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Mutual Fund Style, Characteristic-Matched Performance Benchmarks and Activity Measures: A New Approach*

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Abstract

We propose a new approach for measuring mutual fund style and constructing characteristic-matched performance benchmarks that requires only portfolio holdings and two reference portfolios in each style dimension. The characteristic-matched performance benchmark literature typically follows a bottom-up approach by first matching individual stocks with benchmarks and then obtaining a portfolio's excess return as a weighted average of the excess returns on each of its constituent stocks. Our approach is fundamentally different in that it matches portfolios and benchmarks directly. We illustrate our approach using portfolio holdings of 1183 fund managers over the period 2002-2009. We characterize the cross-section of fund manager styles and show how average style changes over time. The tracking error volatilities of our characteristic-matched benchmarks compare favorably with those of existing methods. Using our benchmarks we explore the link between activity and performance.

Keywords: Performance Measurement; Tailored Benchmark; Characteristic Matching; Size Profile; Growth Profile; Activity; Excess Return
JEL Classification: C43, G11, G23.

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1. Introduction

The measurement of style and performance of managed portfolios is of fundamental importance to the investment industry. Accurate measurement of style allows investors to obtain their desired exposures to particular investment styles.[1] It also is a prerequisite to reliable performance measurement, since the performance of a fund manager should be judged relative to an appropriate style benchmark. For example, a manager focusing on small-cap stocks should be evaluated against a small-cap benchmark, while a manager that follows a growth investing style should have performance compared to a growth benchmark.

Matching of funds and benchmarks based on declared style (as is done for example by Morningstar) can be problematic since declared style does not necessarily match observed style (see Sharpe, 1988, 1992, Carhart, 1997, Brown and Goetzmann, 1997, diBartolomeo and Witkowski, 1997, Chan, Chen and Lakonishok, 2002).

Two main approaches have been used in the literature for ensuring that funds are matched with appropriate benchmarks. The first approach is regression based using past returns. Its point of departure is the capital asset pricing model (CAPM). A tailored benchmark is obtained by taking a linear combination of the risk-free and market benchmark rate of returns, with the weight for each fund determined by its value of beta. The difference between a portfolio's actual performance and its tailored benchmark is referred to as Jensen's alpha (see Jensen, 1967, 1969). Jensen's alpha is calculated as the intercept of an excess return regression. This basic model can be extended by including additional

[1]A pension plan, for example, has to consider the coherence of its overall strategy. It may wish to follow a blend strategy, and try and achieve this goal by giving half the managers value mandates and the other half growth mandates. To achieve the overall desired strategy, it is important to monitor the managers' observed style relative to their assigned mandates. Alternatively, a firm may hope that by hiring a number of managers it is diversifying its overall portfolio. In this case, the objective is to ensure that the styles of the managers are sufficiently diversified. The more accurately a firm can measure what its managers are doing, the better able it will be to achieve its desired overall strategy.

non-CAPM factors such as size, valuation and momentum (see Fama and French, 1992, 1996, and Carhart, 1997). Sharpe (1988, 1992) proposes an alternative regression model based on asset class factors. He uses this to determine the effective mix of a portfolio in terms of the underlying asset classes. A tailored benchmark can then be constructed as a weighted average of these asset class benchmarks, with the weights determined from the regression equation.

Regression-based methods only require data on fund performance, and factors such as size and the price-to-book ratio. To estimate the regression equation, however, a fairly long time series of observations is required. This can be problematic since the style of a fund and the factor parameters could change over time.

The second approach requires portfolio holdings data, but has the advantage that it requires only short time horizons. Expositors of this approach include Daniel, Grinblatt, Titman and Wermers (1997), henceforth DGTW, Chan, Chen and Lakonishok (2002), henceforth CCL, and Chan, Dimmock and Lakonishok (2009), henceforth CDL, (see also Kothari and Warner, 2001). These authors match characteristics at the level of individual stocks. Stocks are first sorted in each style dimension, and then divided into quintile blocks. In two dimensions (say size and value-growth), this generates a total of 25 blocks. CDL show that the resulting benchmarks can be quite sensitive to the way these sorts are done (e.g., whether or not growth is sorted independently of size). DGTW construct a market-cap-weighted benchmark portfolio from each block, while CCL construct equal-weighted portfolios from each block. CDL also show that the choice between market-cap and equal weighting can significantly affect the results. Each stock in a portfolio is matched with the benchmark portfolio with the most similar style characteristics. The excess return on each stock is measured by the difference between its return and its benchmark's return. An overall performance benchmark for a portfolio is then obtained by taking the weighted mean of these excess returns, where each stock is weighted by its dollar share of the portfolio. Each fund, therefore, has its own distinct performance benchmark.

In this paper we develop a new variant on this second approach that differs from DGTW, CCL and CDL in a number of respects. First, our matching of characteristics is done at the level of portfolios, rather than at the level of individual stocks. Second, our benchmarks are defined in a continuous style space, and hence we achieve a direct match between portfolios and benchmarks. By contrast, the DGTW-CCL-CDL approach matches stocks with one of a finite number of benchmark portfolios, and hence these matches are more approximate. Finally, our method is computationally simple and intuitive. For each dimension our method requires only two reference portfolios, which can often be taken off the shelf from index providers. For example, the Russell 3000 Value and Russell 3000 Growth indices could be used in the value-growth dimension.[2]

Our approach begins by determining a portfolio's location in style space. We propose a new formula for doing this which can be applied in each dimension.[3] Our new style measures are of interest in their own right. However, we then show how they can be used to construct performance benchmarks that are tailored specifically to the style of each fund in each style dimension. Our approach has further extensions. First, it can be used to disentangle the effects of different style dimensions on performance. For example, we can assess the impact of changes in size holding the value-growth style fixed. Second, it generates new measures of activity of funds relative to their tailored benchmarks.[4]

We apply our methodology to a US mutual fund data set consisting of 1183 manager portfolios over the period 2002-2009. In the size and value-growth style dimensions, we

[2]By assets benchmarked, Russell's style indices account for more than 98 percent of market share for US equity growth and value oriented products (see Russell Investments, 2008). For illustrative purposes therefore we construct our off-the-shelf portfolios from Russell indices.

[3]CCL also propose a method for locating portfolios in style space. However, their style-space is not continuous. Also, they do not use it to construct performance benchmarks.

[4]There is some ambiguity in the literature regarding the use of the term 'activity'. It is sometimes used to refer to turnover. Here, however, by 'activity' we mean departures from passive tracking of a benchmark.

illustrate the cross-section diversity of styles across managers and the gradual shift in style towards growth stocks from 2002 to 2009. We also show how the size profiles of the Russell 1000 Growth and Russell 1000 Value indices, and the growth profile of the Russell 1000 and index, change over time. We then compute tracking error volatilities for a few variants on our basic characteristic-matched benchmarking method and for the preferred method in CDL. At least some of our benchmarks are competitive in terms of tracking error volatility with CDL's preferred benchmark. Using our new benchmarks, we also explore the link between activity and performance. Our main findings are summarized in the conclusion.

2. Style Profiles and One-Dimensional Tailored Performance Benchmarks

2.1. Style profiles

Portfolios can be classified by style in a number of dimensions. Two of the most commonly considered styles are size and value-growth. We define a style profile $P(w)$ as a function that maps an N-dimensional portfolio w into one style dimension to generate an ordinal ranking of portfolios according to that particular style. For example, suppose $P(w^1) > P(w^2)$. It follows that in this style dimension, portfolio w^1 attains a higher score than portfolio w^2.

Let lg and sm denote two reference portfolios in a given style dimension, such that $P(sm) < P(lg)$ (i.e., lg and sm are abbreviations for reference large and small portfolios in this dimension). For example, in the size dimension lg could be the Russell 3000 portfolio, and sm an equally weighted variant on the Russell 3000 portfolio.[5] In the value-growth dimension, lg could be the Russell 3000 Growth portfolio, and sm the Russell 3000 Value portfolio, etc.

[5]Henceforth although we will frequently refer to the Russell 3000 Equal-Weighted index, it should be noted that the Russell Investment Group does not actually publish such an index. However, this index and its underlying portfolio can easily be constructed.

We propose here a new class of style profiles for a portfolio w that take the following form:

$$P(w) = \frac{\sum_{n=1}^{N}[(lg_n - sm_n)(w_n - sm_n)]}{\sum_{n=1}^{N}(lg_n - sm_n)^2}, \tag{1}$$

where w_n, lg_n and sm_n denote, respectively, the value shares of stock n in portfolios w, lg and sm, and N denotes the total number of stocks in the universe under consideration. It follows that by construction $\sum_{n=1}^{N} w_n = \sum_{n=1}^{N} lg_n = \sum_{n=1}^{N} sm_n = 1$.

From inspection of (1) it can be seen that $P(sm) = 0$ and $P(lg) = 1$. The style profile of a portfolio w is measured relative to these two points of reference. For example, in the size dimension if sm=the Russell 3000 Equal-Weighted portfolio and lg=the Russell 3000 portfolio, then $P(w) < 0$ implies that smaller cap stocks have the largest holdings in w.

Our class of style profiles, therefore, evaluate the style of each portfolio w without ever needing to provide a formal measure of style. Where desired, however, such foundations can be provided. For example, in the size dimension a natural absolute measure of style might be the following:

$$S^*(w) = \sum_{n=1}^{N} w_n mcw_n, \tag{2}$$

where mcw_n denotes the market cap share of stock n, and w_n its value share in portfolio w as noted above. If we make the substitution $lg_n = mcw_n$ and $sm_n = ew_n$ (where $ew_n = 1/N$ are the value shares of an equal-weighted portfolio) in (1), then it can be shown that $P(w)$ is a monotonic increasing function of $S^*(w)$ (see Appendix A).

In the value-growth dimension, one possible absolute measure of style might be the following:

$$G^*(w) = \prod_{n=1}^{N} (p_n/b_n)^{w_n}, \tag{3}$$

where p_n and b_n denote stock n's price and book value per share respectively.[6] Here we

[6]One attractive feature of this formula is that its reciprocal is the market-cap-weighted geometric mean

focus exclusively on price-to-book ratios as a measure of value. Broader measures of value that incorporate earnings, dividends or sales are considered by CDL. Our methodology can be used to construct such broader measures. We return to this issue later.

Making the substitution $sm_n = ew_n = 1/N$ and $lg_n = gw_n$ in (1), where

$$gw_n = \frac{\ln(p_n/b_n)}{\sum_{m=1}^{N} \ln(p_m/b_m)}, \tag{4}$$

we obtain a growth profile that is a monotonic function of $G^*(w)$ and hence has a firm absolute foundation (again see Appendix A).

2.2. One-dimensional tailored benchmark portfolios

We can now construct a new portfolio $\hat{w}$ that is a linear combination of the two reference portfolios lg and sm with respective weights derived directly from w's style profile as follows:

$$\hat{w}_n = P(w) \times lg_n + (1 - P(w)) \times sm_n, \quad \text{for } n = 1, \ldots, N, \tag{5}$$

where $\hat{w}_n$ denotes the value share of stock n.

The portfolio $\hat{w}$ has the important property that it has the same style profile as w itself. That is, $P(w) = P(\hat{w})$. This can be verified by substituting (5) into (1). The portfolio $\hat{w}$ can therefore be interpreted as a tailored benchmark portfolio for w. It is a benchmark because it is itself a linear combination of the two reference portfolios. It is tailored since by construction it has the same style profile as w.

of the book-to-price ratios. That is $1/G^*(w) = \prod_{n=1}^{N} (b_n/p_n)^{w_n}$. Hence the ranking of portfolios does not depend on whether we focus on price-to-book or book-to-price ratios. In this latter case, the growth profile rises as one moves to the left along the growth line. This property is useful since price-to-book and book-to-price ratios contain the same information.

2.3. Tailored benchmark portfolios as distance minimizers

$P(w)$ as defined in (1) and $\hat{w}$ as defined in (5) can also be derived as the solutions to a least squares minimization problem. Let $\tilde{w}$ denote a portfolio formed by taking linear combinations of the reference portfolios lg and sm as follows:

$$\tilde{w}_n = \lambda \times lg_n + (1 - \lambda) \times sm_n \quad \text{for } n = 1, \ldots, N. \tag{6}$$

We can measure the distance between portfolio w and $\tilde{w}$ as follows:

$$D = \sqrt{\sum_{n=1}^{N} (w_n - \tilde{w}_n)^2}. \tag{7}$$

This distance measure is analogous to Euclidean distance.

Substituting (6) into (7), the following expression is obtained:

$$D = \sqrt{\sum_{n=1}^{N} [w_n - (\lambda \times lg_n + (1 - \lambda) \times sm_n)]^2}. \tag{8}$$

Minimizing D with respect to λ yields the following first order condition:

$$\frac{\partial D}{\partial \lambda} = \frac{\sum_{n=1}^{N} [(-mcw_n + ew_n)(w_n - \lambda mcw_n - (1 - \lambda) ew_n]}{\sqrt{\sum_{n=1}^{N} [w_n - (\lambda \times lg_n + (1 - \lambda) \times sm_n)^2]}} = 0. \tag{9}$$

Solving (9) we obtain that $\hat{\lambda} = P(w)$ as defined in (1). Hence it follows that $\hat{w}$ is the linear combination of the reference portfolios lg and sm that most closely approximates in a least squares sense the portfolio w. Again this indicates that it is an appropriate tailored benchmark portfolio for assessing the performance of w.

2.4. Distance as a measure of activity

Returning to the distance measure in (7), if we replace $\tilde{w}$ with $\hat{w}$, we obtain a measure $\hat{D}$ of the distance between a portfolio w and its tailored benchmark $\hat{w}$ as follows:

$$\hat{D} = \sqrt{\sum_{n=1}^{N} (w_n - \hat{w}_n)^2}. \tag{10}$$

We interpret $\hat{D}$ as a measure of a fund manager's activity. A variant on this index (without the square-root sign) has been used previously by Kacperczyk, Sialm and Zheng (2005), henceforth KSZ, and Brands, Brown and Gallagher (2005), henceforth BBG, in a different context.[7] Their variant replaces $\hat{w}$ with the market portfolio. They then compare each portfolio with the market portfolio proxy (e.g., the Russell 3000), and interpret D as a measure of concentration. That is, a portfolio is deemed to have zero concentration if it is identical to the market portfolio. The more it differs from the market portfolio, the more concentrated it is deemed to be relative to the market. KSZ only consider concentration over 10 industry classes, while BBG also calculate it at the level of individual stocks. Both, however, only compare portfolios with the market portfolio, and not with tailored benchmark portfolios.

In our context, $\hat{D}$ is better interpreted as a measure of the activity of a portfolio in a particular style dimension. An active portfolio can be distinguished by its deviation from its passive tailored style benchmark. Funds with a low value of $\hat{D}$ are almost style passive in that dimension in the sense that all the fund manager is effectively doing is allocating money across two passive reference portfolios. Our activity measure $\hat{D}$ is in spirit probably closest to the active share measure of Cremers and Petajisto (2009), henceforth CP. The

[7]The inclusion of the square-root sign in the distance measure allows activity to be compared across portfolios with varying values of N (the stock universe). Otherwise, $\hat{D}$ will tend to systematically fall as N rises.

CP activity measure differs from ours, however, in two important respects. First, it takes 19 reference portfolios (such as the S&P500, Russell 3000, Wilshire 5000), using each in turn as the benchmark and selects for a particular portfolio whichever has the lowest activity measure. In contrast, we construct benchmarks that are specifically tailored to have the same style characteristics as each portfolio, and which minimize the activity measure over a continuous K dimensional style space. Second, the CP activity measure optimizes using mean absolute deviation, while we use least squares.

Later in the paper, we explore the relationship between activity as we have defined it and performance.

2.5. One-Dimensional tailored performance benchmarks

A tailored performance benchmark $Z(w)$ for w in a particular style dimension is obtained from the reference portfolios as follows:

$$Z(w) = P(w) \times R(lg) + [1 - P(w)] \times R(sm), \tag{11}$$

where $R(lg)$ and $R(sm)$ denote respectively the return on holding the lg and sm reference portfolios in that period (e.g., $R(lg) = 1.15$ implies a 15 percent rise in the value of the lg portfolio). The excess return XR of w is then obtained by subtracting the benchmark return Z from the actual return R as follows:

$$XR(w) = R(w) - Z(w).$$

An interesting implication of (11) is that the tailored performance benchmark can be derived directly from the reference indices (which when judiciously chosen should be publicly available) and the portfolio's style profile $P(w)$. It follows that the calculation of excess returns is computationally quite simple.

3. Two-Dimensional Tailored Performance Benchmarks

3.1. Using three reference portfolios

Although the focus here is on two dimensions, the methodology that follows generalizes in a straightforward way to higher dimensions. We distinguish between two style dimensions S and G (e.g., size and value-growth). The style profile of a portfolio w in style dimension S is now denoted by $S(w)$, and in dimension G by $G(w)$.

Three reference portfolios are all that are required to construct tailored benchmark portfolios in S-G space (as long as these portfolios span the space). If we have two reference portfolios in each dimension, the system is overdetermined and hence we can drop one of the portfolios. The resulting tailored benchmark, however, is not invariant to the choice of which portfolio is dropped. One solution to this problem is always to use an equal-weighted portfolio in each dimension as one of the reference portfolios. In the empirical analysis in the next section we experiment with two alternative sets of three reference portfolios. The first set consists of the Russell 3000, Russell 3000 Equal-Weighted, and Russell 3000 Growth portfolios. The second set consists of the Russell 3000, Russell 3000 Equal-Weighted and gw_n portfolios, where the latter is defined in (4). This second set of reference portfolios is of particular interest since it generates benchmark portfolios $\hat{w}$ such that $S^*(\hat{w}) = S^*(w)$ and $G^*(\hat{w}) = G^*(w)$, where S^* and G^* denote absolute measures of size and growth as defined in (2) and (3) respectively.

We use the notation mcw to denote a market-cap-weighted portfolio such as the Russell 3000, ew an equal-weighted portfolio such as the Russell 3000 Equal-Weighted, and gw a growth-weighted portfolio such as the Russell 3000 Growth. Substituting $mcw = lg$ and $ew = sm$ into (1), by construction, we obtain that $S(mcw) = 1$ and $S(ew) = 0$. Similarly, substituting $gw = lg$ and $ew = sm$ in (1), we obtain that $G(gw) = 1$ and $G(ew) = 0$. It is necessary, however, to compute the values of $G(mcw)$ and $S(gw)$. An empirical example for 2008Q4 of the locations of the mcw =Russell 3000, ew =Russell 3000 Equal-Weighted, and gw =Russell 3000 Growth portfolios in S-G space over the Russell 3000 stock universe is provided in Figure 1(a).

Insert Figure 1 Here

To determine the proportions in which our three reference portfolios must be combined to generate a portfolio with the same S and G profiles as w, it is necessary to solve the following system of two simultaneous equations:

$$\begin{pmatrix} S(ew) \\ G(ew) \end{pmatrix} + \hat{\mu}_1 \begin{pmatrix} S(mcw) - S(ew) \\ G(mcw) - G(ew) \end{pmatrix} + \hat{\mu}_2 \begin{pmatrix} S(gw) - S(ew) \\ G(gw) - G(ew) \end{pmatrix} = \begin{pmatrix} S(w) \\ G(w) \end{pmatrix}. \tag{12}$$

The only unknowns in (12) are $\hat{\mu}_1$ and $\hat{\mu}_2$. The terms $S(w)$ and $G(w)$ on the righthand side of (12) denote the S and G profiles of the portfolio w calculated from (1).

Making the substitutions $S(mcw) = G(gw) = 1$ and $S(ew) = G(ew) = 0$, (12) simplifies to the following:

$$\hat{\mu}_1 + \hat{\mu}_2 S(gw) = S(w), \tag{13}$$

$$\hat{\mu}_1 G(mcw) + \hat{\mu}_2 = G(w). \tag{14}$$

These equations yield the following solutions for $\hat{\mu}_1$ and $\hat{\mu}_2$:

$$\hat{\mu}_1 = \frac{S(w) - S(gw)G(w)}{1 - S(gw)G(mcw)}, \tag{15}$$

$$\hat{\mu}_2 = \frac{G(w) - G(mcw)S(w)}{1 - S(gw)G(mcw)}. \tag{16}$$

A tailored benchmark portfolio $\hat{w}$ that has the same S and G profile as the portfolio w can now be derived as follows:

$$\hat{w}_n = \hat{\mu}_1 mcw_n + \hat{\mu}_2 gw_n + (1 - \hat{\mu}_1 - \hat{\mu}_2) ew_n, \quad \text{for } n = 1, \ldots, N, \tag{17}$$

with $\hat{\mu}_1$ and $\hat{\mu}_2$ derived from (15) and (16). That $S(\hat{w}) = S(w)$ can be verified as follows:

$$S(\hat{w}) = \hat{\mu}_1 S(mcw) + \hat{\mu}_2 S(gw) + (1 - \hat{\mu}_1 - \hat{\mu}_2) S(ew) = \hat{\mu}_1 + \hat{\mu}_2 S(gw),$$

since $S(mcw) = 1$ and $S(ew) = 0$. Now substituting for $\hat{\mu}_1$ and $\hat{\mu}_2$ from (15) and (16), we obtain that:

$$S(\hat{w}) = \left[\frac{S(w) - S(gw)G(w)}{1 - S(gw)G(mcw)} \right] + \left[\frac{G(w) - G(mcw)S(w)}{1 - S(gw)G(mcw)} \right] S(gw) = S(w).$$

In the same way it can be shown that $G(\hat{w}) = G(w)$.[8]

The one-dimensional least-squares optimization result for $\hat{w}$ generalizes to higher dimensions. In two dimensions, we define the distance D between the portfolio w and a portfolio $\tilde{w}$ formed by taking linear combinations of the three reference portfolios as follows:

$$D = \sqrt{\sum_{n=1}^{N} (w_n - \tilde{w}_n)^2},$$

where now

$$\tilde{w}_n = \mu_1 \; mcw_n + \mu_2 \; gw_n + (1 - \mu_1 - \mu_2) ew_n. \tag{18}$$

[8] Although we do not do it here, this approach could also be used to construct a growth style benchmark that is matched to multiple dimensions of a portfolio's value-growth characteristics, such as book value, earnings, dividends, sales, etc as recommended by CDL.

Hence we can rewrite D as follows:

$$D = \sqrt{\sum_{n=1}^{N} [w_n - \mu_1 \, mcw_n - \mu_2 \, gw_n - (1 - \mu_1 - \mu_2) ew_n]^2}. \tag{19}$$

Differentiating with respect to μ_1 and μ_2, we obtain first order conditions which on rearrangement are identical to (13) and (14) (see Appendix B). Hence the solutions for μ_1 and μ_2 in this least-squares minimization problem are the same as those given in (15) and (16) above. It follows that $\tilde{w} = \hat{w}$, and hence that $\hat{w}$ is the linear combination of the three reference portfolios that in a least-squares sense most closely approximates the portfolio w, as well as having the same S and G profile as w.

3.2. Using four reference portfolios

Suppose now we have four reference portfolios. The first issue is to ensure that these reference portfolios are mutually compatible. A good example of incompatible portfolios is provided by the Russell 3000, Russell 3000 Value and Russell 300 Growth portfolios. Since these portfolios are not linearly independent (the Russell 3000 portfolio equals the sum of the Russell 3000 Growth and Russell 3000 Value portfolios), this combination of reference portfolios will generate a singular matrix in the system of simultaneous equations below. In the empirical comparisons in the next section, therefore, we use $mcw =$Russell 1000, $ew =$Russell 3000 Equal-Weighted, $vw =$Russell 3000 Value and $gw =$Russell 3000 Growth as our reference portfolios, where our S profiles are obtained by setting $mcw = lg$ and $ew = sm$ in (1) and our G profiles by setting $gw = lg$ and $vw = sm$ in (1). An empirical example of the locations of $mcw =$Russell 1000, $ew =$Russell 3000 Equal-Weighted, $vw =$Russell 3000 Value and $gw =$Russell 3000 Growth portfolios in S-G space is provided in Figure 1(b).

As it stands, the system for constructing tailored benchmark portfolios is overdetermined when there are four reference portfolios. This means that we can generate an infinite number of benchmark portfolios that will be tailored to have the same S and G profiles as a portfolio w.

To choose between them, we can again use least squares optimization. A linear combination of the four reference portfolios takes the following form:

$$\tilde{w}_n = \mu_1 mcw_n + \mu_2 gw_n + \mu_3 vw_n + (1 - \mu_1 - \mu_2 - \mu_3) ew_n, \quad \text{for } n = 1, \ldots, N \tag{20}$$

We now define the distance D between the portfolio w and $\tilde{w}$ as follows:

$$D = \sqrt{\sum_{n=1}^{N} [w_n - \mu_1\ mcw_n - \mu_2\ gw_n - \mu_3\ vw - (1 - \mu_1 - \mu_2 - \mu_3) ew_n]^2}. \tag{21}$$

Differentiating with respect to μ_1, μ_2 and μ_3, the first-order conditions yield a system of three simultaneous equations in three unknowns (see Appendix C). The solutions for μ_1, μ_2 and μ_3 are obtained as follows:[9]

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 1 & S(gw) & S(vw) \\ SG(mcw) & 1 & SG(vw) \\ SV(mcw) & SV(gw) & 1 \end{pmatrix}^{-1} \begin{pmatrix} S(w) \\ SG(w) \\ SV(w) \end{pmatrix}. \tag{22}$$

Substituting the solutions for μ_1, μ_2 and μ_3 obtained from (22) into (20), we obtain the portfolio $\hat{w}$ formed from linear combinations of the four reference portfolios mcw, ew, gw and vw that most closely approximates in a least-squares sense the portfolio w.

It is shown in Appendix D that the size and growth profiles of $\hat{w}$ are the same as those of w (i.e., that $S(\hat{w}) = S(w)$ and $G(\hat{w}) = G(w)$).

[9] The terms $SG(mcw)$, $SG(vw)$, $SV(mcw)$ and $SV(gw)$ in (22) are defined in Appendix C.

4. Disentangling Size and Growth Effects

Changing the parameter λ in (6) may change the style profile in other dimensions as well as the one on we are focusing. For example, the Russell 3000 and Russell 3000 Equal-Weighted portfolios differ in both their S and G profiles in Figure 1(a). To obtain a pure measure of the impact of changes in S on performance holding G fixed, we need to take linear combinations of two reference portfolios with the same G profile. Such reference portfolios can easily be constructed using a variant on our basic method.

Returning to the three reference-portfolio setting, we seek two portfolios $\tilde{w}^0$ and $\tilde{w}^1$ that are linear combinations of our three reference portfolios and have the following properties:

$$S(\tilde{w}^0) = 0, \quad G(\tilde{w}^0) = \breve{G}, \quad S(\tilde{w}^1) = 1, \quad G(\tilde{w}^1) = \breve{G}.$$

The coordinates of $\tilde{w}^0$ and $\tilde{w}^1$ in size-growth space are illustrated in Figure 2.

Insert Figure 2 Here

Setting $S(\tilde{w}^0) = 0$ and $G(\tilde{w}^0) = \breve{G}$ in (15) and (16), the portfolio $\tilde{w}^0$ can be calculated as follows:

$$\mu_1^0 = \frac{-S(gw)\breve{G}}{1 - S(gw)G(mcw)},$$

$$\mu_2^0 = \frac{\breve{G}}{1 - S(gw)G(mcw)}.$$

Using these weights, we obtain that

$$\tilde{w}_n^0 = ew_n + \mu_1^0(mcw_n - ew_n) + \mu_2^0(gw_n - ew_n), \quad \text{for } n = 1, \ldots, N.$$

Similarly, setting $S(\tilde{w}^1) = 1$ and $G(\tilde{w}^1) = \breve{G}$ in (15) and (16), it follows that

$$\mu_1^1 = \frac{1 - S(gw)\breve{G}}{1 - S(gw)G(mcw)},$$

$$\mu_2^1 = \frac{\breve{G} - G(mcw)}{1 - S(gw)G(mcw)},$$

and hence

$$\tilde{w}_n^1 = ew_n + \mu_1^1(mcw_n - ew_n) + \mu_2^1(gw_n - ew_n), \quad \text{for } n = 1, \ldots, N.$$

Having constructed these two portfolios $\tilde{w}^0$ and $\tilde{w}^1$ that have the same G profiles, if we now take linear combinations of $\tilde{w}^0$ and $\tilde{w}^1$, we can vary the S profile while holding the G profile fixed at $\breve{G}$, and hence observe the pure effect of size on our performance benchmarks. This is achieved by varying the parameter θ in the expression below:

$$\breve{w}_n = \theta\tilde{w}_n^0 + (1 - \theta)\tilde{w}_n^1, \quad \text{for } n = 1, \ldots, N.$$

When $\theta = 1$ we are at the coordinates $(0, \breve{G})$ in Figure 2. As θ falls, we move along the horizontal line connecting $\tilde{w}^0$ and $\tilde{w}^1$, until at $\theta = 0$ we arrive at $(1, \breve{G})$.

In an analogous manner we can likewise construct portfolios that allow us to vary the G profile while holding the S profile fixed. This methodology again generalizes to higher dimensions.

5. Conditional Version of Our Methods

CDL demonstrate how characteristic-matched benchmarks constructed by sorting stocks by growth conditional on size tend to generate lower tracking error volatilities than independent sorts on size and growth. Our methods thus far described treat each style dimension independently. Conditional variants, however, could be constructed by eliminating from the reference portfolios in the value-growth dimension (e.g., the Russell 3000 Value and Growth portfolios) all stocks not held in the portfolio w. The remaining stocks in the reference portfolios would then be rescaled so that their shares sum to 1. For example, the growth profile of a small-cap manager would then be calculated from reference portfolios in the value-growth dimension that are themselves by construction also small cap. The reference portfolios in the value-growth dimension therefore would themselves be tailored to each particular portfolio w. This conditional approach should tend to reduce the tracking error volatilities of our methods.

Brands, Brown and Gallagher (2005) (BBG) draw a distinction between two aspects of active management. A manager must first decide which stocks to include in a portfolio, and, second, in what proportions to hold these stocks. BBG refer to these activities as 'stock picking' and 'portfolio construction'. One feature of the conditional version of our method is that it constructs benchmarks that focus exclusively on the latter activity (i.e., portfolio construction). In this sense, our conditional method could provide a useful complement to our unconditional method and existing methods for benchmark construction.

6. An Application to Fund Managers and Indices

6.1. The data set

Our data set consists of a sample of 1183 US institutional fund manager from the Russell database covering the period 2002Q2 to 2009Q3. We select funds for which at least 80 percent of the provided security IDs were matched to the Russell 3000 universe of stocks. Some of the unmatched security IDs contain cash or bond holdings and others are not included in the Russell 3000 listing of stocks. When calculating tracking error volatilities, we only consider managers for which we have at least 12 consecutive quarters of data.

This reduces our sample of managers to 275.[10]

6.2. The cross-section of index and fund style

A scatter plot of fund manager size and growth style profiles provides a useful indication of the range and variability of fund manager behavior. One such example is provided in Figure 3 for the 464 fund managers present in our data set in 2008Q4 (this was the quarter with the most fund managers). The reference portfolios in Figure 3 are *sm*=Russell 3000 Equal-Weighted and *lg*=Russell 1000 in the size dimension, while *sm*=Russell 3000 Value and *lg*=Russell 3000 Growth in the value-growth dimension.

Insert Figure 3 Here

From Figure 3 we can see that not a single fund manager holds a portfolio with a size profile smaller than the Russell 3000 Equal-Weighted index, 40 out of 464 portfolios have larger size profiles than the Russell 1000 index, 2 have smaller growth profiles than the Russell 3000 Value index and 13 have larger growth profiles than the Russell 3000 Growth index. It is also striking how the range of growth profiles fans out as the size profile rises from 0 to 0.8, after which the growth profile range starts to fall again. The variation of growth styles across funds therefore reaches a maximum when the size profile is about 0.8.

Figure 4 provides an equivalent scatter plot for some of the main Russell indices. The reference portfolios in Figure 4 are the same as in Figure 3. Hence by construction, the Russell 3000 Value and Growth indices have growth profile of 0 and 1 respectively, while the Russell 3000 Equal-Weighted and Russell 1000 indices have size profiles of 0 and 1.

[10] The choice of the number of consecutive quarters required for inclusion is somewhat subjective. CDL for example require 16 consecutive quarters. We prefer 12 since the gains in sample size (275 instead of 164 funds) in our opinion outweighs the disadvantages of having a shorter time horizon for some managers. As a robustness check we also calculate results based on the requirement of 16, 20 and 24 consecutive quarters respectively. We find that the results for these alternatives differ only marginally from those obtained for 12 consecutive quarters.

Figure 4 illustrates the problem of distinguishing between small cap value and growth portfolios in a universe that includes large cap stocks. The R2000, R2000V, R2000G, R2500, R2500G, R2500V, RMidcap and RMidcapG are all clustered in the range -0.06 to 0.04 on size and 0.62 to 0.69 on growth.

Insert Figure 4 Here

6.3. The evolution of fund and index style

The evolution of fund style in the size and value-growth dimensions over time is depicted in Figure 5. In panel A it can be seen that the average size profile stays reasonably constant over the period 2002 to 2009, with the suggestion of a slight dip in size from 2007 onwards. Panel B shows a clear upward trend in the growth profile (measured with Russell 3000 Growth and Value as the reference portfolios) over time that reverses slightly starting in 2008.

Insert Figure 5 Here

Figure 6-Panel A shows the evolution of the size profiles of the Russell 1000 Growth and Value indices over the period 1999 to 2009 (measured with the Russell 3000 and Russell 3000 Equal-Weighted indices as the reference portfolios). Over the subperiod 1999Q1 to 2004Q4 the Russell 1000 Growth index has a larger size profile than the Russell 1000 Value index. After 2004Q1 this pattern is reversed. Right at the very end of our sample (2009Q3) there is the indication that another reversal might be about to take place.

Similarly, Figure 6-Panel B shows the evolution of the growth profiles of the Russell 1000 and 2000 indices (measured with the Russell 3000 Growth and Value indices as the reference portfolios). Over the subperiod 1999Q1 to 2004Q4 the Russell 1000 index (which is large cap) has a larger growth profile than the Russell 2000 index (which is small cap). After 2004Q4 this pattern reverses, with the Russell 2000 index now having the larger growth profile. The gap again narrows at the end of the sample.

These findings qualify a standard perception in the literature that large cap stocks tend to have a growth tilt (see for example CDL who discuss this point in the process of explaining why conditional sorts on size should be preferred to independent sorts). While this is true for the first half of our sample, this pattern reverses after 2004.

Insert Figure 6 Here

6.4. Tracking error volatilities of tailored performance benchmarks and excess returns of funds

The tracking error of a benchmark is calculated as the difference between the performance of the portfolio and the performance of its benchmark. Tracking error volatility measured by the annualized standard deviation of tracking error over a sample of quarters is often used in the literature to judge the appropriateness of benchmarks (see for example CDL and CP). A lower tracking error volatility implies that the performance differential between a portfolio and its benchmark has a larger systematic component, thus increasing the usefulness of the benchmark.

Average tracking error volatilities of our tailored performance benchmarks and excess returns of funds are provided in Table 1. In the first column of Table 1 we replicate the value-weight conditional sort (i.e., quarterly size, within-size, BM) method used by CDL. This is their preferred method since it outperforms benchmarks constructed from attribute-matched independent sorts of portfolios, the three-factor time-series model and cross-sectional regressions of returns on stock characteristics.[11] Hence we use the CDL value-weight conditional sort method as a point of reference with which to assess the performance of our tailored benchmarks.

[11]CDL argue also for the use of composite value-growth measures. Our method can be easily extended in this direction by defining more than one dimension in the value-growth domain, and then matching portfolios and benchmarks by style in each dimension. We do not pursue this idea here, however, and hence to improve comparability likewise do not consider CDL's composite value-growth measures either.

Insert Table 1 Here

Seven of our tailored benchmarks are compared with the CDL tailored benchmark and a Russell 3000 benchmark in Table 1. Our seven benchmarks are described below. The first three are absolute benchmarks in the sense that their underlying size and growth profiles are monotonic functions of absolute measures of size (S^{ast}) and growth (G^{ast}) (see section 2.1 and Appendix A). The remaining four benchmarks are relative in the sense that their underlying size and growth profiles are defined relative to two reference portfolios in each style dimension.

1. Abs_S: one-dimensional size benchmark: lg=Russell 3000, sm=Russell 3000 Equal-Weighted
2. Abs_G: one-dimensional growth benchmark: $lg = gw$ (where gw is defined in (4)), sm=Russell 3000 Equal-Weighted
3. Abs_SG: two-dimensional size-growth benchmark: lg_S=Russell 3000, sm_S=Russell 3000 Equal-Weighted, $lg_G = gw$, sm_G=Russell 3000 Equal-Weighted
4. Rel_S: one-dimensional size benchmark: lg=Russell 1000, sm=Russell 3000 Equal-Weighted
5. Rel_G: one-dimensional growth benchmark lg=Russell 3000 Growth, sm=Russell 3000 Value
6. Rel_SG(3): two-dimensional size-growth benchmark: lg_S=Russell 1000, sm_S=Russell 3000 Equal-Weighted, lg_G=Russell 3000 Growth, sm_G=Russell 3000 Equal-Weighted
7. Rel_SG(4): two-dimensional size-growth benchmark: lg_S=Russell 1000, sm_S=Russell 3000 Equal-Weighted, lg_G=Russell 3000 Growth, sm_G=Russell 3000 Value.

Based on median tracking error volatility, the best performer is Rel_G, followed in order by CDL, Abs_SG, Rel_SG(3), Russell 3000, Rel_SG(4), Abs_S, Rel_S and lastly Abs_G. The mean tracking error volatility ranking differs only in that the order of Rel_G and CDL is reversed.[12] We have a number of observations on the results presented in Table 1. First, it is perhaps surprising that Rel_G has a lower tracking error volatility than both Rel_SG(3) and Rel_SG(4) given that the Rel_SG(3) and Rel_SG(4) benchmarks are more closely tailored to the individual funds. The lower tracking error volatility of Rel_G suggests that its tailored benchmarks adjust over time in a way that matches more closely shifts in portfolio holdings, and hence that value/growth orientation is a major driver of shifts in funds' approaches to stock picking. Second, the fact that Rel_SG(3) has a lower tracking error volatility than Rel_SG(4) is also interesting given that by construction the Rel_SG(4) benchmark portfolio must be closer to the underlying portfolio (i.e., it must have a lower distance measure as defined in (10)) than the Rel_SG(3) benchmark portfolio. Again the explanation is probably that the optimization method used by Rel_SG(4) causes the benchmark portfolio to adjust more over time than the portfolio itself. Hence a better fit in each specific period may act to increase tracking error volatility. Third, while Rel_G significantly outperforms Abs_G, we find that Abs_SG has a lower tracking error volatility than either Rel_SG(3) or Rel_SG(4). It is therefore not clear which out of relative and absolute versions of our method should be preferred. Finally, the results in Table 1 demonstrate that at least some of our methods are competitive in terms of tracking error volatility with the best of the methods considered by CDL, while at the same time being conceptually simpler and easier to compute. Furthermore, the fact that the CDL method uses conditional sorts on size in the value-growth dimension while our methods do not, in some sense biases the comparison against our methods. Conditional versions of our methods, constructed in the manner outlined above, may perform even better.

[12] Exactly the same median and mean rankings of methods are obtained when the comparison is restricted to funds present for at least 16 consecutive quarters.

6.5. Activity and performance

A large literature exists on the topic of whether active fund managers on average outperform passive managers (see for example (see for example Wermers, 2000). Cremers and Petajisto (2009), again henceforth CP, go further and consider whether more active managers outperform less active managers. CP distinguish between two notions of activity, which they refer to as stock selection and factor timing. Stock selection is a cross-section concept, which measures the deviation of a portfolio from its benchmark in a particular period. Factor timing can be measured by the tracking error volatility of managers relative to their benchmarks. CP find a positive relationship between stock selection and performance but no clear relationship between factor timing activity and performance.

Here we revisit this issue in a cross-section context using our measure of activity as defined in (10).[13] Activity quintiles and their corresponding average excess returns calculated using three of our tailored benchmarks – Rel_S, Rel_G and Rel_SG(4) – and the Russell 3000 as benchmark are shown in Table 2.

Insert Table 2 Here

The results in Table 2 are striking in that we observe the opposite result to that obtained by CP. That is, we find that more active funds perform worse than less active funds.

There are a number of differences between our study and that of CP that may explain our finding. First, there is very little overlap in our time horizons. Our data set covers the period 2002Q2 to 2009Q3, while the CP's covers the period 1990 to 2003. Second, our time horizon is shorter and includes the financial crisis that started in 2007. Third, our data set consists of institutional fund managers as opposed to mutual fund managers. Hence the lack of overlap in our samples applies to fund managers as well as the time horizon.

[13] We could in principle also use our approach to investigate the link between tracking error volatility and performance.

Fourth, our performance benchmarks are matched in terms of style to each fund manager, while CP only achieve an approximate match by searching over 19 well-known indices to find the one that minimizes their measure of activity and assigning this as the benchmark for that particular manager in that particular period. Finally, our activity measure is least squares based while CP's minimizes the absolute deviation between portfolios and benchmarks.

To determine whether the financial crisis is influencing our results, we try restricting the time span of our data set to 2002Q2-2007Q1. As shown in Table 2, excluding the financial crisis makes the inverse relationship between performance and activity if anything even stronger than before. To assess the impact of better matching of managers and performance benchmarks on the observed link between activity and performance we recalculate activity and excess return for each manager using the Russell 3000 as the benchmark. The use of a common benchmark likewise acts to further accentuate the inverse relationship between activity and performance observed in Table 2. Put another way, the failure to use tailored benchmarks seems to cause the link between activity and performance to be overstated.

Our finding therefore suggests that the relationship between activity and performance may be dependent on the time horizon or sample of fund managers and hence perhaps more complex than previously thought.

6.6. Disentangled Size and Growth Effects

We illustrate the impact on performance of changes in size holding the growth profile fixed in a three-reference-portfolio context. The Russell 3000 Equal-Weighted, Russell 3000 and Russell 3000 Growth indices are used as the reference portfolios. Figure 7 contrasts the impact of varying size by taking linear combinations of our three reference portfolios on total return for each of the four quarters in 2008. The dashed line in each case is obtained by taking linear combinations of the Russell 3000 Equal-Weighted and Russell 3000 reference portfolios. As one moves along this dashed line, both the growth and size profiles change. These effects are disentangled by the three parallel lines in each caption of Figure 7. Each of these lines fixes the growth profile G at a particular value (0, 0.5 or 1), and then allows size to vary. By construction, the slope of these lines is independent of the value of $\bar{G}$.

Insert Figure 7 Here

The most striking feature of Figure 7 is how holding growth fixed can reverse the sign of the slope of the total return line. Over our whole data set of 44 quarters covering the period 1999Q1 to 2009Q4 (our time horizon for the indices is larger than for the fund manager data set) we find that holding growth fixed acts to change the sign of the slope of the total return line for 29 of 44 quarters. By implication, when determining the impact of changes in size on performance it is important to control for changes in style in other dimensions.

A second feature of Figure 7 worth noting is the sensitivity of total return for any given size profile to the reference growth profile. In 2008Q3, total return for any given size profile varies by more than 9 percentage points as the reference growth profile varies between 0 and 1. By contrast, in 2008Q1 and 2008Q4 this difference is less than 2 percentage points, which while much smaller is still not trivial.

7. Conclusion

Characteristic-matched performance benchmarks obtained from portfolio holdings data are typically constructed using a bottom-up approach which first matches individual stocks to one of a number of discrete portfolios with similar style characteristics. The overall benchmark is then calculated by taking a weighted average of the excess returns on each of the individual stocks. We have proposed here an alternative methodology that avoids this bottom-up approach and generates direct matches between portfolios and benchmarks.

Our method also has the advantage of conceptual and computational simplicity. Our tailored performance benchmarks can be easily calculated by taking linear combinations of off-the-shelf indices such as the Russell 3000, Russell 3000 Growth and Russell 3000 Value. It can be applied in multiple style dimensions, and generates tracking error volatilities that are comparable with the best existing methods. In addition, our method provides new measures of style that are of interest in their own right and which shed new light on both the cross-section of index and fund manager style, and the evolution over time of index and fund manager style. It also leads naturally to new measures of activity that focus on deviations of a portfolio's holdings from those of its matched style benchmark, and perhaps a new perspective on the link between performance and activity.

Our new approach to constructing characteristic-matched performance benchmarks provides market participants with a new set of tools for evaluating fund style and performance, and opens up a new direction for research on the construction of characteristic-matched performance benchmarks distinct from the bottom-up approach that has dominated the literature in recent years.

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Appendix

A. Constructing Size/Growth Profiles that are Monotonic Functions of Absolute Measures of Size/Growth

Let mcw and ew denote respectively a market-cap-weighted and equal-weighted portfolio. Setting $lg = mcw$ and $sm = ew$ in (1), we obtain that

$$P(w) = \frac{\sum_{n=1}^{N}[(mcw_n - ew_n)(w_n - ew_n)]}{\sum_{n=1}^{N}(mcw_n - ew_n)^2}, \tag{A.1}$$

where

$$mcw_n = \frac{p_n q_n}{\sum_{m=1}^{N} p_m q_m}, \quad ew_n = \frac{1}{N}, \quad \text{for } n = 1, \ldots, N. \tag{A.2}$$

By construction, $\sum_{n=1}^{N} mcw_n = \sum_{n=1}^{N} ew_n = 1$. In what follows it is assumed that there exist at least two stocks for which $ew_n \neq mcw_n$. Otherwise the style profile $P(w)$ below is not defined.

In this case $P(w)$ is a monotonic (linear) function of the absolute size measure $S^*(w)$ as defined in (2). This can be demonstrated as follows:

$$P(w) = \frac{\sum_{n=1}^{N}[(mcw_n - ew_n)(w_n - ew_n)]}{\sum_{n=1}^{N}(mcw_n - ew_n)^2} = \frac{\sum_{n=1}^{N}(w_n mcw_n) - 1/N}{\sum_{n=1}^{N}(mcw_n^2) - 1/N} = \alpha^S + \beta^S S^*(w), \tag{A.3}$$

where

$$\alpha^S = -\frac{1/N}{\sum_{n=1}^{N}(mcw_n^2) - 1/N}, \qquad \beta^S = \frac{1}{(\sum_{n=1}^{N} p_n q_n)(\sum_{n=1}^{N}(mcw_n^2) - 1/N)}.$$

As long as the same list of stocks is used when computing $P(w)$ for all portfolios, the terms α^S and β^S are constants since they do not depend on w_n. The term $\sum_{n=1}^{N}(mcw_n)^2$ in the denominator of $P(w)$ is the Herfindahl-Hirschman index. It must take a value greater than or equal to $1/N$. The term $\sum_{n=1}^{N}(mcw_n)^2 - 1/N$ can be interpreted as a normalized version of the Herfindahl-Hirschman index, where its minimum value is rescaled to zero rather than $1/N$. In the special case where $\sum_{n=1}^{N}(mcw_n)^2 = 1/N$, there is no size line (since all portfolios have the same size) and $P(w)$ is not defined. This special case aside, it must be the case that $\alpha^S < 0$ and $\beta^S > 0$. It follows that $P(w)$ is an increasing linear (and hence monotonic) function of $S^*(w)$.

Setting $lg = gw$ and $sm = ew$ in (1), where gw is the growth weighted portfolio defined in (3), and now assuming there exist at least two stocks for which $ew_n \neq gw_n$, we obtain that

$$P(w) = \frac{\sum_{n=1}^{N}[(gw_n - ew_n)(w_n - ew_n)]}{\sum_{n=1}^{N}(gw_n - ew_n)^2} = \frac{\sum_{n=1}^{N}(w_n gw_n) - 1/N}{\sum_{n=1}^{N}(gw_n^2) - 1/N} = \alpha^G + \beta^G \ln G^*, \tag{A.4}$$

where

$$gw_n = \frac{\ln(p_n/b_n)}{\sum_{m=1}^{N}\ln(p_m/b_m)}, \quad \text{for } n = 1, \ldots, N, \quad \alpha^G = -\frac{1/N}{\sum_{n=1}^{N}(gw_n^2) - 1/N},$$

$$\beta^G = \frac{1}{[\sum_{n=1}^{N}\ln(p_n/b_n)](\sum_{n=1}^{N}(gw_n^2) - 1/N)}. \tag{A.5}$$

Hence $P(w)$ is now a monotonic (linear) function of the absolute growth measure $G^*(w)$ defined in (3).

For a stock with a relatively high price to book ratio, $gw_n > ew_n$. For a stock with a relatively low price-to-book ratio this inequality is reversed. Again as long as the same list of stocks is used when computing $P(w)$ for all portfolios, the terms α^G and β^G are constants (i.e., they do not depend on w_n). The growth variant of the Herfindahl-Hirschman index $\sum_{n=1}^{N}(gw_n)^2$ must be greater than $1/N$, except in the special case where all stocks have the same price-to-book ratio. In this case, all portfolios have the same growth profile and hence there is no growth line. Also, the growth weights gw_n will be negative for stocks with a price-to-book value of less than one. The presence of negative weights, however, does not create any problems here. The only complication it creates is that it is hence theoretically possible that β^G may be negative. When this happens, $P(w)$ is a monotonically decreasing rather than increasing function of G^*.

B. Demonstration that $\hat{w}$ is the Linear Combination of Three Reference Portfolios in Two Style Dimensions that in a Least-Squares Sense Most Closely Approximates the Portfolio w.

Differentiating (19) with respect to μ_1 and μ_2 generates the following first order conditions:

$$\frac{\partial D}{\partial \mu_1} = \frac{\sum_{n=1}^{N}(ew_n - mcw_n)[w_n - (1-\mu_1-\mu_2)ew_n - \mu_1 mcw_n - \mu_2 gw_n]}{\sum_{n=1}^{N}[w_n - \mu_1\, mcw_n - \mu_2\, gw_n - (1-\mu_1-\mu_2)ew_n]^2} = 0,$$

$$\frac{\partial D}{\partial \mu_2} = \frac{\sum_{n=1}^{N}(ew_n - gw_n)[w_n - (1-\mu_1-\mu_2)ew_n - \mu_1 mcw_n - \mu_2 gw_n]}{\sum_{n=1}^{N}[w_n - \mu_1\, mcw_n - \mu_2\, gw_n - (1-\mu_1-\mu_2)ew_n]^2} = 0.$$

These first order conditions can be rearranged as follows:

$$\sum_{n=1}^{N}(ew_n - mcw_n)(w_n - ew_n) + \mu_1 \sum_{n=1}^{N}(ew_n - mcw_n)(ew_n - mcw_n)$$

$$+\mu_2 \sum_{n=1}^{N}(ew_n - mcw_n)(ew_n - gw_n) = 0,$$

$$\sum_{n=1}^{N}(ew_n - gw_n)(w_n - ew_n) + \mu_1 \sum_{n=1}^{N}(ew_n - mcw_n)(ew_n - mcw_n)$$

$$+\mu_2 \sum_{n=1}^{N} (ew_n - mcw_n)(ew_n - gw_n) = 0.$$

Dividing through the first equation by $\sum_{n=1}^{N}(mcw_n - ew_n)^2$ and the second equation by $\sum_{n=1}^{N}(gw_n - ew_n)^2$ we obtain that

$$S(w) - \mu_1 - \mu_2 \frac{\sum_{n=1}^{N}(mcw_n - ew_n)(gw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2} = 0,$$

$$G(w) - \mu_1 \frac{\sum_{n=1}^{N}(mcw_n - ew_n)(gw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2} - \mu_2 = 0.$$

Noticing now that $\sum_{n=1}^{N}(mcw_n - ew_n)(gw_n - ew_n)/\sum_{n=1}^{N}(gw_n - ew_n)^2 = S(gw)$ and $\sum_{n=1}^{N}(mcw_n - ew_n)(gw_n - ew_n)/\sum_{n=1}^{N}(gw_n - ew_n)^2 = G(mcw)$, the first order conditions reduce to the following:

$$S(w) - \mu_1 - \mu_2 S(gw) = 0, \tag{B.1}$$

$$G(w) - \mu_1 G(mcw) - \mu_2 = 0. \tag{B.2}$$

On closer inspection (B.1) is identical to (13) while (B.2) is identical to (14). Hence the solutions for μ_1 and μ_2 in this least-squares minimization problem are the same as those given in (15) and (16) above. It follows that D is minimized when $\tilde{w}$ equals $\hat{w}$.[14] Hence $\hat{w}$ is the linear combination of the three reference portfolios that in a least-squares sense most closely approximates the portfolio w, as well as having the same S and G profile as w.

[14] D is defined in (7), $\tilde{w}$ in (18) and $\hat{w}$ in (17).

C. Derivation of the 4-reference portfolio simultaneous equation system

Differentiating (D.3) with respect to μ_1, μ_2 and μ_3, after rearrangement we obtain the following first order conditions:

$$\frac{\sum_{n=1}^{N}(mcw_n - ew_n)(w_n - ew_n)}{\sum_{n=1}^{N}(mcw_n - ew_n)^2} - \mu_1 - \mu_2 \frac{\sum_{n=1}^{N}(mcw_n - ew_n)(gw_n - ew_n)}{\sum_{n=1}^{N}(mcw_n - ew_n)^2}$$

$$- \mu_3 \frac{\sum_{n=1}^{N}(mcw_n - ew_n)(vw_n - ew_n)}{\sum_{n=1}^{N}(mcw_n - ew_n)^2} = 0, \tag{C.1}$$

$$\frac{\sum_{n=1}^{N}(gw_n - ew_n)(w_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2} - \mu_1 \frac{\sum_{n=1}^{N}(gw_n - ew_n)(mcw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2} - \mu_2$$

$$- \mu_3 \frac{\sum_{n=1}^{N}(gw_n - ew_n)(vw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2} = 0, \tag{C.2}$$

$$\frac{\sum_{n=1}^{N}(vw_n - ew_n)(w_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2} - \mu_1 \frac{\sum_{n=1}^{N}(vw_n - ew_n)(mcw_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2}$$

$$- \mu_2 \frac{\sum_{n=1}^{N}(vw_n - ew_n)(gw_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2} - \mu_3 = 0. \tag{C.3}$$

These equations can in turn be rewritten as follows:

$$S(w) - \mu_1 - \mu_2 S(gw) - \mu_3 S(vw) = 0, \tag{C.4}$$

$$S(w) - \mu_1 SG(mcw) - \mu_2 - \mu_3 SG(vw) = 0, \tag{C.5}$$

$$SV(w) - \mu_1 SV(mcw) - \mu_2 SV(gw) - \mu_3 = 0, \tag{C.6}$$

where

$$SG(w) = \frac{\sum_{n=1}^{N}(gw_n - ew_n)(w_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2},$$

$$SG(mcw) = \frac{\sum_{n=1}^{N}(gw_n - ew_n)(mcw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2},$$

$$SG(vw) = \frac{\sum_{n=1}^{N}(gw_n - ew_n)(vw_n - ew_n)}{\sum_{n=1}^{N}(gw_n - ew_n)^2},$$

$$SV(w) = \frac{\sum_{n=1}^{N}(vw_n - ew_n)(w_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2},$$

$$SV(mcw) = \frac{\sum_{n=1}^{N}(vw_n - ew_n)(mcw_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2},$$

$$SV(gw) = \frac{\sum_{n=1}^{N}(vw_n - ew_n)(gw_n - ew_n)}{\sum_{n=1}^{N}(vw_n - ew_n)^2}.$$

The terms $SG(w)$, $SG(mcw)$, $SG(vw)$, $SV(w)$, $SV(mcw)$ and $SV(gw)$ in (C.5) and (C.6) require some explanation. $SG(w)$ measures the location of the portfolio w in the one dimensional space with the ew portfolio located at zero and the gw portfolio located at 1. At first glance this seems to be simply the growth profile of w. However, this is not correct since in this relative context the points of reference in growth space are now the gw and vw portfolios rather than the gw and ew portfolios. $SV(w)$ similarly measures the location of the portfolio w in the one dimensional space with the ew portfolio located at zero and the vw portfolio located at 1. $SG(mcw)$ measures the location of the mcw portfolio in the one dimensional space with the ew portfolio located at zero and the gw portfolio located at 1. The other terms can be explained in analogous ways.

The equations (C.4), (C.5) and (C.6) can now be written in matrix notation as stated in (22).

D. Derivation of the Size and Growth Profiles of $\hat{w}$

To verify that $S(\hat{w}) = S(w)$, starting from (20) and using the fact that $S(mcw) = 1$ and $S(ew) = 0$, we obtain that

$$S(\hat{w}) = \mu_1 + \mu_2 S(gw) + \mu_3 S(vw). \tag{D.1}$$

The result now follows directly from a comparison of (C.4) and (D.1).

The fact that $G(\hat{w}) = G(w)$ is less obvious. This requirement is subsumed in (C.5) and (C.6). The easiest way to see that this must be true is to reparameterize the $\tilde{w}$ portfolio as follows:

$$\tilde{w}_n = \mu_1 gw_n + \mu_2 mcw_n + \mu_3 ew_n + (1 - \mu_1 - \mu_2 - \mu_3) vw_n, \quad \text{for } n = 1, \dots, N, \tag{D.2}$$

which in turn implies that the distance between w and $\tilde{w}$ is also reparameterized:

$$D = \sqrt{\sum_{n=1}^{N} [w_n - \mu_1\, gw_n - \mu_2\, mcw_n - \mu_3\, ew - (1 - \mu_1 - \mu_2 - \mu_3) vw_n]^2}. \tag{D.3}$$

Given this reparameterization, one of the first order conditions now reduces to

$$G(w) - \mu_1 - \mu_2 S(mcw) - \mu_3 S(ew) = 0. \tag{D.4}$$

Now starting from (D.2), and using the fact that $G(gw) = 1$ and $G(vw) = 0$, we obtain that

$$G(\hat{w}) = \mu_1 + \mu_2 S(mcw) + \mu_3 S(ew). \tag{D.5}$$

The result now follows directly from a comparison of (D.4) and (D.5).

Table 1: Benchmark Tracking Error Volatilities, Excess Returns and Activity for Funds (2002Q2-2009Q3)

TE Volatility	CDL	Abs_S	Abs_G	Abs_SG	Rel_S	Rel_G	Rel_SG(3)	Rel_SG(4)	R3000
Mean	4.8588	5.7604	7.7792	5.0986	5.7921	4.9016	5.2546	5.5443	5.2965
Median	4.4690	5.5101	7.5945	4.8505	5.5578	4.4663	4.8972	5.1926	4.9538
Stdev.	1.7807	2.1238	1.5361	1.7152	2.1464	1.9764	1.8477	1.9899	2.0942
Min	1.2717	2.1075	4.4849	2.1481	2.0808	1.3451	2.1106	2.1119	1.2159
Max	11.9272	14.6077	13.5771	11.2385	14.6735	12.5606	12.3842	14.3982	12.2965

Excess Return	CDL	Abs_S	Abs_G	Abs_SG	Rel_S	Rel_G	Rel_SG(3)	Rel_SG(4)	R3000
Mean	1.1951	0.0543	-0.4050	0.3926	0.0160	0.7795	-0.1667	-0.3311	1.0611
Median	1.4247	0.3795	-0.2436	0.7309	0.3146	0.8499	-0.0764	-0.2201	1.3077
Stdev.	2.4661	2.5644	2.6829	2.5288	2.5580	2.5497	2.3973	2.4354	2.7175
Min	-7.5056	-9.7014	-9.8622	-11.1001	-9.7231	-10.9438	-10.5787	-10.7257	-9.4097
Max	9.4969	6.4551	6.9226	7.7626	6.3988	7.4485	5.6279	6.4810	9.0072

Activity	CDL	Abs_S	Abs_G	Abs_SG	Rel_S	Rel_G	Rel_SG(3)	Rel_SG(4)	R3000
Mean	0.0976	0.1353	0.1454	0.1352	0.1353	0.1360	0.1326	0.1324	0.1457
Median	0.0949	0.1302	0.1423	0.1302	0.1302	0.1306	0.1278	0.1277	0.1424
Stdev.	0.0300	0.0334	0.0335	0.0333	0.0334	0.0313	0.0323	0.0324	0.0335
Min	0.0469	0.0791	0.0833	0.0789	0.0790	0.0912	0.0788	0.0781	0.0834
Max	0.2944	0.3223	0.3229	0.3222	0.3223	0.3275	0.3219	0.3218	0.3230

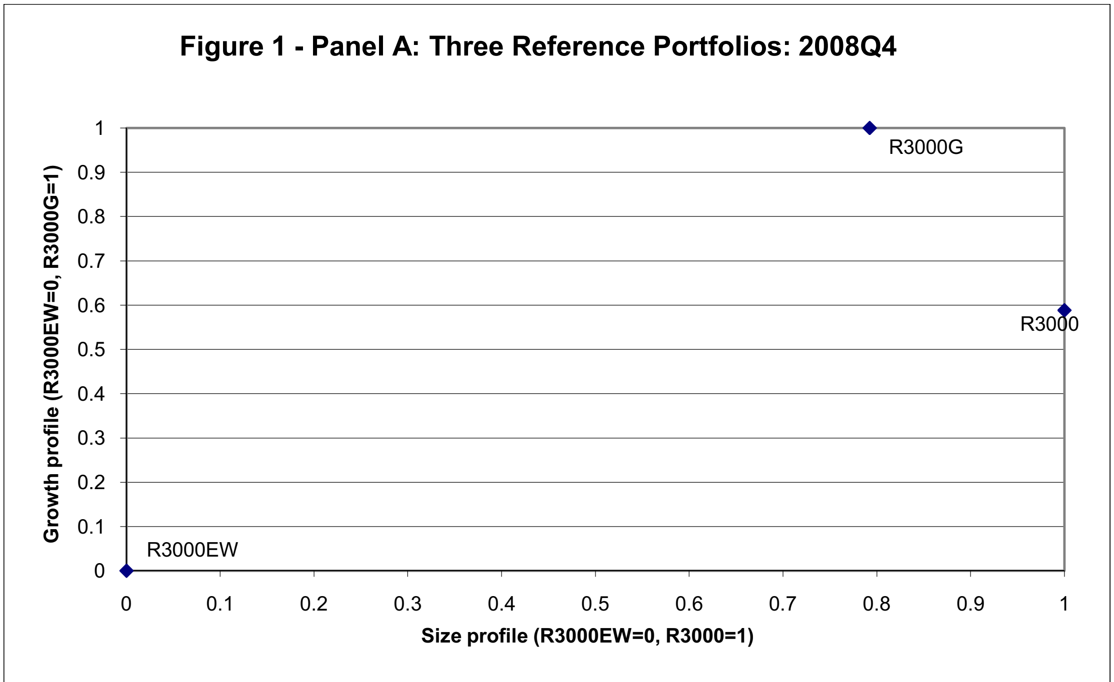
Only funds that were present for at least 12 consecutive quarters are included. At the beginning of each quarter each fund is matched with a tailored performance benchmark portfolio calculated in various ways. A fund's tracking error volatility is the annualized standard deviation of the time series of quarterly differences between the fund's return and its benchmark's return. A fund's excess return is the difference between the annualized return on a fund and its benchmark. A fund's activity is measured by the Euclidean distance between its portfolio holdings and the portfolio holdings of its benchmark. For each method of constructing performance benchmarks, the arithmetic mean, median, standard deviation, maximum and minimum of the tracking error volatilities, excess returns and activity across the 264 funds in the sample are provided. The CDL method replicates a method used by CDL which constructs tailored benchmark portfolios from 28 control portfolios from sorts first by size, and then within each size category, by book-to-market ratio. The next seven methods are all variants on our basic method. Abs_S, Abs_G, Rel_S, and Rel_G construct tailored benchmark portfolios from linear combinations of two reference portfolios that have either the same size or growth profile as a fund. Abs_S and Rel_S are matched on size, while Abs_G and Rel_G are matched on growth style. The 'Abs' benchmarks use reference portfolios that are defined specifically to capture variations in that particular style space. The 'Rel' reference portfolios are taken off-the-shelf. For example, the Rel_G method uses the Russell 3000 Value and Russell 3000 Growth as the reference portfolios in the growth dimension. Abs_SG, Rel_SG(3) and Rel_SG(4) match funds with benchmark portfolios that are tailored to have the same size and growth profiles as each fund. Abs_SG and Rel_SG(3) achieve this by taking linear combinations of three reference portfolios, while Rel_SG(4) uses four reference portfolios. Rel_SG(4) searches over all possible benchmark portfolios with exactly the same size and growth profiles as a fund and selects the one with the lowest activity measure. Finally, tracking error volatilities, excess returns and activity measures are also provided for the case where the Russell 3000 index is used as the benchmark portfolio against which each fund manager is evaluated.

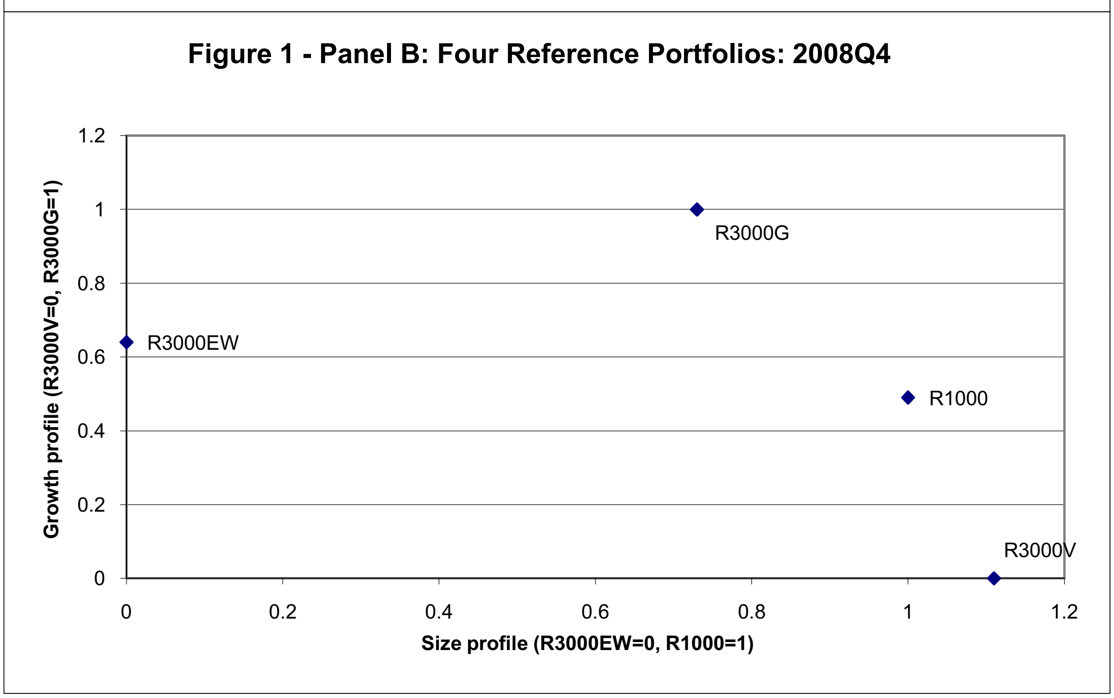
Table 2: Activity versus Performance

2002Q2-2009Q3		Activity Quintiles					
		Lowest	2nd	3rd	4th	Highest	Lowest - Highest
Rel_S	Activity	0.0975	0.1167	0.1314	0.1458	0.1851	
	Excess Return	0.6843	0.2802	-0.0145	0.0543	-0.9242	1.6085
	t-stat	2.4667	1.0636	-0.0403	0.1694	-2.0750	
Rel_G	Activity	0.1025	0.1184	0.1310	0.1445	0.1835	
	Excess Return	1.2289	1.5023	0.9000	0.3678	-0.1014	1.3303
	t-stat	4.4879	5.0670	3.0549	1.1071	-0.2220	
Rel_SG(4)	Activity	0.0966	0.1147	0.1277	0.1420	0.1812	
	Excess Return	0.2647	-0.2380	-0.0884	-0.3409	-1.2531	1.5177
	t-stat	0.8699	-0.9387	-0.3050	-1.0153	-3.0391	
R3000	Activity	0.1062	0.1271	0.1421	0.1572	0.1960	
	Excess Return	1.7956	1.8184	1.0510	0.8411	-0.2004	1.9959
	t-stat	6.2126	6.1721	2.9269	2.4237	-0.4366	

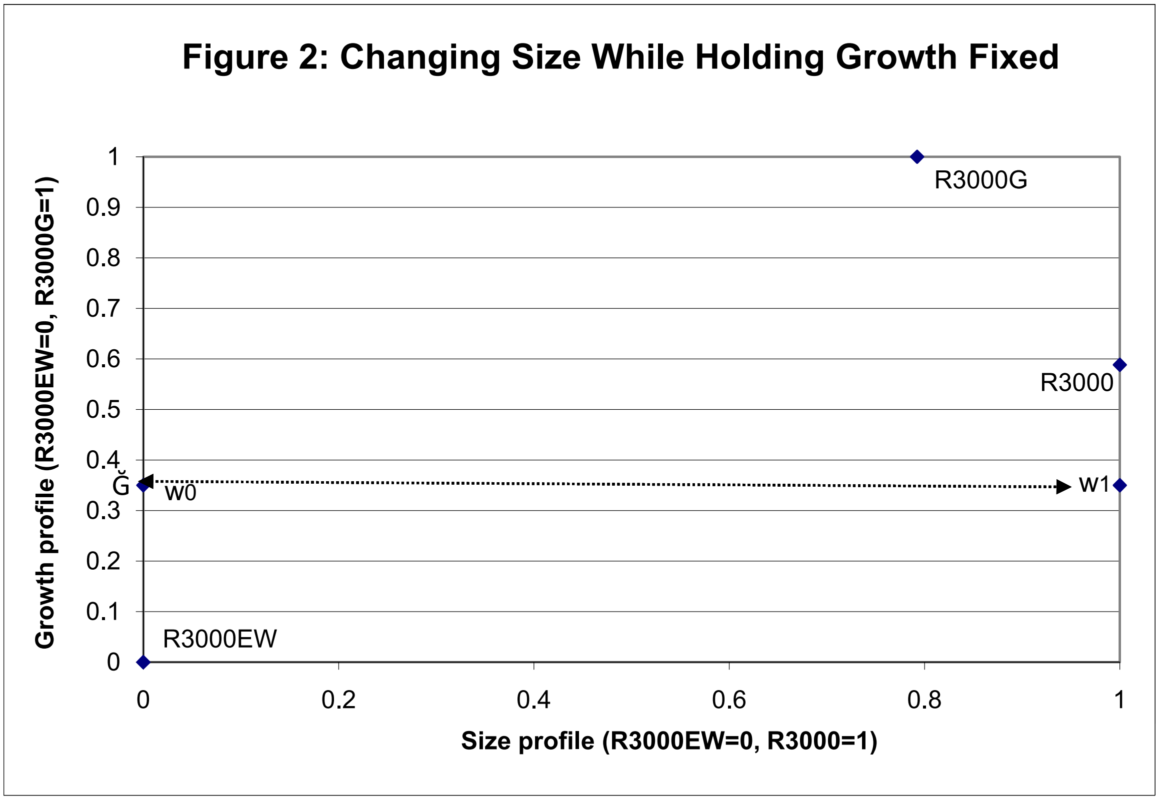
2002Q2-2007Q1		Activity Quintiles					
		Lowest	2nd	3rd	4th	Highest	Lowest - Highest
Rel_S	Activity	0.0972	0.1149	0.1290	0.1437	0.1880	
	Excess Return	1.2120	0.9556	0.3658	-0.1412	-1.7008	2.9128
	t-stat	3.8259	2.4624	0.6720	-0.3132	-3.5788	
Rel_G	Activity	0.1019	0.1170	0.1284	0.1421	0.1870	
	Excess Return	3.0131	2.4941	2.0404	1.5564	0.7360	2.2771
	t-stat	14.0569	6.0247	4.6350	3.8061	1.7128	
Rel_SG(4)	Activity	0.0966	0.1129	0.1252	0.1395	0.1841	
	Excess Return	0.9741	0.6105	0.4494	0.3519	-1.1080	2.0821
	t-stat	2.9212	1.6984	1.0148	0.9449	-2.8661	
R3000	Activity	0.1069	0.1256	0.1404	0.1560	0.1999	
	Excess Return	3.2847	3.1974	1.2576	1.2083	-0.6113	3.8960
	t-stat	10.8802	7.7653	2.1924	2.3387	-1.1968	

A fund's activity is measured as the Euclidean distance between its portfolio holdings and the portfolio holdings of its benchmark. Activity here is averaged across all quarters for each fund. The funds are then divided into quintile blocks based on their average activity. Four different ways of constructing benchmark portfolios are considered. Rel_S and Rel_G construct benchmark portfolios that are linear combinations of two reference portfolios (the Russell 3000 Equal-Weighted and Russell 1000 for Rel_S and the Russell 3000 Value and Russell 3000 Growth for Rel_G). The Rel_S benchmarks are tailored to have the same size profiles as funds, while the Rel_G benchmarks have the same growth profiles as funds. Rel_SG(4) takes linear combinations of the four reference portfolios above. The Rel_SG benchmarks are tailored to have the same size and growth profiles as funds. Activity measures are also calculated using the Russell 3000 as the benchmark. The benchmark in this latter case is not tailored to each individual fund. Excess return is the difference between the annualized return on a fund and its benchmark, averaged across all funds in each quintile block. The t-stats refer to the Excess Returns.

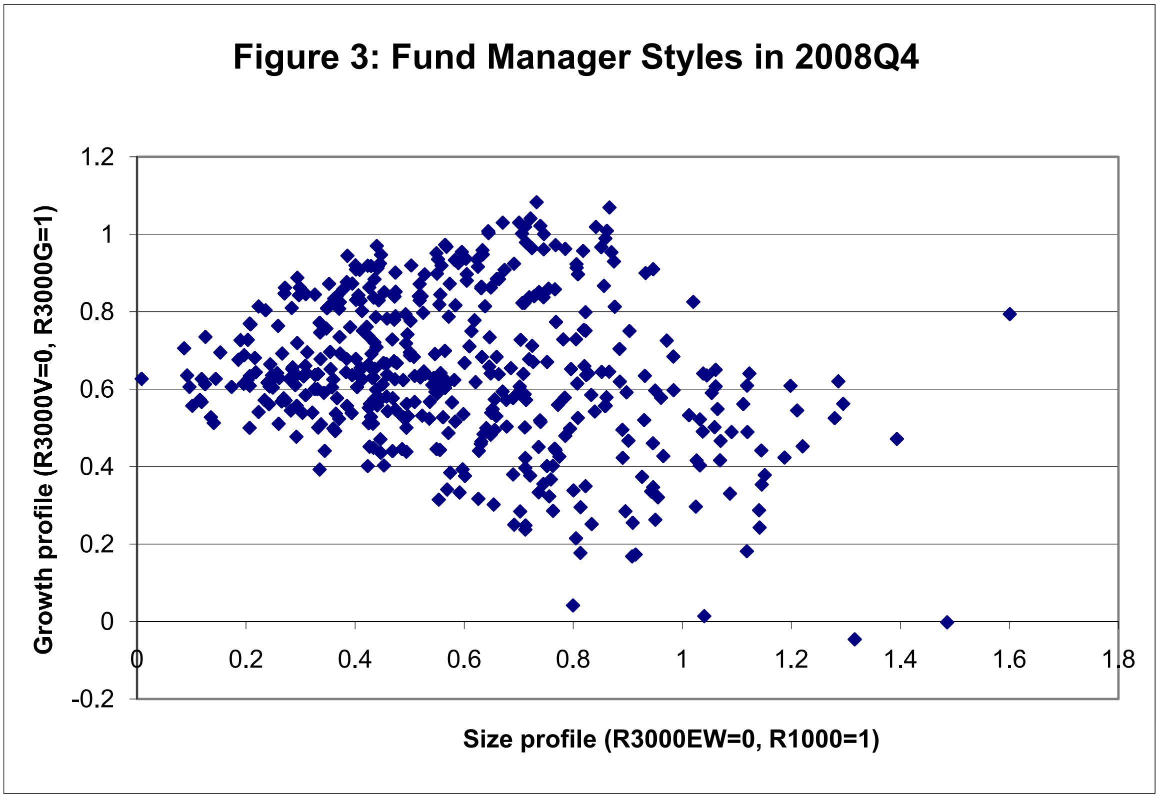




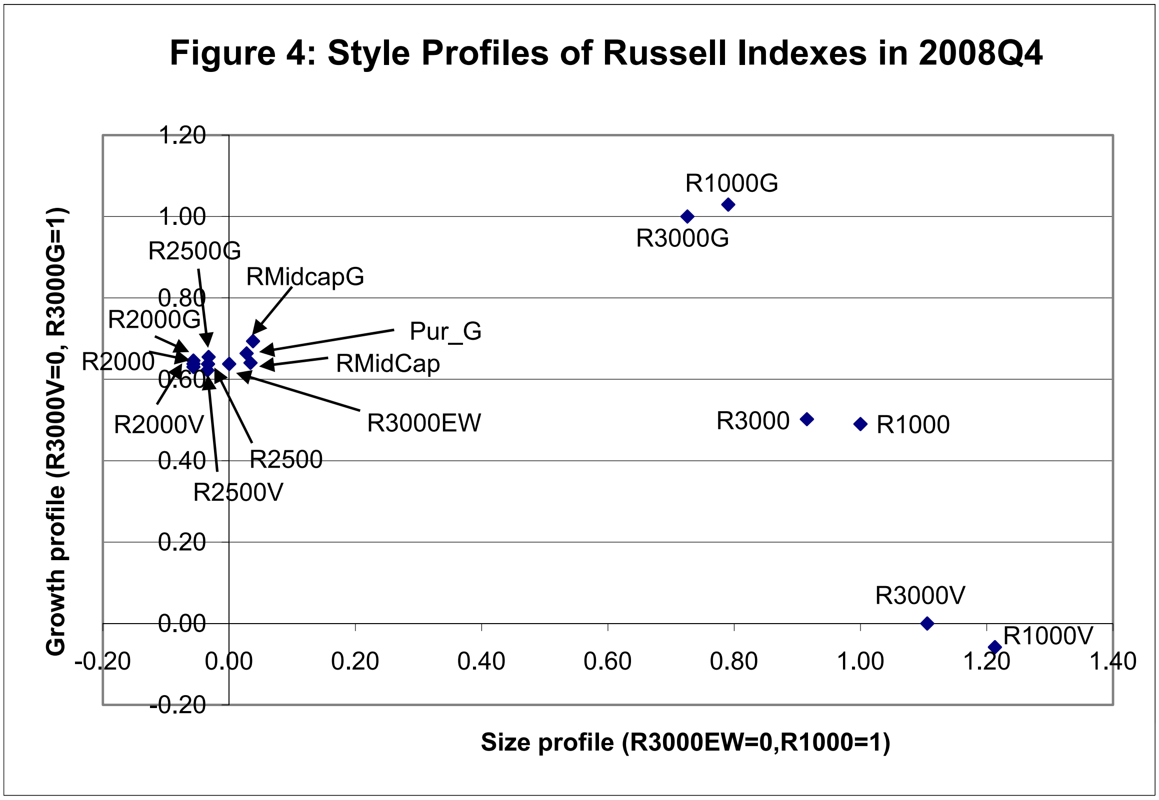
In Panel A, the reference portfolios in the size dimension are provided by the Russell 3000 Equal-Weighted and Russell 3000 indices. In the growth dimension the reference portfolios are provided by the Russell 3000 Equal-Weighted and Russell 3000 Growth indices. By construction it follows that the Russell 3000 Equal-Weighted and Russell 3000 indices have size profiles of 0 and 1, respectively, while the Russell 3000 Equal-Weighted and Russell 3000 Growth indices have growth profiles fo 0 and 1. The issue of interest here is determining the size profile of the Russell 3000 Growth index and the growth profile of the Russell 3000 index. The interpretation of Panel B is similar, except now that the reference portfolios in the growth dimension are provided by the Russell 3000 Value and Russell 3000 Growth indices. The issue of interest now is determining the size profiles of the Russel 3000 Value and Growth indices and the growth profiles of teh Russell 3000 and Russell 3000 Equal-Weighted indices.



By taking linear combinations of the three reference portfolios (here provided by the Russell 3000 Equal-Weighted, Russell 3000 and Russell 3000 Growth indices) it is possible to construct a portfolio w0 that has a size profile of zero and growth profile of Ğ. Similarly a second portfolio w1 can be constructed that has a size profile of 1 and growth profile of Ğ. By now taking linear combinations of w0 and w1 it is possible to vary the size profile while



This figure depicts a cross-section plot of size profiles and growth profiles for 464 fund managers in 2008Q4. The reference portfolios in the size dimension are provided by the Russell 3000 Equal-Weighted and Russell 1000 indices, and in the growth dimension by the Russell 3000 Value and Russell 3000 Growth indices.



This figure depicts a cross-section plot of size profiles and growth profiles for some Russell indices in 2008Q4. The reference portfolios in the size dimension are provided by the Russell 3000 Equal-Weighted and Russell 1000 indices, and in the growth dimension by the Russell 3000 Value and Russell 3000 Growth indices.

Figure 5 - Panel A: The Evolution of Average Fund Manager Size Profiles

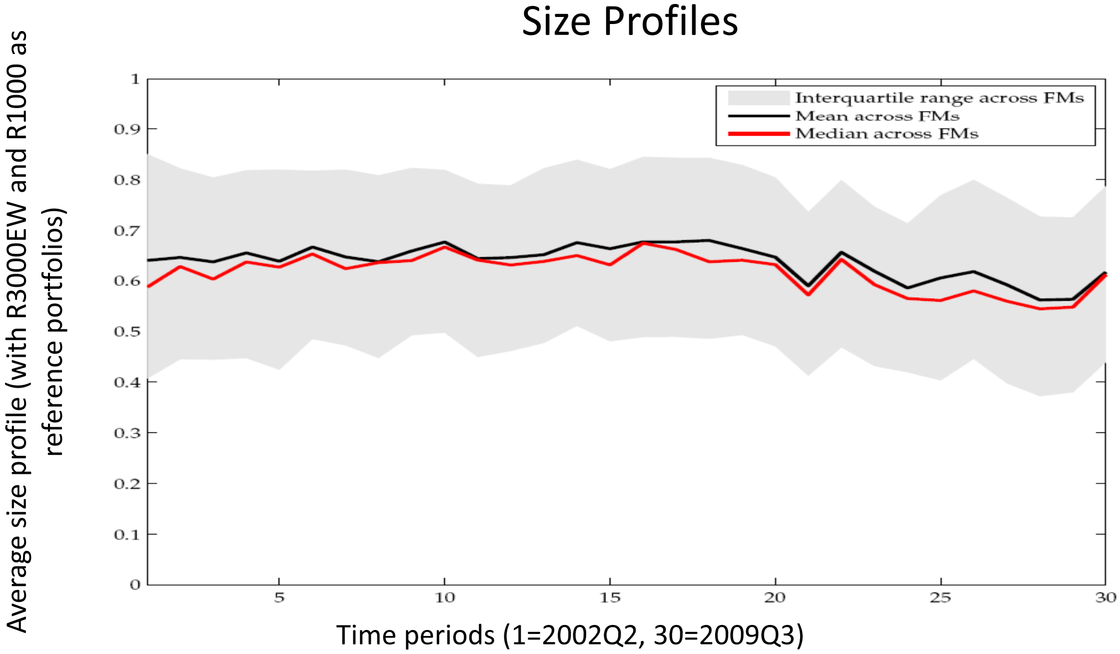
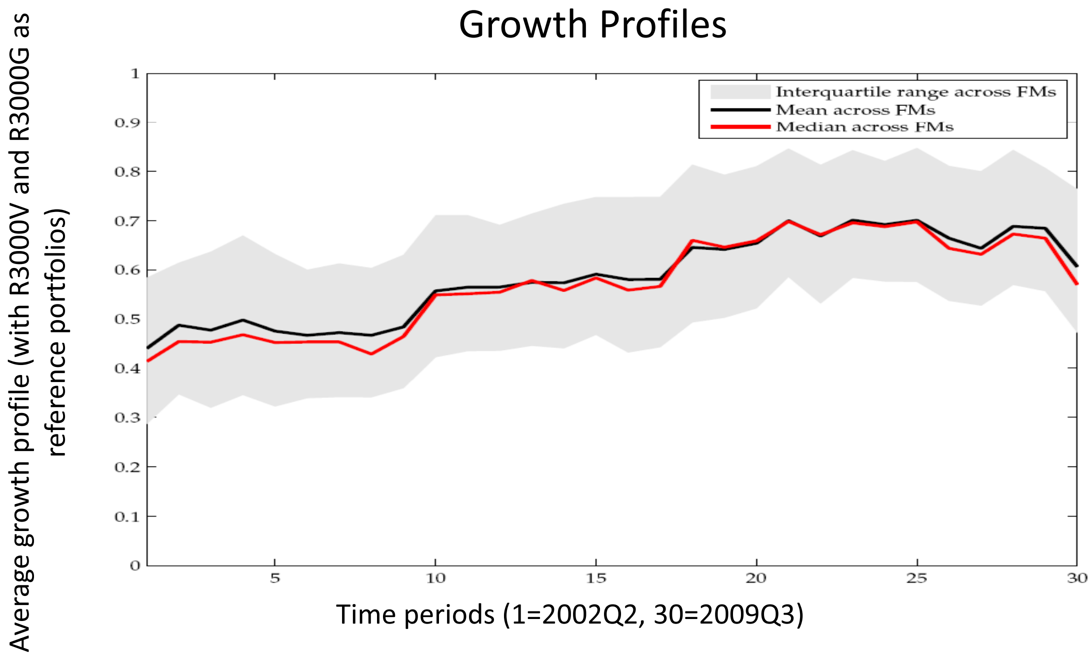


Figure 5 - Panel B: The Evolution of Average Fund Manager Growth Profiles



This figure shows how the average size profile of fund managers was reasonably stable over the period 2002Q2 to 2009Q3, while the average growth profile showed a clear upward trend.

Figure 6 - Panel A: The Evolution of the R1000G and R1000V Size Profiles

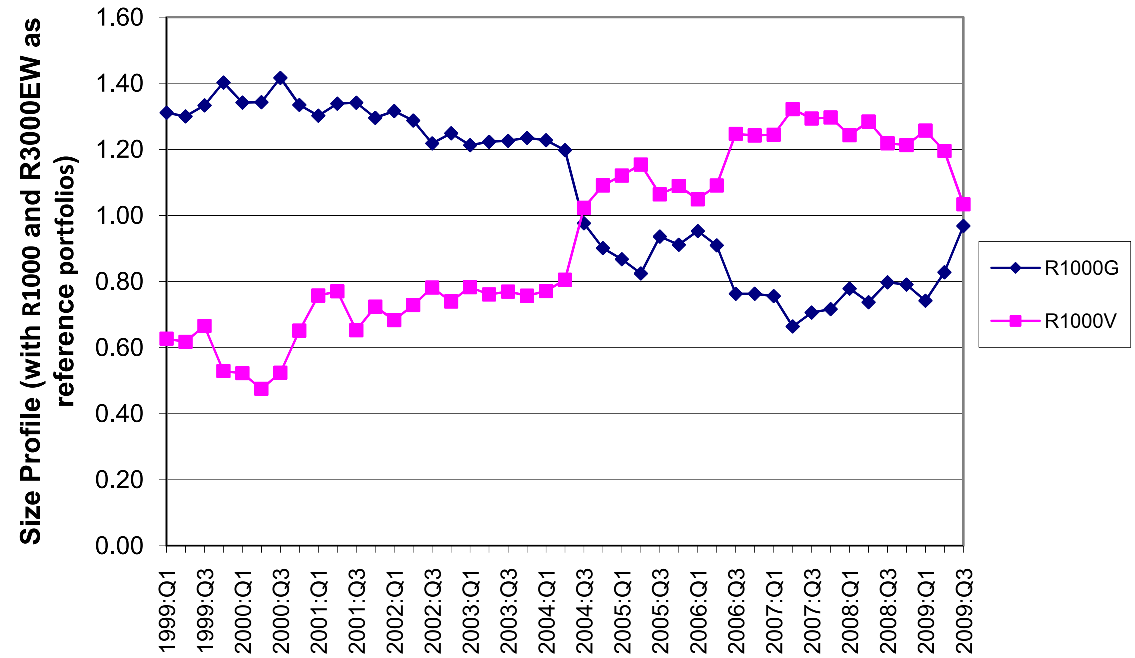
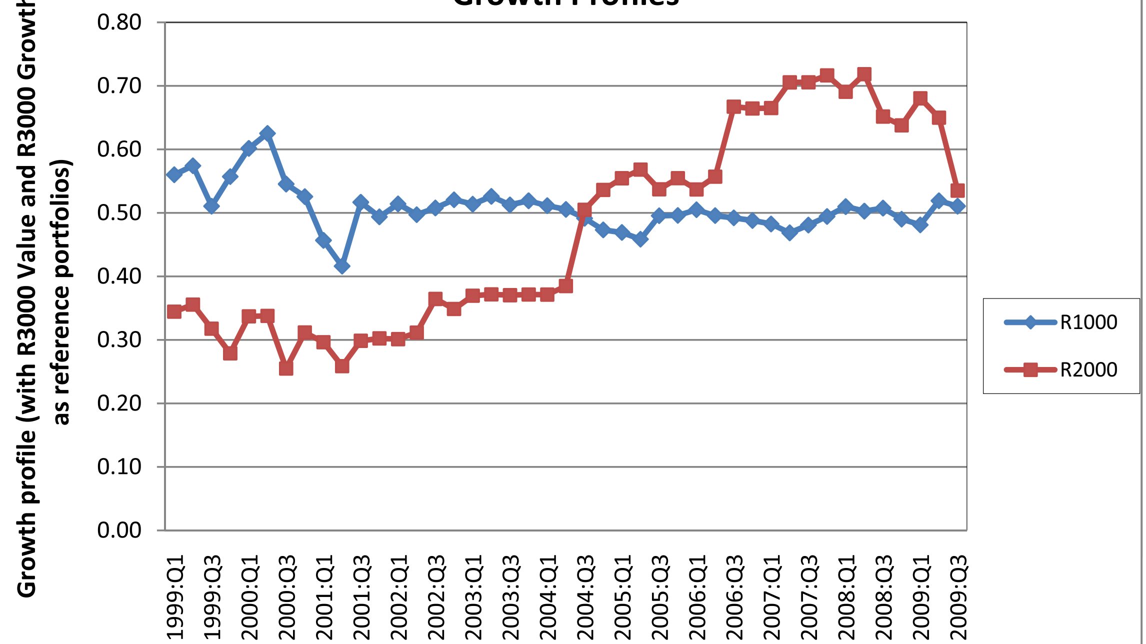
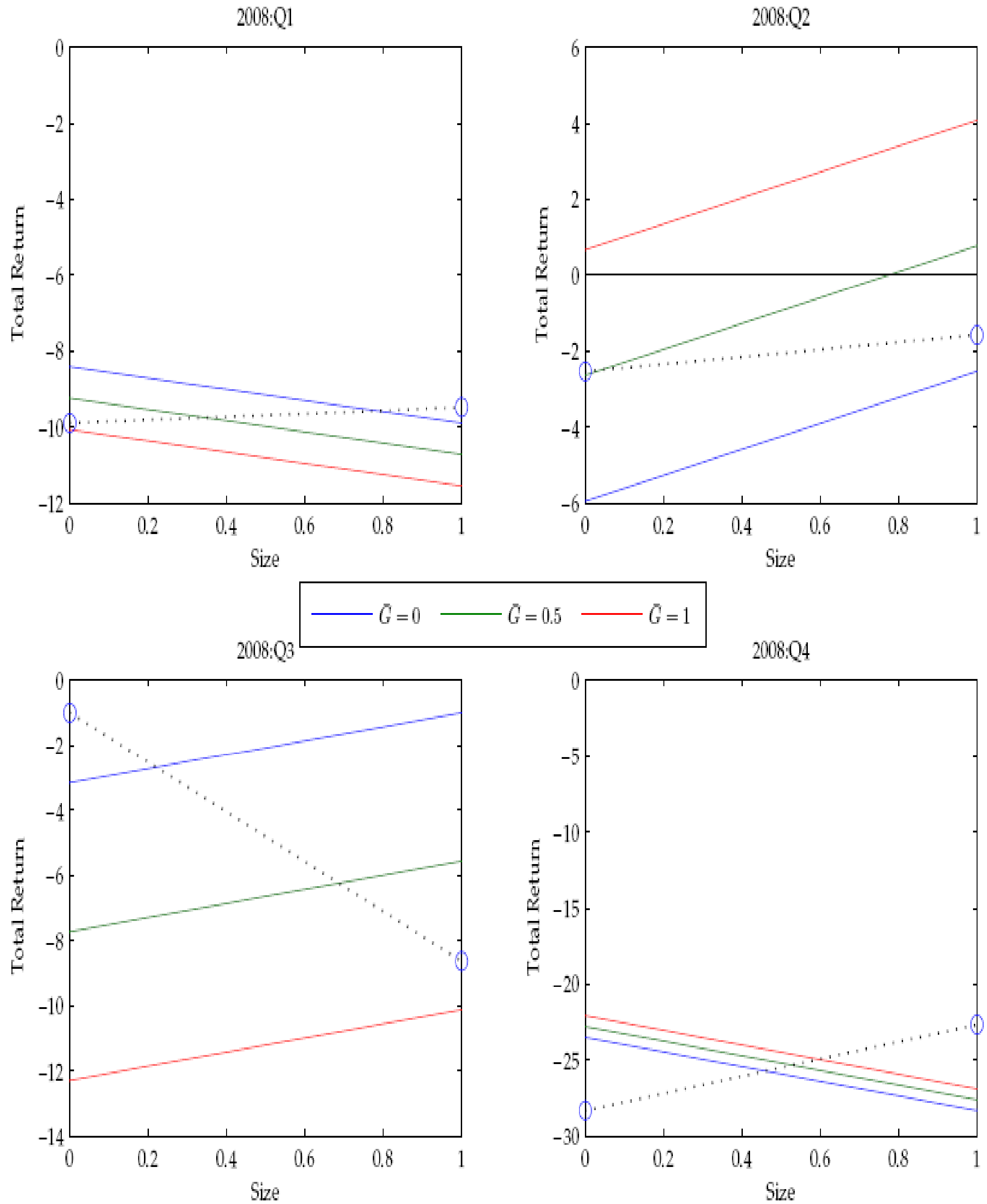


Figure 6 - Panel B: The Evolution of the R1000 and R2000 Growth Profiles



Panel A shows how the size profile of the R1000G index was larger than that of the R1000V index from 1999Q1 until 2004Q3. Thereafter, this pattern reverses. Similarly, Panel B shows that the large cap R1000 index has a higher growth profile than the small cap R2000 index from 1999Q1 until 2004Q3. Thereafter again the pattern reverses.

Figure 7: The Impact of Size on Performance Holding Growth Fixed



The dashed line in each graph shows how total return changes as a function of size without controlling for changes in the growth profile. Size is varied here by taking different linear combinations of the R3000EW and R3000 indices. The blue, green and red lines show how total return varies with size, holding the growth profile fixed. It is possible to vary size holding growth fixed by taking linear combinations of the R3000EW, R3000 and R3000G using the approach outlined in Figure 2. It is striking that in three of the four graphs the impact of increases in size on total return is reversed once one controls for changes in the growth profile.