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Sasan Bakhtiari

School of Economics
Australian School of Business
UNSW Sydney NSW 2052 Australia
http://www.economics.unsw.edu.au

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Outsourcing with Heterogeneous Firms

Sasan Bakhtiari
Australian School of Business
University of New South Wales
Email: s.bakhtiari@unsw.edu.au

Abstract

A general framework for the study of outsourcing is introduced that incorporates dynamics and heterogeneity among both upstream and downstream producers to mimic an exit approach (Hirschman, 1970) to building vertical relations. The environment is one of search friction and incomplete contracts, where final-good producers require a specialized input and, upon matching with a supplier, can only contract the quantity of input. The results imply an assorted matching between producers and suppliers, so that more productive producers pair with more productive suppliers in the long run. It is shown that most efficient producers have some propensity to outsource, but only when there is a thick enough density of highly productive suppliers. Average employment in this model might increase or decrease with outsourcing, which is an observed pattern in the data. Some other diversities in plant-level behavior are also present in the results.

Keywords: Outsourcing, Productivity, Heterogeneity, Search Friction, Incomplete Contracts, Exit Strategy.

JEL Code: D21, D23, L21, L24.

1 Introduction

A decision to outsource is often a complicated and risky one. One major consideration in making this decision involves dealing with uncertainties that shroud a supplier's performance and the cost savings that will eventually follow. In building a new vertical relation, a specialized producer of a final consumption good would make a take-it or leave-it offer to a supplier; a contract that describes the nature of input needed, the required quantity of input and its price, and possibly the length of the contract. Helper (1991) especially finds such contracts a plausible assumption for the U.S. auto industry. For completeness, the contract would also include a clause for a "clean" exit strategy, when the match does not work according to expectations[1]. When an offer is made, a supplier can then decide if she is able to meet the requirements based on her current level of efficiency and costs. The contract should also look attractive and valuable enough to the supplier for a successful match. Serious problems arise in the relationship when a supplier is much larger and much more efficient than the producer[2]. In Hirschman (1970) terminology, both the producer and the supplier exercise an *exit* approach with regard to relation building in a specialized environment: a final good producer drops suppliers that cannot support the level of production the producer wants to achieve and searches for another supplier. In turn, a very large and efficient supplier can also walk away from an unattractive contract that will hold her back from seeking a more valuable contract and larger profits.

This paper tries to mimic the process described above by focusing on the exit approach to building vertical relations among heterogeneous and specialized producers of a final consumption good, when the productivity of middle suppliers is also heterogeneous and unobservable ex ante. This two-layered heterogeneity is particularly useful in generating substantial diversity in outsourcing decisions and patterns that would follow. In this setting, unlike Grossman & Helpman (2002), suppliers do not dictate producers the level of output while creating a serious hold-up problem, but make a decision when offered a take-it or leave-it contract from a producer who decides the scale of her operation independently and optimally.

Future search is also made possible, so both producers and suppliers form their own outside options when evaluating a potential contract. Two certain behaviors can arise from such setting: first, producers and suppliers become selective about whom they want to pair with. This selection process leads to a rough assortment of productivity among producers and suppliers that stay matched together in the long run, so that, on average, more efficient producers pair with more efficient suppliers. Second, producers might hesitate entering the search market in fear of having to put up with a not-so-efficient supplier, or, since search embodies opportunity costs of deviating efforts from production, having to search for a very

[1]For examples see "Avoid Future Trouble: Think Exit with Outsourcing Contracts", *Supplier Selection and Management Report*, Feb.2003, P.5.

[2]A supplier can become a nuisance or exploiting when she thinks she is not getting enough from her contract, according to "How to Manage Suppliers When They Are Much Larger Than You", *Supplier Selection and Management Report*, May 2005, P.9.

long period of time, maybe forever, for an attractive match.

The general framework borrows from the classic model of Grossman & Helpman (2002), where specialized producers enter market freely and compete monopolistically. Each producer needs a specific intermediate input that can be obtained either by internalizing its production (vertical integration) or by forming a one-to-one relation with a specialized middle supplier (outsourcing). Accordingly, the production of final good takes two processes: (i) a core process, "final production" or assembly, that has to be internalized at all times or the producer ceases to exist, and (ii) an intermediate process, "middle production" or parts manufacturing, that can be either internalized or contracted out to a middle supplier[3]. Producers are differentiated in each process by making idiosyncratic draws of their labor productivities, independently of each other and independently for each process within firm. The total labor productivity of an integrated producer is then an aggregation of productivities for the two processes.

Vertical integration distracts focus and requires a producer to use extra management resources to coordinate and plan the whole production process, hence, internalizing is costly. Specialized producers can alternatively form a relation with a middle supplier and procure the required input in that way. In the latter case, the supplier agrees to specialize in the production of the specific input and the producer forfeits her ability to input production. Any break up of the relation after specialization has fully taken place results in both producer and supplier exiting market.

In return, producers get a boost in their productivity on final process by cutting managerial costs. This boost also symbolizes the fact that outsourcing lets producers focus on their core competencies (e.g. quality, unique features and presentation of product) and reinvest in and improve their technology on the final process. Middle suppliers are also assumed, on average, more efficient in the production of intermediate input than producers, to reflect their higher degree of sophistication in their field. This assumption, however, is not the most crucial element in this model, and most results do not depend merely on that.

On the flip side, there are two sources of transaction costs if a match is sought: search friction and uncertainty about the productivity of suppliers. Going on search does not guar-

[3] Antras & Helpman (2004) use "manufacturing" and "headquarter services" to label upstream and downstream processes in an international setting.

antee a match, and a match can be broken by uninterested supplying party. Contractual incompleteness described by Grossman & Hart (1986) is also present, so that producers can only contract the quantity of input they need, but the quality cannot be verified by a third party. Since specializtion practically ties both producer and supplier into a permanent relationship, suppliers have incentive to offer low quality input, with zero value to production, and increase their profit margin. To give a supplier the incentive to deliver high-quality input and make the long-term perspective of the relationship profitable, as in Grossman & Helpman (2002), both parties renegotiate their shares of total revenue through a Nash bargaining after input is ready.

The outcome of matches in this model is equivalent to both producers and suppliers exercising the exit approach. The more efficient producers do not stay matched to less efficient suppliers, mainly because the supplier cannot handle the quantity of input demanded and stay profitable at the same time. More efficient suppliers also drop less efficient producers and go on search again, since their option value is more promising and profitable. This two-sided strategy is summarized in a supplier forming a range of acceptable productivities she should be contented with, and the decision to keep or break a match depends on the productivity of the matched producer falling into that interval. In the long run, on average, more efficient producers and suppliers stay matched, while other matches break, so that the expected productivity of producers increases monotonically with the expected productivity of their suppliers.

As suppliers get very efficient, however, they become less selective and would match to any producer that is reasonably productive, mostly because the density of high-productivity producers declines. The monotonic relation discussed above becomes concave as a result. On the other hand, producers have the option to internalize their input production, and, in the presence of search frictions and uncertainty about suppliers' productivity, the most efficient producers would outsource only if there is a dense enough distribution of high-productivity suppliers, so that the expected search time to forming a successful match is reasonably short for them.

In response to outsourcing, the model manages to make producers either expand or shrink in employment size. Outsourcing encompasses two opposing effects on the size of a producer.

The productivity boost makes them willing to expand, but they are also sending some jobs away. The final direction of change for average employment, as a result, stays ambiguous and directly depends on the relative levels of costs and benefits from outsourcing. On the other hand, average output is most likely higher after outsourcing opens, although the model does not rule out lower average output under some very exceptional circumstances.

The structure of the model also allows for two integrated producers with identical productivities to behave differently, so that one of them stays integrated and the other one goes on search for a supplier. The key to this difference in behavior is the fact that these two producers have the same *total productivity* prior to outsourcing, but the one with higher productivity in its final process benefits greatly from outsourcing and goes on search.

The rest of the paper proceeds as such: Next section reviews the general literature on outsourcing and multi-nationalism. Section 3 sets up the theoretical model and describes the outsourcing mechanism. Section 4 discusses the theoretical results. Section 5 concludes the paper.

2 Related Literature

Earlier studies of outsourcing have been mostly in the context of trade and cross-regional or cross-country differences. Extensions of Heckscher-Ohlin model by Helpman (1984) and Helpman (1985) describe conditions under which inter- and intra-firm trade is possible across countries. Grossman & Helpman (2005) present a north-south model to explain the geographical aspects of domestic versus international outsourcing.

Grossman & Hart (1986) and Hart & Moore (1990) show how contractual incompleteness leads to certain ownership issues within both integrated and outsourcing firms. Based on those results, Grossman & Helpman (2002) relate outsourcing behavior to the costs borne of incomplete contracts and search frictions. Briefly, if benefits from outsourcing surpass its implied costs, the whole industry outsources, and stays integrated otherwise. Antras (2003) and Antras & Helpman (2004) use similar approaches to explain different patterns of integration versus outsourcing. Grossman & Helpman (2004) use ability to observe managerial efforts to get similar results. The effect of trade openness on outsourcing behavior is described by McLaren (2000). He argues that, in a closed economy, suppliers have very weak bargaining power, giving them the incentive to integrate. As trade opens, this position changes and causes many suppliers to choose independence, increasing the pervasiveness of outsourcing. Finally, Naghavi & Ottaviano (2009a) study the effect of offshoring (international outsourcing) on the rate of innovation using a dynamic model and show that offshoring can weaken feedback links from production to R&D and reduce innovation rates.

Antras & Helpman (2004) and Grossman & Helpman (2004) are two papers that incorporate firm heterogeneity into outsourcing decision. Assche & Schwartz (2009) generate similar results by relaxing the input specificity assumption. Antras & Helpman (2006) generalize the framework from Antras & Helpman (2004) to include varying levels of contractability across countries. Naghavi & Ottaviano (2009b) study dynamic implications of having heterogeneity in the production of input. This paper extends some of those results by adding a two-layered heterogeneity in productivities, which helps to study the organization of an industry with outsourcing possibilities in much finer details. Also, the exit approach to forming relations, used in this work, seems to model a managers outsourcing strategy more closely.

3 Theoretical Setup

3.1 Consumers

There is a representative consumer that gains utility over a continuum of consumption good varieties indexed by j. The utility function of this consumer is

$$U = \left(\int_{j \in J} y_j^{\alpha} dj \right)^{1/\alpha}, \tag{1}$$

where J is the set of varieties being produced, and $\alpha < 1$. As a result, the elasticity of substitution among varieties is constant and equal to $1/(1-\alpha) > 1$. Dixit & Stiglitz (1977) show that such economy can be represented by an aggregate output index $Y = U$ and the associated aggregate price index, P, defined as

$$P = \left(\int_{j \in J} p_j^{\frac{\alpha}{\alpha - 1}} dj \right)^{\frac{\alpha - 1}{\alpha}}. \tag{2}$$

The consumer's optimal demand for each variety can then be found as a function of P and Y, which is

$$p_j = A y_j^{-(1-\alpha)}, \tag{3}$$

where $A = PY^{1-\alpha}$ is an aggregate index.

3.2 Integrated Firm

There is one sector, and each firm in this sector produces a distinct variety $j \in J$ of the consumption good. Since the space of varieties is continuous, the probability of two firms producing the same variety is zero, hence, j indexes both variety and the corresponding firm.

To produce variety j, a firm has to first produce a specialized intermediate input and then adapt it to the distinct variety. The production of intermediate input and final good are both constant returns processes with labor as the only input. Specialized producers are differentiated in their efficiency in producing the intermediate input and their efficiency in producing the final good. Specifically, producer j needs $1/\phi_j$ units of labor to produce one unit of input. ϕ_j is randomly drawn from a known cumulative distribution $F(\phi)$ with support $\phi \geq 0$ and is observed after sinking all entry costs. Every unit of input is then transformed into one unit of a specialized final good using $1/\lambda_j$ units of labor. λ_j is also randomly drawn from a known cumulative distribution $G(\lambda)$ with support $\lambda \geq 0$ and observed after entry. The pair (λ_j, ϕ_j) defines the overall production efficiency of firm j.

Total labor required by the integrated producer j to produce one unit of final good can be conveniently described in terms of a *total labor productivity*, defined as[4]

$$\chi_j = \left(\frac{1}{\lambda_j} + \frac{1}{\phi_j} \right)^{-1}. \tag{4}$$

In the remainder, index j will be dropped where it is not causing confusion.

For the moment, focus on the one-period behavior of a specialized producer in steady state. Wage rate for the employed labor is fixed in steady state and, without loss of generality, normalized to one henceforth. I am also abstracting from fixed costs and time variations in productivities to keep the model tractable and help single out important results related to

[4] χ_j is practically the harmonic mean of λ_j and ϕ_j.

outsourcing[5]. Dynamics and entry costs are discussed later in Section 3.4.

Producers decide their production level by maximizing the profit function $\pi_V(\chi) = Ay_j^\alpha - \frac{y}{\chi}$; subscript V referring to vertical integration. Solving the first-order condition gives a producer's optimal output (x_V), price (p_V), and profit (π_V) as

$$x_V(\chi) = (\alpha\chi A)^{\frac{1}{1-\alpha}}, \tag{5}$$

$$p_V(\chi) = 1/(\alpha\chi), \tag{6}$$

$$\pi_V(\chi) = \left(\frac{1}{\alpha} - 1\right)(\alpha A)^{\frac{1}{1-\alpha}}\chi^{\frac{\alpha}{1-\alpha}}, \tag{7}$$

Note that, because of the constant-elasticity demand, producers charge a constant $\frac{1}{\alpha} - 1$ markup over their production costs.

3.3 Outsourcing Firm

A specialized producer can also procure the required input from a middle supplier. Upon finding a supplier, the producer offers her a contract describing the specification and quantity of input needed. The supplier has the option to accept this production plan or to break the match. If plan is accepted, the supplier specializes as needed and production goes ahead with returns described below. The producer then forfeits her ability to input production and relies solely on the supplier for input delivery. Thus, after specialization has fully taken place, the producer and her supplier are practically tied in a permanent relation. Breaking the match at this stage results in both the producer and her supplier exiting market. The input produced in this stage is useless outside the specific relation, and a high quality input is needed for production; low quality (defective) input is cheap to produce but has zero value to production. On the other hand, if the match fails right in the beginning and before specialization, both parties make zero profit in that period and continue search in the next period. For the moment, focus on the one-period operation of producers and suppliers.

Following Grossman & Hart (1986), I consider the relation between a producer and its supplier to be governed by contractual incompleteness. producers can contract the required

[5]Including nonzero fixed costs merely introduces a cutoff productivity that limits the range of "operational" productivities from below. However, the theoretical implications concerning outsourcing behavior would not be affected.

quantity of input, but if they make an ex ante commitment to price, since the quality of the intermediate good is not verifiable by a third party and the producer and supplier are tied in a permanent relationship, the supplier has every incentive to offer a low quality input and increase its own margin of profit. Ex post, however, both parties can renegotiate their claims to final revenue through a Nash bargaining. The supplier has already specialized in an input that is useless outside the relation, hence, she is left with a weak bargaining power at that point. Let the middle supplier's bargaining power be $\omega \in (0, 0.5)$, reflecting this weak position. Middle supplier's nonzero claims to final revenue provides her with the incentive to produce high-quality input.

Suppliers are also heterogeneous in their productivities, ϕ_o. ϕ_o is drawn from a cumulative distribution $H(\phi_o)$ with support $\phi_o \geq 0$ and $E[\phi_o] > E[\phi]$; the intermediate industry is more sophisticated in input production and can offer higher productivity on average. The productivity of an individual supplier is unobservable by final producers until after a match is formed and the possibility of further search in that period is foregone. This uncertainty is an additional risk factor producers should take into account when making decision about their vertical structure.

The benefit from outsourcing is that producers get a boost in their productivity on final process as a porportional increase in λ_j by a factor $\mu (> 1)$. This increase models a reduction in fixed costs as less managers are needed to oversight and coordinate production. It also symbolizes the producer focusing on her core competencies, such as developing unique features that make the product more attractive, and accounts for improvement in final productivity as a result of the producer reinvesting some of the extra revenue to improve her current technology.

An outsourcing decision starts by a specialized producer specifying the quantity of input $x (= y)$ she needs. The profit function for specialized producer, given a match is formed, is $\pi_S(\lambda) = (1 - \omega)Ay^\alpha - \frac{y}{\mu\lambda}$; subscript S referring to specialized producer. Maximizing profit gives the optimal price (p_O) and demand for input (x_O) as

$$x_O(\lambda) = \big((1 - \omega)\alpha A\mu\lambda\big)^{\frac{1}{1-\alpha}}, \tag{8}$$

$$P_O(\lambda) = 1/\big((1 - \omega)\alpha\mu\lambda\big). \tag{9}$$

The optimal profit for specialized producer is

$$\pi_S(\lambda) = \left(\frac{1}{\alpha} - 1\right)\left((1-\omega)A\alpha\right)^{\frac{1}{1-\alpha}}(\mu\lambda)^{\frac{\alpha}{1-\alpha}}. \tag{10}$$

The profit for a middle supplier of productivity ϕ_o matched to a producer of productivity λ is $\pi_M(\lambda, \phi_o) = \omega A x_O(\lambda)^\alpha - \frac{x_O(\lambda)}{\phi}$; subscript M referring to middle supplier. By delivering $x_O(\lambda)$, supplier makes a profit of

$$\pi_M(\lambda, \phi_o) = \left(\frac{\omega}{\alpha(1-\omega)} - \frac{\mu\lambda}{\phi_o}\right)\left((1-\omega)\alpha A\right)^{\frac{1}{1-\alpha}}(\mu\lambda)^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

3.4 Dynamics

The operation of integrated and outsourcing producers specified above repeats in every period, and future values are discounted for both producers and suppliers by a factor δ, where $\delta \in (0, 1)$. In every period, there is a failure rate ξ for a specialized producer which forces the producer, along with the matched supplier if outsourcing, to exit. ξ is assumed non-degenerate and exogenously given. In what follows, ξ mostly behaves like another discount factor, in addition to δ. Therefore, to simplify matters, I define and use $\hat{\delta} = \xi\delta$ where appropriate.

Producers and suppliers find each other through a search process. Let v be the number of producers that enter as vertically integrated, s be the number of producers that enter seeking to outsource, and m be the number of middle suppliers that enter. Number of matches formed in a period is $\eta(s, m)$, which is constant returns in s and m. All producers seeking a supplier are equally likely to find one, hence, the probability of a match is $\eta(s, m)/s$. With constant returns matching, this probability can be stated as $\eta(r) = \eta(1, r)$, where $r = m/s$. On the other hand, the probability of a supplier finding a match is $\eta(s, m)/m$ or $\eta(r)/r$. Once a match is formed, suppliers can decide if they want to keep the match or break it and go on search again.

I assume a long-run situation where the dynamics of the industry has settled on a steady state path. As a result, A, r, v, s, and m are time invariant. Specialized producers enter freely into market, paying a fixed entry cost $c_e > 0$ to cover their start-up costs, such as setting up the physical plant and penetrating market. Suppliers also enter market freely,

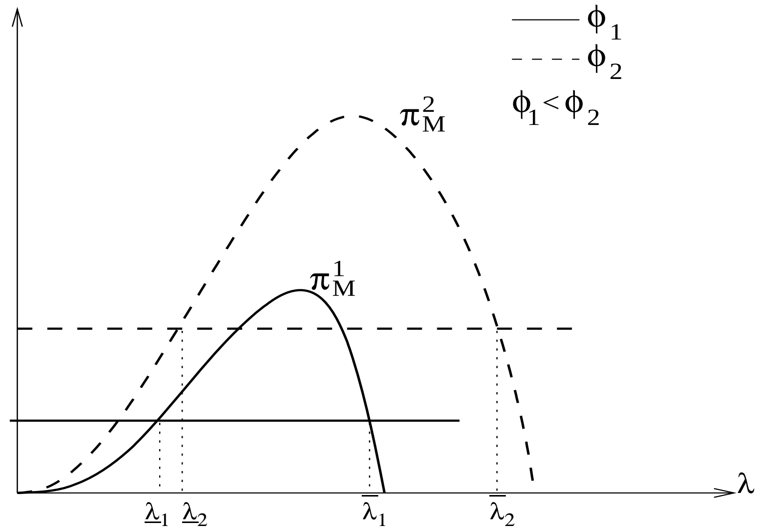

Figure 1: The profit function for a middle supplier and the range of λ productivities for building successful matches. Two profit function π^1_M and π^2_M are plotted corresponding to productivities ϕ_1 and ϕ_2 ($\phi_1 < \phi_2$).

paying a fixed entry cost $c^m_e > 0$. Entry costs for producers and suppliers need not be the same.

3.5 Middle Supplier's Decision

The profit function (11) is not necessarily positive or, with the possibility of further search, in the long-run interest of the supplier. In particular, if supplier expects higher profits from future search, it will break the match. To characterize the nature of this decision, note that, in keeping a match, the present value of supplier is $\pi_M(\lambda, \phi_o)/(1 - \hat{\delta})$. Figure 1 shows a supplier's profit as a function of λ and for two different values of $\phi_o = \phi_1,\ \phi_2\ (\phi_2 > \phi_1)$. Clearly when the producer is too unproductive, relative to supplier, the supplier is better off breaking the match and going on search again for a higher-productivity producer. On the other hand, if the producer is too productive, relative to the supplier, the supplier is unable to keep up with the demanded quantity of input with enough profitability and might even make negative profits. Again, the supplier has incentive to break the match. Let the lower and upper cutoff productivity of λ corresponding to these cases be denoted as $\underline{\lambda}(\phi_o)$ and $\bar{\lambda}(\phi_o)$, respectively, so that a ϕ_o-type supplier is willing to match to producers with $\lambda \in [\underline{\lambda}(\phi_o), \bar{\lambda}(\phi_o)]$.

A supplier that breaks a match receives zero profit in current period and finds a match in next period with $\eta(r)/r$ probability. The probability that this new match is acceptable is $G(\bar{\lambda}(\phi_o)) - G(\underline{\lambda}(\phi_o))$. In this case, the present value of the middle supplier is $\frac{\hat{\delta}}{1-\hat{\delta}}\frac{\eta(r)}{r}\big(G(\bar{\lambda}(\phi_o))-G(\underline{\lambda}(\phi_o))\big)E[\pi_M(\lambda,\phi_o)|\underline{\lambda}(\phi_o)\leq\lambda\leq\bar{\lambda}(\phi_o)]$. But, with $1-\frac{\eta(r)}{r}\big(G(\bar{\lambda}(\phi_o))-G(\underline{\lambda}(\phi_o))\big)$ probability there is no new match or the match fails and the game repeats. Denote the value of going on search after breaking a match by $V_M(\lambda,\phi_o)$, then

$$V_M(\phi_o) = \frac{\hat{\delta}\frac{\eta(r)}{r}\big(G(\bar{\lambda}(\phi_o)) - G(\underline{\lambda}(\phi_o))\big)}{1-\hat{\delta}+\hat{\delta}\frac{\eta(r)}{r}\big(G(\bar{\lambda}(\phi_o)) - G(\underline{\lambda}(\phi_o))\big)}\frac{E[\pi_M(\lambda,\phi_o)|\underline{\lambda}(\phi_o)\leq\lambda\leq\bar{\lambda}(\phi_o)]}{1-\hat{\delta}}.$$

A ϕ_o-type middle supplier breaks a match if the following holds

$$\pi_M(\lambda,\phi_o) < (1-\hat{\delta})V_M(\phi_o), \tag{12}$$

The right-hand side above is constant with respect to λ and is represented by the horizontal lines in Figure 1, whose intersections with profit functions identify the threshold values $\underline{\lambda}$ and $\bar{\lambda}$. Alternatively, these threshold values can be obtained by solving (12) with equality. The behavior of $\underline{\lambda}$ and $\bar{\lambda}$ as functions of ϕ_o is of particular interest and is described by the following series of propositions.

Lemma 1 *For any* $\phi_o > 0$, $\underline{\lambda}(\phi_o)$ *and* $\bar{\lambda}(\phi_o)$ *exist such that* $\underline{\lambda}(\phi_o) > 0$ *and* $\bar{\lambda}(\phi_o) > \underline{\lambda}(\phi_o)$.

Lemma 2 *If* $\hat{\delta}$ *is large enough, then* $d\underline{\lambda}/d\phi_o > 0$ *and* $d\bar{\lambda}/d\phi_o > 0$.

Lemma 3 *As* $\phi_o \to \infty$, $\underline{\lambda}(\phi_o) \to \Lambda$ *and* $\bar{\lambda}(\phi_o) \to \infty$, *where* $0 < \Lambda < \infty$.

Proof for all lemmas and propositions can be found in Appendix A. Lemma 1 says that each supplier with nonzero productivity has incentive to match with a nonempty set of producers. Lemma 2 shows that as suppliers get more productive, they seek more productive producers. But, from Lemma 3 it is clear that as ϕ_o becomes very large, suppliers become less selective about the producers they want to match with, partly because the probability of finding a high-λ producer diminishes. Lemma 2 requires that $\hat{\delta}$ be large enough, emphasizing that the selectiveness of matches among suppliers holds best when future matters and suppliers think long-term.

3.6 Specialized Producer's Decision

The implications of Lemma 2 are far reaching in terms of what kinds of matches between suppliers and producers are successful. The following looks at this issue from the producer's point of view:

Lemma 4 *For a specialized producer with productivity pair* (λ, ϕ) *and for* $\hat{\delta}$ *large enough, there exist* $\underline{\phi}_o(\lambda)$ *and* $\bar{\phi}_o(\lambda)$ *such that the specialized producer can build a lasting match with any supplier of productivity* $\phi_o \in [\underline{\phi}_o(\lambda), \bar{\phi}_o(\lambda)]$.

Lemma 5 *If* $\hat{\delta}$ *is large enough, then* $\frac{d\underline{\phi}_o(\lambda)}{d\lambda} > 0$ *and* $\frac{d\bar{\phi}_o(\lambda)}{d\lambda} > 0$.

Lemma 6 *As* $\lambda \to \infty$, $\underline{\phi}_o(\lambda) \to \infty$ *and* $\bar{\phi}_o(\lambda) \to \infty$.

Lemma 5 says that high-λ producers seek high-productivity suppliers. Lemma 6 shows that, contrary to suppliers, producers stay selective about their suppliers as λ gets very large, mainly because they have the option to internalize.

If a producer decides to outsource in the current period, a successful match happens with $\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)$ probability and the return is $\pi_S(\lambda)$ per each period. With probability $1-\eta(r)\big(H(\bar{\phi}_o(\lambda))-H(\underline{\phi}_o(\lambda))\big)$ there is no match or the match fails. Since nothing has changed for the producer, the decision to search further is still optimal and the game repeats. In total, the value of outsourcing is

$$V_S(\lambda) = \frac{\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)}{1 - \hat{\delta} + \hat{\delta}\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)} \frac{\pi_S(\lambda)}{1 - \hat{\delta}}.$$

On the other hand, if this producer stays fully integrated, its present value would be $V_V(\lambda, \phi) = \pi_V(\chi)/(1 - \hat{\delta})$. A λ-type producer outsources if

$$V_V(\lambda, \phi) < V_S(\lambda). \tag{13}$$

The right-hand side is constant for any given λ, while the left-hand side is monotonically increasing with ϕ. But, $V_V(\lambda, \infty)$ is bounded, and an intersection is not necessarily guaranteed. Especially, if μ is much larger than one, then all producers find it optimal to outsource as the benefits overshadow any cost. To have an industry that is a mix of both vertically integrated and outsourcing producers, the cost and benefit of outsourcing should be in some balance as described below:

Proposition 1 *If* $(1-\omega)\mu^{\alpha} < 1$, *then the equilibrium is a mix of vertically integrated and outsourcing producers.*

In a mixed equilibrium, $V_V(\lambda, \phi)$ and $V_S(\lambda)$ intersect at a $\phi^*(\lambda)$, where a λ-type specialized producer outsources if $\phi < \phi^*(\lambda)$ and stays integrated otherwise.

Proposition 2 *In a mixed equilibrium,* $\phi^*(0) = 0$. *Also, there exists a* $\tilde{\lambda} > 0$ *(with possibility of* $\tilde{\lambda} \to \infty$*) such that* $d\phi^*(\lambda)/d\lambda > 0$ *for* $\lambda \in [0, \tilde{\lambda}]$.

3.7 Closing the Model

Free Entry

Specialized producers enter freely, therefore they should expect to make zero profits in steady state to curb excessive entry. Zero expected profit condition requires that

$$\int_0^\infty \left(\int_{\phi^*(\lambda)}^\infty V_V(\lambda, \phi) dF(\phi) + F(\phi^*(\lambda)) V_S(\lambda) \right) dG(\lambda) = c_e. \tag{14}$$

The left-hand side above is the expected profit of an outsourcing versus integrated producer. This profit is offset by the cost of entry to set it to zero.

Suppliers also enter freely and their zero profit condition can be formulated as

$$\int_0^\infty \int_{\underline{\lambda}(\phi_o)}^{\bar{\lambda}(\phi_o)} V_M(\lambda, \phi_o) dG(\lambda) dH(\phi_o) = c_e^m. \tag{15}$$

Solving equations (14) and (15) together provides the two unknowns A and r.

Labor Clearing

Assume the industry is endowed with a fixed amount of labor L. In steady state the labor demand is equal to labor supply. In equilibrium, let the mass of integrated producers be N and the mass of outsourcing producers be M. Then the condition is

$$\int_0^\infty \left(\int_{\phi^*(\lambda)}^\infty \frac{x_V(\chi)}{\chi} N dF(\phi) + F(\phi^*(\lambda)) \int_0^\infty \frac{x_O(\lambda)}{\chi_o} M dH(\phi) \right) dG(\lambda) = L, \tag{16}$$

where $\chi_o = \left(\frac{1}{\mu\lambda} + \frac{1}{\phi_o}\right)^{-1}$. The first term on the right-hand side is the labor used by all integrated producers, and the second term is the labor used by the matched outsourcing producers and their suppliers. In steady state the fraction of outsourcing firms is

$$\frac{M}{M+N} = \int_0^\infty F(\phi^*(\lambda))dG(\lambda). \tag{17}$$

Equations (16) and (17) together provide the values for N and M.

Entry and Exit

In steady state,

$$v = \xi N, \quad s\eta(r)\int_0^\infty \Big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\Big)dG(\lambda) = \xi M. \tag{18}$$

The first condition sets the number of producers entering as vertically integrated equal to the corresponding number of exits. The second condition is the same thing but for outsourcing producers. $s\eta(r)$ is the number of matches formed in each period, and the integral term is the fraction of those matches that survives. Since r is already known from solving the free entry condition, then $m = rs$.

Definition 1 *A steady state equilibrium for the outsourcing problem with heterogeneous producers and heterogeneous suppliers is the tuplet* $(\underline{\lambda}(\phi_o), \bar{\lambda}(\phi_o), \underline{\phi}_o(\lambda), \bar{\phi}_o(\lambda), \phi^*(\lambda), A, N, M, v, s, m)$ *such that (i)* $\underline{\lambda}(\phi_o)$ *and* $\bar{\lambda}(\phi_o)$ *satisfy* (12) *with equality, (ii)* $\underline{\phi}_o(\lambda)$ *and* $\bar{\phi}_o(\lambda)$ *satisfy* (13) *with equality, and (iii)* (14), (15), (16), (17), *and* (18) *are satisfied, given values of* α, L, c_e, c_e^m, ω, μ, δ, ξ *and the distributions* $F(\phi)$, $G(\lambda)$ *and* $H(\phi_o)$.

4 Theoretical Results

This section discusses the diversity of outsourcing practices among producers in the heterogeneous environment presented in Section 3. Several implications of the model are shown through a series of propositions.

Outsourcing benefits producers by boosting their efficiency in their final good production. Thinking of two producers with the same integrated productivity, χ, the one with higher λ,

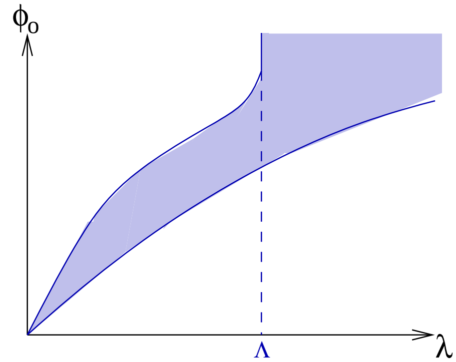

Figure 2: An illustration of suppliers and producers that stay matched in the long run on the productivity plane.

and lower ϕ, multiplies its benefits from outsourcing and would optimally go on search. The firm with lower λ and higher ϕ has more to lose, especially with the prospect of having to match to a supplier with productivity level $\phi^o < \phi$. As a result:

Proposition 3 *In a mixed equilibrium, two integrated producers with the same total productivity can end up in different paths, with one of them outsourcing and the other staying integrated.*

As shown in Lemma 2, not every producer can form a lasting match with a certain supplier. The range of productivities that would stay matched is a window of productivities $[\underline{\lambda}(\phi_o), \bar{\lambda}(\phi_o)]$, that moves towards higher productivities as ϕ_o gets larger. The same reasoning applies to suppliers. The area defined by these two ranges is illustrated in Figure 2 and leads to the following result:

Proposition 4 *On average, specialized producers with higher productivity on their final process form successful and lasting matches with more productive middle suppliers. Conversely, on average, specialized producers with lower productivity on their final process form successful and lasting matches with less productive middle suppliers.*

Lemma 2 especially shows that suppliers become less selective as they become increasingly productive. This is expected as the density of high-λ producers diminishes. On the other hand, high-λ producers have the option to internalize their input production and would outsource only if the expected search time until they find a high-productivity supplier is

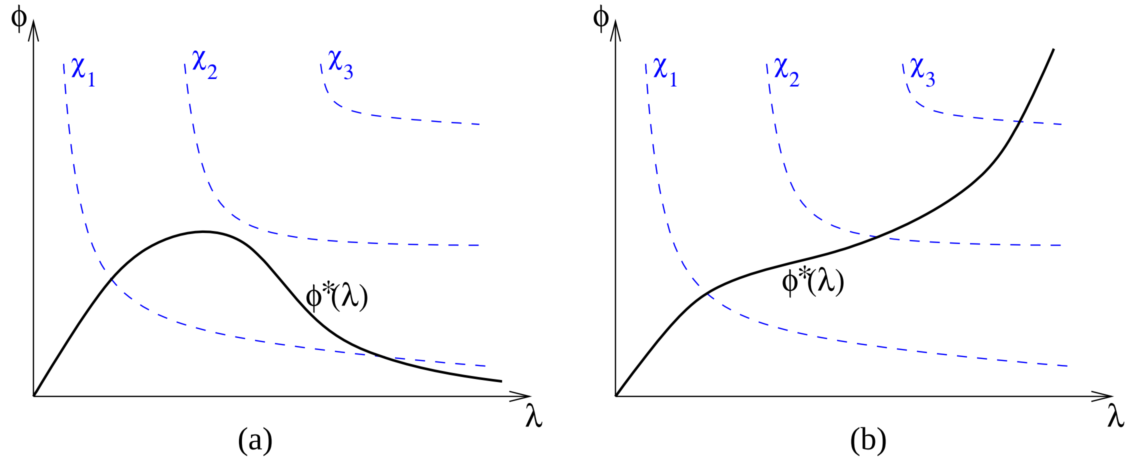

Figure 3: (a) Threshold productivity when the density of high-productivity suppliers is low, and (b) threshold productivity when the density of high-productivity suppliers is high. Three productivity contours for levels $\chi_1 \ll \chi_2 \ll \chi_3$ (low/mid/high productivity) are also illustrated.

reasonably short. The productivity distribution of suppliers is an important factor, and thicker upper tail on that distribution corresponds to shorter expected search time for high-λ producers and lower opportunity cost from going on search. The next proposition formalizes this argument.

Proposition 5 *If finding a high-productivity middle-supplier is probable enough, a nonzero fraction of high-λ producers outsource. If finding a high-productivity middle-supplier is improbable enough, then all high-λ producers stay integrated.*

In particular, this model predicts that some high-productivity integrated producers (with high values of χ) should be outsourcing.

Proposition 6 *If finding a high-productivity middle-supplier is probable enough, the set of high-productivity integrated producers that go on search for a middle supplier is nonempty.*

This porposition comes as a corollary to Proposition 5 because very high-productivity integrated producers are those with high λ's. With the probability of finding a high-productivity supplier large enough, Proposition 5 predicts that a proportion of these producers should be outsourcing (Figure 3).

Outsourcing is generally taken as synonym for downsizing as producers shed off some extra labor in the process. At the same time, an increased efficiency on the production of final good favors labor and output expansion. These two effects work in the opposite directions, and the average size of the industry might move in either direction depending on which effect dominates. The following proposition outlines relevant conditions for expansion or shrinkage of size in response to outsourcing:

Proposition 7 *There exist threshold values M_1 and M_2 ($M_2 > M_1$) such that both M_1 and M_2 are decreasing in $(1-\omega)\mu$ and*

1. *If $\lambda/\phi > M_2$: The outsourcing producer expands in both output and labor size.*
2. *If $M_2 \geq \lambda/\phi > M_1$: The outsourcing producer expands in output but shrinks in labor size.*
3. *If $\lambda/\phi \leq M_1$: The outsourcing producer shrinks in both output and labor size.*

When $(1-\omega)\mu \geq 1$, outsourcing always results in expansion in both output and labor size.

Notice that $(1-\omega)\mu$ is the *net* productivity gain from outsourcing accounting for its transaction cost embodied by ω. The proposition says that the relative efficiency of a producer in its final good and intermediate input processes decides the final direction of size change. If an outsourcing producer is relatively more efficient in producing the output than input, the productivity gain effect dominates. When the producer is relatively more efficient in producing input rather than output, the reverse holds. In a special case, when $(1-\omega)\mu \geq 1$, the productivity gain effect is overwhelming, resulting in all outsourcing producers to expand.

In the light of Proposition 7, the net effect of outsourcing on industry size might be positive or negative. Case 3 in Proposition 7 is an unlikely case, because firms with high values of ϕ would not outsource unless $(1-\omega)\mu$ is very large. As a result, the average industrial output most likely increases with outsourcing, while the direction of change for average size is still ambiguous. The ambiguity can be resolved once the actual distributions of ϕ and λ are known.

5 Conclusion

This paper extends the implications of firm-level heterogeneity and its effect on outsourcing patterns by introducing a model with a two-layered heterogeneity where both upstream suppliers and downstream producers exercise the exit approach Hirschman (1970). The model is meant to mimic a more realistic and managerial approach to forming vertical relations. The structure of the model allows for high-productivity firms to have some propensity to outsource even if offshoring is not an option, but only when the density of high-productivity suppliers in the domestic market is large enough. It also lets firms with the same level of productivity to behave differently, with one of them outsourcing and the other staying vertically integrated. More importantly, this model suggests an assortment in the matches formed between suppliers and producers, where, through a selection mechanism, producers and suppliers roughly match their productivities when forming vertical relations. Some ambiguity, however, is left in determining the direction of change in average employment size after outsourcing opens, because firms adjust their size in a mixed manner. The average output, on the other hand, is mostly predicted will rise. These features are supposed to fit the productivity distribution among outsourcing firms more closely.

A Technical Appendix

Proof of Lemma 1: First, note that matching to some producer is always better than not matching, which results in zero profit. Therefore, the set of producers with whom the match is kept is nonempty. Since $\pi_M(0, \phi_o) = 0$ for any $\phi_o > 0$, clearly $\underline{\lambda}(\phi_o) > 0$. The unimodal form of $\pi_M(\lambda, \phi_o)$ shows that any possible set of solutions will form one connected interval $[\underline{\lambda}(\phi_o), \bar{\lambda}(\phi_o)]$. Let $\underline{\lambda}(\phi_o) = \bar{\lambda}(\phi_o)$ for any ϕ_o; i.e, the set of producers that can have lasting match with this supplier is measure zero. Then the right-hand side of (12) is zero. But, (11) clearly shows that $\pi_M(\lambda, \phi_o) > 0$ for all $\lambda < \frac{\omega\phi_o}{\alpha(1-\omega)\mu}$, meaning that the set $[\underline{\lambda}, \bar{\lambda}]$ must have nonzero measure. This is a contradiction to the initial assumption. Therefore, it must be that $\bar{\lambda}(\phi_o) > \underline{\lambda}(\phi_o)$. ■

Proof of Lemma 2: First note the following preliminary results used in the course of proof:

$$\frac{\partial \pi_M(\lambda)}{\partial \phi_o} > 0, \quad \frac{\partial^2 \pi_M(\lambda, \phi_o)}{\partial \lambda \partial \phi_o} > 0, \quad \frac{\partial \pi_M(\underline{\lambda})}{\partial \underline{\lambda}} > 0, \quad \frac{\partial \pi_M(\bar{\lambda})}{\bar{\lambda}} < 0.$$

I am dropping ϕ_o as argument where obvious to save space. Define

$$B \equiv \frac{r(1-\hat{\delta})}{\hat{\delta}\eta(r)}, \quad \underline{G} \equiv G(\underline{\lambda}), \quad \bar{G} \equiv G(\bar{\lambda}), \quad \underline{g} = \left.\frac{dG(\lambda)}{d\lambda}\right|_{\underline{\lambda}}, \quad \bar{g} = \left.\frac{dG(\lambda)}{d\lambda}\right|_{\bar{\lambda}},$$

and

$$\underline{\pi}_M \equiv \pi_M(\underline{\lambda}), \quad \bar{\pi}_M \equiv \pi_M(\bar{\lambda}).$$

Also let $\hat{E}[()] \equiv \int_{\underline{\lambda}}^{\bar{\lambda}} () dG(\lambda)$.

To start with proof, at $\underline{\lambda}$ and $\bar{\lambda}$ we have

$$\underline{\pi}_M = \bar{\pi}_M = \frac{1}{B+\bar{G}-\underline{G}} \hat{E}[\pi_M(\lambda)].$$

Taking derivatives with respect to ϕ_o and rearranging terms gives

$$\begin{aligned} &\left(\frac{\partial \underline{\pi}_M}{\partial \underline{\lambda}} + \frac{\underline{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\underline{\pi}_M}{B+\bar{G}-\underline{G}}\right)\frac{d\underline{\lambda}}{d\phi_o} - \left(\frac{\bar{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\bar{\pi}_M}{B+\bar{G}-\underline{G}}\right)\frac{d\bar{\lambda}}{d\phi_o} = \\ &\qquad - \frac{\partial \underline{\pi}_M}{\partial \phi_o} + \frac{1}{B+\bar{G}-\underline{G}}\hat{E}\Big[\frac{\partial \pi_M(\lambda)}{\partial \phi_o}\Big]. \end{aligned} \tag{19}$$

and

$$\begin{aligned} &\left(\frac{\underline{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\underline{\pi}_M}{B+\bar{G}-\underline{G}}\right)\frac{d\underline{\lambda}}{d\phi_o} - \left(\frac{\partial \bar{\pi}_M}{\partial \bar{\lambda}} + \frac{\bar{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\bar{\pi}_M}{B+\bar{G}-\underline{G}}\right)\frac{d\bar{\lambda}}{d\phi_o} = \\ &\qquad - \frac{\partial \underline{\pi}_M}{\partial \phi_o} + \frac{1}{B+\bar{G}-\underline{G}}\hat{E}\Big[\frac{\partial \pi_M(\lambda)}{\partial \phi_o}\Big]. \end{aligned} \tag{20}$$

Eliminating $d\bar{\lambda}/d\phi_o$ between (19) and (20) results in $Td\underline{\lambda}/d\phi_o = \underline{S}$, where

$$\begin{aligned} T =& \frac{\partial \underline{\pi}_M}{\partial \underline{\lambda}}\left(\frac{\partial \bar{\pi}_M}{\partial \bar{\lambda}} - \frac{\bar{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} - \frac{\bar{\pi}_M}{B+\bar{G}-\underline{G}}\right) + \frac{\partial \bar{\pi}_M}{\partial \bar{\lambda}}\left(\frac{\underline{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\underline{\pi}_M}{B+\bar{G}-\underline{G}}\right) \\ \underline{S} =& \left(\frac{\partial \underline{\pi}_M}{\partial \phi_o} - \frac{\partial \bar{\pi}_M}{\partial \phi_o}\right)\left(\frac{\bar{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\bar{\pi}_M}{B+\bar{G}-\underline{G}}\right) + \frac{\partial \bar{\pi}_M}{\partial \bar{\lambda}}\left(\frac{1}{B+\bar{G}-\underline{G}}\hat{E}\Big[\frac{\partial \pi_M(\lambda)}{\partial \phi_o}\Big] - \frac{\partial \underline{\pi}_M}{\partial \phi_o}\right). \end{aligned}$$

Based on the preliminary results above, T is negative. Also, based on the preliminary results, $\partial\underline{\pi}_M/\partial\phi_o < \partial\bar{\pi}_M/\partial\phi_o$. Besides, $\hat{E}[\partial\pi_M/\partial\phi_o] > \partial\underline{\pi}_M/\partial\phi_o$, and the last term in $\underline{S}$ will be negative if $\hat{\delta}$ is large enough (making B small enough). In this case, $\underline{S}$ is negative, making $d\underline{\lambda}/d\phi_o > 0$.

Similarly, eliminating $d\bar{\lambda}/d\phi_o$ between (19) and (20) results in $Td\bar{\lambda}/d\phi_o = \bar{S}$, where

$$\bar{S} = \left(\frac{\partial \underline{\pi}_M}{\partial \phi_o} - \frac{\partial \bar{\pi}_M}{\partial \phi_o}\right)\left(\frac{\underline{g}\hat{E}[\pi_M(\lambda)]}{(B+\bar{G}-\underline{G})^2} + \frac{\underline{\pi}_M}{B+\bar{G}-\underline{G}}\right) + \frac{\partial \underline{\pi}_M}{\partial \bar{\lambda}}\left(\frac{1}{B+\bar{G}-\underline{G}}\hat{E}\Big[\frac{\partial \pi_M(\lambda)}{\partial \phi_o}\Big] - \frac{\partial \bar{\pi}_M}{\partial \phi_o}\right).$$

From before, it is given that $T < 0$. Also, given the preliminary results and noting that now $\hat{E}[\partial\pi_M/\partial\phi_o] < \partial\bar{\pi}_M/\partial\phi_o$, for large enough values of $\hat{\delta}$ we have $\bar{S} < 0$. As a result, $d\bar{\lambda}/d\phi_o > 0$. ■

Proof of Lemma 3: Let $\underline{\lambda}(\phi_o) \to \infty$ as $\phi_o \to \infty$. If $\bar{\lambda}(\phi_o)$ stays finite, then it is an immediate contradiction. Hence, assume that $\bar{\lambda}(\phi_o) \to \infty$. Then, the right-hand side in (12) goes to zero. But, $\pi_M(\lambda, \infty)$ is positive for any value of $\lambda > 0$, i.e. supplier is willing to match with any specialized producer, which is a contradiction. Therefore, there should exist a $0 < \Lambda < \infty$, such that $\underline{\lambda}(\phi_o) \to \Lambda$ as $\phi_o \to \infty$.

Now assume that $\bar{\lambda}(\phi_o) \to \hat{\Lambda}$, $\Lambda < \hat{\Lambda} < \infty$, as $\phi_o \to \infty$. As a result, the right-hand side in (12) converges to a constant $K(\Lambda, \hat{\Lambda}) < \infty$. But, the supplier is obviously willing to match with any specialized producer with[6]

$$\lambda > \frac{1}{\alpha(1-\omega)\mu A^{1/\alpha}} \left(\frac{K(\Lambda, \hat{\Lambda})}{\omega} \right)^{\frac{\alpha}{1-\alpha}},$$

which includes any $\lambda > \hat{\Lambda}$, a contradiction to $\hat{\Lambda}$ being finite. Thus $\bar{\lambda}(\phi_o) \to \infty$ as $\phi_o \to \infty$. ■

Proof of Lemma 4: $\bar{\phi}_o$ is the solution to the following

$$\underline{\phi}_o(\lambda) = \min\left\{\phi_o | \bar{\lambda}(\phi_o) \geq \lambda\right\}. \tag{21}$$

Since $\bar{\lambda}(\phi_o)$ is continuous, increasing in ϕ_o, and $\bar{\lambda}(\phi_o) \to \infty$ as $\phi_o \to \infty$, the above solution exists. $\bar{\phi}_o$ is the solution to the following

$$\bar{\phi}_o(\lambda) = \begin{cases} \max\left\{\phi_o | \underline{\lambda}(\phi_o) \leq \lambda\right\} & \text{if } \lambda < \Lambda, \\ \infty & \text{if } \lambda \geq \Lambda. \end{cases} \tag{22}$$

Again, since $\underline{\lambda}(\phi_o)$ is continuous and increasing in ϕ_o, the above solution exists. ■

Proof of Lemma 5: Based on (21) and (22), it is immediately clear that $\underline{\phi}_o(\lambda)$ and $\bar{\phi}_o(\lambda)$ should both be increasing in λ. ■

[6] just set $\pi_M(\lambda, \infty) > K(\hat{\Lambda})$

Proof of Lemma 6: Letting $\lambda > \Lambda$, it is apparent that $\bar{\phi}_o(\lambda) \to \infty$. Also, since $\bar{\lambda}(\phi_o)$ is an increasing and one-to-one mapping, from (21) it is clear that $\lambda \to \infty$ implies that $\underline{\phi}_o(\lambda) \to \infty$. ■

Proof of Proposition 1: Writing an algebraically simplified version of (13) shows that outsourcing producers are those for whom

$$\chi < \left(\frac{\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)}{1 - \hat{\delta} + \hat{\delta}\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)} \right)^{\frac{1-\alpha}{\alpha}} \big((1-\omega)\mu^{\alpha}\big)^{1/\alpha}\lambda.$$

The term in the first parentheses in the right-hand side is less than or equal to one (try $\hat{\delta} = 0$ and $\hat{\delta} = 1$). Consequently, if $(1-\omega)\mu^{\alpha} < 1$, then as $\phi \to \infty$, the inequality is violated, i.e. there are producers that stay integrated. Obviously producers with $\phi_o = 0$ outsource. So there are both types of producers present in equilibrium. ■

Proof of Proposition 2: from (13), $\phi^*(\lambda)$ can be written as $\phi^*(\lambda) = \frac{P(\lambda)\lambda}{1-P(\lambda)}$, where

$$P(\lambda) = \mu(1-\omega)^{\frac{1}{\alpha}} \left(\frac{\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)}{1 - \hat{\delta} + \hat{\delta}\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)} \right)^{\frac{1}{\alpha}-1}.$$

But from (22), it can be inferred that $\bar{\phi}_o(0) = 0$. In turn, $\underline{\phi}(0) \leq \bar{\phi}(0)$ which gives $\underline{\phi}(0) = 0$. As a result, $P(\lambda) \to 0$ as $\lambda \to 0$, causing $\phi^*(\lambda) \to 0$ when $\lambda \to 0$.

In a mixed equilibrium, $\phi^*(\lambda) > 0$ for some $\lambda > 0$, which means that, by continuity of $\phi^*(\lambda)$, there must exist a $\tilde{\lambda} > 0$, such that $d\phi^*(\lambda)/d\lambda > 0$ for $\lambda \in [0, \tilde{\lambda}]$. ■

Proof of Proposition 3: I just need to show that there is a nonempty set of producers with the same total productivity but different paths. Let $\lambda_1 = \kappa\tilde{\lambda}$, where $\kappa < 1$ and $\tilde{\lambda}$ comes from Proposition 2. Define the following integrated producers:

Producer 1: has productivity pair $\left(\lambda_1 - \epsilon, \frac{\lambda_1\phi^*(\lambda_1)(\lambda_1-\epsilon)}{\lambda_1(\lambda_1-\epsilon)-\phi^*(\lambda_1)\epsilon}\right)$.

Producer 2: has productivity pair $\left(\lambda_1 + \epsilon, \frac{\lambda_1\phi^*(\lambda_1)(\lambda_1+\epsilon)}{\lambda_1(\lambda_1+\epsilon)+\phi^*(\lambda_1)\epsilon}\right)$.

Both producers 1 and 2 have the same total productivity equal to $\lambda_1\phi^*(\lambda_1)/\big(\lambda_1 + \phi^*(\lambda_1)\big)$. Note that, $\frac{\lambda_1\phi^*(\lambda_1)(\lambda_1-\epsilon)}{\lambda_1(\lambda_1-\epsilon)-\phi^*(\lambda_1)\epsilon} > \phi^*(\lambda_1) > \phi^*(\lambda_1 - \epsilon)$, for small enough ϵ. The last result comes from Proposition 2 and the fact that $\phi^*(\lambda)$ is increasing at λ_1. Similar reasoning shows that

$\frac{\lambda_1\phi^*(\lambda_1)(\lambda_1+\epsilon)}{\lambda_1(\lambda_1+\epsilon)+\phi^*(\lambda_1)\epsilon} < \phi^*(\lambda_1) < \phi^*(\lambda_1+\epsilon)$, for small enough ϵ. Therefore, for small enough ϵ, producer 1 stays integrated, whereas producer 2 goes on search for a middle supplier. The proposition is thus proved. ■

Proof of Proposition 5: Focus on the right-hand side in (13). As $\lambda \to \infty$, $\bar{\phi}_o(\lambda), \underline{\phi}_o(\lambda) \to \infty$ which lets

$$R = \frac{\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)}{1-\hat{\delta}+\hat{\delta}\eta(r)\big(H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))\big)} \to 0.$$

But, at the same time, $\pi_S(\lambda) \to \infty$. There are two cases:

Case 1: $H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda)) = O(\lambda^n)$, where $n < -\alpha/(1-\alpha)$. In this case, for large enough λ, R goes to zero faster than $\pi_S(.)$ goes to infinity, and the right-hand side in (13) goes to zero. As a result, the fraction of λ-type producers that outsource converges to zero as $\lambda \to \infty$. Notice that $H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda))$, when $\lambda \to \infty$, directly relates to the density of high-productivity suppliers.

Case 2: $H(\bar{\phi}_o(\lambda)) - H(\underline{\phi}_o(\lambda)) = O(\lambda^n)$, where $n > -\alpha/(1-\alpha)$. In this case, $H(.)$ goes to zero more slowly than $\pi_S(.)$ goes to infinity or stays positive, and the right-hand side in (13) goes to infinity. As a result, the fraction of λ-type producers that outsource converges to one as $\lambda \to \infty$. ■

Proof of Proposition 7: Using (5) and (8), outsourcing leads to an expansion in output if $x_O > x_V$, or

$$\frac{\lambda}{\phi} > \frac{1}{(1-\omega)\mu} - 1.$$

Similarly, outsourcing leads to an expansion in labor if $x_O/\lambda > x_V/\chi$, or

$$\frac{\lambda}{\phi} > \frac{1}{\big((1-\omega)\mu\big)^{1/\alpha}} - 1.$$

Define $M_1 = \frac{1}{(1-\omega)\mu} - 1$ and $M_2 = \frac{1}{\big((1-\omega)\mu\big)^{1/\alpha}} - 1$. Obviously, if $(1-\omega)\mu \geq 1$, then $M_1, M_2 \leq 0$ and expansion happens in both output and labor. Now, let $(1-\omega)\mu < 1$. In this case $M_2 > M_1$. Three ranges of values for λ/ϕ can be identified:

1. $\lambda/\phi > M_2$, where expansion happens in both output and labor size.

2. $M_2 \geq \lambda/\phi > M_1$, where expansion happens in output but labor shrinks.

3. $\lambda/\phi \leq M_1$, where shrinkage happens in both output and labor size.

This completes the proof. ∎

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