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Platform Pricing Structure and Moral Hazard

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Abstract

We study pricing by a monopoly platform that matches buyers and sellers in an environment with cross-market externalities. Said platform has no private information, does not set the commodity's price and can only charge trading parties for the transaction. Our innovation consists in introducing moral hazard on the sellers' side and an equilibrium notion of platform reputation in an infinite horizon model. With linear fees the platform can mitigate, but not eliminate, the loss of reputation induced by moral hazard. If lump-sum fees (registration fees) can be levied, moral hazard can be overcome. The upfront payment determines the participation threshold of sellers and extracts them, while (lower) transactions fees provide incentives for good behavior. This breaks the equivalence of lump-sum payments and linear fees (Rochet and Tirole (2006)). We draw implications for the role of subsidies (Caillaud and Jullien (2003)).

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1   Introduction

Marketplaces, stock exchanges and, more recently, internet-based trading platforms bring together sellers and buyers of all stripes. Typically it is difficult to control the behavior of these

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traders, as they transact not with the platform but directly with each other. A trading site such as e-Bay, for example, exerts little or no control on the representations made by sellers to buyers, or on the quality of the items exchanged, the price of which it does not set. Hence opportunistic behavior can reasonably be expected to arise – and evidently does.[1] While the trading parties are the ones directly injured, this is a problem for the platform as it affects the value of the service it offers, and therefore on its ability to extract surplus from traders. That is, platforms face a problem of reputation, which they can govern only indirectly through the behavior of the parties they host.

The object of this paper is to study a platform's response to opportunistic behavior when it has a *limited* set of instruments. It turns out that prices are an easy and effective way of addressing the issue.[2] To capture the problem, we let sellers pick one of a good and a bad action, which affects the buyer's payoff directly. For example, a seller may ship a fake product, or an advertiser may place a misleading ad, which is a waste of the consumer's time. If the good action is socially beneficial (but privately costly), what can a platform do to induce the sellers to do the right thing? Specifically, we observe that some platforms (for example, half.com) charge transaction fees only, while others use both transaction fees and registration fees (such as Yahoo!Stores or MySpaceMyAds.com), and we offer a strong rationale for the latter.

The model is one of repeated interactions over an infinite horizon. At the heart of the paper is the following. Buyers can form an "opinion" of the platform, which corresponds to the belief that a seller they are matched with, will take the good action. This opinion is the *reputation* of the intermediary. The platform's prices generate a behavior on the part of sellers, which buyers anticipate and optimally respond to. Conversely, they also induce a behavior from buyers, that sellers optimally reply to. In equilibrium these behaviors have to be consistent. This defines the

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[1] For example, a German court ordered e-Bay to take preventive measure to guard against the sale of fake Rolex watches in April 2008. In June 2008, a Paris tribunal found e-Bay responsible for counterfeits traded on its site and ordered the site to pay 40 million Euro to LVMH . In contrast, in July 2008, e-Bay was cleared in a lawsuit brought forth by Tiffany and Co., charging it of facilitating the sales of counterfeit goods. Source: NY Times, 15 July 2008.

[2] Other platforms may use a broader set of instruments than we allow for, such as vetting their members, or policing them, as do securities exchanges such as the New York Stock Exchange (NYSE) or the Chicago Mercantile Exchange (CME). For example, the CME regularly audits its own clearing members for financial viability. Furthermore, the NYSE and the National Association of Securities Dealers (NASD) have become a Self-Regulatory Organization (SRO) though their joint enforcement arm called the Financial Industry Regulatory Authority (FINRA). A SRO is able to enact and enforce its own rules, as well as the broader rule of law, and engages in dispute resolution between parties. The FINRA explicitly offers an element of investor protection. Of course, no Internet trading site qualifies as a SRO. In fact, as evidenced by their Terms and Conditions, these intermediaries go to length to point out they do *not* take any responsibility for the deed, claims or statements of their customers. These Terms and Conditions may not be always upheld in a legal challenge.

equilibrium reputation of the platform. A good reputation is helpful in that the buyers' expected value of a trade increases with reputation. We assume that as a punishment, the platform may exclude sellers taking the bad action if they are detected, which occurs with some probability. Hence a cheating seller runs the risk of foregoing future trades. This specific punishment may be extreme, and in some cases may even not be the optimal punishment, but it is quite a natural one.[3] Most trading platforms reserve the right to exclude participants if they are harmful to others, as for example e-Bay, MySpace, Yahoo!Stores, Match.com, GumTree.com.au, or even the more *laissez-faire* craigslist.com.[4]

Our main result states that with up-front payments (to join the platform, say) and linear fees, the moral hazard problem can be entirely overcome. That is, the platform can implement an equilibrium where all sellers choose the good action. This fully replicates (i.e., both in terms of volume of trade and profits) the optimal outcome in the absence of moral hazard, when that outcome involves all sellers choosing the good action. Some papers in the literature (Armstrong (2006), Rochet and Tirole (2006)) have established the equivalence of lump-sum fees and transaction fees, *absent* moral hazard. We show the superiority of the combination of lump-sum and transaction fees – a two-part tariff. Introducing lump sum fees is essential when confronting moral hazard. Facing these, sellers must trade off the upfront payment with the present value of future trade opportunities when making their participation decision. That is, each element of the two-part tariff fulfills a different role: the lump-sum payment is used for surplus extraction – it addresses the participation decision, while the transaction fee solves the incentive problem, *given* the lump sum. More precisely, transactions fees enter *both* the incentive constraint and the participation decision, while the registration fee remains neutral on incentives. With appropriate payments, the critical value defining the measure of participating and non-cheating sellers can be made to exactly coincide, hence no seller takes the bad action in equilibrium. Of course, the marginal seller is completely extracted. In terms of policy, we attract attention to the facts that *(i)* registration and transaction fees are no longer equivalent, unlike in Rochet and Tirole (2006) and Armstrong (2006); and *(ii)* subsidies need to be carefully targeted: subsidizing registrations, in this model, is damaging – in contrast to Caillaud and Jullien (2003).[5] Subsidies are not necessary to attract buyers and they would worsen the sellers' incentive problem. This latter implication is an obvious consequence of moral hazard.

Two-part tariffs are typically used in adverse selection problems as a simple mechanism

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[3]So it may not be *necessary*, but it is certainly sufficient for our purposes.

[4]As in the respective Terms and Conditions of these websites.

[5]Conversely, sellers are essentially subsidized on transaction fee.

to extract surplus without calling on the more involving optimal contract.[6] Similarly, a compensation structure made up of a wage and an output-contingent bonus also implements the optimal contract in some moral hazard problems. Still, our *tariff structure*, to paraphrase Rochet and Tirole (2003), is not a reiteration of a standard two-part tariff, for outcomes *cannot* be contracted on (by assumption). Instead it falls closer to the class of self-enforcing implicit contracts in that it is devoid of explicit agreement and addresses a tension of intertemporal nature. This work also contributes to a broader understanding of moral hazard in the following sense. While the first-best can always be achieved in a standard principal-agent model with appropriate transfers when output is deterministic (as here), in our model the participation of buyers depends on the action of sellers and the incentives of sellers depends on the participation of buyers. That is, whether a seller can be induced to take the good action depends not just on his transfers, but also on those put forth on the buyers' side.

The next section reviews the relevant literature. We then lay out the model in part 3. Section 4 analyzes the moral hazard problem with linear fees only. In part 5 we consider up-front payments and offer a discussion. An example is presented in Section 6. Lastly we conclude. All the proofs are in the Appendix.

2 Literature

This paper lies as the crossroad of the newly developed literature on two-sided markets, and some anterior work on the role of intermediaries. Early on, the two-sided market literature (Rochet and Tirole, 2003 and 2006), Armstrong (2006), Caillaud and Jullien (2003) studied, respectively, the optimal pricing rule of a monopolist under different governance models, on showing the equivalence of up-front fees and linear prices, and on analyzing competition between platforms. Caillaud and Jullien (2003) introduce the notion of "divide-and-conquer strategies", consisting of subsidizing one side and extracting the surplus from the other one. Subsequent papers, such as Hagiu (2006 and 2007), build on this work to investigate other questions. Altering the timing of the game (Hagiu 2006) relaxes the coordination problem but raises the question of commitment to prices. Hagiu (2007) opens up the important question of whether an intermediary should *choose* to operate as a two-sided platform, or as a merchant.[7] Rochet and Tirole's (2003) work suggests that prices on either side may be skewed in one direction,

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[6]See for example Laffont and Tirole (1986), who show that affine contracts can approximate the optimal non-linear schedule.

[7]A merchant takes control of the goods, sets prices and does not faces the problem of indirect externalities.

depending on the "quasi-demand" elasticity on each side. Bolt and Tieman (2008) show this skewness can be so extreme as to set the price at the lower bound of the agents' valuation on one side, and impose a huge mark-up on the other. They draw implications for antitrust analysis: high mark-up may not be necessarily socially bad, nor evidence of a lack of competition. In our case, a high mark-up on the buyers' side is essential to provide incentives on the sellers' side, and consistent with higher welfare. Wright (2004) also highlights the pitfalls of thinking as if the market were one-sided when in fact one faces a two-sided structure. Biglaiser (1993) and Biglaiser and Ma (1992) investigate the role of intermediaries. Their approach endows said intermediary with a superior technology (either innate or acquired at a cost) to inspect goods of varying quality, which provide him with *private information*. The intermediary is long-lived and therefore wants to build a reputation for reliability. In contrast, our platform does not inspect commodities, nor does it take ownership. Therefore it does not set prices for the commodities, nor does it hold information it could signal. Instead, its monitoring technology informs it *ex post* and it can only alter the fees it charges its users. Yet it is able to play a socially beneficial role, not only by matching agents but also by promoting the socially desirable action. It does so because, as in Biglaiser (1993) for example, it builds a good reputation, which is necessary to overcome a market failure induced by asymmetric information.

3   The Model

Our model is based on that of Rochet and Tirole (2003). It departs from the latter by allowing for moral hazard on the sellers' side and because it is cast in a dynamic (infinite horizon) framework.

Agents and primitives   There is a single platform that offers intermediation services to a population of sellers and buyers every period. Sellers and buyers need this intermediation, as they cannot meet by themselves. Both the population of sellers and that of buyers are constituted of a continuum of heterogenous agents. Sellers are indexed by $s \in \mathbb{R}$ and buyers by $b \in \mathbb{R}$ – their respective type. The precise characterization of the populations of sellers and buyers as well as their dynamics will be made below. Time is discrete and indexed by τ.

Matching and payoffs   Conditional on patronizing the platform, a seller is randomly matched in each period with a buyer with exogenous probability $\mu > 0$. If matched, a seller chooses (i) whether to trade and (ii) if trading, an action $a \in \{l, h\}$. A seller of type s values a transaction

according as

$$v(s,a) = \begin{cases} s+d & \text{if } a = l \\ s & \text{if } a = h, \end{cases}$$

where $d > 0$. Action h is costly to the seller as compared to action l. If matched, a buyer faces the sole decision of whether to trade. A buyer of type b values a transaction with a seller taking action a following

$$u(b,a) = \begin{cases} b & \text{if } a = l \\ b+y & \text{if } a = h, \end{cases}$$

where $y > 0$. So a buyer always prefers that the seller take action h over the alternative l. Throughout we impose $y > d$ whence $u(b,h) + v(s,h) > u(b,l) + v(s,l)$ for all s and b: in any transaction the (high-cost) action h is socially beneficial. Agents are risk neutral. Whenever a buyer and a seller complete a trade, they must pay to the platform transaction fees t^b and t^s, respectively, the net payoff per transaction to a seller of type s is $v(s,a) - t^s$ and to a buyer of type b is $u(b,a) - t^b$. Both buyers and sellers get zero when they do not trade.

Information   Each side of the market is uninformed as to the other's type upon transacting. In particular, the buyer does not observe the type of the seller, whose choice of action a may depend on his type. Moreover, the buyer does not observe the seller's action before the transaction is completed. Without this information, the buyer's valuation of a trade depends on his expectation as to the seller's action. Thus, reputation, which is the buyer's perceived probability that the seller will choose action h, plays an important role in the buyer's decision to trade. The platform does not observe sellers and buyers' types either. However, it does have a(n) (imperfect) monitoring technology that provides it with *ex post* information about sellers' actions. Specifically, whenever a seller chooses action l, the platform observes it with probability $\alpha < 1$. The platform never receives a wrong signal: no signal is received by the platform if the sellers chooses action h. This signal structure attempts to reflect imperfections in buyer feedback to the platform (e.g. disputes may be settled directly) or the platform's policy of non-systematic arbitration of dispute.[8] We assume that sellers' actions are not verifiable, or contracting costs are prohibitive compared to the gains from trade. Therefore contracts that are contingent on the sellers' actions are not feasible. When such contracts are feasible there is no role for reputation.

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[8]For example, platforms like MySpace and Match.com explicitly state in their "Terms of use Agreement" that they reserve the right, but have no obligation, to become involved in any way with disputes between members, and in disputes between members and other users.

Rewards and punishment   Intertemporal incentives are required to induce sellers to choose the more costly (and more efficient) action. We consider a natural form of punishment: if detected, sellers who choose action l are excluded from the platform. That is, they are prevented from trading through the platform forever after. Almost all trading platforms or other platforms that offer intermediation services do reserve the right to exclude members whose behavior they deem inappropriate. Websites such as e-Bay, Match.com, Yahoo! Stores and Half.com are only a few examples.[9]

Sellers and buyers population   With probability m a seller leaves the platform ("dies") at the end of each period τ. This probability is exogenous, independent of the sellers' behavior and of his type. Hence in each period sellers may leave the market either because they die or because they were caught cheating and were excluded from the platform. Every period a fixed number (measure) E of new sellers, drawn from a log-concave distribution $F(s)$ with everywhere positive density $f(s)$, enter the market. With these entry and exit movements of sellers, the distribution and number (measure) of sellers in the population may vary from one period to the other. Throughout, we denote the distribution and the measure of sellers in period τ by $F_\tau(s)$ and S_τ, respectively.

Buyers live only one period. Each period, a totally new population of buyers with measure one is drawn from a log-concave distribution $G(b)$ with everywhere positive density $g(b)$. The assumption that buyers live only one period simplifies considerably the analysis and the exposition, but none of our results depend on it. They all remain valid if, similarly to the case of sellers, we assume that at a buyer dies with some probability at the end of each period and a fixed measure of new buyers enters the market.

In this setup we formalize the idea of equilibrium reputation, establish existence of an equilibrium, analyze how fees affect that reputation, and address the question of optimal fees. This analysis is performed in a steady-state, which is characterized by a time-invariant distribution and measure of sellers. Buyers and sellers seek to maximize total discounted utility, while the platform maximizes total discounted profits. To do so, each period buyers simply elect whether to trade if matched; so do sellers, and if trading, which action to take. The platform selects the fees it charges each party for its intermediation service. The common discount factor is $\delta < 1$. The sellers' behavior is the essence of our notion of reputation. We define platform reputation

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[9]For example, Half.com states in its policy and guidelines that: "Sellers are expected to perform in a manner that results in a consistently high level of buyer satisfaction. If a seller's interactions with the Half.com community create unacceptable levels of buyer dissatisfaction, that seller has violated the Seller Non-Performance policy." They also state clearly that a violation of this policy by a seller may result in account suspension.

as the buyers' expectation (i.e., perceived probability) that a seller who trades on the platform chooses action h if a transaction occurs. Of course, in equilibrium this expectation must be consistent with the sellers' behavior. That is, it must be identical to the proportion of sellers who do take action h, among those who trade on the platform.

4 Linear Fees and their Shortcomings

We start with transaction fees only. This allows us to expose the basic mechanics of the model in a simpler setting than with a two-part tariff. Also, this demonstrates the limitations of linear prices under moral hazard, which we will compare to the case where the platform charges both transaction and registration fees (Section 5). Throughout t^s denotes the transaction fee charged to sellers and t^b that to buyers. We begin with the buyers' trading decision given the platform's reputation and the fee charged to buyers, then turn to the sellers' decision given the behavior on the buyer side and the fee charged to sellers. Combining these two enables us to derive the equilibrium reputation and transactions, given the prices charged by the platform.

Buyers' transaction decisions   Let r be the platform's reputation, that is, $r = \Pr(a = h)$ when meeting a randomly drawn seller. If matched, a buyer faces the choice of whether to accept to trade. Conditional on a match and on being of type b, her expected value from the transaction is $ru(b, h) + (1 - r)u(b, l) - t^b = b + ry - t^b$. Hence the buyer accepts the transaction if and only if $b + ry - t^b \geq 0$. Given this, we can define

$$D^b(t^b, r) \equiv 1 - G(t^b - ry) \tag{1}$$

as the proportion of buyers who accept to trade on the platform given the transaction fee t^b and reputation r. We refer to $D^b(t^b, r)$ as *buyer participation*. It can be interpreted as the buyers' demand for transactions. Following the assumptions on $G(b)$, $D^b(t^b, r)$ is continuous, decreasing in t^b and increasing in r. An immediate consequence of this observation is that reputation is valuable to the platform. All things otherwise equal, the higher the platform's reputation, the higher the measure of buyers willing to trade on it.

Sellers' transaction decisions   Fix buyer participation. Let it be X^b in any given period.[10] If matched, a seller has two decisions to make: whether to accept the trade and which action

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[10]Because we are interested in steady state equilibria where all variables are stationary, it is sufficient to study sellers' optimal behavior when buyer participation is the same in every period.

$a \in \{l, h\}$ to take. These decisions depend on the seller's payoff from the current transaction, and on the seller's continuation value of trade. When buyer participation is the same in every period, the seller's problem is stationary and the seller's continuation value of trade is also the same in every period. For a seller of type s, let $V(s)$ denote the present value of the expected future payoffs at the beginning of a period, before he knows whether he will be matched to a buyer in that period. $V(s)$ corresponds to the value of the platform to a seller of type s and satisfies the (Bellman) equation

$$\begin{aligned} V(s) \;=\; & \mu X^b \max\{0 + (1-m)\delta V(s), s + d - t^s + (1-\alpha)(1-m)\delta V(s), s - t^s + (1-m)\delta V(s)\} \\ & + (1 - \mu X^b)(1-m)\delta V(s). \end{aligned} \tag{2}$$

With probability μX^b the seller is matched with a buyer who wishes to trade and faces three options. First, the seller may decide to not trade, in which case he receives 0; he is sure to not be excluded from the platform in the next period, which is worth $V(s)$. Second, he may decide to trade and to choose action l, collect $s + d - t^s$, and to remain on the platform only with probability $1 - \alpha$. Finally, the seller may decide to trade and to select action h, in which case he receives $s - t^s$ and is sure to not be excluded from the platform. In the alternative, with probability $1 - \mu X^b$ the seller is not matched with a buyer willing to trade and therefore can only wait until next period. Throughout a seller dies with probability m. Hence

Lemma 1 *When matched with a buyer, a seller of type s accepts to trade if and only if $s + d \geq t^s$.*

Only sellers with a high-enough valuation trade whereas the others do not. Traders includes all the sellers to whom the net value of a transaction when they choose action l is positive. Given transaction fee t^s, the exact proportion of sellers accepting to trade in period τ when matched, depends on the distribution of sellers in that period F_τ. Let

$$D^s_\tau(t^s) \equiv 1 - F_\tau(t^s - d) \tag{3}$$

denote that proportion. $D^s_\tau(t^s)$ is referred to as *seller participation*. It is decreasing in t^s. For expositional convenience, let $s_l(t^s) \equiv t^s - d$. That is, $s_l(t^s)$ represents the threshold level of s above which sellers accept to trade given transaction fee t^s. Then $D^s_\tau(t^s) = 1 - F_\tau(s_l(t^s))$.

Consider now a seller's choice between actions l and h – this is the *incentive* problem. The seller takes the high-cost and socially beneficial action h if and only if the immediate gain from cheating on the current transaction, d, does not exceed the reduction of his expected

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continuation value of trade if being excluded from the platform. That is, a seller of type s chooses action h if and only if $d \leq \alpha(1-m)\delta V(s)$. The optimal action is characterized as follows.

Lemma 2 *When matched with a buyer, a seller of type s who accepts to trade (and expects buyers' participation to be X^b in the subsequent periods) chooses the high action h if and only if*

$$d \leq \alpha \frac{(1-m)\delta}{1-(1-m)\delta}\mu X^b(s-t^s). \tag{4}$$

When condition (4) holds, $V(s)$ is sufficiently high so that choosing action h is optimal, and conversely if it fails. We see that sellers' incentives to choose action h when transacting depend not only on t^s but also on the buyers' participation on the platform X^b. These cross-market effects are a central feature of two-sided markets.[11] For expositional convenience, let $s_h(t^s, X^b)$ denote the value of s, given t^s and $X^b > 0$, such that (4) binds. It represents the threshold value of s above which sellers choose action h. It satisfies $s_h(t^s, X^b) > s_l(t^s)$ for all t^s and $X^b > 0$. Then we have, given t^s and $X^b > 0$, for

- $s < s_l(t^s)$ sellers do not trade;

- $s \in \left[s_l(t^s), s_h(t^s, X^b)\right)$ sellers trade and choose action l; and

- $s \geq s_h(t^s, X^b)$ sellers trade and choose action h.

Equilibrium With this preliminary analysis we can turn to equilibrium outcomes. The proportion of sellers who trade on the platform and the proportion of sellers who trade and choose action h depends on the distribution of sellers. This distribution may evolve over time as some sellers leave the market and others, not necessarily identical to them, enter it. In particular, excluding sellers in a given period τ in the interval $\left[s_l, (t^s)s_h(t^s, X^b)\right)$ may imply a loss of mass of the distribution F_τ on that fraction of the support. Still we can find a stationary distribution of the population of sellers. To this end, let N, L and H denote the proportions of sellers who do not trade on the platform, trade and choose action l, and trade and choose action h, respectively.

Definition 1 (Equilibrium transactions and reputation) *Given transactions fees t^b and t^s, a steady-state equilibrium is a tuple $\langle X^{b*}, N^*, L^*, H^*, S^*, r^* \rangle$ such that:*

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[11]For example, in Caillaud and Jullien (2003), the sellers' decision to register in the platform depends on the buyers' participation on the platform and vice-versa. Absent registration fees in this model, it is the seller's incentive to choose the high action in a transaction that depends on the buyer's participation on the platform.

1. (steady state conditions) the number of total sellers and the proportions of those sellers who do not trade, trade and choose action l, and trade and choose action h remain constant, i.e.,

   (a) $EF(s_l(t^s)) = mN^*S^*$

   (b) $E[F(s_h(t^s, X^{b*})) - F(s_l(t^s))] = [m + (1-m)\mu X^{b*}\alpha]L^*S^*$

   (c) $E[1 - F(s_h(t^s, X^{b*}))] = mH^*S^*$

   (d) $N^* + L^* + H^* = 1;$

2. (buyer participation is consistent with r^* and t^b) $X^{b*} = D^b(t^b, r^*)$ and

3. (reputation is consistent with seller behavior) $r^* = \frac{H^*}{L^* + H^*}.$

Equilibrium transaction volumes and reputation are then characterized by these six conditions. The first four are necessary for the proportions N, L and H to be stationary, well defined and consistent with seller optimal behavior given t^s and buyer participation X^{b*}; they also determine the (endogenous) measure of active sellers (S^*). Stationarity follows from conditions 1.(a)-1.(c), which require the number of new sellers entering the market to be identical to the number of sellers exiting the market, (a) among those sellers who do not trade on the platform, (b) those who trade and cheat and (c) those who trade and do not cheat. The fifth condition dictates that buyers' transaction decisions be optimal given transaction fee t^b and platform reputation. Last, the sixth condition states that platform reputation be consistent with the sellers' optimal behavior regarding action a. Thus, in equilibrium, platform reputation has to be consistent with sellers' optimal behavior; and sellers' optimal behavior has to be optimal given buyer participation, which in turn depends on reputation. The first result establishes the existence of an equilibrium and qualifies equilibrium reputation under linear prices.

Proposition 1 *For each (t^s, t^b) an equilibrium exists. In any equilibrium $r^* \in (0, 1)$.*

Irrespective of the transaction fees charged by the platform to sellers and buyers, the equilibrium reputation of the platform is always strictly smaller than one. There is always a positive measure of sellers who trade on the platform and choose the low action. These are the sellers in the interval $[s_l(t^s), s_h(t^s, X^{b*}))$.

Although no equilibrium in which reputation is equal to one exists, the value of the trans-action fees does affect the equilibrium reputation.[12] A higher fee t^b for buyers implies that

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[12]In general, one cannot rule out the possibility of multiple equilibria. In what follows, we focus on stable equilibria. We define stable equilibrium as an equilibrium in which, following a small perturbation in the level of reputation, a convergence back to the equilibrium occurs.

fewer of them trade when matched with a seller. This decreases the continuation value $V(s)$, which withers the punishment. The incentive to choose the more costly action h is therefore less powerful and in equilibrium, reputation of the platform drops. Altering the sellers fee t^s has a slightly more intricate impact. A higher fee affects the sellers' participation – this is the direct effect. But a higher transaction fee also depresses the continuation value $V(s)$, so fewer sellers choose action h – this is the incentive effect. Thus a higher t^s implies fewer sellers with low valuation s (who would choose action l), but more cheating among higher valuation sellers. Whether increasing t^s has a positive or negative effect on reputation depends on the magnitude of each of these two effects. In our model, the second effect is dominant, implying that equilibrium reputation decreases with t^s.[13]

Reputational concerns influence the platform's choice of t^s and t^b in the following sense. In a standard model, transaction fees affect the profit per transaction $(t^s + t^b)$ and the number of transactions $\mu D^b D^s S$. Without moral hazard, volumes respond directly to prices t^b and t^s (participation decision). In our model, there is an additional channel: both t^s and t^b also affect the equilibrium reputation, which in turn modifies buyer participation and the number of total sellers in the population. Optimal transaction fees balance all these effects.[14] When the parameter y is high (i.e. buyers are sensitive to reputation), the platform has the incentive to ensure a high reputation. Using transaction fees only, it faces two limitations. First, as stated in Proposition 1, the platform is not able to achieve a reputation of one, which may be optimal (see benchmark case below). Second, to achieve a high reputation, the platform has to charge low transaction fees, which erodes profits. In the next section, we consider the option of charging registration fees in addition to transaction fees and analyze the impact of registration fees, if any, in mitigating these limitations.

5   Registration and Transaction Fees

As evidenced in the preceding analysis, linear fees are not sufficient to alleviate moral hazard. Yet we are about to show that only one additional instrument may be sufficient to completely overcome moral hazard. Let T^s and T^b denote the registration fees for sellers and for buyers, respectively. These are paid only once to access the platform. Only registered traders may use the platform. Before jumping to the main result, we outline a convenient irrelevance result

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[13]For a proof of this result see the Appendix (Lemma 4).

[14]In Section 6, we consider an example and present the optimal linear fees and the platform's profit associated with it.

that speaks to the simplicity of the pricing structure that is used.

5.1 The effect of registration fees on the buyer side

We say that two equilibria are *outcome equivalent* if buyer and seller participation, total number of sellers, platform reputation and platform profit are identical in both equilibria. Now suppose that at least one equilibrium exists, which we establish later.

Proposition 2 (Irrelevance of registration fees on the buyer side) *For each equilibrium when transaction and registration fees are $\langle t_0^b, T_0^b > 0, t_0^s, T_0^s \rangle$, there exists an outcome equivalent equilibrium when transaction and registration fees are $\langle t_1^b, T_1^b = 0, t_0^s, T_0^s \rangle$.*

So using registration fees on the buyer side plays no role on the set of equilibrium outcomes. Thus, when moral hazard is present on the seller's side only, a platform cannot do any better by charging a registration fee on the buyers' side. This is at the heart of the Rochet and Tirole (2006) equivalence result *absent* moral hazard. As we will see below, transaction and registration fees on the *sellers'* side are not equivalent under moral hazard.

Thanks to Proposition 2, we can abstract from the pricing structure on the buyer side without loss of generality. When buyers do not have to register to trade, the buyers' problem depends only on transaction fee t^b and reputation r. So the decision rules are identical to those presented in Section 4. Buyer participation is then given by $D^b(t^b, r)$ as defined by (1).

5.2 Sellers' decisions and equilibrium

Upfront payment may affect participation, so we must begin by considering the sellers' problem.

Sellers' registration and transaction decisions For registered sellers, the registration fee has no *direct* bearing on their decisions whether to accept a trade and which action to select. Therefore $V(s)$ is also not affected directly in the registration fee T^s.[15] As before, given transaction fee t^s and buyer participation $X^b > 0$, if registered, sellers of type *(i)* $s < s_l(t^s)$ never trade, *(ii)* $s \in \left[s_l(t^s), s_h(t^s, X^b) \right)$ trade and choose action l, and *(iii)* $s \geq s_h(t^s, X^b)$ trade

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[15]It may have an influence through the equilibrium reputation of the platform, which at this point we take as given.

and choose action h. Using this, the solution to the Bellman equation (2) reads

$$V(s) = \begin{cases} 0 & \text{if } s < s_l(t^s) \\ \frac{\mu X^b}{1-\delta(1-m)[1-\alpha\mu X^b]}[s+d-t^s] & \text{if } s_l(t^s) \leq s < s_h(t^s, X^b) \\ \frac{\mu X^b}{1-\delta(1-m)}[s-t^s] & \text{if } s_h(t^s, X^b) \leq s. \end{cases}$$

$V(s)$ is continuous and increasing in s. Naturally a seller of type s registers if and only if $V(s) - T^s \geq 0$. Let $s_R(t^s, T^s, X^b)$ denote the value of s such that this latter condition just binds, when sellers' fees are $t^s, T^s > 0$ and buyer's participation is $X^b > 0$. For expositional convenience, let $s_R(t^s, 0, X^b) = s_l(t^s)$. Sellers of type $s \geq s_R(t^s, T^s, X^b)$ register with the platform, whereas the others do not. For buyers, the relevant sellers are those who are registered *and* trade. Given fees t^s and T^s and buyers' participation X^b, a seller does so if and only if $s \geq \max\{s_l(t^s), s_R(t^s, T^s, X^b)\}$. Hence redefining seller participation as the proportion of sellers who register and trade, seller participation in period τ is now given by

$$D_\tau^s(t^s, T^s, X^b) \equiv 1 - F_\tau(\max\{s_l(t^s), s_R(t^s, T^s, X^b)\}).$$

Equilibrium   With these lump-sum fees, the sellers' participation decision depends not only on transaction fees t^s, but also on the registration fee T^s and on buyers' participation. This is because participation requires registration, and the value of being registered depends on the measure of agent's on the other side of the market.[16]   Accordingly, the definition of an equilibrium needs to be adjusted. Without loss of generality, hereafter we focus on instances where T^s and T^b are non-negative.[17]

Definition 2 (Equilibrium with registration) *Given transaction fees t^b and t^s and registration fees T^s, an equilibrium is a tuple $\langle X^{b*}, N^*, L^*, H^*, S^*, r^* \rangle$ such that:*

1. *(steady state conditions) the number of total sellers and the proportions of those sellers who do not trade, trade and choose action l, and trade and choose action h remain constant, i.e.,*

   (a) $EF(s_R(t^s, T^s, X^{b*})) = mN^*S^*$

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[16]In contrast to linear fees, where only the seller's incentive problem is *directly* dependent on the buyers' participation.

[17]Note that it is never optimal for the platform to set a negative registration fee T^b or T^s. That would mean to subsidize registration of buyers or sellers who do not trade, which would not increase buyer or seller participation and consequently would not increase platform's profit.

(b) $E[F(\max\{s_R(t^s, T^s, X^{b*}), s_h(t^s, X^{b*})\}) - F(s_R(t^s, T^s, X^{b*}))] = (m + (1-m)\mu X^{b*}\alpha)L^*S^*$

(c) $E[1 - F(\max\{s_R(t^s, T^s, X^{b*}), s_h(t^s, X^{b*})\}))] = mH^*S^*$

(d) $N^* + L^* + H^* = 1;$

2. *(buyer participation and registration is consistent with r^* and t^b)* $X^{b*} = D^b(t^b, r^*)$

3. *(reputation is consistent with seller behavior)* $r^* = \frac{H^*}{L^* + H^*}.$

The definition remains conceptually the same as before. The main difference is that with registration fees, the definition of equilibrium must reflect the fact that only those sellers who register may trade on the platform. Proposition 3 is the counterpart of Proposition 1.

Proposition 3 *For each (t^b, t^s, T^s) there exists an equilibrium.*

This results departs from Proposition 1 because when agents have to register to trade on the platform, the equilibrium reputation is not necessarily strictly lower than one. As we will see, this will turn out to be an important difference.

Before continuing with the analysis of registration fees on equilibrium outcomes and platform's profits, we develop a benchmark against which forthcoming results will be contrasted.

5.3 A benchmark case with no moral hazard

Suppose that the platform can select directly the reputation r it wants, in addition to the transaction fees (t^s, t^b) it charges. That is, suppose that the platform observes sellers' types and for each seller is able to (*i*) impose the seller's action a in case the seller trades and (*ii*) prevent the seller from trading. The only thing the platform cannot do is force sellers to trade. In this setting there is no moral hazard in the sense that the platform can directly dictate the action of each seller who trades on it.

Since the sellers who eventually choose action l have the consent of the platform, no exclusion of sellers occurs. Thus, the relevant steady state distribution of sellers is F and their number $S = E/m$. Moreover, since the platforms' profit is increasing in the number of transactions (of course, provided $t^s + t^b > 0$), given a transaction fee t^s it is optimal for the platform to allow all sellers with a valuation $s > t^s$ to trade and impose that they choose action h. The platform's problem is then to choose the number of low action sellers that it will permit to trade. Let X^s denote the proportion of sellers who trade and r the reputation. Reputation must satisfy

$r = (1 - F(t^s))/X^s$, which necessarily implies that $X^s = r/(1 - F(t^s))$. We can write the platforms' problem as

$$\max_{t^s, t^b, r} \Pi(t^s, t^b, r) = \mu(E/m) \left[1 - G(t^b - ry) \right] \left[\frac{1 - F(t^s)}{r} \right] (t^s + t^b)$$

subject to

$$\frac{1 - F(t^s)}{1 - F(t^s - d)} \leq r \leq 1.$$

and claim the following.

Lemma 3 *Suppose that F is such that, given $d > 0$, the ratio $[1 - F(x)]/[1 - F(x - d)]$ is bounded away from zero. Then, for y sufficiently large, the solution of the above problem involves $r = 1$ and transaction fees such that $t^s + t^b = [1 - F(t^s)]/f(t^s) = [1 - G(t^b - y)]/g(t^b - y)$.*

The basic trade-off involves increasing reputation, which buyers value because y is high, at the expense of the number of sellers, who also pay t^s – even if they choose l. But when y is large enough, the former effect dominates and leads to a the corner solution: it is worthwhile dropping *all* the low-action sellers.[18] Importantly the platform can always achieve strictly higher profits when it is able to choose reputation directly – contrary to what was studied in Section 4. As a preview of forthcoming results, if the platform can replicate this allocation, it will necessarily increase its profits. Now we are ready to turn to the analysis of the impact of registration fees on equilibrium outcomes.

5.4 The main result

We first note that the equilibrium reputation is increasing in T^s. The lump-sum payment reduces seller participation (those with lower type opt out), while not affecting their incentives with respect to the action a.[19] Since the low-type sellers are precisely those with the lower incentive to choose action h, an increase in registration fees reduces the proportion of cheaters among sellers who trade, increasing reputation. That is, the registration fee works a as way of selecting the sellers that trade on the platform. This selection of sellers can be taken to the

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[18]This result is subject to the qualification that F is a distribution for which, given $d > 0$, the ratio $[1 - F(x)]/[1 - F(x - d)]$ is bounded away from zero. The Logistic and the Extreme are examples of distributions that satisfy this property.

[19]For a proof of this result see the Appendix (Lemma 5). As in the case in which the platform can charge only transaction fees, we cannot rule out the possibility of multiple equilibria. As in that case, we focus here on stable equilibria.

extreme, so that only those sellers who choose action h if they trade, register and trade on the platform. Then we have

Proposition 4 *Given transaction fees t^s and t^b, there exists a registration fee T^s such that in equilibrium the platform's reputation is one.*

By choosing sufficiently high registration fees for sellers, the platform can achieve a perfect reputation; that is, moral hazard can be entirely overcome.[20] With transaction fees only, the equilibrium reputation is always strictly lower than one. This paves the way to our main result.

Proposition 5 *Suppose that it is optimal for the platform to implement a reputation of one in the benchmark case (e.g., y is large enough). Then, there exists a set of fees $\langle t_1^s, T_1^s, t_1^b \rangle$ that generates the same allocation as the optimal allocation in the benchmark case.*

That is, the triplet $\langle t_1^s, T_1^s, t_1^b \rangle$ generates the same buyer participation, the same seller participation and the same profits as can be achieved in the benchmark case when buyers value action h sufficiently highly. Registration fees can be so helpful as to implement the optimal allocation defined in the benchmark case. The reason turns out to be remarkably simple and widely applicable. Linear fees affect both the sellers' participation and their incentives (the choice of action a), while registration fees influence the participation decision only. This allows the platform to appropriately combine transaction and registration fees to provide high incentives for sellers to choose action h *conditional* on participation, and simultaneously to keep low-type sellers out of the platform. With the appropriate mix of fees, the marginal participating seller never take the low action l, and is made to correspond to the marginal seller absent moral hazard.

This outcome cannot be achieved with transaction fees only for the following reasons. First, by lowering transaction fees to increase sellers' incentives to choose action h, the platform induces more low-type sellers to trade. Second, and equally important, registration fees are an efficient instrument for the platform to extract surplus from the sellers. To be more specific, as long as reputation is one, registration fees are as efficient in surplus extraction from sellers as transaction fees. They are perfectly substitutable – it is as if there were no moral hazard, as in Rochet and Tirole (2006).

5.5 Discussion

We have shown that in the presence of moral hazard, transaction and registration fees are no longer generically interchangeable. Moral hazard voids the equivalence result of Rochet and

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[20]Recall that reputation is one when no seller chooses the low action l.

Tirole (2006) for the simple reason that transaction fees and registration fees play a different role. The former enters *both* the incentive constraint and the participation decision, while the latter is neutral on incentives. Indeed, Propositions 4 and 5 imply that Proposition 2 does not have an equivalent on the sellers' side. In the absence of moral hazard of course there is no incentive constraint to satisfy and these two prices substitute perfectly in the participation decision.

On another point, we observe that subsidizing registration to induce agents to trade on the platform worsens the incentive problem by driving a wedge between the participation cut-off s_l and the incentive threshold s_h. As a result, 'divide-and-conquer' strategies advocated by Caillaud and Julien (2003) and Hagiu (2006) (albeit in a model of price competition) need to be carefully considered under moral hazard and platform reputation is important.[21] Subsidizing the side subject to moral hazard may do more harm than good. Moreover, the lump-sum payment is precisely used for surplus extraction, while the linear fee is subsidized to provide incentives.

As mentioned in Footnote 2, there may exist other means to deal with moral hazard, such as *ex post* fines, screening of members, legal remedies and so forth. Our mechanism presents two advantages: *(i)* it is simple, both conceptually and to implement, and *(ii)* it is costless (when the benchmark solution is $r^* = 1$). Its one obvious drawback is that some sellers may be facing a wealth (and credit) constraint that prevents them from participating. It may also be conceivable to let transaction fees vary with a player's history. But, absent registration fees, this can never yield a reputation of one because sellers are not committed to a long-term relationship, so no *ex post* punishment can be imposed on them. Instead, the *ex ante* lump-sum allows the platform to lower transaction fees sufficiently so that all participating sellers take the high action.

6 An Example

To further illustrate the main result, we present a numerical example where we compute optimal fees, maximum platform profits and platform reputation under transaction fees only and when the platform can charge both transaction and registration fees to buyers and sellers.

Buyers and sellers are drawn from logistic distributions. We fix the parameters δ, μ, α, m and d and let y vary. y parametrizes buyers' sensitivity to sellers' action, and therefore to

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[21]DC strategies consist in subsidizing one side to attract it on the platform and extracting surplus from the other.

reputation. Thus, by letting y vary, we can analyze the importance of registration fees as buyers become more sensitive to reputation and consequently reputation becomes more important to the platform. Table 1, shows the results for several selected values of y.

| | Tansaction Fees Only | | | | Transaction and Registration Fees | | | | |
| | Optimal Fees | | | | Optimal Fees | | | | |
y	t^b	t^s	Reputation	Profit	t^b	t^s	T^s	Reputation	Profit
0.51	0.891	0.627	0.82	19.803	0.896	0.626	0.003	0.82	19.811
1	1.136	0.437	0.853	25.966	1.352	0.054	0.968	1	29.051
1.5	1.452	0.217	0.878	33.947	1.671	-0.099	0.975	1	38.53
2	1.759	0.029	0.899	43.968	2	-0.338	1.329	1	50
2.5	2.132	-0.194	0.917	56.197	2.338	-0.720	2.376	1	63.549
3	2.485	-0.383	0.932	70.684	2.685	-1.208	4.066	1	79.207

Table 1: Optimal fees, profits and equilibrium reputation when $\delta = 0.97$, $\mu = 1$, $\alpha = 0.6$, $m = 0.1$ and $d = 0.5$. The values in this table are rounded to the third decimal place.

The difference between the optimal profit under the two regimes increases with y. The more sensitive buyers are to reputation, the more essential are the registration fees. Note that reputation increases with y even when the platform charges only transaction fees. In this case, the platform increases reputation by lowering those fees (on at least one side of the market). In this specific case, the platform reduces the sellers' fees, so it reduces surplus extractions.

The example confirms that platform may find it optimal to guarantee a reputation of one for some values of y. That is, it finds optimal to choose a price structure that completely discourages sellers to cheat. Reputation increases faster in y when the platform uses registrations fees; that is, lump-sum payments allow the platform to achieve higher levels of reputation at a lower cost in terms of surplus extraction. Along the same vein, the difference between the transaction fees charged to buyers and the transaction fees charged to sellers increases with y. This means that reducing the value of the transaction fees charged to sellers is a more efficient tool to increase reputation when lump-sum fees are also charged. So it suggest that in the presence of moral hazard on the seller's side, one should expect prices to be (more) skewed toward the buyer side (see Bolt and Tieman (2008)).

Finally, still according to Table 1, the transaction fees charged to sellers when the platform uses registration fees are lower than those when it does not, and conversely for the buyers' transaction fees. When the platform can charge registration fees to sellers, it needs not use

transaction fees to extract surplus, and reducing transaction fees improves incentives (increases reputation). On the buyers' side, one reason to keep fees low absent registration fees, is to ensure a high reputation. But when the platform can use registration fees, it has a substitute tool. As a consequence, the platform can increase the transaction fee charged to buyers to extract more surplus from them.

7   Conclusion

This paper studies simple pricing strategies available to a monopoly platform in the face moral hazard on one side of the (two-sided) market. It does so by developing a notion of equilibrium resting on rational expectations on the part of both sides of the market. Compared to a moral hazard-free environment, if the platform can only use linear prices it can alleviate, but not overcome, the moral hazard problem with appropriate distortion of its fees. If up-front payments are possible, the platform uniformly improves the outcome. By charging registration fees to sellers, it relieves the tension between ensuring a high reputation and extracting surplus. Registration fees allow the platform to simultaneously extract surplus from sellers and select higher valuation sellers with lower incentives to cheat. Improving on the moral hazard problem is important as it increases surplus extraction from *both* sides. In some cases it can even reach an outcome in which the moral hazard problem is entirely overcome.

The upfront payment extracts surplus from sellers while the on-going transaction fees are used to provide them with the appropriate incentives. Thus the equivalence (Rochet and Tirole (2006), Armonstrong (2006)) of these two forms of payments is broken, as they serve different purposes in our model. In addition, this result stands in contrast with Caillaud and Jullien (2003) or Hagiu (2006), where upfront fees are subsidies and transaction fees are used for surplus extraction.

An obvious extension of the present work is one in which platforms compete. A less evident, but critical one, consists in introducing a proper price-formation mechanism for the good sold, which we so far have abstracted from. Indeed, the manner in which trading parties determine the commodity's price affects whether two-sidedness arise. For example, in Caillaud and Jullien (2003), the intermediary is not a two-sided one without registration fees.

8 Appendix

8.1 Auxiliary Results

Lemma 4 *Suppose the platform can charge only transaction fees to buyers and sellers and let r^* denote the reputation associated with a stable equilibrium. Then, r^* is decreasing in both t^s and t^b.*

Proof. Fix transaction fees t^b and t^s. Fix also buyer participation $X^b > 0$. Given these fees and buyer participation, the proportion of sellers who trade and choose action l (denoted by L) and the proportion of sellers who trade and choose action h (denoted by H) that satisfy the steady state conditions of point 1 of Definition 1 are such that

$$H/(L+H) = \frac{[m + (1-m)\mu X^b \alpha]\{1 - F[s_h(t^s, X^b)]\}}{m[1 - F(s_l(t^s))] + (1-m)\mu X^b \alpha\{1 - F[s_h(t^s, X^b)]\}} \equiv \widetilde{k}(t^s, X^b). \qquad (5)$$

(For a characterization of the proportions N, L and H that satisfy the steady state conditions of point 1 of Definition 1, see the proof of Proposition 1). Next, define $k(t^s, t^b, r) \equiv \widetilde{k}(t^s, D^b(t^b, r))$. From Definition 1, it is clear that r^* is an equilibrium reputation when fees are t^s and t^b only if $k(t^s, t^b, r^*) = r^*$. Furthermore, note that k is differentiable and that if r^* is the reputation associated with a stable equilibrium, then $\partial k(t^s, t^b, r^*)/\partial r < 1$. Thus, to show that the reputation r^* associated with a stable equilibrium decreases with t^s and with t^b, it suffices to show that k increases with t^s and t^b for each $r \in [0, 1]$.

Consider first the case of t^b. Let Num and Den denote the numerator and denominator of $\widetilde{k}$ as given in (5), respectively. Then,

$$\frac{\partial k}{\partial t^b} = \frac{\alpha(1-m)\mu[Den - Num]\{[1 - F(s_h)](\partial D^b/\partial t^b) - f(s_h)D^b(\partial s_h/\partial X^b)(\partial D^b/\partial t^b)\}}{-mf(s_h)(\partial s_h/\partial X^b)(\partial D^b/\partial t^b)}{Den^2}$$

Now note that $\partial D^b/\partial t^b = -g(t^b - ry) < 0$, $\partial s_h/\partial X^b = -d[1 - (1-m)\delta]/(X^b)^2 < 0$ and $Den - Num = m[F(s_h) - F(s_l)] > 0$, which implies that $\partial k/\partial t^b < 0$.

Consider now the case of t^s. Note that $k = \widehat{k} \times [m + (1-m)\mu D^b \alpha]/[m + (1-m)\mu D^b \alpha \widehat{k}]$, where $\widehat{k} = [1 - F(s_h)]/[1 - F(s_l)]$. Since k increases in $\widehat{k}$, it suffices to show that $\widehat{k}$ is decreasing in t^s. We obtain that $\partial \widehat{k}/\partial t^s \leq 0$ if and only if

$$\frac{1 - F(s_h)}{f(s_h)} \frac{f(s_l)}{1 - F(s_l)} \leq \frac{\partial s_h/\partial t^s}{\partial s_l/\partial t^s}. \qquad (6)$$

Because F is log-concave, the hazard rate $f/(1-F)$ is increasing. Therefore, the left-hand-side of (6) is less than one, and (6) is satisfied since $\partial s_l/\partial t^s = \partial s_h/\partial t^s = 1$. ∎

Lemma 5 *Suppose the platform can charge both transaction and registration fees and let r^* denote the reputation associated with a stable equilibrium. Then, r^* is strictly increasing in T^s when $r^* < 1$.*

Proof. This proof follows the same steps of that of Lemma 4. Fix fees t^b, t^s and $T^s \geq 0$. Fix also buyer participation $X^b > 0$. Given these fees and buyer participation, the proportion of sellers who trade and choose action l (denoted by L) and the proportion of sellers who trade and choose action h (denoted by H) that satisfy the steady state conditions of point 1 of Definition 2 are such that

$$
\begin{aligned}
H/(L+H) &= \frac{[m + (1-m)\mu X^b \alpha][1 - F(\max\{s_R(t^s, T^s, X^b), s_h(t^s, X^b)\})]}{m\{1 - F[s_R(t^s, T^s, X^b)]\} + (1-m)\mu X^b \alpha[1 - F(\max\{s_R(t^s, T^s, X^b), s_h(t^s, X^b)\})]} \\
&\equiv \widetilde{k}(t^s, t^b, T^s, X^b).
\end{aligned}
$$

Next, define $k(t^s, t^b, T^s, r) \equiv \widetilde{k}(t^s, T^s, D^b(t^b, r))$. From Definition 2, it is clear that r^* is an equilibrium reputation when fees are t^s, t^b and T^s only if $k(t^s, t^b, T^s, r^*) = r^*$. Furthermore, note that k is differentiable when $k < 1$ and that if $r^* < 1$ is the reputation associated with a stable equilibrium, then $\partial k(t^s, t^b, r^*)/\partial r < 1$. Thus, to show that the reputation $r^* < 1$ associated with a stable equilibrium strictly increases with T^s, it suffices to show that k increases with T^s when $k < 1$. The fact that k increases with T^s when $k < 1$ follows from direct inspection of k, taking into account that (*i*) $s_R < s_h$ when $k < 1$ and (*ii*) $\partial s_R/\partial T^s > 0 \; \forall T^s > 0$ and $\partial s_R/\partial T^s|_{T^s=0^+} > 0$. ∎

8.2 Proofs of the Lemmas and Propositions in the Text

Proof of Lemma 1. First the *only if* part. Suppose that $s + d < t^s$. This implies that also $s < t^s$. Since $V(s)$ is non-negative because the seller can always choose not to trade at any given match, it is clear from direct inspection of each of the seller's payoffs inside the curly brackets on the right-hand side of (2) that not trading dominates both trading and choosing action l and trading and choosing action h.

Now the *if* part. Suppose that $s + d \geq t^s$. If it is also the case that $s \geq t^s$, then regardless of $V(s)$ a matched seller of type s is better off trading and choosing action h than not trading at

all–the third term inside the curly brackets in (2) is greater than the first. So in this case trading and choosing action h clearly dominates not trading. If instead $s < t^s$, the opposite happens, i.e., to trade and choose action h is dominated by not to trade at all. As a consequence, (2) collapses into $V(s) = \mu X^b \max\{0 + \delta V(s), s + d - t^s + (1 - \alpha)\delta V(s)\} + (1 - \mu X^b)\delta V(s)$. When $s + d - t^s \geq 0$, the solution to this equation is $V(s) = \mu X^b[s + d - t^s]/[1 - \delta(1 - \alpha\mu X^b)]$. This implies that $\delta V(s) \leq s + d - t^s + (1 - \alpha)\delta V(s)$, meaning that trading and choosing action l is better than not trading. ∎

Proof of Lemma 2. As a preliminary fact note that the right-hand side of (4) can be written as $\alpha(1 - m)\delta V^{ND}(s)$, where $V^{ND}(s) = [1/(1 - (1 - m)\delta)]\mu X^b[s - t^s]$ corresponds to the present value of the expected payoffs to a seller of type s who trades on the platform and never cheats. We now prove the result in the lemma.

First the *only if* part. We prove the equivalent statement that if condition (4) is not satisfied then the seller's optimal decision when matched is not to choose action h. Suppose that condition (4) is not satisfied. This means that the seller is better off deviating once and then, if not excluded from the platform, choosing the high action forever after. Since the problem is stationary, the seller's optimal decision given a match must be always the same. Thus (always) choosing action h given a match is not optimal.

Now the *if* part. Suppose that (4) holds. Since the right-hand side of (4) is given by $\alpha(1 - m)\delta V^{ND}(s)$ and by definition $V^{ND}(s) \leq V(s)$, this implies that $d \leq \alpha(1 - m)\delta V^{ND}(s) \leq \alpha(1 - m)\delta V(s)$ and therefore taking the low action cannot be optimal to the seller. ∎

Proof of Proposition 1. Consider an arbitrary pair of fees (t^b, t^s). Fix $r \in [0, 1]$. Let $X^b = D^b(t^b, r)$ so that point 2 of Definition 1 is satisfied. Let $x_l = s_l(t^s)$ and $x_h = s_h(t^s, X^b)$. Finally, let

$$N = \frac{[m + (1 - m)\mu X^b \alpha]F(x_l)}{m + (1 - m)\mu X^b \alpha[1 - F(x_h) + F(x_l)]} \tag{7}$$

$$L = \frac{m[F(x_h) - F(x_l)]}{m + (1 - m)\mu X^b \alpha[1 - F(x_h) + F(x_l)]} \tag{8}$$

$$H = \frac{[m + (1 - m)\mu X^b \alpha][1 - F(x_h)]}{m + (1 - m)\mu X^b \alpha[1 - F(x_h) + F(x_l)]} \tag{9}$$

$$S = E\left\{\frac{1}{m} - \frac{(1 - m)\mu X^b \alpha[F(x_h) - F(x_l)]}{m(m + (1 - m)\mu X^b \alpha)}\right\} \tag{10}$$

Given X^b, x_l, and x_h, these values of N, L, H, and S constitute the unique solution to

the system of equations defined by the set of conditions in point 1 of Definition 1. Clearly, $\langle X^b, N, L, H, S, r \rangle$ satisfies all the conditions in Definition 1 if in addition $r = H/(L+H)$.

Now, for any $r \in [0,1]$, let $X^b = D^b(t^b, r)$, $x_l = s_l(t^s)$, $x_h = s_h(t^s, X^b)$ and N, L, H, and S be as in (7)-(10). Continuity of: (i) $D^b(t^b, r)$ in r, (ii) $s_h(t^s, X^b)$ in X^b and (iii) F, implies that $H/(L+H)$ is continuous in r. Moreover, since $x_h > x_l$, then $L > 0$, which implies that $0 < H/(L+H) < 1$. Since $H/(L+H)$ is continuous in r and $0 < H/(L+H) < 1$, it follows by Brower's Fixed Point Theorem that for some $r \in [0,1]$, $r = H/(L+H)$. Furthermore, since $0 < H/(L+H) < 1$, that value of $r \in (0,1)$. ∎

Proof of Proposition 2. The proof consists of four steps. In the first two steps, we characterize buyers' and sellers' registration and transaction decisions. In the third step, we adjust Definition 1 to incorporate the fact that buyers and sellers must register to trade in the platform. In the fourth step, we show the irrelevance of buyers' registration fees for equilibrium outcomes.

Step 1: Buyers' registration and transaction decisions. A buyer registers with the platform if her expected value of trade exceeds the registration fee. Given reputation r, seller participation $X^s > 0$, transaction fee t^b and total number of sellers $S > 0$, that expected value of trade for a buyer of type b is $\mu S X^s [b + ry - t^b]$. The product $\mu S X^s$ corresponds to the probability of being matched with a seller who trades, and $b + ry - t^b$ corresponds to the expected payoff from a transaction. Thus, the buyer registers if and only if $\mu S X^s (b + ry - t^b) - T^b \geq 0$. Let $b_R(t^b, T^b, r, X^s, S) \equiv \min \{ b \in \mathbb{R} : \mu X^s S(b + ry - t^b) - T^b \geq 0 \}$. Buyers of type $b \geq b_R(t^b, T^b, r, X^s, S)$ register whereas others do not. The up-front payment only influences the registration decision, not whether to trade if matched with a seller. A buyer registers *and* trades if and only if $b \geq \max\{t^b - ry, b_R\}$. We then redefine buyer participation as the proportion of buyers who register and accept to trade. Hence, when buyer registration is required, buyer participation is given by $D^b(t^b, T^b, r, X^s, S) \equiv 1 - G(\max\{t^b - ry, b_R(t^b, T^b, r, X^s, S)\})$.

Step 2: Sellers' registration and transaction decisions. A seller registers with the platform if her expected value of trade exceeds the registration fee. That is, if $V(s) - T^s \geq 0$. Conditional on being registered, a seller's decision of whether to trade and of action a is independent of the registration fee. Specifically, a seller of type s: does not trade if $s < s_l(t^s)$; trades and chooses action l if $s_l(t^s) \leq s < s_h(t^s, X^b)$; and trades and chooses action h if $s \geq s_h(t^s, X^b)$. This implies that given t^s and $X^b > 0$, $V(s) = 0 \; \forall s \leq s_l(t^s)$ and $V(s) > 0$ and increasing $\forall s > s_l(t^s)$. Thus, given t^s, $X^b > 0$ and $T^s > 0$, we can define $s_R(t^s, T^s, X^b)$ as the value of s such that $V(s) - T^s \geq 0$. Sellers of type $s \geq s_R(t^s, T^s, X^b)$ register with the platform, whereas the others

do not. For convenience, let $s_R(t^s, T^s = 0, X^b) = s_l(t^s)$. Note that $s_R(t^s, T^s, X^b) > s_l(t^s)$ $\forall T^s > 0$. As in the case of buyer participation, we redefine seller participation as the measure of sellers who register and trade.

Step 3: Equilibrium definition with registration fees. Given transaction fees t^b and t^s and registration fees $T^s \geq 0$ and $T^b \geq 0$, an equilibrium is a tuple $\langle X^{b*}, N^*, L^*, H^*, S^*, r^* \rangle$ such that: (1) the number of total sellers and the proportions of those sellers who do not trade, trade and cheat, and trade and do not cheat remain constant, i.e., (1.a) $EF(\max\{s_l(t^s), s_R(t^s, T^s, X^{b*})\}) = mN^*S^*$, (1.b) $E[F(\max\{s_R(t^s, T^s, X^{b*}), s_h(t^s, X^{b*})\}) - F(\max\{s_l(t^s), s_R(t^s, T^s, X^{b*})\})] = (m + (1-m)\mu X^{b*}\alpha)L^*S^*$, (1.c) $E[1 - F(\max\{s_R(t^s, T^s, X^{b*}), s_h(t^s, X^{b*})\})] = mH^*S^*$ and (1.d) $N^* + L^* + H^* = 1$; (2) buyer participation is consistent with r^* and t^b and T^b, i.e., $X^{b*} = D^b(t^b, T^b, r^*, L^* + H^*, S^*)$; and (3) reputation is consistent with seller behavior, i.e., $r^* = H^*/(L^* + H^*)$.

Step 4: Showing that for any equilibrium when $T^b > 0$, there exists an outcome equivalent equilibrium with $T^b = 0$. Suppose that $\langle X_0^b, N_0, L_0, H_0, S_0, r_0 \rangle$ constitutes an equilibrium when fees are $\langle t_0^b, T_0^b > 0, t_0^s, T_0^s \rangle$. Let X_0^s denote seller participation in this equilibrium, so that $X_0^s = L_0 + H_0$. We next show that $\langle X_0^b, N_0, L_0, H_0, S_0, r_0 \rangle$ is also an equilibrium when fees are $\langle t_1^b, T_1^b = 0, t_0^s, T_0^s \rangle$, where $t_1^b = t_0^b + T_0^b/\mu S_0 X_0^s$. First, note that $b_R(t_1^b, T_1^b = 0, r_0, X_0^s, S_0) = b_R(t_0^b, T_0^b, r_0, X_0^s, S_0)$. Second, note that since $T_0^b > 0$ and $T_1^b = 0$, it follows that $t_0^b - r_0 y < b_R(t_0^b, T_0^b, r_0, X_0^s, S_0)$ and that $t_1^b - r_0 y = b_R^*(t_1^b, T_1^b, r_0, X_0^s, S_0)$. It follows from the two previous observations that $D^b(t_1^b, 0, r_0, X_0^s, S_0) = D^b(t_0^b, T_0^b, r_0, X_0^s, S_0)$. From point (2) of the definition of equilibrium (see Step 3 of this proof) $X_0^b = D^b(t_0^b, T_0^b, r_0, X_0^s, S_0)$. Thus, $X_0^b = D^b(t_1^b, 0, r_0, X_0^s, S_0)$ and satisfies point 2 of the definition of equilibrium when fees are $\langle t_1^b, T_1^b = 0, t_0^s, T_0^s \rangle$. Moreover, given X_0^b, it is clear that the values N_0, L_0, H_0, S_0, and r_0 also satisfy points 1 and 3 of the definition of equilibrium when transaction fees on the seller side are t_0^s and T_0^s. This completes the proof that $\langle X_0^b, N_0, L_0, H_0, S_0, r_0 \rangle$ also constitutes an equilibrium when fees are $\langle t_1^b, T_1^b = 0, t_0^s, T_0^s \rangle$.

It remains to show that the platform's profits are the same in both equilibria. Denote the platform's (expected) period profit in the equilibrium with fees $\langle t_0^b, T_0^b > 0, t_0^s, T_0^s \rangle$ by π_0 and in

the equilibrium with fees $\langle t_1^b, 0, t_0^s, T_0^s \rangle$ by π_1. We obtain that

$$
\begin{aligned}
\pi_1 &= \mu X_0^s X_0^b S_0 \left[t_0^s + t_1^b \right] + X_0^s T_0^s E \\
&= \mu X_0^s X_0^b S_0 [t_0^s + t_0^b + \frac{T_0^b}{\mu X_0^s S_0}] + X_0^s T_0^s E \\
&= \mu X_0^s X_0^b S_0 [t_0^s + t_0^b] + X_0^b T_0^b + X_0^s T_0^s E \\
&= \mu X_0^s X_0^b S_0 [t_0^s + t_0^b] + X_0^b T_0^b + X_0^s T_0^s E = \pi_0.
\end{aligned}
$$

This concludes the proof. ∎

Proof of Proposition 3.   Analogous to that of Proposition 1 and therefore omitted. ∎

Proof of Lemma 3.   Given y, suppose that t^s, t^b and $r < 1$ is a solution to the above problem. We use three necessary conditions for t^s, t^b and $r < 1$ to be an optimum and show that when y is sufficiently large they can hold simultaneously.

1. $\partial \Pi(t^s, t^b, r)/\partial r \le 0$. This is equivalent to $\mu(E/m)(t^s + t^b)\frac{1 - F(t^s)}{r}[g(t^b - ry)y - \frac{1 - G(t^b - ry)}{r}] \le 0$. Since in a any solution $(t^s + t^b)\frac{1 - F(t^s)}{r} > 0$ (otherwise profit would be zero), this implies that

$$
ry \le \frac{1 - G(t^b - ry)}{g(t^b - ry)}. \tag{11}
$$

2. $\partial \Pi(t^s, t^b, r)/\partial t^b = 0$. This is equivalent to

$$
t^s + t^b = \frac{1 - G(t^b - ry)}{g(t^b - ry)}. \tag{12}
$$

3. $\partial \Pi(t^s, t^b, r)/\partial t^s \ge 0$. That is, a decrease in t^s cannot lead to an increase in profits. Note that $[1 - F(t^s)]/[1 - F(t^s - d)]$ increases with t^s because of log-concavity of F, which means that a if t^s and r satisfy the constraint of the problem that constraint will continue to hold of we decrease t^s. The condition $\partial \Pi(t^s, t^b, r)/\partial t^s \ge 0$ is equivalent to $\mu(E/m)[1 - G(t^b - ry)](1/r)[1 - F(t^s) - f(t^s)(t^s + t^b)] \ge 0$. Which implies that

$$
(t^s + t^b) \le \frac{1 - F(t^s)}{f(t^s)}. \tag{13}
$$

Since $[1 - F(x)]/[1 - F(x - d)]$ is bounded away from zero, r is also bounded away from zero. From this and (11) it follows that for y sufficiently high, $t^b - ry$ must be sufficiently low

since $(1 - G(x))/g(x)$ is decreasing in x by log-concavity of G. Combining (11) and (12), we obtain that $ry \leq t^s + t^b$. This is equivalent to $t^s + t^b - ry \geq 0$. From this and the fact that for y high $t^b - ry$ is sufficiently low, it follows that for y high, t^s must be sufficiently high or at least cannot be too low. Now combining (11), (12) and (13), we obtain that $ry \leq [1 - F(t^s)]/f(t^s)$, which means that for y high, t^s must be sufficiently low, since $[1 - F(t^s)]/f(t^s)$ is decreasing by log-concavity of F. Thus, for y sufficiently high, one of the above required conditions must be violated. ∎

Proof of Proposition 4. Consider an arbitrary pair of transaction fees (t^b, t^s). Let T^s be such that $s_R(t^s, T^s, D^b(t^b, 1)) = s_h(t^s, D^b(t^b, 1))$. Denote that value of T^s by T_0^s. Note that T_0^s exists, since (i) $s_R(t^s, T^s, X^b)$ is continuous in T^s, (ii) $s_R(t^s, 0, X^b) = s_l(t^s) < s_h(t^s, X^b)$ $\forall X^b > 0$ and (iii) $\lim_{T^s \to +\infty} s_R(t^s, T^s, X^b) = +\infty$ $\forall X^b > 0$. Next, simply note that if fees are t^b, t^s and T_0^s, then $r = 1$, $X^b = D^b(t^b, 1)$, $N = F(s_R(t^s, T^s, D^b(t^b, 1)))$, $L = 0$, $H = 1 - N$ and $S = E/m$ satisfy all the conditions of Definition 2. ∎

Proof of Proposition 5. Suppose that the solution to the benchmark case is $\langle t_0^s, t_0^b, r = 1 \rangle$. Let s_0 denote the corresponding threshold type of seller above which sellers trade. Because $r = 1$, only sellers who take the high action trade, which implies that $s_0 = t_0^s$. Thus seller participation is given by $X_0^s = 1 - F(t_0^s)$ and in steady state there are E/m sellers. Buyer participation is given by $X_0^b = 1 - G(t_0^b - y)$. The discounted value of the platform's profit associated with the solution of the benchmark case is $\Pi_0 = (1/(1 - \delta))\mu X_0^s X_0^b (t_0^b + t_0^s) E/m$.

Under moral hazard, this allocation can be implemented as follows. Choose the buyers' transaction fee $t_1^b = t_0^b$. Choose the sellers' transaction fee such that sellers of type above s_0 prefer to choose the high action. Using the no-cheating condition (4), this transaction fee, which we denote t_1^s, satisfies

$$d = \alpha \frac{(1 - m)\delta}{1 - (1 - m)\delta} \mu X_0^b [s_0 - t_1^s] \tag{14}$$

and therefore $s_h(t^s, X_0^b) = s_0$. Now select the registration fee T_1^s such that only sellers of type above s_0 trade. That is, choose

$$T_1^s = V(s_0) = \frac{1}{1 - (1 - m)\delta} \mu X_0^b [s_0 - t_1^s] \tag{15}$$

in which case only sellers of type above s_0 register (and trade). Thus seller participation $X_1^s = X_0^s$. All those who trade choose the high action, which implies a reputation of 1 and a

total number of sellers of E/m. Since $t_1^b = t_0^b$ buyer participation also equals the benchmark's: $X_1^b = X_0^b$. All that remains to show is that the platform's profit is the same as in the solution of the benchmark. It consists of the revenues associated with the transaction fees and the revenues associated with the registration fees. Specifically,

$$
\begin{aligned}
\Pi_1 &= \frac{\mu}{1-\delta} X_1^s X_1^b (t_1^b + t_1^s) E/m + T_1^s X_1^s E/m + \sum_{\tau=1}^{\infty} m \delta^\tau T_1^s X_1^s E/m \\
&= \frac{\mu}{1-\delta} X_0^s X_0^b (t_0^b + t_1^s) E/m + T_1^s X_0^s \left(1 + \frac{\delta m}{1-\delta} \right) E/m \\
&= \frac{\mu}{1-\delta} X_0^s X_0^b (t_0^b + t_1^s) E/m + T_1^s X_0^s \left(\frac{1 - \delta(1-m)}{1-\delta} \right) E/m \\
&= \frac{\mu}{1-\delta} X_0^s X_0^b (t_0^b + t_1^s) E/m + \frac{\mu}{1 - \delta(1-m)} X_0^b [s_0 - t_1^s] X_0^s \left(\frac{1 - \delta(1-m)}{1-\delta} \right) E/m \\
&= \frac{1}{1-\delta} \mu X_0^s X_0^b (t_0^b + t_0^s) E/m \\
&= \Pi_0
\end{aligned}
$$

where the second equality follows from the fact that $X_1^s = X_0^s$ and $X_1^b = X_0^b$; the fourth equality follows by substituting T_1^s by its value as given in (15) and noting that $s_0 = t_0^s$. ∎

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