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Bargaining, Tariffs, and Vertical Specialization

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Abstract

How does the bargaining power of firms affect trade policy? We address this question in an international, bilateral oligopoly setting where the Home country specializes in final goods and the Foreign country specializes in intermediate inputs. A matched Home-Foreign pair bargains simultaneously over the input price and the level of output, and competes with other matched pairs in markets. In such environments with vertical specialization, we show that the welfare-maximizing Home tariff rate is strictly decreasing in the bargaining power of Home firms. Surprisingly, we find that an increase in Home bargaining power can also raise Foreign profits. These results hold for fairly general demand function and a number of different procurement mechanisms. In an endogenous market structure setting with free entry and matching, the relationship between the tariff and bargaining power is usually non-monotone. In particular, the relationship is U-shaped (resp. inverted U-shaped) if the demand function is strictly concave (resp. convex). If the demand function is linear, free trade is optimal (i.e., optimal tariff is zero) irrespective of the bargaining power. The relationship between welfare and bargaining power is also explored.

Keywords: Tariffs, Oligopoly, Outsourcing, Bargaining Power, Free Entry
JEL Classification: F12, F13

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1 Introduction

Fragmentation of production chains and the vertical specialization of production have led to rapid growth in intermediate input trade (Hummels, Ishii, and Yi, 2001; Yeats, 2001; Yi, 2003). In recent years, this input trade growth has taken place largely via foreign outsourcing rather than via foreign direct investment (FDI). Hanson, Mataloni, and Slaughter (2005) find that intrafirm trade within U.S. multinationals has grown very rapidly, yet somewhat less than international outsourcing by U.S. firms. Documenting the enormous growth of manufacturing exports from China, Spencer (2005) finds that processing exports (input trade), which occur through international outsourcing between foreign buyers and independent Chinese subcontractors, constitute a large part of this manufacturing exports.

The increasing importance of international outsourcing and FDI has led to new models of vertical specialization which embed organizational structure in an imperfectly competitive framework. These models have matched positive features of reality quite well; however, barring a few exceptions (discussed later in this Introduction), little attention has been paid to welfare implications and trade policy in these models with vertical relationships. For instance, in a survey of recent literature, Antràs and Rossi-Hansberg (2009) note that "although the literature on organizations and trade has been largely concerned with matching positive features of reality ..... much less attention has been given to the normative and policy implications of changes in the international organization of production" (pp. 61). Our paper takes a step toward filling this important gap by explicitly considering tariffs in this environment.

Like horizontal specialization (e.g., in a classical Ricardian world), vertical specialization creates gains from trade. We take the existence of these gains and the pattern of specialization as given. Without loss of generality, we assume that the Foreign country specializes in intermediate inputs and the Home country imports intermediate inputs (from the Foreign country) and specializes in final goodsm production.[1] To facilitate exposition, hereafter, we will use Foreign and Home for the Foreign country and the Home country respectively. Firms first enter incurring a fixed cost and then seek partners from the other stage of production. A matched Foreign-Home pair bargains simultaneously over the input price and the level of output and competes with other matched pairs in a Cournot market.

We examine optimal tariffs, and in particular, the relationship between tariffs and bargaining strength in such environments of vertical specialization. Consider Home government choosing a tariff rate on intermediate imports to maximize welfare. If the bargaining power of its firms is relatively low, will Home government set a high tariff to improve its welfare? Also, is Home's low bargaining power necessarily better for Foreign? When the number of matched pairs is exogenously set, the key results are as follows.

- As bargaining power of Home firms increases, the welfare-maximizing Home tariff

[1] As will be clear from the analysis, whether Home specializes in final goods or intermediate inputs is not important for our results.

rate strictly declines. In particular, if Home firms have no bargaining power, optimal tariff rate for Home is strictly positive, while if Home firms have all the bargaining power, optimal tariff is strictly negative.

- If the number of matched pairs is small, e.g., bilateral monopoly, the low bargaining power of Home can also be bad for Foreign firms because of induced high tariffs. This suggests that an increase in the bargaining power of Home firms might increase Foreign profits as well as Home welfare.

It is important to note how tariffs work in models with vertical relationships. In the absence of vertical relationships, that is if Foreign and Home firms are in the same stage of production, a tariff by Home government benefits Home firms by shifting profits from Foreign to Home. However, when countries are vertically specialized, outputs in two stages are complements and consequently a Home tariff not only hurts Foreign firms but it also hurts Home firms. The only benefit of tariffs is in terms of tariff revenues. This discussion suggests that a tariff by Home government improves Home welfare only when Home firms receive a low share of the profits. In the extreme case, where Home has no bargaining power (Foreign firms make take-it-or-leave-it offers), the adverse effect of tariff on Home profits is absent and we find that a positive tariff improves welfare (unless the demand is too convex). As the bargaining power of Home firms increases, optimal tariff declines since the adverse effect of a tariff on Home profits increases.

The mechanism outlined above is quite general. As long as the number of matched pairs is exogenously given, the results hold for fairly general demand functions, alternative forms of bargaining (e.g., sequential instead of simultaneous bargaining) and alternative means of procurement of inputs (market competition instead of bargaining). The analysis also holds when both Home and Foreign governments choose tariff rates strategically to maximize their respective welfare.

The story is different in the long run when the number of matched pairs is endogenously determined. In the presence of free entry and matching (following entry), the expected profit is zero and so the adverse effect of a tariff on Home profit is not an issue. Nevertheless, bargaining power plays an important role as it affects the thickness of the market—the number of firms entering in each stage of production—and consequently the output. If the bargaining power of any one side is high, fewer firms enter on the other side of the market. Thus compared to a case in which the bargaining power of Foreign and Home firms are similar, matching is worse and output is lower if the bargaining strengths differ significantly. This observation suggests a non-monotone relationship between output and bargaining power which in turn leads to a non-monotone relationship between the optimal tariff and the bargaining power.

- Home's optimal tariff varies non-monotonically with the bargaining power of Home firms. In particular, the relationship between optimal tariff and Home's bargaining power is U-shaped if the demand function is strictly concave and inverted U-shaped if it is strictly convex.

- For the class of demand functions with constant elasticity of slope we find that, irrespective of the bargaining power, the optimal tariff in the long run is zero (resp. strictly positive, strictly negative) if the demand function is linear (resp. strictly concave, strictly convex).

As indicated earlier, there is a rapidly growing strand of literature that focuses on the role of contracts in international vertical specialization. Antràs (2003, 2005), Antràs and Helpman (2004) and Grossman and Helpman (2002, 2003, 2004, 2005) all construct models in which Foreign firms and Home firms bargain over contracts for specialized input. Using monopolistic competition and incomplete contracting, these models consider a rich environment and provide an explanation for the emergence of different organizational forms. However, trade policies are usually absent from these models.

As far as we are aware, the only papers that examine trade policy in the context of trade in intermediate products and contractual incompleteness are Ornelas and Turner (2008, 2011) and Antràs and Staiger (2011). Ornelas and Turner elegantly analyze the effect of tariffs on an incomplete contract setting with outsourcing where foreign firms make relationship-specific investments. Prior to investment by the foreign supplier, the domestic firm decides whether to integrate vertically with the foreign supplier by incurring a fixed cost. By affecting the investment levels and the organizational choice, they show that a reduction in tariff can lead to a significant increase in trade volume, well beyond that which could be explained by standard trade models. Contractual incompleteness and relationship-specific investments are also at the heart of Antràs and Staiger (2011). Taking the offshoring of intermediate inputs as given, they provide the first analysis of trade agreements in the presence of offshoring.

As is clear from the description of our model, offshoring is present in our set up as well. However, to highlight the novel interaction between bargaining power and trade policy, we abstract away from relationship-specific investments and contractual incompleteness. We take a step back, and assume complete contracts. We examine trade policy in a setting where (i) a pair consisting of a foreign firm and a domestic firm bargain over contracting in the price and quantity of the input, and (ii) all pairs engage in oligopolistic interaction in the final goods market. In sections 3 and 4, we show that weak bargaining power can provide a rationale for high tariffs in a wide variety of environments as long as the number of firms is fixed. Section 5 introduces free entry, which is absent in the papers mentioned above. We demonstrate that the monotone relationship between bargaining power and tariff breaks down in the presence of free entry.

A handful of papers have considered trade policy in the context of vertical oligopolies in which Foreign and Home firms are engaged in market competition. Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Chen, Ishikawa and Yu (2004), for instance, analyze the strategic interaction between Foreign firms and Home firms in an international oligopoly, and examine the effect of export subsidies on imported input on the social welfare. These papers, however, do not deal with bargaining in input trade. Moreover, free entry and random matching, which are at the core of our long-run analysis, are not considered in these works. In subsection 3.4.2, by relating market thickness to bargaining power, we discuss how our analysis could be adapted for environments in which intermediate goods are procured in the market.

2 Model

Consider a setting with two countries, Home and Foreign, specializing respectively in a final good and an intermediate input. Foreign has n upstream firms, $F_1, F_2, ..., F_n$. Home has m downstream firms, $H_1, H_2, ..., H_m$; each procures intermediate input from an upstream foreign firm to produce the final good.[2] There are two ways to procure intermediate input: intrafirm trade (FDI) and arm's length trade (outsourcing), and in both cases, contracts are used to specify the delivery of input. Because much of the increase in trade in intermediate inputs can be attributed to an increase in outsourcing, we restrict attention to outsourcing in this paper.[3]

Upon entry, each firm seeks a partner from the other stage of production. Entry cost is K_H for Home firms and K_F for Foreign firms. Matching randomly occurs between Home and Foreign firms. We assume that one-to-one matching takes place in outsourcing.[4] Let $s = s(m, n)$ denote the number of pairs that are formed in this matching process, where $s(m, n) \leq \min\{m, n\}$ and $s(.)$ is increasing in both of its arguments. More properties of this matching function are described in subsection 5.1.

One unit of final good requires one unit of intermediate product. The unit cost of production for the intermediate input is $c(> 0)$. For simplicity, we assume that after procuring intermediate inputs at a negotiated price, Home firms can transform these inputs to final products without incurring additional costs.

The demand for final goods is given by $Q = Q(P)$ where (i) $Q(P) = 0$ for $p \geq \bar{p}(> c)$, (ii) $Q(P)$ is twice continuously differentiable and, (iii) $Q'(P) < 0$ for all $p \in (0, \bar{P})$. These assumptions guarantee the existence of Cournot equilibrium. We will often work with inverse demand functions. These assumptions regarding $Q(P)$ imply that the inverse demand function $P = P(Q)$ is twice continuously differentiable and $P'(Q) < 0$ for all $Q > 0$.

[2]Note that depending on the importing input, Home can represent either North or South. By definition, Home imports intermediate input from Foreign. If the input is a "high-tech" one such as semi-conductors or IC tips, we view Home as a developing country that relies on the import of input from a developed country. Thus, Home is a South country in this case. By contrast, if the input is "low-tech", such as simple assembly or manufacturing, Home can be viewed as a developed country that outsources the production process to a developing country. In this case, Home is a North country.

[3]We assume throughout our analysis that the firms can write complete contracts. The possibility of contractual incompleteness in our setting is discussed in the Conclusion.

[4]Following Grossman and Helpman (2002), we focus on one-to-one matching for tractability. A further attractive feature of one-to-one matching is that there a stable matching always exists in such matching markets. See Roth and Sotomayor (1990) for a textbook treatment.

For sharper characterization we assume that the final good is only consumed in Home. In subsection 3.3, we show that the main results hold in the presence of Foreign consumers. In section 4, we demonstrate that our finding is robust even in the presence of Foreign tariffs.

The timing of the game is as follows. First, Home government sets a specific tariff rate, t, to maximize Home welfare which consists of consumer surplus, aggregate Home profits and tariff revenues. In the short run we assume that the number of matched pairs is fixed and tariffs have no effect on market structure. In the long run analysis, we assume that after observing tariff rates, firms enter the market, search for their partner firms (from another stage of production) and $s = s(m, n)$ matched pairs are formed. Finally, bargaining over the input price and output takes place within a pair and Cournot competition occurs across matched pairs in the Home market.

3 Short-Run Equilibrium

We first consider the short-run equilibrium where entry costs K_H and K_F are sunk and matching has taken place. As indicated earlier, we treat the number of matching pairs $s = s(m, n)$ as fixed and invariant to the tariff rate.

3.1 Bargaining

Consider a third-stage bargaining game. Each pair i consisting of a Home firm H_i and a Foreign firm F_i bargains over (r_i, q_i), where Home firms H_i purchases q_i units of the intermediate product from Foreign firms F_i at the unit price of r_i, and then produces q_i units of the final product.

We characterize the outcome of the bargaining using a generalized Nash bargaining solution in which every Home firm has the same bargaining power denoted by $\beta \in (0, 1)$. The outcome of the third-stage bargaining is s pairs of input prices and quantities $(\hat{r}_i, \hat{q}_i)$ $(i = 1, 2, ..., s)$ which satisfy the following condition: $(r_i, q_i) = (\hat{r}_i, \hat{q}_i)$ is the Nash solution to the bargaining problem between H_i and F_i, given that both expect $(\hat{r}_j, \hat{q}_j)$ $(j \neq i)$ to be agreed upon between H_j and F_j. The relevant utility functions for the analysis of the bargaining are H_i's profit, $\pi_{H_i} \equiv \left[P\left(q_i + \sum_{j\neq i}^{s} q_j\right) - r_i\right] q_i$ and, F_i's profit, $\pi_{F_i} \equiv (r_i - c - t)q_i$. Also, we assume that the disagreement point is zero for both parties.

Then, $(r_i, q_i) = (\hat{r}_i, \hat{q}_i)$ solves the following maximization problem:

$$(\hat{r}_i, \hat{q}_i) = \arg\max_{r_i, q_i} \Bigg[\underbrace{\left\{P\left(q_i + \sum_{j\neq i}^{s} \hat{q}_j\right) - r_i\right\} q_i}_{\pi_{H_i}}\Bigg]^{\beta} \Bigg[\underbrace{(r_i - c - t)q_i}_{\pi_{F_i}}\Bigg]^{1-\beta},$$

subject to

$$\pi_{H_i} \geq 0 \quad \text{and} \quad \pi_{F_i} \geq 0.$$

The assumption below ensures that the solution to the maximization problem is unique.

Assumption 1. The demand function $Q(P)$ is logconcave.

The equivalent assumption (see Appendix A.1) in terms of inverse demand function is:

Assumption 1'. $P'(Q) + QP''(Q) \leq 0$ for all $Q > 0$.

In addition to guaranteeing uniqueness, Assumption 1 or equivalently Assumption 1' ensures that the optimal tariff is non-negative at least for some $\beta \geq 0$.[5]

For any $s \geq 1$, the unique bargaining outcome is given by $\hat{r}_1 = ... = \hat{r}_s \equiv \hat{r}$ and $\hat{q}_1 = ... = \hat{q}_s \equiv \hat{q}$, where $\hat{r}$ (> 0) and $\hat{q}$ (> 0) are determined by (1) and (2) below:

$$\hat{q} = -\frac{P(\hat{Q}) - c - t}{P'(\hat{Q})}, \tag{1}$$

$$\hat{r} = (1-\beta)P(\hat{Q}) + \beta(c+t), \tag{2}$$

and $Q = \hat{Q}$ uniquely solves the following:

$$sP(Q) + P'(Q)Q = s(c+t). \tag{3}$$

Solving the bargaining problem is equivalent to solving the following sequence of decisions. First, each H_i chooses q_i to maximize the joint profit:

$$\pi_i \equiv \pi_{H_i} + \pi_{F_i} = \left[P\left(q_i + \sum_{j \neq i}^{s} \hat{q}_j \right) - c - t \right] q_i.$$

This maximization problem yields (1). Then, each matched pair $i(= 1, 2, ..., s)$ divides this joint profit between themselves according to the bargaining power which implies

$$\left(P(\hat{Q}) - \hat{r}\right)\hat{q} = \beta(P(\hat{Q}) - c - t)\hat{q};$$
$$(\hat{r} - c - t)\hat{q} = (1-\beta)\left(P(\hat{Q}) - c - t\right)\hat{q}.$$

Canceling $\hat{q}$ from both sides of each equation and rewriting it, we get (2). Note that equation (2) can be written as

$$\frac{P(\hat{Q}) - \hat{r}}{\hat{r} - c - t} = \frac{\beta}{1-\beta}, \tag{2'}$$

which implies that the ratio of price-cost margin between Home and Foreign firms is ex-

[5] Deneckre and Kovenock (1999) and Anderson and Renault (2003) show that strict concavity of $(Q(P))^{-1}$ (which is weaker than logconcavity of $Q(P)$) is sufficient to ensure the uniqueness of the Cournot equilibrium. Nevertheless we use Assumption 1 to ensure non-negative tariffs for some $\beta \geq 0$. In their work on dynamic merger review, Nocke and Whinston (2010) use a slightly stronger assumption (than ours): $P'(Q)+QP''(Q) < 0$ for all $Q > 0$.

actly the same as the ratio of their bargaining power. The following lemma records some comparative statics results for future reference.

Lemma 1.

(i) *For a given tariff rate* t*, equilibrium output* $(\hat{Q})$ *and the price of the final good* $\hat{P} \equiv P(\hat{Q})$ *are independent of* β*.*

(ii) *For a given bargaining power* β*, an increase in the tariff rate lowers output and raises prices; i.e.,* $\mathrm{d}\hat{Q}/\mathrm{d}t < 0$*,* $\mathrm{d}\hat{P}/\mathrm{d}t > 0$*, and* $\mathrm{d}\hat{r}/\mathrm{d}t > 0$*.*

Proof: (i) It immediately follows from noting that $\hat{Q}$, i.e., the value of Q that solves (3), does not depend on β.

(ii) Totally differentiating (2) and (3) yields

$$\begin{aligned}
\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t} &= \frac{s/P'(\hat{Q})}{s+1+\epsilon(\hat{Q})}, \\
\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} &= P'(\hat{Q})\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t} = \frac{s}{s+1+\epsilon(\hat{Q})}, \\
\frac{\mathrm{d}\hat{r}}{\mathrm{d}t} &= (1-\beta)\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} + \beta = \frac{s+\beta(1+\epsilon(\hat{Q}))}{s+1+\epsilon(\hat{Q})},
\end{aligned}$$

where

$$\epsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$$

represents the elasticity of the slope of the demand. From Assumption 1' it follows that $\epsilon(Q) \geq -1$, which in turn implies that $s+1+\epsilon(Q) > 0$. The claim directly follows from noting that $P'(.) < 0$ and $s+\beta(1+\epsilon(Q)) > 0$. □

Notes on Elasticity of Slope

From the definition of elasticity of slope, $\epsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$, it follows that

$$\epsilon(Q) \gtreqless 0 \iff P''(Q) \lesseqgtr 0.$$

Thus $\epsilon(Q)$ has a one-to-one relationship with curvature of the inverse demand. The condition $\epsilon(Q) \geq -1$ which follows from logconcavity of $P(Q)$ is sufficient to prove our main results. For sharper characterization—see especially section 5.2—we shall occasionally invoke the following assumption:

Assumption 2. $\gamma(Q) \equiv \frac{\epsilon'(Q)Q}{\epsilon(Q)} \leq 1 \Leftrightarrow \epsilon(Q) = \frac{P''(Q)Q}{P'(Q)} \geq \alpha(Q) \equiv \frac{P'''(Q)Q}{P''(Q)}$.

Assumption 2 implies that the curvature of inverse demand is greater than the curvature of slope of inverse demand for any $Q \geq 0$.[6] Any inverse demand function with constant

[6] Cowan (2007) shows that the ratio of the slope curvature to demand curvature ($\alpha(Q)/\epsilon(Q)$ in our setting) plays an important role in the welfare analysis, and a critical value of this ratio is 1.

elasticity of slope (e.g., linear, constant elasticity, and semi-log among others) satisfies Assumption 2 since $\epsilon(Q) - \alpha(Q) = 1$ holds whenever $\epsilon(Q)$ is constant (and both $\epsilon(Q)$ and $\alpha(Q)$ are well-defined).

3.2 Tariffs

Let $\hat{Q}(t, \beta)$ denote the equilibrium output for a given t and β. Since the equilibrium output does not depend on β in the short run, we use $\hat{Q}(t)$ to denote the equilibrium output. In the first stage, Home government chooses a tariff rate t to maximize Home welfare (W_H) which consists of consumer surplus (CS), aggregate profits for Home firms (Π_H) and tariff revenues (TR):

$$W_H \equiv \underbrace{\left[\int_0^{\hat{Q}(t)} P(y)\mathrm{d}y - P(\hat{Q}(t))\hat{Q}(t)\right]}_{\text{Consumer surplus}} + \underbrace{\beta(P(\hat{Q}(t)) - c - t)\hat{Q}(t)}_{\text{Home profits}} + \underbrace{t\hat{Q}(t)}_{\text{Tariff revenue}}.$$

Both consumer surplus and Home profits decline as the tariff rate increases:

$$\begin{aligned}\frac{\mathrm{d}CS}{\mathrm{d}t} &= -\hat{Q}(t)\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} = -\frac{s\hat{Q}(t)}{s+1+\hat{\epsilon}} < 0,\\ \frac{\mathrm{d}\Pi_H}{\mathrm{d}t} &= -\frac{\beta(2+\hat{\epsilon})}{s}\hat{Q}(t)\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} = -\frac{\beta(2+\hat{\epsilon})\hat{Q}(t)}{s+1+\hat{\epsilon}} < 0.\end{aligned}$$

where $\hat{\epsilon}$ is the value of ϵ at $Q(t) = \hat{Q}(t)$, i.e., $\hat{\epsilon} \equiv \epsilon(\hat{Q}(t))$. While $\frac{\mathrm{d}CS}{\mathrm{d}t} < 0$ holds in general, $\frac{\mathrm{d}\Pi_H}{\mathrm{d}t} < 0$ is specific to vertical specialization. Since the final goods cannot be produced without imported inputs, a tariff on input hurts not only the input producers (Foreign) but also the final goods producers (Home).

The only possible benefit from an increase in tariff rate in our framework is an increase in tariff revenues. Here,

$$\frac{\mathrm{d}TR}{\mathrm{d}t} = \hat{Q}(t) + t\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t},$$

which can be positive or negative depending on t. So, when does the benefit from an increase in tariff revenues outweigh the loss in consumer surplus and profits? Proposition 1 addresses this question in the neighborhood of $t = 0$ (i.e., free trade).

Proposition 1: Welfare effect of a small tariff

Starting from free trade, a small increase in tariff rate raises Home welfare if and only if the bargaining power of Home firms is lower than a critical threshold. More formally,

$$\left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=0} \gtreqless 0 \iff \beta \lesseqgtr \frac{1+\hat{\epsilon}_0}{2+\hat{\epsilon}_0} \equiv \hat{\beta}_0.$$

where $\hat{\epsilon}_0$ is the elasticity of slope of demand evaluated at $t = 0$, i.e., $\hat{\epsilon}_0 \equiv \epsilon(\hat{Q}(0))$.

Proof: Using the expressions for $\frac{\mathrm{d}CS}{\mathrm{d}t}$, $\frac{\mathrm{d}\Pi_H}{\mathrm{d}t}$, and $\frac{\mathrm{d}TR}{\mathrm{d}t}$ from above, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \hat{Q}(t)\left[\frac{1+\hat{\epsilon}-\beta(2+\hat{\epsilon})}{s+1+\hat{\epsilon}}\right] + t\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t}. \tag{4}$$

Evaluating (4) at $t = 0$ gives

$$\begin{aligned} \left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=0} &= \hat{Q}(0)\left[\frac{1+\hat{\epsilon}_0-\beta(2+\hat{\epsilon}_0)}{s+1+\hat{\epsilon}_0}\right] \\ &= \frac{(2+\hat{\epsilon}_0)\hat{Q}(0)}{s+1+\hat{\epsilon}_0}\left[\frac{1+\hat{\epsilon}_0}{2+\hat{\epsilon}_0}-\beta\right]. \end{aligned} \tag{5}$$

Assumption 1 implies that $1+\hat{\epsilon}_0 > 0$ which in turn implies $2+\hat{\epsilon}_0 > 0$ and $s+1+\hat{\epsilon}_0 > 0$. Then the result is immediate from (5). □

The logic underlying Proposition 1 is as follows. Starting from free trade, an introduction of a small tariff generates tariff revenue and increases welfare. The marginal benefit (*MB*) of a tariff at $t = 0$ is given by incremental tariff revenue, i.e.,

$$\textit{MB} \equiv \left.\frac{\mathrm{d}TR}{\mathrm{d}t}\right|_{t=0} = \hat{Q}(0),$$

which is independent of β. The marginal cost (*MC*) of a tariff is given by the sum of the absolute value of (i) the loss in consumer surplus and (ii) the loss in Home profits due to the tariff:

$$\textit{MC} \equiv \left|\frac{\mathrm{d}CS}{\mathrm{d}t}\right|_{t=0} + \left|\frac{\mathrm{d}\Pi_H}{\mathrm{d}t}\right|_{t=0} = \left(\frac{s\hat{Q}(0)}{s+1+\hat{\epsilon}_0}\right) + \left(\frac{\beta(2+\hat{\epsilon}_0)\hat{Q}(0)}{s+1+\hat{\epsilon}_0}\right).$$

Clearly, *MC* is strictly increasing in β since the higher the β the higher the $\left|\frac{\mathrm{d}\Pi_H}{\mathrm{d}t}\right|_{t=0}$. Figure 1 depicts *MB* and *MC*. Observe that if Home's bargaining power is small, *MB* exceeds *MC* and a positive tariff improves welfare, while the opposite is true if Home's bargaining power is high.

Now we turn to the optimal tariff. Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ in (4) equal to zero and simplifying we get:

$$t = -\frac{P'(\hat{Q}(t))\hat{Q}(t)(2+\hat{\epsilon})}{s}\left(\frac{1+\hat{\epsilon}}{2+\hat{\epsilon}}-\beta\right). \tag{6}$$

Let $t = t(\beta)$ denote a solution to (6). Assume that the solution is unique. Then $t = t(\beta)$ is the optimal tariff and $\hat{Q}(.)$ and $\hat{\epsilon}$ in (6) respectively are the aggregate output and slope elasticity of demand evaluated at $t = t(\beta)$. Analyzing (6) further yields the following proposition.

Proposition 2: Optimal tariff and bargaining power

(i) *The optimal tariff is positive if and only if the bargaining power of Home firms is*

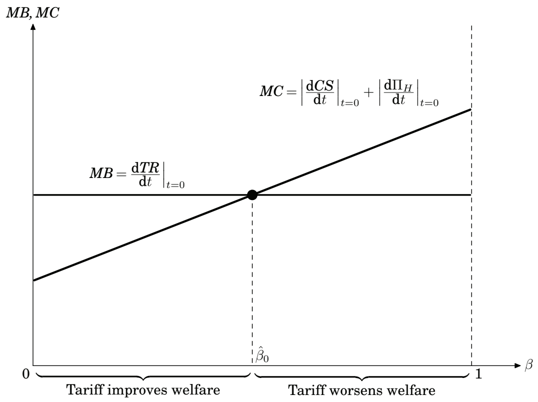

FIGURE 1: WELFARE EFFECT OF A SMALL TARIFF

lower than a critical threshold. More formally, there exists $\hat{\beta}$ such that

$$t(\beta) \gtreqless 0 \iff \beta \lesseqgtr \hat{\beta}.$$

(ii) *The optimal tariff is monotonically decreasing in the bargaining power of Home firms, i.e.,* $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$.

Proof: Consider (ii) first. Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t}\Big|_{t=t(\beta)} = 0$ with respect to β gives:

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -\frac{\frac{\partial^2 W_H}{\partial\beta\partial t}}{\frac{\partial^2 W_H}{\partial t^2}} = -\frac{\frac{\partial \Pi_H}{\partial t}}{\frac{\partial^2 W_H}{\partial t^2}} < 0, \tag{7}$$

where the inequality follows from noting that (a) $\frac{\partial^2 W_H}{\partial t^2} < 0$ (second-order condition) and (b) $\frac{\partial \Pi_H}{\partial t} < 0$ (as proved in the beginning of subsection 3.2).[7] The proof of part (i) follows from combining part (ii) and the following implication of (6): $t(0) > 0$ and $t(1) < 0$. □

The logic underlying Proposition 2 is as follows. When β is close to one, the Home firms' bargaining power is very high and Home captures almost all the profits in the bargaining stage. This situation is like a domestic, single-stage, Cournot oligopoly with s firms. From the industrial organization literature we know that a positive subsidy increases welfare in an oligopoly setup by narrowing the wedge between price and marginal cost. Thus, the optimal policy is an import subsidy when β is high. When β is too low, say close to zero, Home firms have little bargaining power. It is as if that Home welfare is composed only of the consumer surplus and the tariff revenue. It is well-known from the trade literature that in such cases a positive tariff improves welfare (as long as the demand is logconcave) and hence the optimal policy is an import tariff. The discussion suggests $t(\beta) > 0$ when β is low and $t(\beta) < 0$ when β is high. Applying a standard continuity argument (in terms of

[7]Following the standard practice in the literature, we assume that the second-order condition $\frac{\partial^2 W_H}{\partial t^2} < 0$ is satisfied. Another option is to work with the class of inverse demand functions that satisfy Assumption 2 which (together with Assumption 1) implies $\frac{\partial^2 W_H}{\partial t^2} < 0$.

β) it follows that there is a range of values for β such that $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$. Part (ii) of Proposition 2 says that the range is exhaustive. That is, $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$ for all $\beta \in (0, 1)$.

Foreign Consumers

So far we have assumed that all consumers are in the Home country. Now suppose a fraction $\mu \in [0, 1)$ of these consumers reside in Home while the remaining fraction, $(1 - \mu)$, reside in Foreign. All consumers have an identical demand: $Q = Q(P)$, where $Q(P)$ satisfies Assumption 1 and the properties described in section 2. Assume that Home and Foreign markets are segmented. Following Helpman and Krugman (1985, Chapter 5) we can show that if $\hat{P}(t)$ is the equilibrium price in Home it is also the equilibrium price in Foreign.

Let $\hat{Q}(t)$ and $\hat{P}(t)$ respectively denote the aggregate world output and price in a Cournot equilibrium for a given t. We can express Home welfare as

$$W_H = \mu \left(\int_0^{\hat{Q}(t)} P(y)\mathrm{d}y - \hat{P}(t)\hat{Q}(t) \right) + \beta(\hat{P}(t) - c - t)\hat{Q}(t) + t\hat{Q}(t).$$

Differentiating W_H with respect to t and evaluating at $t = 0$, we have

$$\left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=0} = \frac{\hat{Q}(0)(2 + \hat{\epsilon}_0)}{s + 1 + \hat{\epsilon}_0} \left[\frac{1 + \hat{\epsilon}_0 + s(1 - \mu)}{2 + \hat{\epsilon}_0} - \beta \right].$$

As in Proposition 1 we find that a small tariff improves welfare if

$$\beta < \frac{1 + \hat{\epsilon}_0 + s(1 - \mu)}{2 + \hat{\epsilon}_0}.$$

If $\mu = 1$ the threshold value of β is the same as $\hat{\beta}$ in Proposition 1. Observe that lower μ (i.e., fewer Home consumers) makes it more likely for a tariff to improve welfare.

The optimal tariff, $t = t(\beta)$, implicitly solves the following equation:

$$t = -\frac{P'(\hat{Q}(t))\hat{Q}(t)(2 + \hat{\epsilon})}{s} \left(\frac{s(1 - \mu) + 1 + \hat{\epsilon}}{2 + \hat{\epsilon}} - \beta \right).$$

As in Proposition 2 we find that (i) $t(\beta) > 0$ if β is less than a threshold value, and (ii) $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$. The only difference lies in the possibility of $t(\beta) > 0$. The optimal tariff is more likely to be positive under $\mu < 1$ since the negative effect of the tariff on consumer surplus receives less weight.

3.3 Profits and Welfare

Recall that Home and Foreign profits respectively are given by

$$\Pi_H = [P(\hat{Q}) - \hat{r}]\hat{Q} = \beta\Pi, \quad \Pi_F = (\hat{r} - c - t)\hat{Q} = (1 - \beta)\Pi,$$

where $\Pi \equiv [P(\hat{Q}) - c - t]\hat{Q} = -\frac{P'(\hat{Q})\hat{Q}^2}{s}$ is the aggregate joint profits. Differentiating Π_H and Π_F with respect to β yields

$$\begin{aligned} \frac{\mathrm{d}\Pi_H}{\mathrm{d}\beta} &= \quad \Pi \quad + \quad \beta\frac{\partial \Pi}{\partial t} \cdot \frac{\mathrm{d}t}{\mathrm{d}\beta}, \\ \frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} &= \underbrace{-\Pi}_{\text{share effect}} + \underbrace{(1-\beta)\frac{\partial \Pi}{\partial t} \cdot \frac{\mathrm{d}t}{\mathrm{d}\beta}}_{\text{size effect}}. \end{aligned}$$

An increase in β has two effects on Π_i ($i \in \{H, F\}$). First, by increasing the Home firm's share in industry profits, an increase in β reduces Π_H and raises Π_F. We call this the *share effect* which exists even when the tariff is exogenously set. The share effect is positive for Home firms and negative for Foreign firms. Second, an increase in β reduces $t(\beta)$ which in turn leads to higher Π. We call this indirect effect the *size effect* which benefits both Home and Foreign firms. Since both the size effect and the share effect are positive for Home firms, Π_H increases as β increases. Surprisingly, we find that Π_F can increase with an increase in β.

Proposition 3: Higher β, higher foreign profits — a possibility result

An increase in Home firm's bargaining power might lead to higher Foreign profits. For demand functions with constant elasticity of slope (i.e. $\frac{P''(Q)Q}{P'(Q)}$ is constant)

$$\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0 \quad \text{if} \quad \beta < \max\left\{0, 1 - \frac{s}{2+\epsilon}\right\},$$

where ϵ denotes the constant elasticity of slope.

Proof: Since $\Pi_F = s\pi_F$ and s is fixed, we have $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0 \Leftrightarrow \frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} > 0$. We have that

$$\frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} = -\pi + (1-\beta)\frac{\partial \pi}{\partial t} \cdot \frac{\mathrm{d}t}{\mathrm{d}\beta}.$$

Differentiating $\pi = [P(\hat{Q}) - c - t]\hat{q} = -P'(\hat{Q})\hat{q}^2$ with respect to t gives $\frac{\mathrm{d}\pi}{\mathrm{d}t} = -\hat{q} \cdot \left(\frac{2+\epsilon}{s+1+\epsilon}\right) < 0$. In Appendix A.2, we show that, $\frac{\mathrm{d}t}{\mathrm{d}\beta} = \frac{P'(\hat{Q})\hat{Q}\cdot(2+\epsilon)}{1+[(1+\epsilon)-\beta(2+\epsilon)]\left(\frac{1+\epsilon}{s+1+\epsilon}\right)} < 0$. for the class of inverse demand functions with constant ϵ. Substituting $\frac{\mathrm{d}\pi}{\mathrm{d}t}$ and $\frac{\mathrm{d}t}{\mathrm{d}\beta}$ into the above equation, we get

$$\frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} = \left(\frac{(2+\epsilon)\pi}{s + (1-\beta)(\epsilon+1)(\epsilon+2)}\right)\left(\frac{2+\epsilon-s}{2+\epsilon} - \beta\right).$$

The result follows from noting that the value in the first parentheses is strictly positive. □

Proposition 3 suggests that there are some parameterizations such that the size effect dominates the share effect for Foreign firms. In other words, an indirect increase in Foreign profits due to a lower tariff (induced by higher β) might outweigh a direct decrease in

Foreign profit due to a lower share of joint profits (i.e., lower $1-\beta$). This is more likely to hold when the number of matched pairs s is smaller or the the curvature of the inverse demand ϵ is bigger.

To better appreciate Proposition 3, consider $P(Q) = a - Q^b$ for which $\epsilon(Q) = \frac{QP''(Q)}{P'(Q)} = b-1$. If the demand is linear (i.e., $b = 1$) an increase in β leads to higher Π_F if the market structure is a bilateral monopoly (i.e., $s = 1$). This counterintuitive result becomes more likely as b increases. Note that irrespective of the market structure, there always exists b high enough such that $\frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} > 0$ holds.

We conclude this subsection by reporting the relationship between bargaining power and welfare.

Proposition 4: Welfare and bargaining power

Both Home welfare (W_H)and global welfare (W_G) are strictly increasing in the bargaining power of Home firms, i.e., (i) $\frac{\mathrm{d}W_H}{\mathrm{d}\beta} > 0$ and (ii) $\frac{\mathrm{d}W_G}{\mathrm{d}\beta} > 0$.

Proof: Let $W_H(t, \beta)$ denote Home welfare for a given t and β. As before, let $t(\beta)$ denote the optimal tariff for a given β. Now consider β_1 and β_2 that satisfy $0 \leq \beta_1 < \beta_2 \leq 1$. By definition, $W_H(t, \beta_2)$ is a maximum at $t = t(\beta_2)$ which implies

$$W_H(t(\beta_2), \beta_2) \geq W_H(t(\beta_1), \beta_2).$$

Also, we have that

$$W_H(t(\beta_1), \beta_2) > W_H(t(\beta_1), \beta_1),$$

since at a given $t = t(\beta_1)$, (i) Π_H is increasing in β while (ii) CS and TR are invariant with respect to β. Combining the two inequalities we get

$$W_H(t(\beta_2), \beta_2) \geq W_H(t(\beta_1), \beta_2) > W_H(t(\beta_1), \beta_1),$$

implying $\frac{\mathrm{d}W_H}{\mathrm{d}\beta} > 0$.

To prove (ii), write global welfare W_G as:

$$W_G = \int_0^{\hat{Q}(t(\beta))} P(y)\mathrm{d}y - c\hat{Q}(t(\beta)).$$

Differentiating W_G with respect to t, we get:

$$\frac{\mathrm{d}W_G}{\mathrm{d}\beta} = \left[\hat{P}(t(\beta)) - c\right] \frac{\mathrm{d}\hat{Q}(t(\beta))}{\mathrm{d}t} \cdot \frac{\mathrm{d}t(\beta)}{\mathrm{d}\beta}.$$

The result follows from noting that $\frac{\mathrm{d}\hat{Q}(t(\beta))}{\mathrm{d}t} < 0$ (Lemma 1) and $\frac{\mathrm{d}t(\beta)}{\mathrm{d}\beta} < 0$ (Proposition 2). $\square$

3.4 Alternative Means of Procurement

Below we consider environments with alternative means of procurement and show that the negative relationship between tariff and β, and $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0$ hold in those environments as well. For analytical convenience, in this section, we consider demand functions with constant elasticity of slope. Details for subsections 3.4.1 and 3.4.2 are available on request.

3.4.1 Alternative Bargaining

So far we have assumed that each Home-Foreign pair i bargains simultaneously over the input price r_i and the level of output q_i. We have also considered a variant of our model where the bargaining over r and q occurs sequentially. First, each pair $i\,(=1,2,...,s)$ bargains over the input price r_i. Subsequently each H_i chooses q_i taking $\boldsymbol{r} \equiv (r_1, r_2, ..., r_s)$ as given. This sequence is in the spirit of the right-to-manage model in the labor union literature (e.g., Naylor, 2002) where firms and union first bargain over the wage, and each firm then chooses the employment level.

Qualitatively, all our results hold under this alternative bargaining (AB) setup. The key findings are stated below:

(i) Starting from free trade an infinitesimally small tariff improves welfare if and only if $\beta < \hat{\beta}^{AB}$ where $\beta = \hat{\beta}^{AB}$ is the unique solution to the following equation: $1 + \epsilon = \frac{\beta + 1 - \frac{s(1+\beta)+\epsilon}{s(s+1+\epsilon)}}{1-\beta} \equiv B$.

(ii) Optimal tariff is implicitly given by

$$t(\beta) = -\frac{P'(\hat{Q})\hat{Q}}{sB}(-B + 1 + \epsilon),$$

where B is as in (i). It can be shown that $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$.

(iii) Finally,

$$\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0 \quad \text{if } \beta < \max\left\{0, 1 - \frac{s\chi}{2+\epsilon}\right\},$$

where $\chi = \frac{s(s+1+\epsilon)-(s+\epsilon)}{s(s+1+\epsilon)} = 1 - \frac{s+\epsilon}{s(s+1+\epsilon)} \in (0,1)$.

Two comparisons with the original model are worth noting. First, comparing $\hat{\beta}^{AB}$ in (i) with $\hat{\beta}$ in Proposition 1, we find that a tariff is less likely to improve welfare in the alternative bargaining setup. For instance, under linear demand, a tariff raises Home welfare if and only if $\beta < \hat{\beta}^{AB} = \frac{1}{2s+1}$ in the alternative bargaining, whereas it is $\beta < \hat{\beta} = \frac{1}{2}$ in the simultaneous bargaining. Second, comparing (iii) with Proposition 3 reveals that the counterintuitive possibility, i.e., $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0$ is more likely to hold in the alternative bargaining setup.

3.4.2 Vertical Oligopoly

Bargaining, as outlined in the previous sections, is not the only means of procurement of inputs. Final-good producers also buy a range of intermediate inputs from the market as well. To capture such market based procurement, we consider a two-stage vertical oligopoly model with m Home firms producing final goods and n Foreign firms producing intermediate products. Stage 1 involves Cournot competition in the intermediate goods sector sector in which each Foreign firm F_i ($i = 1, 2, ..., n$) chooses x_i units of the intermediate product to maximize its profit, taking other Foreign firms' quantities as given. Stage 2 involves Cournot competition in the final goods sector, in which each Home firm H_k, ($k = 1, 2, ..., m$) chooses its own output, q_k, taking the market price of the intermediate good, r, and other Home firms' output as given. See Ishikawa and Lee (1997) and Ishikawa and Spencer (1999) for applications of vertical oligopoly setup in trade contexts.

In equilibrium of our vertical oligopoly model, the following holds:

$$\frac{P(\tilde{Q}) - \tilde{r}}{\tilde{r} - c - t} = \frac{n}{m + 1 + \epsilon},$$

where $\tilde{Q}$, $P(\tilde{Q})$, $\tilde{r}$ and ϵ are respectively the equilibrium values of aggregate output, price of final good, price of intermediate good, and elasticity of slope of inverse demand function. Recall, in our original bargaining model,

$$\frac{P(\hat{Q}) - \hat{r}}{\hat{r} - c - t} = \frac{\beta}{1 - \beta}.$$

Comparison of the last two equations suggests that we can interpret the ratio of Home and Foreign price-cost margin (in the vertical oligopoly model) as an indicator of the relative bargaining strength of Home and Foreign firms. Under this interpretation, a Home firm's bargaining power increases as the number of Foreign firms (n) increases and it declines as the number of Home firms (m) increases.

We find that an increase in the number of Foreign firms, n, which is equivalent to an increase in the Home firm's bargaining strength (under the above interpretation) leads to lower optimal tariffs. Home's optimal tariff is negative if n is relatively high and positive if n is relatively low. These findings accord well with our findings in Propositions 1 and 2 once we accept that an increase in thickness of one side of the market in the vertical oligopoly model is akin to a decrease in bargaining strength of that side in the original model. Finally, similar to the counterintuitive possibility raised in Proposition 3, we find that an increase in n can lead to higher π_F. An increase in the number of Foreign firms reduces optimal tariff which creates the possibility of higher profit for each Foreign firm.

4 Foreign Tariffs

So far, we have assumed that the Foreign government does not engage actively in trade policy. Here we show that the key result, i.e., the negative relationship between bargaining power and tariffs, is robust under strategic interaction between the two governments. For this purpose, we derive Nash equilibrium tariffs set by Home and Foreign governments and compare these with the tariff examined in the previous section.

Consider an environment in which governments of both countries, Home and Foreign, choose tariff strategically to maximize welfare of their respective countries. To illustrate this strategic interaction in a simple fashion, assume that $\mu = 0$, i.e., all consumers are in the Foreign country. Let t_F denote the Foreign tariff rate on imports of final goods and let t_H denote the Home tariff rate on imports of intermediate input. For a given pair of tariffs (t_H, t_F), let $\hat{Q}(t_H, t_F)$ and $\hat{P}(t_H, t_F) \equiv P(\hat{Q}(t_H, t_F))$ denote the equilibrium aggregate output and price. The welfare of Home and Foreign, denoted by W_H and W_F respectively are:

$$
\begin{aligned}
W_H &= \beta(\hat{P}(t_H, t_F) - c - t_H - t_F)\hat{Q}(t_H, t_F) + t_H \hat{Q}(t_H, t_F); \\
W_F &= \int_0^{\hat{Q}(t_H, t_F)} P(y)\mathrm{d}y - \hat{P}(t_H, t_F)\hat{Q}(t_H, t_F) \\
&\qquad + (1-\beta)(\hat{P}(t_H, t_F) - c - t_H - t_F)\hat{Q}(t_H, t_F) + t_F \hat{Q}(t_H, t_F).
\end{aligned}
$$

Differentiating W_i with respect to t_i for $i \in \{H, F\}$ and evaluating at free trade equilibrium, we get:

$$
\begin{aligned}
\left.\frac{\partial W_H}{\partial t_H}\right|_{(t_H, t_F)=(0,0)} &= \frac{\hat{Q}(0,0)(2+\hat{\epsilon}_0)}{s+1+\hat{\epsilon}_0}\left(\frac{1+\hat{\epsilon}_0}{2+\hat{\epsilon}_0} - \beta\right), \\
\left.\frac{\partial W_F}{\partial t_F}\right|_{(t_H, t_F)=(0,0)} &= \frac{\hat{Q}(0,0)(2+\hat{\epsilon}_0)}{s+1+\hat{\epsilon}_0}\left(\frac{1+\hat{\epsilon}_0}{2+\hat{\epsilon}_0} - (1-\beta)\right),
\end{aligned}
$$

where $\hat{\epsilon}_0 \equiv \epsilon(\hat{Q}(0,0))$. Consistent with Proposition 1, we find that starting from global free trade, $(t_H, t_F) = (0, 0)$, a small Home tariff improves Home welfare if β is smaller than a threshold value. Analogously, a small Foreign tariff improves Foreign welfare if $1 - \beta$ is smaller than a threshold value. Now let us turn to Nash equilibrium tariffs in this two-country setting.

Proposition 5: Nash tariffs and bargaining power

(i) *The country with the lower bargaining power sets a higher tariff. More formally,*

$$
t_H(\beta) \gtreqless t_F(\beta) \iff \beta \lesseqgtr \frac{1}{2}.
$$

where $t_H(\beta)$ and $t_F(\beta)$ respectively denote the optimal tariff for Home and Foreign.

(ii) *Suppose that* $\frac{\partial^2 W_i}{\partial t_i^2} + \frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$ *holds for* $i \neq j \in \{H, F\}$.[8] *Then, the optimal tariff in Home (Foreign) is monotonically decreasing (increasing) in the bargaining power of Home firms, i.e.,*

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} < 0, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} > 0.$$

Proof: Simplifying the first-order conditions, $\frac{\partial W_H}{\partial t_H} = 0$ and $\frac{\partial W_F}{\partial t_F} = 0$, gives:

$$t_H = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2 + \hat{\epsilon})}{s}\left(\frac{1 + \hat{\epsilon}}{2 + \hat{\epsilon}} - \beta\right), \tag{8}$$

$$t_F = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2 + \hat{\epsilon})}{s}\left(\frac{1 + \hat{\epsilon}}{2 + \hat{\epsilon}} - (1 - \beta)\right), \tag{9}$$

where $\hat{\epsilon} \equiv \epsilon(\hat{Q}(t_H, t_F))$. Clearly $t_H = t_H(\beta)$ and $t_F = t_F(\beta)$ satisfy (8) and (9). Subtracting (9) from (8) gives

$$t_H - t_F = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2 + \hat{\epsilon})}{s}(1 - 2\beta).$$

Since $-P'(.)\hat{Q}(.)(2+\hat{\epsilon}) > 0$, the above equation implies (i). For the proof of (ii), see Appendix A.3. □

Many developing countries impose heavy tariffs on imports from developed countries. It is often argued that the tariff protects inefficient domestic producers from competition with foreign rivals. Tariffs are also attractive as a source of revenue for the revenue-constrained governments in the developing countries. Proposition 5 provides a complementary explanation: it says that in the context of vertically related industries, high tariffs can arise from the weak bargaining power of the domestic producers.[9]

An Illustrative Example

To better appreciate Proposition 5, consider the following class of inverse demand functions: $P(Q) = a - Q^b$, where $b > 0$. For this class of demand function, $\epsilon(Q) \equiv \epsilon = b - 1$ which implies that

$$\epsilon \gtreqless 0 \iff b \gtreqless 1.$$

[8] Note that, while $\frac{\partial^2 W_i}{\partial t_i^2} < 0$ from the second-order condition, whether $\frac{\partial^2 W_j}{\partial t_j \partial t_i}$ is positive or negative depends on the strategic relationship between Home and Foreign tariffs. It can immediately be seen that this condition necessarily holds if Home and Foreign tariffs are *strategic substitutes*, i.e., $\frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$.

[9] Indeed, several papers have studied the causes and consequences of little bargaining power of the producers in the developing countries. See, for example, Wes (2000) and the papers cited therein.

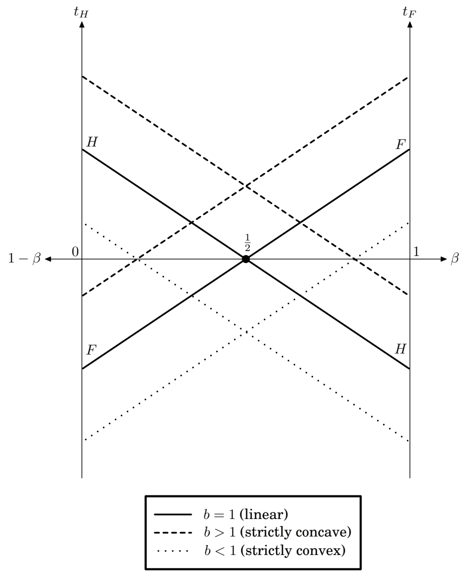

FIGURE 2: t_H, t_F, AND β

Solving (8) and (9) gives

$$
\begin{aligned}
t_H(\beta) &= \frac{(a-c)b(1+b)}{s+b^2}\left(\frac{b}{1+b}-\beta\right), \\
t_F(\beta) &= \frac{(a-c)b(1+b)}{s+b^2}\left(\frac{b}{1+b}-(1-\beta)\right).
\end{aligned}
$$

which are illustrated in Figure 2 above.

First consider linear demand ($b = 1$) for which $\epsilon = b - 1 = 0$. The optimal tariffs for Home and Foreign are shown as bold lines HH and FF respectively. As expected from Proposition 5,

(i) HH lies above (below) FF when the domestic firm has less (more) bargaining power;

(ii) HH and FF are downward sloping in the bargaining power of Home (β) and Foreign ($1-\beta$) respectively.

Both (i) and (ii) are preserved when $\epsilon > 0$ (see the dashed line) as well as when $\epsilon < 0$ (see the dotted line). The only qualitative difference between these cases is in the likelihood of import tariffs and import subsidy. Under linear demand (i.e., $\epsilon = 0$), free trade is optimal if both countries have equal bargaining power. Otherwise, if bargaining power is unequal, the country with less bargaining power sets a positive import tariff while the other one offers an import subsidy. As both HH and FF shift up for $\epsilon > 0$, compared to linear demand, a positive optimal tariff is more likely for strictly concave demand functions ($b > 1$). The opposite is true for strictly convex demand functions ($b < 1$).

5 Long-Run Equilibrium

So far, we have assumed that the number of matched pairs, $s = s(m, n)$, is fixed and in particular, it does not vary with tariff rates. Thus, the market structure is exogenously given. In this section we conduct long-run analysis, in which tariffs affect entry decisions and consequently the market structure.

The timing is as outlined in the last paragraph in section 3. First, the Home government chooses a tariff rate, following which entry occurs. Suppose m Home firms and n foreign firms enter the market. Subsequently, $s = s(m, n)$ pairs are formed via random matching. Firms unable to find a match exit the market. Finally, after matching, each pair i consisting of a Home firm H_i and Foreign firm F_i chooses (r_i, q_i) (to solve the maximization problem stated prior to Assumption 1 in section 3.1).

Let us start with the last stage. The bargaining problem and the Cournot competition works exactly the same as before. Accordingly, as in section 3, the unique equilibrium in this stage is characterized by $\hat{q}_1 = \hat{q}_2 = ... = \hat{q}_s \equiv \hat{q}$ and $\hat{r}_1 = \hat{r}_2 = ... = \hat{r}_s \equiv \hat{r}$, satisfying

$$\begin{aligned} \hat{q} &= -\frac{P(\hat{Q}) - c - t}{P'(\hat{Q})}, \\ \hat{r} &= (1-\beta)P(\hat{Q}) + \beta(c+t), \end{aligned}$$

where $Q = \hat{Q}$ uniquely solves the following:

$$sP(Q) + P'(Q)Q = s(c+t).$$

The third-stage subgame outcome above is identical with that in the short run. Since the number of entrants is endogenous in the long run, the search for their partner firms becomes important. In what follows, we analyze the effect of entry and matching on the equilibrium.

5.1 Entry and Matching

In the second stage, the number of matched pairs $s = s(m, n)$ is endogenously determined by free entry conditions. Recall from section 2 that the entry costs of a Home firm and a Foreign firm respectively are K_H and K_F. Firms are risk-neutral and entry occurs until the post-entry profit equals K_H (K_F) for a Home (Foreign) firm.

Consider first entry by Home firms. If m Home firms enter and only $s = s(m, n)$ firms are matched, the probability of finding a Foreign partner for each Home firm H_i is $\frac{s}{m}$. If successful, H_i receives $\beta(P(\hat{Q}) - c - t)\hat{q}$; otherwise, it receives 0. Thus the expected post-entry profit for a Home firm is $\frac{s}{m}\beta(P(\hat{Q}) - c - t)\hat{q}$. Following the standard practice in the oligopoly literature we treat m as a continuous variable. This implies that in free entry equilibrium, the expected post-entry profit must equal K_H for a Home firm:

$$\frac{\hat{s}}{\hat{m}}\beta(P(\hat{Q})-c-t)\hat{q}=K_H, \tag{10}$$

where $\hat{s}$ and $\hat{m}$ are respectively the equilibrium number of matched pairs and entrants in Home. By analogous reasoning we can establish that the expected post-entry profit must equal K_F for a Foreign firm:

$$\frac{\hat{s}}{\hat{n}}(1-\beta)(P(\hat{Q})-c-t)\hat{q}=K_F,$$

where $\hat{n}$ is the equilibrium number of Foreign firms that enter in Stage 1.

To proceed further, we need to specify the property of the matching function. Following the literature (e.g., Grossman and Helpman 2002), we assume that the matching function satisfies the following properties:

$$s(\lambda m,\lambda n)=\lambda s(m,n), \tag{11}$$

$$\frac{\partial s(m,n)}{\partial m}>0 \text{ and } \frac{\partial s(m,n)}{\partial n}>0, \tag{12}$$

$$\frac{\partial^2 s(m,n)}{\partial m^2}<0 \text{ and } \frac{\partial^2 s(m,n)}{\partial n^2}<0. \tag{13}$$

Equation (11) means constant-returns-to-scale in matching. Furthermore, equations (11)-(13) imply complementarity or supermodularity in matching, i.e., $\frac{\partial^2 s(m,n)}{\partial m \partial n}>0$. Supermodular and log-supermodular functions are routinely used in matching environments (Shimer and Smith, 2000). See Costinot (2009) for application of supermodularity in trade settings.

Letting $\lambda=1/m$ in (11), we have

$$s(m,n)=m\cdot s\left(1,\frac{n}{m}\right)\equiv mS(z),$$

where $z\equiv n/m$. This normalized matching function $S(z)$ satisfies the following properties:

$$S'(z)>0,\quad S''(z)<0,\quad S(z)>zS'(z).^{10} \tag{14}$$

From the two zero-profit conditions mentioned earlier — one for Home firms and one for Foreign firms — it follows that, in equilibrium,

$$\hat{z}=\left(\frac{1-\beta}{\beta}\right)\left(\frac{K_H}{K_F}\right), \tag{15}$$

which implies that the proportion of Foreign firms ($\hat{n}/\hat{m}$) declines as the Home firm's bargaining power increases. Bargaining power thus affects the relative thickness of the market, which affects the probability of finding partner firms and consequently the aggregate output.

[10] These properties of $S(z)$ follow from (10)-(12).

Now we turn to comparative statics with respect to t and β. The new feature in the long run is that the market structure changes with a change in t or β. The effect of an increase in t is qualitatively similar to that in the short run, while the effect of an increase in β is different. Recall that in the short run with a fixed number of firms, an increase in bargaining power has no effect on equilibrium output or final goods' price. Lemma 2 suggests that is no longer the case.

Lemma 2.

(i) *For a given tariff rate t, equilibrium output $(\hat{Q})$ and the price of the final good $\hat{P} \equiv P(\hat{Q})$ have a non-monotone relationship with respect to β.*

(ii) *For given bargaining power β, an increase in tariff rate lowers the number of matched pairs and output, and raises prices in equilibrium; i.e., $\frac{\partial \hat{s}}{\partial t} < 0$, $\frac{\partial \hat{Q}}{\partial t} < 0$, $\frac{\partial \hat{P}}{\partial t} > 0$, and $\frac{\partial \hat{r}}{\partial t} > 0$.*

Proof: (i) In the long run, the comparative statics results depend on both the first-order condition (3) and the Home firms' free entry condition (10). Differentiating these two conditions with respect to β, we get (see Appendix A.4 for details):

$$\frac{\partial \hat{s}}{\partial \beta} = \frac{\hat{s}(\hat{s}+1+\hat{\epsilon})}{\beta(2\hat{s}+\hat{\epsilon})}\left(1-\frac{\sigma(\beta)}{1-\beta}\right), \qquad \frac{\partial \hat{Q}}{\partial \beta} = \frac{\hat{Q}}{\beta(2\hat{s}+\hat{\epsilon})}\left(1-\frac{\sigma(\beta)}{1-\beta}\right),$$

where $\hat{\epsilon} \equiv \epsilon(\hat{Q}(t(\beta),\beta))$ and $\sigma(\beta) = \frac{\hat{z}S'(\hat{z})}{S(\hat{z})} \in (0,1)$ from (14). The result follows from noting that $\lim_{\beta\to 0}\frac{\partial \hat{Q}}{\partial \beta} > 0$ and $\lim_{\beta \to 1}\frac{\partial \hat{Q}}{\partial \beta} < 0$.

(ii) Similar to (i), differentiating (3) and (10) with respect to t yields (see Appendix A.4):

$$\frac{\partial \hat{s}}{\partial t} = \left(\frac{2+\hat{\epsilon}}{2\hat{s}+\hat{\epsilon}}\right)\left(\frac{\hat{s}}{P'(\hat{Q})\hat{q}}\right), \qquad \frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{s}}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})},$$

$$\frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{s}}{2\hat{s}+\hat{\epsilon}}, \qquad \frac{\partial \hat{r}}{\partial t} = (1-\beta)\frac{\partial \hat{P}}{\partial t} + \beta = \frac{2\hat{s}+\beta\hat{\epsilon}}{2\hat{s}+\hat{\epsilon}}.$$

Then, the claim follows from noting that $2\hat{s}+\hat{\epsilon} > 0$, $2\hat{s}+\beta\hat{s} > 0$, and $P'(.) < 0$. $\square$

To understand the non-monotone relationship between β and $\hat{Q}$, suppose $K_H \approx K_F$ and β is low. It follows from (15) that $\hat{m} < \hat{n}$, i.e., there are fewer Home firms. An increase in β encourages entry by Home firms and prompts exit by foreign firms. This improves matching (as there were fewer Home firms to start with) and leads to an increase in aggregate output. On the other hand, if β is high to start with, $\hat{m} > \hat{n}$. By inducing the entry of Home firms and exit of Foreign firms, further increase in β worsens the mismatch which in turn leads to lower output.

5.2 Tariffs

In the first stage, Home government chooses tariff rate t to maximize Home welfare. As profits are zero under free entry, effectively, Home welfare consists of consumer surplus and tariff revenues only. Let $\hat{Q}(t,\beta)$ denote the equilibrium aggregate output for a given t and β.

$$W_H = \underbrace{\left[\int_0^{\hat{Q}(t,\beta)} P(y)\mathrm{d}y - P(\hat{Q}(t,\beta))\hat{Q}(t,\beta)\right]}_{\text{consumer surplus}} + \underbrace{t\hat{Q}(t,\beta).}_{\text{tariff revenue}}$$

Using Lemma 2, we have:

$$\frac{\mathrm{d}CS}{\mathrm{d}t} = -\hat{Q}(t,\beta)\frac{\partial \hat{P}}{\partial t} = -\frac{2\hat{s}(t,\beta)\hat{Q}(t,\beta)}{2\hat{s}(t,\beta)+\hat{\epsilon}} < 0,$$
$$\frac{\mathrm{d}TR}{\mathrm{d}t} = \hat{Q}(t,\beta) + t\frac{\partial \hat{Q}(t,\beta)}{\partial t},$$

where $\hat{s} = \hat{s}(t,\beta)$ is the equilibrium number of matched pairs for a given t and β.

First, as in Proposition 1 we consider a small increase in tariff rate from free trade ($t = 0$). Consumer surplus decreases with tariff while tariff revenues increase for a small increase in tariff rate from $t = 0$. The following proposition shows that whether welfare increases with the imposition of a small tariff depends on demand curvature.

Proposition 6: Welfare effect of a small tariff

Starting from free trade, a small increase in tariff rate raises (lowers) Home welfare if the inverse demand function is strictly concave (convex).

Proof: Using the expressions for $\frac{\mathrm{d}CS}{\mathrm{d}t}$ and $\frac{\mathrm{d}TR}{\mathrm{d}t}$ from above, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \left[\frac{\hat{Q}(t,\beta)}{2\hat{s}(t,\beta)+\hat{\epsilon}}\right]\hat{\epsilon} + t\frac{\partial \hat{Q}(t,\beta)}{\partial t}. \tag{16}$$

Evaluating (16) at $t = 0$ gives

$$\left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=0} = \left[\frac{\hat{Q}(0,\beta)}{2\hat{s}(0,\beta)+\hat{\epsilon}_0}\right]\hat{\epsilon}_0,$$

where $\hat{\epsilon}_0 \equiv \epsilon(\hat{Q}(0,\beta))$. By Assumption 1', $\hat{\epsilon}_0 \geq -1$ which in turn implies $2\hat{s}(0,\beta)+\hat{\epsilon}_0 > 0$. Thus

$$\left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=0} \gtreqless 0 \iff P''(\hat{Q}(0,\beta)) \lesseqgtr 0,$$

which implies the result. □

Proposition 6 suggests that a small import tariff improves welfare only if the inverse demand function is strictly concave. If the inverse demand function is strictly convex, a small import subsidy improves welfare. For linear demand, both tariff and subsidy lower welfare which suggests that free trade is optimal (as we show below).

Now we are ready to derive the sign of optimal tariff in the long run. Let $t(\beta)$ denote the optimal tariff. Then $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ in (16) must equal zero at $t = t(\beta)$ which implies

$$t(\beta) = \left(\frac{-P'(\hat{Q}(t(\beta), \beta))\hat{q}(t(\beta), \beta)}{2} \right) \cdot \hat{\epsilon}. \tag{17}$$

Proposition 7: Optimal tariff, bargaining power and non-monotonicity

(i) *In the long-run equilibrium, the optimal tariff t is strictly positive (negative) if the inverse demand is strictly concave (convex). Note, the optimal tariff is zero if the demand is linear.*

(ii) *Suppose Assumptions 1 and 2 hold. Unless the inverse demand function is linear ($\epsilon = 0$), the optimal tariff t changes non-monotonically with β. More specifically, the following holds:*

$$\operatorname{sgn} \frac{\mathrm{d}t}{\mathrm{d}\beta} = \operatorname{sgn} P''(\hat{Q}(t(\beta), \beta)) \frac{\partial \hat{Q}}{\partial \beta}.$$

Proof: (i) Immediate from (17).

(ii) Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t}\big|_{t=t(\beta)} = 0$ with respect to β gives:

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -\frac{\frac{\partial^2 W_H}{\partial \beta \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

Since $\frac{\partial^2 W_H}{\partial t^2} < 0$ (second-order condition), $\operatorname{sgn}\frac{\mathrm{d}t}{\mathrm{d}\beta} = \operatorname{sgn}\frac{\partial^2 W_H}{\partial \beta \partial t}$. Differentiating (16) with respect to β, we get (see Appendix A.5):

$$\left.\frac{\partial^2 W_H}{\partial \beta \partial t}\right|_{t=t(\beta)} = -\frac{\hat{\epsilon}(\hat{s} - 1 + \hat{\epsilon} - \hat{\alpha})}{2\hat{s} + \hat{\epsilon}} \frac{\partial \hat{Q}}{\partial \beta},$$

where $\alpha(Q) \equiv \frac{QP'''(Q)}{P''(Q)}$ is the curvature of slope of inverse demand. Then, the result follows from noting that $\operatorname{sgn} \epsilon(\hat{Q}) = -\operatorname{sgn} P''(\hat{Q})$ (definition of $\epsilon(Q)$), $2\hat{s} + \hat{\epsilon} > 0$ (Assumption 1), $\hat{s} - 1 + \hat{\epsilon} - \hat{\alpha} > 0$ (Assumption 2), and non-monotonicity of $\partial \hat{Q}/\partial \beta$ (Lemma 2). $\square$

Proposition 7(ii) suggests that the relationship between t and β depends crucially on (a) how aggregate output, $\hat{Q}$, varies with β and (b) the curvatures of inverse demand and its slope. As $\hat{Q}$ varies non-monotonically with with β the relationship between t and β is non-monotone as well. For sharper characterization of the nature of non-monotonicity, Proposition 8 focuses on a class of inverse demand functions.

Proposition 8: More on non-monotonicity of $t(\beta)$

Consider inverse demand functions $P(Q)$ *that satisfy Assumptions 1 and 2, and are either concave for all* $Q \geq 0$ *or convex for all* $Q \geq 0$*. The relationship between* t *and* β *is U-shaped (inverted U-shaped) if the demand function is strictly concave (convex). If the demand function is linear, free trade is optimal (i.e. optimal tariff is zero) irrespective of the bargaining power.*

Proof: Rewrite $\frac{\partial \hat{Q}}{\partial \beta}$ as

$$\frac{\partial \hat{Q}}{\partial \beta} = \frac{\hat{m}\hat{q}}{\beta(2\hat{s}+\hat{\epsilon})}\phi(\beta),$$

where

$$\phi(\beta) \equiv S(\hat{z}) - \frac{\hat{z}S'(\hat{z})}{1-\beta},$$

and $\hat{z} = \hat{z}(\beta)$ is the equilibrium $\frac{\hat{n}}{\hat{m}}$ for a given β (see (15)). It is easy to see from (14) that:

$$\lim_{\beta\to 0}\phi(\beta) > 0, \quad \lim_{\beta\to 1}\phi(\beta) < 0, \quad \phi'(\beta) = \frac{1}{\beta^3}\left(\frac{K_H}{K_F}\right)^2 S''(\hat{z}) < 0.$$

These properties of $\phi(\beta)$ imply that, if $P''(Q) < 0$ ($P''(Q) > 0$) for all $Q \geq 0$, there exists a threshold $\hat{\beta} \in (0,1)$ such that $\frac{\mathrm{d}t}{\mathrm{d}\beta} \gtreqless (\lesseqgtr)\, 0 \;\Leftrightarrow\; \beta \gtreqless \hat{\beta}$, where $\hat{\beta}$ is the unique solution to the following equation: $\phi(\beta) = 0$. If $P''(Q) = 0$ for all $Q \geq 0$, then $\frac{\mathrm{d}t}{\mathrm{d}\beta} = 0,\ \forall\ \beta \in [0,1]$. Combining this observation with sgn$\frac{\mathrm{d}t}{\mathrm{d}\beta}$ = sgn $P''(\hat{Q})\frac{\partial \hat{Q}}{\partial \beta}$ establishes the result. □

An Illustrative Example

Consider once again the class of demand functions given by $P(Q) = a - Q^b$ where $b > 0$. Assume $K_H = K_F$. As in Grossman and Helpman (2002), suppose $s(m,n) = \frac{mn}{m+n}$ is the matching function (which satisfies equations (11)-(13)). The resulting normalized matching function is $S(z) = \frac{z}{1+z}$. Then, according to Proposition 8, for concave demand ($b > 1$), optimal tariff $t(\beta)$ initially declines as β increases, becoming lowest at equal bargaining power (i.e. $\hat{\beta} = \frac{1}{2}$), and then increases as β increases further. For convex demand ($b < 1$), the relationship between β and $t(\beta)$ is exactly the opposite. If the demand is linear (i.e., $b = 1$), $t(\beta) = 0$ for $\beta \in [0,1]$. Figure 3 above illustrates the relationship between β and $t(\beta)$.

Welfare

Unlike the short run, in which both Home and global welfare are increasing in Home firms' bargaining power, the long run relationship between welfare and bargaining power is typically non-monotone (except for linear demand). As in the case of the relationship between optimal tariff and β, this non-monotonicity (of welfare) is due to the non-monotone

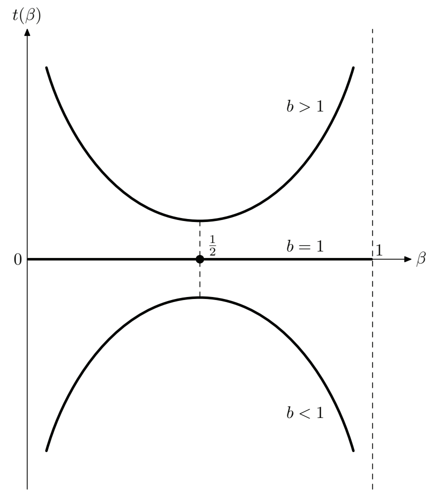

FIGURE 3: NON-MONOTONICITY OF $t(\beta)$

relationship between $\hat{Q}$ and β.

Proposition 9: Welfare and bargaining power

Consider inverse demand functions $P(Q)$ that satisfy Assumptions 1 and 2. For all such demand functions, Home welfare (W_H)and global welfare (W_G) vary non-monotonically with β. Furthermore,

$$\text{sgn}\frac{\mathrm{d}W_G}{\mathrm{d}\beta} = \text{sgn}\frac{\mathrm{d}W_H}{\mathrm{d}\beta} = \text{sgn}\frac{\partial\hat{Q}}{\partial\beta}.$$

Proof: Global welfare (W_G) consists of Home consumer surplus only because Home and Foreign profits are driven to zero by free entry and tariff revenues are simply transfers between Home and Foreign. Home welfare in the long-run is defined as $W_H = W_G + t\hat{Q}$. We have

$$\frac{\mathrm{d}W_G}{\mathrm{d}\beta} = \frac{\partial CS}{\partial Q}\cdot\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\beta} = -P'(\hat{Q})\hat{Q}\cdot\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\beta} = -P'(\hat{Q})\hat{Q}(1+K)\frac{\partial\hat{Q}}{\partial\beta},$$
$$\frac{\mathrm{d}W_H}{\mathrm{d}\beta} = -P'(\hat{Q})\hat{Q}\left(\frac{s+\hat{\epsilon}}{2s}\right)\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\beta} = -P'(\hat{Q})\hat{Q}\left(\frac{s+\hat{\epsilon}}{2s}\right)(1+K)\frac{\partial\hat{Q}}{\partial\beta},$$

where K is as defined in Appendix A.6. Since $-P'(\hat{Q})\hat{Q} > 0$, $s+\hat{\epsilon} > 0$, and $1+K > 0$ (see Appendix A.6), we have that

$$\text{sgn}\frac{\mathrm{d}W_G}{\mathrm{d}\beta} = \text{sgn}\frac{\mathrm{d}W_H}{\mathrm{d}\beta} = \text{sgn}\frac{\partial\hat{Q}}{\partial\beta}.$$

The result follows from noting that $\hat{Q}(t,\beta)$ varies non-monotonically with β (Lemma 2). □

6 Summary and Concluding Remarks

With reduction in trade costs, firms from different countries are increasingly specializing in different but complementary stages of production. In such environments of vertical specialization, tariffs not only hurt foreign producers but also domestic producers. Under what conditions, therefore, might a welfare maximizing government impose a tariff? We show that the weak bargaining power of its firms might prompt a country's government to impose a tariff on foreign producers. Surprisingly, we find that an increase in Home bargaining power not only benefits Home producers but can also benefit Foreign producers by lowering tariff rates. The inverse relationship between bargaining power and the tariff rate breaks down in the long run where a change in bargaining power affects the market structure via matching and entry. We find that in general, the relationship between bargaining strength and optimal tariff is non-monotone. For the special case of linear demand, the optimal tariff is zero, irrespective of the bargaining strength.

Throughout the paper, we have focused on oligopolistic competition and complete contracts to highlight the novel interaction between bargaining strength and trade policy. Conceptually, similar analysis could be applied to models of monopolistic competition and incomplete contracts—two features that have been extensively used in recent works on outsourcing and vertical specialization (see, for example, Grossman and Helpman, 2002; Antràs, 2003). Typically, these models also have relationship-specific investment for which bargaining over revenues (rather than profits) seems more appropriate. Despite this difference, we expect that as long as the relationship specific investment is taken by intermediate goods producers only, the main results will continue to hold. Introducing relationship specific investments by both final goods and intermediate goods producers is likely to affect our results. An increase in Home's bargaining power increases the incentive for investment by Home firms but reduces Foreign firms' incentive to invest. This creates the possibility of a non-monotone relationship between bargaining power and output which in turn can lead to a non-monotone relationship between bargaining power and tariff, even in the absence of entry considerations.

Bargaining structure in our paper is admittedly simplistic. Each Foreign firm bargains with only one Home firm and vice versa. Yet, in reality, a Home firm often negotiates with multiple Foreign firms and similarly a Foreign firm often has supply relationships with multiple Home firms in the global production chains. If we introduce the possibility of bargaining with multiple firms, the outside option of the firms engaged in bargaining will be endogenously determined. In this case, the number of outside options for each firm will be crucial in determining the bargaining power of each firm. While we have briefly addressed the thickness of the market (which can act as a proxy for outside options) in a different context (see subsection 3.4.2), a full-fledged analysis along these lines is challenging and we leave that for future research.

Appendix

A.1 Equivalence between Assumptions 1 and 1'

The assumption $Q(P)$ is logconcave implies

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{\mathrm{d}\ln Q(P)}{\mathrm{d}P}\right] &= \frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{Q'(P)}{Q(P)}\right] \\ &= \frac{Q(P)\cdot Q''(P) - [Q'(P)]^2}{[Q(P)]^2} \leq 0,\end{aligned}$$

which can be expressed as

$$\frac{Q(P)Q''(P)}{[Q'(P)]^2} \leq 1. \tag{A.1}$$

Differentiating $P = P(Q(P))$ with respect to P, we get

$$1 = P'(Q(P))Q'(P).$$

Differentiating this once again with respect to P gives

$$0 = P''[Q'(P)]^2 + P'Q''(P).$$

Rewriting this equation, we get

$$\frac{Q''(P)}{[Q'(P)]^2} = -\frac{P''}{P'}.$$

Substituting this relationship into (A.1), we find that

$$-\frac{QP''(Q)}{P'(Q)} \leq 1,$$

which implies $P'(Q) + QP''(Q) \leq 0$.

A.2 Derivation of $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$ for demand functions with constant elasticity of slope

The optimal tariff $t = t(\beta)$ is given by

$$t = \big\{ - P'(\hat{Q}(t))\hat{Q}(t)\cdot(2+\hat{\epsilon})\big\}\big(\hat{\beta} - \beta\big),$$

where $\hat{\beta} = (1+\epsilon)/(2+\epsilon)$. If ϵ is constant, $\hat{\epsilon} = \epsilon(\hat{Q}) = \epsilon$. Differentiating t with respect to β gives

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = \big\{ - P'(\hat{Q}(t))\hat{Q}(t)(2+\epsilon)\big\}(-1) - \big(\hat{\beta}-\beta\big)(2+\epsilon)\left[P''(\hat{Q})\frac{\hat{Q}(t)}{s} + \frac{P'(\hat{Q}(t))}{s}\right]\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}\beta}, \tag{A.2}$$

where $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}\beta} = \frac{s/P'(\hat{Q}(t))}{s+1+\epsilon}\cdot\frac{\mathrm{d}t}{\mathrm{d}\beta}$ from (3). Substituting this into (A.2) yields

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -(2+\epsilon)\left[\big(\hat{\beta}-\beta\big)\frac{\mathrm{d}t}{\mathrm{d}\beta}\left(\frac{1+\epsilon}{s+1+\epsilon}\right) - P'(\hat{Q}(t))\hat{Q}(t)\right],$$

which gives the following upon rearrangement:

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = \frac{P'(\hat{Q}(t))\hat{Q}(t)\cdot(2+\epsilon)}{1+[(1+\epsilon)-\beta(2+\epsilon)]\left(\frac{1+\epsilon}{s+1+\epsilon}\right)} < 0.$$

A.3 Derivation of $\frac{\mathrm{d}t_H}{\mathrm{d}\beta} < 0$ and $\frac{\mathrm{d}t_F}{\mathrm{d}\beta} > 0$ in the two-country setup

Totally differentiating $\partial W_H/\partial t_H = 0$ and $\partial W_F/\partial t_F = 0$ and rewriting them in a matrix form, we get

$$\begin{bmatrix} \frac{\partial^2 W_H}{\partial t_H^2} & \frac{\partial^2 W_H}{\partial t_H \partial t_F} \\ \frac{\partial^2 W_F}{\partial t_F \partial t_H} & \frac{\partial^2 W_F}{\partial t_F^2} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}t_H}{\mathrm{d}\beta} \\ \frac{\mathrm{d}t_F}{\mathrm{d}\beta} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 W_H}{\partial t_H \partial \beta} \\ -\frac{\partial^2 W_F}{\partial t_F \partial \beta} \end{bmatrix}.$$

Applying Cramer's rule we get:

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} = \frac{\left(-\frac{\partial^2 W_H}{\partial t_H \partial \beta}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2}\right) - \left(\frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)\left(-\frac{\partial^2 W_F}{\partial t_F \partial \beta}\right)}{\Delta}, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} = \frac{\left(\frac{\partial^2 W_H}{\partial t_H^2}\right)\left(-\frac{\partial^2 W_F}{\partial t_F \partial \beta}\right) - \left(-\frac{\partial^2 W_H}{\partial t_H \partial \beta}\right)\left(\frac{\partial^2 W_F}{\partial t_F t_H}\right)}{\Delta},$$

where $\Delta \equiv \left(\frac{\partial^2 W_H}{\partial t_H^2}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2}\right) - \left(\frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)\left(\frac{\partial^2 W_F}{\partial t_F \partial t_H}\right) > 0$ follows from the stability condition. It is easy to check that

$$\frac{\partial^2 W_H}{\partial t_H \partial \beta} = -\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}} < 0, \quad \frac{\partial^2 W_F}{\partial t_F \partial \beta} = \frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}} > 0.$$

Substituting these expressions in $\frac{\mathrm{d}t_H}{\mathrm{d}\beta}$ and $\frac{\mathrm{d}t_F}{\mathrm{d}\beta}$ above we find that

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} = \frac{\left(\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2} + \frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)}{\Delta}, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} = \frac{\left(-\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}}\right)\left(\frac{\partial^2 W_H}{\partial t_H^2} + \frac{\partial^2 W_F}{\partial t_F \partial t_H}\right)}{\Delta}.$$

It follows that $\frac{\mathrm{d}t_H}{\mathrm{d}\beta} < 0$ and $\frac{\mathrm{d}t_F}{\mathrm{d}\beta} > 0$ as long as $\frac{\partial^2 W_i}{\partial t_i^2} + \frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$ for $i \neq j \in \{H, F\}$.

A.4 Derivation of (a) $\frac{\partial \hat{s}}{\partial t} < 0$ and $\frac{\partial \hat{Q}}{\partial t} < 0$, and (b) non-monotonicity of $\frac{\partial \hat{s}}{\partial \beta}$ and $\frac{\partial \hat{Q}}{\partial \beta}$ in the long-run setup

Rewrite Home firm's free entry condition (10) as

$$-\frac{P'(\hat{Q})\hat{Q}^2}{\hat{s}^2} = \frac{K_H}{\beta S(\hat{z})}. \tag{A.3}$$

Consider (a) first. Since equilibrium $\hat{z}$ is independent of t from (15), the right-hand side of (A.3) is also independent of t. Differentiating (A.3) with respect to t gives

$$\frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{q}}{2+\hat{\epsilon}} \cdot \frac{\partial \hat{s}}{\partial t}. \tag{A.4}$$

Furthermore, differentiating $\hat{Q} = \hat{Q}(t, \hat{s}(t))$ with respect to t, we get

$$\frac{\partial \hat{Q}}{\partial t} = \frac{\hat{s}/P'(\hat{Q})}{\hat{s}+1+\hat{\epsilon}} + \frac{\hat{q}}{\hat{s}+1+\hat{\epsilon}} \frac{\partial \hat{s}}{\partial t}, \tag{A.5}$$

where the first and second terms of the right-hand side in (A.5) come from the first-order condition (3). Solving (A.4) and (A.5) for $\frac{\partial \hat{s}}{\partial t}$ and $\frac{\partial \hat{Q}}{\partial t}$ yields the results.

The proof of (b) follows similar steps. Differentiating the first-order condition (3) and free-entry condition (A.3) with respect to β respectively, we get

$$\frac{\partial \hat{Q}}{\partial \beta} = \frac{\hat{q}}{\hat{s}+1+\hat{\epsilon}} \frac{\partial \hat{s}}{\partial \beta}, \quad \frac{\sigma(\beta)}{1-\beta} - 1 = \beta \left(\frac{2+\hat{\epsilon}}{\hat{Q}} \frac{\partial \hat{Q}}{\partial \beta} - \frac{2}{\hat{s}} \frac{\partial \hat{s}}{\partial \beta} \right).$$

Solving the above two equations for $\frac{\partial \hat{Q}}{\partial \beta}$ and $\frac{\partial \hat{s}}{\partial \beta}$ gives the results.

A.5 Derivation of $\frac{\partial^2 W_H}{\partial\beta\partial t}$ in the long-run setup

Differentiating W_H with respect to t and simplifying it, we get:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \frac{2\hat{s}t + P'(\hat{Q})\hat{Q}\hat{\epsilon}}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})}. \tag{A.6}$$

Note from (17) that $2\hat{s}t + P'(\hat{Q})\hat{Q}\hat{\epsilon} = 0$ at $t = t(\beta)$. Differentiating the above equation with respect to β yields:

$$\begin{aligned}
\left.\frac{\partial^2 W_H}{\partial\beta\partial t}\right|_{t=t(\beta)} &= \frac{2t\frac{\partial\hat{s}}{\partial\beta} + \left[\hat{Q}^2 P'''(\hat{Q}) + 2\hat{Q}P''(\hat{Q})\right]\frac{\partial\hat{Q}}{\partial\beta}}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})} && \text{(using } \tfrac{\mathrm{d}t}{\mathrm{d}\beta} = 0 \text{ at } t = t(\beta)\text{)}\\
&= \frac{\left[2t\frac{\hat{s}+1+\hat{\epsilon}}{\hat{q}} + \hat{Q}P''(\hat{Q})(2+\hat{\alpha})\right]}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})}\frac{\partial\hat{Q}}{\partial\beta} && \text{(using } \tfrac{\partial\hat{s}}{\partial\beta} = \tfrac{\hat{s}+1+\hat{\epsilon}}{\hat{q}}\tfrac{\partial\hat{Q}}{\partial\beta} \text{ from Lemma 2)}\\
&= \frac{\left[-\hat{Q}P''(\hat{Q})(\hat{s}+1+\hat{\epsilon}) + \hat{Q}P''(\hat{Q})(2+\hat{\alpha})\right]}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})}\frac{\partial\hat{Q}}{\partial\beta} && \text{(using (17))}\\
&= -\frac{\hat{\epsilon}(\hat{s}-1+\hat{\epsilon}-\hat{\alpha})}{2\hat{s}+\hat{\epsilon}}\frac{\partial\hat{Q}}{\partial\beta}.
\end{aligned}$$

A.6 Derivation of $\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\beta}$ in the long-run setup

We have that

$$\begin{aligned}
\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\beta} &= \frac{\partial\hat{Q}}{\partial\beta} + \frac{\partial\hat{Q}}{\partial t}\frac{\mathrm{d}t}{\mathrm{d}\beta}\\
&= \frac{\partial\hat{Q}}{\partial\beta}(1+K),
\end{aligned}$$

where, using the results from the proof of Proposition 7, we get

$$K \equiv \frac{\partial\hat{Q}}{\partial t}\frac{\hat{\epsilon}\left(\frac{\hat{s}-1+\hat{\epsilon}-\hat{\alpha}}{2\hat{s}+\hat{\epsilon}}\right)}{\frac{\partial^2 W_H}{\partial t^2}}.$$

Substituting the expression for $\frac{\partial\hat{Q}}{\partial t}$ from Lemma 2 and noting from (A.6)

$$\frac{\partial^2 W_H}{\partial t^2} = \frac{2\hat{s}\left[2\hat{s}+\hat{\epsilon}\left(2-\frac{\hat{\epsilon}}{2}+\hat{\alpha}\right)\right]}{(2\hat{s}+\hat{\epsilon})^2 P'(\hat{Q})},$$

we get:

$$K = \frac{\hat{\epsilon}(\hat{s}-1+\hat{\epsilon}-\hat{\alpha})}{2\hat{s}+\hat{\epsilon}\left(2-\frac{\hat{\epsilon}}{2}+\hat{\alpha}\right)}.$$

Since the second-order condition is assumed to hold, $\frac{\partial^2 W_H}{\partial t^2} < 0$ which in turn implies $2\hat{s}+\hat{\epsilon}\left(2-\frac{\hat{\epsilon}}{2}+\hat{\alpha}\right) > 0$. Thus the denominator of K is positive. Then,

$$\begin{aligned}
1+K &= \frac{\hat{s}(2+\hat{\epsilon})+\hat{\epsilon}+\frac{\hat{\epsilon}^2}{2}}{2\hat{s}+\hat{\epsilon}\left(2-\frac{\hat{\epsilon}}{2}+\hat{\alpha}\right)}\\
&\geq \frac{1+\hat{\epsilon}+\frac{\hat{\epsilon}^2}{2}}{2\hat{s}+\hat{\epsilon}\left(2-\frac{\hat{\epsilon}}{2}+\hat{\alpha}\right)} && \text{(since } \hat{s}\geq 1 \text{ and } 2+\hat{\epsilon}\geq 1\text{)}\\
&> 0. && \text{(since } 1+\hat{\epsilon}\geq 0 \text{ and } \hat{\epsilon}^2\geq 0\text{)}
\end{aligned}$$

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