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Knowledge Transfer and Partial Equity Ownership

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Abstract

When firms form an alliance, it often involves one firm acquiring an equity stake in its alliance partner. Such an alliance weakens competition, but induces knowledge transfer between partner firms. We explore oligopoly models that capture the link between knowledge transfer and partial equity ownership (PEO), where alliance partners can choose the level of PEO to connect themselves. PEO, merger and independence are all nested in our model, where PEO can arise in equilibrium and the endogenously determined level of PEO can benefit consumers and/or society. We identify conditions under which antitrust authorities would prohibit, partially permit, or permit PEO.

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1 Introduction

A strategic alliance exists when two or more independent organizations cooperate in the development, manufacturing, or sale of products or services (Barney, 2002). In recent years, the incidence and importance of inter-firm collaborations has substantially increased (Caloghirou, Ioannides and Vonortas, 2003). It has been widely recognized in the management literature that one of the most fundamental objectives of strategic alliances is the transfer of knowledge between partner firms.[1]

Modern economies are becoming increasingly knowledge intensive. Drucker (1993), for example, has argued that in the new economy, knowledge is not just another resource alongside the traditional factors of production—labor, capital, and land—but the only meaningful resource today.[2] Knowledge is often classified into tacit and explicit knowledge.[3] Explicit (or codified) knowledge can be transmitted in formal, systematic language, whereas tacit knowledge is non-verbalizable, intuitive, and unarticulated (Polanyi, 1958, 1966). Because tacit knowledge is difficult to convey, its transfer requires greater effort (Reagans and McEvily, 2003). Tacit knowledge can be transferred through up-close observation, demonstration, hands-on experience, a sharing of feelings and emotions, and face-to-face contact (Hamel, 1991; Nonaka, 1994; Nonaka and Takeuchi, 1995; Cavusgil et al., 2003).

Explicit knowledge can be transferred through licensing and contracting since it is verifiable. However, since tacit knowledge cannot be codified and hence cannot be verified, licensing and contracting play, at best, the limited role in the transfer of tacit knowledge (Mowery, 1983; Pisano, 1990). Here, however, equity ownership plays a critical role in facilitating the transfer of tacit knowledge (Mowery et al., 1996). Using patent citations as a proxy for knowledge flows, Mowery et al. (1996) and Gomes-Casseres et al. (2006) empirically explored the effects of equity ownership between alliance partners on the extent of knowledge flow. Empirical results of both studies supported the hypothesis that equity ownership enhances the extent of knowledge flow between alliance partners.[4] Examples of equity

[1] See Hamel, 1991; Mowery, Oxley and Silverman, 1996; Gomes-Casseres, Hagedoorn and Jaffe, 2006; Oxley and Wada, 2009. Gomes-Casseres et al. (2006), for example, hypothesized that knowledge flows between alliance partners would be greater than flows between pairs of non-allied firms, and found empirical results that are consistent with this hypothesis.

[2] See Quinn (1992) and Toffler (1990) for similar arguments.

[3] See Polanyi (1958, 1966) for a seminal work on tacitness of knowledge. See also Nonaka (1994), Nonaka and Takeuchi (1995), Grant (1996), Inkpen and Dinur (1998), and Adler (2001), among others.

[4] Both studies used Cooperate Agreements and Technology Indicators (CATI) database developed by the Maastricht Economic Research Institute in Technology (MERIT) to identify alliances of firms. Mowery et

strategic alliances listed below also demonstrate the connection between equity ownership and knowledge transfer.[5]

• In March 2007, Citigroup and Nikko Cordial announced the formation of a strategic alliance. Citigroup, after acquiring a 4.9% stake in Nikko, will help its partner develop a strategic solution by transferring considerable know-how and expertise in principal investments and private equity.

• In December 2000, Vodafone announced that it would acquire a 15% stake in Japan Telecom for Yen 249 billion. The Chairman of Japan Telecom said that, through this strategic alliance, his company would benefit from Vodafone's global leadership in mobile communications, and access to world-wide technology, content, and expertise.

• In April 2010, Groupe Aeroplan Inc. announced that it would acquire a stake of less than 20% of AeroMexico's frequent flyer program, Club Premier. Aeroplan's Chief Executive Officer said that Club Premier would benefit from Aeroplan's know-how and develop the necessary skill sets that will be critical to its successful transformation into a profitable coalition program.

• In April 2004, Harvey World Travel announced its plan to take an initial 11% equity holding in Webjet, an internet travel business specialist. Webjet's Managing Director, David Clarke, said that the arrangement would provide Webjet with a strategic development partner which would enhance Webjet's ability to capitalize on opportunities in a rapidly changing travel market in the Australian region.

Partial equity ownership (PEO) induces knowledge transfer between alliance partners. This paper explores an oligopoly model that captures this important link between PEO

al. (1996) focused on bilateral alliances that involved at least one U.S. firm and which were established during 1985 and 1986, and the patent data were drawn from the Micropatent database, which contains all information recorded on the front page of every patent granted in the U.S. since 1975. Gomes-Casseres et al. (2006) matched the firms in the CATI database to the NBER Patent Citations Data File. They used only alliance and patent data from information technology sectors, and analyzed citations on an annual basis from 1975 to 1999. See also Ono, Nakazato, Davis, and Alley (2004) for empirical evidence from the Japanese automobile industry. They also put forth a theoretical framework incorporating partial ownership arrangement and technology transfer, but they did not derive the equilibrium of the model by solving it.

[5]Details are available, respectively, from http://www.citigroup.com/citigroup/press/2007/070306a.htm, http://www.vodafone.com/start/media_relations/news/group_press_releases/2000/press_release20_12.html, http://m.theglobeandmail.com/globe-investor/aeroplan-buying-stake-in-aeromexico-loyalty-program/article1547079/?service=mobile, and http://www.webjet.com.au/about/webjet-to-enter-major-commercial-and-equity-alliance-with-harvey-world-travel-limited/

and knowledge transfer. We consider an industry consisting of $n + 2$ firms that produce a homogeneous product and compete in quantities. Compared to firm 2, firm 1 has a cost advantage due to its superior knowledge that firm 2 does not have. Constant marginal costs of firms 1 and 2 are $c - x$ and c, respectively. They have an option of forming an equity strategic alliance in which firm 1 owns a fraction $\theta \in [0, 1]$ of firm 2's share. Given θ, firm 1 decides whether to transfer its superior knowledge to firm 2. Firm 1's knowledge transfer increases firm 2's competitive position by decreasing firm 2's cost from c to $c - x$, and this in turn reduces firm 1's profitability. Firm 1 therefore would not transfer its knowledge without monetary return. We assume that the knowledge is tacit and not verifiable, and hence contracts or licensing agreements cannot guarantee monetary return for firm 1. In this context, PEO can play an important role in facilitating the transfer of knowledge. We assume that other n firms' marginal costs are all c, where they are independent and cannot form an alliance.

A key innovation of our paper is that the level of PEO, θ, is endogenously determined. At the beginning of the model, firms 1 and 2 jointly determine the level of θ ($\in [0, 1]$) to maximize their joint profit in the subsequent equilibrium, and the monetary terms of the equity transfer through efficient bargaining. Then, once θ is chosen, firm 1 determines, based on its own profit-maximizing motive, whether or not to transfer its knowledge to firm 2. By transferring its knowledge, firm 1 loses its competitive advantage over firm 2, but a fraction θ of firm 2's incremental profit due to knowledge transfer belongs to firm 1. A sufficiently high level of θ induces firm 1 to transfer its knowledge to firm 2, and weakens the degree of competition between firms 1 and 2. This works to the alliance partners' advantage. However, weaker competition between firms 1 and 2 induces other n firms outside the alliance to use more aggressive strategy, and this works to the alliance partners' disadvantage.

The trade-off just mentioned determines the equilibrium level of PEO endogenously. Because of the latter disadvantageous effect of a PEO, when firms 1 and 2 intend to induce knowledge transfer they choose the lowest possible level of PEO that is still sufficient for firm 1 to transfer its knowledge.[6] This is referred to as *the minimum PEO for knowledge transfer* and denoted $\hat{\theta}$ (> 0). The disadvantageous effect of PEO also implies that, when firms 1 and 2 do not intend to induce knowledge transfer, they choose $\theta = 0$. Under the general downward-sloping inverse demand function, we identify a sufficient condition under which firms 1 and 2 choose $\theta = \hat{\theta}$ in the equilibrium.

Firms 1 and 2 choose the level of PEO to maximize their joint profitability. Would equity

[6] This is not always the case under differentiated oligopoly. See Section 4 for details.

alliances formed by profit-maximizing motives benefit or hurt consumers and/or the society? PEO weakens the degree of competition in the industry, and this negatively affects consumer surplus and total surplus. At the same time, PEO can improve firm 2's production efficiency by inducing knowledge transfer, and this positively affects them. This trade-off suggests a possibility that PEO may benefit consumers and/or the society.

Similar trade-offs were previously explored in the context of horizontal mergers, and we present a discussion on our contribution to this literature in Section 2. By exploring PEO arrangements as a way for firms to connect themselves, we identify a new trade-off associated with the size of knowledge to be transferred and the minimum PEO for transferring the knowledge, and explore welfare implications of the trade-off. We also find that antitrust implications of horizontal mergers can change in significant ways when PEO is introduced as an option to the merger/non-merger dichotomy (see the last 2 pages of subsection 3.3 for details).

In order to obtain sharper results on equilibrium characterization and welfare/policy implications, we need to specify the functional form of the inverse demand function, and we use a linear inverse demand function in our analysis. We can then identify necessary and sufficient conditions for PEO to emerge endogenously in the equilibrium. We find that firms 1 and 2 choose independence ($\theta = 0$) when x is small, PEO ($\theta \in (0, 1)$) when x is in an intermediate range, and merger ($\theta = 1$) when x is large. We also find that, when x is in the intermediate range, the equilibrium level of PEO increases as x increases. All three options—independence, PEO, and merger—are nested in our model, and one of these options is chosen by the alliance partners depending on the significance of the superior knowledge (captured by x).

Concerning welfare consequences of PEO, we find that the endogenously determined level of PEO can increase both consumer surplus and total surplus, decrease consumer surplus but increase total surplus, or decrease both of them, depending on the significance of the superior knowledge captured by x, and the extent of overall competition in the industry captured by n. The finding that PEO can increase consumer surplus is worth noting because it has often been argued that antitrust authorities maximize consumer surplus rather than total surplus (see, for example, Whinston, 2007). We find that, in our framework, PEO arrangements can never increase consumer surplus without knowledge transfer, indicating the importance of knowledge transfer for improving consumer surplus (see the remark presented after Proposition 4 in subsection 3.3 for details).

PEO arrangements have been previously studied under static oligopoly models as well as repeated oligopoly models. However, the level of PEO in these models is exogenously given rather than endogenously determined. Also, to the best of our knowledge, no previous papers have explicitly analyzed the process by which PEO induces knowledge transfer between competing firms. See Section 2 for details on previous theoretical analyses of PEO.

In the United States, cases of PEO have gone mostly unchallenged by antitrust agencies (see Gilo (2000) for details). However, the U.S. antitrust agencies have recently begun to pay increasing attention to the possible antitrust harms of PEO. For example, Deborah Platt Majoras, the then Deputy Assistant Attorney General of the Antitrust Division of the U.S. Department of Justice, mentioned in her speech (given in April 2002) that PEO can raise antitrust issues when the two companies or their subsidiaries are competitors. Also, several legal scholars have argued that PEO, even if it is not accompanied by control/influence rights, results in antitrust harms in oligopolistic industries by reducing quantities and raising prices (Gilo, 2000; O'Brien and Salop, 2000, 2001). These arguments are based on the idea that PEO weakens competition and decreases welfare.

Our analysis yields richer antitrust implications of PEO arrangements when PEO is linked to knowledge transfer. We consider an antitrust authority that maximizes the equilibrium total surplus by announcing a maximum permissible level of PEO before firms 1 and 2 choose the level of θ. Consider a range of parameterizations in which firms 1 and 2 would choose $\theta = \hat{\theta}$ if no restriction is imposed on θ. If knowledge transfer induced by $\theta = \hat{\theta}$ increases total surplus, the authority has no need to impose any restrictions because firms 1 and 2 choose the minimum PEO for knowledge transfer anyway. In contrast, if knowledge transfer induced by $\theta = \hat{\theta}$ decreases total surplus, the authority prohibits PEO (i.e., forces $\theta = 0$). Hence the authority's relevant options are to impose no restrictions on PEO or to prohibit PEO in our base model. The qualitative nature of our result would remain unchanged when the authority maximizes consumer surplus.

A partial permission of PEO can be another relevant option for the authority in our framework. We demonstrate this possibility in Section 4 under a differentiated oligopoly model. We find that firms 1 and 2 may choose a level of PEO that is strictly above the minimum PEO for knowledge transfer, and that they may also choose to merge. In such cases, the antitrust authority can increase consumer and total surplus by setting the maximum permissible level of PEO equal to the minimum PEO required for knowledge transfer. This is a partial permission of PEO, because the authority permits PEO only up to a certain level that imposes a binding constraint on the alliance partners' profit-maximizing motive. Our analysis therefore suggests that the antitrust authority's optimal policy is prohibition, partial permission, or permission of PEO, depending on the significance of the knowledge to be transferred, the extent of competition in the industry, and the degree of product differentiation.

The remainder of the paper is organized as follows. Section 2 discusses this study's relationship to the literature. Section 3 analyzes a model that captures the link between PEO and knowledge transfer under linear homogeneous demand, explores its welfare consequences and antitrust implications, and discusses the robustness of our findings. Section 4 demonstrates, using a differentiated oligopoly model, that partial permission of PEO can be the optimal antitrust policy. Section 5 explores how far our analysis can go under a general inverse demand function. Section 6 concludes the paper.

2 Relationship to the literature

PEO

PEO arrangements among firms alter their competitive incentives. The competitive effect of PEOs has been previously studied in the context of static oligopoly as well as repeated oligopoly models. In these models, the levels of PEO are exogenously assumed rather than endogenously determined.

Reynolds and Snapp (1986), in their seminal contribution to theoretical analyses of PEOs, analyzed a modified Cournot oligopoly model consisting of n firms that produce the homogeneous product with the same constant marginal cost c. The n firms are linked by PEO, where each firm i holds ownership interest v_{ik} in firm k. Ownership interests are not accompanied by any decision-making rights in the sense that each firm i determines the amount of its own production under any PEO structure. The levels of PEO (the values of v_{ik}, $i = 1, ..., n$, $k = 1, ..., n$, $i \neq k$) are exogenously given.[7] Under the model outlined above, Reynolds and Snapp showed that, if one or more Cournot competitors increase the level of ownership links with rival firms, equilibrium market output will decline. In other words, they demonstrated that, in markets where entry is difficult, PEO could result in less output and higher prices because PEO arrangements reduce the degree of competition among participants by linking their profitability.[8]

[7]More precisely, Reynolds and Snapp make a distinction between firms and plants, where firms are profit-maximizing decision-making units that control plants. Each firm i owns the decision-making right of plant i. In addition to its own share (which is $1 - \sum_{k \neq i} v_{ki}$) of plant i's profit, each firm i also receives v_{ik} share of plant k's ($k \neq i$) profit.

[8]Drawing on the work of Reynolds and Snapp (1986), Bresnahan and Salop (1986) devised a Modified

Could firms increase their profitability by linking themselves through a PEO? Consider a PEO between firms 1 and 2 under the modified Cournot oligopoly model analyzed by Reynolds and Snapp (1986), where other $n-2$ firms have no PEO arrangements. As pointed out by Reitman (1994), the equilibrium joint profit of firms 1 and 2 declines under any levels of PEO for all $n \geq 3$, where the result is similar to the finding of Salant, Switzer and Reynolds (1983) for merger.[9] Reitman (1994) showed, using a conjectural variations model, that with conjectures that are more rivalrous than Cournot, there exist individually rational PEO arrangements with any number of firms in the industry.

Farrell and Shapiro (1990a) showed that, under the Cournot oligopoly model with homogeneous goods, PEO arrangements between two firms can increase their equilibrium joint profit if their production efficiency is different. Suppose that firm 1 holds a share, α, of the stock of firm 2, and each firm j (=2, ..., n) holds no PEO in other firms. As firm 1 increases its stake α in firm 2, firm 1 reduces its output while all other firms increase their outputs in the equilibrium. This is because, as α increases, firm 1 is increasingly willing to sacrifice profits at its own facility in order to augment profits at firm 2. Farrell and Shapiro found that, if firm 1 is smaller than firm 2 (that is, if firm 1 is less cost efficient than firm 2), then a certain level of PEO α increases the joint equilibrium profit of firms 1 and 2, because a larger fraction of their output is produced under a more cost-efficient production facility, firm 2. Note, however, that in practice we often see the reverse: a big firm buys part of a smaller firm, as acknowledged by Farrell and Shapiro.

Malueg (1992) and Gilo, Moshe and Spiegel (2006) analyzed the competitive effect of the PEO under repeated oligopoly models, focusing on the ability of firms to engage in tacit collusion. Malueg (1992) considered a repeated symmetric Cournot duopoly model in which the firms hold identical stakes in one another (cross ownership), and showed that increasing the degree of cross ownership may decrease the ease or likelihood of collusion. Investigating a family of demand functions, Malueg found that the curvature of the demand function can alter the possibilities for collusion. Gilo et al. (2006) considered a repeated Bertrand

Herfindahl-Hirshman Index (MHHI) to quantify the competitive effects of horizontal joint ventures under a number of alternative financial interest and control arrangements. Also, Kwoka (1992) analyzed the output and profit effects of horizontal joint ventures under a conjectural variations model in which a joint venture constitutes a new producer rather than replaces an existing firm.

[9] Salant, Switzer and Reynolds showed, under a Cournot oligopoly model with a linear demand and symmetric constant marginal costs across all n firms, that horizontal mergers can be profitable only if more than 80 percent of the firms merge. Then, mergers between two firms are not profitable for any $n \geq 3$. See Perry and Porter (1985) for a related analysis.

oligopoly model consisting of n firms in which firms and/or their controllers acquire some of their rivals' non-voting shares. The n firms need not have similar stakes in one another in their model. Gilo et al. established necessary and sufficient conditions for PEO arrangements to facilitate tacit collusion, and also examined how tacit collusion is affected when firms' controllers make direct passive investments in rival firms.

In the literature on theoretical analyses of PEO arrangements, several papers have pointed out the link between PEO and knowledge transfer. For example, Reynolds and Snapp (1986) pointed out that a PEO offers a means for appropriating the returns to technology transfer. Also, Reitman (1994) argued that PEO arrangements may be beneficial to society if they encourage firms to exchange expertise or assets that would otherwise not be made available. However, to the best of our knowledge, no previous papers have explicitly analyzed the process by which a PEO induces knowledge transfer between competing firms. The present paper fills this important gap in the literature by exploring a model in which the level of PEO is endogenously determined through the link between PEO and knowledge transfer.

Welfare effects of horizontal mergers

PEO induces knowledge transfer in our model. PEO itself reduces competition, but induced knowledge transfer improves productivity. The trade-off between reduction in competition and productivity improvement associated with a merger (or other combination) was first articulated in the economics literature by Williamson (1968). This trade-off was then explored in game-theoretic models of market competition in the context of horizontal merger. See Whinston (2007) for a recent survey of the literature.

Farrell and Shapiro (1990b) analyzed the welfare effects of horizontal mergers in a Cournot oligopoly model. In their model, firms have different production efficiencies, and hence, a merger may increase welfare by shifting output to the facility with the lower marginal cost. Farrell and Shapiro found general conditions under which horizontal mergers raise prices (i.e. reduce consumer surplus), and showed that any merger not creating synergies (economies of scale or learning) raises price. They then developed a procedure for analyzing the effect of a merger on rivals and consumers (referred to as external effects) and thus provided sufficient conditions for a profitable merger to raise welfare. See Levin (1990) and McAfee and Williams (1992) for related analyses.

Horizontal mergers, however, are not the sole context in which the trade-off between reduction in competition and productivity improvement arises. For example, Williamson's (1968) seminal contribution, mentioned above, starts by stating, "Suppose that merger (*or other combination*) is proposed that yields economies but at the same time increases market power [italics added].” Our contribution to this literature is to explore welfare consequences and antitrust implications of PEO, which we believe is an important class of “other combinations”. We find that antitrust implications of horizontal mergers can change in significant ways by introducing PEO as an option to the merger/non-merger dichotomy (see Subsection 3.3 for details).

Transfer of knowledge (or learning) is an important channel through which horizontal mergers achieve synergies. Farrell and Shapiro (1990b), for example, pointed out, “A merger may enhance efficiency at some or all of the merging facilities: one facility may learn from its partner's patents, management expertise, etc.” When firms merge and the merger enables learning, it is reasonable to assume that the learning actually occurs because it improves the merged firm's profitability. In contrast, when firms are connected through PEO, the presence of learning opportunity does not automatically guarantee the occurrence of learning. In our PEO model, a firm with superior knowledge (firm 1) transfers its knowledge to its alliance partner (firm 2) only if firm 1 owns a sufficiently high level of PEO in firm 2. And an increase in the significance of the superior knowledge leads to an increase in the minimum PEO for knowledge transfer. Hence, an increase in the level of PEO does not necessarily hurt consumers or decrease welfare because the increase is driven by an increase in the amount of knowledge transferred. In fact, we find that PEO benefits consumers in equilibrium when the level of PEO is relatively high. This counterintuitive result arises due to the trade-off between an increase in the size of learning and the corresponding reduction in competition, which is uniquely captured by our analysis of PEO.

3 Endogenous PEO and its welfare effects

3.1 The model

Consider an industry consisting of $(n+2)$ firms $(n \geq 1)$ that produce a homogenous product. The industry faces a linear inverse demand given by $P(Q) = a - dQ$ $(a > 0,\ d > 0)$ where Q denotes the industry output (in Section 5 we explore how far our analysis can go under the general downward-sloping inverse demand). Let p_i and q_i denote each firm i's $(i = 1, 2, ..., n)$ price and quantity, respectively. Each firm i's cost for producing q_i (> 0) units of the product is $c_i q_i$ where c_i (> 0) denotes firm i's constant marginal cost.

Compared to firm 2, firm 1 has a cost advantage due to its superior knowledge that firm 2 does not have. Firms 1 and 2 have an option to form an equity alliance. In particular, they negotiate and jointly choose the level of firm 1's ownership in firm 2's equity, denoted θ ($0 \leq \theta \leq 1$), and the monetary terms of the equity transfer. Given θ, firm 1 decides whether to transfer its superior knowledge to firm 2. Assume that $c_1 = c - x < c_2 = c$ without knowledge transfer ($c > x > 0$), where x captures firm 1's cost advantage over firm 2 due to its superior knowledge. If firm 1 transfers its knowledge to firm 2, firm 2's marginal cost is reduced to $c_2 = c - x$. Regarding other firms' marginal costs, we assume $c_3 = ... = c_{n+2} = c$. We assume that firms $3, ..., n+2$ are independent and cannot form alliances.

We consider the three-stage game described below:

Stage 1 [Alliance formation]: Firms 1 and 2 negotiate and jointly choose the level of firm 1's ownership in firm 2's equity, denoted θ ($0 \leq \theta \leq 1$), and the monetary terms of the equity transfer through efficient bargaining. The level of θ becomes common knowledge.

Stage 2 [Knowledge transfer]: Firm 1 determines whether or not to transfer its knowledge to firm 2. Firms' constant marginal costs are given by $c_1 = c_2 = c - x < c_3 = ... = c_{n+2} = c$ if knowledge is transferred, and by $c_1 = c - x < c_2 = c_3 = ... = c_{n+2} = c$ otherwise, where $c > x > 0$.

Stage 3 [Cournot competition]: Whether or not firm 1 transferred its knowledge to firm 2 becomes common knowledge, and hence every firm knows $(c_1, c_2, c_3, ..., c_{n+2})$. If $\theta \in [0, \frac{1}{2}]$, each firm i simultaneously and non-cooperatively chooses q_i to maximize its profit. If $\theta \in (\frac{1}{2}, 1]$, firm 1 chooses q_1 and q_2 and firm m $(= 3, ..., n+2)$ chooses q_m, simultaneously and non-cooperatively, to maximize their own profits.[10]

Remarks

• Our model assumes that firm 1's possible alliance partner is firm 2 and does not allow firm 1 to transfer its knowledge to other firms. This assumption makes the analysis tractable and focused, and can be justified as follows. In reality, firms have different sets of strengths and weaknesses. When forming an alliance, a firm would choose an alliance partner whose weakness can be most effectively improved by knowledge transfer from the partner. Our model reflects this reality in a reduced form by assuming that firm 2 is the only firm whose production efficiency can be improved by knowledge transfer from firm 1.

• At Stage 1, firms 1 and 2 jointly determine the level of θ to maximize their joint profit in the subsequent equilibrium. The joint surplus is shared between firms 1 and 2 through the monetary terms of the equity transfer, which is determined by efficient bargaining. Dis-

[10]More generally, we can assume that firm 1 chooses q_1 and q_2 if $\theta \in (\zeta, 1]$ where $\zeta \in (0, 1)$. The qualitative nature of our main results would remain unchanged as long as ζ is sufficiently large.

tribution of the surplus between firms 1 and 2 does not affect the results of the paper. See footnote 12 for a related discussion.

• We have considered the robustness of our results under the following two alternative modelling choices. First, our model does not allow firm 1 to transfer a part of its knowledge. We have considered a variant of our model that allows partial transfer of knowledge. Second, firm 2 cannot hold ownership in firm 1's equity in our model. We have considered a variant of our model that allows cross ownership. See subsection 3.4 for details.

3.2 Equilibrium characterization

We derive Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model described above. Define variable $k \in \{0, 1\}$ by $k = 1$ if firm 1 transferred its knowledge to firm 2 at Stage 2 and $k = 0$ otherwise. Note that proofs of lemmas and propositions are presented in the Appendix.

Stage 3 subgames

Every Stage 3 subgame can be represented by (θ, k). Each firm i's $(i = 1, 2, ..., n + 2)$ profit, denoted by $\pi_i(\theta, k, q_1, q_2, ..., q_{n+2})$, is given by

$$\begin{aligned} \pi_1(\theta, k, q_1, q_2, ..., q_{n+2}) &= [P(Q) - (c - x)]q_1 + \theta[P(Q) - (c - kx)]q_2, \\ \pi_2(\theta, k, q_1, q_2, ..., q_{n+2}) &= (1 - \theta)[P(Q) - (c - kx)]q_2, \\ \pi_m(\theta, k, q_1, q_2, ..., q_{n+2}) &= [P(Q) - c]q_m, \end{aligned} \tag{1}$$

where $m = 3, ..., n + 2$. Throughout the analysis in this section, we make the following assumption.

Assumption 1: $x < min\{\frac{a-c}{2}, c\} \equiv \bar{x}$.

This is the necessary and sufficient condition for each firm i to produce a strictly positive amount of the product in the equilibrium of the Stage 3 subgame for all (θ, k, n) where $\theta \in [0, \frac{1}{2}]$ (see Claim 1 in the Appendix). In what follows, we focus on the case of $c > \frac{a-c}{2} \Leftrightarrow c > \frac{a}{3}$ (so that $\bar{x} = \frac{a-c}{2}$), to simplify description of the results and reduce the number of cases to be considered. See footnote 13 for the equilibrium characterization results when $c < \frac{a}{3}$.

Consider equilibria of Stage 3 subgames. Let $q_i^*(\theta, k, n)$ and $\pi_i^*(\theta, k, n)$ respectively denote firm i's quantity and profit in the equilibrium of the Stage 3 subgame represented by (θ, k). Suppose $\theta \in [0, \frac{1}{2}]$. Then, at Stage 3, each firm i simultaneously and non-cooperatively chooses q_i to maximize $\pi_i(\theta, k, q_1, q_2, ..., q_{n+2})$. Through a standard analysis of Cournot competition, we find that the equilibrium is unique for any given (θ, k, n), where the equilibrium quantities are given by

$$\begin{aligned} q_1^*(\theta,k,n) &= \frac{(1-\theta)(a-c)+[n+2-(1+(n+1)\theta)k]x}{d(n+3-\theta)}, \\ q_2^*(\theta,k,n) &= \frac{a-c-(1-(n+2)k)x}{d(n+3-\theta)}, \\ q_m^*(\theta,k,n) &= \frac{a-c-(1+(1-\theta)k)x}{d(n+3-\theta)}, \end{aligned} \tag{2}$$

where $m = 3, ..., n+2$, and each firm i's equilibrium profit is given by

$$\pi_i^*(\theta,k,n) = \pi_i(\theta,k,q_1^*(\theta,k,n),q_2^*(\theta,k,n),...,q_{n+2}^*(\theta,k,n)). \tag{3}$$

Next consider Stage 3 subgames with $\theta \in (\frac{1}{2}, 1]$. In this class of Stage 3 subgames, firm 1 chooses q_1 and q_2 to maximize $\pi_1(\theta, k, q_1, q_2, ..., q_{n+2})$. Note that firm 2 cannot be more cost effective than firm 1. Then, since firms 1 and 2 produce a homogeneous product, firm 1 cannot be strictly better off by producing a strictly positive quantity at firm 2. To be more specific, suppose firm 1 chooses $q_1 = \alpha > 0$ and $q_2 = \beta > 0$. Firm 1 would be strictly better off by choosing $q_1 = \alpha + \beta$ and $q_2 = 0$ if $\theta \in (\frac{1}{2}, 1)$, and, if $\theta = 1$, firm 1 is indifferent across any (q_1, q_2) as long as $q_1 + q_2 = \alpha + \beta$. Given this, for expositional simplicity we make a tie-breaking assumption that, if firm 1 is indifferent between shutting down and not shutting down firm 2, firm 1 chooses to shut down firm 2. We then find that the equilibrium of any Stage 3 subgame with $\theta \in (\frac{1}{2}, 1]$ is unique, where the equilibrium quantities are given by $q_i^*(\theta, k, n) = q_i^*(0, 0, n-1)$ for $i = 1, 3, ..., n+2$ and $q_2^*(\theta, k, n) = 0$, and each firm i's equilibrium profit is given by (3).

"PEO paradox"

Let $\pi_{12}^*(\theta, k, n) \equiv \pi_1^*(\theta, k, n) + \pi_2^*(\theta, k, n)$ denote the joint profit of firms 1 and 2 in the equilibrium of the Stage 3 subgame. Lemma 1 tells us that $\pi_{12}^*(\theta, k, n)$ is decreasing in θ for all $\theta \in [0, \frac{1}{2}]$. That is, holding everything else constant, PEO decreases the joint profitability of firms 1 and 2. This result can be viewed as a PEO version of the merger paradox (a well-known result in the industrial organization literature) as discussed below. We find that the qualitative nature of the result remains mostly unchanged under the general demand function (see Lemma G1 in Section 5).

Lemma 1: For any given k $(\in \{0, 1\})$, $\pi_{12}^*(\theta, k, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$, and constant for all $\theta \in (\frac{1}{2}, 1]$. Furthermore, for any given $\theta' \in (\frac{1}{2}, 1]$, (i) $\pi_{12}^*(\frac{1}{2}, 1, n) >$

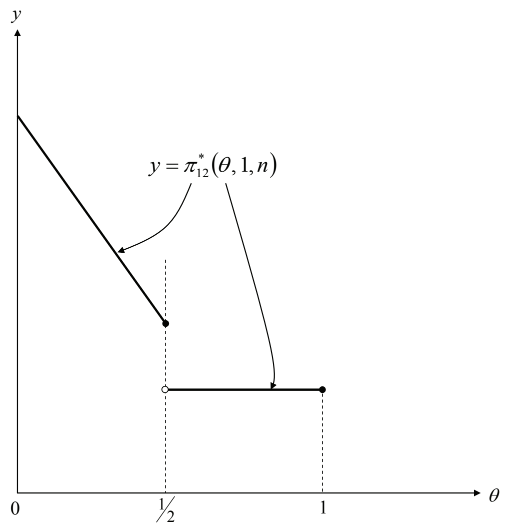


Figure 1-1

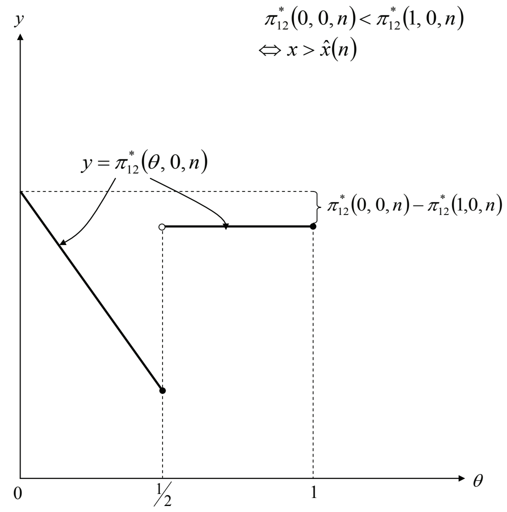


Figure 1-2

$\pi^*_{12}(\theta', 1, n)$ holds for all $x \in (0, \bar{x})$, and (ii) $\pi^*_{12}(0, 0, n) > (=, <)\ \pi^*_{12}(\theta', 0, n)$ holds if $x < (=, >)\ \hat{x}(n) \equiv \frac{n^2+2n-1}{3n^2+2n+11}(a-c)$, where $\hat{x}(n) < \bar{x}$ holds for all $n \geq 1$.

First consider $\pi^*_{12}(\theta, 1, n)$, where $k = 1$ so that $c_1 = c_2 = c - x$. PEO reduces the degree of competition between firms 1 and 2 and eliminates competition when $\theta \in (\frac{1}{2}, 1]$. This effect works in the direction of increasing the joint profit of firms 1 and 2 as θ increases. At the same time, weaker competition between firms 1 and 2 induces other n firms to use a more aggressive strategy, and this effect works in the direction of decreasing the joint profit of firms 1 and 2 as θ increases. Lemma 1 tells us that the latter effect dominates the former effect for all $\theta \in (0, 1]$, so that $\pi^*_{12}(\theta, 1, n)$ is decreasing in θ as depicted in Figure 1-1.

A similar trade-off was identified in the theoretical analyses of mergers. Salant, Switzer, and Reynolds (1983) analyzed a symmetric Cournot oligopoly model with linear demand and constant marginal costs and found that mergers are not typically profitable for the merging parties unless they include the vast majority of industry participants ("merger paradox").[11] In their model, mergers between two firms out of N (≥ 3) firms are never profitable. Although the merger eliminates competition between two merging firms, it induces outsiders to take more aggressive actions. The latter effect dominates the former so that mergers between two firms reduce their equilibrium joint profits. Lemma 1 tells us that this basic trade-off extends to firms connected by PEOs.

Next consider $\pi^*_{12}(\theta, 0, n)$, where $k = 0$ so that $c_1 = c - x < c_2 = c$. In addition to

[11]See Perry and Porter (1985) for a related analysis.

the two effects mentioned above, there is another effect at work due to cost asymmetry. That is, an increase in θ within the interval $[0, \frac{1}{2}]$ shifts outputs from cost-efficient firm 1 to cost-inefficient firm 2. This effect works in the direction of reducing the joint profit, and, combined with the two effects mentioned above, yields that $\pi^*_{12}(\theta, 0, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$ as depicted in Figure 1-2. When $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down the operation of firm 2. All output is then shifted to the cost efficient firm (firm 1), resulting in a discontinuous increase of the joint profit $\pi^*_{12}(\theta, 0, n)$ when θ increases from $\frac{1}{2}$ to $\theta \in (\frac{1}{2}, 1]$. The improvement of cost efficiency due to firm 2's shut down becomes more substantial as x increases. This implies that the distance between $\pi^*_{12}(0, 0, n)$ and $\pi^*_{12}(1, 0, n)$ (note that $\pi^*_{12}(\theta, 0, n)$ is constant for all $\theta \in (\frac{1}{2}, 1]$) increases as x increases (see Figure 1-2), and we find that $\pi^*_{12}(1, 0, n)$ exceeds $\pi^*_{12}(0, 0, n)$ when x is large enough to satisfy $x > \hat{x}(n)$.

Stage 2 subgames: The minimum PEO for knowledge transfer

We next consider firm 1's optimal decision at Stage 2 in Stage 2 subgames. Given θ, firm 1 decides whether or not to transfer its knowledge to firm 2 (that is, $k = 1$ or 0) in order to maximize its profit in the subsequent Stage 3 subgame $\pi^*_1(\theta, k, n)$. If $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down firm 2 in the equilibrium of the subsequent Stage 3 subgame as mentioned above, and hence firm 1 is indifferent between transferring its knowledge and not transferring it.

Now suppose $\theta \in [0, \frac{1}{2}]$. Firm 1 transfers its knowledge to firm 2 if and only if $\pi^*_1(\theta, 1, n) - \pi^*_1(\theta, 0, n) \geq 0$. When does this condition hold? Proposition 1 provides a precise characterization.

Proposition 1 [Knowledge transfer]: Suppose $\theta \in [0, \frac{1}{2}]$. Consider firm 1's decision at Stage 2 in the equilibrium of the Stage 2 subgame. There exists a threshold $x_{max}(n) \equiv \frac{2n+1}{6n+10}(a - c) < \bar{x}$, which is increasing in n, with the following properties.
(i) Suppose $0 < x \leq x_{max}(n)$. Then there exists a unique value $\hat{\theta}(x, n) \in (0, \frac{1}{2}]$ such that firm 1 transfers knowledge to firm 2 if and only if $\theta \geq \hat{\theta}(x, n)$, where $\hat{\theta}(x, n)$ is strictly increasing in x and strictly decreasing in n for all $x \in (0, x_{max}(n))$ and $n \geq 1$.
(ii) Suppose $x_{max}(n) < x < \bar{x}$. Then firm 1 does not transfer knowledge to firm 2 for any $\theta \in [0, \frac{1}{2}]$.

Firm 1 has a cost advantage over firm 2. By transferring its knowledge, firm 1 loses its competitive advantage over firm 2, but a fraction θ of firm 2's incremental profit due to knowledge transfer belongs to firm 1. Proposition 1 tells us that the latter effect dominates the former effect when θ is sufficiently high so that firm 1 chooses to transfer its knowledge

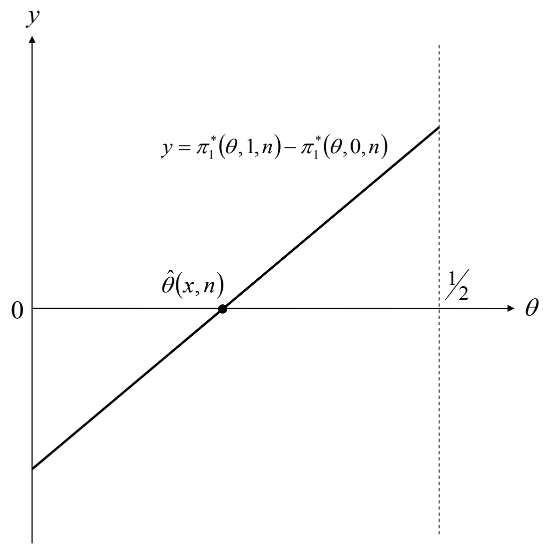


Figure 2

if $\theta \geq \hat{\theta}(x,n)$. See Figure 2 for a diagrammatic representation of Proposition 1. In what follows, we call $\hat{\theta}(x,n)$ the *minimum PEO for knowledge transfer.* In Section 5, we show the existence of the minimum PEO for knowledge transfer under the general demand function (see Proposition G1).

As x increases, firm 1's loss of its competitive advantage through knowledge transfer becomes more substantial. Hence, in order to induce firm 1 to transfer its knowledge, a higher level of PEO is necessary to compensate firm 1 for the loss of its competitive advantage, implying that $\hat{\theta}(x,n)$ is increasing in x. And, once x exceeds $x_{max}(n)$, firm 1 does not transfer its knowledge even when $\theta = \frac{1}{2}$. Also, as n increases, firm 1's loss of its competitive advantage by transferring its knowledge to firm 2 becomes less substantial, since firm 2 is one of the $n+1$ other firms for firm 1. This means that a lower level of PEO is sufficient to compensate firm 1 for the loss of its competitive advantage, implying that $\hat{\theta}(x,n)$ is decreasing in n. This also means that, for any given $\theta \in [0, \frac{1}{2}]$, an increase in n increases the maximum amount of competitive advantage firm 1 is willing to lose, implying that $x_{max}(n)$ $(\equiv \frac{2n+1}{6n+10}(a-c))$ is increasing in n.

Equilibrium characterization

We are now ready to derive the equilibrium of the entire game. At Stage 1, firms 1 and 2 jointly determine the level of θ to maximize their joint profit in the subsequent equilibrium.[12]

[12]Let η^* denote the amount of money transferred from firm 1 to firm 2 in the equilibrium, and let θ^*

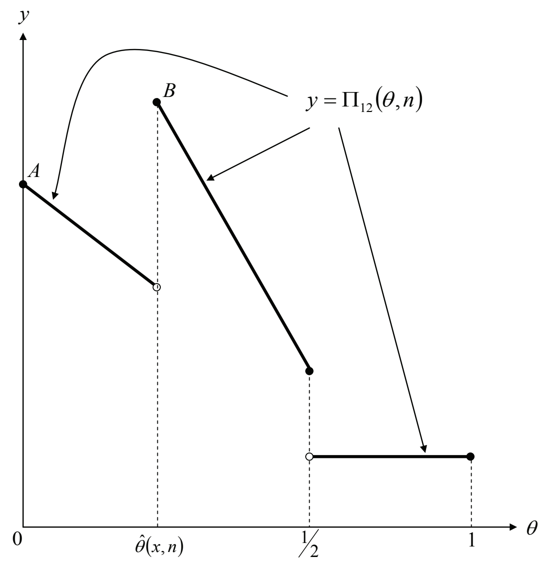


Figure 3

First, suppose $x \leq x_{max}(n)$. Proposition 1 tells us that knowledge transfer does not occur if $\theta \in [0, \hat{\theta}(x,n))$ and does occur if $\theta \in [\hat{\theta}(x,n), \frac{1}{2}]$. If $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down firm 2, and so $\pi^*_{12}(\theta, k, n)$ is constant for all $\theta \in (\frac{1}{2}, 1]$ and $k \in \{0, 1\}$. That is, the level of PEO and whether or not knowledge is transferred do not matter when $\theta \in (\frac{1}{2}, 1]$. Let $\Pi_{12}(\theta, n)$ denote the joint profit of firms 1 and 2 in the equilibrium of Stage 2 subgames. We then find that $\Pi_{12}(\theta, n) = \pi^*_{12}(\theta, 0, n)$ for all $\theta \in [0, \hat{\theta}(x,n))$ and $\Pi_{12}(\theta, n) = \pi^*_{12}(\theta, 1, n)$ for all $\theta \in [\hat{\theta}(x,n)), 1]$. See Figure 3 for a diagrammatic representation of $\Pi_{12}(\theta, n)$ as a function of θ (notice that Figure 3 is a combination of Figures 1-1 and 1-2).

In order to induce knowledge transfer, firms 1 and 2 must choose at least $\theta = \hat{\theta}(x,n)$, the minimum PEO for knowledge transfer. Since $\Pi_{12}(\theta, n)$ is decreasing in θ for all $\theta \geq \hat{\theta}(x,n)$, their optimal choice is $\theta = \hat{\theta}(x,n)$ for the knowledge-transfer option. At the same time, they have an option of not inducing knowledge transfer. Under the no-knowledge-transfer option, firms 1 and 2 choose $\theta = 0$ since $\Pi_{12}(\theta, n)$ is decreasing in θ for all $\theta \in [0, \hat{\theta}(x,n))$. Comparison between these two relevant options $\theta = 0$ or $\hat{\theta}(x,n)$ (points A or B in Figure

and k^* respectively denote equilibrium values of θ and k. If the monetary transfer is determined by the generalized Nash bargaining process in which firm 1's bargaining power is given by $\beta \in (0, 1)$, then η^* is determined by $\pi^*_1(\theta^*, k^*, n) - \eta^* = \pi^*_1(0, 0, n) + \beta[\pi^*_{12}(\theta^*, k^*, n) - \pi^*_{12}(0, 0, n)]$. It can be shown that the value of η^* is negative under a certain range of parameterizations. The case of negative η^* can be interpreted as the case in which a part of firm 2's equity is transferred to firm 1 at a price below the on-going market price.

3) leads us to Proposition 2 (i) and (ii). In Section 5, we show, under the general demand function, that relevant options for firms 1 and 2 are $\theta = 0$ or $\hat{\theta}(x, n)$ when x is relatively small, and we identify a sufficient condition for firms 1 and 2 to choose $\theta = \hat{\theta}(x, n)$ in the equilibrium.

Next suppose $x > x_{max}(n)$. Proposition 1 tells us that knowledge transfer does not occur for any given $\theta \in [0, \frac{1}{2}]$. This implies that $\Pi_{12}(\theta, n) = \pi^*_{12}(\theta, 0, n)$ for all $\theta \in [0, 1]$. Then, given that $x_{max}(n) > \hat{x}(n)$ holds for all $n \geq 1$, Lemma 1 tells us (see also Figure 1-2) that firms 1 and 2 choose $\theta = 1$ (merger) for any $x > x_{max}(n)$, yielding Proposition 2 (iii).

Proposition 2 [Equilibrium characterization]: For any given $x \in (0, \bar{x})$ and $n \geq 1$, there exists a unique value $\theta^*(x, n)$ and a threshold $x_{min}(n) \in (0, x_{max}(n))$ such that actions taken by firms 1 and 2 in the unique equilibrium of the game are described by (i) - (iii) below.
(i) Suppose $0 < x \leq x_{min}(n)$. Then firms 1 and 2 choose $\theta = \theta^*(x, n) \equiv 0$ at Stage 1 and firm 1 does not transfer its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_{min}(n) < x \leq x_{max}(n)$. Then firms 1 and 2 choose $\theta = \theta^*(x, n) \equiv \hat{\theta}(x, n)$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2, where $\theta^*(x, n)$ is strictly positive and strictly increasing in x for all $x \in (x_{min}(n), x_{max}(n)]$ with $\theta^*(x_{max}(n), n) = \frac{1}{2}$.
(iii) Suppose $x_{max}(n) < x < \bar{x}$. Then firms 1 and 2 choose $\theta = \theta^*(x, n) \equiv 1$ (merger) at Stage 1.[13]

The logic behind Proposition 2 can be explained as follows. Suppose firms 1 and 2 choose $\theta = 0$ ($< \hat{\theta}(x, n)$) at Stage 1. Then, firm 1's knowledge is not transferred to firm 2 (that is, $k = 0$) at Stage 2, and their joint profit is $\pi^*_{12}(0, 0, n)$ in the subsequent equilibrium. In order to induce knowledge transfer, the level of PEO should be increased from 0 to $\hat{\theta}(x, n)$. Firms 1 and 2 prefer $\theta = \hat{\theta}(x, n)$ to $\theta = 0$ if $\pi^*_{12}(\hat{\theta}(x, n), 1, n) - \pi^*_{12}(0, 0, n) \geq 0$, where $\pi^*_{12}(\hat{\theta}(x, n), 1, n) - \pi^*_{12}(0, 0, n)$ can be decomposed as follows:

$$\pi^*_{12}(\hat{\theta}(x, n), 1, n) - \pi^*_{12}(0, 0, n) = \underbrace{[\pi^*_{12}(\hat{\theta}(x, n), 1, n) - \pi^*_{12}(\hat{\theta}(x, n), 0, n)]}_{\text{knowledge transfer effect}} + \underbrace{[\pi^*_{12}(\hat{\theta}(x, n), 0, n) - \pi^*_{12}(0, 0, n)]}_{\text{PEO effect}}.$$

The knowledge transfer effect is positive. That is, holding the level of PEO fixed at $\theta = \hat{\theta}(x, n)$, knowledge transfer increases the joint profit of firms 1 and 2. The PEO effect can

[13] Suppose $c < \frac{a}{3}$ so that $\bar{x} = c$. If $x_{max}(n) < c$, Proposition 2 would remain unchanged. If $x_{min}(n) < c \leq x_{max}(n)$, Proposition 2 consists of (i) and (ii) only, where $x_{max}(n)$ in (ii) is replaced by $\bar{x}$. If $c \leq x_{min}(n)$, Proposition 2 would be (i) only, where $x_{min}(n)$ in (i) is replaced by $\bar{x}$.

be regarded as the cost that firms 1 and 2 jointly incur in order to induce knowledge transfer, since an increase of θ from 0 to $\hat{\theta}(x,n)$ holding $k=0$ fixed decreases their joint profit. That is, the PEO effect is negative. We find that $\lim_{x\to 0}\hat{\theta}(x,n)>0$. This implies that the PEO effect does not approach zero as x approaches zero, where $\lim_{x\to 0}[\pi_{12}^*(\hat{\theta}(x,n),0,n)-\pi_{12}^*(0,0,n)]<0$. In contrast, the knowledge transfer effect decreases as x decreases, and approaches zero as x approaches zero. Hence, when x is small enough, we have $\pi_{12}^*(\hat{\theta}(x,n),1,n)-\pi_{12}^*(0,0,n)<0$ and so firms 1 and 2 prefer $\theta=0$ to $\theta=\hat{\theta}(x,n)$.

An increase in x increases the knowledge transfer effect, and it also increases the magnitude of the PEO effect because $\hat{\theta}(x,n)$ is increasing in x. We find that the knowledge transfer effect dominates the PEO effect if x is large enough to satisfy $x\in(x_{min}(n),x_{max}(n)]$, where firms 1 and 2 choose $\theta=\hat{\theta}(x,n)$ over $\theta=0$. Once x exceeds $x_{max}(n)$, no PEO $\theta\in[0,\frac{1}{2}]$ can induce knowledge transfer, leaving $\theta=0$ or 1 as relevant options for firms 1 and 2. They choose $\theta=1$ in this case.

3.3 Welfare consequences and antitrust implications

Let $CS(\theta,n)$ and $TS(\theta,n)$ respectively denote consumer surplus and total surplus in the equilibrium of the Stage 2 subgame represented by θ. We first compare $CS(\theta^*(x,n),n)$ and $CS(0,n)$. That is, we compare consumer surplus at the endogenously determined level of PEO $\theta=\theta^*(x,n)$ with consumer surplus at $\theta=0$. We then compare $TS(\theta^*(x,n),n)$ and $TS(0,n)$. We have that $CS(\theta^*(x,n),n)=CS(0,n)$ and $TS(\theta^*(x,n),n)=TS(0,n)$ for all $x\in(0,x_{min}(n)]$, because $\theta^*(x,n)=0$ in the equilibrium for all $x\in(0,x_{min}(n)]$. Given this, we focus on the case of $x\in(x_{min}(n),\bar{x})$ in what follows.

Proposition 3 [Consumer surplus]:
(A) For $n=1$, $CS(\theta^*(x,1),1)<CS(0,1)$ holds for all $x\in(x_{min}(1),\bar{x})$. That is, the endogenously determined level of PEO decreases consumer surplus for all relevant x when $n=1$.

(B) For any given $n\geq 2$, there exists a threshold value $x_{CS}(n)\in[x_{min}(n),x_{max}(n))$ with the properties (i) - (iii) below. Furthermore, $x_{CS}(n)$ is strictly decreasing in n for all $n\geq 2$ whenever $x_{CS}(n)>x_{min}(n)$.
(i) Suppose $x_{min}(n)<x\leq x_{CS}(n)$. Then $CS(\theta^*(x,n),n)\leq CS(0,n)$ holds, where equality holds if and only if $x=x_{CS}(n)$.
(ii) Suppose $x_{CS}(n)<x\leq x_{max}(n)$. Then $CS(\theta^*(x,n),n)>CS(0,n)$ holds.
(iii) Suppose $x_{max}(n)<x<\bar{x}$. Then $CS(\theta^*(x,n),n)<CS(0,n)$ holds, where $\theta^*(x,n)=1$.

When $x \in (x_{min}(n), x_{max}(n)]$, firms 1 and 2 choose $\theta = \theta^*(x, n) = \hat{\theta}(x, n)$ to induce knowledge transfer, which reduces firm 2's cost. The cost reduction works in the direction of increasing the industry output, but the chosen PEO $\theta = \hat{\theta}(x, n)$ weakens competition in the industry, working in the direction of reducing the industry output. Hence the net impact of the PEO $\hat{\theta}(x, n)$ on consumers is not immediately obvious. When $n = 1$, the latter negative effect dominates the former positive effect, so that the endogenously determined level of PEO decreases consumer surplus for all relevant x (Proposition 3 (A)). When $n \geq 2$, the negative effect still dominates the positive effect when x is relatively small, but, as x increases, the positive effect associated with the cost reduction becomes increasingly more significant than the negative effect associated with the PEO, and the net effect on consumers becomes positive once x exceeds a threshold $x_{CS}(n)$. Consequently, the endogenously determined level of PEO increases consumer surplus when $x \in (x_{CS}(n), x_{max}(n)]$.

Proposition 3 (B) (i) and (ii) tell us that PEO increases consumer surplus when the level of PEO is relatively high. This rather counterintuitive result arises in our model because for all $x \in (x_{min}(n), x_{max}(n)]$, an increase in the equilibrium level of PEO $\theta^*(x, n)$ $(= \hat{\theta}(x, n))$ is driven by an increase in the amount of knowledge to be transferred, x.

When x exceeds $x_{max}(n)$, two firms merge. Proposition 3 (B) tells us that the merger reduces consumer surplus. It is straightforward to show that, if $\theta = 0$ or $\theta = 1$ were the only options (merger/non-merger dichotomy), firms 1 and 2 would choose $\theta = 1$ when x is large enough, and the merger necessarily reduces consumer surplus. The option for firms 1 and 2 to connect themselves by PEO can not only increase their profits but also benefit consumers since PEO can serve as more efficient vehicle for knowledge transfer than the merger.

Proposition 3 (B) also tells us that PEO is more likely to benefit consumers as n increases in the sense that the interval $(x_{CS}(n), x_{max}(n)]$ gets larger as n increases (recall that $x_{max}(n)$ is increasing in n from Proposition 1). To understand the logic, recall that $\hat{\theta}(x, n)$ is decreasing in n (Proposition 1). This implies that, holding x fixed, knowledge transfer is induced at a lower level of PEO in the equilibrium as n increases, indicating that consumers are more likely to benefit from PEO as n increases.

Proposition 4 [Total surplus]:
(A) When $n = 1$, there exist values $x'_{TS} \approx 0.0260(a - c)$ and $x''_{TS} \approx 0.226(a - c)$, $x_{min}(1) < x'_{TS} < x_{max}(1) < x''_{TS} < \bar{x}$, with the following properties:
(i) Suppose $x_{min}(1) < x \leq x'_{TS}$. Then $TS(\theta^*(x, 1), 1) \leq TS(0, 1)$ holds, where equality holds if and only if $x = x'_{TS}$.
(ii) Suppose $x'_{TS} < x \leq x_{max}(1)$. Then $TS(\theta^*(x, 1), 1) > TS(0, 1)$ holds.

(iii) Suppose $x_{max}(1) < x \leq x''_{TS}$. Then $TS(\theta^*(x,1),1) \leq TS(0,1)$ holds where $\theta^*(x,1) = 1$, and equality holds if and only if $x = x''_{TS}$.
(iv) Suppose $x''_{TS} < x < \bar{x}$. Then $TS(\theta^*(x,1),1) > TS(0,1)$ holds where $\theta^*(x,1) = 1$.

(B) For any given $n \geq 2$, there exists a value $x_{TS}(n) \in [x_{min}(n), x_{CS}(n)]$ such that $TS(\theta^*(x,n),n) > TS(0,n)$ holds for all $x \in (x_{TS}(n), \bar{x})$, where $x_{TS}(n) < x_{CS}(n)$ whenever $x_{min}(n) < x_{CS}(n)$.

Whenever PEO increases consumer surplus, it also increases total surplus. To see this, pick any $x \in (x_{CS}(n), x_{min}(n)]$ so that the endogenously determined level of PEO $\theta^*(x,n) = \hat{\theta}(x,n)$ increases consumer surplus. This means that the industry output is higher under $\theta = \theta^*(x,n)$ than under $\theta = 0$. Also, when $\theta = \theta^*(x,n)$, knowledge is transferred from firm 1 to firm 2 and hence a larger fraction of industry output is produced at the lower cost $c - x$ in the equilibrium. This implies that total surplus is also larger under $\theta = \theta^*(x,n)$ than under $\theta = 0$. Propositions 3 and 4 tell us that PEO increases total surplus under a broader range of parameterizations than the range of parameterizations in which PEO increases consumer surplus. In fact, we found that for any $n = 2, 3, ..., 100$, $x_{TS}(n) = x_{min}(n)$ holds, implying that PEO increases total surplus for all $x \in (x_{min}(n), x_{max}(n)]$. We conjecture that this is true for all $n \geq 2$, but we were unable to prove it.

When x exceeds $x_{max}(n)$, two firms merge. The merger reduces the industry output and hence reduces consumer surplus (Proposition 3 (B) (iii)), but a larger fraction of industry output is produced in the cost efficient firm 1 under merger. Proposition 4 (B) tells us that the latter positive effect dominates the former negative effect so that the merger increases total surplus for all $x \in (x_{max}(n), \bar{x})$ and $n \geq 2$.

Remark: Propositions 3 and 4 tell us that the endogenously determined level of PEO can increase not only total surplus but also consumer surplus. This result is important because it has often been argued that antitrust authorities maximize consumer surplus rather than total surplus. For example, Whinston (2007) pointed out, "Although many analyses of antitrust policy in the economics literature focus on an aggregate surplus standard, enforcement practice in most countries (including the U.S. and the E.U.) is closest to a consumer surplus standard" (p. 2374). Our finding suggests that an antitrust authority may allow PEO even when its objective is to maximize consumer surplus.

Can consumer surplus be improved by PEO when that PEO does not induce knowledge transfer? To provide an answer to this question, suppose that firm 1 cannot transfer its knowledge to firm 2, and that firms 1 and 2 can connect themselves through any levels of partial cross ownership represented by θ (defined as in our original model) and ψ ($\in$

$[0,1]$), where ψ denotes the level of firm 2's ownership in firm 1's equity. We find that such PEO arrangements can never raise aggregate output or consumer surplus, which in turn indicates the importance of knowledge transfer for PEO to improve consumer surplus.[14] Farrell and Shapiro (1990b) showed in their analyses of horizontal mergers, that mergers can increase consumer surplus only if they create synergies. Our analysis suggests that Farrell and Shapiro's finding is robust for PEO because the endogenously determined level of PEO can increase consumer surplus only if the PEO induces knowledge transfer.

Antitrust implications

We now turn to the antitrust implications of PEO arrangements. Propositions 3 and 4 suggest that antitrust authorities might allow, rather than prohibit, PEO arrangements between competitors to benefit consumers and/or maximize welfare. To make this idea precise, let us consider an antitrust authority whose objective is to maximize total surplus under the following simple extension of the model, in which everything is the same as in the original model except for the following. At Stage 0, the antitrust authority can announce a maximum permissible level of PEO, denoted $\tilde{\theta} \in [0,1]$. Then, firms 1 and 2 jointly choose θ at Stage 1, where $\theta \in [0, \tilde{\theta}]$ must be satisfied. Assume that the antitrust authority announces $\tilde{\theta}$ only if the authority can strictly increase the equilibrium total surplus by doing so.

Let $ts^*(\theta, k, n)$ denote total surplus in the equilibrium of the Stage 3 subgame. We find that, for any given $n \geq 2$, $ts^*(\theta, 1, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$, and $ts^*(\frac{1}{2}, 1, n) > ts^*(\theta, 1, n)$ holds for all $\theta \in (\frac{1}{2}, 1]$. That is, given $c_2 = c - x$ (since $k = 1$), an increase in PEO decreases equilibrium total surplus by weakening competition in the industry. Then, from the antitrust's standpoint, knowledge should be transferred at $\theta = \hat{\theta}(x, n)$, the minimum PEO for knowledge transfer. Under the linear homogeneous demand, the antitrust's preference matches with the choice made by firms 1 and 2, since they choose $\theta = \hat{\theta}(x, n)$ whenever they intend to induce knowledge transfer (this result is

[14] Consider a standard Cournot oligopoly model with $n+2$ firms. The demand and cost structure are the same as described in subsection 3.1. Suppose $\theta \leq \frac{1}{2}$ and $\psi \leq \frac{1}{2}$. Then firm 1 chooses q_1 to maximize $(1-\psi)[P(Q)-(c-x)]q_1 + \theta[P(Q)-c]q_2$ while firm 2 chooses q_2 to maximize $\psi[P(Q)-(c-x)]q_1 + (1-\theta)[P(Q)-c]q_2$. Profit maximization problems of firms $3, ..., n+2$ remain the same as before. Let $Q(\theta, \psi)$ denote the aggregate output in the unique Cournot equilibrium. We find that $Q(\theta, \psi) = \frac{(n+2-\psi-\theta)(a-c)+(1-\psi)x}{d(n+3-\psi-\theta)}$ attains its maximum at $(\theta, \psi) = (0, 0)$ implying that PEO lowers aggregate output and consumer surplus in the absence of knowledge transfer. If either $\theta > \frac{1}{2}$ or $\psi > \frac{1}{2}$ then the inefficient firm, i.e. firm 2, is shut down and the aggregate equilibrium output is same as the aggregate equilibrium output under merger between firm 1 and firm 2. From Farrell and Shapiro (1990b), we know that such mergers that reshuffle production but do not create synergies cannot raise consumer surplus.

robust under more general inverse demand functions; see Section 5). Hence the antitrust authority's relevant option is either to impose no restrictions on PEO or to prohibit PEO. The authority imposes no restrictions on PEO if $TS(\theta^*(x,n),n) \geq TS(0,n)$ and prohibits PEO otherwise (see Proposition 4).

The qualitative nature of the result remains unchanged when the antitrust authority's objective is to maximize consumer surplus. We find that the antitrust authority's relevant option is either to impose no restrictions on PEO or to prohibit PEO, and that the authority imposes no restrictions on PEO if $CS(\theta^*(x,n),n) \geq CS(0,n)$ and prohibits PEO otherwise (see Proposition 3).

PEO vs. the merger/non-merger dichotomy

Merger and non-merger (independence) are both nested in our model in the sense that firms 1 and 2 can choose independence ($\theta = 0$), PEO, or merger ($\theta = 1$) at Stage 1. Models without PEO have been previously explored in the literature on merger. By introducing PEO as an option, welfare and antitrust implications based on the merger/non-merger dichotomy change in significant ways. To see this, let us consider what would happen in our framework if merger and independence were the only options for firms 1 and 2 at Stage 1. In what follows, this model is referred to as the merger/non-merger dichotomy model, and our original model is referred to as the PEO model to make the distinction clear. In the dichotomy model, we find that there exists a threshold value $\hat{x}(n) \equiv \frac{n^2+2n-1}{3n^2+12n+11}(a-c) \in (0,\bar{x})$ such that firms 1 and 2 choose to merge if $x > \hat{x}(n)$ and remain independent if $x < \hat{x}(n)$. We also find that there exists a threshold value $\tilde{x}_{TS}(n) \equiv \frac{2n+5}{2n^2+12n+17}(a-c) \in (0,\bar{x})$ such that equilibrium total surplus is higher under merger than under non-merger if and only if $x > \tilde{x}_{TS}(n)$. We find that $\hat{x}(n) < \tilde{x}_{TS}(n)$ if $n = 1$ or 2, and $\tilde{x}_{TS}(n) < \hat{x}(n)$ if $n \geq 3$.

When PEO is introduced as an option, firms 1 and 2 choose $\theta = \hat{\theta}(x,n) \in (0, \frac{1}{2}]$ for all $x \in (x_{min}(n), x_{max}(n)]$ as shown in Proposition 2, whereas they choose either $\theta = 0$ or $\theta = 1$ in the dichotomy model. The choice of PEO $\theta = \hat{\theta}(x,n)$ increases not only the alliance partners' joint profit but also the total surplus, except for the case of $n = 1$ and $x \in (x_{min}(1), x'_{TS})$. In other words, the introduction of PEO as an option would benefit alliance partners as well as society.

In order to compare policy implications of the dichotomy model and those of the PEO model, we compare equilibrium outcomes of the two models with and without the antitrust authority, where the authority maximizes equilibrium total surplus by announcing a maximum permissible level of PEO at Stage 0. Table 1 presents the comparison of equilibrium outcomes for $x \in (x_{min}(n), x_{max}(n)]$. Recall that in the PEO model, firms 1 and 2 choose

- $n = 1$

Range of x		PEO model	Dichotomy model
$(\hat{x}(1), x_{max}(1))$	No Antitrust	PEO	Merger
	Antitrust	PEO	Non-merger
$(x'_{TS}, \hat{x}(1))$	No Antitrust	PEO	Non-merger
	Antitrust	PEO	Non-merger
$(x_{min}(1), x'_{TS})$	No Antitrust	PEO	Non-merger
	Antitrust	Non-PEO	Non-merger

- $n = 2$

Range of x		PEO model	Dichotomy model
$(\tilde{x}_{TS}(2), x_{max}(2))$	No Antitrust	PEO	Merger
	Antitrust	PEO	Merger
$(\hat{x}(2), \tilde{x}_{TS}(2))$	No Antitrust	PEO	Merger
	Antitrust	PEO	Non-merger
$(x_{min}(2), \hat{x}(2))$	No Antitrust	PEO	Non-merger
	Antitrust	PEO	Non-merger

- $n = 3$

Range of x		PEO model	Dichotomy model
$(\hat{x}(3), x_{max}(3))$	No Antitrust	PEO	Merger
	Antitrust	PEO	Merger
$(\tilde{x}_{TS}(3), \hat{x}(3))$	No Antitrust	PEO	Non-merger
	Antitrust	PEO	Merger
$(x_{min}(3), \tilde{x}_{TS}(3))$	No Antitrust	PEO	Non-merger
	Antitrust	PEO	Non-merger

Comparison of equilibrium outcomes: PEO model and dichotomy model

: PEO reconciles conflict of interests between alliance partners and the antitrust authority

: PEO reveals conflict of interests between alliance partners and the antitrust authority

Table 1

$\theta = 0$ if $\theta \in (0, x_{min}(n)]$ and $\theta = 1$ if $\theta \in (x_{max}(n), \bar{x})$, and hence equilibrium outcomes in the PEO model are the same as those in the dichotomy model if x is not in the range $(x_{min}(n), x_{max}(n)]$.

We find that PEO can reconcile conflict of interests between alliance partners and the antitrust authority that may arise under the merger/non-merger dichotomy. Suppose $n = 1$ or 2 so that $\hat{x}(n) < \tilde{x}_{TS}(n)$, and suppose $x \in (\hat{x}(n), \tilde{x}_{TS}(n))$. Then firms 1 and 2 would choose to merge under the dichotomy model in the absence of the antitrust authority. In the presence of the authority, however, the authority would prohibit the merger because the merger reduces total surplus. This conflict of interests between the firms and the authority can be resolved by introducing PEO as an option. Consider the $n = 2$ case. We find that $x_{min}(2) < \hat{x}(2) < \tilde{x}_{TS}(2) < x_{max}(2)$ holds, implying that the following property holds for all $x \in (\hat{x}(2), \tilde{x}_{TS}(2))$: Firms 1 and 2 would choose $\theta = \hat{\theta}(x, 2)$ in the absence of the authority, and the authority would impose no restrictions on PEO. Hence there is no conflict between the firms and the authority, and the equilibrium joint profit of firms 1 and 2 and the equilibrium total surplus are both higher in the PEO model than in the dichotomy model. When $n = 1$, analogous property holds when $x \in (\hat{x}(1), x_{max}(1))$, where we find that $x_{min}(1) < \hat{x}(1) < x_{max}(1) < \tilde{x}_{TS}(1)$ holds.

Now suppose $n \geq 3$ so that $\tilde{x}_{TS}(n) < \hat{x}(n)$, and suppose $x \in (\tilde{x}_{TS}(n), \hat{x}(n))$. Then firms 1 and 2 would choose to remain independent under the dichotomy model in the absence of the authority. The authority, however, would prefer they merge because the merger increases total surplus. This conflict of interests can be resolved by introducing PEO as an option. Consider the $n = 3$ case. We find that $x_{min}(3) < \tilde{x}_{TS}(3) < \hat{x}(3) < x_{max}(3)$ holds. This means that, in the PEO model, firms 1 and 2 would choose $\theta = \hat{\theta}(x, 3)$ in the absence of the authority, and the authority would impose no restrictions on PEO for all $x \in (\tilde{x}_{TS}(n), \hat{x}(n))$. The equilibrium joint profit of firms 1 and 2 and the equilibrium total surplus are both higher in the PEO model than in the dichotomy model.[15]

We also find that, when $n = 1$ and $x \in (x_{min}(1), x'_{TS})$, the introduction of PEO reveals a conflict of interests that is buried under the merger/non merger dichotomy. In the PEO model, firms 1 and 2 would choose $\theta = \hat{\theta}(x, 1)$ if $x \in (x_{min}(1), x'_{TS})$, but the authority would prohibit PEO because total surplus is higher under $\theta = 0$ than under $\theta = \hat{\theta}(x, 1)$. This conflict of interests is buried in the dichotomy model because firms 1 and 2 would choose to remain independent and the authority would also prefer independence to a merger.

3.4 Robustness of the results

We end this subsection by making two points regarding the robustness of our results. Details of the analyses can be found in Supplementary Note A.

First, in our model, firm 1 decides whether or not to transfer its knowledge to firm 2, but cannot transfer a part of its knowledge. Our results would remain unchanged in a variant of our model that allows partial transfer of knowledge. To see this, suppose that at Stage 2, firm 1 chooses the extent of knowledge transfer, denoted $k \in [0, 1]$, which reduces firm 2's

[15] Our numerical simulation suggests that the qualitative nature of the result would remain unchanged for all $n \geq 3$.

marginal cost from $c_2 = c$ to $c_2 = c - kx$. That is, firm 1 can choose to transfer its entire knowledge ($k = 1$), a part of it ($k \in (0,1)$), or none of it ($k = 0$). Given θ, firm 1 chooses k to maximize its profit in the subsequent Stage 3 subgame $\pi_1^*(\theta, k, n)$. Suppose $\theta \in [0, \frac{1}{2}]$ and let k^* denote the value of k that maximizes $\pi_1^*(\theta, k, n)$. In Supplementary Note A we show that $\pi_1^*(\theta, k, n)$ is strictly convex in k, implying $k^* \notin (0, 1)$. That is, partial knowledge transfer is never optimal. Firm 1 either transfers its knowledge in full ($k^* = 1$) or transfers no knowledge ($k^* = 0$).[16] Hence our results remain unchanged in this variant of the model.

Second, in our model, firm 1 can hold ownership in firm 2's equity, but firm 2 cannot hold ownership in firm 1's equity. We have considered a variant of the model in which firm 2 can also hold ownership in firm 1's equity. Suppose that at Stage 1, firms 1 and 2 choose θ and ψ, where ψ ($0 \leq \psi \leq 1$) denotes the level of firm 2's ownership in firm 1's equity, and the monetary terms of the equity transfer through efficient bargaining. Everything else is the same as in the original model. We have found that the qualitative nature of the results remains mostly unchanged in this variant of the model. As in the original model, we find that there exists a threshold $\tilde{x}_{min}(n) \in (0, x_{max}(n))$ with the following property. If $0 < x \leq \tilde{x}_{min}(n)$, knowledge is not transferred at Stage 2 in the equilibrium. At Stage 1 firms 1 and 2 choose $\theta = 0$ and $\psi = max(\frac{(3n+5)x-(n-1)(a-c)}{a-c+(2n+3)x}, 0) \equiv h(x, n)$, where $h(x, n) < \frac{1}{2}$ holds for all $x \in (0, \tilde{x}_{min}(n)]$ and $n \geq 1$. Here $\psi = h(x, n)$ plays the role of increasing firms 1 and 2's joint profit by shifting some output from the cost-inefficient firm (firm 2) to the cost-efficient firm (firm 1) when $h(x, n) > 0$. If $\tilde{x}_{min}(n) < x \leq x_{max}(n)$, knowledge is transferred at Stage 2 in the equilibrium. At Stage 1 firms 1 and 2 choose $\theta = \hat{\theta}(x, n)$ (where $\hat{\theta}(x, n)$ is as defined in the original model) and $\psi = 0$. That is, firm 2's ownership in firm 1's equity plays no role in facilitating knowledge transfer from firm 1 to firm 2. If $x_{max}(n) < x < \bar{x}$, firms 1 and 2 choose to merge as in the original model. The qualitative nature of the welfare consequences of PEO also remains unchanged in this variant.

We have found that partial cross ownership does not occur in the equilibrium even though it is allowed in this variant of the model. Our conjecture is that partial cross ownership would occur if firm 2 also has unique knowledge that can be transferred to firm 1, because ψ would then play a role of facilitating knowledge transfer from firm 2 to firm 1. Enrichment of the model in this direction is beyond the scope of this paper and left to future research.

[16]Similar results exist in the licensing literature. In a linear duopoly with fixed fee as the payment mechanism, Kabiraj and Marjit (1993) show that either the best technology is licensed or no technology is licensed at all. Creane and Konishi (2009) show that licensing the best technology is most profitable (if any) in an oligopoly as well, provided the inverse demand function is weakly concave.

4 Partial permission of PEO as an antitrust policy

In the previous section, we have shown that the antitrust authority would either impose no restrictions on PEO or completely prohibit PEO to maximize consumer surplus and/or total surplus. In this section, we demonstrate that a partial permission of PEO (that is, the authority permits PEO only up to a certain level that imposes a binding constraint on the alliance partners' profit-maximizing motive) can also be an optimal action of the authority. We illustrate this possibility by extending our model to differentiated oligopoly.

In this extension, everything is the same as in the original model except for the demand structure and the focus on the case of three firms. Consider an economy consisting of an imperfectly competitive sector with three firms, each producing a symmetrically differentiated product, and a competitive numeraire sector whose output is denoted by q_0. Each firm i (=1,2,3) produces product i, and let p_i and q_i denote respectively the price and quantity of product i.

There is a continuum of consumers of the same type, and the representative consumer's preferences are described by the utility function $U(q_1, q_2, q_3) + q_0$, where

$$U(q_1, q_2, q_3) \equiv a(q_1 + q_2 + q_3) - \frac{q_1^2 + q_2^2 + q_3^2}{2} - b(q_1q_2 + q_2q_3 + q_3q_1),$$

$a > 0$ and $b \in (0, 1]$. This yields linear inverse demands:

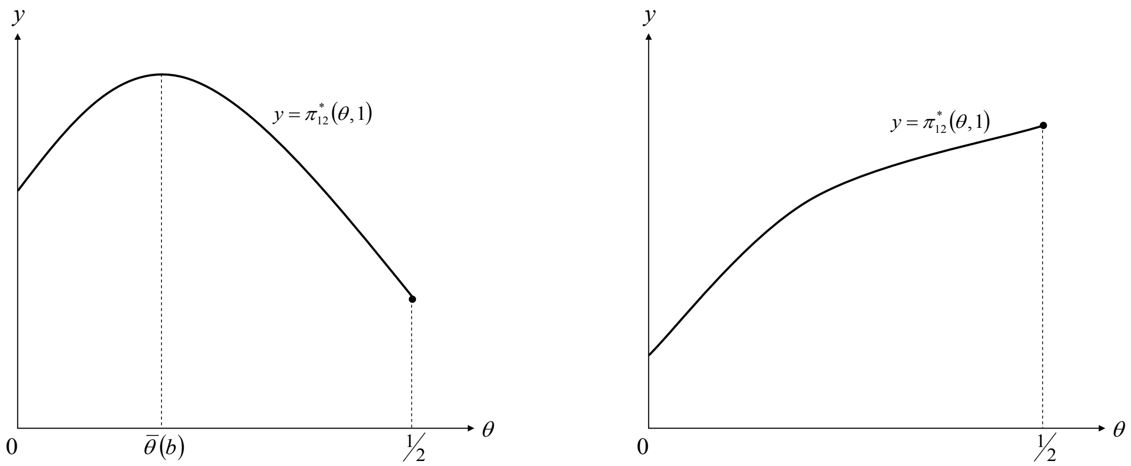
$$p_i = a - q_i - b(q_j + q_k), \qquad i, j, k \in \{1, 2, 3\}; i \neq j \neq k. \tag{4}$$

This is a standard specification of the representative consumer model, where the consumer prefers product variety (see, for example, Vives, 1999). The term b captures the degree of product differentiation in the market. As b increases, the degree of product differentiation decreases. Note that the case of the linear homogenous demand is nested as a special case of $b = 1$.

As in the previous section, we derive Subgame Perfect Nash Equilibria (SPNE) in pure strategies. Analysis of the extended model is similar to the one presented in the previous section, and so we present just the outline of the analysis in the text with some more analytical details presented in the Appendix. Analogous to Assumption 1, we make Assumption 2.[17]

Assumption 2: $x < min\{\frac{2-b}{2b}(a - c), c\} \equiv \bar{x}$.

[17]Each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for all (θ, k) where $\theta \in [0, \frac{1}{2}] \Leftrightarrow x < \bar{x}$, where the proof is analogous to the proof of Claim 1 presented in the Appendix.



(a) $b > b'$

(b) $b < b'$

Figure 4

In what follows, we focus on the case of $c > \frac{2-b}{2b}(a-c) \Leftrightarrow c > \frac{2-b}{2+b}a$ as in the previous section. Let $q_i^*(\theta, k)$ and $\pi_i^*(\theta, k)$ respectively denote firm i's quantity and profit in the equilibrium of the Stage 3 subgame represented by (θ, k), where $\pi_i^*(\theta, k) = \pi_i(\theta, k, q_1^*(\theta, k), q_2^*(\theta, k), q_3^*(\theta, k))$. Also, let $\pi_{12}^*(\theta, k) \equiv \pi_1^*(\theta, k) + \pi_2^*(\theta, k)$ denote the joint profit of firms 1 and 2 in the equilibrium of the Stage 3 subgame.

We find that $\pi_{12}^*(\theta, 1)$ is increasing in θ and $\pi_{12}^*(\theta, 1) \geq \pi_{12}^*(\theta, 0)$ for all $\theta \in (\frac{1}{2}, 1]$, implying that $(\theta, k) = (1, 1)$ maximizes $\pi_{12}^*(\theta, k)$ when θ is constrained to $\theta \in (\frac{1}{2}, 1]$ (see Claims 4 and 5 in Appendix). For $\theta \in [0, \frac{1}{2}]$, Lemma 2 studies the property of the equilibrium joint profit $\pi_{12}^*(\theta, 1)$ as a function of θ.

Lemma 2: There exists a value $\bar{\theta}(b) \equiv min\{\frac{4-6b+2b^2}{4-3b^2}, \frac{1}{2}\} \geq 0$ such that $\pi_{12}^*(\theta, 1)$ is strictly increasing in θ for all $\theta \in [0, \bar{\theta}(b))$ and strictly decreasing in θ for all $\theta \in (\bar{\theta}(b), \frac{1}{2}]$. Furthermore, $\bar{\theta}(b) < \frac{1}{2}$ holds if $b > b' \equiv \frac{6-2\sqrt{2}}{7} \approx 0.453$ and $\bar{\theta}(b) = \frac{1}{2}$ holds otherwise.

See Figure 4 for a diagrammatic representation of Lemma 2. The property of $\pi_{12}^*(\theta, 1)$ is qualitatively different between the case of homogeneous products (that is, the case of $b = 1$) and the case of $b < 1$. When $b = 1$, $\pi_{12}^*(\theta, 1)$ is monotone decreasing in θ for all $\theta \in [0, \frac{1}{2}]$ (Lemma 1 in the previous section). This result is nested in Lemma 2 since $\bar{\theta}(1) = 0$. In contrast, when $b \in (b', 1)$, $\pi_{12}^*(\theta, 1)$ is increasing in θ when θ is relatively small, achieving the maximum value when $\theta = \bar{\theta}(b)$, and decreasing in θ when θ exceeds $\bar{\theta}(b)$. This is true for any small x including the case of $x = 0$. Also, when $b < b'$, $\pi_{12}^*(\theta, 1)$ is increasing in θ for all $\theta \in [0, \frac{1}{2}]$. In the previous section, we found that the PEO paradox, a PEO version of the merger paradox, occurs when products are homogeneous. In contrast, Lemma 2 tells us that the PEO paradox does not necessarily occur when products are differentiated.

Next we turn to the analysis of Stage 2 subgames. For any given $\theta \in (\frac{1}{2}, 1]$, we find that $\pi_1^*(\theta, 1) \geq \pi_1^*(\theta, 0)$ holds (see Claim 5 (i) in Appendix). Given this, for expositional simplicity we assume that firms 1 and 2 choose $k = 1$ when $\theta \in (\frac{1}{2}, 1]$. Now suppose $\theta \in [0, \frac{1}{2}]$, and consider firm 1's incentive to transfer its knowledge to firm 2. Proposition 5, which is qualitatively similar to Proposition 1, tells us that firm 1 transfers its knowledge to firm 2 if θ is greater than or equal to $\hat{\theta}(x)$, the minimum PEO for knowledge transfer.

Proposition 5 [Knowledge transfer]: Suppose $\theta \in [0, \frac{1}{2}]$. Consider firm 1's decision at Stage 2 in the equilibrium of the Stage 2 subgame. There exists a threshold $x_{max}(b) \leq \bar{x}$ with the properties (i) and (ii) listed below, where $x_{max}(b) < \bar{x}$ if $b > b'' \approx 0.683$ and $x_{max}(b) = \bar{x}$ otherwise.
(i) Suppose $0 < x \leq x_{max}(b)$. Then there exists a unique value $\hat{\theta}(x) \in (0, \frac{1}{2}]$ such that firm 1 transfers knowledge to firm 2 if and only if $\theta \geq \hat{\theta}(x)$, where $\hat{\theta}(x)$ is strictly increasing in x for all $x \in (0, x_{max}(b))$.
(ii) Suppose $x_{max}(b) < x < \bar{x}$. Then firm 1 does not transfer knowledge to firm 2 for any $\theta \in [0, \frac{1}{2}]$.

We now demonstrate that partial permission of PEO can be the antitrust authority's optimal action. To see this possibility, suppose that $0 < \hat{\theta}(x) < \bar{\theta}(b) < \frac{1}{2}$ holds. Then, when firms 1 and 2 intend to induce knowledge transfer through PEO, they would choose $\theta = \bar{\theta}(b)$ that is above the minimum PEO $\hat{\theta}(x)$, because $\theta = \bar{\theta}(b)$ is the level of PEO that maximizes their joint profit conditional upon knowledge transfer. The authority, in turn, may impose a binding constraint on the level of PEO by announcing $\tilde{\theta} = \hat{\theta}(x)$, because the total surplus is higher when knowledge is transferred at the minimum PEO for knowledge transfer (see Claim 6 in Appendix). The announcement of $\tilde{\theta} = \hat{\theta}(x)$ is partial permission of PEO because firms 1 and 2 would like to choose $\theta = \bar{\theta}(b)$ but the authority allows them to form PEO only up to $\hat{\theta}(x)$.

We find that $\lim_{x\to 0} \hat{\theta}(x) < \bar{\theta}(b)$ holds if and only if $b < \tilde{b} \approx 0.738$ (see Claim 7 in Appendix). This finding suggests that $\hat{\theta}(x) < \bar{\theta}(b)$ can hold if $b < \tilde{b}$. We have identified values $b_1 \cong 0.454$ and $b_2 \cong 0.732$ with the following properties. For any given $b \in (b_1, b_2)$, there exists an open interval $\chi \subset (0, \bar{x})$ such that if $x \in \chi$, partial permission of PEO is the authority's optimal action. More specifically, if $x \in \chi$, (i) firms 1 and 2 would choose $\theta = \bar{\theta}(b)$ $(< \frac{1}{2})$ in the absence of the authority, and (ii) the authority announces $\tilde{\theta} = \hat{\theta}(x)$

($< \bar{\theta}(b)$), firms 1 and 2 choose $\theta = \hat{\theta}(x)$, and firm 1 transfers its knowledge to firm 2 in the equilibrium. Also, for any given $b < b_1$, there exists a value $\breve{x} \in (0, \bar{x})$ such that if $x > \breve{x}$, "prohibit merger and encourage PEO" is the authority's optimal action. More specifically, if $x > \breve{x}$, (i) firms 1 and 2 would merge in the absence of the authority, and (ii) the authority announces $\tilde{\theta} = \hat{\theta}(x)$ ($< \frac{1}{2}$), firms 1 and 2 choose $\theta = \hat{\theta}(x)$, and firm 1 transfers its knowledge to firm 2 in the equilibrium. Regarding complete prohibition of PEO, for any $b < \tilde{b}$ there exists a threshold for x such that the authority announces $\tilde{\theta} = 0$ in the equilibrium if x is less than the threshold. Details of the analysis are presented in Supplementary note A. In order to fully characterize the equilibrium of the model with and without the authority, however, we need to specify a value of b. Below we present full-characterization results under $b = 0.6$ ($\in (b_1, \tilde{b})$) and $b = 0.4$ ($< b_1$).

The case of* $b = 0.6$*: Partial permission of PEO

Under $b = 0.6$, we find that $\hat{\theta}(x) < \bar{\theta}(b)(\approx 0.384)$ holds if and only if $x \in (0, x_1)$ where $x_1 \approx 0.707(a-c)$. In this case, firms 1 and 2 induce knowledge transfer by choosing $\theta = \bar{\theta}(b)$ ($> \hat{\theta}(x)$) in the equilibrium. Knowledge transfer increases their joint profit, and the PEO further increases it. Hence the condition for knowledge transfer, $\theta \geq \hat{\theta}(x)$, does not impose a binding constraint on firms 1 and 2's choice of θ. Consequently, knowledge transfer with $\theta = \bar{\theta}(b)$ occurs in the equilibrium for all $x \in (0, x_1)$. In contrast, if $x \in (x_1, \bar{x})$, the condition does impose a binding constraint. This leads us to Proposition 6.

Proposition 6 [Equilibrium characterization]: For any given $x \in (0, \bar{x})$, there exists a unique value $\theta^*(x)$ such that actions taken by firms 1 and 2 in the unique equilibrium of the game are described as follows:
(i) Suppose $0 < x < x_1$ ($\approx 0.707(a-c)$). Then firms 1 and 2 choose $\theta = \theta^*(x) \equiv \bar{\theta}(b) \approx 0.384$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_1 \leq x < \bar{x}$. Then firms 1 and 2 choose $\theta = \theta^*(x) \equiv \hat{\theta}(x)$ at Stage 1 and firm 1 transfers its knowledge to firm 2 at Stage 2, where $\theta^*(x)$ is strictly positive and strictly increasing in x for all $x \in (x_1, \bar{x})$.

We now turn to the antitrust implications. Suppose that at Stage 0, the antitrust authority can announce a maximum permissible level of PEO, denoted $\tilde{\theta} \in [0, 1]$, with the objective of maximizing total surplus.[18] As in the previous section, the authority announces $\tilde{\theta}$ only if

[18]The qualitative nature of our results remain unchanged under an alternative assumption that the authority's objective is to maximize the equilibrium consumer surplus.

it can strictly increase the equilibrium total surplus by doing so. Let $ts^*(\theta, k)$ denote total surplus in the equilibrium of the Stage 3 subgame. We find that, for any given $k = \{0, 1\}$, $ts^*(\theta, k)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$ (see Claim 6 in Appendix). An increase in θ weakens competition in the industry, decreasing the equilibrium total surplus. This result and Proposition 6 lead to the following proposition.

Proposition 7-1 (b=0.6) [Antitrust implications]: There exists a value $x_2 \approx 0.0118(a-c)$ such that actions taken by the antitrust authority and firms 1 and 2 in the unique equilibrium are described as follows:
(i) Suppose $0 < x < x_2$. Then the antitrust authority announces $\tilde{\theta} = 0$ at Stage 0, firms 1 and 2 choose $\theta = 0$ at Stage 1, and firm 1 does not transfer its knowledge to firm 2 at Stage 2.
(ii) Suppose $x_2 \leq x < x_1$. Then the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ $(< \bar{\theta})$ at Stage 0, firms 1 and 2 choose $\theta = \tilde{\theta}$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.
(iii) Suppose $x_1 \leq x < \bar{x}$. Then the antitrust authority imposes no restrictions at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.

Proposition 7-1 tells us that the antitrust authority's optimal policy is to prohibit PEO if $x \in (0, x_2)$, partially permit PEO by imposing a binding constraint $\tilde{\theta} = \hat{\theta}(x)$ if $x \in [x_2, x_1)$, or permit any levels of PEO without imposing any restrictions if $x \geq x_1$.

The logic behind this result can be explained as follows. First, consider the case of $0 < x < x_1$. Without any restrictions imposed, firms 1 and 2 would choose $\theta = \bar{\theta}(b) > \hat{\theta}(x)$ to induce knowledge transfer. The authority's preference is that knowledge be transferred at $\hat{\theta}(x)$, since $ts^*(\theta, k)$ is strictly decreasing in θ. We find that, if the authority announces $\tilde{\theta} = \hat{\theta}(x)$, firms 1 and 2 would choose $\theta = \hat{\theta}(x)$ and knowledge is transferred in the subsequent equilibrium for all $x \in (0, \bar{x})$. The authority then announces $\tilde{\theta} = \hat{\theta}(x)$ if $ts^*(\hat{\theta}(x), 1) > ts^*(0, 0)$ holds (that is, if knowledge transfer with $\theta = \hat{\theta}(x)$ increases total surplus), where we find that $ts^*(\hat{\theta}(x), 1) > ts^*(0, 0) \Leftrightarrow x > x_2$. The announcement of $\tilde{\theta} = \hat{\theta}(x)$ is partial permission of PEO. If $x < x_2$, then $ts^*(0, 0) > ts^*(\hat{\theta}(x), 1)$ holds, and hence the authority prohibits PEO entirely by announcing $\tilde{\theta} = 0$.

Second, consider the case of $x_1 \leq x < \bar{x}$. Firms 1 and 2 would then choose $\theta = \hat{\theta}(x)$ to induce knowledge transfer even without any restrictions imposed, and this is the desirable outcome for the authority. Hence the authority does not impose any restrictions in this case.

The case of $b = 0.4$***: Prohibit merger and encourage PEO***

The equilibrium characterization result when $b = 0.4$ is that for any given $x \in (0, \bar{x})$, in the unique equilibrium of the game firms 1 and 2 choose $\theta = 1$ (i.e, they choose to merge) at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2. They choose to merge in the equilibrium even though any levels of PEO $\theta \in [0, 1]$ are possible.

Now we turn to the antitrust implications by introducing the antitrust authority that can impose a maximum permissible level of PEO, $\tilde{\theta}$.

Proposition 7-2 (b=0.4) [Antitrust implications]: There exists a value $x_2' \approx 0.00663(a-c)$ such that actions taken by the antitrust authority and firms 1 and 2 in the unique equilibrium are described as follows:

(i) Suppose $0 < x < x_2'$. Then the antitrust authority announces $\tilde{\theta} = 0$ at Stage 0, firms 1 and 2 choose $\theta = 0$ at Stage 1, and firm 1 does not transfer its knowledge to firm 2 at Stage 2.

(ii) Suppose $x_2' \leq x < \bar{x}$. Then the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1, and firm 1 transfers its knowledge to firm 2 at Stage 2.

Proposition 7-2 tells us that the optimal antitrust policy is to "prohibit merger and encourage PEO" unless x is very small. That is, although the merger can improve welfare, the antitrust authority can further increase welfare by prohibiting firms 1 and 2 to merge and instead inducing them to choose the minimum PEO for knowledge transfer by announcing $\tilde{\theta} = \hat{\theta}(x)$ as the maximum permissible level of PEO.

Notice that, if the antitrust policy were based on permission or prohibition of merger, the authority would permit a merger to induce knowledge transfer unless x were very small. The introduction of the possibility for alliance partners to be linked through PEO substantially changes the optimal antitrust policy because the authority can maximize the total surplus by following the "prohibit merger and encourage PEO" strategy.

5 General demand function

In this section we explore how far our analysis can go under the homogeneous oligopoly model with a general inverse demand function $P(Q)$ ($Q \equiv \sum_{i=1}^{n+2} q_i$ denotes the industry output), where $P(Q)$ is continuously differentiable as often as is required and $P'(Q) < 0$ for all $Q > 0$. Assume that there exists a choke price $\bar{P} > 0$ such that quantity demanded is zero for all $P \geq \bar{P}$. Furthermore, following Nocke and Whinston (2010), we consider logconcave inverse demand functions, i.e., the inverse demand functions satisfying

$$P'(Q) + QP''(Q) < 0 \tag{5}$$

for all $Q > 0$, which ensures both the existence and the uniqueness of a Cournot equilibrium. Everything else is the same as in the model considered in Section 3.

The basic approach to the analysis of the model is the same as the one under the linear inverse demand function analyzed in Section 3. We will focus on our main findings in the text below, and present analytical details in Supplementary Note B.

First consider the PEO paradox. Let $\pi_{12}^*(\theta, k, n) \equiv \pi_1^*(\theta, k, n) + \pi_2^*(\theta, k, n)$ denote the joint profit of firms 1 and 2 in the equilibrium of the Stage 3 subgame. Lemma 1 told us that $\pi_{12}^*(\theta, k, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$ under the linear demand. Lemma G1 tells us that the result is similar, although somewhat weaker, under the general demand. As explained in Section 3, this result is referred to as the PEO paradox because the underlying logic is similar to the merger paradox in the theoretical analysis of merger.

Lemma G1 [PEO paradox]: Suppose that (i) $n \geq 2$, or (ii) $n = 1$ and $P''(Q) \leq 0$ for all $Q > 0$ hold. Then, for any given k ($\in \{0, 1\}$), $\pi_{12}^*(\theta, k, n)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$.[19]

Next consider firm 1's incentive to transfer its knowledge to firm 2 at Stage 2. If $\theta \in (\frac{1}{2}, 1]$, firm 1 shuts down firm 2 in the equilibrium of the subsequent Stage 3 subgame and hence firm 1 is indifferent between transferring its knowledge and not transferring it, as in Section 3 under the linear demand.

Suppose $\theta \in [0, \frac{1}{2}]$. Firm 1 transfers its knowledge to firm 2 if and only if

$$\pi_1^*(\theta, 1, n) - \pi_1^*(\theta, 0, n) \geq 0 \tag{6}$$

holds. This leads us to the following proposition.

Proposition G1 [Knowledge transfer]: Suppose $\theta \in [0, \frac{1}{2}]$. For any given $n \geq 1$, there exists a threshold $x_{max}(n) > 0$ with the following property: For any given $x < x_{max}(n)$, there exists a unique value $\hat{\theta}(x, n) \in (0, \frac{1}{2}]$ such that $\pi_1^*(\theta, 1, n) - \pi_1^*(\theta, 0, n) \leq 0$ holds for all $\theta \in [0, \hat{\theta}(x, n)]$, where the weak inequality holds with equality only when $\theta = \hat{\theta}(x, n)$.

[19] Suppose $n = 1$ and $P''(Q) > 0$ for some $Q > 0$. Then, there exists $\theta = \theta_0 \in (0, \frac{1}{2})$ such that $\pi_{12}^*(\theta, k, n)$ is decreasing in θ for all $\theta \in (\theta_0, \frac{1}{2})$.

We call $\hat{\theta}(x,n)$ the minimum PEO for knowledge transfer as in Section 3, because firm 1 transfers the knowledge if $\theta = \hat{\theta}(x,n)$ but does not transfer it if $\theta < \hat{\theta}(x,n)$. Notice that compared with Proposition 1, Proposition G1 is somewhat weaker because it does not rule out the possibility that $\pi_1^*(\theta,1,n) - \pi_1^*(\theta,0,n) < 0$ holds with a certain $\theta > \hat{\theta}(x,n)$.[20] However, this possibility does not affect the level of θ chosen by firms 1 and 2 at Stage 1, as long as $\pi_{12}^*(\theta,k,n)$ is strictly decreasing in θ for all $\theta \in [0,\frac{1}{2}]$ (that is, as long as $n \geq 2$ or $P'' \leq 0$ holds).

Let us now consider the level of θ that firms 1 and 2 choose at Stage 1 in the equilibrium to maximize their joint profit. We focus on cases in which $n \geq 2$ or $P'' \leq 0$ holds. If the minimum PEO $\hat{\theta}(x,n)$ exists, then one of their relevant options is to induce knowledge transfer by choosing $\theta = \hat{\theta}(x,n)$. Since $\pi_{12}^*(\theta,1,n)$ is strictly decreasing in θ, there is no incentive for them to induce knowledge transfer by choosing any $\theta > \hat{\theta}(x,n)$. If firms 1 and 2 do not intend to induce knowledge transfer, they choose either $\theta = 0$ or 1, given that $\pi_{12}^*(\theta,0,n)$ is strictly decreasing in θ for all $\theta \in [0,\frac{1}{2}]$. Then, firms 1 and 2 choose $\theta = \hat{\theta}(x,n)$ in the equilibrium (that is, PEO arises as an equilibrium outcome) if and only if $\pi_{12}^*(\hat{\theta}(x,n),1,n) \geq \max\{\pi_{12}^*(0,0,n), \pi_{12}^*(1,0,n)\}$ holds. Proposition G2 identifies a sufficient condition for this.

Proposition G2 [PEO as an equilibrium outcome]: There exists a range of parameter values for x, denoted $X \subset \Re$, with the following property: For any given $x \in X$, there exists a value $n(x)$ such that firms 1 and 2 choose $\theta = \hat{\theta}(x,n) \in (0,\frac{1}{2}]$ at Stage 1 in the equilibrium if $n \geq n(x)$.

As we showed in Section 3, we can obtain a sharper equilibrium-characterization result under a linear inverse demand function. We can compute the minimum PEO $\hat{\theta}(x,n)$, explicitly compare $\pi_{12}^*(\hat{\theta}(x,n),1,n)$ and $\max\{\pi_{12}^*(0,0,n), \pi_{12}^*(1,0,n)\}$, and identify a necessary and sufficient condition (in terms of x) for PEO to arise as an equilibrium outcome as presented in Proposition 2.

Finally, we consider the welfare consequences and policy implications of PEO. Let $ts^*(\theta,k,n)$ denote total surplus in the equilibrium of the Stage 3 subgame. We find that, for any given (k,n), $ts^*(\theta,k,n)$ is strictly decreasing in θ for all $\theta \in [0,\frac{1}{2}]$, and $ts^*(\frac{1}{2},k,n) > ts^*(\theta,k,n)$

[20] We also find that, for any given $x < x_1(n)$, there exists a unique value $\tilde{\theta}(x,n) \in [\hat{\theta}(x,n),\frac{1}{2}]$ such that $\pi_1^*(\theta,1,n) - \pi_1^*(\theta,0,n) \geq 0$ holds for all $\theta \in [\tilde{\theta}(x,n),\frac{1}{2}]$. Under the linear inverse demand function, we have that $\hat{\theta}(x,n) = \tilde{\theta}(x,n)$ holds. However, under the general inverse demand function, we were unable to rule out the possibility of $\hat{\theta}(x,n) < \tilde{\theta}(x,n)$.

holds for all $\theta \in (\frac{1}{2}, 1]$. Then, if firms 1 and 2 choose $\theta = \hat{\theta}(x,n)$ in the equilibrium, the PEO $\hat{\theta}(x,n)$ improves welfare if $ts^*(\hat{\theta}(x,n),1,n) \geq ts^*(0,0,n)$ holds. Regarding policy implications, an antitrust authority whose objective is to maximize total surplus would impose no restrictions on PEO if $ts^*(\hat{\theta}(x,n),1,n) \geq ts^*(0,0,n)$, and completely prohibit PEO otherwise. Analogous property holds for consumer surplus.

Again, as we showed in Section 3, we can obtain sharper results under a linear inverse demand function by computing the minimum PEO $\hat{\theta}(x,n)$ and explicitly comparing $ts^*(\hat{\theta}(x,n),1,n)$ and $ts^*(0,0,n)$ (and also $cs^*(\hat{\theta}(x,n),1,n)$ and $cs^*(0,0,n)$). This allows us to identify a necessary and sufficient condition (in terms of x) for the equilibrium PEO to improve total surplus and/or consumer surplus as presented in Propositions 3 and 4.

6 Conclusion

The incidence and importance of equity strategic alliances have substantially increased in recent years. PEO induces the transfer of tacit knowledge between alliance partners. We have explored oligopoly models that capture this important link between PEO and knowledge transfer, where PEO, merger and independence are all nested in one model. Unlike previous theoretical analyses in which levels of PEO are exogenously given, the level of PEO is endogenously determined in our model through strategic interactions among alliance partners and firms outside the alliance. We have found that the endogenously determined level of PEO can increase both consumer surplus and total surplus, decrease consumer surplus but increase total surplus, or decrease both of them.

PEO itself reduces competition, but induced knowledge transfer improves productivity. We have explored the trade-off between reduction in competition and productivity improvement in the context of PEO. Firm 2 can learn from firm 1's superior knowledge. Unlike in models of horizontal mergers, the presence of learning opportunities does not automatically guarantee the occurrence of learning in our PEO model because firm 1 transfers its knowledge to firm 2 only if it owns a sufficiently high level of PEO in firm 2. We have found that, under the linear homogeneous inverse demand function, the endogenously determined level of PEO increases consumer surplus and/or total surplus when the level of PEO is relatively high. This counterintuitive result arises because a higher level of PEO chosen in equilibrium implies that the significance of the knowledge to be transferred is also high.

Given the growing antitrust interest in PEO, we have explored the antitrust implications of our analysis by considering an antitrust authority that can announce a maximum permissible level of PEO. When knowledge transfer is desirable for the authority, it prefers that the transfer be induced at the minimum PEO for knowledge transfer, because PEO itself reduces welfare by weakening competition in the industry. When products are homogeneous, the alliance partners choose the minimum PEO whenever they intend to induce knowledge transfer, and hence the authority's relevant options are to either prohibit PEO or impose no restrictions on PEO. When products are differentiated, however, the alliance partners' profit maximizing level of PEO may be higher than the minimum PEO. The authority's optimal policy can then be partial permission of PEO, including the possibility of "prohibit merger and encourage PEO." We have identified conditions under which one of these three policy options—prohibit, partially permit, or permit PEO—is optimal.

We have found that antitrust implications of horizontal mergers can change in significant ways by introducing PEO as an option to the merger/non-merger dichotomy. In our framework, if merger or independence were the only options for firms 1 and 2, they would choose to merge if x is relatively large and choose to remain independent if x is relatively small. When PEO is introduced as an option, firms 1 and 2 would choose PEO when x is in an intermediate range. We have shown that PEO can reconcile a conflict of interests between the alliance partners and the antitrust authority that may arise under the merger/non-merger dichotomy. When $n = 1$ or 2, firms 1 and 2 would choose to merge but the authority would prohibit merger if x is in an intermediate range. Also, when $n \geq 3$, firms 1 and 2 would remain independent but the authority would like them to merge if x is in an intermediate range. We have found that PEO reconciles these two different types of conflicts. That is, when PEO is an option and x is in the intermediate range, the optimally chosen level of PEO maximizes not only the joint profit of firms 1 and 2 but also total surplus. Hence the authority has no incentives to intervene in the formation of PEO arrangements driven by the alliance partners' profit-maximizing motives.

Transfer of knowledge within strategic alliances is becoming increasingly important in the knowledge economy. In this paper we have focused on the transfer of tacit knowledge between alliance partners that are horizontally competing against each other. In reality, knowledge transfer also occurs between vertically related alliance partners (for example, between manufacturers and suppliers), and between joint venture companies and their parent firms. In future research, we plan to study the economic implications of the link between these types of knowledge transfer and PEO.

Appendix (For Online Publication)

Proofs for Section 3

We first establish the following claim (see Assumption 1 in the text).

Claim 1: Each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for all (θ, k, n) where $\theta \in [0, \frac{1}{2}] \Leftrightarrow x < min\{\frac{a-c}{2}, c\}$.

Proof: Each firm i's profit function given by (1) in the text is strictly concave in q_i when $P(Q) = a - dQ$. Suppose each firm i produces a strictly positive amount in the equilibrium of the Stage 3 subgame for a given (θ, k, n) where $\theta \in [0, \frac{1}{2}]$. The necessary first order conditions are:

$$\begin{aligned}
\frac{\partial \pi_1(\theta, k, q_1, q_2, ..., q_{n+2})}{\partial q_1} &\equiv [P(Q) - (c - x)] + P'(Q)q_1 + \theta P'(Q)q_2 = 0, \\
\frac{\partial \pi_2(\theta, k, q_1, q_2, ..., q_{n+2})}{\partial q_2} &\equiv (1 - \theta)[P(Q) - (c - kx) + P'(Q)q_2] = 0, \\
\frac{\partial \pi_m(\theta, k, q_1, q_2, ..., q_{n+2})}{\partial q_m} &\equiv P(Q) - c + P'(Q)q_m = 0.
\end{aligned}$$

where $m = 3, 4, ..., n + 2$.

Substituting $P(Q) = a - dQ$ and $P'(Q) = -d$ and solving these equations imply (2) in the text. For any given (θ, k, n) where $\theta \in [0, \frac{1}{2}]$, $q^*_m(\theta, k, n) > 0$ $(m = 3, ..., n + 2) \Rightarrow q^*_i(\theta, k, n) > 0$ for all $i = 1, 2, ..., n + 2$. Also, $q^*_m(\theta, k, n) > 0$ for all (θ, k, n) where $\theta \in [0, \frac{1}{2}]$ $\Rightarrow x < min\{\frac{a-c}{2}, c\}$. This proves the necessity part of the claim, and the proof of sufficiency is analogous. *Q.E.D.*

Proof of Lemma 1: First let $k = 1$. We find that $\frac{\partial}{\partial \theta}\pi^*_{12}(\theta, 1, n) = -\frac{(n-1+\theta)(a-c+(n+1)x)^2}{d(n+3-\theta)^3} \leq 0$ for all $\theta \in [0, \frac{1}{2}]$ where the inequality holds with strict inequality for all $\theta \in (0, \frac{1}{2}]$. Also, for any given $n \geq 1$, $q^*_i(\theta, 1, n)$ is constant for all $\theta \in (\frac{1}{2}, 1]$, and hence $\pi^*_{12}(\theta, 1, n)$ is also constant for all $\theta \in (\frac{1}{2}, 1]$. We find that $\pi^*_{12}(\frac{1}{2}, 1, n) - \pi^*_{12}(\theta', 1, n) = \frac{(2n^2+4n-1)(a-c+(n+1)x)^2}{(2n+5)^2(n+2)^2 d} > 0$ for any $\theta' \in (\frac{1}{2}, 1]$. This proves the lemma for $k = 1$.

Next let $k = 0$. We find that $\frac{\partial}{\partial \theta}\pi^*_{12}(\theta, 0, n) = -\frac{(a-c-x)[(n-1+\theta)(a-c)+x(n^2+(3-\theta)n+4-2\theta)]}{d(4-\theta)^3} < 0$ for all $\theta \in [0, \frac{1}{2}]$. We also find that $\pi^*_{12}(\theta, 0, n)$ is constant for all $\theta \in (\frac{1}{2}, 1]$, and that for any $\theta' \in (\frac{1}{2}, 1]$, $\pi^*_{12}(0, 0, n) - \pi^*_{12}(\theta', 0, n) = \frac{(a-c-x)}{(n+3)^2(n+2)^2 d}[(n^2 + 2n - 1)(a - c) - (3n^2 + 12n + 11)x]$. Hence $\pi^*_{12}(0, 0, n) - \pi^*_{12}(\theta', 0, n) > (=, <)\ 0$ holds if $x < (=, >)\ \hat{x}(n) \equiv \frac{n^2+2n-1}{3n^2+12n+11}(a - c)$, where $\hat{x}(n) < \bar{x}$ holds for all $n \geq 1$. *Q.E.D.*

Proof of Proposition 1: Define $\Delta(\theta, n) \equiv \pi_1^*(\theta, 1, n) - \pi_1^*(\theta, 0, n)$, where we find that $\Delta(\theta, n) = \frac{x}{d(n+3-\theta)^2}[[(n+1)\theta - (2n+3)]x - [\theta^2 - (n+5)\theta + 2](a-c)]$. Firm 1 transfers its knowledge to firm 2 in the equilibrium if and only if $\Delta(\theta, n) \geq 0$ holds. We have that $\Delta(0, n) = -\frac{x}{d(n+3)^2}[2(a-c) + (2n+3)x] < 0$, and $\frac{d}{d\theta}\Delta(\theta, n) = \frac{x}{d(n+3-\theta)^3}[(a-c-x)(n^2 - (n+1)\theta)) + (2n^2-3)x + 8(a-c)n + 11(a-c)]$. We find that $\frac{d}{d\theta}\Delta(\theta, n) > 0$ holds for all $n \geq 1$. Also, $\Delta(\frac{1}{2}, n) = \frac{x}{d(2n+5)^2}[(2n+1)(a-c) - (6n+10)x]$, where $\Delta(\frac{1}{2}, n) > (=, <)\ 0$ holds if $x < (=, >)\ x_{max}(n) \equiv \frac{2n+1}{6n+10}(a-c)$ (which is decreasing in n for all $n \geq 1$), and $x_{max}(n) < \bar{x}$ holds for all $n \geq 1$. This implies that for any given $x \in (0, x_{max}(n))$, there exists a unique value $\hat{\theta}(x, n)$ such that $\Delta(\theta, n) = 0 \Leftrightarrow \theta = \hat{\theta}(x, n)$. This proves (i) and (ii) except for the property that $\hat{\theta}(x, n)$ is strictly increasing in x and strictly decreasing in n for all $x \in (0, x_{max}(n))$ and $n \geq 1$. Suppose $x \in (0, x_{max}(n))$. We have that $\Delta(\theta, n) = 0 \Leftrightarrow x = \frac{\theta^2-(n+5)\theta+2}{(n+1)\theta-(2n+3)}(a-c) \equiv \tilde{x}(\theta, n)$, where $\frac{d}{d\theta}\tilde{x}(\theta, n) = \frac{(n+1)\theta^2-2(2n+3)\theta+2n^2+11n+13}{((n+1)\theta-(2n+3))^2}(a-c) > 0$ and $\frac{d}{dn}\tilde{x}(\theta, n) = \frac{(4-\theta)(1-\theta)^2}{[(n+1)\theta-(2n+3)]^2} > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. This implies the desired property of $\hat{\theta}(x, n)$. *Q.E.D.*

Proof of Proposition 2: Note that we have $x_{max}(n) - \hat{x}(n) = \frac{5n^2+20n+21}{2(3n+5)(3n^2+12n+11)} > 0$. Proposition 1 and Lemma 1 together imply that at Stage 1, firms 1 and 2 have following three relevant options if $x \in (0, x_{max}(n)]$: (i) Choose $\theta = \hat{\theta}(x, n)$, the minimum PEO that induces knowledge transfer, (ii) choose $\theta = 0$, or (iii) choose $\theta = 1$. Consider $\pi_{12}^*(\hat{\theta}(x, n), 1, n) - \pi_{12}^*(0, 0, n) \equiv g1(x, n)$. By definition of $\tilde{x}(\theta, n)$ (see the proof of Proposition 1), we have that $\theta = \hat{\theta}(x, n) \Leftrightarrow x = \tilde{x}(\theta, n)$, where $\frac{d}{d\theta}\tilde{x}(\theta, n) > 0$ for all $\theta \in [0, \frac{1}{2}]$. Given this, for algebraic convenience we substitute $x = \tilde{x}(\theta, n)$ in $g1(x, n)$ so that it becomes a function of θ. We find that $g1(\tilde{x}(\theta, n), n) = \frac{1-\theta}{d(n+3)^2(n+3-\theta)^2(2n+3-(n+1)\theta)^2}f1(\theta, n)(a-c)^2$ where $f1(\theta, n) \equiv (n^2+4n+5)\theta^5 + (n^4+4n^3-7n^2-52n-64)\theta^4 + (-2n^5-19n^4-54n^3+9n^2+266n+310)\theta^3 + (n^6+14n^5+73n^4+140n^3-106n^2-736n-716)\theta^2 + (10n^4+124n^3+559n^2+1090n+777)\theta - (4n^4+44n^3+180n^2+324n+216)$. We now establish the following claim.

Claim 2: There exists a unique value $x_{min}(n) \in (0, x_{max}(n))$ such that $g1(x, n) > 0$ if $x \in (x_{min}(n), x_{max}(n)]$, $g1(x, n) = 0$ if $x = x_{min}(n)$, and $g1(x, n) < 0$ if $x \in (0, x_{min}(n))$.
Proof: We have that $f1(0, n) < 0$ and $f1(\frac{1}{2}, n) > 0$. Also, $\frac{d}{d\theta}f1(\theta, n) = 2\theta n^6 + (-6\theta^2 + 28\theta)n^5 + (4\theta^3 - 57\theta^2 + 146\theta + 10)n^4 + (16\theta^3 - 162\theta^2 + 280\theta + 124)n^3 + (5\theta^4 - 28\theta^3 + 27\theta^2 - 212\theta + 559)n^2 + (20\theta^4 - 208\theta^3 + 798\theta^2 - 1472\theta + 1090)n + 25\theta^4 - 256\theta^3 + 930\theta^2 - 1432\theta + 777 > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. Hence there exists a unique value $\theta'' \in (0, \frac{1}{2})$ such that $f1(\theta, n) > (=, <)\ 0$ if $\theta > (=, <)\ \theta''$. We find that $\tilde{x}(0, n) < 0 < \tilde{x}(\frac{1}{2}, n) = x_{max}(n)$ for all $n \geq 1$, which implies that $\hat{\theta}(0, n) > 0$. We then have that $g1(0, n) = \pi_{12}^*(\hat{\theta}(0, n), 1, n) - \pi_{12}^*(0, 1, n) < 0$,

which implies the result. *Q.E.D.*

Finally, noting that $\pi_{12}^*(1,0,n) = \pi_1^*(0,0,n-1)$, we find that $\pi_{12}^*(\hat{\theta}(\tilde{x}(\theta,n),n),1,n) - \pi_{12}^*(1,0,n) = \frac{[(n+1)\theta^2-(n+1)(n+4)\theta-1]^2(\theta+n^2+2n-1)(1-\theta)}{d(n+2)^2[(n+1)\theta-(2n+3)]^2[\theta-(n+3)]^2}(a-c)^2 > 0$. This and Claim 2 along with Lemma 1 imply Proposition 2 except for the property of $x_{min}(n)$. Then Claim 3 below completes the proof.

Claim 3: $x_{min}(n)$ is strictly increasing in n for all $n \geq 1$.
Proof: First note that $g1(0,n) < 0$ for all $n \geq 1 \Rightarrow f1(\hat{\theta}(0,n),n) < 0$ for all $n \geq 1$. We find by using mathematical software Maple that $f1(\hat{\theta}(0,n),n+1) - f1(\hat{\theta}(0,n),n) = -\frac{1}{2}n^6 - \frac{13}{2}n^5 - \frac{59}{2}n^4 - 50n^3 + \frac{63}{2}n^2 + 217n + 205 + (\frac{1}{2}n^5 + 4n^4 + \frac{23}{2}n^3 + \frac{13}{2}n^2 - 20n - 19)(n^2+10n+17)^{\frac{1}{2}} > 0$ for all $n \geq 1$. Furthermore, we have that $\frac{d^2}{dnd\theta}f1(\theta,n) = 12\theta n^5 + 5(-6\theta^2+28\theta)n^4 + 4(4\theta^3 - 57\theta^2 + 146\theta + 10)n^3 + 3(16\theta^3 - 162\theta^2 + 280\theta + 124)n^2 + 2(5\theta^4 - 28\theta^3 + 27\theta^2 - 212\theta + 559)n + 20\theta^4 - 208\theta^3 + 798\theta^2 - 1472\theta + 1090 > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. This implies the result. *Q.E.D.*

Proof of Proposition 3: Consumer surplus can be expressed as $cs(q_1, q_2, ..., q_{n+2}) \equiv \frac{1}{2}d(q_1 + q_2 + ... + q_{n+2})^2$. Consumer surplus in the equilibrium of the Stage 3 subgame represented by (θ, k) is given by $cs(q_1^*(\theta,k,n), q_2^*(\theta,k,n), ..., q_{n+2}^*(\theta,k,n)) \equiv cs^*(\theta,k,n)$. We have that $cs^*(1,0,n-1) - cs^*(1,0,n) = -\frac{(a-c-x)[2(a-c)n^2+(8(a-c)+2x)n+7(a-c)+5x]}{2d(n+2)^2(n+3)^2} < 0$ for all $n \geq 1$, implying that merger between firms 1 and 2 reduce consumer surplus compared to non-merger. Next consider $cs^*(\hat{\theta}(x),1,n) - cs^*(0,0,n) \equiv g2(x,n)$. As in the proof of Proposition 2, we substitute $x = \tilde{x}(\theta,n)$ in $g2(x,n)$. We find that $g2(\tilde{x}(\theta,n),n) = \frac{1-\theta}{2d(n+3)^2(n+3-\theta)^2[2n+3-(n+1)\theta]^2}f2(\theta,n)f3(\theta,n)(a-c)^2$, where $f2(\theta,n) \equiv 2(2-\theta)n^3 + (\theta^2 - 13\theta + 26)n^2 + (\theta^3 - 5\theta^2 - 12\theta + 48)n + (4\theta^3 - 24\theta^2 + 26\theta + 18)$, and $f3(\theta,n) \equiv -(n+2)\theta^2 + (n^2+6n+10)\theta - (2n+6)$. We find that $f2(\theta,n) > 0$ for all $\theta \in [0, \frac{1}{2}]$ and $n \geq 1$. We also find that $f3(\theta,1) < 0$ for all $\theta \in [0, \frac{1}{2}]$, and this implies Proposition 3 (A). Next consider the case of $n \geq 2$. Let θ_{root1} and θ_{root2}, $\theta_{root1} < \theta_{root2}$, denote two roots of $f3(\theta,n) = 0$. We find that $\theta_{root1} \in (0, \frac{1}{2})$ and $\theta_{root2} > 1$ for all $n \geq 2$, and that $\theta_{root1} = \frac{4(n+3)}{n^2+6n+10+\sqrt{n^4+12n^3+48n^2+80n+52}}$ is strictly decreasing in n for all $n \geq 2$. This implies Proposition 3 (B). *Q.E.D.*

Proof of Proposition 4: Total surplus in the equilibrium of the Stage 3 subgame represented by (θ,k,n) is $ts^*(\theta,k,n) + \pi_1^*(\theta,k,n) + \pi_2^*(\theta,k,n) + ... + \pi_{n+2}^*(\theta,k,n) \equiv ts^*(\theta,k,n)$. Consider $ts^*(\hat{\theta}(x,n),1,n) - ts^*(0,0,n) \equiv g3(x,n)$. When $n = 1$, we find that $g3(\tilde{x}(\theta,1),1) = \frac{(1-\theta)(5-\theta)}{32d(5-2\theta)^2(4-\theta)^2}f4(\theta)(a-c)^2$ where $f4(\theta) \equiv 25\theta^4 - 242\theta^3 + 477\theta^2 + 536\theta - 256$ and that $f4(\theta) > (=,<)\ 0$ holds if $\theta < (=,>)\ \check{\theta} \cong 0.375$ where $\tilde{x}(\check{\theta},1) \equiv x'_{TS} \approx 0.0260(a-c)$.

Also, $ts^*(1,0,1) - ts^*(0,0,1) = -\frac{(a-c-x)[7(a-c)-31x]}{288d} > 0\ (=,<)\ 0$ holds if $x > (=,<)$ $x''_{TS} \cong 0.226(a-c)$. This implies Proposition 4 (A).

Now consider $n \geq 2$. Let $x \in (x_{max}(n), \bar{x})$ where $\theta^*(x,n) = 1$. We find that $TS(\theta^*(x,n),n) - TS(0,n) = ts^*(0,1,n-1) - ts^*(0,0,n) = \frac{(a-c-x)((2n^2+12n+17)x-(2n+5)(a-c))}{2d(n+2)^2(n+3)^2}$ which is strictly positive if and only if $x > \frac{(2n+5)(a-c)}{2n^2+12n+17}$. Since $x_{max}(n) - \frac{(2n+5)(a-c)}{2n^2+12n+17} = \frac{(4n^3+14n^2-4n-33)(a-c)}{2(3n+5)(2n^2+12n+17)} > 0$ for all $n \geq 2$ we have that $TS(\theta^*(x,n),n) - TS(0,n) > 0$ for all $x \in (x_{max}(n), \bar{x})$. Next consider $x \in [x_{CS}(n), x_{max}(n)]$. By Proposition 3, for those values of x, $CS(\theta^*(x,n),n) - CS(0,n) = cs^*(\theta^*(x,n),1,n) - cs^*(0,0,n) \geq 0$ or equivalently $Q^*(\theta^*(x,n),1,n) - Q^*(0,0,n) \geq 0$ where $Q^*(\theta,k,n) = \sum_{i=1}^{n+2} q_i^*(\theta,k,n)$. Express $ts^*(\theta^*(x,n),1,n) - ts^*(0,0,n)$ as

$$\int_{Q^*(0,0,n)}^{Q^*(\theta^*(x,n),1,n)} (P(y)-c)dy + x[q_1^*(\theta^*(x,n),1,n) + q_2^*(\theta^*(x,n),1,n) - q_1^*(0,0,n)].$$

Since $Q^*(\theta^*(x,n),1,n) \geq Q^*(0,0,n)$ and $q_1^*(\theta^*(x,n),1,n) + q_2^*(\theta^*(x,n),1,n) - q_1^*(0,0,n) = \frac{(n+3-(n+2)\theta^*(x,n))(a-c)+[(1-\theta^*(x,n))(2n^2+7n)+(3-4\theta^*(x,n))]x}{d(n+3)(n+3-\theta^*(x,n))} > 0$ we have that $ts^*(\theta^*(x,n),1,n) - ts^*(0,0,n) > 0$ for all $x \in [x_{CS}(n), x_{max}(n)]$. As the inequality is strict, standard continuity argument implies that there exists a $x_{TS}(n) < x_{CS}(n)$ such that $TS(\theta^*(x,n),n) - TS(0,n) = ts^*(\theta^*(x,n),1,n) - ts^*(0,0,n) > 0$ for all $x \in (x_{TS}(n), x_{max}(n)]$. *Q.E.D.*

Proofs for Section 4

Claim 4: Suppose $\theta \in (\frac{1}{2}, 1]$. Then $\pi_{12}^*(\theta,1)$ is increasing in θ for all $\theta \in (\frac{1}{2}, 1]$. More precisely, $\pi_{12}^*(\theta,1)$ is constant for all $\theta \in (\frac{1}{2}, \frac{b}{2-b}]$ and strictly increasing in θ for all $\theta \in (\max\{\frac{b}{2-b}, \frac{1}{2}\}, 1]$.

Proof: For any given $\theta \in (\frac{1}{2}, 1]$, firm 1 chooses q_1 and q_2 to maximize $\pi_1(\theta, k, q_1, q_2, q_3)$ given. Firm 3's maximization problem is same as before. Assume $k = 1$. Solving the first-order conditions corresponding to the profit maximization problem gives the equilibrium quantities. If $\theta \in (\frac{b}{2-b}, 1]$ the equilibrium quantities are as follows: $q_1^*(\theta,1) = \frac{\theta[2-b(1+\theta)][(2-b)(a-c)+2x]}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$, $q_2^*(\theta,1) = \frac{[(2-b)\theta-b][(2-b)(a-c)+2x]}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$, and $q_3^*(\theta,1) = \frac{4\theta[(1-b)(a-c)-bx]+b^2x(1+\theta^2)]}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$.[21] Else, if $\theta \in (\frac{1}{2}, \frac{b}{2-b})$, $q_1^*(\theta,1) = \frac{(2-b)(a-c)+2x}{4-b^2}$, $q_2^*(\theta,1) = 0$, and $q_3^*(\theta,1) = \frac{(2-b)(a-c)-bx}{4-b^2}$. Note the interval $(\frac{1}{2}, \frac{b}{2-b}]$ is empty for $b < \frac{2}{3}$. Thus shutting down firm 2 is profitable for firm 1 if and only if $b > \frac{2}{3}$ and $\theta \in (\frac{b}{2-b}, 1]$. Each firm i's equilibrium profit is given by $\pi_i^*(\theta,1) = \pi_i(\theta, 1, q_1^*(\theta,1), q_2^*(\theta,1), q_3^*(\theta,1))$.

[21]The denominator, i.e. $\theta(8-8b^2+2b^3) - (1+\theta^2)b^2(2-b)$, is strictly positive whenever $\theta \in (\frac{b}{2-b}, 1]$.

That $\pi_{12}^*(\theta,1)$ is constant for all $\theta \in (\frac{1}{2}, \frac{b}{2-b}]$ follows from noting that $q_i^*(\theta,k)$, $i=1,2,3$, is independent of θ whenever $\theta \in (\frac{1}{2}, \frac{b}{2-b}]$. For all $\theta \in (\max\{\frac{b}{2-b}, \frac{1}{2}\}, 1]$,

$$\frac{\partial}{\partial \theta}\pi_{12}^*(\theta,1) = \frac{A_1[(1+\theta^2)(8-6b^2+b^3)-8\theta(1-b+b^2)]}{[\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)]^3} > 0$$

where $A_1 \equiv 2(1-\theta)(1+\theta)(1-b)b^2((2-b)(a-c)+2x)^2 > 0$. *Q.E.D.*

Claim 5: For any given $\theta \in (\frac{1}{2}, 1]$,
(i) $\pi_1^*(\theta,1) \geq \pi_1^*(\theta,0)$,
(ii)$\pi_{12}^*(\theta,1) \geq \pi_{12}^*(\theta,0)$.

Proof: For $\theta \in (\max\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, \frac{1}{2}\}, 1]$, we find that $q_1^*(\theta,0) = \frac{\theta(2-b)[(2-b(1+\theta))(a-c)+(2+b)x]}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$, $q_2^*(\theta,0) = \frac{\theta(2-b)((2-b)(a-c)-bx)-b((2-b)(a-c)+2x)}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$, and $q_3^*(\theta,0) = \frac{4\theta(1-b)(a-c)+(b^2+\theta b^2-2b\theta)x}{\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)}$. Else, if $\theta \in (\frac{1}{2}, \min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, 1\}]$, $q_1^*(\theta,0) = \frac{(2-b)(a-c)+2x}{4-b^2}$, $q_2^*(\theta,0) = 0$, and $q_3^*(\theta,0) = \frac{(2-b)(a-c)-bx}{4-b^2}$. Each firm i's equilibrium profit corresponding to $(\theta,k) = (\theta,0)$ is given by $\pi_i^*(\theta,0) = \pi_i(\theta,0,q_1^*(\theta,0),q_2^*(\theta,0),q_3^*(\theta,0))$.

Consider (i) first. Note that

$$\begin{aligned}
\pi_1^*(\theta,1) - \pi_1^*(\theta,0) &= \int_0^1 [q_2^*(\theta,k) + \sum_{i=1,2,3} \frac{\partial \pi_1(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))}{\partial q_i}\frac{\partial q_i^*(\theta,k)}{\partial k}]dk, \\
&= \int_0^1 [q_2^*(\theta,k) + \frac{\partial \pi_1(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))}{\partial q_3}\frac{\partial q_3^*(\theta,k)}{\partial k}]dk,
\end{aligned}$$

where the second equality follows from the first-order conditions: $\frac{\partial \pi_1(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))}{\partial q_i} = 0 (i=1,2)$. For any $k \in [0,1]$, we find that either $q_2^*(\theta,k)=0$, $\frac{\partial q_3^*(\theta,k)}{\partial k} = 0$ or else, $q_2^*(\theta,k) > 0$ and $\frac{\partial q_3^*(\theta,k)}{\partial k} = -\frac{bx\theta(2-b-b\theta)}{[\theta(8-8b^2+2b^3)-(1+\theta^2)b^2(2-b)]} < 0$. We also have that $\frac{\partial \pi_1(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))}{\partial q_3} = -b(q_1^*(\theta,k)+\theta q_2^*(\theta,k)) < 0$. Hence, $\frac{\partial \pi_1(\theta,k,q_1^*(\theta,k),q_2^*(\theta,k),q_3^*(\theta,k))}{\partial q_3}\frac{\partial q_3^*(\theta,k)}{\partial k} \geq 0$ which in turn implies (i).

Now we turn to the comparison between $\pi_{12}^*(\theta,1)$ and $\pi_{12}^*(\theta,0)$. Consider $\theta \in (\frac{1}{2}, \max\{\frac{1}{2}, \frac{b}{2-b}\}]$. For all $\theta \in (\frac{1}{2}, \frac{b}{2-b}]$ we have that $\pi_{12}^*(\theta,1) = \pi_{12}^*(\theta,0) = \frac{((2-b)(a-c)+2x)^2}{(4-b^2)^2}$. Next, consider $\theta \in (\max\{\frac{b}{2-b}, \frac{1}{2}\}, \min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, 1\}]$ for which we continue to have $\pi_{12}^*(\theta,0) = \frac{((2-b)(a-c)+2x)^2}{(4-b^2)^2}$. Since (a) $\pi_{12}^*(\frac{b}{2-b},1) = \frac{((2-b)(a-c)+2x)^2}{(4-b^2)^2}$, and (b)$\pi_{12}^*(\theta,1)$ is continuous and strictly increasing in θ for all $\theta \in (\frac{1}{2},1]$, it follows that $\pi_{12}^*(\theta,1) > \pi_{12}^*(\theta,0)$ for all $\theta \in (\max\{\frac{b}{2-b}, \frac{1}{2}\}, \min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, 1\}]$. This completes the proof if $\min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, 1\} = 1$. Else, if $\min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, 1\} = \frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}$, the proof follows from noting that for all $\theta \in (\min\{\frac{b((2-b)(a-c)+2x)}{(2-b)((2-b)(a-c)-bx)}, \frac{1}{2}\}, 1]$, $\pi_{12}^*(\theta,1) - \pi_{12}^*(\theta,0) = A_2 + B_2 x > 0$ where $A_2 = 2b^3 - b^4 - (8b^2-6b^3)\theta + (16-16b-4b^2+4b^3)\theta^2 + 2b^3\theta^3 - (4b^2-2b^3+b^4)\theta^4$, $B_2 =$

$4b^3-2b^4-(8b^2-8b^3+2b^4-b^5)\theta+(16-32b+4b^2+12b^3-4b^4)\theta^2+(4b^3-2b^4+b^5)\theta^3-\theta^4(4b^2-4b^3)$.[22]
Q.E.D.

Proof of Lemma 2: For any given $\theta \in [0, \frac{1}{2}]$, we have that $q_1^*(\theta, k) = \frac{[2-b(1+\theta)](a-c)+[2+b-(b+\frac{2b}{2-b}\theta)k]x}{4+2b-(2+\theta)b^2}$, $q_2^*(\theta, k) = \frac{(2-b)(a-c)-[b-(2+b)k]x}{4+2b-(2+\theta)b^2}$, and $q_3^*(\theta, k) = \frac{(2-b)(a-c)-[b+\frac{b}{2-b}[2-(1+\theta)k]]x}{4+2b-(2+\theta)b^2}$. Then, differentiating $\pi_{12}^*(\theta, 1) \equiv \pi_1^*(\theta, 1)+\pi_2^*(\theta, 1)$ with respect to θ yields that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta, 1) = \frac{b^2(4-3b^2)[(2-b)(a-c)+2x]^2}{(2-b)(4+2b-(2+\theta)b^2)^3}$ $(\frac{4-6b+2b^2}{4-3b^2} - \theta) > 0(=, <)$ 0 if and only if $\theta < (=, >)\frac{4-6b+2b^2}{4-3b^2}$. Define $\bar{\theta}(b) = \min\{\frac{4-6b+2b^2}{4-3b^2}, \frac{1}{2}\}$ and note that $\frac{4-6b+2b^2}{4-3b^2} < \frac{1}{2} \Leftrightarrow 4 - 12b + 7b^2 < 0$. For $b \in (0, 1)$, $4 - 3b^2 > 0$, and $4 - 12b + 7b^2 < 0 \Leftrightarrow b < \frac{6-2\sqrt{2}}{7} \approx 0.453$. This implies the result. *Q.E.D.*

Proof of Proposition 5: We find that $\pi_1^*(\theta, 1)-\pi_1^*(\theta, 0) = \frac{x}{(2-b)^2(4+2b-(2+\theta)b^2)^2}(A_3\theta^2+B_3\theta+C_3)$ where $A_3 = -2b^2[(8(1-b)+b^3)(a-c)+4(1-b)x] < 0$, $B_3 = (2-b)[2(2+b)(2-b)^2(a-c) + ((2+b)(2-b)^2 - b^4)x] > 0$, $C_3 = -b(2-b)^2[(4-2b)(a-c) + (4+b)x] < 0$. Define $\Delta(\theta) = A_3\theta^2+B_3\theta+C_3$. Firm 1 transfers its knowledge to firm 2 in the equilibrium if and only if $\Delta(\theta) \geq 0$ holds. We have that $\Delta(0) = C_3 < 0$. Given $A_3 < 0$, $\Delta'(\theta) = 2A_3\theta+B_3 \geq A_3+B_3$ for all $\theta \in [0, \frac{1}{2}]$. Since $A_3 + B_3 = 2(2-b)(8-4b-6b^2+3b^3+b^4)(a-c) + [8(1-b)(2-b^2) + 4b^3(1-b) + (b^4+b^5)]x > 0$, we have that $\Delta'(\theta) > 0$, for all $\theta \in [0, \frac{1}{2}]$. Furthermore, we find that

$$\Delta(\frac{1}{2}) = (16-32b+22b^2-6b^3+b^4-\frac{b^5}{4})(a-c) + (8-24b+11b^2+3b^3-\frac{5b^4}{2}+\frac{b^5}{2})x > 0$$

for all $x \leq \bar{x} = \frac{2-b}{2b}(a-c)$ as long as $b \leq b'' \approx 0.683$. Else if $b > b''$, $\Delta(\frac{1}{2}) > 0 \Leftrightarrow x < -\frac{16-32b+22b^2-6b^3+b^4-\frac{b^5}{4}}{8-24b+11b^2+3b^3-\frac{5b^4}{2}+\frac{b^5}{2}} \equiv x''(b)$, where $x''(b) \in (0, \bar{x})$.

Define $x_{max}(b) = \bar{x}(x''(b))$ when $b \leq (>)b'' \approx 0.683$. Observe that for all $x \leq x_{max}(b)$, $\Delta(0) < 0$, $\Delta'(\theta) > 0$ and $\Delta(\frac{1}{2}) > 0$. This implies that for all $x \leq x_{max}(b)$ there exists a unique value $\hat{\theta}(x) \in (0, \frac{1}{2})$ such that knowledge transfer occurs, i.e. $\Delta(\theta) > 0$ if and only if $\theta \in [\hat{\theta}(x), \frac{1}{2}]$. If $x \in (x_{max}(b), \bar{x})$, $\Delta(\frac{1}{2}) < 0$ which implies $\Delta(\theta) < 0$ for all $\theta \in [0, \frac{1}{2}]$ (since $\Delta'(\theta) > 0$) and knowledge transfer does not occur.

From $\Delta(\theta) = 0$ we get that $x = \frac{(2-b)[-b^2(4-2b-b^2)\theta^2+(16-8b-4b^2+2b^3)\theta-2b(4-4b+b^2)]}{4b^2(1-b)\theta^2-(16-16b+4b^3-3b^4+b^5)\theta+b(16-12b+b^3)}(a-c) \equiv \tilde{x}(\theta)$ where $\tilde{x}'(\theta) = \frac{b(2-b)^2[b^4(4-4b+b^2+b^3)\theta^2-2b^2(16-16b^2+4b^3+b^4)\theta+64-64b^2+16b^3++20b^4-12b^5+2b^6]}{(4b^2(1-b)\theta^2-(16-16b+4b^3-3b^4+b^5)\theta+b(16-12b+b^3))^2}(a-c) > 0$ for all $\theta \in [0, \frac{1}{2}]$. This implies that $\hat{\theta}(x)$ is strictly increasing in x. *Q.E.D.*

Claim 6: Let $ts^*(\theta, k)$ denote total surplus in the equilibrium of the Stage 3 subgame. For any given $k = \{0, 1\}$, $ts^*(\theta, k)$ is strictly decreasing in θ for all $\theta \in [0, \frac{1}{2}]$.

[22] Though B_2 can be positive or negative, $A_2 > 0$. Clearly $A_2 + B_2x > 0$ if $B_2 > 0$. We find that even if $B_2 < 0$, $A_2 + B_2\bar{x} > 0$ which implies $A_2 + B_2x \geq A_2 + B_2\bar{x} > 0$ for all $x < \bar{x}$.

Proof: For any given (θ, k) we have that

$$ts^*(\theta,k) = U(q_1^*(\theta,k), q_2^*(\theta,k), q_3^*(\theta,k)) - c(q_1^*(\theta,k)+q_2^*(\theta,k)+q_3^*(\theta,k)) + x(q_1^*(\theta,k)+kq_2^*(\theta,k)).$$

Suppose $k = 1$. Substituting $q_i^*(\theta,1)$ from the proof of Claim 4 in the above expression and subsequently differentiating $ts^*(\theta,1)$ with respect to θ we find that for all $\theta \in [0, \frac{1}{2}]$, $\frac{dts^*(\theta,1)}{d\theta} = -\frac{[(2-b)(a-c)+2x][(4-2(2-\theta)b-b^2(1+\theta)-\theta b^3)(a-c)+(8-4(1-\theta)b+2(2-\theta)b^2-(2+3\theta)b^3)x]}{(2-b)(4+2b-b^2(2+\theta))^3} < 0$. The proof of $\frac{dts^*(\theta,0)}{d\theta} < 0$ is analogous. *Q.E.D.*

Claim 7: $\lim_{x\to 0} \hat{\theta}(x) < \bar{\theta}(b)$ holds if and only if $b < \tilde{b} \approx 0.738$

Proof: From Lemma 2 we know that $\bar{\theta}(b) < \frac{1}{2}$ holds if $b > b' \approx 0.453$. The minimum PEO for knowledge transfer, $\hat{\theta}(x)$, is given by the smaller root of the quadratic equation $A_3\theta^2 + B_3\theta + C_3 = 0$ where A_3, B_3 and C_3 are as in the proof of Proposition 5. We have that

$$\lim_{x\to 0} \hat{\theta}(x) = \frac{(2-b)(8-2b^2-2\sqrt{16-8b^2+5b^4+2b^5-8b^3})}{2b^2(4-2b-b^2)},$$

which is strictly less than $\bar{\theta}(b)$ if and only if $b < \tilde{b} \approx 0.738$. *Q.E.D.*

Proof of Proposition 6: We have that $\frac{\partial}{\partial\theta}\pi_{12}^*(\theta,0) = -\frac{45(7(a-c)-3x)}{(112-9\theta)^3}[(73(a-c)-39x)\theta - (28(a-c)-108x)]$, where $112-9\theta > 0$ holds for all $\theta \in [0, \frac{1}{2}]$, and $7(a-c)-3x > 0$, $73(a-c)-39x > 0$, and $73(a-c)-39x > 28(a-c)-108x$ hold for all $x \in (0, \bar{x})$. Noting that $28(a-c)-108x > (=,<)\ 0$ if $x < (=,>)\ x_3 \cong 0.259(a-c)$, define $\breve{\theta}(x)$ by $\breve{\theta}(x) = \frac{28(a-c)-108x}{73(a-c)-39x}$ if $x \in (0, x_3)$, and 0 if $x \in [x_3, \bar{x})$. Then, $\theta = \breve{\theta}(x)$ maximizes the value of $\pi_{12}^*(\theta,0)$ when $\theta \in [0, \frac{1}{2}]$. We find that $\bar{\theta}(0.6) > \hat{\theta}(x)$ for all $x \in (0, x_1)$ and $\bar{\theta}(0.6) \leq \hat{\theta}(x)$ for all $x \in [x_1, \bar{x})$, where $x_1 \cong 0.707(a-c)$. Given this, we define $\theta^*(x)$ by $\theta^*(x) = \bar{\theta}(0.6)$ if $x \in (0, x_1)$ and $\theta^*(x) = \hat{\theta}(x)$ if $x \in [x_1, \bar{x})$. We find that $\pi_{12}^*(\theta^*(x), 1) > \pi_{12}^*(1,1)$ for all $x \in (0, \bar{x})$, and hence Claim 5 implies that $\pi_{12}^*(\theta^*(x), 1) > \pi_{12}^*(\theta, k)$ for all $\theta \in (\frac{1}{2}, 1]$, $k = \{0, 1\}$, and $x \in (0, \bar{x})$. We now establish Claim 8.

Claim 8: $\pi_{12}^*(\theta^*(x), 1) - \pi_{12}^*(\breve{\theta}(x), 0) > 0$ for all $x \in (0, \bar{x})$.

Proof: We first find that,

(i) if $x \in (0, x_3)$, $\pi_{12}^*(\theta^*(x), 1) - \pi_{12}^*(\breve{\theta}(x), 0) = (5/110936)[6482(a-c) + 619x]x > 0$, and

(ii) if $x \in [x_3, x_1)$, $\pi_{12}^*(\theta^*(x), 1) - \pi_{12}^*(\breve{\theta}(x), 0) = (45/36224)(a-c)^2 + (35825/126784)(a-c)x + (82325/1774976)x^2 > 0$.

As in the proof of Proposition 1, we have that $\theta = \hat{\theta}(x) \Leftrightarrow x = \tilde{x}(\theta) \equiv \frac{7(549\theta^2-6370\theta+1470)}{-1800\theta^2+217280\theta-16905}(a-c)$, where $\tilde{x}'(\theta) = \frac{4998(648\theta^2-18585\theta+106085)}{(1800\theta^2-217280\theta+16905)^2}(a-c) > 0$ for all $\theta \in [0, \frac{1}{2}]$. Define $\pi_{12}^{**}(y,1)$ and $\pi_{12}^{**}(y,0)$ by $\pi_{12}^{**}(\theta^*(\tilde{x}(y)), 1) \equiv \pi_{12}^{**}(y,1)$ and $\pi_{12}^{**}(\theta^*(\tilde{x}(y)), 0) \equiv \pi_{12}^{**}(y,0)$. Also, define y_1 and

y_{max} by $\tilde{x}(y_1) = x_1$ and $\tilde{x}(y_{max}) = \bar{x}$, where we find that $y_1 \approx 0.384$ and $y_{max} \approx 0.446$. We then find that,

(iii) if $y \in [y_1, y_{max}) \Leftrightarrow x \in [x_1, \bar{x})$, $\pi_{12}^{**}(y,1) - \pi_{12}^{**}(y,0) = -(15/128)\ [(13181827815y^6 - 332117373420y^5 + 2589364899282y^4 - 7940837629788y^3 + 25636640848735y^2 - 25642413008800y + 4698430464000)/((1800y^2 - 21728y + 16905)^2(112-9y)^2)](a-c)^2$, where $\pi_{12}^{**}(y,1) - \pi_{12}^{**}(y,0) > 0$ holds for all $y \in [y_1, y_{max})$. (i), (ii) and (iii) together imply Claim 8. *Q.E.D.*

Then, Proposition 5, Lemma 2, and Claim 8 together imply the desired result. *Q.E.D.*

Proof of Proposition 7-1: Let $ts^*(\hat{\theta}(x), 1) - ts^*(0,0) \equiv \psi(x)$. As in the proof of Proposition 6, we substitute $x = \tilde{x}(y)$ in $\psi(x)$ and find $\psi(\tilde{x}(y)) = -(15/512)\ [(3422829717y^6 + 149538030012y^5 - 4750899194874y^4 + 28881577705036y^3 + 16957636927245y^2 - 94616152065120y + 21251362406400)/((1800y^2 - 21728y + 16905)^2(112-9y)^2)](a-c)^2$. We find that there exists a unique threshold value $y_{t0} \approx 0.239$ in the relevant range of y such that $\psi(\tilde{x}(y)) < (=, >)\ 0$ if $y < (=, >)\ y_{t0}$, where $\tilde{x}(y_{t0}) \equiv x_2 \approx 0.0118(a-c)$. Claim 8 implies that, if the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1 in the subsequent equilibrium for all $x \in [x_2, \bar{x})$. Then Claim 6 and Proposition 6 together imply (ii) and (iii) of the proposition. Now suppose $x \in (0, x_2)$ so that $ts^*(0,0) > ts^*(\hat{\theta}(x), 1)$. From the proof of Proposition 6, we have $\frac{\partial}{\partial\theta}\pi_{12}^*(0,0) > 0$ for all $x \in (0, x_2)$. This implies that, for all $x \in (0, x_2)$, the antitrust authority must announce $\tilde{\theta} = 0$ in order to induce firms 1 and 2 to choose $\theta = 0$ at Stage 1 in the subsequent equilibrium. This implies (i) of the proposition. *Q.E.D.*

Proof of Propositions 7-2: It can be shown that $ts^*(\hat{\theta}(x), 1) > ts^*(\theta, k)$ for all $\theta \in (\frac{1}{2}, 1]$, $k = \{0, 1\}$, and $x \in (0, \bar{x})$ (see Claim C in Supplementary Note A). We substitute $x = \tilde{x}(y) \equiv \frac{8(19y^2 - 480y + 80)}{-75y^2 + 1912y - 880}(a-c)$ in $ts^*(\hat{\theta}(x), 1)$ and $ts^*(0,0)$ to find that $ts^*(\hat{\theta}(\tilde{x}(y)), 1) - ts^*(0,0) = (-5/196)[(545087y^6 - 15553288y^5 - 250838624y^4 + 7407812096y^3 + 2972174080y^2 - 32346388480y + 5338726400)/((75y^2 - 1912y + 880)^2(y-28)^2](a-c)^2 < (=, >)\ 0$ if $y < (=, >)$ $y_{TS} \cong 0.169$, where $\tilde{x}(y_{TS}) \equiv x_2' \approx 0.00663(a-c)$. As in the proof of Proposition 7-1, we find that $\pi_{12}^*(\hat{\theta}(x), 1) - \pi_{12}^*(\theta, 0) > 0$ holds for all $\theta \in [0,1]$ for any given $x \in [x_2', \bar{x})$. Hence, if the antitrust authority announces $\tilde{\theta} = \hat{\theta}(x)$ at Stage 0, firms 1 and 2 choose $\theta = \hat{\theta}(x)$ at Stage 1 in the subsequent equilibrium for all $x \in [x_2', \bar{x})$. Claim 6 then implies (ii) of the proposition. We also find that $\frac{\partial}{\partial\theta}\pi_{12}^*(0,0) > 0$ holds for all $x \in (0, 0.00663(a-c))$, which implies that, for all $x \in (0, x_2')$, the antitrust authority must announce $\tilde{\theta} = 0$ in order to induce firms 1 and 2 to choose $\theta = 0$ at Stage 1 in the subsequent equilibrium. Claim 6 then implies (i) of the proposition. *Q.E.D.*

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