device is quite small. A comparison of tables 11 and 13 reveals that higher replacement rates decrease LL and LH type's expected utilities more in Case 4 when replacement rates are relatively small. Surprisingly, the presence of temptation does not mitigate but exaggerate the adverse welfare implications of a PAYG system for poor households for smaller replacement rates. If the replacement rates are big enough (bigger than 0.3 for LL types and bigger than 0.4 for LH types) then the presence of temptation does mitigate the adverse effects of social security. This non-linear effects are due to the structure of social security benefit function and general equilibrium effects. One should note that our social security benefit formula is more progressive towards the low-skill retired parents (namely, low-income households).

4.4 Sensitivity analysis

We conduct a sensitivity analysis to test robustness of our results. We increase the values of temptation parameters in the benchmark model to .1572 and to 0.2. In each experiment, we re-calibrate the benchmark model when $\Psi = 0.4$ to hit our target moment. We then vary the replacement rates and report the effects on aggregate variables and welfare in tables from 14 to 17. We find that our results are quite robust. When we increase the value of temptation parameter, the self-control cost becomes more severe. It means that the role of social security as a self-control cost reducing device becomes more important. Consequently, the adverse effect of PAYG system on welfare is mitigated further.

5 Conclusion

We study the effects of the PAYG social security in an environment where agents are altruistic and have self-control problems. We first conduct our analytical analysis in a simple partial equilibrium OLG model. Next, we extend to a large scale general equilibrium OLG model with altruistic individuals. We calibrate our large OLG model to the US data to conduct quantitative analysis. We find that the presence of altruism mitigates the adverse effects of temptation on savings but magnifies the severity of temptation and the self-control costs. In such an environment PAYG system not only provides consumption smoothing and risk-sharing but also provides an additional benefit: a channel that reduces the severity of temptation. Therefore, the adverse welfare effects of the PAYG system futher mitigated.

In short, in this paper we make two contributions to the literature. First, we develop an analytical framework to explore how the interaction between altruism and temptation affects individuals' inter-temporal allocations and welfare. Second, we analyze the roles and welfare implications of a PAYG social security system when altruism and temptation are presented at the same time.

In this analysis we focus on steady state analysis. We abstract from the short-run effects of social security reforms. In addition, we aslo assume away interaction between self-control problems and liquidity constraints. We think that these are interesting issues for future research.

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Appendix A: Solution for the analytical model

We use the backward induction method to solve the household's problem. In particular we first solve the household's second period problem and then use that solution to solve the household's first period problem. In each period we solve the problem in two steps.

Starting from the second period, we first we solve the sub-problem of $\max\limits_{\widetilde{c_2}} v\left(\widetilde{c_2}, \widetilde{b_2}\right)$ to obtain the maximized values of hypothetical temptation consumption and bequest. Then we solve the sub-problem of $\max\limits_{\widetilde{c}, \widetilde{b}} \{u(c,b) + v(c,b)\}$ by using the maximized values we obtained in the first step. More specifically, the second period problem is given by

$$V_2(s_1) = \max_{c_2, b_2} \left\{ \frac{c_2^{1-\sigma}}{1-\sigma} + \gamma \frac{b_2^{1-\sigma}}{1-\sigma} + \lambda \left(\frac{c_2^{1-\sigma}}{1-\sigma} + \kappa\gamma \frac{b_2^{1-\sigma}}{1-\sigma} \right) \right\} - \max_{\widetilde{c_2}, \widetilde{b_2}} \left\{ \lambda \left(\frac{\widetilde{c_2}^{1-\sigma}}{1-\sigma} + \kappa\gamma \frac{\widetilde{b_2}^{1-\sigma}}{1-\sigma} \right) \right\}$$

subject to

$$\begin{aligned} c_2 + b_2 &= Rs_1 + T; \\ \widetilde{c_2} + \widetilde{b_2} &= Rs_1 + T. \end{aligned}$$

In the first step, we solve the temptation sub-problem to find the optimal level of temptation consumption and bequests

$$\max_{\widetilde{c_2}, \widetilde{b_2}} \left\{ \lambda \left(\frac{\widetilde{c_2}^{1-\sigma}}{1-\sigma} + \kappa\gamma \frac{\widetilde{b_2}^{1-\sigma}}{1-\sigma} \right) : \widetilde{c_2} + \widetilde{b_2} = Rs_1 + T \right\}.$$

Then, we solve the remaining commitment sub-problem while taking the maximized value of the temptation consumption as given:

$$\max_{c_2, b_2} \left\{ \frac{c_2^{1-\sigma}}{1-\sigma} + \gamma \frac{b_2^{1-\sigma}}{1-\sigma} + \lambda \left(\frac{c_2^{1-\sigma}}{1-\sigma} + \kappa\gamma \frac{b_2^{1-\sigma}}{1-\sigma} \right) : c_2 + b_2 = Rs_1 + T \right\}.$$

The parameter κ determines the strength of the future temptation utility. In order to make our analysis tractable we assume agents are tempted to consume all available resources. It implies that $\kappa = 0$. Hence, the second period maximized value of the temptation consumption is simplified to the following:

$$\widetilde{c_2} = Rs_1 + T.$$

Hence, the individual's utility maximization problem becomes the following:

$$\begin{aligned} V_2(s_1) &= \max_{c_2, b_2} \left\{ (1+\lambda) \frac{c_2^{1-\sigma}}{1-\sigma} + \gamma \frac{b_2^{1-\sigma}}{1-\sigma} \right\} \\ \text{s.t. } c_2 + b_2 &= Rs_1 + T. \end{aligned}$$

Note that since the level of temptation utility $\widetilde{c_2}$ has no effect on the levels of consumption c_2

and bequest b_2, we leave it out. The FOCs deliver the following two equations:

$$\begin{aligned} \frac{(1+\lambda)}{c_2^{\sigma}} &= p_2, \\ \frac{\gamma}{b_2^{\sigma}} &= p_2, \end{aligned}$$

where p_2 is the Lagrangian multiplier. The combination of these two equations yields the following relationship between the level of consumption and bequest: $b_2 = \left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1}{\sigma}} c_2$. By plugging it into the budget constraint we can get the following decision rules:

$$c_2^* = \frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} \left(Rs_1 + T\right), \tag{10}$$

$$b_2^* = \frac{\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} \left(Rs_1 + T\right). \tag{11}$$

The maximum of the value function in the second period is given by

$$\begin{aligned} V_2(s_1) &= \left\{(1+\lambda)\frac{(c_2^*)^{1-\sigma}}{1-\sigma} + \gamma\frac{(b_2^*)^{1-\sigma}}{1-\sigma}\right\} - \left\{\lambda\frac{\widetilde{c}_2^{\,1-\sigma}}{1-\sigma}\right\} \\ &= \left((1+\lambda)\frac{(c_2^*)^{1-\sigma}}{1-\sigma} + \gamma\frac{\left(\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1}{\sigma}} c_2^*\right)^{1-\sigma}}{1-\sigma}\right) - \left(\lambda\frac{\widetilde{c}_2^{\,1-\sigma}}{1-\sigma}\right) \\ &= \left((1+\lambda)\frac{1}{1-\sigma} + \gamma\frac{\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}\right)(c_2^*)^{1-\sigma} - \left(\lambda\frac{\widetilde{c}_2^{\,1-\sigma}}{1-\sigma}\right) \\ &= \left((1+\lambda)\frac{1}{1-\sigma} + \gamma\frac{\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}\right)\left(\frac{(Rs_1+T)}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} - \left(\lambda\frac{(Rs_1+T)^{1-\sigma}}{1-\sigma}\right) \\ &= \left[\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma} - \lambda\right]\frac{(Rs_1+T)^{1-\sigma}}{1-\sigma}. \end{aligned}$$

The self-control cost in the second period is given by

$$
\begin{aligned}
SCC_2 &= \lambda\left(\frac{(c_2^*)^{1-\sigma}}{1-\sigma}-\frac{(\widetilde{c_2})^{1-\sigma}}{1-\sigma}\right) \\
&= \lambda\left[\frac{\left(\frac{Rs_1+T}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}}{1-\sigma}-\frac{(Rs_1+T)^{1-\sigma}}{1-\sigma}\right] \\
&= \lambda\left[\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}-1\right]\frac{(Rs_1+T)^{1-\sigma}}{1-\sigma}.
\end{aligned}
$$

Now we can solve the first period maximization problem. We first set the following value function:

$$
V_1(b_1)=\max_{c_1,s_1}\left\{\left(\frac{c_1^{1-\sigma}}{1-\sigma}+\beta V_2(s_1)\right)+\lambda\left(\frac{c_1^{1-\sigma}}{1-\sigma}+\kappa\beta V_2(s_1)\right)\right\}-\max_{\widetilde{c_1},\widetilde{s_1}}\left\{\lambda\left(\frac{\widetilde{c_1}^{1-\sigma}}{1-\sigma}+\kappa\beta V_2(\widetilde{s_1})\right)\right\}
$$

We solve the individual's problem in two steps as in the above. In the first step we get the following temptation consumption level:

$$
\widetilde{c_1}=(1-\tau)\,w+b_1.
$$

In the second step, the utility maximization problem is written in terms of the the Bellman equation:

$$
V_1(b_1)=\max_{c_1,s_1}\left\{\begin{array}{c}(1+\lambda)\frac{c_1^{1-\sigma}}{1-\sigma}+\beta V_2(s_1) \\ \text{s.t. } c_1+s_1= \quad (1-\tau)\,w+b_1\end{array}\right\}.
$$

Taking the FOCs lead to the following Euler's equation:

$$
\frac{1+\lambda}{c_1^{\sigma}}=\beta\frac{\partial V_2(s_1)}{\partial s_1},
$$

where $\frac{\partial V_2(s_1)}{\partial s_1}$ is the marginal value function that represents the marginal utility of saving. Taking the derivative of the value function $V_2(s_1)$ with respect to s_1 yields

$$
\frac{\partial V_2(s_1)}{\partial s_1}=\frac{\left[\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}-\lambda\right]R}{(Rs_1+T)^{\sigma}}.
$$

Combining the above two equations yields that

$$\begin{aligned}
\frac{1+\lambda}{c_1^{\sigma}} &= \frac{\left[\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}-\lambda\right]\beta R}{(Rs_1+T)^{\sigma}}, \\
(1+\lambda)(Rs_1+T)^{\sigma} &= \psi\beta R c_1^{\sigma},
\end{aligned}$$

where $\psi = \left(\left(1+\lambda+\gamma\left(\frac{\gamma}{(1+\lambda)}\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}-\lambda\right)$. This, in turn, delivers the followings:

$$\begin{aligned}
c_1 &= \left(\frac{1+\lambda}{\psi R}\right)^{\frac{1}{\sigma}}(Rs_1+T), \\
s_1 &= \left[\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}c_1-T\right]\frac{1}{R}.
\end{aligned}$$

Using the expression for s_1 in the budget constraint we can calculate the following expressions:

$$\begin{aligned}
c_1+\left[\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}c_1-T\right]\frac{1}{R} &= (1-\tau)w+b_1, \\
\left[1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}\right]c_1-\frac{T}{R} &= (1-\tau)w+b_1
\end{aligned}$$

After manipulating the above two equations, we can get individuals' decision rules:

$$c_1^* = \frac{1}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)w_1+b_1+\frac{T}{R}\right), \tag{12}$$

$$s_1^* = \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}((1-\tau)w_1+b_1)+\left(\frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}-1\right)\frac{T}{R}. \tag{13}$$

The self-control cost in period 1 is given by

$$\begin{aligned}
SCC_1 &= \lambda\left(\frac{(c_1^*)^{1-\sigma}}{1-\sigma}-\frac{(\widetilde{c}_1)^{1-\sigma}}{1-\sigma}\right) \\
&= \lambda\left[\frac{\left(\frac{\left((1-\tau)w_1+b_1+\frac{T}{R}\right)}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\right)^{1-\sigma}}{1-\sigma}-\frac{((1-\tau)w_1+b_1)^{1-\sigma}}{1-\sigma}\right].
\end{aligned}$$

Optimal choices in the second period in terms of wage income, bequest and pension income are

$$c_2^* = \frac{1}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} y_2, \tag{14}$$

$$b_2^* = \frac{\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\gamma}{1+\lambda}\right)^{\frac{1}{\sigma}}} y_2, \tag{15}$$

where, y_2 is the income available at beginning of the second period.

$$y_2 = Rs_1^* + T = \frac{R\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}\left((1-\tau)\,w + b_1\right) + \frac{\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}}{1+\left(\frac{\psi\beta R}{(1+\lambda)}\right)^{\frac{1}{\sigma}}} T.$$

Appendix B: Solution algorithm for quantitative model

General procedure to solve a general equilibrium problem

1. Discretize state space of asset $[a_0, a_1, ..., a_{\max}]$.
2. Guess initial factor prices R, w and endogenous government policy variables.
3. Solve household problem to obtain decision rules of consumption, savings and labor supplies (See algorithm 3 for more details).
4. Obtain a stationary distribution across states (See algorithm 4 for more details).
5. Clear factor markets to get new factor prices and balance government budget to pin down endogenous government variables.
6. Check a relative change in aggregate capital stocks after each iteration and stop algorithm when the change is relatively small (10^{-4} percent). Otherwise, repeat step from 3 to 6.

Solving household problem to obtain decision rules

1. Guess initial value function and marginal value function of the next household in the dynasty.
2. Use backward induction method to solve for decision rules, value function and marginal value function of the current household from period J back to period 1.
3. Use value function and marginal value function at the first period of the current household to update value function and marginal value function of the next household.
4. Repeat 2 and 3 until value function converges. In other words, if a relative difference between value functions of two consecutive iterations is relatively small, $100\frac{\|V^{i+1}-V^i\|}{\|V^i\|} \leq \varepsilon = 10^{-4}$, then stop.

Stationary measures

1. Guess a distribution of bequests or initial asset holdings of households in period 1 (e.g. uniform distribution) .
2. Iterate this distribution forward to obtain the distribution of assets from period 2 to J given decision rules and Markov transition probabilities.
3. Use the distribution of savings in the last period J and Markov transition probability of skill transmission to update the distribution of bequests.
4. Keep repeating steps 1 to 3 until the distribution of assets converges. In other words, if the relative difference between the distributions of two consecutive iterations is relatively small, for example $100\frac{\|\mu^{i+1}-\mu^{i}\|}{\|\mu^{i}\|} < \varepsilon = 10^{-8}$, then stop.

Appendix C: Tables

Ψ	Y	C	K	K/Y	R
0.4	0.2269	0.7443	0.569	2.5075	1.059

Table 2: Agggregate variables in the economy with no altruism and no temptation

λ	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.757	1.053
0.025	98.740	98.924	96.540	2.696	1.054
0.037	98.127	98.400	94.883	2.666	1.055
0.056	97.232	97.634	92.499	2.623	1.056
0.084	95.953	96.545	89.159	2.562	1.058
0.127	94.122	94.978	84.513	2.476	1.060

Table 3: Aggregate variables in the non-altruistic framework with different degree of temptation and no social security: $\theta = 0$, $\Psi = 0$ and $\beta = 1.0126$

Ψ	Y	C	K	K/Y	R
0.4	0.22684	0.7147	0.5674	2.5013	1.0592

Table 4: Agggregate variables in the economy with altruism but no temptation

λ	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.663	1.055
0.025	98.283	98.651	95.302	2.583	1.057
0.037	97.472	97.988	93.135	2.545	1.058
0.056	96.267	97.011	89.972	2.489	1.059
0.084	94.618	95.643	85.756	2.414	1.062
0.127	92.374	93.843	80.224	2.313	1.065

Table 5: Aggregate variables in the altruistic framework with social security and different degree of temptation: $\theta = 1$, $\Psi = 0$ and $\beta = 0.949372$

Ψ^I	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.757	1.053
0.100	98.537	98.929	95.990	2.686	1.054
0.200	97.188	97.944	92.384	2.621	1.056
0.300	95.943	97.039	89.132	2.561	1.058
0.400	94.803	96.223	86.223	2.508	1.059
0.500	93.766	95.468	83.627	2.459	1.060
0.600	92.754	94.748	81.144	2.412	1.062
0.700	91.797	94.070	78.840	2.368	1.063
0.800	90.888	93.429	76.691	2.326	1.064
0.900	90.036	92.831	74.709	2.288	1.065
1.000	89.237	92.274	72.882	2.252	1.067

Table 6: Aggregate effects of social security in the non-altruistic framework with no temptation: $\theta = 0$, $\lambda = 0$ $\beta = 1.0126$

Ψ^I	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	99.599	98.911	99.914	99.125	107.161	104.997	96.796	96.758	99.482
0.200	99.179	97.783	99.788	98.237	113.003	109.268	93.540	93.463	98.937
0.300	98.743	96.628	99.617	97.337	117.957	112.988	90.224	90.107	98.369
0.400	98.305	95.466	99.414	96.435	122.181	116.251	86.854	86.696	97.791
0.500	97.867	94.314	99.181	95.537	125.846	119.143	83.428	83.229	97.208
0.600	97.375	93.105	98.873	94.584	129.039	121.721	79.863	79.621	96.566
0.700	96.852	91.873	98.514	93.602	131.852	124.030	76.187	75.900	95.888
0.800	96.298	90.616	98.099	92.589	134.357	126.108	72.387	72.053	95.175
0.900	95.716	89.343	97.632	91.548	136.603	128.003	68.464	68.081	94.429
1.000	95.106	88.053	97.117	90.479	138.635	129.738	64.409	63.976	93.652

Table 7: Welfare effects of social security in the non-altruistic framework with no temptation: $\theta = 0$, $\lambda = 0$ and $\beta = 1.0126$

Ψ^I	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.746	1.053
0.100	98.502	98.895	95.893	2.674	1.055
0.200	97.148	97.907	92.276	2.609	1.056
0.300	95.886	96.987	88.987	2.549	1.058
0.400	94.748	96.146	86.082	2.495	1.059
0.500	93.637	95.367	83.308	2.443	1.061
0.600	92.620	94.644	80.819	2.396	1.062
0.700	91.652	93.958	78.495	2.352	1.063
0.800	90.737	93.310	76.336	2.310	1.065
0.900	89.866	92.698	74.319	2.271	1.066
1.000	89.044	92.124	72.447	2.234	1.067

Table 8: Aggregate effects of social security in the non-altruistic framework with temptation: $\theta = 0$ and $\lambda = 0.0786$, and $\beta = 1.0201$

Ψ^I	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	99.625	98.937	99.963	99.147	107.323	105.147	96.773	96.733	99.511
0.200	99.256	97.860	99.900	98.302	113.331	109.565	93.520	93.437	99.017
0.300	98.868	96.747	99.786	97.442	118.422	113.402	90.198	90.068	98.496
0.400	98.491	95.641	99.651	96.593	122.773	116.787	86.839	86.531	97.978
0.500	98.061	94.472	99.442	95.694	126.540	119.773	83.347	82.985	97.401
0.600	97.631	93.314	99.200	94.796	129.841	122.457	79.787	79.373	96.819
0.700	97.170	92.131	98.907	93.867	132.771	124.868	76.110	75.640	96.202
0.800	96.684	90.930	98.561	92.913	135.390	127.058	72.312	71.786	95.553
0.900	96.166	89.707	98.162	91.927	137.750	129.056	68.381	67.796	94.869
1.000	95.619	88.468	97.710	90.909	139.903	130.895	64.311	63.667	94.152

Table 9: Welfare effects of social security in the non-altruistic framework with temptation:$\theta = 0$ and $\lambda = 0.0786$, and $\beta = 1.0201$

Ψ^I	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.661	1.055
0.100	98.765	98.984	96.607	2.603	1.056
0.200	97.760	98.189	93.902	2.556	1.058
0.300	97.020	97.618	91.940	2.522	1.059
0.400	96.548	97.305	90.702	2.500	1.059
0.500	96.321	97.213	90.112	2.490	1.059
0.600	96.259	97.269	89.949	2.487	1.060
0.700	96.325	97.437	90.123	2.490	1.059
0.800	96.522	97.719	90.635	2.499	1.059
0.900	96.859	98.122	91.517	2.515	1.059
1.000	97.345	98.655	92.798	2.537	1.058

Table 10: Aggregate effects of social security in the altruistic framework with no temptation: $\theta = 1$ and $\lambda = 0$, and $\beta = .949372$

Ψ^I	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	99.170	98.500	99.947	99.371	105.376	103.624	97.671	97.763	99.182
0.200	98.783	97.739	99.921	98.790	109.979	106.809	95.416	95.592	98.746
0.300	98.510	97.059	99.928	98.282	114.010	109.642	93.251	93.494	98.406
0.400	98.334	96.465	99.967	97.852	117.622	112.212	91.169	91.460	98.152
0.500	98.262	95.947	100.027	97.492	120.921	114.589	89.154	89.473	97.986
0.600	98.252	95.449	100.089	97.170	123.976	116.825	87.154	87.487	97.868
0.700	98.285	94.952	100.147	96.873	126.831	118.953	85.147	85.482	97.781
0.800	98.345	94.459	100.202	96.602	129.529	121.001	83.129	83.453	97.714
0.900	98.431	93.981	100.255	96.361	132.102	122.995	81.098	81.396	97.668
1.000	98.537	93.523	100.312	96.156	134.454	124.956	79.055	79.312	97.641

Table 11: Welfare effects of social security in the altruistic framework with no temptation: $\theta = 1$ and $\lambda = 0$, and $\beta = .949372$

Ψ	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.639	1.056
0.100	98.820	99.016	96.757	2.584	1.057
0.200	97.912	98.293	94.306	2.542	1.058
0.300	97.303	97.841	92.686	2.514	1.059
0.400	97.006	97.664	91.903	2.500	1.059
0.500	96.894	97.648	91.610	2.495	1.059
0.600	96.946	97.794	91.745	2.497	1.059
0.700	97.148	98.080	92.276	2.507	1.059
0.800	97.446	98.445	93.065	2.520	1.059
0.900	97.868	98.932	94.190	2.540	1.058
1.000	98.404	99.514	95.631	2.565	1.057

Table 12: Aggregate effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.0786$, and $\beta = .95512$

Ψ	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	99.145	98.598	99.977	99.447	105.791	103.791	97.648	97.821	99.187
0.200	98.599	97.350	99.996	98.955	110.770	107.133	95.387	95.718	98.609
0.300	98.488	96.606	100.057	98.544	115.172	110.120	93.232	93.687	98.389
0.400	98.768	96.410	100.045	98.069	120.162	113.102	91.679	91.901	98.487
0.500	98.840	95.990	100.149	97.784	123.719	115.617	89.684	89.942	98.442
0.600	98.934	95.559	100.253	97.531	127.044	118.003	87.700	87.971	98.413
0.700	99.093	95.112	100.357	97.311	130.198	120.302	85.722	85.982	98.429
0.800	99.196	94.666	100.447	97.100	133.207	122.534	83.712	83.943	98.406
0.900	99.291	94.222	100.532	96.916	136.023	124.727	81.684	81.865	98.379
1.000	99.376	93.768	100.611	96.751	138.441	126.901	79.625	79.733	98.345

Table 13: Welfare effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.0786$, and $\beta = .95512$

Ψ	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.617	1.056
0.100	98.825	99.019	96.770	2.562	1.058
0.200	98.052	98.400	94.683	2.527	1.058
0.300	97.639	98.115	93.579	2.508	1.059
0.400	97.472	98.034	93.135	2.500	1.059
0.500	97.451	98.092	93.078	2.499	1.059
0.600	97.595	98.302	93.462	2.506	1.059
0.700	97.895	98.665	94.263	2.519	1.059
0.800	98.228	99.060	95.154	2.535	1.058
0.900	98.776	99.645	96.637	2.560	1.058
1.000	99.332	100.247	98.156	2.586	1.057

Table 14: Aggregate effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.1572$, and $\beta = .959588$

Ψ	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	98.859	98.594	99.963	99.360	105.901	103.898	97.625	97.844	98.986
0.200	98.462	96.592	100.000	98.913	110.991	107.302	95.417	95.800	98.442
0.300	98.557	96.509	100.089	98.583	115.535	110.364	93.351	93.839	98.440
0.400	98.525	95.944	100.356	98.520	118.720	112.963	90.977	91.796	98.341
0.500	98.720	95.488	100.472	98.292	122.703	115.684	88.979	89.837	98.381
0.600	98.875	95.088	100.588	98.093	126.470	118.290	86.994	87.857	98.404
0.700	99.090	94.687	100.710	97.918	130.104	120.830	85.018	85.848	98.469
0.800	99.173	94.214	100.820	97.741	133.552	123.313	82.974	83.763	98.435
0.900	99.310	93.750	100.946	97.571	136.644	125.802	80.964	81.657	98.440
1.000	99.304	93.276	101.053	97.403	139.135	128.231	78.867	79.461	98.343

Table 15: Welfare effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.1572$, and $\beta = .959588$

Ψ	Y	C	K	K/Y	R
0.000	100.000	100.000	100.000	2.605	1.056
0.100	98.834	99.056	96.795	2.552	1.058
0.200	98.111	98.476	94.840	2.519	1.059
0.300	97.821	98.280	94.063	2.505	1.059
0.400	97.730	98.269	93.820	2.501	1.059
0.500	97.729	98.342	93.819	2.501	1.059
0.600	97.830	98.512	94.088	2.506	1.059
0.700	98.239	98.968	95.186	2.524	1.059
0.800	98.589	99.373	96.131	2.540	1.058
0.900	99.015	99.844	97.289	2.560	1.058
1.000	99.583	100.448	98.846	2.586	1.057

Table 16: Aggregate effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.2$, and $\beta = .9616$

Ψ	G1:L,L	G1:L,H	G1:H,L	G1:H,H	G2:L	G2:H	G3:L	G3:H	Average
0.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
0.100	99.294	99.017	99.979	99.284	106.142	104.017	97.606	97.881	99.316
0.200	99.343	96.871	100.023	98.864	111.470	107.533	95.415	95.875	99.065
0.300	99.582	96.998	100.114	98.597	116.245	110.696	93.399	93.959	99.188
0.400	99.837	96.683	100.231	98.376	120.635	113.635	91.419	92.033	99.286
0.500	100.060	96.305	100.350	98.121	124.721	116.444	89.421	90.070	99.353
0.600	100.222	95.765	100.465	97.902	128.592	119.139	87.403	88.063	99.364
0.700	100.435	95.376	100.607	97.754	132.420	121.794	85.476	86.061	99.435
0.800	100.491	94.889	100.714	97.582	135.961	124.402	83.432	83.959	99.382
0.900	100.516	94.354	100.813	97.389	138.970	126.978	81.341	81.787	99.298
1.000	100.518	93.757	100.924	97.174	141.406	129.504	79.236	79.557	99.190

Table 17: Welfare effects of social security in the altruistic framework with temptation: $\theta = 1$ and $\lambda = 0.2$, and $\beta = .9616$