Australian School of Business Research Paper No. 2014 ECON 16

The Risk Return Relationship: Evidence from Index Return and Realised Variance Series

Minxian Yang

The Risk Return Relationship: Evidence from Index Return and Realised Variance Series

Minxian Yang
School of Economics
UNSW Australia
(The University of New South Wales)
m.yang@unsw.edu.au

Abstract

The risk return relationship is analysed in bivariate models for return and realised variance (RV) series. Based on daily time series from 21 international market indices for more than 13 years (January 2000 to February 2013), the empirical findings support the arguments of risk return tradeoff, volatility feedback and statistical balance. It is reasoned that the empirical risk return relationship is primarily shaped by two important data features: the negative contemporaneous correlation between the return and RV, and the difference in the autocorrelation structures of the return and RV.

Keywords: risk premium, volatility feedback, return predictability, realised variance model, statistical balance

JEL Classification: C32, C52, G12, G10

First Version: February 2014
This Version: March 2014

1. Introduction

We argue that the empirical risk return relationship in portfolio return and realised variance (RV) series is largely conveyed by two salient data features: (a) the contemporaneous correlation (CC) between the return and RV is negative; and (b) the RV has much stronger autocorrelations than the return. Feature (a) implies that high volatilities are associated with price falls or negative returns, which leads to a negative term in the expected return (i.e., the conditional mean return). Hence, a positive risk premium is required to compensate the expected loss from holding the portfolio for a high-volatility period. Feature (b) implies that the conditional volatility of the return also has strong autocorrelations and cannot have predictive power for the weakly-autocorrelated return (see Christensen and Nielsen (2007)). Consequently, in the expected return, the positive risk premium must precisely offset the negative effect induced by the CC. The above argument is tested in our empirical analysis, where econometric models explicitly accommodate data features (a) and (b).

We examine the risk return relationship in daily and weekly index return and RV series by using bivariate normal variance-mean mixture models. The excess returns (referred to as returns hereafter) and RVs of 21 international market indices, from 2000-01-03 to 2013-02-05, in the *Realised Library* of Heber, Lunde, Shephard and Sheppard (2009) are analysed. The data features (a) and (b) are prominent for all indices considered, see Tables 1 and 4. Our estimation results support the argument outlined in the previous paragraph. Specifically, for almost all of 21 markets in the data set, we find that in the expected return: (i) there is a significantly positive risk premium effect; (ii) there is a significantly negative effect induced by the CC between the returns and RVs; (iii) the conditional volatility does not have predictive power; and (iv) the short-memory component of the volatility does not have predictive power. Finding (i) supports the risk return tradeoff implied by the intertemporal capital asset pricing model of Merton (1973) in that the risk premium effect is formulated in terms of the conditional volatility (variance or standard deviation) itself. Finding (ii) is a reflection of data feature (a) and can be interpreted as the volatility feedback effect, see Yang (2011). Finding (iii) conforms to the statistical balance argument that a strongly-autocorrelated variable (e.g., volatility) does not predict a weakly-autocorrelated variable (e.g., return), see Christensen and Nielsen (2007). Finding (iv) is in contrast to the positive relationship found in the expected S&P 500 return and the lagged short-memory component of the VIX (implied volatility), see Christensen and Nielsen (2007) and Bollerslev, Osterrieder, Sizova and Tauchen (2013). Our findings are qualitatively insensitive to variations in econometric models (two bivariate models are considered), in functional forms of the short-memory component of volatility in the expected mean (two functional forms are considered), and in sampling frequencies (daily and weekly frequencies are considered).

In the literature, while the importance of this risk return relationship has attracted many empirical investigations, the evidence from time series data is still mixed. In the earlier studies with return series, the relationship between the expected return and the conditional volatility is found to be positive by some authors but insignificant or negative by others, depending on data and model specifications, see the references in Ghysels, Santa-Clara and Valkanov (2005) and Lundblad (2007) among others.

More recently, Ghysels et al (2005) argue that conflicting empirical results from earlier studies are attributable to the difficulties in quantifying the conditional volatility and propose that the monthly conditional variance is estimated as a weighted average of squared daily returns in the previous month. Using this approach, they find that the expected return is positively related to the conditional variance for the monthly CRSP value-weighted market return series. Lundblad (2007) reasons that the empirical findings are mixed because the samples used are too small to allow for reliable inference. He demonstrates by simulation that the GARCH-type models cannot lead to reliable conclusions unless a long series (with at least 2000 monthly observations) is used. He then finds a positive effect of the conditional variance on the expected return by using GARCH-type models with a long monthly U.S. market return series.

Christensen and Nielsen (2007) point out that the conditional-volatility-in-mean-type models are not statistically balanced because returns are of short memory while volatilities are typically of long memory. They suggest that the risk return relationship be specified in terms of the short-memory component of the volatility (i.e., the shock to the volatility) and find that the expected S&P 500 return is positively related to the lagged short-memory component of the VIX index. The same positive relationship is reported by Bollerslev et al (2013), who also find a positive relationship between the expected S&P 500 return and the lagged difference between the VIX and RVs. However, they detect a negative relationship between the expected S&P 500 return and the lagged short-memory component of the RV. The approaches of Christensen and Nielsen (2007) and Bollerslev et al (2013) have the merit of statistical balance, while the risk-return-tradeoff specifications of Ghysels et al (2005) and Lundblad (2007), which are expressed in terms of the conditional variance itself, are consistent with the theoretical form suggested by Merton (1973).

With univariate GARCH-type models that have normal variance-mean mixture distributions, Yang (2011) shows that when the return is contemporaneously correlated with its volatility, the expected return is subject to the CC effect[1] in addition to the conventional risk premium effect. He finds that the two effects, which are significant with opposite signs, are nullified in the expected return for the CRSP value-weighted portfolio return series at daily frequency. Wang and Yang (2013) substantiate the results of Yang (2011) with the G7 market return series. Additionally, they document that there is little evidence in the G7 data for non-monotone relationships between the expected return and the conditional volatility (see Backus and Gregory (1993) and Rossi and Timmermann (2010)).

Building on the above literature, the current study also borrows from the recent development in the joint models of the return and RV (Hansen, Huang and Shek (2012) and Corsi, Fusary and Vecchia (2013)) and takes advantage of the availability of RV data (Heber et al (2009)). The bivariate models we consider utilise the intraday information (via RV) to improve the accuracy in quantifying the conditional variance since the RV is much more informative about the volatility than the realised return itself (see Andersen, Bollerslev, Diebold and Labys (2003) among others). Important data features, as described in the first paragraph of this section, are accounted for in our models. Specifically, the normal variance-mean mixtures (see Yang (2011) for univariate models and Corsi et al (2013) for a bivariate model) are used to acknowledge the CC between the return and RV. The HAR model of Corsi (2009) is adopted to deal with the strong autocorrelations in the RV. As a result, the idea that the risk premium is associated with the short-memory component of the volatility (Christensen and Nielsen (2007)) is readily incorporated in our models.

As the volatility is tangible via the RV, our bivariate models provide an ideal framework to accommodate Yang's (2011) argument that the expected return is influenced by both the risk premium and the CC between the return and the volatility. Indeed, in an efficient market, the joint effect of the risk premium and the CC on the expected return should be zero from the viewpoint of either market efficiency or statistical balance. Both require that the weakly-autocorrelated return be unpredictable by the strongly-autocorrelated volatility that is based on public information. Empirically, we find that the hypothesis of the joint effect of the risk premium and the CC on the expected return being zero cannot be rejected for almost all 21 market indices considered in this study. Part of the appeal of our approach is that the risk

[1]Yang (2011) interprets the effect of the CC between the return and volatility as the volatility feedback of French, Schwert and Stambaugh (1987), which describes the phenomenon that bad news (price fall or negative return) is contemporaneously associated with high volatility.

premium effect is defined in terms of the conditional volatility level (compatible with Merton's (1973) theoretical form) on the one hand and the expected return is allowed to be unaffected by the conditional volatility (compatible with statistical balance) on the other. Our approach, which has not been used in the literature for studying the risk return relationship in the bivariate context of return and RV series, provides an alternative angle to explain and interpret the conflicts in the time series evidence on the risk return relationship.

Limited by sample sizes, our empirical findings are based on daily and weekly series and are short term in nature. Our findings, born out of two data features discussed in the first paragraph of this section, may shed light on the risk return relationships at lower frequencies. For instance, if both data features (a) and (b) are present at monthly frequency, similar conclusions are expected to hold. A key point is that both features (a) and (b), if present, need to be accounted for in modelling the risk return relationship. We note that our short-term analysis at daily and weekly frequencies has an advantage in mitigating the impact of variations in the investment opportunity set[2].

The rest of the paper is organised as follows. Section 2 details the two models used in this study. Section 3 describes data. Estimation results and inferences are reported in Section 4. Concluding remarks are contained in Section 5. References, tables and figures are at the end of this paper.

2. Models

Let x_t be the daily close-to-close return of an asset in excess of the risk-free interest rate (simply return hereafter) and y_t be the daily open-to-close realised variance (RV) of the return at the end of day t. The observable information set generated by $\{x_t, y_t; x_{t-1}, y_{t-1}; \dots\}$ is denoted by $\mathcal{I}_t$. The RV y_t is known as an estimate of the integrated variance. Because no trading is recorded overnight, y_t generally under-estimates the daily close-to-close integrated variance when it is an unbiased estimate of the open-to-close integrated variance. In what follows, $y_{w,t} = \frac{1}{4}\sum_{i=1}^{4} y_{t-i}$ and $y_{m,t} = \frac{1}{17}\sum_{i=5}^{21} y_{t-i}$ are called weekly and monthly RVs. Two well-known empirical characteristics are of interest for jointly modelling (x_t, y_t), see Andersen, Bollerslev, Diebold and Labys (2003) and Andersen, Bollerslev, Frederiksen and

[2] Merton (1973) derives a theoretical relationship that links the conditional mean return to the conditional variance and the conditional covariance with variation in the investment opportunity set. Most studies in this literature implicitly assume that the investment opportunity set does not change (hence the covariance term drops from the conditional mean). Arguably, the covariance term can no longer be ignored for long horizons.

Nielsen (2010) among others. First, y_t has long memory in the sense that its autocorrelation decays to zero slowly. Second, the distribution of $x_t/y_t^{1/2}$ is much closer to a normal distribution than that of x_t. In what follows, we consider two normal variance-mean mixture models for the pair (x_t, y_t). These models are capable of capturing the contemporaneous correlation between x_t and y_t and the strong autocorrelations in y_t. As the purpose of this paper is to examine the risk return relationship in the bivariate models of (x_t, y_t), the RV is treated as an observable that is intimately connected to the conditional variance of x_t. However, no effort is made to separate the continuous and jump components of the RV.

2.1 Non-central Gamma Model

This is an extended version of the model of Corsi et al (2013), where the conditional distribution of the realised variance is assumed to be the autoregressive (AR) Gamma model of Gourieroux and Jasiak (2006). Specifically,

$$(1) \qquad x_t|\mathcal{I}_{t-1}, y_t \sim \mathrm{N}(\mu_t + \beta y_t, \psi y_t) , \qquad \psi > 0 ,$$

$$y_t|\mathcal{I}_{t-1} \sim \mathrm{NG}(\delta, \lambda_t, c) , \qquad \delta > 0 , \qquad c > 0 ,$$

$$\lambda_t = a_1 y_{t-1} + a_2 y_{w,t-1} + a_3 y_{m,t-1} + a_4 l_{t-1} , \qquad a_i \geq 0 ,$$

where μ_t and λ_t are functions of the information set $\mathcal{I}_{t-1}$; $l_{t-1} = y_{t-1}$ if $x_{t-1} < 0$ and 0 otherwise; $\mathrm{NG}(\delta, \lambda_t, c)$ is the non-central gamma distribution with δ, λ_t and c being the shape, non-centrality and scale parameters respectively; $(\beta, \psi, \delta, c, a_1, a_2, a_3, a_4)$ are constant parameters. The non-central gamma distribution in (1) implies

$$(2) \qquad \mathrm{E}(y_t|\mathcal{I}_{t-1}) = c\delta + c\lambda_t \qquad \text{and} \qquad \mathrm{var}(y_t|\mathcal{I}_{t-1}) = c^2\delta + 2c^2\lambda_t$$

(see Gourieroux and Jasiak (2006)). The inclusion of $y_{w,t-1}$ and $y_{m,t-1}$ in λ_t is a pragmatic way to explain the strong autocorrelations of y_t (see the HAR model of Corsi (2009) and Andersen, Bollerslev, and Diebold (2007) among others). The presence of l_{t-1} in λ_t captures the leverage effect (i.e., negative x_{t-1} leads to greater conditional volatility than positive x_{t-1}). While μ_t is a constant in Corsi et al (2013), it is extended here as a function of $\mathcal{I}_{t-1}$ to account for the risk return tradeoff effect

$$(3) \qquad \mu_t = m_0 + m_1\lambda_t + m_2\eta_{t-1} + \varphi x_{t-1} , \qquad \eta_{t-1} = y_{t-1} - (c\delta + c\lambda_{t-1}) ,$$

where (m_0, m_1, m_2, φ) are constant parameters. Specifically, m_1 is the effect of the traditional risk premium and m_2 the effect of the short-memory component of y_{t-1}. The lagged return x_{t-1} is included in μ_t to account for the return's autocorrelation that is not caused by the volatility-related measurements λ_t or η_{t-1}. As x_t is the close-to-cloes return and y_t is the open-to-close realised variance, the specification $\mathrm{var}(x_t|\mathcal{I}_{t-1}, y_t) = \psi y_t$ allows the instantaneous variance $\mathrm{var}(x_t|\mathcal{I}_{t-1}, y_t)$ to differ from y_t. Clearly, when $\psi = 1$, $\mathrm{var}(x_t|\mathcal{I}_{t-1}, y_t)$ reduces to that of Corsi et al (2013).

The return in (1) may be alternatively written as

(4) $$x_t = \mu_t + \beta y_t + \psi^{1/2} y_t^{1/2} \xi_t \ ,$$

where $\xi_t \sim iid\ \mathrm{N}(0,1)$ and independent of y_t. Given $\mathcal{I}_{t-1}$, the quantity $(x_t - \mu_t)$ carries new information. The contemporaneous correlation (CC) between the return and RV is captured by the parameter β that determines the sign of the CC. In the presence of the risk premium effect, the CC between the return and the volatility may be interpreted as the volatility feedback effect of French et al (1987), see Yang (2011). It can be verified that

(5) $$\mathrm{E}(x_t|\mathcal{I}_{t-1}) = \mu_t + \beta c\delta + \beta c\lambda_t = \varphi x_{t-1} + (m_0 + \beta c\delta) + (m_1 + \beta c)\lambda_t + m_2\eta_{t-1} \ ,$$

$$\mathrm{var}(x_t|\mathcal{I}_{t-1}) = (\beta^2 c + \psi)c\delta + (2\beta^2 c + \psi)c\lambda_t \ ,$$

i.e., the conditional mean is linearly related to the conditional variance, consistent with Merton (1973). The impact of λ_t (or $\mathrm{var}(x_t|\mathcal{I}_{t-1})$) on the conditional mean, $m_1 + \beta c$, is the sum of the risk premium effect m_1 and the volatility feedback effect βc. Note that the CC has the same sign as β: $\mathrm{corr}(x_t, y_t|\mathcal{I}_{t-1}) = \beta[\mathrm{var}(y_t|\mathcal{I}_{t-1})/\mathrm{var}(x_t|\mathcal{I}_{t-1})]^{1/2}$. Hence the joint effect $m_1 + \beta c$ is identified (or signalled) by variations in the conditional mean of x_t whereas βc by contemporaneous co-variations between x_t and y_t. To be consistent with data features, neither m_1 nor βc can be dropped because the latter captures the CC while the former is required to establish the statistical balance. To examine the risk return relationship, the main parameters of interest are βc, m_1, m_2 and $m_1 + \beta c$.

The non-central gamma distribution $\mathrm{NG}(\delta, \lambda, c)$ is in fact a mixture of (centred) Gamma distributions, $\mathrm{Gamma}(\delta + k, 1)$, with Poisson probability weights $p_k = e^{-\lambda}\lambda^k/k!$ for $k = 0,1,2, \ldots$. The probability density function (PDF) of y being $\mathrm{NG}(\delta, \lambda, c)$ is given by

(6) $$\mathrm{pdf}_{\mathrm{NG}}(y|\delta, \lambda, c) = \frac{1}{c}\left(\frac{y}{c}\right)^{\delta-1} \exp\left(-\frac{y}{c} - \lambda\right) \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(\delta+k)} \left(\frac{y}{c}\lambda\right)^k \ ,$$

where $\Gamma(\cdot)$ is the gamma function. Let $\mathrm{pdf}_{\mathrm{N}}(\cdot)$ be the PDF of $\mathrm{N}(0,1)$. Then the joint conditional PDF of (x_t, y_t) given $\mathcal{I}_{t-1}$ can be expressed as

(7) $$\mathrm{pdf}(x_t, y_t|\mathcal{I}_{t-1}) = \mathrm{pdf}(x_t|y_t, \mathcal{I}_{t-1})\mathrm{pdf}(y_t|\mathcal{I}_{t-1})$$

$$= \mathrm{pdf_N}\big(\xi_t(\theta)\big)\mathrm{pdf_{NG}}(y_t|\delta,\lambda_t,c)|J_t|$$

$$= \mathrm{pdf_N}\big(\xi_t(\theta)\big)\mathrm{pdf_{NG}}(y_t|\delta,\lambda_t,c)\big(\psi^{1/2}y_t^{1/2}\big)^{-1} \ ,$$

where θ is the vector of parameters to be estimated, $\xi_t(\theta) = (x_t - \mu_t - \beta y_t)/(\psi^{1/2}y_t^{1/2})$, and $J_t = \big(\psi^{1/2}y_t^{1/2}\big)^{-1}$ is the Jacobian of the transformation from x_t to $\xi_t(\theta)$. As the functional form of (7) is known, the maximum likelihood (ML) can readily be carried out to estimate θ. The infinite sum in (6) needs to be truncated in computing the log likelihood. Corsi et al (2013) suggest truncating terms with $k > 90$. The empirical results reported in Section 4.1 of this paper are based on truncating terms with $k > 299$.

2.2 Log Normal Model

This model may be viewed as a further extension of Corsi et al (2013) to the cases where the RV is conditionally log normal. The model can be expressed as

(8) $$x_t = \mu_t + B_t\sigma_t^2 + \sigma_t\xi_t \ , \qquad \xi_t \sim iid \ \mathrm{N}(0,1) \ ,$$

$$\ln(y_t) = \psi_0 + \psi_1 \ln(h_t^2) + \eta_t \ , \qquad \eta_t \sim iid \ \mathrm{N}(0,\gamma) \ , \qquad \gamma > 0 \ ,$$

$$\ln(\sigma_t^2) = \rho_0 + \rho_1 \ln(h_t^2) + \rho_2\eta_t \ ,$$

where $h_t^2 = \mathrm{var}(x_t|\mathcal{I}_{t-1})$, μ_t and B_t are functions of $\mathcal{I}_{t-1}$, σ_t^2 is the instantaneous variance of the return, ξ_t is independent of $(\mathcal{I}_{t-1}, y_t)$, and η_t is independent of $\mathcal{I}_{t-1}$. Similar to Corsi et al (2013), the returns is the normal variance-mean mixture $x_t|(\mathcal{I}_{t-1}, y_t) \sim \mathrm{N}(\mu_t + B_t\sigma_t^2,\ \sigma_t^2)$. Differing from Corsi et al (2013), the conditional distribution of the RV y_t is log-normal. Similar to Hansen et al (2013), the RV y_t is specified to be a linear function of the log conditional variance of x_t and the volatility shock η_t that represents news arrivals. The parameters (ψ_0, ψ_1) remedies the discrepancy that y_t is the open-to-close RV whereas h_t^2 is the conditional variance of the close-to-close return x_t. The instantaneous variance σ_t^2 is the counterpart of "ψy_t" in Section 2.1. Being a simple combination of $\ln(h_t^2)$ and η_t (or equivalently y_t), σ_t^2 is also conditionally log-normal. Obviously, σ_t^2 reduces to y_t when $(\rho_0, \rho_1, \rho_2) = (\psi_0, \psi_1, 1)$. In general, as both y_t and σ_t^2 are subject to the same news about the volatility, $\rho_2 > 0$ holds. That σ_t^2 is different from y_t affords certain flexibility in standardising the return x_t. Andersen et al (2010) document that majority of the standardised returns of 30 DJIA stocks do not reject the normality when the effects of jumps and return-volatility correlations are accounted for. In our setting, where jumps are not separately treated, the flexibility in σ_t^2 is expected to improve the empirical fit of the normality assumption for the standardised shock ξ_t.

The fact that h_t^2 is the conditional variance of x_t places some restrictions on the parameters (B_t, ρ_0, ρ_1). To see these, the conditional variance of x_t is expressed as

(9) $$h_t^2 = \mathrm{var}(x_t|\mathcal{I}_{t-1}) = e^{\bar{\gamma}}(e^{\bar{\gamma}} - 1)e^{2\rho_0}B_t^2 h_t^{4\rho_1} + e^{0.5\bar{\gamma}}e^{\rho_0}h_t^{2\rho_1},$$

where $\bar{\gamma} = \rho_2^2\gamma$. Clearly, the following restrictions must hold:

(10) $$B_t = \beta/h_t^{\rho_1}, \qquad \rho_1 = 1, \qquad e^{\bar{\gamma}}(e^{\bar{\gamma}} - 1)\beta^2 e^{2\rho_0} + e^{0.5\bar{\gamma}}e^{\rho_0} = 1,$$

where β is a constant. Let (β, γ, ρ_2) be free parameters. Then, e^{ρ_0} must be the positive root of the last equation, i.e.,

(11) $$e^{\rho_0} = \left[-e^{0.5\bar{\gamma}} + \sqrt{e^{\bar{\gamma}} + 4\beta^2 e^{\bar{\gamma}}(e^{\bar{\gamma}} - 1)}\right]/[2\beta^2 e^{\bar{\gamma}}(e^{\bar{\gamma}} - 1)]$$

if both $\bar{\gamma} > 0$ and $\beta \neq 0$ and $e^{\rho_0} = e^{-0.5\bar{\gamma}}$ if either $\bar{\gamma} = 0$ or $\beta = 0$. Given these restrictions, the model can be expressed as

(12) $$x_t = \mu_t + \beta\sigma_t^2/h_t + \sigma_t\xi_t\,, \qquad \xi_t \sim \mathrm{N}(0,1)\,,$$

$$\ln(y_t) = \psi_0 + \psi_1 \ln(h_t^2) + \eta_t\,, \qquad \eta_t \sim iid\,\mathrm{N}(0,\gamma)\,, \quad \gamma > 0\,,$$

$$\sigma_t^2 = e^{\rho_0}h_t^2 e^{\rho_2\eta_t}\,,$$

where e^{ρ_0} is a function of (β, γ, ρ_2) as defined by (11). To close the model, the functional forms for μ_t and h_t^2 are specified as

(13) $$\mu_t = m_0 + m_1 h_t + m_2 e^{\eta_{t-1}} + \varphi x_{t-1}\,,$$

$$\ln h_t^2 = b_0 + b_1 \ln h_{t-1}^2 +$$

$$a_1 \ln y_{t-1} + a_2 \ln y_{w,t-1} + a_3 \ln y_{m,t-1} + a_4 x_{t-1} + a_5|x_{t-1}|\,,$$

where m_1 is the effect of the conventional risk return tradeoff effect, m_2 is the effect of the short-memory part of the RV, φx_{t-1} captures the return's autocorrelation caused by factors other than h_t and η_{t-1}, (a_4, a_5) provide a measure for the leverage effect, and (a_1, a_2, a_3) are the HAR parameters (see Corsi (2009)) that account for the RV's strong autocorrelations. It follows that the conditional mean of the return is

(14) $$E(x_t|\mathcal{I}_{t-1}) = m_0 + (m_1 + \beta c_1)h_t + m_2 e^{\eta_{t-1}} + \varphi x_{t-1}\,,$$

where $c_1 = e^{\rho_0 + 0.5\bar{\gamma}}$. Similar to the non-central gamma model, the effect of h_t on the expected return is the sum of the effects of the risk premium (m_1) and the CC between the return and the RV (βc_1). Again, $m_1 + \beta c_1$ is identified by variations in the conditional mean of x_t whilst βc_1 is identified by contemporaneous co-variations between x_t and y_t. When x_t and y_t are of short and long memory respectively, $m_1 + \beta c_1 = 0$ is required to maintain statistical balance. It can be shown that

(15) $$\operatorname{cov}(x_t, y_t | \mathcal{I}_{t-1}) = \beta\left(e^{0.5(\rho_2+1)^2\gamma} - e^{0.5(\rho_2^2+1)\gamma}\right)e^{\rho_0+\psi_0} h_t^{1+2\psi_1} ,$$

i.e., the sign of the contemporaneous covariance between the return and realised variance is determined by the sign of β when $\rho_2 > 0$ (which is true for the empirical results in Section 4). To examine the risk return relationship, the main parameters of interest are βc_1, m_1, m_2 and $m_1 + \beta c_1$.

As the distribution of $\ln y_t | \mathcal{I}_{t-1}$ is $\mathrm{N}(\psi_0 + \psi_1 \ln h_t^2 ,\ \gamma)$, the conditional PDF of (x_t, y_t) for given $\mathcal{I}_{t-1}$ can be written as

(16) $$\begin{aligned} \operatorname{pdf}(x_t, y_t | \mathcal{I}_{t-1}) &= \operatorname{pdf}(y_t | \mathcal{I}_{t-1}) \operatorname{pdf}(x_t | y_t, \mathcal{I}_{t-1}) \\ &= \operatorname{pdf}_{\mathrm{N}\gamma}\big(\eta_t(\theta)\big) \operatorname{pdf}_{\mathrm{N}}\big(\xi_t(\theta)\big) |J_t(\theta)| \\ &= \operatorname{pdf}_{\mathrm{N}\gamma}\big(\eta_t(\theta)\big) \operatorname{pdf}_{\mathrm{N}}\big(\xi_t(\theta)\big) \left|\frac{1}{\sigma_t(\theta) y_t}\right| , \end{aligned}$$

where $\operatorname{pdf}_{\mathrm{N}\gamma}$ and $\operatorname{pdf}_{\mathrm{N}}$ are the densities of $\mathrm{N}(0, \gamma)$ and $\mathrm{N}(0, 1)$ respectively, θ is the vector of all parameters to be estimated,

$$\xi_t(\theta) = (x_t - \mu_t - \beta \sigma_t^2(\theta)/h_t)/\sigma_t(\theta) ,$$

$$\eta_t(\theta) = \ln(y_t) - \psi_0 - \psi_1 \ln(h_t^2) ,$$

$$\sigma_t^2(\theta) = e^{\rho_0} h_t^2 e^{\rho_2 \eta_t(\theta)} ,$$

$J_t(\theta)$ is the Jacobian of the transformation from (x_t, y_t) to $(\xi_t(\theta), \eta_t(\theta))$. Based on (16), the parameters can readily be estimated by the maximum likelihood method.

3. Data

The index returns and realised variances are obtained from the *Realised Library* of Heber et al (2009). The data include 21 indices ranging from 2000-01-03 to 2013-02-05, with some indices having shorter ranges (S&P-CNX and S&P-TSX). The interest rates used to calculate excess returns are obtained from Datastream. The interest rates are mainly local 3-month rates from the countries where the indices are measured. The excess return x_t is measured as the difference between the daily log return (close-to-close) and the daily interest in daily percentages. The realised variances (RV) are the kernel estimates (see Barndorff-Nielsen, Hansen, Lunde and Shephard (2008)), scaled as squared daily percentages. For the FT-Straits-Times index, as the observations of the two months between 2007-12-28 and 2008-03-03 are missing, the close-to-close return on 2008-03-03, being the difference between the log close prices of 2008-03-03 and 2007-12-28, is adjusted by a division of 44 (the number of days in the gap).

The summary statistics of the excess returns and the associated log RVs are given in Table 1. For all indices, the contemporaneous correlation between the excess return and the log RV is negative and significant (judged by the Bartlett's bands $\pm 2T^{-1/2}$). Further, consistent with previous findings (see Andersen et al (2003) and Corsi (2009) among others), all log RVs exhibit strong autocorrelation or long memory indicated by enormous Ljung-Box Q-statistics. While all return series also have sizeable autocorrelations indicated by Q-statistics, they are much weaker than those of the log RVs. As argued in Section 1, the risk return relationship is primarily embedded in these important data features, which our models will accommodate.

In Table 1, additional characteristics in the return series include: near-zero mean, large standard deviation, negative skewness, large kurtosis. These are consistent with the well-known features for asset return series (see Bollerslev, Engle and Nelson (1994) among others). Moreover, for each log RV (except IBEX35), while the kurtosis is typically not far from 3, the skewness is positive and large.

4. Results

The estimation results for all 21 indices are presented in Tables 2 to 5. Each table is divided into three panels (a, b and c), roughly in accordance with the geographical location of each index.

4.1 Results for Non-central Gamma Model

The estimation results for the Non-central Gamma (NG) model are reported in Table 2. In (5), the effects of the conditional variance and the lagged short-memory part of the RV on the expected return are summarised by the key parameters $m_1 + \beta c$ and m_2 respectively.

First, the estimates of $m_1 + \beta c$ are statistically zero at the 5% level for all indices except S&P-CNX, whilst the estimates of m_1 and βc are all statistically significant. Here the risk premium effect ($m_1 > 0$) offsets the volatility feedback effect ($\beta c < 0$). This confirms the requirement of statistical balance: $m_1 + \beta c = 0$. The magnitudes of $m_1 + \beta c$ are typically much smaller than those of either m_1 or βc.

Second, the estimates of m_2 are statistically zero at the 5% level for all indices (except Hang Seng), providing little support for the hypothesis that the lagged short-memory part of the RV, defined as $\eta_{t-1} = y_{t-1} - E(y_{t-1}|\mathcal{I}_{t-2})$, has a positive effect on the expected return. However, the insignificance of η_{t-1} could be a consequence of the remaining autocorrelations in η_t.

Third, for all market indices, η_t have substantial autocorrelations that are summarised by the Ljung-Box $Q_{30}(\eta)$ statistics, although they are much smaller than the Q_{30} statistics of the RVs (the former range from 1.3% to 6.1% of the latter). Because η_t by definition should be a martingale difference process, the remaining autocorrelations in η_t is an indication of certain misspecifications in the RV equation in (1). For this reason, the results from the log normal model in Section 4.2 are preferable. As the autocorrelations in the standardised shock ξ_t, measured by the $Q_{30}(\xi)$ statistics, are small, the return equation in (1) appears to be reasonably adequate for this data set.

Additionally, the estimates of ψ are statistically greater than one for all series, signalling that it is beneficial to adjust the open-to-close realised variance for the purpose of standardising the close-to-close return. The estimates of a_4 in the HAR specification for λ_t are all significantly positive, confirming the presence of the leverage effect.

4.2 Results for Log Normal Model

The estimation results for the log normal model are presented in Table 3. The results are largely consistent with the findings in the previous section, whilst the log normal model fits data better than the non-central gamma model (judged by the remaining autocorrelations in standardised shocks). The parameters of interest are $m_1 + \beta c_1$ and m_2 in (14).

First, the estimates of $m_1 + \beta c_1$ are statistically insignificant at the 5% level for all indices except FTSE-MIB and S&P-CNX. The magnitudes of $m_1 + \beta c_1$ estimates are negligible in comparison with the estimates of m_1 and βc_1, which are both statistically significant, for all indices. These estimates are consistent with the arguments of risk return tradeoff ($m_1 > 0$), volatility feedback ($\beta c_1 < 0$) and statistical balance ($m_1 + \beta c_1 = 0$). An interpretation is that the risk premium for holding a portfolio in a high volatility day precisely compensates the expected price fall associated with high volatility.

Second, the estimates of m_2 are statistically insignificant at the 5% level for all indices except FTSE100, Swiss and IBEX35. For these three exceptions, the m_2 estimates are positive with magnitudes comparable to those of $m_1 + \beta c_1$, but much smaller than those of m_1. Hence, there is little supporting evidence for the argument that the risk premium effect is rendered by the short-memory part of the volatility in this data set with the log normal model.

Third, if the model fits data perfectly, the shocks ξ_t and η_t will have no autocorrelations by definition. Indeed, the autocorrelations in ξ_t and η_t are small as their Ljung-Box Q_{30} statistics are much smaller than those of x_t and $\ln(y_t)$ for all indices. For example, the $Q_{30}(\eta)$ statistics range from 0.074% to 0.229% of the Q_{30}statistics of $\ln(y_t)$. In this sense, the log normal model generally fits the data well and captures the major dynamic features of the returns and the log RVs.

Additionally, the estimates of ρ_2 are all positive and the estimates of (ψ_0, ψ_1) are statistically different from (0, 1) at the 5% level for all indices, highlighting the difference between $\mathrm{E}(\ln(y_t)|\mathcal{I}_{t-1})$ and $\ln(\mathrm{var}(x_t|\mathcal{I}_{t-1}))$. The estimates of a_4 in the specification of $\ln(\mathrm{var}(x_t|\mathcal{I}_{t-1}))$ are all significantly negative at the 5% level, confirming the presence of the leverage effect in all indices. Further, Figure 1 presents the histograms for ξ_t and η_t of the S&P 500 index, where their distributions are visually close to normality. In fact, other histograms (not presented) suggest that the distribution of ξ_t is closer to normality than that of η_t for all indices considered.

Overall, the in-sample fit of the log normal model is superior to that of the non-central gamma model, in the sense of capturing the dynamic features of the data (judged by the autocorrelations remained in the standardised residuals ξ_t and η_t). Given that both models lead to the same conclusion about the risk return relationship, our results appear to be robust to choices between the two models considered. In what follows, we further consider a variation in the functional form of η_{t-1} and a variation in sampling frequency respectively for the log normal model, which is our preferred model.

4.3 Quadratic Short-Memory Volatility in Mean

In addition to (13), an alternative version of μ_t, which includes a quadratic function of the short memory volatility η_{t-1}

$$\mu_t = m_0 + m_1 h_t + m_2 \eta_{t-1} + m_3 \eta_{t-1}^2 + \varphi x_{t-1},$$

is also estimated as a robustness check. The estimation results lead to the same conclusions as in Section 4.2 (hence the details are not presented). Of interest are the estimates of (m_2, m_3), which are only jointly statistically significant at the 5% level for Nasdaq100, Swiss and FT-Straits-Times with the Wald statistic p-values being 0.034, 0.0038 and 0.024 respectively. The estimates of (m_2, m_3) are both positive for Swiss and FT-Straits-Times, whereas they have opposite signs for Nasdaq100. For all three indices, the magnitudes of (m_2, m_3) are much smaller than those of m_1. Hence the conclusion in Section 4.2 that η_{t-1} has little effect on the expected return appears to be insensitive to the variations in functional forms considered (exponential η_{t-1} versus quadratic η_{t-1}).

4.4 Weekly Data

The log normal model is also estimated for the same data set at the weekly frequency (based on the end-of-Friday observations). While the model specifications in (11)-(13) are valid, the symbols $(y_t, y_{w,t}, y_{m,t})$ now represent the (weekly, monthly, quarterly) RVs respectively. The weekly RV is defined as the sum of the daily RVs within the week. The monthly and quarterly RVs are defined respectively as the averages of the current and 3 and 15 previous weekly RVs. Also, x_t represents the weekly (Friday-close to Friday-close) excess return.

The descriptive statistics of the weekly returns and log realised variances are given in Table 4. The data characteristics summarised in Section 3 are all present in Table 4. For all indices, the CC between the return and log(RV) is negative and the autocorrelation in the return is much weaker than that of the RV. The autocorrelations of the weekly returns appear to be weaker than those of the daily returns. According to the Q15 statistics in Table 4, thirteen of the 21 weekly returns reject the null of no autocorrelation at the 5% level of significance, whereas nineteen of the 21 daily returns reject according to the Q30 statistics in Table 1.

The conclusions based on Table 3 are all supported by the estimation results presented in Table 5. In particular, at the 5% level of significance, m_1 is significantly positive for all but Nikkei and $m_1 + \beta c_1$ is insignificant for all but IBEX35, βc_1 is significantly negative for all indices, and m_2 is insignificant for all indices. Further, the dynamic features of the data are well captured by the model in that there is little autocorrelation remaining in the standardised residuals (shocks) ξ_t and η_t. Hence the conclusions reached in Section 3.3 appear to be robust to moderate variations in the sampling frequency.

5. Conclusion

Using bivariate models, we provide empirical evidence for the risk return relationship in the daily and weekly return and RV series from 21 international market indices. Our findings conform to the arguments of risk return tradeoff, volatility feedback, as well as statistical balance. These hold pervasively for almost all indices considered. We argue that the major data features (the negative CC between the return and RV, and the different autocorrelations in the return and RV) contain crucial information about the risk return relationship. The price fall associated with high volatility (owing to negative CC) needs to be compensated by a positive risk premium in the expected return, whilst the different autocorrelation structures of the return and RV prevent the conditional volatility from having predictive power for the return (owing to statistical balance). Future research will be directed to examining the risk premium of jumps in return and RV series. In particular, the model of Chrisoffersen, Jacobs and Ornthanalai (2012) can be extended for this purpose.

The computation of the empirical results is carried out in R version 2.15.3 of R Core Team (2013). The function "optim" with the BFGS algorithm is used for maximising the log likelihoods.

6. References

Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys (2003), Modeling and forecasting realized volatility, *Econometrica*, 71, 579-625

Andersen, T.G., T. Bollerslev, and F.X. Diebold (2007), Roughing It Up: Including Jump Components in the Measurement, Modeling and Forecasting of Return Volatility, *Review of Economics and Statistics*, 89(4), 701-20.

Andersen, T.G., T. Bollerslev, P. Frederiksen, and M. O. Nielsen (2010), Continuous-time models, realized volatilities, and testable distributional implications for daily stock returns, *Journal of Applied Econometrics*, 25, 233-261

Backus, D.K. and A.W. Gregory (1993), Theoretical relations between risk premiums and conditional variances, *Journal of Business and Economic Statistics*, 11(2), 177-185

Barndorff-Nielsen, O.E., P.R. Hansen, A. Lunde and N. Shephard (2008), Designing Realized Kernels to Measure the ex post Variation of Equity Prices in the Presence of Noise, *Econometrica*, 76(6), 1481-1536

Barndorff-Nielsen, O.E. (1997), Normal inverse Gaussian distributions and stochastic volatility modelling, *Scandinavian Journal of Statistics*, 24(1), 1-12

Bollerslev, T., R.F. Engle, and D.B. Nelson (1994), ARCH Models, *Handbook of Econometrics*, Edited by R.R Engle and D.L. McFadden, Volume 4, Chapter 49

Bollerslev, T., D. Osterrieder, N. Sizova and G. Tauchen (2013), Risk and return: long-run relations, fractional cointegration and return predictability, *Journal of Financial Economics*, 108, 409-424

Corsi, F. (2009), A simple approximate long-memory model of realized volatility, *Journal of Financial Econometrics*, 7(2), 174-196.

Corsi, F., N. Fusari and D.L. Vecchia (2013), Realizing smiles: options pricing with realized volatility, *Journal of Financial Economics*, 107, 284-304.

Christensen, B.J. and M.Ø. Nielsen (2007), The effect of long memory in volatility on stock market fluctuations, *Review of Economics and Statistics*, 89, 684-700.

Christoffersen, P. K. Jacobs and C. Ornthanalai (2012), Dynamic jump intensities and risk premium: evidence from S&P500 returns and options, *Journal of Financial Economics*, 106, 447-472.

French, K.R., G.W. Schwert and R.F Stambaugh (1987), Expected stock returns and volatility," *Journal of Financial Economics*, 19, 3-29.

Ghysels, E., P. Santa-Clara and R. Valkanov (2005), There is a risk return tradeoff after all, *Journal of Financial Economics,* 76, 509-548.

Glosten, L., R. Jagannathan and D. Runkle (1993), On the relation between expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, 48, 1779-1801.

Gourieroux, C. and J. Jasiak (2006), Autoregressive Gamma processes, *Journal of Forecasting*, 25, 129-152

Hansen, P.R., Z. Huang and H.H. Shek (2012), Realized GARCH: a joint model for returns and realized measures of volatility, *Journal of Applied Econometrics*, 27, 877-906

Heber, G., A. Lunde, N. Shephard and K. K. Sheppard (2009), Oxford-Man Institute's Realized Library, Version 0.2, Oxford-Man Institute, University of Oxford

Jensen, M.B. and A. Lunde (2001), The NIG-S&ARCH model: a fat-tailed, stochastic and autoregressive conditional heteroskedastic volatility model, *Econometrics Journal*, 4, 319-342.

Lundblad, C.(2007), The risk return tradeoff in the long run: 1836-2003, *Journal of Financial Economics*, 85, 123-150.

Maheu, J. and T. McCurdy (2004), News arrival, jump dynamics and volatility components for individual stock returns, *Journal of Finance*, 59, 755-793.

Merton, R. (1973), An intertemporal capital asset pricing model, *Econometrica*, 41, 867-887.

Nelson, D. (1991), Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, 59, 347-370.

R Core Team (2013), R: A language and environment for statistical computing, R Foundation for Statistical Computing, Vienna, Austria, URL http://www.R-project.org/.

Rossi, A. and A. Timmermann (2010), What is the shape of the risk return relation? Working Paper, University of California, San Diego

Wang, J. and M. Yang (2013), On the risk return relationship, *Journal of Empirical Finance*, 21, 132-141.

Yang, M. (2011), Volatility feedback and risk premium in GARCH models with generalized hyperbolic distributions, *Studies in Nonlinear Dynamics & Econometrics*, 15(3), 124-142, Article 6 (http://www.bepress.com/snde/vol15/iss3/art6).

7. Tables and Figures

Table 1. Summary Statistics of Returns and Log Realised Variances

Here, Q30 is the Ljung-Box Q statistic at lag 30 and nObs is the number of observation used for estimating models. Corr is the contemporaneous correlation between the the excess return and the log realised variance.

Table 1a	S&P 500	DJIA	Nasdaq 100	Russel 2000	S&P TSX	IPC Mexico	Bovespa
Return							
Mean	-0.004	0.002	-0.015	0.011	0.014	0.039	0.037
Stdev	1.339	1.250	1.788	1.655	1.161	1.431	1.915
Skewness	-0.121	-0.033	0.119	-0.283	-0.660	0.039	-0.209
Kurtosis	10.103	10.185	8.349	7.084	10.398	7.675	7.655
Min	-9.689	-8.615	-10.240	-12.461	-9.065	-8.303	-15.406
Max	10.641	10.530	13.264	8.755	7.521	10.419	13.360
Q30	101.9	101.7	94.8	77.2	109.0	87.7	64.8
log RV							
Mean	-0.349	-0.390	-0.145	-0.296	-1.198	-0.847	0.584
Stdev	1.022	0.989	1.059	0.945	1.065	0.911	0.783
Skewness	0.537	0.639	0.469	0.578	0.834	0.556	0.634
Kurtosis	3.480	3.741	2.964	3.957	4.000	3.474	4.702
Min	-3.029	-2.958	-3.211	-3.667	-3.930	-3.657	-2.579
Max	4.534	4.514	4.200	4.163	3.568	3.225	4.427
Q30	39666.2	39131.9	50056.4	29824.1	38245.9	33325.0	17556.1
Corr	-0.096	-0.089	-0.127	-0.103	-0.157	-0.074	-0.113
nObs	3241	3243	3246	3244	2659	3247	3169

Table 1b.	FTSE 100	Euro STOXX	DAX	CAC 40	AEX	FTSE MIB	Swiss	IBEX 35
Return								
Mean	-0.009	-0.026	-0.005	-0.021	-0.025	-0.037	-0.002	-0.018
Stdev	1.241	1.564	1.594	1.533	1.531	1.559	1.260	1.539
Skewness	-0.156	-0.007	-0.042	-0.011	-0.116	-0.092	-0.065	0.065
Kurtosis	9.225	7.407	8.596	7.424	8.694	8.051	9.814	7.697
Min	-8.936	-8.743	-11.065	-8.537	-9.133	-9.187	-8.709	-9.555
Max	9.480	10.539	12.013	10.425	9.565	10.750	10.781	12.870
Q30	101.3	105.1	100.5	105.0	125.4	79.2	74.9	68.3
log RV								
Mean	-0.610	0.024	0.110	-0.062	-0.255	-0.277	-0.614	-0.094
Stdev	1.055	1.028	1.029	1.010	1.047	1.084	0.943	1.053
Skewness	0.324	0.305	0.378	0.220	0.429	0.165	0.805	-0.050
Kurtosis	2.944	3.287	3.106	3.035	3.070	2.695	3.405	2.669
Min	-3.232	-4.262	-3.097	-3.217	-3.471	-3.447	-2.519	-3.182
Max	3.483	4.693	4.164	3.818	3.679	3.762	3.202	3.581
Q30	49697.9	41608.2	48056.3	46857.8	46874.2	47382.5	52360.6	52737.3
Corr	-0.134	-0.126	-0.141	-0.133	-0.149	-0.155	-0.127	-0.125
nObs	3261	3285	3293	3310	3309	3276	3255	3275

Table 1c.	Nikkei 225	KOSPI	Hang Seng	S&P CNX	FT Straits Times	All Ordinaries
Return						
Mean	-0.019	0.011	0.007	0.054	0.013	0.000
Stdev	1.575	1.729	1.751	1.631	1.219	0.977
Skewness	-0.445	-0.569	-2.025	-0.241	-0.335	-0.709
Kurtosis	9.624	8.319	51.828	12.777	9.867	9.333
Min	-12.113	-12.838	-32.407	-13.806	-9.635	-7.280
Max	13.232	11.229	13.397	16.438	8.913	4.514
Q30	54.5	32.1	118.0	72.5	74.3	39.3
log RV						
Mean	-0.251	-0.008	-0.457	0.094	-0.812	-1.273
Stdev	0.864	0.944	0.839	0.921	0.759	0.995
Skewness	0.381	0.355	0.541	0.689	0.723	0.542
Kurtosis	3.788	3.163	3.886	3.845	3.950	3.459
Min	-2.728	-2.511	-3.002	-2.371	-2.779	-4.446
Max	3.647	4.171	3.798	4.616	3.322	2.745
Q30	30685.6	41955.4	34493.5	24692.5	36716.9	34085.1
Corr	-0.120	-0.152	-0.144	-0.186	-0.093	-0.121
nObs	3145	3204	2941	2583	3204	3257

Table 2. Estimation Results for the Non-central Gamma Model

The model estimated is defined by the equations (1) and (3). The standard errors, obtained from the sandwich form of the variance matrix estimate, are given in parentheses. In the table, $Q_{30}(\xi)$ and $Q_{30}(\eta)$ are the Ljung-Box Q-statistics at lag 30 computed from the ξ_t and η_t series based on the estimated parameters. The standard errors for the estimates of βc and $m_1 + \beta c$ are computed by the "delta" method. The estimates of m_2 and $m_1 + \beta c$ that are statistically significant at the 5% (or less) level are indicated with "**".

Table 2a	S&P 500	DJIA	Nasdaq 100	Russel 2000	S&P TSX	IPC Mexico	Bovespa
ψ	1.201	1.133	1.973	2.237	2.265	2.879	1.369
	(0.028)	(0.028)	(0.051)	(0.053)	(0.063)	(0.084)	(0.037)
δ	1.162	1.201	1.189	1.279	1.136	1.269	1.594
	(0.053)	(0.059)	(0.048)	(0.052)	(0.052)	(0.045)	(0.086)
c	0.254	0.238	0.260	0.235	0.102	0.168	0.436
	(0.034)	(0.034)	(0.031)	(0.023)	(0.010)	(0.015)	(0.031)
β	-0.331	-0.288	-0.536	-0.389	-0.931	-0.240	-0.229
	(0.035)	(0.036)	(0.048)	(0.048)	(0.089)	(0.060)	(0.026)
a_1	1.233	1.230	1.219	1.071	3.196	1.117	0.611
	(0.290)	(0.279)	(0.224)	(0.218)	(0.626)	(0.257)	(0.089)
a_2	1.103	1.288	0.933	1.234	2.661	1.485	0.698
	(0.221)	(0.272)	(0.186)	(0.208)	(0.591)	(0.251)	(0.100)
a_3	0.302	0.309	0.448	0.326	0.785	0.994	0.147
	(0.148)	(0.180)	(0.141)	(0.154)	(0.381)	(0.217)	(0.080)
a_4	0.776	0.783	0.942	1.223	2.592	1.057	0.445
	(0.197)	(0.175)	(0.151)	(0.136)	(0.353)	(0.209)	(0.075)
φ	-0.059	-0.052	-0.028	0.000	0.007	0.068	0.004
	(0.015)	(0.015)	(0.016)	(0.016)	(0.019)	(0.017)	(0.016)
m_0	0.076	0.066	0.189	0.118	0.134	0.080	0.114
	(0.021)	(0.020)	(0.027)	(0.031)	(0.021)	(0.029)	(0.046)
m_1	0.088	0.073	0.132	0.094	0.093	0.043	0.119
	(0.013)	(0.012)	(0.016)	(0.015)	(0.010)	(0.015)	(0.018)
m_2	-0.061	-0.063	-0.011	-0.019	-0.099	-0.066	-0.022
	(0.040)	(0.039)	(0.038)	(0.047)	(0.077)	(0.040)	(0.026)
βc	-0.084	-0.069	-0.139	-0.091	-0.095	-0.040	-0.100
	(0.010)	(0.009)	(0.013)	(0.011)	(0.008)	(0.010)	(0.012)
$m_1 + \beta c$	0.004	0.004	-0.007	0.002	-0.002	0.002	0.019
	(0.007)	(0.007)	(0.008)	(0.010)	(0.006)	(0.012)	(0.012)
$\log(L)$	-6863.2	-6542.8	-8425.1	-7968.9	-3032.1	-5958.5	-10809.9
$Q_{30}(\xi)$	40.9	34.3	41.4	50.9	46.3	39.2	48.8
$Q_{30}(\eta)$	577.9	646.8	451.7	602.8	462.8	486.9	843.6

Table 2b.	FTSE 100	Euro STOXX	DAX	CAC 40	AEX	FTSE MIB	Swiss	IBEX 35
ψ	1.543	1.209	1.159	1.363	1.449	1.654	1.488	1.443
	(0.043)	(0.029)	(0.028)	(0.032)	(0.036)	(0.045)	(0.040)	(0.035)
δ	1.124	1.173	1.188	1.208	1.181	1.088	1.405	1.150
	(0.040)	(0.051)	(0.047)	(0.043)	(0.040)	(0.033)	(0.060)	(0.037)
c	0.171	0.331	0.312	0.239	0.209	0.226	0.107	0.219
	(0.016)	(0.043)	(0.033)	(0.019)	(0.017)	(0.018)	(0.009)	(0.017)
β	-0.685	-0.369	-0.429	-0.517	-0.643	-0.687	-0.828	-0.550
	(0.069)	(0.035)	(0.033)	(0.034)	(0.040)	(0.041)	(0.057)	(0.040)
a_1	1.906	0.872	1.297	1.323	1.674	1.413	3.467	1.536
	(0.292)	(0.215)	(0.185)	(0.208)	(0.216)	(0.203)	(0.496)	(0.219)
a_2	1.673	0.872	0.694	1.182	1.363	1.321	2.837	1.288
	(0.286)	(0.150)	(0.167)	(0.199)	(0.226)	(0.201)	(0.473)	(0.200)
a_3	0.665	0.252	0.353	0.419	0.397	0.439	0.561	0.476
	(0.201)	(0.125)	(0.113)	(0.132)	(0.130)	(0.136)	(0.279)	(0.140)
a_4	0.880	0.690	0.506	0.926	0.960	0.851	1.873	0.985
	(0.193)	(0.122)	(0.120)	(0.132)	(0.143)	(0.127)	(0.254)	(0.148)
φ	-0.066	-0.049	-0.030	-0.051	-0.016	-0.058	-0.010	-0.014
	(0.020)	(0.016)	(0.016)	(0.016)	(0.017)	(0.017)	(0.017)	(0.017)
m_0	0.101	0.093	0.158	0.115	0.107	0.143	0.115	0.129
	(0.020)	(0.024)	(0.024)	(0.024)	(0.023)	(0.022)	(0.021)	(0.023)
m_1	0.121	0.128	0.133	0.126	0.139	0.152	0.089	0.119
	(0.016)	(0.016)	(0.015)	(0.012)	(0.013)	(0.013)	(0.009)	(0.011)
m_2	-0.065	-0.029	-0.036	-0.026	-0.044	-0.010	0.006	-0.023
	(0.045)	(0.030)	(0.033)	(0.037)	(0.042)	(0.041)	(0.062)	(0.039)
βc	-0.117	-0.122	-0.134	-0.124	-0.134	-0.155	-0.089	-0.120
	(0.013)	(0.013)	(0.012)	(0.010)	(0.010)	(0.011)	(0.007)	(0.009)
$m_1 + \beta c$	0.004	0.005	-0.001	0.002	0.005	-0.003	0.001	-0.002
	(0.006)	(0.008)	(0.007)	(0.006)	(0.007)	(0.007)	(0.004)	(0.006)
$\log(L)$	-5870.4	-8667.4	-8848.6	-8181.3	-7401.9	-7577.5	-5128.1	-7946.1
$Q_{30}(\hat{\xi})$	40.2	50.5	39.7	64.8	56.5	37.3	51.2	57.1
$Q_{30}(\hat{\eta})$	300.3	712.1	299.7	368.7	350.1	207.5	342.8	251.3

Table 2c.	Nikkei 225	KOSPI	Hang Seng	S&P CNX	FT Straits Times	All Ordinaries
ψ	2.018	1.912	2.436	1.301	2.056	1.599
	(0.050)	(0.054)	(0.080)	(0.039)	(0.056)	(0.033)
δ	1.393	1.295	1.457	1.327	1.653	1.129
	(0.064)	(0.051)	(0.083)	(0.068)	(0.096)	(0.037)
c	0.204	0.229	0.176	0.413	0.083	0.125
	(0.016)	(0.020)	(0.022)	(0.056)	(0.009)	(0.011)
β	-0.440	-0.698	-0.641	-0.427	-0.561	-0.346
	(0.044)	(0.049)	(0.069)	(0.038)	(0.087)	(0.060)
a_1	1.541	1.848	1.620	0.898	4.510	0.597
	(0.205)	(0.203)	(0.368)	(0.143)	(0.548)	(0.260)
a_2	1.387	1.203	1.937	0.403	2.962	3.141
	(0.255)	(0.194)	(0.368)	(0.113)	(0.588)	(0.378)
a_3	0.396	0.310	0.485	0.161	1.642	1.011
	(0.191)	(0.145)	(0.256)	(0.081)	(0.453)	(0.328)
a_4	0.812	0.398	0.243	0.449	0.756	1.919
	(0.147)	(0.137)	(0.213)	(0.089)	(0.349)	(0.286)
φ	-0.029	-0.026	0.014	0.073	-0.004	-0.011
	(0.017)	(0.018)	(0.019)	(0.021)	(0.018)	(0.014)
m_0	0.138	0.212	0.199	0.161	0.098	0.075
	(0.034)	(0.034)	(0.038)	(0.037)	(0.028)	(0.015)
m_1	0.082	0.161	0.106	0.214	0.045	0.035
	(0.014)	(0.017)	(0.019)	(0.029)	(0.010)	(0.010)
m_2	0.029	-0.017	**0.114	-0.055	0.039	0.023
	(0.045)	(0.051)	(0.055)	(0.039)	(0.083)	(0.043)
βc	-0.090	-0.160	-0.113	-0.176	-0.046	-0.043
	(0.012)	(0.013)	(0.016)	(0.018)	(0.007)	(0.009)
$m_1 + \beta c$	-0.007	0.001	-0.007	**0.038	-0.001	-0.009
	(0.009)	(0.008)	(0.012)	(0.016)	(0.006)	(0.007)
$\log(L)$	-7521.9	-8590.8	-6456.8	-7294.5	-4510.1	-3136.9
$Q_{30}(\hat{\xi})$	38.7	26.2	42.7	35.5	32.8	22.3
$Q_{30}(\hat{\eta})$	482.8	342.7	347.3	105.7	445.5	360.3

Table 3. Estimation Results for the Log-Normal Model

The model estimated is defined by the equations (11), (12) and (13). In the table, $Q_{30}(\hat{\xi})$ and $Q_{30}(\hat{\eta})$ are the Ljung-Box Q-statistics at lag 30 estimated from the ξ_t and η_t series based on the estimated parameters. The standard errors for the estimates of βc and $m_1 + \beta c$ are computed by the "delta" method. The estimates of m_2 and $m_1 + \beta c$ that are statistically significant at the 5% (or less) level are indicated by "**".

Table 3a	S&P 500	DJIA	Nasdaq 100	Russel 2000	S&P TSX	IPC Mexico	Bovespa
ψ_0	-0.377	-0.316	-0.722	-0.928	-0.965	-1.264	-0.525
	(0.023)	(0.024)	(0.028)	(0.031)	(0.033)	(0.034)	(0.055)
ψ_1	0.886	0.897	0.926	1.005	1.102	1.075	1.043
	(0.020)	(0.022)	(0.024)	(0.027)	(0.033)	(0.046)	(0.046)
ρ_2	1.134	1.129	0.621	0.838	0.854	1.118	1.040
	(0.048)	(0.051)	(0.055)	(0.045)	(0.053)	(0.087)	(0.057)
γ	0.250	0.243	0.202	0.278	0.250	0.290	0.252
	(0.008)	(0.009)	(0.007)	(0.009)	(0.008)	(0.011)	(0.008)
β	-0.463	-0.406	-1.343	-0.492	-0.925	-0.123	-0.363
	(0.047)	(0.045)	(0.149)	(0.056)	(0.088)	(0.031)	(0.049)
b_0	0.110	0.077	0.380	0.352	0.368	0.413	0.251
	(0.020)	(0.019)	(0.034)	(0.038)	(0.045)	(0.074)	(0.035)
b_1	0.604	0.627	0.361	0.540	0.398	0.540	0.419
	(0.037)	(0.038)	(0.045)	(0.045)	(0.050)	(0.082)	(0.046)
a_1	0.297	0.287	0.382	0.270	0.256	0.224	0.261
	(0.021)	(0.021)	(0.025)	(0.018)	(0.021)	(0.020)	(0.021)
a_2	0.044	0.034	0.120	0.058	0.133	0.041	0.104
	(0.028)	(0.029)	(0.029)	(0.029)	(0.030)	(0.037)	(0.030)
a_3	0.057	0.050	0.108	0.068	0.091	0.101	0.077
	(0.011)	(0.011)	(0.015)	(0.012)	(0.014)	(0.025)	(0.014)
a_4	-0.132	-0.124	-0.076	-0.092	-0.097	-0.053	-0.056
	(0.008)	(0.009)	(0.005)	(0.006)	(0.008)	(0.006)	(0.005)
a_5	0.032	0.032	0.068	0.033	0.076	0.058	0.066
	(0.010)	(0.010)	(0.009)	(0.008)	(0.012)	(0.014)	(0.007)
φ	-0.052	-0.045	-0.021	-0.007	0.004	0.065	0.007
	(0.014)	(0.014)	(0.016)	(0.016)	(0.019)	(0.017)	(0.016)
m_0	0.022	0.020	0.040	0.014	0.059	0.073	-0.133
	(0.034)	(0.034)	(0.054)	(0.068)	(0.048)	(0.058)	(0.100)
m_1	0.417	0.377	1.112	0.495	0.751	0.108	0.440
	(0.050)	(0.051)	(0.131)	(0.070)	(0.083)	(0.058)	(0.080)
m_2	-0.007	-0.005	0.054	-0.033	0.009	-0.006	0.014
	(0.022)	(0.019)	(0.038)	(0.031)	(0.027)	(0.012)	(0.041)
βc_1	-0.430	-0.384	-1.189	-0.469	-0.805	-0.122	-0.349
	(0.040)	(0.039)	(0.124)	(0.050)	(0.069)	(0.030)	(0.045)
$m_1 + \beta c_1$	-0.013	-0.007	-0.077	0.026	-0.055	-0.014	0.091
	(0.038)	(0.041)	(0.042)	(0.055)	(0.057)	(0.053)	(0.067)
$\log(L)$	-5494.1	-5172.1	-6897.3	-6970.6	-1931.8	-4789.0	-10070.3
$Q_{30}(\hat{\xi})$	32.7	26.9	31.7	45.4	39.8	39.1	48.1
$Q_{30}(\hat{\eta})$	29.4	29.5	38.0	47.0	41.1	43.0	38.4

Table 3b.	FTSE 100	Euro STOXX	DAX	CAC 40	AEX	FTSE MIB	Swiss	IBEX 35
ψ_0	-0.550	-0.355	-0.320	-0.468	-0.518	-0.642	-0.547	-0.533
	(0.027)	(0.025)	(0.026)	(0.026)	(0.025)	(0.027)	(0.026)	(0.029)
ψ_1	1.041	0.937	0.959	0.976	0.922	0.948	0.966	1.005
	(0.028)	(0.023)	(0.024)	(0.025)	(0.022)	(0.024)	(0.028)	(0.025)
ρ_2	0.682	0.982	0.971	0.865	0.793	0.646	0.906	0.908
	(0.070)	(0.054)	(0.056)	(0.052)	(0.053)	(0.050)	(0.068)	(0.060)
γ	0.215	0.256	0.215	0.219	0.222	0.246	0.146	0.200
	(0.009)	(0.011)	(0.007)	(0.008)	(0.008)	(0.009)	(0.005)	(0.007)
β	-1.244	-0.633	-0.874	-0.892	-1.101	-1.458	-1.145	-0.888
	(0.167)	(0.069)	(0.086)	(0.086)	(0.107)	(0.136)	(0.117)	(0.087)
b_0	0.121	0.075	0.076	0.119	0.165	0.206	0.146	0.193
	(0.021)	(0.018)	(0.018)	(0.022)	(0.028)	(0.028)	(0.028)	(0.031)
b_1	0.619	0.591	0.600	0.565	0.475	0.486	0.586	0.445
	(0.051)	(0.051)	(0.053)	(0.053)	(0.053)	(0.045)	(0.062)	(0.062)
a_1	0.260	0.256	0.282	0.263	0.322	0.287	0.289	0.308
	(0.018)	(0.022)	(0.023)	(0.021)	(0.023)	(0.020)	(0.024)	(0.022)
a_2	0.010	0.055	0.025	0.055	0.092	0.090	0.044	0.103
	(0.027)	(0.032)	(0.030)	(0.033)	(0.037)	(0.029)	(0.038)	(0.037)
a_3	0.055	0.062	0.060	0.067	0.074	0.088	0.045	0.081
	(0.011)	(0.012)	(0.012)	(0.011)	(0.012)	(0.012)	(0.011)	(0.013)
a_4	-0.080	-0.092	-0.072	-0.077	-0.071	-0.075	-0.079	-0.069
	(0.007)	(0.006)	(0.005)	(0.006)	(0.006)	(0.006)	(0.007)	(0.006)
a_5	0.055	0.061	0.048	0.067	0.089	0.092	0.059	0.074
	(0.011)	(0.010)	(0.008)	(0.010)	(0.010)	(0.009)	(0.011)	(0.010)
φ	-0.041	-0.050	-0.030	-0.049	-0.012	-0.051	-0.005	-0.008
	(0.017)	(0.017)	(0.015)	(0.016)	(0.016)	(0.017)	(0.017)	(0.017)
m_0	0.008	-0.019	0.060	-0.012	-0.009	0.042	-0.036	0.031
	(0.036)	(0.049)	(0.046)	(0.047)	(0.043)	(0.039)	(0.046)	(0.040)
m_1	1.053	0.545	0.693	0.752	0.923	1.147	0.945	0.727
	(0.145)	(0.068)	(0.076)	(0.077)	(0.092)	(0.110)	(0.101)	(0.077)
m_2	**0.020	0.025	0.025	0.039	0.021	0.013	**0.084	**0.033
	(0.006)	(0.025)	(0.025)	(0.023)	(0.021)	(0.015)	(0.026)	(0.012)
βc_1	-1.089	-0.574	-0.760	-0.792	-0.952	-1.223	-0.999	-0.789
	(0.136)	(0.056)	(0.067)	(0.070)	(0.085)	(0.106)	(0.094)	(0.071)
$m_1 + \beta c_1$	-0.036	-0.029	-0.068	-0.040	-0.029	**-0.076	-0.055	-0.062
	(0.043)	(0.043)	(0.040)	(0.041)	(0.041)	(0.038)	(0.048)	(0.038)
$\log(L)$	-4372.6	-7470.6	-7520.2	-7037.1	-6178.0	-6353.5	-3678.8	-6742.1
$Q_{30}(\hat{\xi})$	31.7	38.9	36.1	56.8	55.4	36.0	45.8	47.9
$Q_{30}(\hat{\eta})$	62.5	72.0	81.6	107.3	107.4	57.9	72.6	85.3

Table 3c.	Nikkei 225	KOSPI	Hang Seng	S&P CNX	FT Straits Times	All Ordinaries
ψ_0	-0.882	-0.712	-0.908	-0.410	-0.895	-0.751
	(0.038)	(0.036)	(0.035)	(0.038)	(0.087)	(0.024)
ψ_1	1.110	0.986	0.859	0.993	0.832	0.924
	(0.040)	(0.032)	(0.034)	(0.037)	(0.042)	(0.022)
ρ_2	0.870	0.446	0.741	0.759	1.183	1.164
	(0.053)	(0.055)	(0.060)	(0.081)	(0.202)	(0.040)
γ	0.239	0.196	0.220	0.235	0.163	0.343
	(0.008)	(0.007)	(0.008)	(0.009)	(0.005)	(0.010)
β	-0.493	-2.108	-0.723	-0.998	-0.356	-0.186
	(0.058)	(0.278)	(0.092)	(0.175)	(0.076)	(0.032)
b_0	0.304	0.335	0.435	0.177	0.437	0.159
	(0.037)	(0.034)	(0.064)	(0.033)	(0.071)	(0.031)
b_1	0.531	0.357	0.495	0.234	0.405	0.683
	(0.050)	(0.047)	(0.066)	(0.056)	(0.050)	(0.053)
a_1	0.272	0.388	0.301	0.371	0.349	0.164
	(0.018)	(0.023)	(0.023)	(0.024)	(0.025)	(0.018)
a_2	0.026	0.075	0.077	0.142	0.119	0.068
	(0.027)	(0.029)	(0.049)	(0.038)	(0.035)	(0.042)
a_3	0.068	0.100	0.145	0.116	0.138	0.067
	(0.013)	(0.014)	(0.027)	(0.016)	(0.025)	(0.016)
a_4	-0.053	-0.033	-0.027	-0.059	-0.037	-0.136
	(0.005)	(0.005)	(0.007)	(0.007)	(0.009)	(0.012)
a_5	0.039	0.092	0.043	0.114	0.112	0.033
	(0.008)	(0.008)	(0.009)	(0.012)	(0.016)	(0.016)
φ	-0.029	0.006	0.022	0.074	-0.008	-0.017
	(0.017)	(0.018)	(0.017)	(0.020)	(0.017)	(0.014)
m_0	0.065	0.032	0.062	-0.056	0.045	0.075
	(0.070)	(0.064)	(0.067)	(0.070)	(0.051)	(0.028)
m_1	0.376	1.794	0.607	1.007	0.295	0.111
	(0.072)	(0.238)	(0.094)	(0.145)	(0.084)	(0.047)
m_2	0.043	0.027	0.043	-0.031	0.018	-0.008
	(0.034)	(0.034)	(0.035)	(0.036)	(0.034)	(0.009)
βc_1	-0.472	-1.828	-0.680	-0.885	-0.345	-0.183
	(0.052)	(0.234)	(0.082)	(0.143)	(0.072)	(0.030)
$m_1 + \beta c_1$	-0.096	-0.034	-0.073	**0.122	-0.051	-0.071
	(0.057)	(0.048)	(0.054)	(0.062)	(0.050)	(0.042)
$\log(L)$	-6572.3	-7344.8	-5374.8	-6115.1	-3519.7	-2032.4
$Q_{30}(\hat{\xi})$	38.2	26.1	42.3	48.7	33.3	27.1
$Q_{30}(\hat{\eta})$	34.2	32.7	38.8	38.2	37.4	39.7

Table 4. Summary Statistics of Returns and Log Realised Variances, Weekly

Here, Q15 is the Ljung-Box Q statistic at lag 15 and nObs is the number of observation used for estimating models. Corr is the contemporaneous correlation between the the excess return and the log realised variance.

Table 4a	S&P 500	DJIA	Nasdaq 100	Russel 2000	S&P TSX	IPC Mexico	Bovespa
Return							
Mean	-0.039	-0.003	-0.092	0.042	0.094	0.145	0.193
Stdev	2.755	2.602	3.876	3.531	2.461	3.359	4.278
Skewness	-0.682	-0.872	-0.578	-0.591	-1.096	-0.502	-0.716
Kurtosis	8.810	9.958	11.254	6.519	9.635	8.575	6.795
Min	-19.533	-18.969	-29.335	-18.065	-15.910	-18.078	-24.956
Max	11.332	11.092	22.822	15.171	11.290	18.430	16.246
Q15	23.6	24.6	28.5	12.4	27.9	28.7	28.2
log RV							
Mean	1.320	1.279	1.502	1.383	0.477	0.840	2.267
Stdev	0.956	0.924	0.997	0.860	1.013	0.831	0.706
Skewness	0.699	0.805	0.563	0.819	0.935	0.632	0.845
Kurtosis	3.485	3.871	2.810	4.167	4.127	3.332	5.111
Min	-0.556	-0.607	-0.921	-0.893	-1.537	-1.152	0.422
Max	5.108	5.106	4.738	4.866	4.615	3.996	5.608
Q15	39666.2	39131.9	50056.4	29824.1	38245.9	33325.0	17556.1
Corr	-0.199	-0.175	-0.219	-0.215	-0.248	-0.148	-0.176
nObs	642	642	643	643	527	636	619

Table 4b.	FTSE 100	Euro STOXX	DAX	CAC 40	AEX	FTSE MIB	Swiss	IBEX 35
Return								
Mean	-0.068	-0.151	-0.044	-0.128	-0.145	-0.201	-0.021	-0.098
Stdev	2.648	3.387	3.563	3.252	3.409	3.521	2.853	3.402
Skewness	-1.139	-0.984	-0.822	-1.044	-1.165	-0.983	-1.067	-0.818
Kurtosis	14.205	9.894	9.283	10.378	11.467	10.027	16.146	8.035
Min	-23.197	-26.612	-26.126	-26.331	-28.383	-24.511	-25.249	-24.272
Max	12.550	13.655	16.200	11.925	13.513	19.228	15.655	12.357
Q15	47.0	43.2	41.7	37.1	35.1	36.7	52.8	25.7
log RV								
Mean	1.040	1.715	1.781	1.612	1.415	1.402	1.030	1.577
Stdev	1.020	0.961	0.973	0.952	0.991	1.020	0.925	1.006
Skewness	0.331	0.488	0.494	0.319	0.561	0.196	0.833	-0.019
Kurtosis	2.924	3.118	3.195	2.886	2.993	2.647	3.350	2.504
Min	-1.898	-0.713	-1.212	-0.654	-0.842	-1.295	-1.201	-0.859
Max	4.441	5.277	5.066	4.783	4.607	4.777	4.184	4.449
Q15	49697.9	41608.2	48056.3	46857.8	46874.2	47382.5	52360.6	52737.3
Corr	-0.187	-0.197	-0.213	-0.188	-0.201	-0.200	-0.175	-0.177
nObs	643	639	642	648	649	639	640	639

Table 4c.	Nikkei 225	KOSPI	Hang Seng	S&P CNX	FT Straits Times	All Ordinaries
Return						
Mean	-0.106	0.052	0.016	0.253	0.045	-0.030
Stdev	3.262	3.907	3.429	3.468	2.982	2.172
Skewness	-1.098	-0.617	-0.138	-0.656	-0.720	-1.064
Kurtosis	11.817	7.212	6.773	6.032	11.248	9.636
Min	-27.901	-23.665	-17.898	-18.244	-18.512	-16.714
Max	13.067	17.370	17.294	14.160	18.626	7.573
Q15	15.9	20.1	15.9	34.8	14.4	18.8
log RV						
Mean	1.421	1.627	1.172	1.796	0.835	0.436
Stdev	0.792	0.889	0.776	0.884	0.713	0.916
Skewness	0.487	0.362	0.765	0.656	0.786	0.630
Kurtosis	3.904	3.183	4.566	3.610	3.966	3.489
Min	-0.524	-0.859	-1.289	-0.323	-0.690	-2.584
Max	4.820	5.260	5.175	5.140	4.080	3.736
Q15	30685.6	41955.4	34493.5	24887.9	36716.9	34085.1
Corr	-0.215	-0.187	-0.194	-0.203	-0.117	-0.231
nObs	625	638	569	501	627	638

Table 5. Estimation Results for the Log-Normal Model, Weekly

The model estimated is defined by the equations (11), (12) and (13). In the table, $Q_{15}(\hat{\xi})$ and $Q_{15}(\hat{\eta})$ are the Ljung-Box Q-statistics at lag 15 estimated from the ξ_t and η_t series based on the estimated parameters. The standard errors for the estimates of βc and $m_1 + \beta c$ are computed by the "delta" method. The estimates of m_2 and $m_1 + \beta c$ that are statistically significant at the 5% (or less) level are indicated by "**".

Table 5a	S&P 500	DJIA	Nasdaq 100	Russel 2000	S&P TSX	IPC Mexico	Bovespa
ψ_0	-0.180	-0.236	-0.647	-1.197	-1.091	-1.364	-0.860
	(0.092)	(0.103)	(0.130)	(0.191)	(0.149)	(0.190)	(0.337)
ψ_1	0.972	1.026	0.990	1.154	1.239	1.072	1.150
	(0.049)	(0.059)	(0.054)	(0.077)	(0.089)	(0.082)	(0.121)
ρ_2	0.773	0.801	0.643	0.900	0.986	1.121	1.027
	(0.107)	(0.124)	(0.108)	(0.109)	(0.119)	(0.102)	(0.108)
γ	0.221	0.205	0.198	0.226	0.199	0.217	0.240
	(0.018)	(0.018)	(0.016)	(0.017)	(0.016)	(0.017)	(0.019)
β	-1.407	-1.255	-1.824	-1.139	-1.171	-0.396	-0.615
	(0.286)	(0.272)	(0.412)	(0.187)	(0.225)	(0.094)	(0.131)
b_0	0.184	0.224	0.488	0.788	0.650	1.043	0.989
	(0.064)	(0.074)	(0.106)	(0.120)	(0.104)	(0.211)	(0.235)
b_1	0.367	0.304	0.342	0.301	0.227	0.204	0.103
	(0.098)	(0.107)	(0.098)	(0.084)	(0.086)	(0.144)	(0.100)
a_1	0.419	0.443	0.390	0.350	0.364	0.349	0.357
	(0.051)	(0.060)	(0.046)	(0.045)	(0.051)	(0.052)	(0.051)
a_2	0.064	0.085	0.066	0.068	0.070	0.204	0.149
	(0.066)	(0.061)	(0.063)	(0.049)	(0.044)	(0.083)	(0.060)
a_3	0.068	0.055	0.120	0.087	0.123	0.084	0.092
	(0.021)	(0.021)	(0.026)	(0.023)	(0.026)	(0.036)	(0.040)
a_4	-0.065	-0.059	-0.041	-0.038	-0.036	-0.033	-0.020
	(0.009)	(0.009)	(0.006)	(0.006)	(0.008)	(0.006)	(0.005)
a_5	0.025	0.024	0.018	0.023	0.027	0.017	0.025
	(0.013)	(0.012)	(0.007)	(0.008)	(0.011)	(0.009)	(0.007)
φ	-0.113	-0.089	-0.043	-0.082	-0.103	-0.042	-0.090
	(0.039)	(0.040)	(0.041)	(0.042)	(0.042)	(0.039)	(0.037)
m_0	0.119	0.018	0.345	0.079	0.588	0.913	-0.356
	(0.204)	(0.220)	(0.306)	(0.422)	(0.270)	(0.360)	(0.782)
m_1	1.046	1.025	1.284	0.889	0.783	0.157	0.716
	(0.224)	(0.232)	(0.317)	(0.203)	(0.193)	(0.154)	(0.265)
m_2	0.083	0.065	0.197	0.129	-0.093	-0.091	-0.032
	(0.100)	(0.096)	(0.159)	(0.171)	(0.149)	(0.151)	(0.266)
βc_1	-1.147	-1.058	-1.483	-0.938	-0.948	-0.378	-0.560
	(0.204)	(0.206)	(0.301)	(0.137)	(0.156)	(0.084)	(0.107)
$m_1 + \beta c_1$	-0.100	-0.033	-0.199	-0.050	-0.165	-0.221	0.156
	(0.107)	(0.118)	(0.108)	(0.155)	(0.152)	(0.140)	(0.236)
$\log(L)$	-2591.2	-2531.4	-2887.8	-2863.0	-1583.0	-2454.3	-3489.8
$Q_{15}(\hat{\xi})$	6.2	11.5	8.5	7.5	9.2	7.0	16.9
$Q_{15}(\hat{\eta})$	6.6	4.8	7.1	9.2	19.2	9.8	13.9

Table 5b.	FTSE 100	Euro STOXX	DAX	CAC 40	AEX	FTSE MIB	Swiss	IBEX 35
ψ_0	-0.666	-0.223	-0.311	-0.362	-0.569	-0.570	-0.560	-0.664
	(0.126)	(0.119)	(0.120)	(0.131)	(0.121)	(0.123)	(0.147)	(0.152)
ψ_1	1.157	0.994	1.029	1.036	1.025	1.006	1.055	1.112
	(0.073)	(0.055)	(0.052)	(0.061)	(0.057)	(0.056)	(0.081)	(0.067)
ρ_2	0.722	0.774	0.736	0.784	0.865	0.776	0.711	0.778
	(0.099)	(0.094)	(0.089)	(0.097)	(0.102)	(0.096)	(0.118)	(0.097)
γ	0.207	0.220	0.207	0.175	0.187	0.223	0.160	0.186
	(0.017)	(0.020)	(0.019)	(0.016)	(0.015)	(0.018)	(0.015)	(0.017)
β	-1.572	-1.592	-1.704	-1.630	-1.471	-1.554	-1.865	-1.426
	(0.322)	(0.300)	(0.326)	(0.322)	(0.273)	(0.292)	(0.438)	(0.302)
b_0	0.355	0.223	0.245	0.299	0.429	0.499	0.322	0.511
	(0.087)	(0.085)	(0.076)	(0.095)	(0.091)	(0.099)	(0.133)	(0.113)
b_1	0.387	0.393	0.457	0.314	0.304	0.182	0.440	0.209
	(0.123)	(0.142)	(0.113)	(0.129)	(0.098)	(0.082)	(0.200)	(0.106)
a_1	0.372	0.434	0.388	0.457	0.474	0.447	0.438	0.422
	(0.054)	(0.066)	(0.050)	(0.069)	(0.065)	(0.050)	(0.080)	(0.050)
a_2	0.023	0.045	0.008	0.042	0.032	0.146	-0.032	0.110
	(0.061)	(0.075)	(0.065)	(0.057)	(0.051)	(0.059)	(0.091)	(0.065)
a_3	0.068	0.050	0.056	0.081	0.076	0.106	0.051	0.097
	(0.020)	(0.022)	(0.018)	(0.023)	(0.020)	(0.028)	(0.021)	(0.022)
a_4	-0.045	-0.048	-0.041	-0.042	-0.038	-0.034	-0.048	-0.037
	(0.008)	(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.010)	(0.006)
a_5	0.028	0.014	0.017	0.023	0.031	0.039	0.023	0.028
	(0.011)	(0.011)	(0.008)	(0.010)	(0.011)	(0.008)	(0.011)	(0.009)
φ	-0.095	-0.088	-0.058	-0.125	-0.085	-0.074	-0.034	-0.059
	(0.039)	(0.039)	(0.040)	(0.038)	(0.038)	(0.041)	(0.051)	(0.041)
m_0	0.258	-0.014	0.295	0.292	0.052	0.364	0.219	0.383
	(0.204)	(0.249)	(0.281)	(0.254)	(0.258)	(0.232)	(0.353)	(0.254)
m_1	1.179	1.110	1.148	1.124	1.060	1.037	1.264	0.966
	(0.255)	(0.209)	(0.236)	(0.237)	(0.206)	(0.212)	(0.324)	(0.231)
m_2	-0.083	0.212	0.217	0.092	0.127	-0.046	0.271	0.137
	(0.071)	(0.141)	(0.145)	(0.129)	(0.140)	(0.137)	(0.262)	(0.113)
βc_1	-1.279	-1.245	-1.341	-1.312	-1.169	-1.221	-1.508	-1.187
	(0.233)	(0.199)	(0.217)	(0.223)	(0.188)	(0.197)	(0.315)	(0.218)
$m_1 + \beta c_1$	-0.100	-0.134	-0.193	-0.188	-0.109	-0.184	-0.244	**-0.221
	(0.110)	(0.104)	(0.109)	(0.103)	(0.103)	(0.099)	(0.132)	(0.101)
$\log(L)$	-2380.6	-2950.6	-3017.6	-2849.4	-2750.3	-2760.2	-2295.5	-2853.5
$Q_{15}(\hat{\xi})$	14.3	18.2	13.2	22.8	12.8	14.7	18.6	10.5
$Q_{15}(\hat{\eta})$	15.4	18.8	11.5	19.1	22.7	12.1	16.8	9.2

Table 5c.	Nikkei 225	KOSPI	Hang Seng	S&P CNX	FT Straits Times	All Ordinaries
ψ_0	-1.313	-0.702	-0.595	-0.677	-0.756	-0.596
	(0.326)	(0.190)	(0.158)	(0.255)	(0.194)	(0.076)
ψ_1	1.315	0.993	0.837	1.123	0.886	0.954
	(0.143)	(0.073)	(0.077)	(0.108)	(0.074)	(0.051)
ρ_2	0.966	1.093	0.778	0.846	1.786	1.001
	(0.125)	(0.106)	(0.116)	(0.114)	(0.205)	(0.091)
γ	0.203	0.218	0.220	0.264	0.156	0.220
	(0.021)	(0.018)	(0.022)	(0.023)	(0.012)	(0.017)
β	-0.721	-0.881	-0.808	-0.647	-0.351	-0.778
	(0.180)	(0.149)	(0.227)	(0.176)	(0.078)	(0.143)
b_0	1.024	0.635	0.743	0.866	0.676	0.434
	(0.600)	(0.145)	(0.309)	(0.207)	(0.209)	(0.075)
b_1	0.058	0.208	0.085	-0.099	0.230	0.283
	(0.459)	(0.094)	(0.326)	(0.123)	(0.129)	(0.086)
a_1	0.343	0.403	0.374	0.400	0.451	0.391
	(0.062)	(0.052)	(0.072)	(0.054)	(0.059)	(0.054)
a_2	0.211	0.141	0.372	0.281	0.176	0.150
	(0.241)	(0.071)	(0.282)	(0.091)	(0.106)	(0.065)
a_3	0.050	0.112	0.157	0.088	0.080	0.117
	(0.027)	(0.031)	(0.068)	(0.043)	(0.036)	(0.031)
a_4	-0.020	-0.021	-0.031	-0.027	-0.024	-0.076
	(0.007)	(0.006)	(0.010)	(0.008)	(0.007)	(0.011)
a_5	0.024	0.047	0.034	0.057	0.049	0.018
	(0.011)	(0.009)	(0.016)	(0.013)	(0.010)	(0.015)
φ	-0.022	-0.090	-0.022	0.027	-0.026	-0.023
	(0.038)	(0.041)	(0.043)	(0.045)	(0.037)	(0.038)
m_0	0.690	0.402	0.844	0.266	0.248	0.427
	(0.404)	(0.313)	(0.420)	(0.426)	(0.286)	(0.181)
m_1	0.368	0.667	0.523	0.690	0.283	0.529
	(0.204)	(0.147)	(0.250)	(0.209)	(0.123)	(0.138)
m_2	0.077	-0.047	-0.161	-0.212	-0.053	-0.125
	(0.063)	(0.150)	(0.291)	(0.265)	(0.232)	(0.108)
βc_1	-0.656	-0.738	-0.744	-0.599	-0.327	-0.687
	(0.147)	(0.106)	(0.192)	(0.150)	(0.067)	(0.110)
$m_1 + \beta c_1$	-0.289	-0.072	-0.221	0.091	-0.044	-0.158
	(0.158)	(0.107)	(0.143)	(0.182)	(0.123)	(0.112)
$\log(L)$	-2762.3	-3017.3	-2413.0	-2499.0	-2194.1	-1882.0
$Q_{15}(\hat{\xi})$	21.5	15.7	7.9	20.7	14.7	7.8
$Q_{15}(\hat{\eta})$	22.1	11.9	15.1	17.4	17.5	10.1

Figure 1. Histograms of ξ_t and η_t for S&P 500 Index

The thick curve is the normal density function with the sample mean and variance of either ξ_t or η_t. The thin curve is a kernel density estimate.