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A Dynamic Model of Search and Intermediation

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Abstract

This paper develops a dynamic model of an economy with search frictions in which homogeneous agents choose between specializing as producers or as merchants, and can change occupation at any time. Merchants operate alongside a decentralized search market and provide immediacy in trade in return for a price. Agents who know the location of a merchant have the option of paying the merchant's price and avoiding search. We characterize equilibria in symmetric Markov strategies, and derive conditions under which merchants and their clients form a repeated relationship. We analyze welfare and explore conditions for the endogenous rise of an institution of intermediation.

Keywords: Search, endogenous intermediation, repeated interaction.

JEL Classification: D02, D51, D83.

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1 Introduction

Merchants and traders—agents who mediate the transfer of goods and services between producers and consumers—are central to the economic process. Specialization, the source of the wealth of nations, cannot proceed unless some agents mediate exchange. Thus Hicks, who considered the merchant trader the "principal character" in economic history, wrote that "it is specialization upon trade that is the beginning of the new world" (Hicks, 1969, p.25). In an economy with any degree of specialization, agents must trade to acquire goods that they wish to consume in exchange for goods that they produce. Searching for trading partners can be costly and time-consuming. Some agents recognize that there is profit in facilitating exchange and specialize as intermediaries or merchants. They reduce search costs and provide immediacy in exchange. Further, merchants derive profits from repeat business with returning clients rather than from random encounters with one-time clients.

The present paper focuses on these characteristics of the merchant. Our objective is a parsimonious but self-contained model of the merchant trader that reflects some essential features of an institution of intermediation in its "early and rude state". We are especially interested in the prospect of an endogenous emergence of specialist merchant traders. We establish equilibria in which merchants price their services to encourage repeat business, and their clients prefer to return to them in future periods rather than search for other trading partners. We show that such intermediation is profitable for some ranges of values of critical technological and institutional parameters; in other ranges the economy may harbor brigands but not *bona fide* merchants. We fully characterize two classes of intermediation equilibria that may exist, as well as a class of equilibria with brigandry. In a closed form example with reasonable search technologies, we show the existence of a unique equilibrium of each class.

The model is a variant of Diamond's (1982) tropical island economy. In our version homogeneous agents choose to specialize as producers or merchants, and can change occupation at any time. In each period a producer must exchange his output before he can consume. Exchanges are made with other producers or with merchants—the producer finds these partners through search. A merchant sets up a trading post; producers arriving at her post can trade by paying a commission that the merchant sets each period. Locating a merchant through search may not be *a priori* easier than locating another producer. However, a merchant can predict her own location from one period to the next, whereas a producer cannot. A producer who succeeds in finding a merchant may thus return to her in the next period, avoiding search. The viability of specialized intermediation is predicated on the ability of intermediaries to form such ongoing relationships with their clients.

Producers sometimes forget the locations of their merchants, so an unmediated search market remains active in parallel. A producer may therefore credibly decline to return to his merchant and choose instead to search anew for a trading partner. The continued existence of a search market also affords incipient merchants a pool from which they can draw clients. The process by which a new merchant acquires clients is a part of the dynamics of the model.

An equilibrium determines the occupational choice of each agent, and the commissions charged by merchants. We find that there are three classes of equilibria in symmetric Markov strategies. In *bandit equilibria*, merchants act as bandits and claim the entire output of their clients as "commission". In this case, producers understandably never return to these merchants, but search for trading partners in each period. This of course bears no resemblance to an institution of intermediation.[1]

Of greater interest to us are *intermediation equilibria*, in which merchants charge a commission that induces existing clients to return in succeeding periods. Intermediation equilibria exist only if producers remember the location of their merchants with sufficiently high probability. There are two distinct classes of intermediation equilibria. In one class, the price charged by merchants is the highest that is compatible with repeated interaction: were the price any higher, clients of merchants would be better off searching for trading partners each period. In the other class, the price is the lowest that is compatible with repeated interaction: were the price any lower, merchants would be better off acting as bandits instead of intermediaries that encourage clients to return.

An agent who specializes as a merchant facilitates exchange, but does not produce output. Moreover, as the measure of merchants increases, the unmediated search market gets thinner. The optimal measure of merchants in the economy must balance these effects. We find that, in general, an equilibrium is not optimal. We give conditions under which an intermediation equilibrium improves welfare compared to an economy with no merchants.

Finally, we discuss conditions under which an institution of intermediation can be expected to arise endogenously when the *status quo ante* is an economy with no merchants. We provide a heuristic interpretation of the parameters in our model in terms of technological and socio-political conditions that determine when intermediation rises and flourishes and when it declines under threat of brigandry and disorder.

The investigation of the role of intermediaries in speeding up search was initiated by Rubinstein and Wolinsky (1987).[2] In their model, buyers, sell-

[1] It is perhaps no accident that, in many historical contexts, merchants and brigands possessed similar enforcement capabilities. Even today unscrupulous merchants may, if they choose, defraud an unsuspecting client once with ease.

[2] Various other functions of intermediaries have been investigated in the literature:

ers, and intermediaries are randomly matched: trading with an intermediary is not a choice. In equilibrium, intermediaries are active if buyers and sellers encounter an intermediary at least as often as they encounter each other.

Gehrig (1993) presents a static model in which buyers and sellers, who differ in valuations and costs that are private information, can choose to search for trading partners and negotiate price, or access intermediaries whose locations and prices are publicly observable. Yavas (1994) allows heterogeneous agents to choose the intensity of private search, or to opt for the service of an intermediary. Spulber (1996) presents a dynamic model in which buyers, sellers, and intermediaries are heterogeneous in several respects, and intermediaries and their prices have to be found through search; there is no parallel unmediated search market in which buyers and sellers can trade directly. The focus is on deriving the equilibrium bid-ask spread and comparing the outcome with Walrasian prices. Rust and Hall (2003) extend the model of Spulber (1996) by adding a second type of intermediaries who post publicly observable prices. Howitt (2005) examines the role of fiat money in a search market where merchants organize exchange.

The papers cited above can accommodate rich heterogeneity among agents, but intermediaries are exogenously present in the economy and do not choose their calling. In contrast, a key concern of the present paper is to generate an endogenous distribution of occupational assignments in equilibrium starting from a homogeneous population.

We are aware of only a few papers that explicitly model endogenous occupational choice between production and intermediation. Li (1998) does so in the context of a friction quite different from ours: the function of intermediaries is to assess the quality of goods that are traded. In Bhattacharya and Hagerty (1987), producers may trade only with intermediaries; thus, the viability of intermediation is never in question. In Hellwig (2002) and Shevchenko (2004), the role of intermediaries is to resolve the problem of double coincidence of wants. Intermediaries achieve this by complementing money in Hellwig's model, and by stocking a variety of goods in Shevchenko's model. In both these models, there is also an unmediated search market, as in ours. Prices set by intermediaries in Hellwig's model are publicly observable; terms of trade with an intermediary are determined by Nash bargaining in Shevchenko's model.

Our paper is closely related to Masters (2007). Masters also investigates the endogenous emergence of intermediaries in the context of Diamond's tropical island economy. In his model, intermediaries enter the market with a unit of a good they have acquired after exchange. This gives them an advantage in Nash bargaining with producers because the intermediary has the option of consuming the good she holds whereas the producer does not. He finds that, when all producers have identical production costs, interme-

Spulber (1999) has an extensive survey.

diaries uniformly reduce welfare in the economy; however, when production costs are *ex ante* unequal intermediation can increase welfare. In contrast, the advantage of intermediaries in our model comes from the potential of repeated trade which Masters does not allow. As a consequence, in our model, intermediation can improve welfare even though all producers face identical costs (which we normalize to zero).

Intermediaries do not establish durable links with their clients in any of the papers above. In our paper, the benefit of establishing a trading link with an intermediary is that search costs can be avoided in *future* periods. At any intermediation equilibrium, clients and their merchants form ongoing relationships.

Durable client relations is accommodated in some papers that investigate price-setting by sellers in a market where consumers search for prices. The pricing component of the model here is similar to Benabou (1997), but simpler as our agents are homogeneous while Benabou allows heterogeneous agents. Burdett and Coles (1997) presents a model of noisy search in which a searching producer can observe more than one price with positive probability.

The next section sets out the model. Equilibrium is derived in Sections 3 and 4 for general matching functions. We then adopt a specific form for the matching functions in the remainder of the paper. In Section 5 we derive closed-form characterizations of equilibria of each of class and show these to be unique. Welfare is analyzed in Section 6. Section 7 discusses the possibility of an endogenous rise of merchants *ab initio*. Section 8 concludes with comments. Longer proofs have been placed in the appendix.

2 Model

2.1 Context

The setting is a highly stylized model of production, search, and exchange, adapted from Diamond (1982). The economy operates over an infinite succession of discrete periods. Agents are homogeneous and risk-neutral; they live for ever; the set of agents is a continuum of unit measure.

Each agent has access to a technology for production that generates one unit of a homogeneous, divisible good at no cost in each period. A taboo against consuming the output of one's own production ensures that exchange must precede consumption. This artifice allows, within a one-commodity framework, a representation of the reality that agents in an economy consume very little of their own output and the need for specialization and trade is paramount.[3]

[3]Although the model is in effect one of pure exchange, it will be convenient for termi-

The search for trading partners has to be undertaken anew after each episode of production (this may happen, for example, if the pursuit of production takes producers to random and unpredictable locations). Thus, every period each agent sets out to trade his output with any other agent he may encounter. The probability that a given agent will meet a trading partner during the period is $\lambda \in (0, 1)$. We interpret λ as a measure of the efficacy of unmediated search.[4] Once two agents meet, the units are traded one for one,[5] consumption takes place, and the agents are free to return to production which will generate another unit of the good next period. Throughout the paper we assume that untraded good cannot be carried as inventory.[6] Thus, an agent unsuccessful in effecting exchange foregoes consumption and returns in the next period with a newly produced unit.

We normalize payoffs so that the utility of consuming x units in a period is x. Letting $\delta \in (0, 1)$ represent the common discount factor of the agents, the present value of expected payoff of any agent is given by $v = \lambda + \delta v$, so that

$$v = \frac{\lambda}{1 - \delta}. \tag{1}$$

Is there scope in such a setting for some agents to set up as specialist intermediaries and offer the service of immediate trade in return for a price?

2.2 Producers and Merchants

We next allow each agent, in every period t, the choice of specializing either as a *producer*, or as a *merchant*. A specialist producer can access the production technology described above to produce output, but cannot commit to be available for trade at a specified location. In contrast, a specialist merchant cannot produce output, but can commit to be available for trade at a specified location. A specialist merchant operates a *trading post* where producers can exchange their output. For this service, a merchant charges a price that she sets each period.[7]

Agents can switch occupation between periods. At the end of each period, each producer may attempt to switch occupation and become a producer in the next period, and correspondingly a merchant may attempt

nological clarity to interpret it as a special model with production.

[4]More generally, $1 - \lambda$ may be interpreted as a measure of any transaction cost associated with coordinating trades without an organized market. In the language of search models the transaction cost is a lost trading opportunity.

[5]As agents are symmetric, any reasonable bargaining solution would prescribe an equal share of the gains from trade.

[6]Alternatively, we could assume that production cannot be carried out with a unit of the good already in hand. What we need is that a producer is not in possession of more than one unit at any given time.

[7]Since units are traded one for one in the unmediated search market, the merchant's price is also her commission or bid-ask spread.

to become a producer. A change of occupation, however, is a significant decision that may be temporarily constrained by cost and feasibility considerations. We model this as a friction in the implementation of the decision; for each given agent the attempt to switch occupations is unsuccessful with a probability $\alpha \in (0, 1]$, with realizations identically and independently distributed across agents and periods. With probability $1 - \alpha \in [0, 1)$, the agent finds that the decision to change occupation can indeed be implemented in that particular period. In the analysis, this friction rules out a class of implausible equilibria that are characterized at the end of Section 4. An arbitrarily small positive α is sufficient.

A merchant's location or price is not public information and must be initially found by a producer during search. Merchants may be peripatetic, but they are not subject to random displacements in location. A merchant decides each period where she will locate in the following period and communicates this information to her clients. Thus, once a producer makes contact with a merchant, it opens for him the prospect of a long-term relationship for future trade without further search.

If a producer trades with a merchant in period t, then he learns where the merchant will locate in period $t + 1$. With probability $\gamma \in (0, 1)$ he remembers this information at $t + 1$; with the complementary probability $1 - \gamma$ he forgets this information before $t + 1$. If a producer has not traded with a merchant in period t, then he does not know the location of any merchant at $t + 1$.

The non-persistence of memory embodied in $\gamma \in (0, 1)$ is meant to reflect the unavoidable frictions in continuing business relations, perhaps more pervasive in a nascent market than in a mature one. For example, the pursuit of production may take a producer too far from his merchant. In the model the assumption ensures that an unmediated search market remains viable.

An agent is *informed* in period t if he knows the location of a merchant's trading post at the beginning of t, and *uninformed* if he does not.

An uninformed producer must search for a trading partner. The search may yield one of three outcomes in a given period: either he does not meet a trading partner, or he meets another producer who is also searching, or he finds a trading post. If he fails to find a partner, he cannot trade. If he finds another producer, units are exchanged one for one. If he comes upon a trading post, he concludes trade at the merchant's set price, and learns where the merchant will locate at $t + 1$.

An informed producer has two options: he may proceed directly to his merchant's trading post, pay the merchant's commission, and immediately conclude trade; alternatively, he may search anew for a trading partner. In the latter case, he forsakes the knowledge of his erstwhile merchant and is exactly in the same position as an uninformed producer. An informed producer may also return to find that her merchant is absent because she has switched occupation. Such a producer proceeds to the unmediated search market as well. We assume that informed producers on their way to a merchant are unavailable to other producers searching for a partner.

Note that a producer can meet at most one trading partner in a period. Thus, if he meets a merchant, he may as well trade—regardless of how adverse the price set by the merchant is—since his current unit of the good will become obsolete after the present period. However, in equilibrium a producer will not return to a merchant who offers an unacceptable price even if he remembers her location in the following period.

Let m_t denote the measure of agents who specialize as merchants in period t and s_t denote the measure of producers in the search market. For a producer who searches for a trading partner, the probability of success is governed by two *matching functions*, λ^m and λ^s: $\lambda^m(m_t, s_t)$ is the probability that he finds a merchant and $\lambda^s(m_t, s_t)$ the probability that he meets another producer who is also searching in period t. Thus, the probability that he is able to trade within the period is given by $\lambda^m(m_t, s_t) + \lambda^s(m_t, s_t)$. Throughout the paper we assume that λ^m (resp. λ^s) is (weakly) increasing in m (resp. s); and the following conditions obtain:

$$\begin{gathered}\lambda^m(m,s) + \lambda^s(m,s) \in (0,1), \\ \lambda^m(0,1) = 0, \quad \lambda^m(m,s) > 0 \quad \text{for } m > 0, \\ \lambda^s(0,1) = \lambda, \quad \lambda^s(m,s) > 0 \quad \text{for } s > 0.\end{gathered}$$

Note that when $m = 0$ all agents are producers who search, i.e., $s = 1$, which is the primitive economy described in Section 2.1. In what follows, we often suppress the arguments of the functions λ^m and λ^s unless we are evaluating them at a particular point. No confusion should arise.

Merchants may meet and serve multiple clients within a period. The clients of a given merchant in a given period t come from two sources: informed clients from period $t-1$ who choose to return, and new clients—producers who discover her trading post during period t in the course of search. Merchants do not meet other merchants; such meetings are inconsequential in this model.

A merchant must start a period with sufficient inventory to conduct the first trade. She funds each subsequent trade out of the proceeds of the previous one. A merchant who plans to set a price p_t in period t must therefore carry from period $t-1$ an inventory of $1-p_t$ unit to offer her first client at t in return for the client's single unit.

Since untraded output cannot be carried in inventory, only producers who have effected an exchange in the previous period can exercise the option of becoming merchants. A continuing merchant can of course carry inventory as needed. Thus pricing and occupational choice decisions must be made one period in advance. An agent who was a producer in $t-1$ and wishes to switch occupation and become a merchant at t must carry over the necessary inventory by foregoing $1 - p_t$ units of consumption. Correspondingly, a merchant who decides to switch to production in period t can enjoy an extra $1 - p_{t-1}$ units of consumption in $t - 1$.

In an alternative formulation, the merchant could be modeled as a market-maker who organizes exchange between producers. Producers who show up at her trading post pay a commission to the merchant to trade with each other. A merchant then would not need to carry inventory between periods; nor would a new merchant need capital to start up business. Then all decisions—in particular, occupational choice and pricing—pertaining to period t could be made at the start of that period. The two formulations lead to almost identical results with only small differences in the explicit expressions for equilibrium values of some variables.

2.3 Solution Concept

The interaction among the agents in this economy over time is modeled as a stochastic game.[8] At the beginning of period t, each agent knows his *information state*—whether he is informed, or uninformed—and his occupation—a producer or a merchant. The sequence of events unfolds as follows.

At the beginning of period t, Each merchant μ sets a price $p_t^\mu \in [0, 1]$ which she has decided upon previously. Producers will learn this price only when (and if) they arrive at the merchant's post. Each informed producer (having produced output) decides whether to return to the merchant that he traded with in the previous period (choice R) or to undertake search for a trading partner (choice S). An uninformed producer has no option but to search. Informed producers whose erstwhile merchants have changed occupation also search. The outcomes of the search processes are realized; and trade takes place. Each producer who trades with a merchant forgets where the merchant will locate at $t + 1$ with probability $1 - \gamma$; the ones who do not will be the only informed agents in period $t + 1$. Next, agents choose their occupation for period $t + 1$. Decisions to change occupation are implemented subject to the friction α described earlier. Agents who will be merchants in the following period $t + 1$ choose the prices they will charge in $t + 1$. Finally consumption takes place.

For a producer, the period-t (Bernoulli) payoff is 1 if at t he trades with another producer, $1 - p_t$ if he trades with a merchant who charges a price p_t, and 0 if he fails to execute a trade at t. The period-t payoff of a merchant μ who sets a price p_t^μ and serves k_t^μ clients is $p_t^\mu k_t^\mu$.

In any period t, agents have imperfect information of the history of play up to t. The *personal history observed* by an agent in period t consists of

[8]The description here is informal: we do not furnish the measure-theoretic structure to make it entirely precise; but the details are standard.

(a) the prices she set and the size of the clientele she served in every period up to $t-1$ that she operated as a merchant, (b) the prices he paid in every period up to $t-1$ that he was a producer and traded with a merchant, and (c) the outcome in every period up to $t-1$ that he searched for a trading partner.[9]

An agent's *strategy* is a sequence of functions (indexed by t) that prescribe, for every period t, an action in A^i or A^u as a function of the personal history observed by the agent at t and the agent's information state at the beginning of t. We call an agent's strategy *Markov* if the sequence of functions is time-invariant, and if the action it prescribes in period t is determined entirely by the outcomes observed by the agent at $t-1$ and by the agent's information state at the beginning of t. In particular, for a merchant μ following a Markov strategy, the choice of price at t, p_t^μ, can depend only on the size of her clientele k_{t-1}^μ and her price p_{t-1}^μ at $t-1$; for an informed producer following a Markov strategy, the decision of whether to return to his merchant at t can depend only on the price the merchant had charged at $t-1$.

A profile of Markov strategies is *symmetric* if the function prescribing the pricing rule is the same for every merchant and the function prescribing the return-decision rule is the same for every informed producer. The occupational choice decisions may vary across agents. We look for equilibria in symmetric Markov strategies that produce steady-state outcomes as defined below.

Definition 2.1. An *equilibrium* is a profile of symmetric Markov strategies such that, given the strategy choices of other agents,

- the occupational choice of each agent in every period t is optimal;
- the price set by each merchant in every period t maximizes that merchant's expected continuation payoff at t;
- for each informed producer, the return decision in every period t maximizes his expected continuation payoff at t;
- $m_t = m$, $s_t = s$ for every period t.

We do not *a priori* restrict an agent's set of strategies to be symmetric or Markov. However, we are able to completely characterize only the set of equilibria in which agents' strategy choices are symmetric and Markov.

The focus of the paper is the class of equilibria in which each merchant and her (informed) clients form a repeated relationship: we call these intermediation equilibria.

Definition 2.2. An *intermediation equilibrium* with $m^* \in (0,1)$ merchants is an equilibrium such that an informed producer has no incentive to search

[9] In particular, a merchant does not observe the identity of an individual customer and thus cannot give discounts to returning customers. Merchants do not observe the history of prices set by other merchants. Producers do not observe the client size of any merchant.

for a trading partner: his (weakly) optimal choice in every period t is to return to the merchant he had dealt with at $t-1$.

2.4 Preliminary Observations

Observation 2.1. It is important to note that the size of the clientele that a given merchant μ serves in period t, k_t^μ, is stochastic. It is perfectly possible that most—or even all—of the clients served by a particular merchant at $t-1$ forget her location at t. The realization of k_t^μ can vary across merchants at t, and over time for the same merchant μ.

Observation 2.2. At an intermediation equilibrium, by definition, only uninformed producers search. Thus, at an intermediation equilibrium with $m \in (0,1)$ merchants, the expected size of clientele for a given merchant μ, who had set a price p_{t-1}^μ and served k_{t-1}^μ clients at $t-1$, evolves according to the equation

$$E\big(k_t^\mu | k_{t-1}^\mu, p_{t-1}^\mu, m) = \gamma k_{t-1}^\mu + \frac{\lambda^m s}{m}, \tag{2}$$

where E is the expectation operator. A fraction γ of a merchant's clients from period $t-1$ retain the knowledge of her location and return to deal with her. Moreover, each of the s producers in the search market discovers a merchant with probability λ^m. Since there are m merchants, the expected size of searching producers that arrive at the trading post of a given merchant in any period t is $\lambda^m s/m$. This yields equation (2).

The above equation also shows how an incipient merchant—an erstwhile producer who sets up as a merchant in period t—can start from a base of $k_{t-1}^\mu = 0$ and acquire clients over time.

Observation 2.3. At an intermediation equilibrium with m merchants, the expected continuation value at t of a merchant μ who sets a constant price p each period and who had k_{t-1} clients at $t-1$ can be written as

$$V_t^\mu(k_{t-1}, p, m) = \sum_{\tau=t}^{\infty} \delta^{\tau-1} p \gamma^\tau k_{t-1} + V^\mu(0, p, m) \tag{3}$$

$$= \frac{p\,\gamma k_{t-1}}{1-\gamma\delta} + V^\mu(0, p, m). \tag{4}$$

The first term reflects the discounted stream of profits from the pool of returning clients from $t-1$: of the k_{t-1} clients, a fraction γ returns at t, of whom a fraction γ return at $t+1$ and so on. The second term reflects the (time-invariant) continuation value from clients who come upon merchant μ at t for the first time: as argued in Observation 2.2, their measure is $\lambda^m(m,s)s/m$.

By a similar argument, $V(0,p,m)$ can be written as

$$V^\mu(0,p,m) = \frac{p\,\lambda^m s}{(1-\gamma\delta)m} + \delta V^\mu(0,p,m), \tag{5}$$

so that we have

$$V^\mu(0,p,m) = \frac{p\,\lambda^m s}{(1-\delta)(1-\gamma\delta)m}. \tag{6}$$

Observation 2.4. At an intermediation equilibrium with m merchants, the expected measure of searching producers is given by

$$s = (1-\gamma)\big(1-m-s\big) + (1-\gamma\lambda^m\big)s. \tag{7}$$

In each period, a fraction $(1-\gamma)$ of the $(1-m-s)$ informed producers forget the location of their merchants and return as searchers. Of the s searching producers, a fraction λ^m discover a merchant's trading post; of those, a fraction γ return as informed; all others return as searchers. This gives (7), which simplifies to

$$s = \frac{(1-\gamma)(1-m)}{1-\gamma(1-\lambda^m)}. \tag{8}$$

The next two sections develop the analysis of the class of equilibria in which the occupational choices of agents result in a measure $m \in (0,1)$ of merchants.

3 Pricing

In this section, we take a fixed stationary occupational assignment with $m \in (0,1)$ merchants as given, and characterize the sequence of prices that the merchants set and the decision rules that the informed producers follow in any equilibrium. Section 4 analyzes the occupational choices in equilibrium.

In any given period t, a merchant must determine the price to charge, and an informed producer must correspondingly decide whether to return to the merchant he dealt with at $t-1$ or to search for a trading partner. We want to look for equilibria in which these decisions are based only on the information observed by the corresponding decision maker in period $t-1$. Moreover, we look for equilibria in which the decision rules of the agents are time-invariant (by Markov restriction) and symmetric (by assumption).

Thus, in equilibrium, the decision rule of any informed producer can be represented as a time-invariant partition $\{P^R, P^S\}$ of $[0,1]$: a producer returns to his merchant in period t if and only if the price charged by that merchant in period $t-1$ was in P^R; otherwise, the producer searches for a trading partner. We refer to this loosely as an informed producer's return strategy.

Lemma 3.1. *In any equilibrium, P^S is nonempty, and* $\sup P^S = 1$*. In an intermediation equilibrium, if one exists, P^R is nonempty,* $\sup P^R < 1$ *and* $\sup P^R \in P^R$.

Proof. In any period, there are at least $(1-\gamma)(1-m)$ agents searching. Hence a producer's per period expected payoff from search is at least $\lambda^s\big(m,(1-\gamma)(1-m)\big) > 0$. Thus, a price greater than $1-\lambda^s\big(m,(1-\gamma)(1-m)\big)$ cannot be in P^R. It follows that $\big(1-\lambda^s\big(m,(1-\gamma)(1-m)\big),\ 1\big] \subset P^S$, and $\sup P^S = 1$.

By definition of an intermediation equilibrium, P^R is non-empty. From the argument in the previous paragraph, $\sup P^R \leq 1-\lambda^s\big(m,(1-\gamma)(1-m)\big)$.

If $\sup P^R \notin P^R$, then a merchant charging $p \in P^R$ can increase her current period profit by raising her price slightly without affecting the return-decision of any client. □

Our primary interest is in intermediation equilibria with an ongoing relationship between a merchant and her informed clients. First, we dispense with a case in which all merchants always charge a price of unity and producers never return. This is really a description of a search equilibrium with *bandits*, not of an institution of intermediation. These bandits live off appropriating the entire endowments of any searching producers who unluckily encounter them; the producers obviously do not return to trade with them.

Lemma 3.2 (Bandit Pricing)**.** *Each merchant always setting a price of* 1 *and each producer always choosing to search, even when informed, constitute mutual best responses for any given measure of merchants.*

Proof. Since clients do not return, it is never optimal for a merchant to set a price below 1. Even if a merchant deviates and sets a price below 1 in some period, the strategy profile calls for her to revert to a price of 1 in the following period. Hence, it is never optimal for an informed producer to return to his merchant.[10] □

The next lemma describes a merchant's best response to a given (symmetric, time-invariant) return strategy P^R on the part of producers. A merchant will set her price at 1 or the highest price at which clients return ($\sup P^R$) depending on whether it is more profitable to act as a bandit and take her clients' entire endowment, or to induce her clients to return when they remember her location.

[10] A producer is indifferent between trading at a price of 1 and declining trade and foregoing consumption. However, an outcome in which a producer declines trade cannot be sustained as an equilibrium since his merchant would be better off lowering the price slightly.

Lemma 3.3 (Optimal Pricing). *Given a return decision rule for the informed producers, characterized by P^R, a merchant's best response is to set price in each period t as follows:*

$$p_t = \begin{cases} \sup P^R & \text{if} \quad \sup P^R > 1 - \gamma\delta \\ \sup P^R \text{ or } 1 & \text{if} \quad \sup P^R = 1 - \gamma\delta \\ 1 & \text{if} \quad \sup P^R < 1 - \gamma\delta \text{ or } P^R = \emptyset. \end{cases} \tag{9}$$

Proof: See Appendix.

The intuition behind the lemma is transparent. First, it cannot be optimal for a merchant to set a price other than $\sup P^R$ or 1 since she can increase her current period profit, without affecting the return-decision of any client, by raising her price slightly. Next, suppose a merchant deviates from the constant price sequence of $\sup P^R$ for one period and charges a price of 1. She would gain $1 - \sup P^R$ from each client in that period, but lose her client base. The expected discounted loss on each client is $1/(1-\gamma\delta)$. The lemma follows from a comparison of these magnitudes.

By Lemma 3.3, at an intermediation equilibrium, $\sup P^R \geq 1 - \gamma\delta$; and if $\sup P^R > 1-\gamma\delta$, merchants set a constant price $p = \sup P^R$ and informed producers return to their merchants.

We next compute the optimal return decision rule for informed producers. Consider a configuration (p, m) in which all m merchants charge a constant price p in each period. Let $V^r(p,m)$ denote the expected continuation payoff of an informed producer who returns to his merchant and let $V^s(p,m)$ denote the corresponding expected continuation value of a producer who searches. Then, $V^r(p,m)$ and $V^s(p,m)$ are given by

$$V^r(p,m) = (1-p) + \delta\big[(1-\gamma)V^s(p,m) + \gamma V^r(p,m)\big] \tag{10}$$

$$V^s(p,m) = \lambda^s(1 + \delta V^s(p,m)) + \lambda^m V^r(p,m) + (1 - \lambda^m - \lambda^s)\delta V^s(p,m). \tag{11}$$

A returning producer concludes trade immediately with his merchant at the price p, consumes $1-p$, and returns again with another unit next period with probability γ. With probability $1-\gamma$ he forgets his information and must search. This yields equation (10). A searcher encounters another searcher with probability λ^s, trades units one for one, and will again search in the following period. With probability λ^m he finds a trading post with a merchant charging price p, and then is precisely in the same position as a returning producer. With probability $1-\lambda^m-\lambda^s$ he does not find a trading partner during the period, receives no utility, and returns as a searcher next period. This yields equation (11).

The solutions to equations (10) and (11) yield the continuation values for the informed and uninformed producers:

$$V^r(p,m) = \frac{(1-\gamma)\delta\lambda^s + [1-\delta(1-\lambda^m)](1-p)}{(1-\delta)\big[1-\gamma\delta(1-\lambda^m)\big]} \tag{12}$$

$$V^s(p,m) = \frac{(1-\gamma\delta)\lambda^s + \lambda^m(1-p)}{(1-\delta)\big[1-\gamma\delta(1-\lambda^m)\big]}. \tag{13}$$

Let $p^*(m)$ denote the price at which $V^r(p,m)$ equals $V^s(p,m)$. Then we have

$$p^*(m) = \frac{1-\lambda^m-\lambda^s}{1-\lambda^m}. \tag{14}$$

We provide a precise interpretation of $p^*(m)$ below.

Suppose an individual merchant sets a price p' every period while all other merchants charge a price $p \in P^R$. Then the continuation value of an informed client who decides to return to this merchant in each period that he is informed is:

$$\bar{V}^r(p'|p,m) = (1-p') + \delta\big[(1-\gamma)V^s(p,m) + \gamma\bar{V}^r((p'|p,m)\big] \tag{15}$$

from which we can solve for $\bar{V}^r(p'|p,m)$ in terms of $V^s(p,m)$ and p'. $\bar{V}^r$ is decreasing in p' as expected.

Define $f(p,m)$ to be the value of p' which equates $\bar{V}^r((p'|p,m)$ and $V^s(p,m)$. Some algebra yields

$$f(p,m) = \frac{(1-\gamma\delta)(1-\lambda^m-\lambda^s) + p\lambda^m}{1-\gamma\delta(1-\lambda^m)}. \tag{16}$$

For any p' lower than $f(p,m)$ the client is better off returning to the merchant, while at a higher price he would do better to search. The client is exactly indifferent between returning and searching when $p' = f(p,m)$. When all merchants set the same price p, an informed producer is indifferent between searching and returning to his merchant if $p = f(p,m)$. From (16), this is solved by $p^*(m)$ given in (14).

Note that

$$\frac{\partial f(p,m)}{\partial p} = \frac{\lambda^m}{1-\gamma\delta(1-\lambda^m)} = \frac{\lambda^m}{\lambda^m + (1-\gamma\delta)(1-\lambda^m)} \in (0,1)$$

Thus $p \gtreqless p^*(m) \Rightarrow p \gtreqless f(p,m)$, which leads to the following observation.

Observation 3.1. Suppose there are m merchants and each merchant charges a price p. Informed producers prefer to return to their merchants rather than search if and only if $p \leq p^*(m)$. Further, if $p < p^*(m)$ then an individual merchant can raise her price slightly without affecting this return decision.

Lemma 3.4 (Intermediation Pricing)**.** *With fixed occupational choices and a given measure m of merchants, an intermediation equilibrium exists if and only if*

$$\gamma\delta \geq \frac{\lambda^s}{1-\lambda^m}. \tag{17}$$

Moreover, only two stationary price paths can arise as outcomes of intermediation equilibria:

$$p = 1-\gamma\delta, \quad \text{and } p = \quad p^*(m) = \frac{1-\lambda^m-\lambda^s}{1-\lambda^m}.$$

proof: See Appendix.

The above lemmata establish that only three stationary price paths can arise as outcomes along an equilibrium path when occupational choice is fixed. One is the static pure monopoly price of 1 that we have termed the *bandit price*. Another is the lowest (symmetric) price compatible with repeated interaction: $1-\gamma\delta$, where the clients appropriate the entire information rent. A merchant is better off acting as a bandit than charging a lower price to attract repeat clients. We refer to $1-\gamma\delta$ as the *competitive intermediation price*. The third is the highest (symmetric) price compatible with repeated interaction: $p^*(m)$. At this price, the merchant appropriates the entire information rent from her clients. At any higher price, informed producers are better off searching than returning to their merchants. This can be thought of as the monopoly price in the context of a repeated relationship: we refer to it as the *monopoly intermediation price*.

The analysis of the monopoly price extends the classic model of search by Diamond (1971) to a repeated environment and incorporates a parallel search market. As in Diamond's model, each merchant enjoys local monopoly; but here the monopoly power is tempered by the coexistence of the search market. The highest price at which a merchant has returning clients is less than the static pure monopoly price of unity, which would be the equilibrium in Diamond's framework.[11]

We have modeled the merchants as price-setters. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer as in Rubinstein and Wolinsky (1987) or Masters (2007). Each party would then retain some of the gains from reduced search costs.

[11]The model of competition among merchants here is qualitatively similar to models of sequential search without recall. In models of sequential search, an agent who encounters an unacceptable price simply defers consumption and continues to search—the cost of additional search may be a delay in consumption or represented as a fixed amount. In our model, the cost of a bad search outcome is a reduction in current period consumption. The agent (producer) trades at the unfavourable price, consumes, and resumes search next period with a new unit of the good.

4 Occupational Choice and Equilibria

4.1 Occupational Choice

Assume a given stationary configuration (p, m) in which a measure m of merchants set a constant price $p \in P^R$. An agent who was a producer at $t-1$, and decides to start up as a merchant at t, will begin with no established client base and must acquire clients over time. (Some of the searching population in period t, $t+1, \dots$ will chance upon her trading post.) A continuing merchant may also find herself with no clients, since it is possible that all her clients may forget her location. The continuation value of such an *incipient merchant* is $V^\mu(0, p, m)$, defined in Observation 2.3.

A new merchant must start trading with an inventory of $1-p$ that she uses to fund her first trade. Since a merchant does not produce, this inventory must come from unconsumed output carried over from the previous period. Similarly, a merchant in $t-1$ who decides to become a producer in t can consume an extra $1-p$ in period $t-1$ which she would otherwise have carried as inventory.

The next lemma identifies the restrictions imposed by the requirement that the occupational choices of agents are optimal in equilibrium.

Lemma 4.1 (Occupational Choice)**.** *At a stationary configuration* (p, m)*, the occupational choice of each agent is optimal at every period* t *if and only if*

$$\delta V^\mu(0, p, m) = \delta V^s(p, m) + (1-p). \tag{18}$$

Proof. A producer who decides at $t-1$ to start up as a merchant at t can obtain the continuation value $\delta V^\mu(0, p, m)$, but he must sacrifice $1-p$ of consumption at $t-1$. Correspondingly, a merchant who decides at $t-1$ to switch to production at t will start as an uninformed producer and obtain the continuation value $\delta V^s(p, m)$, but she can consume at $t-1$ her current inventory of $1-p$ that would have funded her next trade.

Thus if $\delta V^\mu(0, p, m) - (1-p) > \delta V^s(p, m)$, a positive measure of uninformed producers will want to start up as merchants but no merchant will want to switch to production.[12] Conversely, if $\delta V^\mu(0, p, m) < \delta V^s(p, m) + (1-p)$, a positive measure of merchants will have an incentive to switch to production, but no producer will have an incentive to become a merchant. In either case, the measure of merchants cannot remain constant. Condition (18) negates these two possibilities, and ensures that each agent's occupational choice is optimal at any t. □

[12]It is not necessary to explicitly consider switches by informed producers, since these must have a continuation value bounded below by V^s.

4.2 Equilibria

Proposition 4.1 below combines Lemma 3.4 and Lemma 4.1 to provide a complete characterization of all equilibria in which some but not all agents are merchants.

Definition 4.1. We define three strategy profiles that will feature in the characterization of equilibria. The three profiles differ in their specifications of pricing and return decision rules. They share in common the part of the profile concerning occupational choice. For all three profiles,

($\star$) agents in a subset of measure $m \in (0, 1)$ always specialize as merchants; the remaining agents always specialize as producers.

A *bandit strategy profile* is one in which

- ($\star$) holds,
- each producer always searches (i.e., P^R is empty),
- each merchant always sets the price $p_t = 1$ in every period t.

A *monopoly intermediation strategy profile* is one in which

- ($\star$) holds,
- the informed producers' return decision rule is given by $P^R = [0, p^*(m)]$,
- a merchant's pricing rule is given by (9).

A *competitive intermediation strategy profile* is one in which

- ($\star$) holds,
- the informed producers' return decision rule is given by $P^R = [0, 1 - \gamma\delta]$,
- each merchant sets $p_t = 1 - \gamma\delta$ if she had set $p_{t-1} \leq 1 - \gamma\delta$, and sets $p_t = 1$ otherwise.

Proposition 4.1 (Equilibria). (a) *A bandit strategy profile constitutes an equilibrium if and only if*

$$\frac{\lambda^m(m, 1-m)}{\lambda^s(m, 1-m)} = \frac{(1-\delta)m}{1-m}. \tag{19}$$

(b) *A monopoly intermediation strategy profile constitutes an equilibrium if and only if* (17) *holds and*

$$1 - \gamma\delta = \frac{\delta\,(1 - \lambda^m - \lambda^s)\,\lambda^m\,s}{\lambda^s\,m}. \tag{20}$$

(c) *A competitive intermediation strategy profile constitutes an equilibrium if and only if* (17) *holds and*

$$\frac{\lambda^m s}{m} = \frac{\gamma\delta\lambda^m + (1-\gamma\delta)\lambda^s}{(1-\gamma\delta)\,(1+\lambda^m)} + (1-\delta)\gamma \tag{21}$$

proof: See Appendix.

The intuition behind Proposition 4.1 is as follows. If occupational choices remain invariant between periods, then m is constant over time. By Lemma 3.3 there are then exactly three prices that can be supported in equilibrium. By Lemma 3.4, two of these prices—the "monopoly" and "competitive" prices that yield intermediation equilibria—are incentive compatible for merchants if and only if Condition (17) is satisfied. Finally, Condition (18) ensures that continuing with invariant occupational choices is incentive-compatible for each individual agent. Thus occupational choices do remain invariant, and Lemma 3.3 can be applied.

In Proposition 4.1, the bandit equilibrium is determined by substituting the bandit price $p = 1$ into Condition (18) and imposing the additional condition that all producers search irrespective of their information state. Thus modified, Condition (18) then implicitly determines the measure of merchants m in a bandit equilibrium. The monopoly equilibrium and the competitive equilibrium are determined by substituting the appropriate prices $p^*(m)$ and $1 - \gamma\delta$ into Condition (18) together with the requirement that informed producers return to their merchants. In each case, the modified Condition (18) then implicitly determines the measure of merchants. Further, Condition (17) must hold in the intermediation equilibria.

Finally, at each intermediation equilibrium price there is one set of agents that is indifferent between the equilibrium action and an alternative action. In monopoly, informed producers are indifferent between return and search, and in the competitive equilibrium merchants are indifferent between charging the equilibrium price and the bandit price. This indifference is used to support the appropriate actions off the equilibrium path in the equilibrium strategy profiles.

The friction in the decision to change occupation, embodied in the probability $\alpha > 0$ introduced earlier, did not play any part in the results presented in this or earlier sections. In fact, its role is to rule out a class of equilibria that we find less than convincing. These are precisely equilibria in which the price lies strictly between the the competitive price and the monopoly price. Below we establish the unique strategy profile that can support these prices in equilibrium, and show that these equilibria vanish when the friction α is positive.

Consider a pair $(\bar{p}, \bar{m})$ that satisfies the condition (18), which must necessarily hold in equilibrium. Define the strategy profile $\bar{\sigma}$ as follows:

- Producers set $P^R = [0, \bar{p}]$,
- Merchants set $p_t = \bar{p}$ in each period t; if a merchant sets $p_t > \bar{p}$ in any t, then in $t + 1$ the merchant changes occupation and becomes a producer.

Lemma 4.2. *Suppose $\bar{p} > 1-\gamma\delta$, informed producers use the return decision* *rule $P^R = [0, \bar{p}]$, and $\bar{m}$ is such that $(\bar{p}, \bar{m})$ satisfies condition (18). Then it is optimal for each agent who is a merchant in period t to set $p_t = \bar{p}$, regardless of history.*

Proof: See Appendix.

Lemma 4.3. *Suppose $(\bar{p}, \bar{m})$ satisfies condition (18), $\bar{p} < p^*(\bar{m})$, and each agent who is a merchant in period t sets $p_t = \bar{p}$. Then it is optimal for informed producers to set $P^R = [0, \bar{p}]$ only if merchants that deviated in period t and set $p_t > \bar{p}$ change occupation and become producers in period $t+1$.*

Proof: See Appendix.

Thus if a price strictly between the competitive and monopoly prices is supported in equilibrium, then the equilibrium strategy profile must be $\bar{\sigma}$ described above. Proposition 4.2 below shows that such a price can be supported as an equilibrium if and only if the decision to change occupation is frictionless.

Proposition 4.2. *A configuration $(\bar{p}, \bar{m})$ that solves (18) can be supported as an equilibrium if and only if $\alpha = 0$.*

proof: See Appendix 1.

Thus equilibria that support prices in the interval between the competitive price and the monopoly price are not robust in the sense that they vanish if there is any friction in the decision to change occupations. We ignore these equilibria in the remainder of the paper.

Proposition 4.3. *There are at most three classes of equilibria in the economy with $\alpha > 0$. These are the ones defined in Definition 4.1.*

Proof. By Lemma 3.3 It is never optimal for merchants to charge a price strictly less than $1 - \gamma\delta$, regardless of the return rule used by producers. Given m, producers will not return at prices higher than $p^*(m)$, hence it is also suboptimal to charge prices in the interior of $(p^*(m), 1)$. Thus the only candidates for equilibrium prices are prices in the interval $[1 - \gamma\delta, p^*(m)]$, and unity. The proof then follows from Propositions 4.1 and 4.2. □

5 Closed-Form Solutions

In our analysis so far, the matching functions λ^m and λ^s were quite arbitrary. In the remainder of the paper, we focus on a specific pair of matching functions in the interest of gaining further insight and obtaining closed-form expressions for the equilibrium values of some key variables. The matching functions we focus on are given by

$$\lambda^m(m,s) = \frac{\lambda\, m^{1/2}}{m^{1/2}+s^{1/2}} \tag{22}$$

$$\lambda^s(m,s) = \frac{\lambda\, s^{1/2}}{m^{1/2}+s^{1/2}}, \qquad \lambda \in (0,1). \tag{23}$$

Observe that merchants and producers are treated symmetrically by the matching technology: merchants enjoy no *a priori* advantage in this respect. The matching functions are also homogeneous of degree 0 so that there are no thick-market externalities.

Using the search functions in (22) and (23), we find existence conditions for each of the three types of equilibria outlined in Proposition 4.1 in terms of the parameters of the model (γ, δ, and λ). We also determine the equilibrium price at these equilibria and the equilibrium measure of merchants at the bandit and monopoly intermediation equilibria.[13]

Proposition 5.1 (Closed-Form Solutions)**.** *Let the matching functions λ^m and λ^s be given by* (22) *and* (23)*. Then,*

(a) *A unique bandit equilibrium always exists. At this equilibrium, $p = 1$, and the measure of agents specializing as merchants is given by $m = 1/2$.*

(b) *A monopoly intermediation equilibrium exists if and only if*

$$\gamma\delta^2(1-\lambda)^2 \geq (1-\gamma\delta)(\lambda-\gamma\delta). \tag{24}$$

If it exists, it is unique. The measure of agents specializing as merchants is given by

$$m^* = \frac{(1-\gamma)(1+a^{1/2})}{(1-\gamma)(1+a^{1/2})(1+a)+\gamma\lambda a}, \tag{25}$$

$$\text{where} \quad a = \frac{(1-\gamma\delta)^2}{\delta^2(1-\lambda)^2}; \tag{26}$$

and the price set by each merchant in every period is given by

$$p^* = \frac{\delta(1-\lambda)^2+(1-\lambda)(1-\gamma\delta)}{\delta(1-\lambda)^2+(1-\gamma\delta)}. \tag{27}$$

(c) *If $\gamma\delta \geq \lambda$, a competitive intermediation equilibrium exists. If it exists, it is unique. At this equilibrium, $p = 1-\gamma\delta$ and $m \in (0, \frac{1}{2})$.*

[13]We omit the long and uninformative closed-form expression for m in the competitive equilibrium.

Proof: See Appendix.

The proof consists of using the particular matching functions (22) and (23) in the conditions derived in Proposition 4.1—in particular, inequality (17), and the equations(19), (20), (21).

Observation 5.1. (i) There is a threshold value $\lambda^* \in (\gamma\delta, 1)$ such that the economy has a monopoly intermediation equilibrium if and only if $\lambda \leq \lambda^*$.

(ii) The condition in part (c) is sufficient but not necessary.

(iii) It follows from parts (b) and (c) of Proposition 5.1 that both a monopoly and a competitive intermediation equilibrium exist when $\gamma\delta \geq \lambda$.

Proof of (i). Rewrite (24) as

$$\phi(\lambda) \equiv \gamma\delta^2(1-\lambda)^2 - (1-\gamma\delta)(\lambda - \gamma\delta) \geq 0 \tag{28}$$

Inequality (28) is strict for $\gamma\delta \geq \lambda$. Also, we have $\phi' < 0$ for all $\lambda \in [0, 1]$, and $\phi(1) = -(1-\gamma\delta)^2 < 0$. Since ϕ is continuous, the existence of λ^* in the remark is assured. □

The intuition is transparent: for intermediation equilibria to exist, the rate at which searchers can be found cannot be too high compared to the rate at which former clients return to their merchants. Otherwise, it would be more profitable for merchants to act as bandits than to induce clients to return.

6 Welfare

Merchants in this model provide a beneficial trading externality: an encounter with a merchant opens up the prospect of a long-term relationship for future trade, and potentially reduces search costs. However, specialization by merchants in the service of exchange comes at the expense of the production of the physical good. Merchants also create negative externalities—as the measure of merchants rises, the search market gets thinner affecting the trade prospects of the searching population. Also, the clientele of a new merchant in steady-state is not drawn entirely from the hitherto searching population; some of her clients would otherwise have been clients of the merchants already in the market.

A natural measure of social welfare here is the expected aggregate consumption per period. How does welfare at an equilibrium with merchants compare with an economy with no merchants? What is the optimal measure of merchants in the economy? Is the equilibrium measure of merchants optimal? This section addresses these questions in the context of the closed-form economy described in Section 5.

It is obvious that the bandit equilibrium outcome is worse for welfare than an economy with no merchants: bandits do not produce; nor do they reduce search cost for other agents.

Suppose that the economy is in steady-state with m merchants who set a price $p \in P^R$. Then, the size of the set of uninformed agents is given by equation (8). With m merchants, $1 - m$ units are produced in a period; of these, a fraction $(1 - \lambda^m - \lambda^s)s$ fails to get traded and consumed. Letting W denote the welfare, we have

$$W(m) = 1 - m - (1 - \lambda^m - \lambda^s)\, s(m). \tag{29}$$

Proposition 6.1 and Observation 6.1 demonstrate that the welfare associated with an intermediation equilibrium may be higher or lower than the welfare associated with a pure-search economy.

Proposition 6.1 (Welfare)**.** *Let the matching functions be given by* (22) *and* (23)*.*

(a) *Welfare is maximized at an interior measure* $\tilde{m}$ *of merchants. At* $\tilde{m}$*,*

$$s'(\tilde{m}) = -\frac{1}{1-\lambda}. \tag{30}$$

(b) *The welfare associated with the monopoly intermediation equilibrium characterized in Proposition* 5.1(b) *is greater than the welfare in the pure-search economy if and only if*

$$\gamma(1-\gamma\delta)^2 > (1-\gamma)[(1-\gamma\delta) + (1-\lambda)]. \tag{31}$$

proof: See Appendix.

Observation 6.1. Condition (31) holds for a wide range of parameter values that are consistent with Proposition 5.1 (b). For example, try $\gamma = 9/10,\ \delta = 5/8,\ \lambda = 9/16$. Note that $\lambda = \gamma\delta$, so the existence condition is satisfied. Similarly, there are also ranges of values for which an intermediation equilibrium exists, but condition (31) does not hold. Thus in general there is no correspondence between equilibria and optima, or even a presumption that welfare at an equilibrium is necessarily greater than in the pure-search economy.

7 The Rise of Merchants

Suppose that we start with an economy in which all agents specialize as producers. Under what conditions can we expect an institution of intermediation to endogenously arise in this economy? We show below that, if γ is sufficiently large, then it will be strictly profitable for an arbitrarily small positive measure of producers to deviate and set up as merchants. Below we interpret an increase in γ as a consequence of increasing maturity and stability in civil society. In this interpretation, as society progresses from its "early and rude state", intermediation arises endogenously and merchants replace bandits and pirates.

Consider therefore an economy in which each agent specializes as a producer every period, and searches for trading partners. Each agent's per-period payoff is $\lambda^s(0,1) = \lambda$. Let each producer's return-decision-rule, when informed, be given by $P^R = [0, 1-\lambda]$.

Now, suppose a small measure of agents deviates, starts up as merchants, and sets a price less than $1-\lambda$. Producers who come upon their trading posts will want to return. Thus the merchants, beginning with a client base of zero, will acquire clients over time. Proposition 7.1 below shows that this deviation is profitable provided that a merchant's retention rate of clients, γ, is sufficiently high relative to λ.

Proposition 7.1 (Emergence of Merchants)**.** *Let the matching functions be specified by* (22) *and* (23)*, and let $\gamma\delta > \lambda$. Suppose that each agent's strategy is to specialize as producer in every period and, if informed, use the return-rule $P^R = [0, 1-\lambda]$.*

(a) *There exists $\bar{m} \in (0,1)$ such that, for all $m' \in (0, \bar{m})$, any subset of agents of measure m' would find it profitable to start up as merchants and set a price $\tilde{p} \in [1-\gamma\delta, 1-\lambda]$.*

(b) *The payoffs of deviating merchants increases without bound as $m' \to 0$.*

Proof. Let a subset of agents of measure m simultaneously start up as merchants and set a price $\tilde{p} \in [1-\gamma\delta, 1-\lambda]$ every period. Since $\gamma\delta > \lambda$, $\tilde{p} \in P^R$. Then, the continuation value of this deviation for an individual merchant is given by

$$
\begin{aligned}
V^\mu(0, \tilde{p}, m) &= \frac{\tilde{p}\ \lambda^m(m, 1-m)\,(1-m)}{(1-\delta)(1-\gamma\delta)\,m}, \quad \text{[see (36)]} \\
&= \frac{\tilde{p}\ \lambda\,(1-m)}{(1-\delta)(1-\gamma\delta)\left[\,m + m^{1/2}(1-m)^{1/2}\,\right]}, \quad \text{by (22) and (23)} \\
&\geq \frac{\lambda\,(1-m)}{(1-\delta)\left[\,m + m^{1/2}(1-m)^{1/2}\,\right]}, \ \text{since } \tilde{p} \geq 1-\gamma\delta. \qquad (32) \\
&> \frac{\lambda}{1-\delta}, \quad \text{for } m < 1/4, \ \text{since } m^{1/2}(1-m)^{1/2} \leq 1/2. \qquad (33)
\end{aligned}
$$

The condition $\gamma\delta > \lambda$ also ensures that $\sup P^R > 1-\gamma\delta$; thus, the payoff for the deviating agents who become merchants is higher than their payoff if they were to become bandits (see Lemma 3.3). This, in conjunction with inequality (33), establishes part (a).

Moreover, from (32), $V^{\mu}(0, \tilde{p}, m)$ increases without bound as $m \to 0$ which is part (b). □

Observation 7.1. If $\tilde{p} \in (1-\gamma\delta, 1-\lambda)$, the deviation makes all agents—not only those in the deviating subset—strictly better off.

The prospect of an endogenous rise of an institution of intermediation *ab initio* thus depends on the relative values of the parameters γ and λ (for a fixed δ). These parameters, in turn, are arguably determined by social and technological conditions.

Exchange for personal consumption between producers has occurred since prehistory within local circles, and formed the basis for division of labor and specialization in village economies. The ambit of such exchange, for which λ is a proxy, is likely to remain limited and evolve slowly over time.

The parameter γ, which captures the ability of merchants to communicate with their clients and of the clients to return to their merchants, is likely to be more sensitive to social, political, and technological conditions. Communication and commerce may be rendered impossible between one period and the next by natural calamities or bandits or unreliable transportation; rulers may prevent access or impose tolls; local wars may intervene. Viewed in this way, γ is likely to rise with improvements in law and order and in the technology of communication and transport. Thus, farsighted merchants are unlikely to thrive in primitive and unmoderated societies; they appear only when some modicum of public security has already been established. When order deteriorates in established societies, disrupting communication and transportation networks, even erstwhile reliable merchants may turn to banditry; but professionally mediated trade arises again as the rule of law is restored and communication improves.[14]

8 Conclusion

Intermediaries perform many roles in facilitating trade. They may variously exploit advantages in the technology of transaction and trade, costs of storing inventory, aggregating information, assessing quality of goods or some attributes of agents or a market, matchmaking, and so forth. We focused on only two aspects that are interrelated in our model—reducing the cost of search, and fostering long-term trading relationships with clients.

[14] This interpretation is not inconsistent with European history. In the second half of the first millennium AD there was a general decline of law and order, accompanied by a contraction of trade. As stability was re-established and the rule of law gained ascendancy in the second millennium, professional merchants flourished and trade expanded as well, both within Europe and across the Mediterranean.

Our primary objective was to develop a self-contained, if rudimentary, account of an emergent institution of intermediation. Thus, the important modeling concern was to start with a homogeneous population, endogenize the choice between the two occupations of production and intermediation, and investigate the configuration of parameters that predicate the rise of intermediation as a sustainable occupation.

In focusing on these, we have marginalized several other concerns that may legitimately claim attention in the context of this paper. We briefly comment on some of these below.

We suppose that the price at which a producer trades with a merchant is set by the merchant. In our model, this results in either the merchant or the producer extracting all the rent in equilibrium. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer, with the consequence that both parties would retain some of the gains from reduced search costs.

Our treatment of competition among merchants is minimalist. In particular, a producer knows at most one merchant; he cannot maintain his link with a merchant and simultaneously search for a better price. It may be of interest to investigate the consequences of allowing producers to randomly observe a second price, as in Burdett and Coles (1997). It is worth reiterating, however, that even the simple model elaborated here incorporates the full extent of competition that is afforded by standard models of sequential search without recall (see Footnote 11).

The present model is one of pure exchange: the production process is entirely mechanistic in that it involves no choice variable. As Diamond (1982) has shown in a model of search, reducing anticipated delays in exchange can influence production decisions. In future work, we plan to extend the present model by incorporating production to yield richer general equilibrium interactions. This would also provide the bridge between the analysis of the microstructure of exchange and the formulation of macroeconomic policy, which was the intention of Diamond's original article.

Appendix: Proofs

Proof of Lemma 3.3. Fix a return decision rule P^R. No price other than $\sup P^R$ or 1 can optimal: a merchant can increase her current period profit, without affecting the return-decision of any client, by raising her price slightly.

We start with the price sequence $p_t = \sup P^R$ for every period t, and show that the merchant cannot gain by deviating from this sequence if $\sup P^R > 1 - \gamma\delta$.

STEP 1: Consider a one-period deviation in some period τ in which the merchant sets $p_\tau = 1$ (the best one-period deviation). The merchant's net gain from this deviation discounted to period τ is:

$$\begin{aligned}\Delta_1 \;&\equiv\; k_\tau\big(1 - \sup P^R\big) \;-\; \gamma\delta k_\tau \sup P^R\big(1 + \gamma\delta + \ldots\big) \\ &= k_\tau\left(1 - \frac{\sup P^R}{1-\gamma\delta}\right).\end{aligned}$$

$\Delta_1 < 0$ since $\sup P^R > 1 - \gamma\delta$. Thus if there is a profitable deviation from the constant sequence $\sup P^R$, then the price must deviate from $\sup P^R$ in at least two periods.[15]

STEP 2: So let τ be a period in which $p_\tau = 1$ and $\tau' = \tau + n$ the next period such that $p_{\tau'} = 1$, with $p_t = \sup P^R$ for the intermediate periods $\tau < t < \tau'$ (there may be no such intermediate periods).

Now replace p_τ with $\sup P^R$, and calculate the change in the merchant's profit. The merchant loses $k_\tau(1 - \sup P^R)$ in period τ owing to the lower price she charges. However, of these k_τ producers, the ones that remain informed return in the periods $\tau+1, \ldots, \tau+n$, which they would not have done if the merchant had charged p_τ. The merchant's net gain discounted to period τ is:

$$\begin{aligned}\Delta_2 &= k_\tau(1 - \sup P^R) \;+\; \sum_{t=\tau+1}^{\tau+n-1}(\gamma\delta)^{t-\tau}\;\sup P^R k_\tau + \;(\gamma\delta)^n k_\tau \\ &= -k_\tau[1-(\gamma\delta)^n] \;+\; \sup P^R k_\tau\left(\sum_{t=0}^{n-1}(\gamma\delta)^t\right) \\ &= -k_\tau[1-(\gamma\delta)^n] \;+\; \sup P^R k_\tau\left(\frac{1-(\gamma\delta)^n}{1-\gamma\delta}\right),\end{aligned}$$

which is positive since $\sup P^R > 1 - \gamma\delta$. Thus the merchant's profit increases if she sets $p_\tau = \sup P^R$.

[15]We consider multi-period deviations below: it is not obvious that the single-deviation property applies to this game.

By Step 1 and Step 2, if $\sup P^R > 1 - \gamma\delta$, then the merchant's profit is maximized by setting $p_t = \sup P^R$ in each period t.

Using an analogous argument, when $\sup P^R < 1 - \gamma\delta$ it is optimal to set $p_t = 1$ in each period. When $\sup P^R = 1 - \gamma\delta$, the merchant, in each period, is indifferent between setting $p_t = \sup P^R$ and setting $p_t = 1$. □

Proof of Lemma 3.4. Consider an intermediation equilibrium with price p. From Observation 3.1 we know that clients will find it suboptimal to return to their merchants if $p > f(p, m)$, thus we must have $p \leq f(p, m) \iff p \leq p^*(m)$. By Lemma 3.3 we know that merchants strictly prefer to charge a price of unity rather than $p < 1 - \gamma\delta$. Hence $p \geq 1 - \gamma\delta$. Thus we must have $p^*(m) \geq 1 - \gamma\delta$ which is restated as condition (17). This establishes necessity.

By Lemma 3.3, we must have $\hat{p} = \sup P^R \geq 1 - \gamma\delta$ in an intermediation equilibrium.

If $\sup P^R = 1 - \gamma\delta$, then each merchant is indifferent between setting $p = \sup P^R$ and $p = 1$ in each period. Let the return strategy of the producer be given by $P^R = [0,\ 1 - \gamma\delta]$ and the (symmetric) pricing rule of each merchant be to set $p_t = 1 - \gamma\delta$ if she had set $p_{t-1} \leq 1 - \gamma\delta$, and $p_t = 1$ if she had set $p_{t-1} > 1 - \gamma\delta$. This is a best response to the producers' return strategy P^R, and the return strategy is a best response to the pricing rule.[16]

Suppose all merchants charge a price p in the interior of the interval $(1 - \gamma\delta,\ p^*(\hat{m}))$. By Observation 3.1, $p \neq \sup P^R$, since an informed producer will prefer to return to his merchant even if that merchant slightly raises her price. Thus a price $p \in (1 - \gamma\delta, p^*(\hat{m}))$ cannot obtain in an intermediation equilibrium with given $\hat{m}$.[17]

Finally consider $p = p^*(m)$. If all merchants charge this price then an informed producer is indifferent between return and search; at any higher price he will search. So let each informed producer return if his merchant charges $p \leq p^*(m)$, and search otherwise. Note that this is weakly optimal even if the merchant is expected to revert to $p^*(m)$ after a deviation. Let merchants set $p = p^*(m)$. This is a best response (strictly so if $p^*(m) > 1 - \gamma\delta$) to the producers' return strategy P^R, and the return strategy is a best response to the pricing rule.

This establishes sufficiency and the specific equilibria, and confirms that there are no other equilibria. □

[16] Note that if $1 - \gamma\delta < p^*(\hat{m})$, then in equilibrium the merchant's strategy cannot specify reversion to $1 - \gamma\delta$ after a deviation, for then the clients will find it optimal to return.

[17] This is *not* true if the friction α vanishes, and agents can change occupation (see Proposition 4.2)

Proof of Proposition 4.1. Lemma 3.4 shows that the pricing decisions of merchants and the return decisions of producers given in (a)–(c) of Definition 4.1 are the only ones that are consistent with equilibrium if occupational choices remain invariant between periods. Condition (18) ensures that no agent can benefit by changing occupation, so that invariant occupational choices remain weakly optimal. The proof combines these two conditions.

(a) If agents follow the bandit strategy profile, we have $s = 1-m$ at every t, and a merchant extracts a price of unity from each of the $\lambda^m(m, 1-m)[1-m]/m$ of searching producers who come upon her trading post. Further, since informed clients don't return, every merchant is in the same position as one who served no clients in the previous period. Thus, for any merchant μ at every t

$$V^\mu(0, 1, m) = \frac{\lambda^m(m, 1-m)(1-m)}{(1-\delta)m}. \tag{34}$$

Further, at the given strategy profile, the continuation value at any t for any producer, whether informed or uninformed, is given by

$$V^s(1, m) = \frac{\lambda^s(m, 1-m)}{1-\delta}. \tag{35}$$

Noting that in the bandit strategy $p = 1$ and applying Lemma 4.1, we obtain (19).

(b) If agents follow the monopoly intermediation strategy profile, using (14) in (6), we have

$$V^\mu\big(0, p^*(m), m\big) = \frac{(1-\lambda^m-\lambda^s)\lambda^m s}{(1-\delta)(1-\gamma\delta)(1-\lambda^m)m}. \tag{36}$$

It follows from equations (11) and (14) that

$$V^s\big(p^*(m), m\big) = \frac{\lambda^s}{(1-\delta)(1-\lambda^m)}. \tag{37}$$

Condition (20) is obtained by substituting the continuation values from (36) and (37) in Condition (18). Condition (17) follows from equation (14) and the requirement that $p \geq 1-\gamma\delta$ in an intermediation equilibrium (Lemma 3.3).

(c) Condition (21) is obtained by substituting $1-\gamma\delta$ for the price in equation (6) and equation (13), and using the resulting continuation values in Condition (18). Condition (17) ensures that the price $1-\gamma\delta$ does not exceed the monopoly price derived in (14), for otherwise informed producers would choose not to return to their merchants. □

Proof of Lemma 4.2. Clearly it is never optimal to set $p_t < \bar{p}$. If a merchant charges $p_t > \bar{p}$, then it is optimal to charge $p_t = 1$. If she does so, then she will have no returning clients in $t + 1$, so she is indifferent between continuing as a merchant in period $t + 1$ (with no returning clients) and becoming a producer (by condition (18)). Hence it is sufficient to show that charging $p_t = \bar{p}$ and continuing as a merchant dominates charging $p_t = 1$ and continuing as a merchant. By Lemma 3.3, this follows from $\bar{p} > 1 - \gamma\delta$ and the fact that the merchant expects a strictly positive number of clients $E(k_t) \geq \frac{s\lambda^m}{\bar{m}} > 0$ in period t, regardless of history.

□

Proof of Lemma 4.3. Suppose informed producers set $P^R = [0, \bar{p}]$. Then by Lemma 4.2 it is strictly optimal for each merchant in period t to set $p_t = \bar{p}$, even if she had deviated in the previous period. Since $\bar{p} < p^*(\bar{m})$, it is therefore strictly optimal for an informed producer to return to his merchant in each period that he expects the merchant to be present at her trading-post, even if the merchant deviated in the previous period and set $p > \bar{p}$. Hence $P^R = [0, \bar{p}]$ cannot be optimal. □

Proof of Proposition 4.2. By Lemmas 4.2 and 4.3, the only subgame-perfect strategy profile that is consistent with an equilibrium configuration $(\bar{p}, \bar{m})$, with $\bar{p} \in (1 - \gamma\delta, p^*(\bar{m}))$, is one in which merchants change occupation after charging a price higher than $\bar{p}$, and clients do not return if the merchant charges a price higher than $\bar{p}$ because they expect the merchant to cease operating as a merchant. The clients' return rule must of course be correspondingly specified. This gives precisely the strategy profile $\bar{\sigma}$.

However, suppose that the strategy profile is $\bar{\sigma}$, and $\alpha > 0$. consider a merchant who charged $p_t > \bar{p}$. With probability α, she will be unable to change occupation and will continue as a producer in $t + 1$. Even though she does not expect her former clients to return, by Lemma 4.2 it is optimal for her to charge $p_{t+1} = \bar{p}$.

Now consider her former clients. In period $t + 1$ they can go directly to the search market, which yields expected continuation value $V^s(\bar{p}, \bar{m})$. Alternatively, they can first visit the merchant's post and then proceed to the search market if the merchant is indeed not present, which yields expected value $\alpha(1 - \bar{p}) + (1 - \alpha)V^s(\bar{p}, \bar{m})$. But since $\bar{p} < p^*(\bar{m})$, it follows that $(1 - \bar{p}) > V^s(\bar{p}, \bar{m})$, hence the second strategy is strictly optimal.

Thus if $\alpha > 0$ it is optimal for an informed producer to return to his merchant's post in $t+1$ even if the merchant set $p_t > \bar{p}$, thus $P^R = [0, \bar{p}]$ cannot be optimal for informed producers, and $\bar{\sigma}$ cannot be an equilibrium.

□

Proof of Proposition 5.1. (a) Using (22) and (23) in (19) and recognizing that $s = 1 - m$ at a bandit equilibrium, we get

$$\frac{\lambda\, m^{1/2}(1-m)}{m^{1/2} + (1-m)^{1/2}} = \frac{\lambda\,(1-m)^{1/2} m}{m^{1/2} + (1-m)^{1/2}},$$

which reduces to $m = 1/2$. This proves part (a).

(b) Simplify(20) to obtain

$$\delta(1 - \lambda^s - \lambda^m)\lambda^m s - (1 - \gamma\delta)\lambda^s m = 0 \tag{38}$$

Substitute (22) and (23) in (38) to get

$$\frac{m^{1/2}}{s^{1/2}} = \frac{\delta(1-\lambda)}{1-\gamma\delta}. \tag{39}$$

Using (22) and (23) in (17), we get

$$\gamma\delta \geq \frac{\lambda\, s^{1/2}}{(1-\lambda)\, m^{1/2} + s^{1/2}}, \qquad \text{or,}$$

$$\frac{m^{1/2}}{s^{1/2}} \geq \frac{\lambda - \gamma\delta}{\gamma\delta(1-\lambda)}. \tag{40}$$

Combining (39) and (40) gives (24).

By (39),

$$\frac{s}{m} = \frac{(1-\gamma\delta)^2}{\delta^2(1-\lambda)^2}. \tag{41}$$

Defining $a = \frac{s}{m}$ gives the value of a in (26).

Using $a = \frac{s}{m}$, (22) reduces to

$$\lambda^m = \frac{\lambda}{1 + a^{1/2}}, \tag{42}$$

and (8) becomes

$$am = \frac{(1-\gamma)(1-m)}{1 - \gamma\left(1 - \dfrac{\lambda}{1+a^{1/2}}\right)},$$

which yields the value m^* in terms of the parameters given in (25). It is easily seen that $m^* \in (0,1)$.

Finally, (27) follows from substituting (22), (23) and (39) in (14).

(c) Since $\lambda = \lambda^s + \lambda^m$ and hence $\lambda \geq \lambda^s$, we have

$$\begin{aligned} \frac{1-\lambda}{\lambda^s} &\geq \frac{1-\lambda}{\lambda} = \frac{1}{\lambda} - 1 \\ \Rightarrow \frac{1-\lambda+\lambda^s}{\lambda^s} &\geq \frac{1}{\lambda} \\ \Rightarrow \frac{1-\lambda^m}{\lambda^s} &\geq \frac{1}{\lambda} \\ \Rightarrow \lambda &\geq \frac{\lambda^s}{1-\lambda^m}. \end{aligned}$$

Thus, whenever $\gamma\delta \geq \lambda$, (17) is satisfied.

Using (22) and (23) and letting $b = \frac{m^{1/2}}{s^{1/2}}$, (21) reduces to

$$\frac{\lambda}{b(b+1)} - \frac{\gamma\delta\lambda b + (1-\gamma\delta)\lambda}{(1-\gamma\delta)(b+1) + \gamma\delta\lambda b} - \gamma(1-\delta) = 0. \tag{43}$$

Which is a function of b and the parameters. Cross-multiplying and collecting terms, (43) can be written as

$$\psi(b) \equiv Ab^3 + Bb^2 + Cb + D = 0, \tag{44}$$

where

$$\begin{aligned}
A =& -\left[\gamma\delta\lambda + (1-\gamma\delta)(1-\delta)\gamma + (1-\delta)\gamma^2\delta\lambda\right] \ < \ 0,\\
B =& -\left[(1-\gamma\delta)\lambda + \gamma\delta\lambda + 2(1-\gamma\delta)(1-\delta)\gamma + (1-\delta)\gamma^2\delta\lambda\right] \ < 0,\\
C =& \gamma\delta\lambda^2 - (1-\delta)(1-\gamma\delta)\gamma,\\
D =& \lambda(1-\gamma\delta) \ > 0.
\end{aligned}$$

Since $\psi''(b) = 6Ab + 2B < 0$ for $b \geq 0$, ψ is strictly concave for $b \geq 0$. Moreover, $\psi(0) = D > 0$, and it is easy to verify that $\psi(1) < 0$. It follows that (44) has a unique solution for $b \geq 0$, and the solution occurs in the range $b \in (0, 1)$. Observe that in this range, $0 < m < s(m) \leq (1-m)$ which implies that $m \in (0, \frac{1}{2})$. Thus, (21) always has a unique positive solution which occurs for some $m \in (0, \frac{1}{2})$. Further, $\gamma\delta \geq \lambda$ is sufficient to ensure that this is a competitive equilibrium. □

Proof of Proposition 6.1. (a) Using equations (22) and (23) in (29),

$$W(m) = 1 - m - (1-\lambda)\, s(m). \tag{45}$$

$W(0) = \lambda$ and $W(m)$ must fall below λ for values of m in excess of $1-\lambda$: at least λ units of output must be produced in the economy for welfare to exceed λ. Since W is continuous in m, it attains a maximum over $[0, 1-\lambda]$. We now verify that the derivative of $W(m)$ is positive at $m = 0$.
From (45), we have

$$W'(m) = -1 - (1-\lambda)\, s'(m)$$

so that $W'(m) > 0$ if $s'(m)$ is negative and larger in absolute value than $1/(1-\lambda)$. Some tedious algebra yields

$$s'(m) = -\frac{(1-\gamma)\,(m^{1/2} + s^{1/2})^{\,2} + (1/2)\,\gamma\lambda s^{2/3} m^{-1/2}}{(1-\gamma)\,(m^{1/2} + s^{1/2})^2 + \gamma\lambda\, m^{1/2}\,(m^{1/2} + s^{1/2}) + (1/2)\,\gamma\lambda m^{1/2} s^{1/2}},$$

which is negative for all values of s and m between 0 and 1. Using the fact that $s \to 1$ as $m \to 0$, we find that $s'(m)$ increases without bound in absolute value as $m \to 0$. Thus, W attains an interior maximum and (30) follows from the first-order condition.

(b) For welfare in the monopoly intermediation equilibrium with m^* merchants, identified in Proposition 5.1(b), to be greater than that in the pure-search economy, we need

$$1 - m^* - (1-\lambda)s(m^*) > \lambda,$$

or,

$$\frac{1}{m^*} - \frac{s(m^*)}{m^*} > \frac{1}{1-\lambda}. \tag{46}$$

Recalling that we defined $a = \frac{s}{m}$ and substituting the value of m^* from (25), (46) reduces to

$$\gamma a(1-\lambda) > (1-\gamma)(1 + a^{1/2}). \tag{47}$$

Substituting $a = \frac{(1-\gamma\delta)^2}{(1-\lambda)^2}$ from equation (39) in (47) and simplifying yields condition (31). □

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