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A Spillover-Based Theory of Credentialism

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A Spillover-Based Theory of Credentialism*

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Abstract

I propose a model in which credentials, such as diplomas, are intrinsically valuable; a situation described as *credentialism*. The model overcomes an important criticism of signaling models by mechanically tying a worker's wages to their productivity. A worker's productivity is influenced by the skills of their coworkers, where such skills arise from an ability-augmenting investment that is made prior to matching with coworkers. A worker's credentials allow them to demonstrate their investment to the labor market, thereby allowing workers to match with high-skill coworkers in equilibrium. Despite the positive externality associated with a worker's investment, I show how over-investment is pervasive in equilibrium.

Keywords: Credentialism, Matching, Spillovers, Signaling
JEL Codes: D80, I20, J24, C78

1 Introduction

There are at least two interpretations of the dramatic increase in educational attainment experienced in the post-war period in many countries. The first is that it represents a natural response to the changing nature of work, including the implementation of new technologies that require high skilled labor. A second perspective asserts that educational attainment is fueled, in part, by *credentialism*, whereby workers have an intrinsic demand for credentials. Proponents of this perspective assert that "credentialing, not educating, has become the primary business of North American universities", that a "resume without

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one or more degrees from a respected institution will not be taken seriously enough even to be considered, no matter how able or informed the applicant may be", and that economic forces have made "credentials the object rather than the byproduct of educational achievement".[1]

These two interpretations are by no means mutually exclusive, and it is of great policy relevance to understand the relative contribution of each. Despite decades of intensive research in labor economics, the literature is yet to reach a solid consensus on the relative magnitudes of each explanation. I believe this is due, in no small part, to a tension that exists between the plausibility of credentialism on the one hand, and the absence of a compelling underlying theory of credentialism on the other. The development of such a theory is the primary objective of this paper.

Economists have typically used models of signaling (and/or screening) to understand credentialism.[2] These models focus on the worker-firm relationship, where credentials inherit an intrinsic value because they affect the beliefs, and therefore the willingness to pay wages, of the firm. The relevance of this mechanism has been placed in serious doubt in recent years, as criticisms have been levied on signaling theory's heavy reliance on an unrealistic assumption: that a worker's wages are largely insensitive to their performance. The criticism is summed up nicely as the idea "that companies rather quickly discover the productivity of employees who went to college, whether a Harvard or a University of Pheonix. Before long, their pay adjusts to their productivity rather than to their education credentials" (Becker (2006)). The assumption that wages are insensitive to performance is not only intuitively unpalatable, it is not in accordance with the evidence regarding the prominence of explicit performance pay (Lemieux *et al*. (2009)), nor with the estimates of the speed of employer learning (Lange (2007), and Arcidiacono *et al*. (2008)).

Although this type of argument is damaging for the practical relevance of traditional signaling theory in the labor context, it raises the issue of what we are to make of the widely expressed credentialist sentiment[3], and of the empirical evidence that has amassed over the decades in support of signaling.[4] Rather than dismiss the practical relevance of credentialism altogether, I propose an alternative model that addresses the above criticism.

[1]The first two quotes are from Jane Jacobs' book, *Dark Age Ahead* (Jacobs (2004) [pp. 44-45]), while the third quote is from David F. Labaree's book, *How to Succeed in School Without Really Learning* (Labaree (1997), [p. 253]).

[2]Seminal work of course includes Spence (1973), Spence (1974), Rothschild and Stiglitz (1976) and Arrow (1973).

[3]For example, as expressed in the sociological literature (e.g. Labaree (1997), Brown (1995), Collins (1979) and Berg (1970)) and in the popular press (e.g. Jacobs (2004)).

[4]For example, Tyler, Murnane, and Willet (2000), Lang and Kropp (1986), Bedard (2001), as well as others surveyed in Weiss (1995), Ferrer and Riddell (2002), and Lange and Topel (2006).

Specifically, I model an economy in which workers with heterogeneous abilities make skill-enhancing investments prior to joining N-worker firms. A worker's marginal productivity is shaped in part by this skill, but also in part by the skill of their coworkers. Once in a firm, each worker exerts labor effort in order to produce a verifiable, individual output. That is, there are no effort externalities, such as team production. The wage received by a worker is mechanically tied to their productivity (e.g. output-contingent wage contracts). This assumption is made in order to most cleanly address the above criticism of signaling models. To be sure, output-contingent wage contracts imply that a worker's productivity is effectively observed immediately by the firm. Thus, in contrast to signaling, credentials play no role in influencing beliefs within the *worker-firm* relationship. Instead, credentials play a role in shaping the composition of coworker groups, thereby exerting influence on the *worker-coworker* relationship.

The assignment of workers to firms - i.e. to other coworkers - unfolds on the basis of workers' investments (as opposed to skills). This feature reflects a situation in which skills are 'soft' - prohibitively difficult to describe, quantify, and communicate - relative to investments. For example, it is much easier to accurately communicate one's educational history on a resume than it is to communicate one's creativity, punctuality, or honesty. Given this, workers tend to be hired, and therefore exposed to coworkers, on the basis of their quantifiable investment history rather than on loose claims of being 'skilled'.

Since higher types make higher investments in equilibrium, all workers find those with higher investments more desirable as coworkers. All stable matches will therefore involve positive assortative matching on the investment dimension. Following the literature (e.g. Peters (2007a), Hoppe, Moldovanu and Sela (2009), Peters and Siow (2002)) I model the matching process by imposing positive assortative matching. This approach imposes disciplines the analysis; not only does it produce a stable matching outcome in equilibrium, it also imposes concrete consequences associated with making off-equilibrium investments.[5]

Given these basic ingredients, the mechanism is simple: credentials acquire an intrinsic value because they grant access to groups of higher-skilled coworkers. After establishing the existence and uniqueness of separating equilibria, I show how over-investment in education is pervasive in equilibrium. I then explore how changes in the degree of spillovers influences investment, output, and payoffs, and demonstrate that the results are robust

[5] For example, stability alone does not rule out a situation in which the very lowest types are convinced to make very high investments because of a concern that any lower investment would cause them to match with very undesirable coworkers. This is unreasonable because no such undesirable coworkers exist: workers of the lowest type are already the *most* undesirable workers in the matching market. See Peters (2007a) for an elaboration on this issue.

to a generalization of the function describing spillovers. In the discussion that follows, I demonstrate the robustness of the theory to a situation in which workers are able to replace undesirable coworkers prior to the commencement of production, as well as discuss how the theory could be tested against related theories. I conclude with the hope that the framework studied here provides a useful basis from which to evaluate the significance of credentialism in the labor market.

1.1 Relationship to the Literature

The possibility that education infers positive externalities has long been recognized, but standard treatments are 'global' in the sense that all agents in a given region benefit from some measure of aggregate educational investment (e.g. Lucas (1988), and Moretti (2004)). In contrast, spillovers in this model are 'local' in that they occur within the boundaries of the firm.[6] This approach opens the possibility that individuals influence who they match with by choosing appropriate investments. This is a central feature of the model and one that distinguishes it from models of matching with exogenous characteristics (e.g. two-sided matching; Becker (1973), Roth and Sotomayor (1990), and Legros and Newman (2002) and (2007), search and matching; Shimer and Smith (2000), Smith (2006), and Burdett and Coles (1997), and assignment; Sattinger (1993) and Kremer (1993)) and models with endogenous characteristics where potential partners are encountered randomly (Burdett and Coles (2001)). I show how this 'competition for coworkers' reverses the standard intuition that positive spillovers imply under-investment and that greater spillovers reduce investment.

The model is designed to capture the feature that there is often a disconnect between the characteristics used to form matches and the characteristics that potential partners are interested in. This aspect distinguishes the model from models of pre-marital investment, where an agent's attractiveness as a partner is completely captured by their observable investment (e.g. Peters (2007b), Peters and Siow (2002), Cole, Mailath, and Postlewaite (2001), Felli and Roberts (2002), Gall, Legros, and Newman (2006)).

An extreme form of this feature is prominent in a class of models in which the investment is unproductive and acts purely as a signal of an underlying characteristic (Hoppe, Moldovanu, and Sela (2009), Rege (2008), Damiano and Li (2007), and Bidner (2010)). The analysis here complements these models by exploring the consequences of productive investment. Although this type of extension is largely trivial in standard signaling models, it produces a number of additional insights and subtleties in the present context. For in-

[6] In this way, firms take on some key properties of 'clubs' in the sense of Buchanan (1965).

stance, I show that separating equilibria involve a continuous investment function only if investment is unproductive. Furthermore, productive investment allows one to drop the assumption of complementary interaction between workers, and therefore removes the central trade-off studied in the case of unproductive investments (investments are wasteful but facilitate efficient matching). However, a new trade-off emerges in which the over-investment inherent in a separating equilibrium is compared with the under-investment inherent in random matching. Finally, having productive investment allows one to study the impact of spillovers on aggregate variables such as output and inequality, whereas such impacts are absent when investment is unproductive since they are determined by the exogenous distribution of types.[7]

In a similar setting, Hopkins (2005) studies an economy in which workers make potentially productive investments prior to being paired with a firm. The papers share the feature that workers invest in order to secure a job in a 'good' firm, but differ in the account of why firms are heterogeneous. There, firms are exogenously heterogeneous, whereas here firms are ex-ante homogeneous and are only differentiated in equilibrium because of the human resources they house.

1.2 Foundations of Key Assumptions

Far from ignoring the relevance of the 'changing nature of work' interpretation of recent educational attainment, the theory developed here is built upon salient features of the modern workplace. Technical progress has not only increased the physical proximity of workers, but has also changed the types of tasks performed, the ways in which labor is managed, and the nature of the relationship a worker has with their coworkers. These changes have altered the types of skills that are valuable in the labor market. For instance, in compiling a list of these 'new basic skills', Murnane and Levy (1996) write:

> A surprise in the list of New Basic Skills is the importance of soft skills. These skills are called "soft" because they are not easily measured on standardized tests. In reality, there is nothing soft about them. Today more than ever, good firms expect employees to raise performance continually by learning from each other through written and oral communication and by group problem solving.

The earlier part of this quote inspires the assumption that skills are 'soft', whereas the latter part inspires the assumption of skill spillovers. Extending this theme, Autor, Levy, and Murnane (2003) and Levy and Murnane (2004) argue that computers can not easily

[7] Although, see Bidner (2008) and Cole, Mailath, and Postlewaite (1995) for models in which the underlying characteristics being signaled are endogenous.

perform non-routine tasks, and document the growing importance of tasks related to *expert thinking* ("solving problems for which there are no rule-based solutions"), and *complex communication* ("interacting with humans to acquire information, to explain it, or to persuade others of its implications for action"). A worker's ability to perform such tasks is reasonably sensitive to the skills of their coworkers: expert thinking is often a collaborative process that involves input from multiple people and perspectives, whereas complex communication is an inherently inter-personal process.

Skill spillovers may also arise as the result of changes in the way a workforce is managed. Ichniowski and Shaw (2003) describe recent trends in Human Resource Management, such as the importance of "pay-for-performance plans like gain-sharing or profit-sharing, problem-solving teams, broadly defined jobs, cross-training for multiple jobs, employment security policies and labor-management communication procedures". Many of these practices enhance the interconnectedness of employees. For example, Gant, Ichniowski, and Shaw (2002) provide evidence that worker productivity is improved following the introduction of innovative work practices because of stronger social capital developed between workers, and Drago and Garvey (1998) find that 'helping effort' is more readily expended by workers engaged in a large variety of tasks.

The model is presented in Section 2 and analyzed in Section 3. The model's robustness is examined in Section 4 and some comments on empirical implications are made in Section 5. Conclusions are drawn in Section 6. Proofs are contained in the appendix.

2 Model

There is a continuum of workers, indexed by $i \in [0,1]$. Each worker draws an ability, θ_i, from some absolutely continuous distribution, F, with support $\Theta \equiv [\theta_0, \theta_1]$ where $0 < \theta_0 \leq \theta_1 < \infty$. Having observed their ability, workers make an investment, $x \geq 0$, at a cost of cx. This investment produces a *skill* of:

$$s(x,\theta) = \theta \cdot g(x),$$

where g is a strictly increasing and concave function which satisfies the usual regularity conditions: $\lim_{x\to 0} g'(x) = \infty$ and $\lim_{x\to\infty} g'(x) = 0$. That is, all types face diminishing marginal returns to investment. The key assumption embodied in this specification is that the marginal return to investment is greater for higher types. This type of property arises naturally when ability is interpreted as an 'aptitude for comprehension' or a 'capacity to learn' (as opposed to an endowment of knowledge), and is commonly employed in the empirical literature.[8]

Given an investment of x_i, and a consumption of w_i, agent i simply obtains a payoff of $w_i - c \cdot x_i$. Of course, the amount available for an agent to consume depends upon their investment and the details of this relationship are now explored.

2.1 The Matching Stage

Workers take their investments to the matching stage, in which they are assigned to firms (and therefore coworkers). An assignment of workers to firms links each worker with exactly one firm, in such a way that each firm has either zero or N workers linked to it. The N workers linked to a particular firm are said to be *matched*. Following Peters (2007a), Hoppe, *et al*. (2009) and Peters and Siow (2002), I assume matching is *positive assortative on investment*. This means that for all pairs of workers (i, j), if $x_i \geq x_j$ then the lowest investment made by those workers matched with i is at least as great as the largest investment made by those workers matched with j. The definition of positive assortative matching is expanded upon below once we focus on separating equilibria.

2.2 The Production Stage

Consider a particular firm that hires N workers. Each worker produces a verifiable individual output, Y_i, by exerting effort. Specifically, given an effort of e_i and a productivity of y_i, worker i produces a total output of

$$Y_i = y_i \cdot e_i. \tag{1}$$

Importantly, production is individualistic in the sense that a worker's output is not affected by the effort choices of other workers: there is no team production, sabotage, production line technology, or anything like that. I assume that effort is inelastically supplied at one unit to avoid making the analysis needlessly complicated. However, it is useful to keep in mind the fact that there are no effort externalities with firms. Instead, the salience of coworkers comes solely from the feature that a worker's productivity, y_i, is derived in part from the skill of coworkers. In particular, I assume:

$$y_i = y(s_i, \overline{s}_{-i}) \equiv (1 - \phi) \cdot s_i + \phi \cdot \overline{s}_{-i}, \tag{2}$$

where $\overline{s}_{-i} \equiv (N-1)^{-1} \cdot \sum_{j \neq i} s_j$ is the average skill of i's coworkers within the firm, and $\phi \in (0, 1]$ parameterizes the degree of skill spillovers. This parameterization is convenient

[8]For example, multiplicative separability is implicitly assumed whenever a log wage equation is additively separable in schooling and ability. Evidence of investment-ability complementarity (i.e. a demonstration that returns to education are increasing in ability) is surveyed in Weiss (1995).

for at least three reasons. First, it ensures that spillovers are neutral in the sense that the average productivity across workers is independent of ϕ. Second, it stresses the point that complementarity interaction (i.e. y having a positive cross-partial derivative) is not necessary once we allow for productive investment. Third, it allows for a clean comparison with related literature (Peters and Siow (2002)). I offer a generalization of this form in section 4, where I show that separating equilibria under the more general formation are exactly the same as the separating equilibrium arising in the more specific setting.

In order to address the criticism of standard signaling models as cleanly as possible, I assume that workers are simply paid their output: $w_i = Y_i$.[9] This is equivalent to firms offering worker i a wage per unit of effort that is *mechanically* tied to i's marginal productivity of effort. Standard signaling models require that such contracts are infeasible.

2.3 Separating Equilibrium

From here, I focus on separating equilibria. Besides providing a natural comparison with the existing literature, I show that separating equilibria are more robust than pooling equilibria when workers are able to replace undesirable coworkers, as explained in Section 4.2 below. A separating equilibrium is a pair of objects - a strictly increasing investment function and a return function. The investment function is required to reflect optimizing behavior given the return function, and the return function is required to be consistent with the distribution of equilibrium investments and positive assortative matching. Each of these aspects are considered in turn.

2.3.1 Optimality

If a worker of type θ invests x and has coworkers with an average skill of $\bar{s}$, then their payoff is:

$$U(x, \theta, \bar{s}) = (1 - \phi) \cdot s(x, \theta) + \phi \cdot \bar{s} - c \cdot x. \tag{3}$$

These preferences are depicted, in the left panel of Figure 1, in $(x, \bar{s})$ space as U-shaped indifference curves. The U-shape comes from the fact that at relatively low investment levels the marginal return to investment is greater than the marginal cost, implying that a marginally higher investment raises the payoff - which requires a lower average coworker

[9]Since there are no effort externalities, this contract maximizes the total surplus available within a firm. That is, firms would have no incentive to offer wage contracts that were also conditioned on the output of others in the firm (e.g. team performance contracts) since such contracts can not increase the total available surplus, and may actually distort effort decisions. Competition from other firms would ensure that the entire output is returned to the worker in the form of wages.

skill to retain indifference. The reverse is true for relatively high investment levels. In addition, the fact that the marginal return to investment is higher for higher types implies a single-crossing property (also indicated in the left panel of Figure 1, where $\theta' > \theta$).[10]

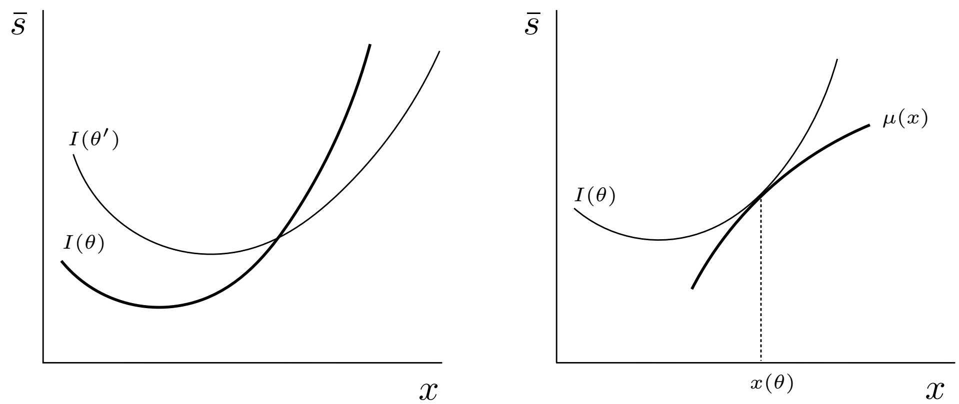

Figure 1: Preferences and Optimality

When making investment decisions, workers understand that positive assortative matching will imply that the average skill of their coworkers will be sensitive to their investment. Workers conjecture that an investment of x leads them to be assigned to a firm in which the average skill of their coworkers is $\mu(x)$. Given μ, workers choose their investments optimally:

$$\sigma(\theta) = \arg\max_{x \geq 0} \; U(x, \theta, \mu(x)). \tag{4}$$

The optimality of investments is indicated in the right panel of Figure 1. Given the return function, μ, workers simply choose the investment level that places them on the highest possible indifference curve.

2.3.2 Matching

Given that all others invest according to a strictly increasing function of type, $\sigma : \Theta \to R_+$, positive assortative matching implies that firms are segregated in equilibrium. That is, all firms hire workers of at most one type. Thus we can speak of 'type θ' firms, and note that feasibility implies that type θ firms exist with a density of $F'(\theta)/N$. When considering the implication of making a particular investment, agent i takes this segregation as given as realizes that they will end up at a segregated firm for any investment they choose. The type of firm that a worker is assigned to will of course depend on the investment made.

[10] The U-shapedness and single-crossing properties are easily verified by consulting the explicit expression for an indifference curve: $I(\theta) = (1/\phi) \cdot [u + c \cdot x - (1 - \phi) \cdot s(x, \theta)]$.

To get at this, let the distribution of investment be given by G. Positive assortative matching requires that an investment of x leads i to be matched with a segregated firm containing workers of type $t(x)$, where the position of x in the investment distribution equals the position of $\sigma(t(x))$ in the investment distribution. Given that investment is a strictly increasing function of type, the latter equals the position of $t(x)$ in the type distribution. Thus, positive assortative matching requires that for each x, $t(x)$ satisfy

$$G(x) = F(t(x)). \tag{5}$$

To see that this uniquely defines $t(x)$ for each $x \in \mathbb{R}$, consider the distribution of investment, G, as depicted in Figure 2. Since the lowest investment arising in equilibrium is $\sigma(\theta_0)$, we have that $G(x) = 0$ for all x lower than this amount. Similarly, the highest investment arising in equilibrium is $\sigma(\theta_1)$, so we have $G(x) = 1$ for investments higher than this amount. If we consider an investment that happens to arise in equilibrium, say $x = \sigma(\theta)$, then we know that $G(x) = F(\theta)$ since investment is a strictly increasing function of type. Finally, if the investment function exhibits a discontinuity, say at θ_d, then G will be flat on an interval above or below (or both) $\sigma(\theta_d)$ as indicated.

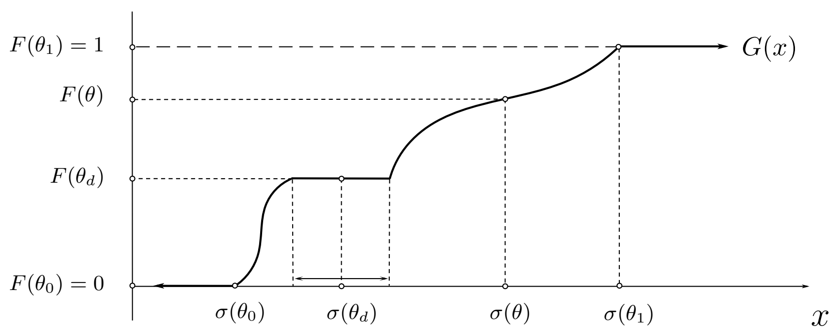

Figure 2: Positive Assortative Matching

For any value of x we see that there exists a unique value of θ such that $G(x) = F(\theta)$. This value of θ, by definition, is $t(x)$. This has immediate consequences for choosing an equilibrium investment: if $x = \sigma(\theta)$, then $t(x) = \theta$. That is, if a worker invests an amount that happens to be an equilibrium investment for some type, then the worker is matched with coworkers of that type. However, positive assortative matching also has implications for off-equilibrium investments. For instance, investing anything less than the lowest equilibrium investment implies that a worker is to be matched with workers of the lowest type. Similarly, investing more than the highest equilibrium investment implies a worker is to be matched with workers of the highest type. The only remaining possibility is investing in the neighborhood of a discontinuity, such as the region around $\sigma(\theta_d)$ as depicted in Figure 2. In this case, positive assortative matching requires that workers be matched with coworkers of type θ_d.

Positive assortative matching therefore places restrictions on the equilibrium return function; by investing x a worker matches with coworkers of type $t(x)$, and since such coworkers invest $\sigma(t(x))$ in equilibrium, they have a skill of $s(\sigma(t(x)), t(x))$. Thus, given a strictly increasing investment function, σ, we say that the return function, μ, satisfies positive assortative matching if

$$\mu(x) = s(\sigma(t(x)), t(x)), \tag{6}$$

for all $x \in \mathbb{R}$, where $t(x)$ satisfies (5).

Definition 1. *A separating equilibrium is a strictly increasing investment function, $\sigma : \Theta \to R_+$, and a return function, $\mu : R_+ \to R_+$, such that i) σ is optimal given μ, and ii) μ satisfies positive assortative matching given σ. That is, equations (4) and (6) hold.*

2.4 Benchmarks

Before turning to the derivation of separating equilibria, it is useful to present some benchmark investment levels. First, consider the investment made by type θ if they took as given the fact that they were to be matched with coworkers who were also of type θ. This investment level is called the *Nash* investment for type θ workers as it represents the Nash equilibrium of the game in which N workers of the same type simultaneously choose an investment level and obtain payoffs according to (3). By taking the average skill of coworkers as fixed, workers effectively face a flat return function. Workers then choose the investment that places them on the highest indifference curve that lies on this return function. An investment is a Nash investment for some type if the skill generated by this investment coincides with the average skill initially conjectured. This is represented by point N in Figure 3, where $x_N(\theta)$ is the Nash investment for type θ.

The Nash investment is relevant because workers of the lowest type, θ_0, must make their Nash investment in equilibrium. The standard reasoning applies: they can do no worse than be matched with type θ_0 coworkers and therefore must find their Nash investment to be a best response. Thus:

$$\sigma(\theta_0) = x_N(\theta_0). \tag{7}$$

The Nash investment is inefficient, and as expected, represents an under-investment because each of the N workers are better off if each of them invest a little more. Naturally, this arises because the workers do not take into account the fact that their investment

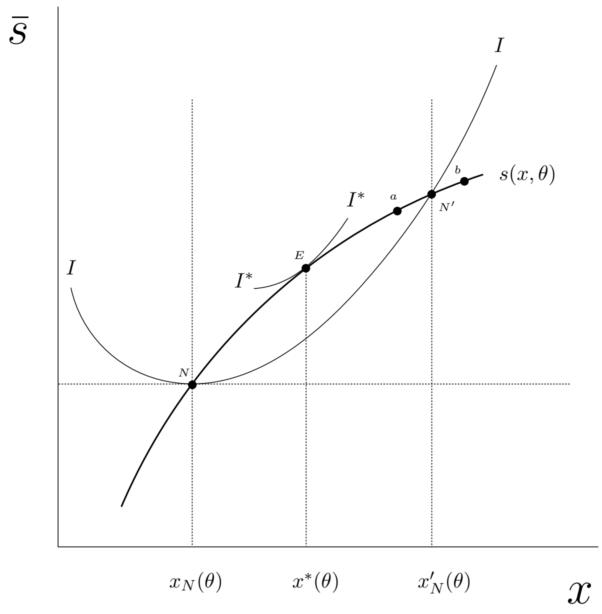

Figure 3: Benchmark Investments

benefits the others. The *efficient* investment for workers of type θ can be found as that investment that places workers on the highest indifference curve on the skill production function. This is represented by point E in Figure 3, where $x^*(\theta)$ is the efficient investment for type θ.

As workers invest amounts increasingly greater than the efficient level, their payoffs decrease. At some point there will be an investment level that gives workers a payoff equal to that arising with Nash investment. This is referred to as the *upper-Nash* investment, and is represented by point N' in Figure 3, where $x'_N(\theta)$ is the upper-Nash investment for type θ.

The significance of the upper-Nash investment level is that it places restrictions on the types of investments that can be made by workers with types marginally above θ_0. Specifically, it can not be the case that $\lim_{\theta\to\theta_0}\sigma(\theta) < x'_N(\theta_0)$. If this were true, e.g. at a point like a in Figure 3, then workers of type θ_0 would prefer to deviate by mimicking a type marginally above θ_0 (since a lies on a higher indifference curve). Similarly, it can not be the case that $\lim_{\theta\to\theta_0}\sigma(\theta) > x'_N(\theta_0)$. If this were true, e.g. at a point like b, then types marginally above θ_0 would prefer to deviate by mimicking type θ_0 workers (since b lies on a lower indifference curve). Thus, we are left with the conclusion that

$$\lim_{\theta\to\theta_0}\sigma(\theta) = x'_N(\theta_0). \tag{8}$$

With this minimal structure, we already have the following.

Result 1. *If investment is productive, then there does not exist a separating equilibrium with a continuous investment function. Specifically, the equilibrium investment function must be discontinuous at θ_0.*

Simply, productive investment implies $x_N(\theta) < x'_N(\theta)$.[11] But if this is true, (7) and (8) together imply that the investment function can not be continuous at θ_0. This result suggests a way in which equilibrium behavior is qualitatively affected by considering productive investment.

2.5 Solving

Having placed restrictions on investment behavior for the lowest types, we are ready to explore how all other types invest. Since $t(z)$ is a non-decreasing function, so too is μ. Thus, μ is almost everywhere differentiable. In fact, μ must be differentiable at all investments made by interior types.[12] Thus, for all $\theta \in (\theta_0, \theta_1)$, the first-order condition for optimal investment implies that:

$$(1-\phi)\cdot s_x(x,\theta) + \phi \cdot \mu'(x) = c \tag{9}$$

at $x = \sigma(\theta)$. Since (6) holds uniformly for $x \in (x'_N(\theta_0), \sigma(\theta_1))$, we can differentiate both sides of (6) to get:

$$\mu'(x) = s_x(x, t(x)) + s_\theta(x, t(x)) \cdot t'(x), \tag{10}$$

which, combined with the fact that $\theta = t(x)$ at $x = \sigma(\theta)$, implies that the inverse investment function must satisfy:

$$t'(x) = \frac{c - s_x(x, t(x))}{\phi \cdot s_\theta(x, t(x))} = \frac{c - t\cdot g'(x)}{\phi \cdot g(x)} \equiv \Gamma(t, x). \tag{11}$$

Thus, we have that $\sigma(\theta) = t^{-1}(\theta)$ for $\theta \in (\theta_0, \theta_1)$, where t is a solution to the initial values problem defined by $t'(x) = \Gamma(t,x)$ and $(\theta_0, x'_N(\theta_0))$, where the initial conditions follow from the analysis of the previous section.

Proposition 1. *A unique separating equilibrium exists. In this equilibrium, $x(\theta_0) = x_N(\theta_0)$, and $\sigma(\theta) = t^{-1}(\theta)$ for $\theta \in (\theta_0, \theta_1]$, where t is a solution to the initial values problem defined by $t'(x) = \Gamma(t,x)$ and $(\theta_0, x'_N(\theta_0))$.*

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[11] This follows from the observation that $x_N(\theta) < x^*(\theta)$ and $x^*(\theta) < x'_N(\theta)$ if (and only if) investment is productive.

[12] Intuitively, if μ we not differentiable at $\sigma(\theta)$ then there would exist a neighborhood of types around θ that would also prefer to invest $\sigma(\theta)$.

The unique separating equilibrium is illustrated from two vantage points in Figure 4. The left panel depicts the equilibrium return function in $(x, \bar{s})$ space. The return function is flat for low investments, reflecting the fact that such investments all lead to a match with type θ_0 coworkers. There comes a point at which the return function makes a discrete jump (i.e. at $x = x'_N(\theta_0)$). This reflects the fact that investing above this point allows workers to match with some higher type who also makes a higher investment (and is therefore of a higher skill). The return function increases continuously from this point, until the highest equilibrium investment is reached. Beyond that point, the return function is once again flat. Importantly, each type finds their indifference curve to be tangential to the return function at unique point, and this point lies on that type's skill production function (which is required by (6)).

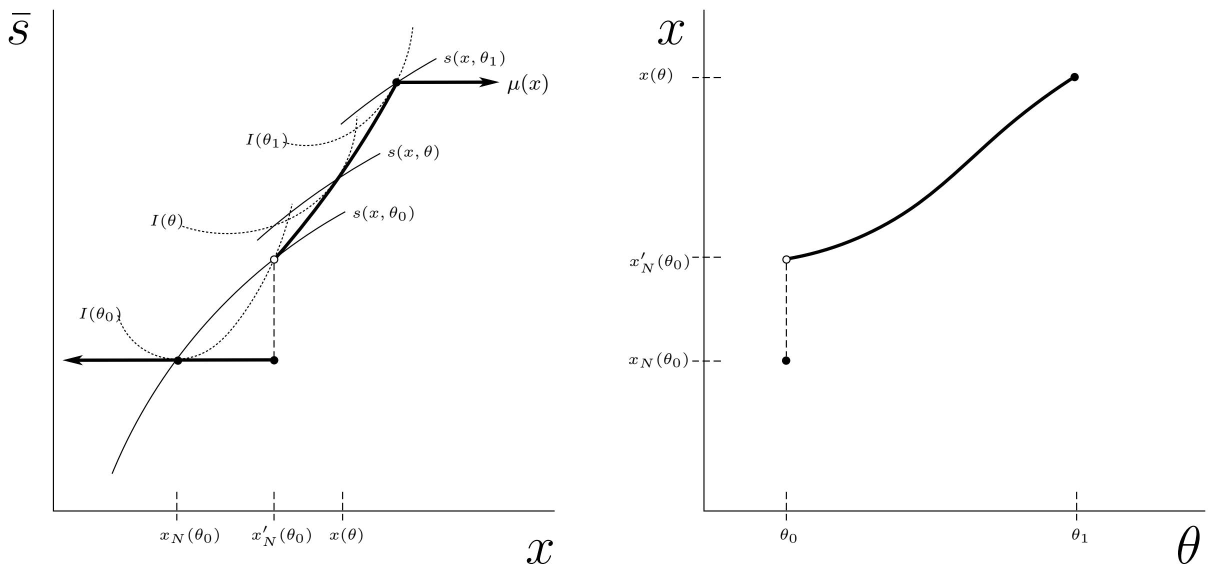

Figure 4: The Separating Equilibrium

The right panel of Figure 4 depicts the equilibrium investment function in (θ, x) space. The function has a discontinuity at θ_0, but is strictly increasing and continuous otherwise. The continuous section is the inverse of the solution to the initial values problem. The separable specification for the skill production function is convenient, since it allows us to derive an explicit solution for the initial values problem (see appendix for a proof):

$$t(x) = \left[t_0 \cdot g(x_0)^{\frac{1}{\phi}} + \frac{c}{\phi} \int_{x_0}^{x} g(z)^{\frac{1-\phi}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}}, \tag{12}$$

where $t_0 = \theta_0$ and $x_0 = x'_N(\theta_0)$.

3 Analysis

3.1 Efficiency

A higher investment not only raises a worker's productivity, but also the productivity of their coworkers. In this sense, investment entails a positive externality. If workers' exposure to spillovers were insensitive to their investment, as in models of global spillovers such as Lucas (1988), then this naturally leads to underinvestment. Arguments along these lines support the public subsidization of education. However, when investments allow workers to join higher skilled firms, this implication no longer holds (for almost all workers).

Proposition 2. *Almost all workers over-invest in equilibrium:* $\sigma(\theta) > x^*(\theta)$ *for* $\theta \in \Theta/\theta_0$.

Notice that the under-investment intuition still holds for workers of the lowest type, since they always make their Nash investment. But such workers exist in zero measure, and in this sense, over-investment is *pervasive* in equilibrium. This observation points out one value in modeling heterogeneous types, because the efficiency properties of equilibrium are quite different if there is a single (and therefore, lowest) type.

The intuition for over-investment is clarified by observing that workers perceive the net marginal return to investment to be the sum of two components:

$$\frac{\partial}{\partial x}\{y(s(x,\theta),\mu(x)) - cx\} = [(1-\phi)\cdot s_x - c] + [\phi \cdot \mu'(x)]. \tag{13}$$

The first bracketed term is the 'private' component and the second is the 'credential' component. Given segregation, the efficient investment for type θ workers arises when workers perceive the net marginal return to be the net marginal *social* return, $s_x - c$. The private component alone clearly produces too little incentive to invest, since the return suffers a shortfall of $\phi \cdot s_x$. This arises because the private return does not take into account the external benefit on coworkers. To explore the extent to which the credential component provides added incentive to invest, note that (6) implies

$$\mu(x) = s(x, t(x)) \tag{14}$$

for all $x = \sigma(\theta)$ for some $\theta \in (\theta_0, \theta_1)$. Since this holds uniformly in x on $(\sigma(\theta_0), \sigma(\theta_1))$, we can differentiate both sides of (14) and multiply by ϕ to get

$$\phi \cdot \mu'(x) = \phi \cdot s_x + \phi \cdot s_\theta \cdot t'(x). \tag{15}$$

Thus, the credential component contributes the required shortfall in the net marginal return, $\phi \cdot s_x$. However, it contributes even more than this when $\phi \cdot s_\theta \cdot t'(x)$ is positive. But, this term is always positive (when $\phi > 0$ and $s_\theta > 0$) because of simple fact that higher types make higher investments. Thus, the model not only captures the feature that credentials have an intrinsic value, as captured by $\mu(x)$, but also verifies that this leads to over-investment.

The fact that agents invest efficiently in models of pre-marital investment with complete information (such as Peters and Siow (2002)) can be seen partly as a consequence of the fact that 'skill' in those models depends only on investment, and not on type (so that $s_\theta = 0$).

3.2 The Effect of Spillovers

Spillovers are 'neutral' in equilibrium in the sense that if we changed ϕ while holding investment fixed, no worker's productivity would change. Given this, the following result points to a mechanism through which spillovers raise productivity without making existing skills more productive *per se*.

Proposition 3. *An increase in spillovers increases the equilibrium investment, and therefore productivity, of almost all workers (those with types $\theta \in \Theta/\theta_0$).*

Just as in the case where exposure to spillovers is insensitive to investment, higher spillovers leads to lower welfare. This is not surprising given that spillovers are the source of inefficiency in both constructions, but the mechanisms are quite different. For example, here spillovers induce a *negative* relationship between output and welfare.

The following result indicates that spillovers are an indispensable component of the mechanism producing over-investment.

Proposition 4. *Investment becomes efficient as spillovers disappear: $\sigma(\theta) \to x^*(\theta)$ as $\phi \to 0$.*

Investment is clearly efficient when there are no spillovers, so this result establishes a certain continuity with that scenario. A corollary of this is that the investment function becomes continuous in the limit as spillovers disappear.

4 Robustness

4.1 Skill Production Function Generalization

One may be concerned that the linear specification of the skill production function is responsible for the main results. In this section I argue that this is not true by generalizing this function to the CES class, and demonstrating that separating equilibria are completely unaffected.

Proposition 5. *Generalize the skill production function to:*

$$y(s, \bar{s}) = [(1-\phi) \cdot s^{\rho} + \phi \cdot \bar{s}^{\rho}]^{\frac{1}{\rho}}, \tag{16}$$

where $\rho \in (-\infty, 1]$. *For any* ρ, *a unique separating equilibrium exists. Furthermore, it is exactly the same equilibrium for all* ρ.

The linear case considered above corresponds to $\rho = 1$, and this result tells us that (literally) nothing would change if we had instead used a form that incorporates complementarity from the CES class. For example, the Cobb-Douglas and Leontief specifications correspond to $\rho \to 0$ and $\rho \to -\infty$, respectively. This highlights the point that it is the weighting parameter, ϕ, and not the substitution parameter, ρ, that is important. The key properties of CES forms that are responsible for this result are $y(z, z) = z$, $y_s(z, z) = (1-\phi)$, and $y_{\bar{s}}(z, z) = \phi$ for all z.

4.2 The Observability of Skill

The explanation for credentialism developed here avoids an important criticism of signaling models by allowing for output-contingent wages. This can be thought of as effectively capturing an extreme version of employer learning, since firms need not commit to wage payments prior to observing productivity. Since the mechanism relies on the fact that workers can not feasibly be sorted into firms on the basis of their skill, it is reasonable that one may foster a degree of skepticism regarding the plausibility of the proposed explanation on the basis that a worker's skill may also be learned quickly.

If only *the firm* learns their workers' skills then nothing would change. This is simply because firms are only interested in skill insofar as it affects productivity, and output contingent wages allows firms to act as if they observed productivity. On the other extreme, if *the market* observes worker skill then the environment is more appropriately modeled as in the spirit of Peters and Siow (2002), Peters (2007b), or Cole *et al*. (2001) - e.g. have matches formed positive assortative along the skill dimension. Investment loses its credential quality in this setting, thereby invalidating the explanation proposed here.

However, the assumption that the market observes worker skills is highly demanding. For instance, a worker's skill can not be inferred from information on investment and output alone, since coworkers exert an influence on output.[13] Even if matching were positive

[13] Even if coworker skill can be inferred from equilibrium behavior, many equilibria can potentially be supported because of the flexibility in assigning off-equilibrium beliefs.

assortative on some aggregate of investment and output (e.g. in a repeated setting) investment still retains a credential quality since this helps raise observed output via spillovers. Thus, a situation in which skill is observed by the market seems quite implausible because of the quite detailed information on all of a worker's coworkers that is required. This implausibility is only heightened if output is observed with noise or subject to external shocks.

4.3 Option to Re-match

The static nature of the model may arouse suspicion that the results are reliant on the fact that workers are, in effect, bonded to their coworkers. That is, the model does not capture the reality that the "credential is not a passport to a job, as naive graduates sometimes suppose. It is more basic and necessary: a passport to consideration for a job", Jacobs (2004). That is, one may conjecture that the extent of over-investment will be reduced if workers can be rejected from firms if they are found to not be of a suitable skill. Intuitively, the possibility of rejection lowers incentives to invest since a high-skill firm is no longer bound to accept a masquerading worker.

To explore this, I now consider a simple extension of the above model in which matching occurs in two stages. Potential firms are formed in the first stage according to positive assortative on investment as before. These are only *potential* firms because of two reasons - i) a small proportion of potential firms are exogenously dissolved at the end of the first stage, and ii) workers can decide to reject potential coworkers for an arbitrarily small cost, $\kappa > 0$. A worker is rejected from the firm if all of their coworkers decide to reject them. In the second stage, all workers from dissolved firms are once again matched positive assortative on investment, some of who will fill the positions left by rejected workers. The rejected workers are not rematched (e.g. they are stigmatized). All other workers remain with their first-stage matches. A worker's coworkers are those that they are matched with at the end of the second stage.

If we hold fixed the original equilibrium investment function, then no worker has an incentive to eject any of their coworkers since an ejected worker would simply be replaced by an identical worker in the second stage, but a positive cost is incurred. However, the point is that the option to eject undesirable coworkers could potentially alter investment incentives. Specifically, the over-investment observed in the original equilibrium could be drastically reduced because of lowered incentives to masquerade as a higher type; masqueraders will be ejected by their potential coworkers for small enough switching costs.

To explore this possibility, consider a firm that consists of workers that invest x and are of type $t(x)$. Suppose that workers in this firm, which have a skill of $s(x, t(x))$, are confronted with a deviating worker who instead has a skill of $\tilde{s}$. Each of the non-deviating workers therefore have an average coworker skill of

$$\bar{s}(\tilde{s}, x) \equiv \frac{1}{N-1} \cdot \tilde{s} + \frac{N-2}{N-1} \cdot s(x, t(x)), \tag{17}$$

and will almost surely be re-matched with a coworker with skill $s(x, t(x))$ if they reject the deviating worker. Thus, workers will not reject the deviating worker as long as $\tilde{s} \geq s^*(x)$, where $s^*(x)$ satisfies

$$U\left(\sigma(t(x)), t(x), \bar{s}(s^*(x), x)\right) - \kappa = U\left(\sigma(t(x)), t(x), s(x, t(x))\right). \tag{18}$$

That is, workers will not punish the deviation if the induced change in average coworker skill does not justify the switching cost.

We are interested in determining the conditions under which the original equilibrium remains an equilibrium in this new setting. To this end, suppose all other workers are investing according to the original equilibrium σ. The optimal investment problem now is

$$\max_{x \geq 0} \; U(x, \theta, s(x, t(x))), \;\; \text{s.t.} \;\; s(x, \theta) \geq s^*(x). \tag{19}$$

If we ignore the constraint, then the solution to this problem is, by definition, $\sigma(\theta)$. Thus, $\sigma(\theta)$ must still be an equilibrium if we can be sure that the constraint is satisfied at $x = \sigma(\theta)$:

$$s(\sigma(\theta), \theta) \geq s^*(\sigma(\theta)). \tag{20}$$

But, from the definition of $s^*(x)$ and the fact that $\kappa > 0$, we know that $s^*(x) < s(x, t(x))$. Using the fact that $x = \sigma(\theta)$ in this inequality implies that the constraint indeed does always hold. I conclude that the original equilibrium remains an equilibrium when workers can reject undesirable potential coworkers at an arbitrarily small cost.

Intuitively, suppose workers invest according to some arbitrary strictly increasing function, $\tilde{\sigma}$. For any positive switching cost there is a set of investments lying above $\tilde{\sigma}(\theta)$ that they can make without being rejected by the resulting match. If $\tilde{\sigma}$ is to be an equilibrium investment function, then it must ensure that workers have no incentive to marginally increase their investment. Workers are never rejected when making lower investments, and therefore there must also be no incentive to decrease investment. The constraint therefore has no effect on these marginal conditions, and therefore if $\tilde{\sigma}$ is an equilibrium it must also be an equilibrium in the case where the constraint is ignored. Since σ is the unique separating equilibrium, it is also the unique separating equilibrium in this extended setting.

From another perspective, allowing workers to reject potential coworkers places no additional consequences on cutting investment. For example, if a worker of type $\theta > \theta_0$ cuts their investment, they will find that they are matched with coworkers of a lower type. These coworkers will be glad to accept such a deviating worker because the deviator's type is higher than expected for their investment level. If workers of the lowest type could be convinced to invest more than their Nash investment, then the upper-Nash investment decreases and marginally higher types find that they can invest less without being matched with θ_0 workers. This logic applies to all higher types also, and in the end all workers invest less (i.e. more efficiently). However, type θ_0 workers can not be convinced to invest more than their Nash level: if they did, there would exist a profitable downward deviation by cutting investment by a small enough amount that rejection is avoided. Thus, the capacity to observe the skill of potential coworkers and reject them if desired has no effect on the behavior of the lowest types, and therefore also has no effect on any higher type.

A conclusion from this exercise is that the results are not being driven by the static nature of the model: i.e. the same results emerge in settings where workers agents are not forever bonded to their coworkers. Notice that this would not be true of pooling equilibria, where potential coworkers all make the same investment but will generally be of different types. For sufficiently low switching costs, there will always be incentives to reject the lowest type workers within a group of potential coworkers. For this reason, it seems that the focus on separating equilibria is appropriate.

5 Empirical Aspects

There are many existing explanations for the observed positive relationship between education and earnings. For this reason, it is important to be clear about the empirical implications of this model, and how these differ from existing explanations.

A central prediction is that there is a *causal* relationship between credentials and i) wages, and ii) productivity, for any given actual investment level. This is because credentials determine which group of coworkers a worker belongs to, thereby influencing the worker's productivity via spillovers. Neither of these causal relationships are present in human capital models, since wages depend on productivity, which in turn depend on a worker's actual investment.[14] The latter relationship is not present in signaling models; although credentials influence beliefs and therefore wages, productivity is unaffected by credentials. This suggests an approach that can, in principal, allow us to discriminate be-

[14]Similarly, neither of the relationships are operational in assignment models with observed skill (e.g. such as Kremer (1993), Sattinger (1993), Peters and Siow (2002)) for similar reasons.

tween theories: treat one of two otherwise similar groups of workers with pure credentials (e.g. phony BA degrees), and examine the resulting differences in wages and productivity between groups.[15]

This empirical prediction is also shared by models i) in which investment is unproductive, as in Hoppe, *et al*. (2009), and ii) in which firms are *ex-ante* heterogeneous, as in Hopkins (2005). These explanations can potentially be disentangled by using worker-firm matched data, as this allows for the estimation of firm fixed effects.

The issue of whether education is productive is illuminated by examining the extent to which educational attainment remains significant after controlling for firm fixed effects. The issue of whether firms are ex-ante heterogeneous is illuminated by examining the extent to which coworker characteristics can explain the firm fixed effects (as opposed to characteristics such as capital per worker).

Observing a panel of matched worker-firm data allows us to go one step further and identify worker fixed effects. In this setting, Abowd et al. (1999) demonstrate how average worker effects are much more important than average firm effects in explaining industry effects. Explicitly analyzing the effect of coworkers, Lopes de Melo (2008) reports that workers' fixed effects are not correlated with firm fixed effects, but are positively correlated with the average fixed effect of their coworkers. This type of evidence is supportive of the notion that it is the quality of employees that really sets firms apart. The latter result in Lopes de Melo (2008) provides valuable evidence of the type of labor market sorting stressed in this model.

6 Conclusions

The model of credentialism developed here is based on two key assumptions regarding skill. First, that there skill spillovers in the workplace; a worker's productivity is influenced by the skill of their coworkers. Second, that skills are soft information; it is infeasible for workers to match on the basis of their skill. Instead, by allowing matches to form on the basis of investment, I propose that investment inherits a credential quality as it acts as a ticket to desirable coworkers. Despite similarities to models with unproductive invest-

[15]This is the type of variation used in Tyler, Murnane, and Willett (2000), who exploit the fact that U.S. states have different test score requirements in order to be awarded the GED credential. Their key finding - that the GED credential causes higher wages - is supportive of standard signaling models, but is also consistent with the mechanism proposed here. Data on productivity were not available, so it is difficult to distinguish the two interpretations of the data. However, it is interesting to note that the credential effects do not appear until five years after the test was taken. This weighs against the signaling interpretation to the extent that one finds it implausible that discrepancies between productivity and wages can last five years.

ment, I show that the equilibrium investment function must be discontinuous. Despite the positive spillover, I show that over-investment is pervasive in equilibrium. Despite the neutral nature of spillovers, I show that investment and productivity are increasing in the degree of spillovers (but welfare is decreasing).

The model addresses a highly relevant and potent criticism of standard signaling models by eliminating any role for learning on the firms' part through the mechanical binding of wages to productivity. Thus, in understanding credentialism, the model developed here stresses the importance of workplace spillovers rather than an inability to write productivity-contingent wage contracts.

APPENDIX

A Proofs

Proof of Proposition 1

Proof. The proof establishes that i) the necessary conditions for an equilibrium investment function - namely, that $\sigma(\theta_0) = x_N(\theta_0)$, $\sigma(\theta) = t^{-1}(\theta)$ for $\theta \in (\theta_0, \theta_1)$ where t solves the initial values problem, along with the optimal behavior of type θ_1 - are satisfied by a unique function, and ii) this function constitutes an equilibrium investment function.

The solution to the initial values problem is unique, and strictly increasing for $x \geq x_0$. Γ is a linear ordinary differential equation, and therefore a unique solution is guaranteed to exist for *any* set of initial values (t_0, x_0) such that $x_0 > 0$. The analysis of the previous section revealed that the initial conditions must be $(\theta_0, x'_N(\theta_0))$, and since $x'_N(\theta_0) > 0$, a unique solution to the initial values problem exists. Furthermore, this solution is strictly increasing for $x > x_0$ as required because the denominator of Γ is always positive and the numerator is an increasing function at all x such that $s_x(x, \theta) < c$ (since s is concave in x). Thus, it is sufficient to check that this holds at $x = x_0$. But this is guaranteed by the fact that $x'_N(\theta_0) > x^*(\theta_0)$.

The investment of the highest type: Finally, we need to determine the investment made by workers of the highest type, θ_1. The fact that σ is strictly increasing already implies that $\sigma(\theta_1) \geq t^{-1}(\theta_0)$, where t is the solution to the initial values problem just described. These workers are free to choose a higher investment but will not be compensated by being matched with a higher type (since they are already matched with coworkers of the highest possible type). Such an upward deviation is profitable if and only if $\phi \cdot s_x(x, \theta_1) > c$ at $x = t^{-1}(\theta_0)$. However this can not be the case because $\Gamma(t, x) > 0$ for all $t \geq t_0$ (including $t = \theta_1$). Thus, it must be the case that $\sigma(\theta_1) = t^{-1}(\theta_0)$. This type of argument can be applied to verify that there can never be an equilibrium investment function with a discontinuity at some $\theta \in (\theta_0, \theta_1)$.

First-order conditions are sufficient: Given that a unique function satisfies the necessary conditions, I establish that the function indeed represents an equilibrium by showing that the first-order necessary conditions for the workers' maximization problem are also sufficient. Note that no worker prefers to invest above $\sigma(\theta_1)$ or below $\sigma(\theta_0)$. This follows from single crossing, since θ_1 does not prefer to invest any higher than $\sigma(\theta_1)$, and θ_0 does not prefer to invest any lower than $\sigma(\theta_0)$. No worker prefers to invest in $(x_N(\theta_0), x_N'(\theta_0)]$: holds by construction θ_0 workers, and can not be true for a worker of some higher type because they would prefer to invest marginally above $x_N'(\theta_0)$. It is therefore sufficient to verify that workers do not prefer to choose some other equilibrium investment. This is proved in two parts; first that equilibrium investments are locally optimal, and second that μ can not cross a worker's equilibrium indifference curve (implying that local optima are also global optima). First, we show that the second-order sufficient condition holds at equilibrium investments:

$$(1-\phi) \cdot g''(\sigma(\theta)) \cdot \theta + \phi \cdot \mu''(\sigma(\theta)) < 0 \tag{21}$$

for all $\theta \in (\theta_0)\theta_1)$. The first-order condition can be written as

$$(1-\phi) \cdot g'(\sigma(\theta)) \cdot \theta + \phi \cdot \mu'(\sigma(\theta)) = c, \tag{22}$$

which holds uniformly in $\theta \in (\theta_0, \theta_1)$. Differentiating this w.r.t. θ, dividing by $\sigma'(\theta)$, and re-arranging gives:

$$(1-\phi) \cdot g''(\sigma(\theta)) \cdot \theta + \phi \cdot \mu''(\sigma(\theta)) = -(1-\phi) \cdot \frac{g'(\sigma(\theta))}{\sigma'(\theta)} < 0 \tag{23}$$

for all $\theta \in (\theta_0, \theta_1]$.

Second, suppose μ crosses the equilibrium indifference curve of some type above that type's equilibrium investment. Since μ is differentiable on the set of equilibrium investments, the slope of μ at such a crossing point must be greater than the slope of the type's equilibrium indifference curve. But this then implies that the indifference curve that is tangent to μ at the crossing point belongs to a lower type, contradicting strictly increasing investments. The analogous argument holds for the case where μ crosses the equilibrium indifference curve of some type below that type's equilibrium investment. □

Proof of Proposition 2

Proof. Efficient investments satisfy $s_x(x^*(\theta), \theta) = c$. Using this in the expression for $t'(x)$ evaluated at $x = \sigma(\theta)$ gives:

$$t'(\sigma(\theta)) = \frac{s_x(x^*(\theta), \theta) - s_x(\sigma(\theta), \theta)}{\phi \cdot s_\theta(\sigma(\theta), \theta)}. \tag{24}$$

The fact that investment is a strictly increasing function implies the left side is positive, which implies that the numerator of the right side is positive (since the denominator is always positive). But the concavity of s in x means that $s_x(x^*(\theta), \theta) > s_x(\sigma(\theta), \theta)$ implies $\sigma(\theta) > x^*(\theta)$ as required. □

Proof of Proposition 3

Proof. An increase in ϕ raises the Nash investment of the lowest type. This lowers their Nash payoff, thereby increasing the upper-Nash investment. Thus, higher spillovers increase $x'_N(\theta_0)$. Furthermore, Γ is strictly decreasing in ϕ. Together these facts imply that the solution to the initial values problem, $t(x)$, is shifted to the right when ϕ increases (i.e. it starts at a point to the right, and never crosses the original solution since the slope is everywhere flatter). Since $t(x)$ is the inverse investment function for $\theta \in \Theta/\theta_0$, we have that such types invest strictly more when spillovers are higher. □

Proof of Proposition 4

Proof. First, the Nash investment approaches the efficient investment as ϕ goes to zero. As the efficient investment produces the greatest possible payoff, the upper-Nash investment also approaches the efficient investment as ϕ goes to zero. Thus, the lowest type invests efficiently in the limit.

Suppose that for some θ, we had $\sigma(\theta) > x^*(\theta)$ in the limit as $\phi \to 0$. Since both functions are continuous, this must also hold for set of neighboring types. But then from (24) there must be a (non-empty) set of types for which $t'(\sigma(\theta))$ goes to infinity as $\phi \to 0$ (since the denominator goes to zero while the numerator goes to a positive value). Thus, by making ϕ sufficiently close to zero we can make the investments of two distinct types arbitrarily close to each other. This is not possible in equilibrium since the lower of the types will always have an incentive to mimic the higher of the types since a marginally higher investment would produce a discrete increase in coworker skill. □

Proof of Proposition 5

Proof. I show that the analysis above continues to apply under this generalization since ρ changes neither the Nash investment, nor the initial values problem (i.e. neither the upper-Nash investment, nor Γ).

The Nash investment satisfies:

$$y_s(s_N(\theta), s_N(\theta)) \cdot s_x(x_N(\theta), \theta) = c, \tag{25}$$

where $s_N(\theta) \equiv s(x_N(\theta), \theta)$. But since $y_s(z,z) = (1-\phi)$ for all z, we have that $x_N(\theta)$ is unaffected by ρ.

The upper-Nash investment satisfies

$$y(s'_N(\theta), s'_N(\theta)) - c \cdot x'_N(\theta) = y(s_N(\theta), s_N(\theta)) - cx_N(\theta), \tag{26}$$

where $s'_N(\theta) \equiv s(x'_N(\theta), \theta)$. But since $y(z,z) = z$ for all z, both sides are independent of ρ and we conclude that $x'_N(\theta)$ is also independent of ρ.

Finally, the first-order condition for optimal investment is

$$y_s(s,s) \cdot s_x + y_{\bar{s}}(s,s) \cdot [s_x + s_\theta \cdot t'(x)] = c. \tag{27}$$

which uses the fact that $\mu'(x) = s_x + s_\theta \cdot t'(x)$. But since $y_{\bar{s}}(z,z) = \phi$ for all z, and recalling that $y_s(z,z) = (1-\phi)$ for all z, the differential equation describing $t'(x)$ will be independent of ρ. □

Derivation of Solution to Initial Values Problem

Proof. Let $a(x) \equiv \frac{-g_x(x)}{\phi g(x)}$ and $b(x) \equiv \frac{c}{\phi g(x)}$, so that we can write:

$$t'(x) = a(x) \cdot t(x) + b(x).$$

The solution to this first-order linear differential equation is:

$$t(x) = \left[K + \int^x b(z) \cdot \exp\left(-\int^z a(t)dt\right) dz\right] \cdot \exp\left(\int^x a(z)dz\right),$$

where K is a constant that adjusts so that the initial condition is satisfied and the notation $\int^x f(z)dz$ represents the indefinite integral of $f(x)$. Note that $\int^x a(z)dz = -(1/\phi)\ln(g(x))$, so that $\exp(\int^x a(z)dz) = g(x)^{-1/\phi}$, which lets us write:

$$\begin{aligned} t(x) &= \left[K + \int^x b(z) \cdot g(z)^{\frac{1}{\phi}} dz\right] \cdot g(x)^{-\frac{1}{\phi}} \\ &= \left[K + \frac{1}{\phi}\int^x c \cdot g(z)^{\frac{1-\phi}{\phi}} dz\right] \cdot g(x)^{-\frac{1}{\phi}} \end{aligned}$$

For any given $\{t_0, x_0\}$, we know that:

$$K = t_0 \cdot g(x_0)^{\frac{1}{\phi}} - \frac{c}{\phi}\int^{x_0} g(z)^{\frac{1-\phi}{\phi}} dz,$$

which gives the stated result once substituted. □

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