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ON PRICING IN THE PRESENCE OF BUDGET CONSTRAINED CONSUMERS

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ABSTRACT. We look at imperfectly competitive markets where some consumers might be budget-constrained. We find that the equilibrium price under budget constrained demand (say, p_B) is often higher than the equilibrium price under standard demand (say, p_A). The relationship between p_B and p_A depends on the elasticity of the standard demand (at p_A), technology, and market structure. Lack of competition and inefficient technology make $p_B > p_A$ more likely.

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1. INTRODUCTION

How does the equilibrium price change with the introduction of budget-constrained consumers? Our results indicate that the price often increases, especially when the markets lack competition and the production technology is inefficient. This conclusion is remarkably robust: it holds for fairly general (i) demand functions, (ii) budget distribution, and (iii) market structure (with and without free entry). Our finding suggests that, contrary to what one might expect, prices can be higher in poor regions where consumers have limited purchasing power. Since the poor have less purchasing power, they demand less. As the aggregate demand shifts inward, the equilibrium price is expected to be lower as well. This intuitive argument implicitly assume perfect competition and it does not necessarily go through when markets are imperfectly competitive.

Key to pricing under imperfect competition is demand elasticity. Introducing budget-constrained consumers can make the demand less elastic and lead to higher equilibrium price. For example, in a monopoly, we find that the equilibrium price is always higher under budget-constrained demand. In the first part of the paper we focus on arbitrary but exogenous market structure. We characterize the necessary and sufficient condition for equilibrium price to be higher in the presence of budget constrained consumers. Subsequently, we analyze pricing in a free entry environment where the market structure is endogenously determined. While the underlying mechanisms are somewhat different, in both cases,—exogenous and endogenous market

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structure—lack of competition and inefficient technology generate higher equilibrium price under budget constrained demand.

That inward demand shift can have ambiguous effect on equilibrium price has been noted before. Baldenius and Rachelstein [6] have shown that a parallel shift in inverse demand lowers price for all log-concave inverse demand function but raises it for constant elasticity demand function. Effect of parallel demand shifts in context of monopoly price discrimination is also discussed in Cowan [10]. For demand shifts under oligopoly, see Dixit [13], Quirmbach [19], and Hamilton [15]. While insightful, the demand shifts in these papers are often well behaved and captured by a change in the demand parameter. Furthermore, budget distribution has little role to play.

Demand shift in our framework is induced by a change in budget distribution and is rarely well-behaved. The shift does not necessarily preserve curvature or even log-concavity (see section 2). Furthermore, rather than a change in demand parameter, the demand shift in our framework is better viewed as resulting from a change in environment: from one, where no consumer is budget constrained, to another, where some consumers are budget constrained.

Demand shifts inward due to exogenous reduction in budget m to the extent that $x(p)$—amount of a good x that unconstrained consumers demand at price p—becomes unaffordable for some consumers (i.e., $px(p) > m$). Consumers who cannot afford $x(p)$ at price p are referred to as budget constrained consumers (at p). As we illustrate in section 2, a consumer might be budget constrained for some prices but not for others. Some consumers might always be budget constrained and yet some others might never be budget constrained in the relevant range of prices. Accordingly, the set of budget constrained consumers and the magnitude of demand shift vary endogenously with p.

Let $D_A(p)$ and $D_B(p)$ respectively denote the standard demand and the budget-constrained demand respectively. While the relationship between $D_A(p)$ and $D_B(p)$ can be quite arbitrary, the relationship between the elasticities of $D_A(p)$ and $D_B(p)$ is neat (Proposition 1):

- At any given price p, if the standard demand function $D_A(p)$ is elastic, the budget-constrained demand $D_B(p)$ is elastic as well but less elastic than $D_A(p)$. On the other hand if $D_A(p)$ is inelastic, then $D_B(p)$ is inelastic, but it is less inelastic than $D_A(p)$.

The elasticity relationship covers some distance towards the comparison of equilibrium prices under $D_A(p)$ and $D_B(p)$. However, it does not go far enough. Translating elasticity ranking to equilibrium price ranking implicitly relies on the condition that a firm's marginal revenue function is decreasing in its output—a condition that holds for all logconcave demand functions. For fairly standard budget distribution and linear demand, we find that the induced budget-constrained demand $D_B(p)$ is not logconcave since the demand from budget-constrained consumers, i.e. $x = \frac{m}{p}$, is not logconcave. This in turn suggests that equilibrium price under $D_B(p)$ is not necessarily unique. Let p_i $(i = A, B)$ denote the equilibrium price in economy i. In principle, multiple p_B can generate ambiguity in price ranking as one p_B might be higher than p_A while another p_B might be lower than p_A. Such ambiguity in price ranking does not arise in our framework as Propositions 2 and 3 establish that exactly one of the following relationships holds: (i) all p_B are greater than p_A, (ii) all p_B are less than p_A, and (iii) $p_B = p_A$. Which one among (i)—(iii) holds and when?

- Equilibrium price is higher (lower) under budget-constrained demand, i.e., $p_B > p_A$ holds, *if and only if* the standard demand is elastic (inelastic) at p_A (Proposition 4).
- Lack of competition and inefficient technology makes it more likely that $p_B > p_A$. In particular, $p_B > p_A$ always holds in a monopoly (Proposition 5).

An attractive feature of the *if and only if* condition is that it does not involve properties of $D_B(p)$ or even budget distribution. As noted earlier, $D_B(p)$ might not be well-behaved. The *if and only if* condition says that it is not necessary to know the properties of $D_B(p)$.

We endogenize the market structure by introducing free entry (of firms) in section 8 and revisit the price comparison. As with an exogenous market structure, we find that (i) $p_B > p_A$ holds if the elasticity of $D_A(p)$ at p_A is greater than a threshold value and (ii) lack of competition—modeled as high entry cost—and high unit cost make $p_B > p_A$ more likely. In contrast to the exogenous market structure, however, the threshold (mentioned in (i)) is strictly lower than unity. The threshold is $\frac{1}{2}$ if the budget distribution is uniform, irrespective of the demand function considered. For suitably concave budget distribution, the threshold is arbitrarily close to zero.

An important outcome of the analysis based on endogenous market structure is that $p_B > p_A$ could hold even under inelastic demand. Thus budget-constrained consumers can end up paying more for necessities which have relatively inelastic demand. Recent empirical works suggest that the poor often pay more for food and other necessities (Li [17], Attanasio [4]). Introducing variety-specific fixed costs in a monopolistic competition model with CES preferences, Li [17] demonstrates that the price index for a poor individual's consumption basket might be higher since she cannot buy as many varieties as the rich consumers. In the context of homogenous products, one possible explanation is that the poor cannot avail of the bulk discounts that are offered for purchases beyond a certain threshold level (Attanasio [4]). Consequently they pay a higher price per unit. We show that even in the canonical models of imperfect competition (e.g., monopoly, oligopoly) with linear pricing, the possibility of higher prices in poor regions can arise once we introduce the notion of poor/budget-constrained consumers in a natural way.

In the concluding section we discuss how $D_A(p)$ and $D_B(p)$ can be interpreted as demand for two different regions and also how the two scenarios, fixed number of firms and free entry, can respectively be interpreted as trade and autarky. Under these interpretations we argue that trade liberalization makes $p_B > p_A$ less likely. The concluding section also discusses price discrimination and non-linear pricing.

2. An illustrative example

To illustrate the implications of our results, we present a simple example with quadratic utility function and uniform budget distribution. Consider economy A with unit mass of consumers with identical preferences over x and y:

$$V(x, y) = x - \frac{x^2}{2} + y.$$

The price of x and y are p and 1 respectively. Consumers differ in the overall budget (m) that can be spent on these two goods. Budget (m) is uniformly distributed in $[1, 2]$. Each consumer demands $x = 1 - p$ irrespective of m. Since there is a unit mass of consumers

$$D_A(p) = 1 - p,$$

where $D_A(p)$ denotes aggregate demand of x. As $p(1 - p) \leq \frac{1}{4}$ and minimum m is 1, no consumer is budget-constrained in economy A.

Suppose there are n identical firms producing good x at a constant marginal cost c. Assuming Cournot competition, it is straightforward to show that the equilibrium price and output in the economy are, respectively:

$$p_A = \frac{1 + nc}{n + 1}, \qquad D_A(p_A) = \frac{n(1 - c)}{n + 1}. \tag{1}$$

Now consider economy B which is identical to economy A in all respects but budget. Budget (m) in economy B is distributed uniformly on $[0, 1]$. We say that a consumer m is budget-constrained at p if she could not afford $x = (1 - p)$ or equivalently if $p(1 - p) > m$. Observe that a consumer with

- $m \geq \frac{1}{4}$ is never budget-constrained for any price (since the maximum value of px for $p \in [0, 1]$ is $\frac{1}{4}$);
- $m = 0$ is budget constrained for all $p \in (0, 1)$;
- $m = \frac{3}{16}$ is budget-constrained only for $p \in (\frac{1}{4}, \frac{3}{4})$ but not for $p \in [0, \frac{1}{4}]$ or for $p \in [\frac{3}{4}, 1]$.

A budget-constrained consumer with $m < px(p)$ buys $\frac{m}{p}$, while others with $m > px(p)$ buy $1 - p$. Summing up the demand for all consumers gives the aggregate demand for the budget-constrained economy:

$$D_B(p) = \int_{p(1-p)}^{1} (1-p)dm + \int_{0}^{p(1-p)} \frac{m}{p}dm = \frac{1}{2}(2 - 3p + 2p^2 - p^3).$$

Observe that $D_B(p) - D_A(p)$ or $\frac{D_B(p)}{D_A(p)}$ are not constant. Thus, as Figure 1 shows, the inward demand shift in this simple example is neither parallel nor proportional. The shift does not preserve curvature either, as $D_B''(p)(= 2 - 3p)$ is strictly positive for $p < \frac{2}{3}$ and strictly negative for $p > \frac{2}{3}$.

(Figure 1 to be inserted here)

Fortunately, this particular $D_B(p)$ is logconcave, which guarantees unique Cournot equilibrium. In general, $D_B(p)$ is not logconcave since the demand from budget-constrained consumers, i.e. $x = \frac{m}{p}$, is not logconcave.[1] Also, since $D_B(p)$ is a third-order polynomial even for quadratic utility and uniform distribution of budget, computing equilibrium p_B seems complicated in general, let alone the comparison between p_A and p_B.

The following condition, formally established in Proposition 4, suggests that computing p_B and/or analyzing the properties of $D_B(p)$ are not necessary for comparing p_A and p_B:

$$p_B > (=, <) p_A \Leftrightarrow \epsilon_A(p_A) > (=, <) 1,$$

where $\epsilon_A(p_A) \equiv -\frac{p_A D_A'(p_A)}{D_A(p_A)}$ denotes the elasticity of $D_A(p)$ at equilibrium price p_A. For the example considered here,

$$p_B > p_A \Leftrightarrow \epsilon_A(p_A) > 1 \Leftrightarrow \frac{p_A}{1-p_A} > 1 \Leftrightarrow c + \frac{1}{2n} > \frac{1}{2}. \tag{2}$$

Observation 1. Equilibrium price is higher under budget-constrained demand if the number of firms (n) is small and/or unit cost c is high. In particular, if the market structure is a monopoly ($n = 1$), $p_B > p_A$ holds for all $c > 0$. Similarly, if $c > \frac{1}{2}$, $p_B > p_A$ holds irrespective of the market structure.

In section 8 we endogenize the number of firms by allowing entry. Let k denote the entry cost for each firm. Setting $\frac{(p_A - c)D_A(p_A)}{n} - k = 0$ and solving for n gives the free entry number of firms: $n_A = \frac{1-c-\sqrt{k}}{\sqrt{k}}$. Substituting $n = n_A$ in (1) gives

$$p_A = c + \sqrt{k}. \tag{3}$$

[1]That $x = \frac{m}{p}$ is not logconcave follows from noting that $\frac{d^2 ln \frac{m}{p}}{dp^2} = \frac{1}{p^2} > 0$. Consider the quadratic utility function and $F(m) = m^\alpha$. For all $\alpha < 1$ we find that there always exists positive prices such that $\frac{d^2 ln D_B(p)}{dp^2} < 0$. Details are available on request.

For quadratic utility function and uniform distribution of m in $[0,1]$ we find that

$$p_B > p_A \Leftrightarrow \epsilon_A(p_A) > \frac{1}{2} \Leftrightarrow \frac{p_A}{1-p_A} > \frac{1}{2} \Leftrightarrow c + \sqrt{k} > \frac{1}{3} \tag{4}$$

Observation 2. The key implication of (4) is qualitatively the same as that of (2). $p_B > p_A$ holds if $\epsilon_A(p_A)$ is greater than a threshold and this condition is satisfied when unit cost c is high and/or competition is low. Since n is endogenous in this formulation, low competition is captured by high entry cost k. A new but important feature under free entry is that the threshold of $\epsilon_A(p_A)$ below which $p_B > p_A$ holds is strictly lower than unity. Thus $p_B > p_A$ can hold even when the demand is inelastic.

3. Preliminaries

Consider an economy with two sectors: a competitive sector producing good y and an oligopolistic sector with $n(\geq 1)$ firms producing a homogenous good x at a constant marginal cost $c > 0$. There is a unit mass of consumers with identical preferences. A consumer chooses x and y to maximize her utility:

$$V(x,y) \equiv U(x) + y,$$

subject to the budget constraint

$$px + y \leq m,$$

where y's price is 1, p denotes the price of x and m denotes the consumer's budget (i.e., spending limit) on x and y. Budget m is distributed according to a strictly increasing, twice differentiable cumulative distribution function $F(m)$ over an interval $[\underline{m}, \overline{m}]$. Furthermore $F(\underline{m}) = 0$ and $F(\overline{m}) = 1$.

Assume that (i) $U(x)$ is at least thrice continuously differentiable, (ii) $U'(0) \equiv \bar{p} < \infty$ and furthermore, (iii) there exists a $\overline{X} > 0$ (but not necessarily finite) such that $U'(x) > 0$ and $U''(x) < 0$ for all $x \in (0, \overline{X})$.[2] Utility maximization yields the following direct demand:

$$x(p,m) = \begin{cases} 0 & \text{if } \; p \geq \bar{p}, \\ x(p) & \text{if } \; p \in (U'(\frac{m}{p}), \bar{p}), \\ \frac{m}{p} & \text{if } \; p \leq U'(\frac{m}{p}). \end{cases} \tag{5}$$

where $x(p)$ is the unique value of x that maximizes $U(x) - px$.

Equation (1) says that no consumer buys x if $p \geq \bar{p}$. For lower values of p, a consumer buys $x(p)$ or $\frac{m}{p}$ depending on her budget m. If m is high (and consequently $U'(\frac{m}{p})$ is low) a consumer buys $x(p)$. Else, if $m < px(p)$ (or equivalently $p \leq U'(\frac{m}{p})$) the consumer spends her entire budget on x and buys $\frac{m}{p}$ units. We say this consumer

[2]Finite $U'(0)$ guarantees the existence of Cournot equilibrium. See, for example, Anderson and Renault [1]

is budget-constrained as she cannot afford $x(p)$. To examine cleanly the impact of budget-constrained consumers on pricing we consider two scenarios:

(A) $\underline{m} > \max_{p\in[c,\bar{p}]} px(p)$,
(B) $\underline{m} = 0$.[3]

Consider an economy where (A) holds. Call this the economy A. As the consumer with minimum income $\underline{m}$ can afford $x(p)$ for any p, no consumer is budget-constrained in economy A. This description corresponds to the standard partial equilibrium setting with quasi-linear preferences where all consumers can afford $x(p)$ and there is no income effect. On the other hand, if (B) holds, there are always some consumers who cannot afford $x(p)$ for some $p > 0$. An economy, where (B) holds, is referred to as the budget-constrained economy or in short, economy B.

4. Demand and elasticity

4.1. **Aggregate demand.** Consider economy A first. Each consumer demands $x(p)$ in economy A for $p < \bar{p}$. Since there is a unit mass of consumers, $x(p)$ denotes the aggregate demand as well. In economy B, a consumer with income $m \geq px(p)$ demands $x(p)$, while a consumer with income $m < px(p)$ demands $\frac{m}{p}$. Thus the aggregate demand in economy B is

$$\int_{px(p)}^{\overline{m}} x(p)f(m)dm + \int_{0}^{px(p)} \frac{m}{p}f(m)dm = x(p) - \frac{1}{p}\int_{0}^{px(p)} F(m)dm,$$

where the equality follows from expanding the first integral, applying integration by parts for the second and simplifying subsequently. To summarize, we have

$$D_i(p) = \begin{cases} x(p), & \text{if} \quad \text{i = A}, \\ x(p) - \frac{1}{p}\int_{0}^{px(p)} F(m)dm, & \text{if} \quad \text{i = B}, \end{cases} \tag{6}$$

where $D_i(p)$ denotes the aggregate demand function for x in economy $i(= A, B)$.

4.2. **Price elasticity of demand.** Define $\epsilon_A(p) \equiv -\frac{pD_A^{'}(p)}{D_A(p)}$ and $\epsilon_B(p) \equiv -\frac{pD_B^{'}(p)}{D_B(p)}$ as the price elasticity of demand corresponding to the demand functions $D_A(p)$ and $D_B(p)$ respectively. Since $D_A(p) = x(p)$,

$$\epsilon_A(p) = -\frac{pD_A^{'}(p)}{D_A(p)} = -\frac{px^{'}(p)}{x(p)}.$$

[3]We can state (B) as $\underline{m} \leq \min_{p\in[c,\bar{p}]} px(p)$. Since $x(\bar{p}) = 0$, $\min_{p\in[c,\bar{p}]} px(p) = 0$. The only $m(\geq 0)$ that satisfy the requirement is $\underline{m} = 0$.

Recall from (6),

$$D_B(p) = x(p) - \frac{1}{p} \int\limits_{\underline{m}}^{px(p)} F(m)dm.$$

Multiplying both sides by p and taking the total derivative with respect to p we get

$$pD'_B(p) + D_B(p) = px'(p) + x(p)\,[1 - F(px(p))].$$

Rearranging this equation and rewriting it in terms of elasticities we get

$$|\epsilon_B(p) - 1| = |\epsilon_A(p) - 1| \frac{x(p)(1 - F(px(p)))}{D_B(p)}, \tag{7}$$

where $\frac{x(p)(1-F(px(p)))}{D_B(p)}$ is the share of unconstrained consumers demand in economy B's aggregate demand for x and $|.|$ denotes absolute value.

Proposition 1. For all $p \in (0, \bar{p})$,
(i) $\epsilon_B(p) > (=, <)1$ if and only if $\epsilon_A(p) > (=, <)1$.
(i) $\epsilon_B(p) > (=, <)\epsilon_A(p)$ if and only if $\epsilon_A(p) < (=, >)1$.

Proof: Part (i) is immediate from (7). Rearranging (7) gives

$$\frac{\epsilon_B(p) - \epsilon_A(p)}{1 - \epsilon_A(p)} = \frac{x(p)(1 - F(px(p)))}{D_B(p)}$$

which implies (ii), once we note that $\frac{x(p)(1-F(px(p)))}{D_B(p)} \in (0, 1)$ for the relevant range of p. □

Proposition 1 says that if $x(p)$ is elastic then the budget constrained demand is elastic as well but less elastic than $x(p)$. On the other hand if $x(p)$ is inelastic then the budget constrained demand is inelastic as well but less inelastic than $x(p)$. To see why, note that the price elasticity of demand corresponding to $x = \frac{m}{p}$ is 1 while that corresponding to $x = x(p)$ is $-\frac{px'(p)}{x(p)} = \epsilon_A(p)$. Since $\epsilon_B(p)$ is a weighted average of $\epsilon_A(p)$ and 1, we have that

$$\min\{\epsilon_A(p), 1\} < \epsilon_B(p) < \max\{\epsilon_A(p), 1\}$$

when $\epsilon_A(p) \neq 1$ and $\epsilon_A(p) = \epsilon_B(p) = 1$ otherwise. Proposition 1 follows from expanding the above inequality separately for the two cases—$\epsilon_A(p) < 1$ and $\epsilon_A(p) > 1$.

5. Pricing

Suppose there are $n(\geq 1)$ firms producing the homogeneous product x at constant marginal cost $c \in (0, \bar{p})$. Product market competition is Cournot. That is, firms compete in quantities.

Consider economy A first where the industry demand is $D_A(p) \equiv x(p)$. Each firm $i \in \{1, 2, ..., n\}$ chooses output x_i to maximize its profit,

$$\pi_i \equiv (p(x) - c)x_i,$$

where $p = p(x)$ is the inverse industry demand function corresponding to $x = x(p)$.

The standard first-order conditions are

$$p'(x)x_i + p(x) = c \quad \text{for } i = 1, ..., n.$$

Summing up these conditions yields

$$p'(x)x + np(x) = nc \tag{8}$$

where $x = \sum_{i=1}^{n} x_i$ denote the aggregate output. In Cournot equilibrium, each firm i chooses $x_i = \frac{x_A}{n}$ where x_A is a solution to (8). The following condition —now standard in the oligopoly literature—guarantees the existence and uniqueness of x_A.

$$2p'(x) + xp''(x) < 0. \tag{9}$$

Remark 1. All logconcave inverse demand functions satisfy equation 9. In terms of direct demand function the equivalent condition is that for all $p < \bar{p}$, $(x(p))^{-1}$ is strictly concave (Deneckre and Kovenock [12]). Anderson and Renault [1] provide an elegant characterization of direct demand functions in terms of ρ concavity. Using their terminology the condition for uniqueness can be stated as the direct demand function is (-1)- strictly concave.

The unique x_A that solves (8) and $p_A = p(x_A)$ respectively are the equilibrium output and price of x in economy A. Using $x_A = x(p_A)$ and $p'(x_A) = \frac{1}{x'(p_A)}$, rewrite (8) as follows:

$$\frac{-x\,(p_A)}{x'\,(p_A)} + np_A = nc.$$

Divide both sides by n and rearrange to obtain the pricing equation,

$$p_A \left(1 - \frac{1}{n\epsilon_A(p_A)}\right) = c, \tag{10}$$

where the left-hand side of (10) is a firm's marginal revenue (in Cournot equilibrium) expressed in terms of p and the right-hand side of (10) is marginal cost. We record the following results for future reference.

Lemma 1. (i) $p(1 - \frac{1}{n\epsilon_A(p)})$ is increasing in p; (ii) p_A is strictly decreasing in n and increasing in c.

Proof: See Appendix.

Now consider the budget-constrained economy where the industry demand is $D_B(p)$. The relevant pricing equation in economy B, i.e., the counterpart of (10) in economy

A is:

$$p_B \left(1 - \frac{1}{n\epsilon_B(p_B)}\right) = c, \tag{11}$$

where $\epsilon_B(p)$, as defined earlier, is $-\frac{pD'_B(p)}{D_B(p)}$. Using the limiting values, i.e.,

$$\lim_{p \to c} p \left(1 - \frac{1}{n\epsilon_B(p)}\right) < c, \qquad \lim_{p \to \bar{p}} p \left(1 - \frac{1}{n\epsilon_B(p)}\right) > c,$$

we can show that there exists an equilibrium with $x_i = \frac{D(p_B)}{n}$ for all $i \in \{1, 2, ..., n\}$ where $p_B \in (c, \bar{p})$ solves (11).

We remain agnostic about the uniqueness of p_B because, as we have argued before (see footnote 1), budget-constrained demand is not necessarily logconcave. This suggests that $p(1 - \frac{1}{n\epsilon_B(p)})$ might be non-monotone and the solution to (11) might not be unique.

6. Comparison of equilibrium prices

Instead of assuming unique p_B, or imposing restriction on $U(x)$ and $F(m)$ (to ensure uniqueness), we take the road less travelled: allow for multiple p_B and compare p_A with all possible p_B that satisfy (11).

Proposition 2. Define

$$\mathcal{P} = \{p \in R_+ : p \left(1 - \frac{1}{n\epsilon_B(p)}\right) = c\}$$

as the set of possible equilibrium prices in the budget-constrained economy and let $p_B \in \mathcal{P}$. Then,

$$p_B > (=, <) p_A \Rightarrow \epsilon_A(p_A) > (=, <) 1.$$

Proof: We show that $p_B > p_A \Rightarrow \epsilon_A(p) > 1$. The proofs of (i) $p_B < p_A \Rightarrow \epsilon_A(p_A) < 1$ and (ii) $p_B = p_A \Rightarrow \epsilon_A(p_A) = 1$ are analogous and hence omitted.

Suppose not. That is, suppose $p_B > p_A$ and yet $\epsilon_A(p_A) \leq 1$. From the pricing equations, (10) and (11), $p_B > p_A \Rightarrow \epsilon_A(p_A) > \epsilon_B(p_B)$, which, together with $\epsilon_A(p_A) \leq 1$ imply $\epsilon_B(p_B) < 1$. By Proposition 1, $\epsilon_B(p_B) < 1 \Rightarrow \epsilon_B(p_B) > \epsilon_A(p_B)$. Consequently

$$p_B \left(1 - \frac{1}{n\epsilon_B(p_B)}\right) > p_B \left(1 - \frac{1}{n\epsilon_A(p_B)}\right). \tag{12}$$

Moreover, we have

$$p_B \left(1 - \frac{1}{n\epsilon_A(p_B)}\right) > p_A \left(1 - \frac{1}{n\epsilon_A(p_A)}\right) = c. \tag{13}$$

The inequality in (13) follows from noting that $p\left(1 - \frac{1}{n\epsilon_A(p)}\right)$ is strictly increasing in p (by Lemma 1). The equality is due to the pricing equation (10). Combining (12) and

(13) gives

$$p_B \left(1 - \frac{1}{n\epsilon_B(p_B)}\right) > c,$$

which is a contradiction because

$$p_B \in \mathcal{P} \Rightarrow p_B \left(1 - \frac{1}{n\epsilon_B(p_B)}\right) = c.$$

□

Proposition 2 says that a necessary condition for $p_B > p_A$ is that $x(p)$ is elastic at $p = p_A$. Observe that the condition does not involve $e_B(p)$ or $D_B(p)$, which implies no further restriction on budget distribution $F(m)$ is necessary. Furthermore, a closer look at Proposition 2 reveals the following, which drastically simplifies the price comparison.

Proposition 3. Suppose $p_B^0 \in \mathcal{P}$.

$$p_A < (=, >) p_B^0 \quad \Rightarrow \quad p_A < (=, >) p_B \quad \text{for all} \quad p_B \in \mathcal{P}.$$

Proof: First, we prove that $p_A < p_B^0 \Rightarrow p_A < p_B$ for all $p_B \in \mathcal{P}$. Suppose not. That is, suppose $p_A < p_B^0$ and there exists $p_B^1 \in \mathcal{P}$ such that $p_A \geq p_B^1$. By Proposition 2,

$$p_A < p_B^0 \Rightarrow \epsilon_A(p_A) > 1,$$
$$p_A \geq p_B^1 \Rightarrow \epsilon_A(p_A) \leq 1,$$

which leads to contradiction since both $\epsilon_A(p_A) > 1$ and $\epsilon_A(p_A) \leq 1$ cannot hold simultaneously. The proof of other parts are analogous and hence omitted. □

To understand the value of Proposition 3 note that in presence of multiple p_B, it is possible to have a p_B strictly higher than p_A while another p_B strictly lower. In that case, comparison of equilibrium prices between the standard demand and the budget-constrained demand will depend on the selection of p_B. Proposition 2 implies that this possibility never arise since exactly one of the following holds: (i) all $p_B \in \mathcal{P}$ are lower than p_A, (ii) all $p_B \in \mathcal{P}$ are strictly higher than p_A, or (iii) $p_B = p_A$. Combining the results from Propositions 2 and 3 we obtain a key characterization result of this paper.

Proposition 4. Let p_A denote the equilibrium price under standard demand. Similarly let $\mathcal{P}$ denote the set of equilibrium prices under budget-constrained demand.

$$\epsilon_A(p_A) > (=, <) 1 \Leftrightarrow p_A < (=, >) p_B,$$

for all $p_B \in \mathcal{P}$.

Proof: We show that

$$\epsilon_A(p_A) > 1 \Leftrightarrow p_A < p_B,$$

for all $p_B \in \mathcal{P}$. Proofs of the other two if and only if claims are analogous and hence omitted. That $\epsilon_A(p_A) > 1 \Leftarrow p_A < p_B$ for all p_B follows directly from Propositions 2 and 3. We prove the other side (i.e., sufficiency) by contradiction. Suppose $\epsilon_A(p_A) > 1$ and yet there exists $p_B \in \mathcal{P}$ such that $p_B \le p_A$. By Proposition 2, a necessary condition for $p_B \le p_A$ is $\epsilon_A(p_A) \le 1$ which leads to contradiction (since we started with the supposition $\epsilon_A(p_A) > 1$). □

Proposition 4 says that $\epsilon_A(p_A) > 1$ is necessary and sufficient for $p_B > p_A$. The condition does not involve $D_B(p)$ or p_B which makes it easy to verify. Before we turn to the the implications of Proposition 4 for different market structures, two remarks are in order—both regarding costs.

Remark 2. Incorporating cost asymmetry in our framework is straightforward. Suppose there are n firms. Let c_i denote firm i's marginal cost and without loss of generality assume that $c_1 \le c_2 \le ... \le c_n$. Define $\overline{c} \equiv \frac{\Sigma_{i=1}^n c_i}{n}$. Proceeding as in section 5, one can show that the pricing equations for economies A and B—which lie at the heart of our analysis—are effectively the same as in (10) and (11) except that c is substituted by $\bar{c}$. This innocuous substitution does not affect the results.

Remark 3. Departure from constant marginal costs partially affects our results. While the possibility of $p_B > p_A$ remains, $\epsilon_A(p_A) > 1$ is no longer necessary and sufficient for $p_B > p_A$. In particular we find that $\epsilon_A(p_A) > 1$ is sufficient but not necessary for $p_B > p_A$ if marginal cost is decreasing in output. Similarly, if marginal cost is increasing in output, $\epsilon_A(p_A) > 1$ is necessary but not sufficient for $p_B > p_A$. In other words, decreasing marginal costs increases the possibility of $p_B > p_A$ while the opposite is true for increasing marginal costs.

The main idea, i.e., $p_B > p_A$ if $\epsilon_A(p_A)$ is greater than a certain threshold holds even if marginal cost is not constant. The assumption of constant marginal cost helps to provide sharper characterization. As the main action in this paper is in the demand side, we continue with constant marginal cost following much of the recent literature in homogenous and differentiated products oligopoly (see, for example, Anderson and Renault [1], Hackner[14], Qiu [18], Vives [22]).[4]

7. Market Structure

Let us start with monopoly ($n = 1$). Under monopoly, $\epsilon_A(p_A) > 1$ as long as $c > 0$. From Proposition 4 we know that $\epsilon_A(p_A) > 1 \Leftrightarrow p_B > p_A$. Thus for monopoly, equilibrium price is always higher under budget constrained demand for all $c > 0$. When $n > 1$, competition reduces p_A and $\epsilon_A(p_A)$ may or may not be greater than unity and consequently $p_B > p_A$ may or may not hold. For sharper characterization

[4]An exception is Dastidar [11]. In context of Bertrand-Cournot comparison, he examines a homogenous products oligopoly framework with increasing and asymmetric marginal costs.

of results under $n > 1$ and free entry (in the next section), hereafter, we restrict our attention to logconcave demand functions (for economy A).[5]

Assumption 1. $x(p)$ is logconcave in p for all $p \leq \bar{p}$.

Lemma 2. Suppose Assumption 1 holds. Then
(i)) $\epsilon_A(p) \equiv -\frac{p_A D_A^{'}(p)}{D_A(p)}$ is strictly increasing p;
(ii) $\epsilon_A(p_A)$ is strictly decreasing in n and strictly increasing in c.

Proof: Recall $D_A(p) \equiv x(p)$. Differentiating $\epsilon_A(p) \equiv -\frac{p_A D_A^{'}(p)}{D_A(p)} = -p_A \frac{d \ln D_A(p)}{dp}$ yields:

$$\frac{d\epsilon_A(p)}{dp} = -\frac{d \ln D_A(p)}{dp} - p_A \frac{d^2 \ln D_A(p)}{dp^2}.$$

The result then follows from noting that $\frac{d \ln D_A(p)}{dp} = \frac{D_A^{'}(p)}{D_A(p)} < 0$ and $\frac{d^2 \ln D_A(p)}{dp^2} < 0$ (by Assumption 1). Part (ii) follows immediately from noting that (a) $\epsilon_A(p)$ is strictly increasing p (part (i)) and (b) p_A is strictly decreasing in n and strictly increasing in c (Lemma 1). □

Now we are ready to state and prove the result which links competition, costs and the likelihood of $p_B > p_A$. Hereafter, following the standard practice in the oligopoly literature, we treat the number of firms (n) as a continuous variable.

Proposition 5. Equilibrium price under budget constrained demand, p_B, is strictly higher than the equilibrium price under standard demand, p_A, if competition is low or unit cost is high. More formally, for all $c \in (0, \bar{p})$, there exists a unique $n(c) > 1$ such that

$$p_B > (=, <) p_A \Leftrightarrow n < (=, >) n(c).$$

Furthermore, $n(c)$ is strictly increasing in c and $n(c) = \infty$ for all $c \geq \bar{c}$ where $\epsilon_A(\bar{c}) = 1$.

Proof: Proposition 4 says that $p_B > p_A \Leftrightarrow \epsilon_A(p_A) > 1$. Since $\epsilon_A(p_A) > 1$ for $n = 1$ and $\epsilon_A(p_A)$ is strictly decreasing in n (Lemma 2 (ii)) it follows that there exists $n = n(c)(> 1)$ such that $\epsilon_A(p_A) > 1 \Leftrightarrow n < n(c)$. Thus we have

$$p_B > p_A \Leftrightarrow \epsilon_A(p_A) > 1 \Leftrightarrow n < n(c).$$

It is easy to extend the above to the following: $p_B > (=, <) p_A \Leftrightarrow n < (=, >) n(c)$.

Let $p_A(n, c)$ denote the equilibrium p for a given n and c. From the above argument it follows that $\epsilon_A(p_A(n(c), c)) = 1$. Thus

$$n'(c) = -\frac{\frac{\partial p_A}{\partial c}}{\frac{\partial p_A}{\partial n}} > 0.$$

[5]Without logconcavity of $x(p)$, we can establish that for all $c \in (0, \bar{p})$, there exists a unique $n(c) > 1$ such that $p_B > p_A$ holds *if* $n < n(c)$. Logconcavity allows us to replace the *if* by *if and only if*.

where the inequality follows from Lemma 1(ii): p_A is decreasing in n and increasing in c.

Finally, consider high unit cost c. In particular, $c \geq \bar{c}$ where $\epsilon_A(\bar{c}) = 1$. Since $p_A > c$ holds in equilibrium (for all finite n) and $c \geq \bar{c}$ we have $p_A \geq \bar{c}$. Applying Lemma 2 (ii) gives $p_A \geq \bar{c} \Leftrightarrow \epsilon_A(p_A) > \epsilon_A(\bar{c}) = 1$ which in turn implies $p_B > p_A$ (Proposition 4). Note that the argument does not rely on n. Thus $p_B > p_A$ holds for all n whenever $c \geq \bar{c}$. □

Lack of competition makes $p_B > p_A$ more likely. Proposition 5 makes this idea precise. It says that the $p_B > p_A$ holds if the degree of competition—measured by number of firms—is lower than a threshold level. Furthermore, as unit cost increases, the threshold level of competition increases. Thus, $p_B > p_A$ becomes more likely under higher cost. Finally, the last bit, i.e., $n(c) = \infty$ for all $c \geq \bar{c}$, says that if cost is high enough, $p_B > p_A$ holds irrespective of the degree of competition since if $c \geq \bar{c}$, p_A is always in the elastic part of the demand curve.

8. Free Entry

So far we have assumed that the number of firms, n, is fixed and the same for two economies, A and B. Since $D_A(p) \neq D_B(p)$, the number of firms is likely to be different if market structure is endogenously determined. Suppose that each firm incurs fixed cost $k > 0$ to enter and entry occurs until net profit equals zero. In this environment, the equilibrium price and the number of firms corresponding to $D_i(p)$, denoted by n_i and p_i respectively, solve the pricing equation,

$$p_i\left(1 - \frac{1}{n_i\epsilon_i(p_i)}\right) = c, \tag{14}$$

and the following zero-profit condition,

$$\frac{(p_i - c)D_i(p_i)}{n_i} = k. \tag{15}$$

Hereafter, we focus on the price comparison between the two economies A and B where (14) and (15) hold. Consider economy A first. We already know that there exists a unique $p = p(n)$ that solves (14) for a given n. Furthermore, since industry profit $(p(n) - c)D_A(p(n))$ is strictly decreasing in n there exists a unique $n = n_A$ that solves (15).[6] Thus, in the unique equilibrium of economy A, $n = n_A$ and $p = p(n_A) = p_A$. Lemma 3 records the properties of p_A and n_A.

Lemma 3. The equilibrium number of firms in economy A, n_A, is strictly decreasing in c and k while the equilibrium price, p_A, is strictly increasing in c and k.

Proof: See Appendix. □

[6]We prove this formally in the Appendix.

Concerning economy B, we cannot ascertain the uniqueness of (n_B, p_B) unless we impose some additional conditions. Later, we provide a sufficient condition for uniqueness of (n_B, p_B). The following result holds for comparison between the unique p_A and any p_B in economy B.

Proposition 6. For a given cdf $F(.)$, there exists a threshold $e^F \in (0,1)$ such that

$$\varepsilon_A(p_A) > e^F \Rightarrow p_B > p_A$$

Proof: It suffices to show that $\varepsilon_A(p_A) \geq 1 \Rightarrow p_B > p_A$. Suppose not. That is, suppose $\varepsilon_A(p_A) \geq 1$ and yet $p_A \geq p_B$. We have that

$$p_A \geq p_B \Rightarrow \varepsilon_A(p_A) \geq \varepsilon_A(p_B) \geq \varepsilon_B(p_B) \tag{16}$$

where the first inequality follows from Lemma 2 (i) and the second inequality follows from Proposition 1. The pricing equation (14) implies that

$$p_A \geq p_B \Rightarrow n_A\varepsilon_A(p_A) \leq n_B\varepsilon_B(p_B) \tag{17}$$

In addition we have that

$$(p_A - c)D_A(p_A) > (p_B - c)D_A(p_B) \geq (p_B - c)D_B(p_B) \tag{18}$$

where the first inequality is due to the strict concavity of $(p-c)D_A(p)$ and the second inequality follows from noting that $D_A(p_B) \geq D_B(p_B)$.

From (16) and (17) it follows that $n_A \leq n_B$. Equation (18), together with $n_A \leq n_B$ implies that

$$\frac{(p_A - c)D_A(p_A)}{n_A} - k > \frac{(p_B - c)D_B(p_B)}{n_B} - k.$$

which is contradiction since $\frac{(p_i-c)D_i(p_i)}{n_i} - k = 0$ for both $i = A, B$. □

As before, we find that $p_B > p_A$ holds if $\varepsilon_A(p_A) > 1$. However, Proposition 6 shows that $\varepsilon_A(p_A) > 1$ is no longer necessary for $p_B > p_A$. Later we show that for $e^F = \frac{1}{2}$ for linear demand and $e_F \leq \frac{1}{2}$ for concave demand. Since $e^F < 1$, $p_B > p_A$ can occur even in the inelastic region of the demand curve. Thus $p_B > p_A$ is more likely under free entry. Can we say something more? To do so, hereafter we focus on a class of $F(m)$ which satisfy the following:

$$\frac{d}{dz}\left(\frac{\int_0^z tf(t)dt}{z(1-F(z))}\right) \geq 0 \tag{19}$$

A number of standard distributions including uniform and Pareto among others satisfy (19). Lemma 4 highlights an important benefit of (19).

Lemma 4. Suppose $F(m)$ satisfies (19). Then (n_B, p_B) is unique.

Proof: See Appendix. □

Uniqueness of (n_B, p_B) greatly facilitates the characterization of the necessary and sufficient condition for $p_B > p_A$.

Proposition 7. Consider the class of $F(m)$ that satisfy (19). The necessary and sufficient condition for $p_B > p_A$ is:

$$\epsilon_A(p_A) > \frac{\int_0^{p_A x(p_A)} m f(m) dm}{p_A x(p_A) F(p_A x(p_A))} \equiv e^F.$$

Proof: First consider n_A and p_A which respectively denote the number of firms and price in the free entry equilibrium of economy A. Choose $n = n'$ such that $p_B(n') = p_A$ and compute profit per firm in economy B, i.e, $\frac{(p_A - c)D_B(p_A)}{n'} - k$. Now note that

$$\frac{(p_A - c)D_B(p_A)}{n'} - k < 0 \Leftrightarrow p_B > p_A.^7$$

That is, $p_B > p_A$ if and only if firms incur losses at $p = p_A$, because in that case firms exit which reduces n below n' and drives p_B up, in particular, higher than p_A. Note that

$$\begin{aligned} \frac{(p_A - c)D_B(p_A)}{n'} - k &= \frac{(p_A - c)D_B(p_A)}{n'} - \frac{(p_A - c)D_A(p_A)}{n_A}, \\ &= (p_A - c)[\frac{D_B(p_A)}{n'} - \frac{D_A(p_A)}{n_A}], \\ &= -(p_A - c)^2[D_B^{'}(p_A) - D_A^{'}(p_A)], \end{aligned} \tag{20}$$

where the last equality exploits a rearranged version of (14):

$$-(p_i - c)D_i^{'}(p_i) = \frac{D_i(p_i)}{n_i}.$$

From (6) we have $D_B(p) - D_A(p) = -\frac{1}{p} \int\limits_0^{px(p)} F(m)dm$ which upon differentiation gives:

$$D_B^{'}(p_A) - D_A^{'}(p_A) = -\frac{p_A(p_A x'(p_A) + x(p_A))F(p_A x(p_A)) - \int\limits_0^{p_A x(p_A)} F(m)dm}{p_A^2}.$$

Simplifying the right-hand side of the above equation gives

$$D_B^{'}(p_A) - D_A^{'}(p_A) = \frac{x(p_A)}{p_A}\left(\epsilon_A(p_A) - \frac{\int\limits_0^{p_A x(p_A)} m f(m)dm}{p_A x(p_A) F(p_A x(p_A)}\right). \tag{21}$$

[7]This $\Leftrightarrow$ implicitly uses the following property: each firm's equilibrium profit is declining in n in economy B. Proof of this property is immediate if the marginal revenue function in economy B, i.e., $p(1 - \frac{1}{n\epsilon_B(p)})$ is increasing in p. See the proof of Lemma 4 in the Appendix where we establish that $p(1 - \frac{1}{n\epsilon_B(p)})$ is indeed increasing in p.

Combining (20) and (21) we have

$$\begin{aligned} p_B > p_A &\Leftrightarrow \frac{(p_A - c)D_B(p_A)}{n'} - k < 0, \\ &\Leftrightarrow D_B^{'}(p_A) - D_A^{'}(p_A) > 0, \\ &\Leftrightarrow \epsilon_A(p_A) > \frac{\int\limits_0^{p_A x(p_A)} m f(m) dm}{p_A x(p_A) F(p_A x(p_A)}. \end{aligned}$$

□

As in the case of fixed number of firms, we find that $p_B > p_A$ holds if and only if elasticity of the standard demand function at $p = p_A$ is greater than a threshold e^F. To better appreciate Proposition 7 consider the following budget distribution for which e^F is constant:

(22) $$F(m) = \left(\frac{m}{\bar{m}}\right)^{\alpha}, \qquad \text{if} \qquad m \in [0, \bar{m}]$$

where $\alpha \in (0,1]$, and $F(m) = 1(0)$ if $m > \bar{m}(m < 0)$. Since $e^F \equiv \frac{\int_0^{p_A x(p_A)} m f(m) dm}{p_A x(p_A) F(p_A x(p_A))} = \frac{\alpha}{1+\alpha}$,

$$p_B > p_A \Leftrightarrow \epsilon_A(p_A) > \frac{\alpha}{1+\alpha}.$$

Thus if $F(m)$ is uniform (i.e., $\alpha = 1$) as in illustrative example, equilibrium price in the budget-constrained economy is higher if and only if $\epsilon_A(p_A) > \frac{1}{2}$. Two features are worth noting.

- While $e^F = \frac{\alpha}{1+\alpha}$ or $e^F = \frac{1}{2}$ seem special as they correspond to specific forms of $F(m)$, note that the same e^F applies for all logconcave demand functions.
- Observe that no matter how inelastic the demand is, there always exists α low enough such that $p_B > p_A$. Thus a good might be highly inelastic, as is often the case for necessities, and yet the equilibrium price in budget-constrained economy might be higher.

An advantage of working with $F(m) = (\frac{m}{\bar{m}})^{\alpha}$ is that the threshold value e^F does not depend on p_A. While, in general e^F will generally depend on p_A, for concave $F(m)$ we find an upper bound: $e_F \leq \frac{1}{2}$. This finding implies that if $F(m)$ is concave and $\epsilon_A(p_A) > \frac{1}{2}$, equilibrium price is higher in the budget constrained economy. To understand how we obtain this upper bound, write

$$e_F = \frac{\int\limits_0^{p_A x(p_A)} m f(m) dm}{p_A x(p_A) F(p_A x(p_A)} = \frac{p_A x(p_A) F(p_A x(p_A) - \int\limits_0^{p_A x(p_A)} F(m) dm}{p_A x(p_A) F(p_A x(p_A)}.$$

Now consider Figure 2.

(Figure 2 to be inserted here)

In Figure 2, $e_F \equiv \frac{p_A x(p_A) F(p_A x(p_A)) - \int_0^{p_A x(p_A)} F(m)dm}{p_A x(p_A) F(p_A x(p_A))}$ can be expressed as

$$\frac{(\text{Area A} + \text{Area B}) - \text{Area B}}{\text{Area A} + \text{Area B}} = \frac{\text{Area A}}{\text{Area A} + \text{Area B}} = \frac{1}{1 + \frac{\text{Area B}}{\text{Area A}}}.$$

Concavity of $F(m)$ implies $\text{Area B} \geq \text{Area A}$ which in turn implies $e_F \leq \frac{1}{2}$.

Finally we conclude our analysis by examining the link between cost, competition and the possibility of $p_B > p_A$. Since n is endogenous, we use entry cost, k, as a proxy measure for competition. In particular, we interpret an increase in k as a decrease in the degree of competition.

Proposition 8. Suppose, in addition to the condition stated in Proposition 7, the following holds: $\frac{\int_0^z tf(t)dt}{zF(z)}$ is weakly decreasing in z for all $z \in (0, \bar{m})$. Then $p_B > p_A$ holds if competition is low, or equivalently, if entry cost k is high enough. More formally, for all c there exists $k(c)$ such that $p_B > p_A$ if $k > k(c)$. Furthermore, $k(c)$ is strictly decreasing in c.

Proof: See Appendix. □

The essence of Proposition 8 is same as Proposition 5: the low level of competition and high unit cost makes $p_B > p_A$ more likely. The analysis is clearcut for $F(m) = (\frac{m}{\bar{m}})^\alpha$ for which the threshold $e^F = \frac{\alpha}{1+\alpha}$ is constant:

$$p_B > p_A \Leftrightarrow \epsilon_A(p_A) > \frac{\alpha}{1+\alpha}.$$

Since $\epsilon_A(p_A)$ is increasing in p_A and p_A is increasing k, it follows that higher k, or equivalently, lower competition makes $p_B > p_A$ more likely. In Appendix we show that the additional restriction, $\frac{\int_0^z tf(t)dt}{zF(z)}$ is weakly decreasing in z, helps to prove Proposition 8 when e_F is not constant.

9. Summary and Concluding Remarks

Introducing budget-constrained consumers in an oligopoly setting, we have shown that the equilibrium price under budget-constrained demand (p_B) can often be higher than that under standard demand (p_A). This holds even though the budget-constrained demand is lower than standard demand at all relevant prices. Despite the non-standard nature of the demand shifts and the possibility of multiple p_B, we were able to identify necessary and sufficient conditions for $p_B > p_A$. If the market structure is exogenous, the necessary and sufficient condition for $p_B > p_A$ is that the elasticity of the standard demand (at p_A) is greater than unity. The condition is easy to verify as it does not depend on the details of the budget-constrained demand or budget distribution. Under endogenous market structure (i.e., free entry), price-elasticity greater than unity is sufficient but not necessary for $p_B > p_A$. For both exogenous market and endogenous market structures we found that the lack of competition and inefficient technology make $p_B > p_A$ more likely.

Throughout the paper we refrained from offering any interpretation of the two economies, A and B. Here we offer one which could be useful in the context of urban economics and/or international trade. Think of the two economies A and B as two different regions with segmented markets. The two regions, A and B, might be two different suburbs in the same city, two different cities in the same country, or even two different countries. Outcomes arising from the analysis of an endogenous market structure can then be viewed as autarky outcomes. In autarky, each firm in region i incurs fixed cost k and serves region i only. The equilibrium values of p_i and n_i, $i = A, B$, are given by solution to the pricing equation (14) and the zero-profit condition (15) respectively. Now consider a trade regime where by incurring the same fixed cost k each firm can serve both A and B. Then the pricing equations, i.e. $p_i\left(1 - \frac{1}{n\epsilon_i(p_i)}\right) = c$, still apply as long as the markets are segmented but the relevant zero-profit condition is

$$\frac{(p_A - c)D_A(p_A)}{n} + \frac{(p_B - c)D_B(p_B)}{n} = k.$$

Our analysis in sections 3-7 with exogenous market structure (i.e., n is given and is the same for A and B) can be viewed as the trade regime where the common n is given by the solution to the above equation. Since $p_B > p_A$ holds for a smaller range of elasticities under exogenous market structure, loosely speaking, our analysis suggests that $p_B > p_A$ is less likely under the trade regime.

That p_B and p_A are different under the trade regime might sound odd. However, the difference in equilibrium prices (across markets) arise from the assumption of segmented markets which is fairly standard in the imperfect competition models of trade (see, for example, Brander and Krugman [7], Venables [21], Bagwell and Staiger [5]). Is welfare higher when firms treat the markets as integrated rather than segmented? In other words, is welfare higher under uniform pricing? This question lies at the heart of the industrial organization literature on third degree price discrimination (see Varian [20], Holmes [16], and Armstrong and Vickers [3] among others). Cowan[10] and Aguirre, Cowan, and Vickers [2] study the welfare effects of third-degree price discrimination under monopoly. Using the curvature and slope of demand functions, they show that uniform pricing delivers higher welfare for a large class of demand functions. Unlike Cowan [10] and Aguirre et al. [2], we do not restrict our attention to monopoly. Whether uniform pricing raises welfare in our oligopoly framework remains an open question. Here we abstract from the welfare question to focus on the issue at hand: comparison of equilibrium prices under standard demand and the budget-constrained demand.

A limitation of our analysis is that we consider only linear pricing. Che and Gale [9] study a mechanism-design problem where a monopoly seller offers a good to a buyer who may be budget-constrained. In the framework they consider, the optimal mechanism for a monopolist may involve the use of nonlinear pricing, declining price sequence, and financing. The pricing mechanism is richer in their work than in our contribution. However, focusing on linear pricing allow us to examine a richer and broader set of environments. Che and Gale [9] consider a pure monopoly with unit demand and linear preferences. We consider monopoly and oligopoly and examine free entry environment in addition to the canonical set up with exogenous market structure. Preferences are fairly general and consumers can buy multiple units in our framework. Analysis of non-linear pricing in oligopoly environments with budget constrained consumers is left for future research.[8]

10. APPENDIX

Proof of Lemma 1: (i) Using $\epsilon_A(p) = -\frac{px'(p)}{x(p)} = -\frac{p(x)}{xp'(x)}$ we get

$$p\left(1 - \frac{1}{n\epsilon_A(p)}\right) = p(x(p)) + \frac{x(p)p'(x(p))}{n} = g(x(p))$$

Since $x'(p) < 0$, $p\left(1 - \frac{1}{n\epsilon_A(p)}\right)$ is increasing in $p \Leftrightarrow g'(x) < 0$ where

$$g'(x) \quad = \quad p'(x) + \frac{xp''(x)}{n} + \frac{p'(x)}{n} = \frac{(n-1)p'(x)}{n} + \frac{2p'(x) + xp''(x)}{n}$$

Note that (a) $p'(x) < 0$ and (b) $2p'(x) + xp''(x) < 0$ (equation 9). Together (a) and (b) imply $g'(x) < 0$.

(ii) From (8) we have

$$p'(x_A)x_A + np(x_A) = nc, \tag{23}$$

where x_A and $p(x_A) = p_A$ respectively denote the aggregate output and price in Cournot equilibrium. Differentiating (23) with respect to n and c respectively gives

$$\frac{dx_A}{dn} = -\frac{p(x_A) - c}{(n+1)p'(x_A) + x_A p''(x_A)}, \qquad \frac{\partial x_A}{\partial c} = \frac{n}{(n+1)p'(x_A) + x_A p''(x_A)}.$$

By (9), $(n+1)p'(x_A) + x_A p''(x_A) < 0$ which imply (a) $\frac{dx_A}{dn} > 0$ and (b) $\frac{dx_A}{dc} < 0$. Since $p'(x_A) < 0$, (a) and (b) imply that

$$\frac{dp_A}{dn} = p'(x_A)\frac{dx_A}{dn} < 0; \qquad \frac{dp_A}{dc} = p'(x_A)\frac{dx_A}{dc} > 0.$$

[8] A framework with budget constrained consumers and non-linear pricing naturally invites two further possibilities: (i) a group of consumers might want to resell the product to other consumers, (ii) consumers might want to form coalitions in order to exploit bulk discounts. See Calzolari and Pavan [8] for an analysis of the intricacies associated with monopoly pricing when buyers are expected to resell.

Proof of Lemma 3: First we prove that (n_A, p_A) is unique. Let $x_A(n)$ and $p_A(n) = p_A(x_A(n))$ respectively denote the aggregate output and price in Cournot equilibrium for a given $n \geq 1$. To prove uniqueness of (n_A, p_A) it is sufficient to show that

$$\frac{(p_A - c)D_A(p_A)}{n} \equiv \frac{(p(x_A(n)) - c)x_A(n)}{n} \equiv \widetilde{\pi}(n)$$

is strictly decreasing in n. Note that

$$\begin{aligned}\frac{d}{dn}\left[(p(x_A) - c)x_A\right] &= \left[p(x_A) - c + x_A p'(x_A)\right]\frac{dx_A}{dn} \\ &= \left[p(x_A) - c + \frac{x_A}{n}p'(x_A)\right]\frac{dx_A}{dn} + \frac{(n-1)x_A}{n}p'(x_A)\frac{dx_A}{dn}\end{aligned}$$

First-order condition in section 5 imply $p(x_A) - c + \frac{x_A}{n}p'(x_A) = 0$. The result then follows from noting that $p'(x_A) < 0$, $\frac{dx_A}{dn} > 0$.

(ii) First we prove that $\frac{dn_A}{dk} < 0$ and $\frac{dp_A}{dk} > 0$. Differentiating $\widetilde{\pi}(n_A) \equiv k$ and rearranging gives $\frac{dn_A}{dk} = \frac{1}{\widetilde{\pi}'(n_A)}$. From the proof of part (i) we know that $\widetilde{\pi}'(n_A) < 0$ which imply $\frac{dn_A}{dk} < 0$. This finding together with $\frac{\partial p_A(n_A)}{\partial n} < 0$ [Lemma 1(ii)] imply that $\frac{dp_A}{dk} = \frac{\partial p_A(n_A)}{\partial n}\frac{dn_A}{dk} > 0$.

Now we prove that $\frac{dn_A}{dc} < 0$. Differentiating the free entry condition $\widetilde{\pi}(n_A) \equiv k$ we get:

$$\frac{dn_A}{dc} = -\frac{\frac{\partial \pi(.)}{\partial c}}{\frac{\partial \pi(.)}{\partial n}}.$$

Since $\frac{\partial \pi(.)}{\partial n} < 0$, $\frac{dn_A}{dc} < 0 \Leftrightarrow \frac{\partial \pi(.)}{\partial c} < 0$. Differentiating $\widetilde{\pi}(n) = \frac{(p(x_A(n))-c)x_A(n)}{n} = -\frac{p'(x_A(n))x_A^2(n)}{n^2}$ with respect to c and rearranging subsequently we get

$$\frac{\partial \widetilde{\pi}(n)}{\partial c} = -\frac{x_A}{n^2}\left[2p'(x_A(n)) + x_A(n)p''(x_A(n))\right]\frac{\partial x_A}{\partial c} < 0$$

where the inequality follows from noting (a) $2p'(x_A(n)) + x_A(n)p''(x_A(n)) < 0$ and (b) $\frac{\partial x_A}{\partial c} < 0$ (see the proof of Lemma 1(ii)).

Finally note that $\frac{dp_A(n_A)}{dc} = \frac{\partial p_A(n_A)}{\partial c} + \frac{\partial p_A(n_A)}{dn}\frac{dn_A}{dc}$. By Lemma 1(ii), (a) $\frac{\partial p_A(n_A)}{\partial c} > 0$ and (b) $\frac{\partial p_A(n_A)}{dn} < 0$. We have already proved that (c) $\frac{dn_A}{dc} < 0$. Together, (a) - (c) imply $\frac{dp_A(n_A)}{dc} > 0$. $\square$

Proof of Lemma 4: For economy A, we have shown that $2p'(x) + xp''(x) < 0$ or its equivalent, i.e., $p(1 - \frac{1}{n\epsilon_A(p)})$ is increasing in p, is sufficient to prove that (n_A, p_A) is unique. To show (n_B, p_B) is unique it suffices to establish that $p(1 - \frac{1}{n\epsilon_B(p)})$ is increasing in p. Below we prove $\epsilon_B(p)$ is increasing in p which implies $p(1 - \frac{1}{n\epsilon_B(p)})$ is increasing in p.

Expand $D_B(p)$ as follows:

$$\begin{aligned}D_B(p) &= x(p)(1 - F(px(p)) + \int_0^{px(p)} \frac{m}{p}f(m)dm, \\ &= x(p)(1 - F(px(p))\left[1 + \frac{\int_0^{px(p)} mf(m)dm}{px(p)(1 - F(px(p)))}\right]\end{aligned}$$

Using the above expression for $D_B(p)$, (7), after some rearrangement can be written as

$$|\epsilon_B(p) - 1| = \frac{|\epsilon_A(p) - 1|}{1 + \frac{\int_0^{px(p)} mf(m)dm}{px(p)(1-F(px(p)))}} \tag{24}$$

Suppose $\epsilon_A(p) > 1$ which imply $\frac{dpx(p)}{dp} = px'(p) + x(p) = x(p)(1 - \epsilon_A(p)) < 0$. Think of $px(p)$ as z. Condition (19) then imply (a) $1 + \frac{\int_0^{px(p)} mf(m)dm}{px(p)(1-F(px(p)))}$ is decreasing in p. Lemma 1(ii) imply (b) $\epsilon_A(p) - 1$ is increasing in p. Together with equation (24), (a) and (b) imply that $\epsilon_B(p)$ is increasing in p.

For $\epsilon_A(p) < 1$ rewrite (24) as

$$1 - \epsilon_B(p) = \frac{1 - \epsilon_A(p)}{1 + \frac{\int_0^{px(p)} mf(m)dm}{px(p)(1-F(px(p)))}}.$$

Here, $1 - \epsilon_A(p)$ is decreasing in p while $1 + \frac{\int_0^{px(p)} mf(m)dm}{px(p)(1-F(px(p)))}$ is increasing in p. Thus $1 - \epsilon_B(p)$ is decreasing in p which in turn imply $\epsilon_B(p)$ is increasing in p. □

Proof of Proposition 8: Define $h(p_A) \equiv \epsilon_A(p_A) - e^F = \epsilon_A(p_A) - \frac{\int_0^{p_A x(p_A)} mf(m)dm}{p_A x(p_A) F(p_A x(p_A))}$. Applying Proposition 7 gives:

$$p_B > (=, <) p_A \Leftrightarrow h(p_A) > (=, <) 0.$$

If $\epsilon_A(p_A) \geq 1$, $h(p_A) = \epsilon_A(p_A) - e^F > 0$ since $e^F < 1$. If $\epsilon_A(p_A) < 1$, $p_A x(p_A)$ is increasing in p_A. Think $p_A x(p_A) = z$. Since $\frac{\int_0^z tf(t)dt}{zF(z)}$ is weakly decreasing in z and z is decreasing in p_A it follows that $e^F = \frac{\int_0^{p_A x(p_A)} mf(m)dm}{p_A x(p_A) F(p_A x(p_A))}$ is decreasing in p_A. This finding, together Lemma 2(i) imply that $h(p_A)$ is increasing in p_A whenever $\epsilon_A(\hat{p}_A) < 1$. Thus there exists a critical $\hat{p}_A$ satisfying $\epsilon_A(\hat{p}_A) < 1$ such that

$$h(p_A) > 0 \Leftrightarrow p_A > \hat{p}_A.$$

The proof then follows from noting that p_A is increasing in c and k (Lemma 3). □

REFERENCES

[1] Anderson, S.P., Renault,R., 2003. Efficiency and surplus bounds in Cournot competition, Journal of Economic Theory 113, 253-264.

[2] Aguirre,I., Cowan,S., Vickers, J., 2010. Monopoly price discrimination and demand curvature, American Economic Review 100, 1601-1615.

[3] Armstrong, M., Vickers, J., 2001. Competitive price discrimination, The RAND Journal of Economics 32, 579-605.

[4] Attanasio, O.P., Frayne, C., 2006. Do the poor pay more? mimeo.

[5] Bagwell, K., Staiger, R.W., 2009. Delocation and trade agreements in imperfectly competitive markets, mimeo, Stanford University.

[6] Baldenius, T., Rachelstein,S., 2000. Comparative statics of monopoly pricing, Economic Theory, Economic Theory 16, 465-469.

[7] Brander, J., Krugman, P., 1983. A reciprocal dumping model of international trade, Journal of International Economics 15, 313-321.

[8] Calzolari, G., Pavan, A., 2006. Monopoly with resale, RAND Journal of Economics 37, 362-375.

[9] Che,Y-K., Gale,I., 2000. The optimal mechanism for selling to a budget-constrained buyer, Journal of Economic Theory 92, 198-233.

[10] Cowan,S., 2007. The welfare effects of third-degree price discrimination with non-linear demand functions, RAND Journal of Economics 38, 419-428.

[11] Dastidar, K. G., 1997. Comparing Cournot and Bertrand in a homogeneous product market. Journal of Economic Theory 75, 205-212.

[12] Deneckre,R., Kovenock,D., 1999. Direct demand-based Cournot existence and uniqueness conditions, Working paper, University of Wisconsin.

[13] Dixit,A., 1986. Comparative statics for oligopoly, International Economic Review 27, 107-122.

[14] Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. Journal of Economic Theory 93, 233-239.

[15] Hamilton, S.F., 1999. Demand shifts and market structure in free-entry oligopoly equilibria, International Journal of Industrial Organization 17, 259-275.

[16] Holmes, T., 1989. The effect of third-degree price discrimination in oligopoly. American Economic Review 79, 244-250.

[17] Li, N., 2010. An Engel curve for variety, Job Market Paper (2010) available at http://works.bepress.com/nicholasli/35.

[18] Qiu, L., 1997. On the dynamic efficiency of Bertrand and Cournot equilibria. Journal of Economic Theory 75, 213-229.

[19] Quirmbach, H., 1988. Comparative statics for oligopoly: demand shift effects. International Economic Review 29, 451-459.

[20] Varian, H., 1985. Price discrimination and social welfare. American Economic Review 75, 870-875.

[21] Venables,A., 1985. Trade and trade policy with imperfect competition: the case of identical products and free entry. Journal of International Economics 19, 1-20.

[22] Vives, X., 1999. Oligopoly pricing: old ideas and new tools. MIT Press, Cambridge and London.

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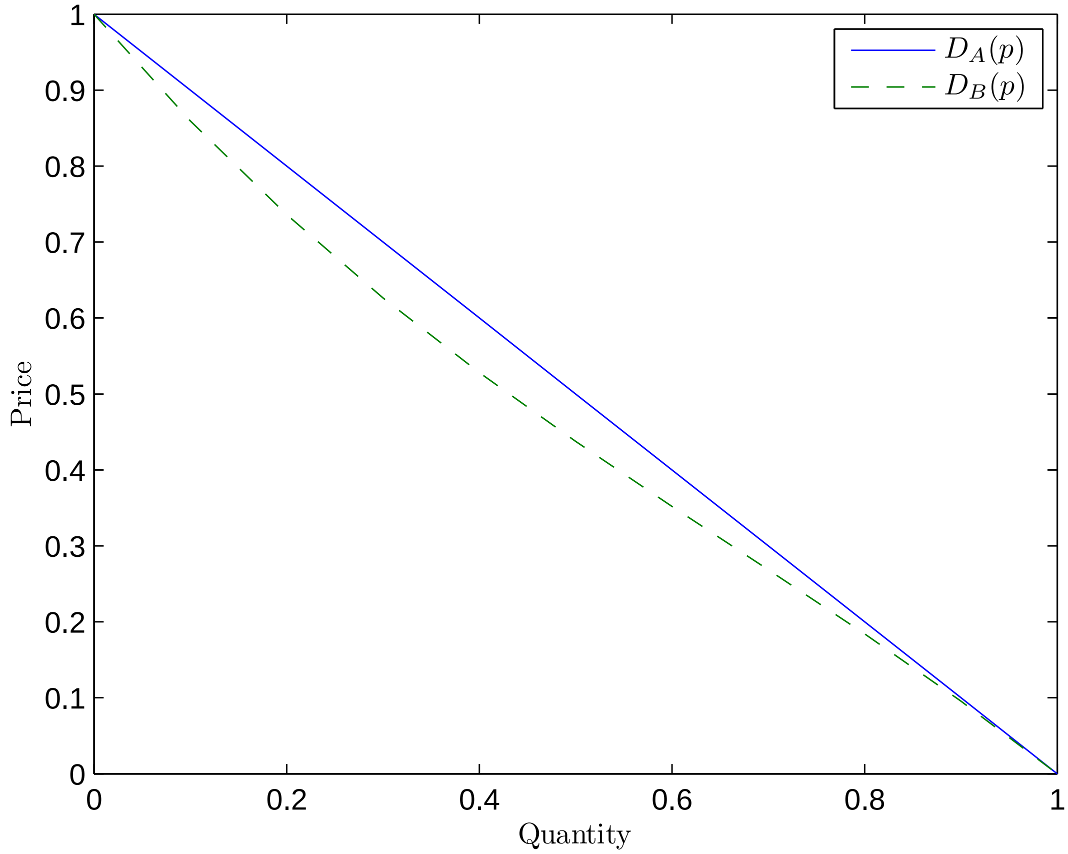

Figure 1: $D_A(p)$ and $D_B(p)$

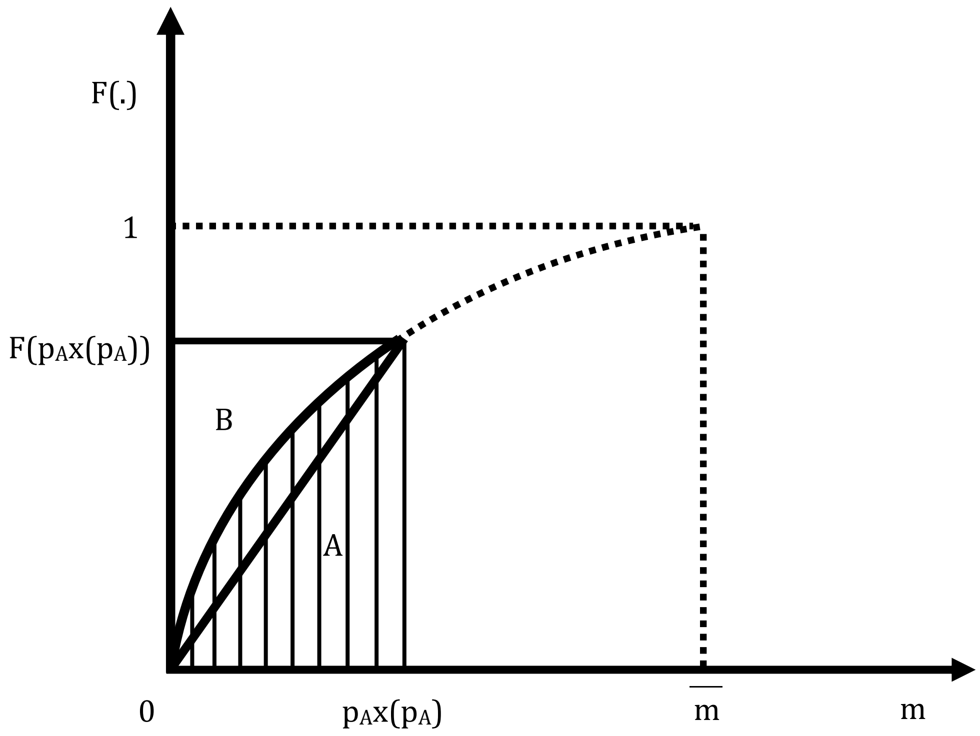

Figure 2: Concave F(m)