Australian School of Business Research Paper No. 2014 ECON 18

Endogenous Comparative Advantage, Gains From Trade and Symmetry-Breaking

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November 2013

Abstract

Similar countries often choose very different policies and specialize in very distinct industries. This paper proposes a mechanism to explain policy diversity among similar countries from an open economy perspective. I study optimal policies in a two country model when policies affect determinants of trade patterns. I show that welfare gains from trade can provide sufficient incentive for asymmetric equilibrium policies, even if the two countries have identical economic fundamentals. Any asymmetric equilibrium exhibits greater production specialization than the autarky optimum; this is the source of welfare gains. For this same reason, a more asymmetric Nash equilibrium Pareto dominates a less asymmetric one. All equilibria are asymmetric if aggregate income is sufficiently convex in policy, under suitable restrictions on technology and preferences. As an application, I consider a model where skill distribution is the determinant of trade patterns and the policy in question is education policy. When heterogeneous agents choose their skill level optimally, optimal skill function is convex in government policy. In this application, symmetry-breaking in optimal education policy requires that the education cost of agents is relatively inelastic with respect to skill. (JEL Classification: F11, E62.)

Keywords: Symmetry-breaking, Endogenous comparative advantage, Gains from trade, Education policy.

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1 Introduction

Why do similar countries choose very different domestic policies?[1] Existing theories (Benabou and Tirole (2006), Alesina and Angeletos (2005)) explain this by modeling the optimal policy problem in a closed economy setting. They rely on coordination failure as the cause for policy diversity. However, studying each country separately, coordination failure can not rule out similarity of equilibrium policies as an equally plausible outcome. This is why Matsuyama (2002) notes, "coordination failure offers no compelling reason" why we should expect to observe diversity across space, time, or groups.

Instead, Matsuyama (2002, 2004) proposes symmetry-breaking as an explanation of observed diversity. The logic of symmetry-breaking relies on modelling interdependence between the relevant space, time, or groups. In the symmetric strategic setup of Matsuyama (2002, 2004), symmetry-breaking happens when asymmetry is the only stable equilibrium outcome.

In this paper, I apply the logic of symmetry-breaking to explain why similar countries may choose different domestic policies in equilibrium. I model the policy problem in an open economy where two symmetric countries choose a domestic policy that affects their comparative advantage. I show that equilibrium policy diversity arises because it allows countries to gain from international specialization and trade.[2] However, in the optimal policy problem any symmetric equilibrium is also stable. Thus, I follow Amir, Garcia, and Knauff (2010) in ruling out existence of any symmetric equilibrium to ensure equilibrium policy diversity.

What kind of policies may affect international comparative advantage? International trade literature predicts pattern of trade on the basis of differences in domestic factor endowments (Heckscher-Ohlin) , skill distribution (Grossman and Maggi (2000), Bougheas and Riezman (2007)), sector-specific technologies (Ricardo) or institutions (Costinot (2009), Levchenko (2007)). Typically, these differences are treated as entirely exogenous.[3] However, these determinants of trade pattern are affected by choice of domestic education policy, national policies on savings and capital accumulation, sector-specific R&D policies, and labor-market policies or credit market reforms.[4] In this paper, I endogenize the source of comparative advantage by incorporating a comparative-advantage

[1] IMF World Economic Outlook (2006), Bosworth and Collins (2008), Panagariya (2006), and He-Kuijis (2007) highlight important policy differences between India and China. Arora and Gambardella (2005, Chapter 3) discuss differences in economic policies between Ireland and Greece. Important differences in economic policies between USA and Europe have received considerable attention in the literature (Alesina, Glaeser and Sacerdote (2001), Krueger and Kumar (2004)).

[2] Similar countries specialize in distinct industries in the open economy. This observation is the key motivation behind Grossman and Maggi (2000). Baumol and Gomory (2000) document that for the largest economies (Germany, Japan, and US) cross-industry pattern of specialization is remarkably stable.

[3] Notable exceptions are Clarida and Findlay (1991, 1992) and Deardorff (1997).

[4] In fact, policymakers in many countries emphasize international comparative advantage they derive from domestic policies in an increasingly integrated world. For example, international competitiveness of the knowledge-intensive service sector is an important factor in formulating education policies in service-exporting nations, like India or Ireland. The Indian task force report on Human Resource Development in Information Technology (2000) makes 47 specific recommendations with a view "to create a sustainable competitive advantage" in the knowledge-led businesses. A very similar picture of the education policy of Ireland emerges from the Human Capital Priority program of Ireland's National Development Plan (2007).

motive of domestic policy.

To formulate a policy that enhances comparative advantage, policy makers of a country need to take into consideration relevant policies of its trade partners.Thus, as opposed to each country choosing in isolation what policy is best for itself, countries interact in the open economy in their optimal design of national policies that affect international comparative advantage.

My general setup consists of a two-good, two-factor, two-country model in which both good and factor markets are perfectly competitive. The planner in each country chooses a single policy that affects productivity differently for different goods. For example, the policy in question affects relative technological progress across sectors or factor composition of a country. Because the countries are otherwise identical, a difference in government policy is the only potential source of comparative advantage.

I model the optimal policy problem as a non-cooperative optimization in which each country chooses a policy to maximize aggregate welfare. The optimal policy problem in this setting is a symmetric submodular game. Can these symmetric countries choose asymmetric policies in a pure strategy Nash equilibrium (PSNE) ? I show that the countries face a clear trade-off. An asymmetric policy choice gives rise to endogenous comparative advantage and gains from trade. But it also means that at least one of the identical countries is choosing a policy that would be suboptimal in the absence of trade opportunities. Thus the gains from trade needs to be weighed against the loss in welfare due to a suboptimal domestic policy. Naturally, an asymmetric equilibrium exists if the gains from trade are large enough.

In fact, if an asymmetric equilibrium exists, it is associated with higher aggregate welfare for both countries compared to the common autarky optimum. In an asymmetric equilibrium ex-ante identical countries are ex-post different with endogenous comparative advantage in different industries. This leads to the welfare gains in asymmetric equilibrium. More generally, a more asymmetric PSNE Pareto dominates a less asymmetric PSNE.

But when are the gains from trade due to asymmetric policy choice large enough to ensure symmetry-breaking in optimal policy? In the optimal policy problem, a symmetric equilibrium is always stable. Thus, condition for symmetry-breaking of Matsuyama (2002) does not apply in this setting. I rely on quasiconvexity in the welfare function a la Amir, Garcia, Knauff (2010) for ruling out symmetric equilibrium.[5] This quasiconvexity in welfare arises due to social increasing return to policy, viz. convexity in aggregate income with respect to policy. I show that sufficiently large social increasing returns to policy can give rise to large gains from trade and ensure existence of asymmetric equilibrium.

Interestingly, the social increasing returns to policy arises despite a constant returns scale in production. The key to understanding this is the concept of envelop production possibility frontier. Constant returns to scale technologies imply that for any given policy, the production possibility set is a convex set. By varying the policy, I can define a production possibility set of an economy as an

[5] This approach to symmetry-breaking has many applications in the industrial organization literature (for example, Amir (1996, 2000), Amir and Wooders (2000), Amir, Garcia and Knauff (2008)) in the context of R&D investment and capacity choice under demand uncertainty by ex-ante identical firms.

upper envelope of various production possibility sets, each corresponding to a different policy choice. The upper envelope of different convex sets is not necessarily a convex set. This nonconvexity in the envelope production possibility set implies quasiconvexity in the welfare function which is crucial for symmetry-breaking in optimal policy.[6] In this framework nonconvexity of production possibility set arises due to influence of government policy on the supply side of the economy. I illustrate existence of social increasing returns to policy in a specific application to education policy.

In the application, the social planner allocates a fixed education budget between two categories of education.[7] The agents are endowed with heterogeneous ability. They choose skill enrolling in one of the two education categories and incur an education cost specific to that category. The total and marginal cost of education increase in skill and decrease in ability. The education choice is intermediated by government education policy. Apart from the endogenous skill choice by agents and education policy choice by the government, the model is standard Heckscher-Ohlin economy.

I show that endogenous skill choice of agents imply an increasing returns to education policy. The degree of the social increasing returns is governed by the elasticity of education cost in skill. Previous literature on education policy (Benabou (2002)) identifies this elasticity as the determinant of progressiveness of education system. This elasticity is the key institutional parameter for symmetry breaking in this application, provided technologies of the two sectors are sufficiently different and consumer preference is reasonably diversified.

To summarize, this paper makes three contributions. First, it provides a new answer (along the line of Matsuyama (2002)) to the well-researched question, why do similar countries choose very different policies (Benabou and Tirole (2006), Alesina and Angeletos (2005)), from an open-economy perspective in a simple, tractable and general framework. It shows that when comparative advantage is endogenous to domestic policy, gains from trade can be an explanation of equilibrium policy diversity among similar countries. Second, I exploit properties of a standard general equilibrium model of international trade to characterize the conditions for and welfare-implications of asymmetric equilibrium in a symmetric optimal policy problem. I show that conditions of symmetry-breaking relying on payoff nonconcavities in a submodular game (Amir, Garcia, and Knauff(2010)) is related to nonconvexity in the envelop production possibility set (Baumol and Bradford (1972)) or the related concept of social increasing return to policy. I also establish that greater equilibrium asymmetry gives rise to larger welfare gains. Third, I contribute to the trade and education policy literature (Chang and Huang (2012), Bougheas, Kneller and Riezman (2009)) by showing endogenous skill choice by agents, in presence of simple, linear education subsidies, can be a source of social increasing returns to policy and symmetry-breaking in optimal policy.

In the context of this application to education policy, I briefly discuss three extensions. First,

[6] Similar concept of nonconvex social production possibility set has been studied in general equilibrium models of trade in presence of environmental externality (see for example, Copeland and Taylor (1999) and Baumol and Bradford (1972)). Following up on the pioneering work of Ethier (1982), national external economies at the industry level is enjoying renewed interest in trade theory literature in models of Bertrand competition with a continuum of industries (for example, Grossman and Rossi-Hansberg (2010), Lyn and Rodriguez-Clare (2013)). Here, I restrict attention to a perfectly competitive framework with two industries.

[7] Climent and Mukhopadhyay (2013) study implications of a similar budget allocation problem for Indian economy.

I show that in an economy with Grossman and Maggi (2000) production structure, submodularity in production can give rise to social increasing return to education policy and hence equilibrium policy diversity. This resonates with the key result of Chang and Huang (2012). Using a different specification of education policy in a Grossman-Maggi setup, Chang and Huang (2012) show that Nash equilibrium choice of education systems by two countries interacting strategically are necessarily more divergent than their autarky choices. In the second extension allowing for initial differences among the two countries in the Heckscher-Ohlin framework, I show that in the open economy these countries optimally choose to magnify initial differences by investing relatively more in their respective areas of comparative advantage.[8] However, I illustrate that in order to explain *substantial* policy diversity among *similar* countries, social increasing return to policy is still essential. I also briefly discuss the case when in addition to comparative advantage (and aggregate welfare), policy makers also care about redistributive implications of policy choice.[9]

The rest of the paper follows a simple organization. In section 2, I present the general framework and the optimal policy problem, and establish the conditions for and welfare implications of symmetry- breaking in optimal policy. I analyze the application to education policy in section 3. I briefly discuss the three extensions in section 4. Section 5 concludes.

2 Optimal Policy in a General Framework

I consider a two-good, two-factor, perfectly competitive world comprising two identical large economies. Let F_i denote aggregate production of good i and C_i denote the aggregate demand of good i. I denote autarky variables by a superscript A and trade variables by a superscript T. All foreign country variables are designated by an asterisk.

Let $u(C_1, C_2)$ denote the direct utility function, p the relative price of good 1 and Y the aggregate income in the country. Aggregate indirect utility is denoted by V. I assume homothetic preferences which ensure that aggregate demand of each good and indirect utility is linearly homogenous in income,

$$C_i(p,\ \gamma) \quad = \quad f_i(p)Y \tag{1}$$
$$V(p, \gamma) \quad = \quad f(p)Y \tag{2}$$

where $f(p) \equiv u(f_1(p),\ f_2(p))$.

Each government chooses a domestic policy γ that is the only source of comparative advantage in the open economy. Without loss of generality, let an increase in γ confer a comparative advantage in good 1. I interpret γ as a policy that affects sector-specific technologies or the relative factor endowments. Government policy γ affects equilibrium quantities directly through its effects on

[8] Similar question regarding optimal education policy and trade has been studied in a small open economy context in Bougheas, Kneller and Riezman (2009), and more recently by Deardroff (2013).

[9] Derivations and numerical solutions for all the extensions are provided separetely.

economic fundamentals and indirectly through the equilibrium price.

For any given γ, one can define the production possibility frontier as the set of maximum output vector (F_1, F_2) that can be produced using the economy's available technologies and factor inputs. Let us denote this set as PPF(γ). Profit maximization by competitive final good producers ensure that at any point (F_1, F_2) on PPF(γ) for any γ and any p, aggregate value of production $(pF_1(p, \gamma) + F_2(p, \gamma))$ is maximized such that p equals marginal rate of transformation,

$$p = \left| \frac{dF_2}{dF_1} \right| .$$

From the equality of aggregate value of production and aggregate income,

$$Y(p,\ \gamma) = pF_1(p,\ \gamma) + F_2(p,\ \gamma). \tag{3}$$

Thus, $Y(p, \gamma)$ is same as the national income function in Copeland and Taylor (2003) or revenue function in Dixit and Norman (1980).

In the competitive equilibrium of the closed economy,

$$C_i(p^A,\ \gamma) = F_i(p^A,\ \gamma)\ ,\ i = 1, 2, \tag{4}$$

and hence the equilibrium price, p^A, depends only on own policy γ.

In the open economy both countries' policies determine the equilibrium terms-of-trade $p^T(.)$ from goods market clearing of the world,

$$C_i(p^T,\ \gamma) + C_i^*(p^T,\ \gamma^*) = F_i(p^T,\ \gamma) + F_i^*(p^T,\ \gamma^*) \tag{5}$$

and the foreign policy γ^* affects domestic welfare through the terms-of-trade externality. Below I define the competitive equilibrium of the economy for a given policy parameter γ.

Definition 1 *For any given policy γ, the competitive equilibrium is defined by a sequence of equilibrium price and quantities $\{p^j,\ C_i(p^j, \gamma), F_i(p^j, \gamma), C_i^*(p^j, \gamma^*), F_i^*(p^j, \gamma^*), Y(p^j, \gamma), Y^*(p^j, \gamma^*), i = 1, 2, j = A, T\}$ such that*

1. *$C_1(p^j, \gamma), C_2(p^j, \gamma)$ solve the consumer's utility maximization problem in the home country*

$$\max_{C_1, C_2} u(C_1, C_2)\ s.t.\ p^j C_1 + C_2 = Y$$

Similarly, $C_i^*(p^j, \gamma^*)$ solve the consumer optimization problem in the foreign country.

2. *$F_1(p^j, \gamma), F_2(p^j, \gamma)$ solve producer's profit maximization problem which implies that aggregate income is maximized subject to the technology and factor endowment constraints of the economy*

$$\max_{F_1,F_2} Y = p^j F_1 + F_2 \ s.t. \ (F_1, F_2) \in PPF(\gamma)$$

Similarly, $F_i^*(p^j, \gamma^*)$ solve the producer optimization problem in the foreign country.

3. *Markets clear.*

$$\text{Autarky: } C_i(p^A, \gamma) = F_i(p^A, \gamma), i = 1, 2.$$

$$\text{Open economy: } C_i(p^T, \gamma) + C_i^*(p^T, \gamma^*) = F_i(p^T, \gamma) + F_i^*(p^T, \gamma^*).$$

I denote aggregate welfare under autarky by $U^A(\gamma)$ and aggregate welfare under trade as $U^T(\gamma, \gamma^*)$. By definition of indirect utility function,

$$\begin{aligned} U^A(\gamma) &\equiv V(p^A(\gamma), Y(p^A(\gamma), \gamma)), \\ U^T(\gamma, \gamma^*) &\equiv V(p^T(\gamma,\ \gamma^*), Y(p^T(\gamma,\ \gamma^*), \gamma)). \end{aligned}$$

I assume that $U^A(\gamma)$ and $U^T(\gamma,\ \gamma^*)$ are differentiable upto second order. The world welfare is denoted by $W(.)$,

$$W(\gamma, \gamma^*) \equiv U^T(\gamma, \gamma^*) + U^T(\gamma^*, \gamma).$$

Suppose that policy γ lies in a bounded policy space $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$. A change in γ changes the production possibility frontier. Let us define the upper envelope of PPF(γ) as the PPF. Since γ is a choice variable of the government, PPF describes the frontier of the true production possibilities of a country from the point of view of the government. Below I define the government's optimization problem:

Definition 2 *In the closed economy each government chooses γ to maximize aggregate welfare, $U^A(\gamma)$. In the non cooperative optimal policy problem in the open economy, each government chooses own policy to maximize $U^T(\gamma,\ \gamma^*)$ taking the other country's policy as given. In the cooperative optimal policy problem in the open economy the social planner chooses (γ, γ^*) to maximize the world welfare, $W(\gamma, \gamma^*)$.*

The countries are identical in terms of all economic fundamentals. When both countries choose the same policy ($\gamma = \gamma^*$), countries are endogenously in autarky. This gives us the first insightful property of the welfare function which I use throughout the paper:

$$U^T(\gamma, \gamma) = U^A(\gamma). \tag{6}$$

A similar property applies to equilibrium price under trade and autarky:

$$p^T(\gamma,\gamma) = p^A(\gamma).$$

Also, in the open economy the aggregate welfare function satisfies the gains from trade property. By this property, a country, for any given own policy, can gain in welfare terms by trading with a partner with a different policy,

$$U^T(\gamma,\ \gamma^*) \geqslant U^T(\gamma,\ \gamma)\ \forall\gamma^*,\ \forall\gamma, \tag{7}$$

with equality at $\gamma = \gamma^*$. From the gains from trade property, for any two arbitrary policies γ_i and γ_j , the relative welfare of choosing γ_i compared to γ_j , conditional on the trading partner choosing γ_j , improves in open economy over that in autarky.[10] In this sense trade favors asymmetry in policy. Below I summarize these properties of the welfare function.

Remark 3 *By symmetry of the setup*

$$\begin{aligned} U^T(\gamma,\gamma) &= U^A(\gamma), \\ p^T(\gamma,\gamma) &= p^A(\gamma), \\ \frac{\partial p^T(\gamma,\gamma)}{\partial\gamma} &= \frac{1}{2}\frac{dp^A(\gamma)}{d\gamma}. \end{aligned}$$

Remark 4 *By gains from trade property of the welfare function*

$$U^T(\gamma,\gamma^*) \geqslant U^T(\gamma,\gamma)\forall\gamma^*,\forall\gamma,$$

with equality at $\gamma = \gamma^*$.

When autarky optimum policy is not unique, using (7), it is straightforward to show that at least one asymmetric equilibrium exists in both the non cooperative and cooperative optimal policy problem of the open economy. Hence, I restrict attention to the case where the autarky problem has a unique optimum, $\widetilde{\gamma}$.

When do these symmetric countries choose asymmetric policies in the open economy if the autarky problem has a unique optimum? To answer this question, it is important to understand an important property of asymmetric equilibrium. Suppose that $(\gamma',\ \gamma'^*)$ is an asymmetric pure strategy Nash equilibrium. By gains from trade (7),

$$U^T(\widetilde{\gamma},\ \widetilde{\gamma}) \leq U^T(\widetilde{\gamma},\ \gamma'^*).$$

But since γ' is the best response to γ'^*,

[10] From (7), we know that

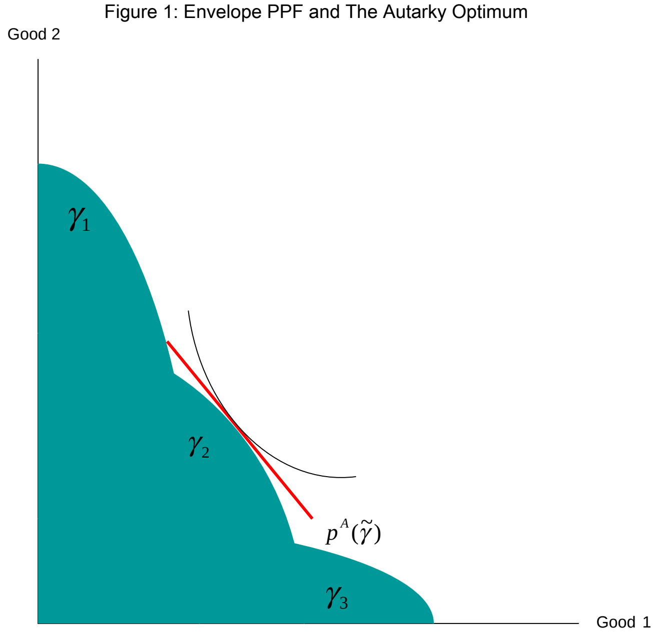
$$\frac{U^T(\gamma_i,\gamma_j)}{U^T(\gamma_j\ ,\gamma_j\ )} > \frac{U^A(\gamma_i)}{U^A(\gamma_j\ )}.$$

$$U^T(\tilde{\gamma},\ \gamma'^*) \leq U^T(\gamma',\ \gamma'^*).$$

Thus, at any asymmetric PSNE both countries gain in aggregate welfare compared to the common autarky optimum. Note that this result is independent of the countries experiencing any price change at the open economy asymmetric equilibrium compared to the autarky optimum. Thus, the welfare gain in the asymmetric equilibrium is due to increase in production specialization, which allows for an expansion of the consumption possibility frontier through trade.

First, I illustrate graphically conditions for equilibrium policy diversity in the light of this welfare-improving nature of asymmetric equilibrium. In Figure 1 for illustration purposes I consider 3 possible policy options $\gamma_1 < \gamma_2 < \gamma_3$. The shaded region denotes the envelope PPF. For the set of consumer preferences described in Figure 1, the autarky optimal policy $\tilde{\gamma}$ is given by γ_2.

Figure 1: Envelope PPF and The Autarky Optimum



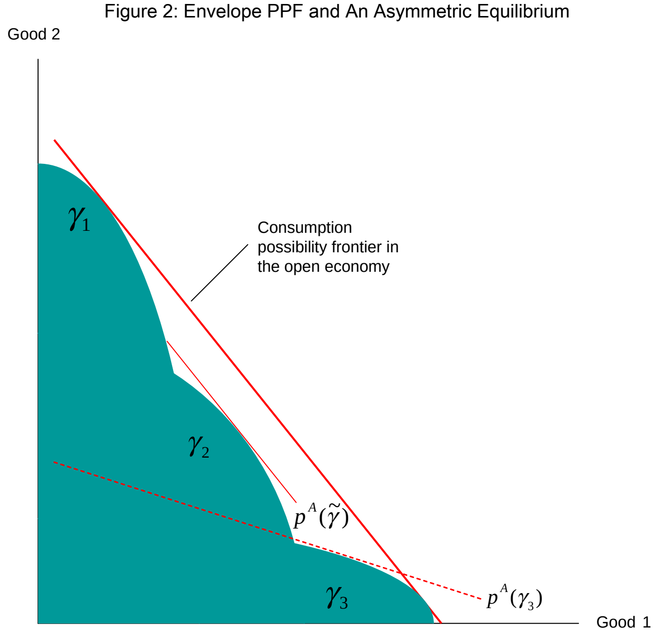
In this framework with identical homothetic demand between countries, comparative-advantage in any industry arises from the supply side. Thus, the role of policy γ in affecting production possibilities of a country is crucial for existence of an asymmetric equilibrium with endogenous comparative advantage in different industries. In a perfectly competitive world with constant returns to scale technologies, the production set for any given γ, PPF(γ), is a convex set. However, constant returns to scale technologies do not imply that the production set described by the envelope PPF is a convex set. If the production set described by PPF is a convex set, it is straightforward to prove that an asymmetric PSNE does not exist.[11]

But when can $(\gamma_1,\ \gamma_3)$ be a Pareto-improving asymmetric NE in the open economy? For illustration purposes, I completely shut down the traditional channel of gains from trade due to price

[11] If the envelop PPF is a convex set, no two production points on the boundary satisfy p=MRT condition.

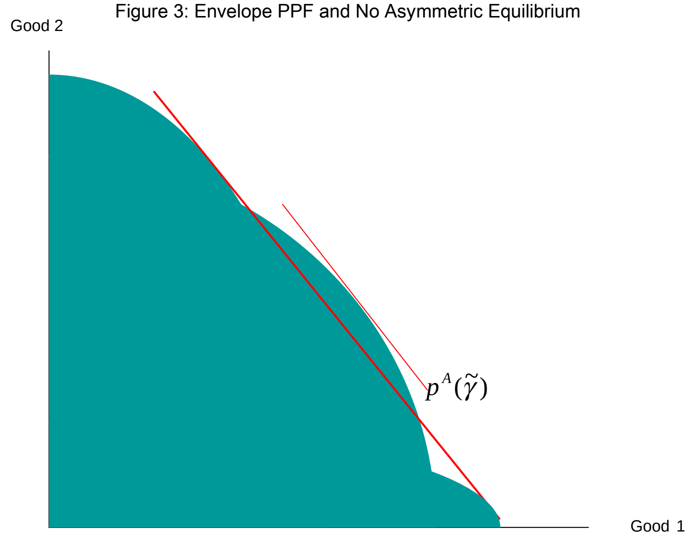
movement from autarky to free trade. Hence, the free trade price at the asymmetric equilibrium is same as the autarky price at $\widetilde{\gamma}$. In Figure 2, a Pareto-improving asymmetric NE exists since the production possibilities described by the PPF is sufficiently nonconvex set. In the open economy a country that chooses $\gamma_3 > \gamma_2$, does not suffer a major adverse terms-of-trade movement, since in the open economy equilibrium price is less responsive to any country's policy movements.[12] Thus by specializing in two distinct industries each country observes an expansion in its consumption possibility frontier. This opens the door for welfare gains in an asymmetric equilibrium in the open economy. In contrast, in Figure 3 when the production possibility set does not show enough nonconvexity, no such asymmetric PSNE exists.

Figure 2: Envelope PPF and An Asymmetric Equilibrium



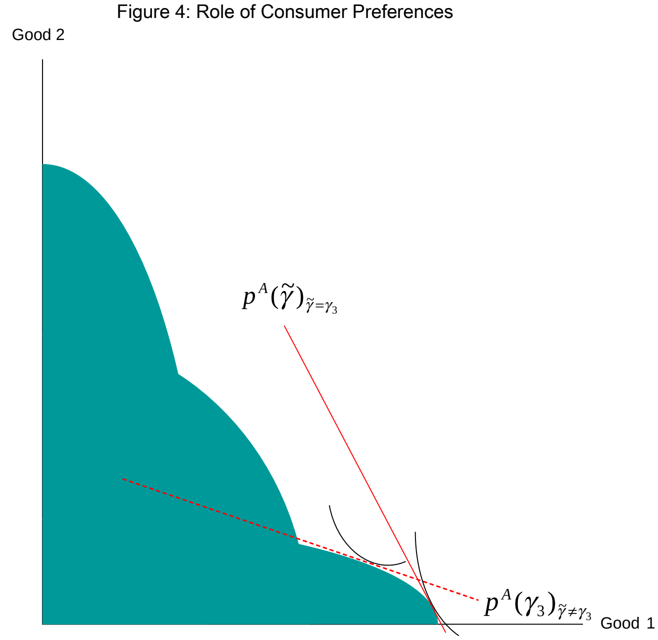
[12]This is due to two reasons. First, $\frac{\partial p^T}{\partial \gamma} = .5 \frac{dp^A}{d\gamma}$ at a point of symmetry. Thus change in own policy affects equilibrium price relatively less in the open economy. Moreover, in the asymmetric equilibrium the foreign country's choice of policy actually improves the TOT for the home country.

Figure 3: Envelope PPF and No Asymmetric Equilibrium



However, sufficiently nonconvex production possibility set alone is not sufficient for existence of an asymmetric PSNE. In Figure 4, I consider a situation where the PPF is same as in Figure 2, and consumer preferences are significantly biased towards good 1. This makes γ_3 the relevant autarky optimal policy. Presence of significant bias in consumption implies that even by choosing a higher γ, the relative price of good 1 is not very low. The dotted line corresponds to $p^A(\gamma_3)_{\tilde{\gamma}\neq\gamma_3}$ under consumer preferences in Figure 2, and the solid line corresponds to $p^A(\gamma_3)_{\tilde{\gamma}=\gamma_3}$ under consumer preferences biased towards good 1. Note that for such biased consumer preferences there is no asymmetric equilibrium in the open economy, and γ_3 becomes the dominant strategy.

Figure 4: Role of Consumer Preferences



The figures offer a great tool for illustration. More generally, I consider a bounded policy space

$\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ to relate conditions for equilibrium asymmetry using non-convexities in the production set to conditions for symmetry-breaking in the literature (Matsuyama (2002) and Amir, Garcia and Knauff (2010)). This allows me to derive properties of the aggregate income function that are crucial for equilibrium policy diversity and to establish welfare properties of asymmetric equilibria. As before, $\widetilde{\gamma}$ denote the unique autarky optimal policy.

I focus on the non cooperative optimal policy problem. Each government simultaneously chooses its policy to maximize aggregate welfare. The optimal policy of a country depends on the policy of the trading partner. I restrict attention to the pure strategy Nash equilibrium (PSNE), and assume that an equilibrium exists in the policy game.[13]

Welfare properties of different asymmetric PSNEs follow from a simple generalization of the gains from trade property. Not only a country gains in welfare by trading with a partner who has a different policy, but a country gains more from trade, given her own policy, more different is the trading partner.[14] I summarize this property of the welfare function in the following lemma.

Lemma 5 (Greater the Difference, Greater the Gains) *For any given own policy, the welfare of a country increases with an increase in the foreign policy, if the foreign policy is greater than the own policy. Thus,*

$$\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} > 0 \ for \ \gamma^* > \gamma.$$

Similarly,

$$\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} < \ 0 \ for \ \gamma^* < \gamma.$$

Proof. See Appendix. ■

This property of the welfare function simply says that given a country's own policy, greater is the difference with the trading partner, greater are the welfare gains. This generic property of comparative-advantage driven trade and the definition of NE allow us to rank various asymmetric PSNEs in terms of associated welfare. Consider two asymmetric PSNEs (γ_1, γ_1^*) and (γ_2, γ_2^*) such that the home country is an exporter of good 1 in both of these equilibria ($\gamma_i > \gamma_i^*$, $i = 1, 2$). Also, the countries are more different in the first PSNE than in the second one, $\gamma_1^* < \gamma_2^* < \gamma_2 < \gamma_1$. Both countries attain a higher welfare in (γ_1, γ_1^*) compared to (γ_2, γ_2^*). Hence, an equilibrium with greater asymmetry increases the welfare of both countries. I describe this property in Proposition 6. Proposition 6 provides a welfare-ranking of multiple asymmetric PSNEs on any given side of the diagonal of the strategy space.

Proposition 6 (Welfare Ranking of Asymmetric PSNEs) *Consider two pairs of asymmetric PSNEs – $(\gamma_1,\gamma_1^*), (\gamma_1^*,\gamma_1)$ and $(\gamma_2,\gamma_2^*), (\gamma_2^*,\gamma_2)$ such that $\gamma_1^* < \gamma_2^* < \gamma_2 < \gamma_1$. Both countries have a higher welfare at (γ_1,γ_1^*) compared to at (γ_2,γ_2^*) and at (γ_1^*,γ_1) relative to at (γ_2^*,γ_2).*

[13] In an application in the next section, the optimal policy of a country decreases in the policy of the other country. Such strategic substitutability ensures the existence of at least one PSNE in the optimal policy problem by Topkis (1978).

[14] Ethier (2008) highlights a similar result "the greater the differences , the greater the gains" in comparative advantage-driven trade.

Proof. See Appendix. ■

But when does an asymmetric PSNE exist? As a corollary of Proposition 6, whenever an asymmetric PSNE exists the Pareto optimum is asymmetric. It is straightforward to show that $(\widetilde{\gamma}, \widetilde{\gamma})$ is the only candidate for a symmetric Pareto optimum. Alternatively, if $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique Pareto optimum, a Pareto-improving asymmetric equilibrium cannot exist. If the world welfare $W(.,\ .)$ is strictly quasiconcave, the symmetric strategy profile at $(\widetilde{\gamma}, \widetilde{\gamma})$ is the unique Pareto optimum. Hence, If the world welfare $W(.,\ .)$ is strictly quasiconcave, an asymmetric equilibrium does not exist.

Next, I investigate properties of the welfare function sufficient for the existence of an asymmetric PSNE. The policy game does not satisfy the sufficient conditions for the existence of an asymmetric PSNE in a symmetric game in the literature. In Matsuyama (2002) an asymmetric PSNE exists if the symmetric PSNE is Cournot unstable. In this two-good model with the terms-of-trade externality as the only source of strategic interaction, the symmetric equilibrium, if it exists, is stable. Stability of the symmetric equilibrium follows from the substitutability of the two goods in consumption.[15] Amir, Garcia and Knauff (2010) rule out any symmetric equilibrium since in their game the payoff function does not satisfy the necessary condition for an interior optimum at any interior point of symmetry. In this case by the gains from trade property the welfare function, $U^T(\gamma, \gamma^*)$, has slope 0 at $(\widetilde{\gamma}, \widetilde{\gamma})$.[16] Hence the exact sufficient condition outlined in Amir, Garcia and Knauff (2010) is not satisfied in my framework. However, similar to Amir, Garcia and Knauff (2010), I rely on some form of payoff nonconcavity to rule out a symmetric equilibrium in this case.

In this game the unique autarky optimum policy $\widetilde{\gamma}$ is also the unique candidate for a symmetric equilibrium. Any unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$ comes with a gain from a trade component and a loss in autarky utility component. If the home country can profitably deviate to a $\gamma' \neq \widetilde{\gamma}$ when the partner is choosing $\widetilde{\gamma}$, only asymmetric PSNEs exist in this game. Existence of such a profitable deviation implies

$$\underbrace{U^T(\gamma',\ \widetilde{\gamma}) - U^A(\gamma')}_{\text{Gains from trade}} > \underbrace{U^A(\widetilde{\gamma}) - U^A(\gamma')}_{\text{Loss in autarky utility}}.$$

Thus the symmetric PSNE at $(\widetilde{\gamma}, \widetilde{\gamma})$ is ruled out if the gains from trade at the strategy profile $(\gamma',\ \widetilde{\gamma})$ exceeds the loss in autarky utility from choosing γ'.

In general, the problem of ruling out any symmetric PSNE is technically equivalent to finding a global maximum of $U^T(\gamma, \widetilde{\gamma})$ at a $\gamma' \neq \widetilde{\gamma}$, even though $\widetilde{\gamma}$ is a local maximum. The welfare function in the open economy, $U^T(\gamma, \widetilde{\gamma})$, is quasi-convex in own policy over some part of the action space for this separation of the global maximum (γ') from the autarky optimum. In fact if $U^T(\gamma, \gamma^*)$ is strictly quasiconvex in own policy and $\widetilde{\gamma}$ is an interior optimum in autarky, there is a unique pair

[15] See appendix for a proof of stability of any symmetric PSNE.

[16] The necessary first order condition of maximization of $U^T(\gamma, \widetilde{\gamma})$ is satisfied at $\widetilde{\gamma}$ since by gains from trade and endogenous autarky properties of the welfare function, $U^T(\gamma, \widetilde{\gamma})$ is an upper envelop of $U^A(\gamma)$ with equality $\gamma = \widetilde{\gamma}$, where $\widetilde{\gamma}$ is the critical point of $U^A(\gamma)$.

of asymmetric NE given by the extremes $(\underline{\gamma},\ \overline{\gamma})$.and $(\overline{\gamma},\ \underline{\gamma})$.[17, 18] By welfare ranking of asymmetric PSNE in proposition 6, these extreme PSNEs are also the best in terms of welfare.

This quasiconvexity in the welfare function arises in presence of social increasing return to policy, i.e. if aggregate income is convex in government policy. I show in the appendix that if $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$, $W(\gamma,\gamma^*)$ is strictly quasiconcave and there is no asymmetric equilibrium in the open economy. On the other hand, given an interior $\widetilde{\gamma}$, if $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ is positive and sufficiently large, we can have a situation where $U^T(\gamma,\ \gamma^*)$ is quasiconvex at $(\widetilde{\gamma},\ \widetilde{\gamma})$, even though $U^A(\gamma)$ is quasiconcave at $\widetilde{\gamma}$. This imply that all equilibria in the open economy are asymmetric. I summarize these conditions for existence of asymmetric equilibrium in the next proposition.

Proposition 7 (Symmetry- Breaking and Social Increasing Return) *If aggregate income is linear in policy* ($\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$), *no asymmetric PSNE exist in the optimal policy problem. If aggregate income is sufficiently convex in policy (given an interior* $\widetilde{\gamma}$, *if* $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ *is positive and sufficiently large), all PSNE in the open economy are asymmetric. Whenever an asymmetric PSNE exist, Pareto optimum is also asymmetric.*

Proof. See Appendix. ■

Since convexity in income with respect to policy plays such an important role for ensuring equilibrium asymmetry, I explore this property of the economy in detail in an application. I show, in an application to education policy in a standard Heckscher-Ohlin model, that such social increasing return in aggregate production may arise in presence of optimal endogenous skill choice of agents even though technologies are constant returns to scale.

3 An Application to Education Policy

I consider a specific application of the abstract general policy problem to education policy. Governments allocate a fixed education budget, financed by lumpsum taxation, between higher and primary education.

I consider a simple model of endogenous skill choice by heterogenous agents. In each country there are two types of labor. High types are born with ability h and low types are born with ability l, $h > l$. There are n_L low type workers and n_H high type workers. A positively skewed skill distribution implies $n_L > n_H$.

[17] Because the game is symmetric, any asymmetric PSNE appears in pairs. When there are only asymmetric PSNEs in this game, the total number of PSNEs is even. In a game with continuum action space, usually there are an odd number of PSNEs by Wilson's Oddness Theorem (1971). This result is based on the degree theory and requires continuity of the best response form. Ruling out symmetric equilibrium in this game involves a robust jump of the best replies across the diagonal of the strategy space as in Amir, Garcia, Knauff (2010). Hence, my results are consistent with the Wilson's Oddness Theorem (1971).

[18] In this setup a symmetric mixed strategy NE always exists, by the Folk Theorem (Dasgupta and Maskin 1986). However, in a game characterized by strategic substitutability, MSNE is usually unstable. Also, in my case countries attain greater welfare in any asymmetric PSNE compared to a symmetric MSNE. Moreover, it is standard in the policy literature to focus on the PSNE as the relevant solution concept.

Each type of worker chooses a skill level incurring cost of education (measured in welfare terms). Both total and marginal cost of education rise in skill and decrease in ability. Cost of education is affected by allocation of education budget by the government. Let h^e denote skill chosen by high type workers and l^e denote skill chosen by low type workers. Total effective endowment of high-skilled and low-skilled labor is given by

$$H^e = h^e n_H \text{ and } L^e = l^e n_L.$$

As before, u denotes the direct utility function,

$$u(C_1, C_2) = C_1^{\mu} C_2^{1-\mu},$$

where μ is the expenditure share of good 1 and C_i is aggregate consumption of good i. Here, C_i is given by

$$n_H c_i^H + n_L c_i^L$$

where c_i^H and c_i^L denote individual consumption of high and low type agents. Good 2 is the numeraire and p is the relative price of good 1. I assume that factor price equalization holds over the entire strategy space at the equilibrium price for the specified set of parameters and functional forms.

High types choose skill, h^e and consumption, c_1^H and c_2^H, to maximize utility

$$\begin{aligned} u(c_1^H, c_2^H) - \beta_H(\frac{h^e}{h})^{\epsilon},\ \beta_h &> 0, \epsilon > 1, \\ s.t.\ pc_1^H + c_2^H &\leq w_H h^e, \end{aligned}$$

given equilibrium wage, price and educational institutional parameters. Here, w_H (w_L) refers to the wage of high- and low-type workers. Low type workers solve a similar optimization problem.

Welfare cost of education is captured by $\beta_H(\frac{h^e}{h})^{\epsilon}$. Both total and marginal cost of education rise in the ability h and fall in the skill h^e. Note that a fall in β_t, $t = H, L$ reduces the cost of education. I define β_t as the quality of educational institutions. Here, ϵ is the elasticity of cost of education with respect to skill. Note that given an ability h, the relative cost of acquiring higher skill $h'' > h'$, $\frac{\beta_H(\frac{h''}{h})^{\epsilon}}{\beta_H(\frac{h'}{h})^{\epsilon}}$, increases in ϵ. I define ϵ as the progressivity of the education system, following the interpretation in Benabou (2002). The condition $\epsilon > 1$ ensures that the second order condition of optimality of the agents' skill choice is satisfied.

The optimal skill choice function is given by

$$
\begin{aligned}
H^e &= c_{h'}(\frac{H^\epsilon p^{\frac{1-a}{\eta_1-\eta_2}}}{n_H\beta_H})^{\frac{1}{\epsilon-1}},\ c_{h'} = f(\eta_1,\eta_2,\mu,\epsilon),\ a = \mu\eta_1 + (1-\mu)\eta_2, \qquad (8) \\
L^e &= c_{l'}(\frac{L^\epsilon p^{\frac{-a}{\eta_1-\eta_2}}}{n_L\beta_L})^{\frac{1}{\epsilon-1}},\ c_{l'} = f(\eta_1,\eta_2,\mu,\epsilon).
\end{aligned}
$$

where H $(= hn_H)$ and L $(= ln_L)$ denote original endowments of skill in the economy.

Producers maximize profit given the equilibrium wage, price and effective endowments of skill. Producers of the two goods employ high- and low-skilled labor with different intensities

$$F_i(H_i^e, L_i^e) = H_i^{e\eta_i} L_i^{e1-\eta_i}$$

where η_i denotes the share of high-skilled labor per unit cost of good i. Good 1 is relatively more intensive in high-skilled labor, $\eta_1 > \eta_2$.

In equilibrium all the optimization conditions hold and both labor and goods markets clear. Below I define competitive equilibrium of this economy.

Definition 8 *A competitive equilibrium is defined by a sequence* $\{p, w_H, w_L,\ h^e, l^e,\ c^H_{i=1,2},\ c^L_{i=1,2}, h^{e*}, l^{e*}, c^{H*}_{i=1,2}, c^{L*}_{i=1,2}\}$ *s.t.*

1. *Given goods and factor prices* $\{p, w_H, w_L\}$*, high (low) type workers optimally choose* h^e *and* $c^H_{i=1,2}$ (l^e *and* $c^L_{i=1,2}$) *to maximize utility from consumption net of education cost, subject to usual budget constraint in each country.*

2. *Given goods and factor prices* $\{p, w_H, w_L\}$ *and effective endowment of factors* (H^e, L^e)*, producers in sector i choose optimal allocation of factors* (H_i^e, L_i^e) *to maximize profit in each country.*

3. *Markets clear. This imply, in each country*

$$
\begin{aligned}
H_1^e + H_2^e &= H^e \\
L_1^e + L_2^e &= L^e
\end{aligned}
$$

Under autarky

$$C_i = F_i(H_i^e, L_i^e)$$

Under trade

$$C_i + C_i^* = F_i(H_i^e, L_i^e) + F_i(H_i^{e*}, L_i^{e*})$$

The government has a total education budget $T = 1$, and chooses a fraction $\gamma \in [0, 1]$ to spend on higher education. The remainder goes to primary education. The government expenditure on higher education, γ, improves the higher educational institutions,

$$\beta_H = g(\gamma),\ g' < 0.$$

Similarly, $\beta_L = g(1-\gamma)$.[19] For simplicity, I assume that initially higher education and basic education institutions have same quality ($\beta_H = \beta_L = b > 1$), and government policy affects the institutions in a simple linear fashion,

$$\begin{aligned}\beta_H &= b - c\gamma,\ b > c > 0,\\ \beta_L &= b - c(1-\gamma).\end{aligned}$$

Note that the government always has the option of improving both types of institutions equally ($\gamma = .5$), but may choose to attach different priorities to different institutions. From optimal skill choice function, (8), the condition $\epsilon > 1$ ensures that H^e is a convex function of γ, given p. Thus when agents choose their skill levels optimally, optimal skill function in the economy is convex in government policy, even though government policy affects education costs in a simple linear fashion. This ensures that necessary condition for symmetry-breaking viz. aggregate income is convex in policy, is satisfied in this framework.

The government takes the optimal response of the agents and market clearing conditions as given and chooses allocation of the education budget, γ, to maximize the aggregate indirect utility. As before, in the closed economy each government chooses γ to maximize aggregate welfare, $U^A(\gamma)$. In the non cooperative optimal policy problem in the open economy, each government maximizes $U^T(\gamma, \gamma^*)$ taking the other country's policy as given. In the cooperative optimal policy problem in the open economy the social planner maximizes the world welfare, $W(\gamma, \gamma^*)$.

Let us define the relative welfare under a policy γ' compared to γ'', given that the foreign country is choosing γ^*, as

$$r^T(\gamma',\ \gamma'',\ \gamma^*) = \frac{U^T(\gamma', \gamma^*)}{U^T(\gamma'', \gamma^*)}. \tag{9}$$

Using a similar notation,

$$r^A(\gamma',\ \gamma'') = \frac{U^A(\gamma')}{U^A(\gamma'')}, \tag{10}$$

stands for the relative welfare under a policy γ' compared to γ'' in autarky. For any $\gamma' > \gamma''$, (10) increases in μ and η_i . Hence, the autarky optimal policy ($\widetilde{\gamma}$) monotonically increases in the expenditure share of the skill intensive good and in the skill intensities of production. This is intuitive since γ is the fraction of education budget spent on higher education. For any $\gamma' > \gamma''$, (9) increases in μ and η_1, which ensures that the non - cooperative best reply correspondence in the open economy, BR(γ^*), shifts up in μ and η_1. I summarize this result in the next lemma.

[19] The assumptions of education system essentially mean that education is publicly funded and government decides which type of institutions to emphasize relatively more.

Lemma 9 *For any $\gamma' > \gamma''$, (10) increases in μ and η_i . The autarky optimal policy $\widetilde{\gamma}$ is increasing in μ, η_i. For any $\gamma' > \gamma''$, (9) increases in μ and η_1. In the open economy $BR(\gamma^*)$ shifts up in μ and η_1.*

Proof. See Appendix. ■

These comparative static properties of (9) help us to understand the comparative static properties of an asymmetric PSNE. For any $\gamma' > \gamma''$ (9) is increasing in both μ and η_1. For given η_1, let μ' be the minimum value of μ such that $\gamma' > \widetilde{\gamma}$ is a profitable unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$. An increase in η_1 increases $r^T(\gamma', \widetilde{\gamma}, \widetilde{\gamma})$ and ensures that γ' is a profitable unilateral deviation from $(\widetilde{\gamma}, \widetilde{\gamma})$ for values of $\mu < \mu'$. In general, if $\underline{\mu}$ and $\overline{\mu}$ are respectively the minimum and maximum values of μ for which only asymmetric PSNEs exist, then both $\underline{\mu}$ and $\overline{\mu}$ decline in η_1. Note that η_1 is the high-skill intensity in the production of good 1, and μ is the expenditure-share of the more skill-intensive good 1. Thus, given a sufficiently large social increasing return, only asymmetric PSNEs exist if consumer preferences and production technologies are not biased towards the same factor of production.

The following proposition summarizes the condition for equilibrium policy diversity in the open economy. Note that in this model,

$$\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2} = w_H(p)s_H(p)\frac{\partial \beta_H^{\frac{1}{1-\epsilon}}}{\partial \gamma^2} + w_L(p)s_L(p)\frac{\partial \beta_L^{\frac{1}{1-\epsilon}}}{\partial \gamma^2}, \tag{11}$$

where $s_t(p)$, $t = H, L$, are given from (8). Given $\epsilon > 1$, a fall in ϵ increases the convexity of (11) provided $b \leq 1$. Hence, if ϵ is sufficiently low, $(\eta_1 - \eta_2)$ is sufficiently high and consumers do not prefer either of the goods too strongly, an asymmetric PSNE exists.

Proposition 10 *If ϵ is sufficiently low and $b \leq 1$, $(\eta_1 - \eta_2)$ is sufficiently high and consumers do not prefer either of the goods very strongly, an asymmetric PSNE exists in the open economy optimal policy problem.*

Proof. See Appendix. ■

4 Discussion

The application to education policy in the previous section provides a setting where aggregate income is convex in government policy. This leads to asymmetric equilibrium policy choice for symmetric countries to encourage trade. In presence of standard constant returns to scale technology in a Heckscher-Ohlin set up, this social increasing return to policy arises due to endogenous skill choice of agents. I consider an alternative specification which shares the production structure of Grossman and Maggi (2000), but has a similar specification of consumer preferences and government policy. For simplicity I assume allocation of education budget by the government affects effective endowment of skill directly. The submodular production technology implies that even when government policy affects skills in a linear fashion, aggregate production and income can be sufficiently convex in policy and this gives rise to symmetry-breaking in optimal education policy in the open economy.[20]

This symmetry-breaking result resonates with the key result of Chang and Huang (2012). Using a novel specification of education policy in a Grossman-Maggi setup, Chang and Huang (2012) show that Nash equilibrium choice of education systems by two countries interacting strategically are necessarily more divergent than their autarky choices. Chang and Huang (2012) model the two-way causal relationship between trade and education systems. A country's education system determines its talent distribution and comparative advantage; the possibility of trade by raising the returns to the sector of comparative advantage in turn induces countries to further differentiate their education systems and reinforces the initial pattern of comparative advantage. In my setup with symmetric countries, I isolate the effect of trade on optimal choice of education policy. Even in absence of any preexisting differences, possibility of trade is sufficient for symmetry-breaking in optimal education policy because of the social increasing return to policy introduced by the submodular production specification, provided consumer preferences are reasonably diversified.

But how important is this social increasing returns in explaining policy diversity among similar (but not identical) countries? In this case, as in Chang and Huang (2012), the relevant question is whether countries optimally choose to magnify pre-existing national differences by investing relatively more in their respective areas of comparative advantage. I consider the simple specification of Heckscher-Ohlin model in section 3. For simplicity I assume allocation of education budget by the government affects effective endowment of skill directly.

A natural way to introduce initial differences is to consider countries that have different initial factor endowments. In this setting if the government policy is sufficiently effective in increasing skill, countries optimally amplify initial sources of comparative advantage in the open economy equilibrium. In such a PSNE both countries attain larger aggregate welfare compared to their respective autarky optima.[21]

[20] Note that the intuition for this result is quite general. To see this, let us consider the general framework in section 2. In the general framework say ϖ is the original determinant of trade and γ is the relevant policy,

$$\frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2} = (p^T \frac{\partial^2 F_1(p^T,\gamma)}{\partial \gamma^2} + \frac{\partial^2 F_2(p^T,.)}{\partial \gamma^2})$$

plays the crucial role for existence of asymmetric equilibrium. But note that

$$\frac{\partial^2 F_i(p^T,\gamma)}{\partial \gamma^2} = \frac{\partial^2 F_1(p^T,\gamma)}{\partial \varpi^2}(\frac{\partial \varpi}{\partial \gamma})^2 + \frac{\partial F_1(p^T,\gamma)}{\partial \varpi}(\frac{\partial^2 \varpi}{\partial \gamma^2}).$$

In the presence of submodularity in production essentially $\frac{\partial^2 F_2(p^T,\gamma)}{\partial \varpi^2} > 0$, and hence $\frac{\partial^2 \varpi}{\partial \gamma^2} > 0$ is not required for the convexity in production.

[21] However, for the relatively skill abundant country, an increase in the higher education investment leads to a terms-of-trade deterioration. When these terms-of-trade considerations are very important, an inefficient symmetric non cooperative outcome may exist. In such a symmetric PSNE, the world welfare improves if each country invests more in their areas of comparative advantage. This possible inefficiency of the PSNE raises the same concern of international policy cooperation as in the familiar case of tariff/tax policies (see for example, Deardorff (1997)). The difference is that in this case the countries should focus in their relative areas of comparative advantage to attain a

Thus, if countries are initially different, social increasing return is no longer necessary for amplification of initial comparative advantage in the open economy optimal policy outcome. However, I Illustrate in a series of numerical simulation, in absence of such increasing return to policy one can explain significant policy differences among countries via welfare gains from trade only if the countries are originally significantly different. To explain *significant* policy diversity among *similar* countries, one would still need to rely on presence of an increasing return to policy.

A natural extension of the pure welfare maximizing optimal policy problem is to allow governments to also care for redistributive equity. In the general framework it is straightforward to show that symmetric autarky equilibrium continues to remain the only candidate for a symmetric Pareto optimum. Even in presence of redistributive concerns, quasiconvexity in the welfare function and convexity in aggregate income remain important for existence of an asymmetric equilibrium.[22] In the presence of political concerns it is no longer true that both countries attain higher social welfare at an asymmetric PSNE compared to any symmetric PSNE.[23] Also, allowing for endogenous comparative advantage in a political economy framework, political preferences of the government can be a source of comparative advantage in the open economy.

5 Conclusion

In this paper I show that similar countries may choose different policies because these policies allow them to specialize in different industries and gain from international trade. I find that even identical countries may optimally choose different policies in an open economy, and both of these symmetric countries gain in aggregate welfare in any asymmetric equilibrium compared to the autarky optimum. In an application in the competitive economy an asymmetric equilibrium arises if education policy affects the determinants of trade, namely effective endowments of skill, in a strong convex fashion. I construct a model where agents optimally choose their skill levels given government policy, and show that optimal skill is a convex function of government policy. I also study countries that are similar but not identical and find that these countries may optimally choose to invest more in their respective areas of comparative advantage to magnify initial differences.

This paper outlines a general mechanism that applies to many different policies which can potentially affect comparative advantage in the open economy. For any particular application, it is important to model the domestic economic environment more carefully. For example, education policy is an important policy in encouraging trade, but there are several reasons why education

Pareto improvement. When comparative advantage is at least partly endogenous to national policy, an international policy coordination requires the countries to agree to disagree.

[22] Note that if the production possibility set described by the envelope PPF is a convex set, an asymmetric equilibrium does not exist under free trade, independent of the nature of government preferences.

[23] For example, if social welfare differs from aggregate welfare due to a political concern for increases in inequality, it is possible that one of the countries prefer a symmetric PSNE over an asymmetric PSNE because of the increase in inequality under trade. Using numerical simulation in the simple Hecksher-Ohlin economy I illustrate how the gain in social welfare in the asymmetric PSNE varies with changes in economic fundamentals. Also, I demonstrate numerically the parameter space for which an asymmetric equilibrium exists and how key convexity parameters affect the optimal policy outcome. Details are available upon request.

policy is important for the domestic economy itself. Since future human capital is typically not accepted as collateral, availability of credit for financing educational expenditure is limited. This aspect of the education policy has received attention in both trade and macro policy literature (Benabou (2002), Ranjan (2000), Chesnokova and Krishna (2008)). Also, skills learnt in the earlier stages of academic development are complementary in acquiring advanced skills. In future work I intend to incorporate these aspects of education in a more complete application to study the interplay between the domestic and the international motives of optimal policy, and to study the implications for inequality.

Acknowledgement *This paper is based on the first chapter of my Ph.D. thesis, "Why Do Similar Countries Choose Different Policies? Endogenous Comparative Advantage and Welfare Gains." I am indebted to Gene Grossman and Esteban Rossi-Hansberg for invaluable guidance and encouragement. I am also grateful to Avinash Dixit and Robert Staiger for very helpful comments. I would like to thank Roland Benabou, Saroj Bhattarai, Arnaud Costinot, Arghya Ghosh, Kala Krishna, Devashis Mitra, Vinayak Tripathi, Alan Woodland and participants of various workshops and seminars for their comments and suggestions. Most of this research has been conducted at International Economics Section, Princeton University. I thank the International Economics Section, Princeton University, Social Policy Program, Woodrow Wilson School and Australian School of Business, University of New South Wales for their financial support. All remaining errors are mine.*

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6 Appendix

6.1 Derivation and Proofs for Section 2

From homothetic demand aggregate demand for good i C_i is linearly homogenous in income,

$$C_i = f_i(p) * Y.$$

Here Y is aggregate income,

$$Y = pF_1 + F_2,$$

where F_i stands for aggregate production of good i. The aggregate indirect utility is given by,

$$V(.,.) = U(f_1(p) * y,\ f_2(p) * y) = Yf(p),$$

where $f(p) = U(f_1(p),\ f_2(p))$. From Roy's identity,

$$\frac{f'(p)Y}{f(p)} = -C_1 = -f_1(p)Y. \tag{12}$$

The marginal effect of p on indirect utility is given by,

$$\frac{\partial V}{\partial p} = f'(p)Y + f(p)F_1 + f(p)(p\frac{dF_1}{dp} + \frac{dF_2}{dp}).$$

The condition $p = MRT = \frac{-dF_2}{dF_1}$ implies

$$\frac{\partial V}{\partial p} = f(p)(F_1 + \frac{f'(p)}{f(p)})Y).$$

From (12),

$$f(p)(F_1 + \frac{f'(p)}{f(p)}Y) = f(p) * (F_1 - f_1(p)Y) = f(p) * (F_1 - C_1).$$

Hence from (4), $\frac{\partial V}{\partial p} = 0$ in autarky. Assume that the policy instrument γ provides absolute advantage in good 1 and absolute disadvantage in good 2. Since total value of consumption equals income,

$$f_1(p) * p + f_2(p) = 1. \tag{13}$$

Proof of Lemma 5. Since γ^* affects welfare of the home country through terms-of-trade,

$$\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} = \frac{\partial V}{\partial p}\frac{\partial p^T}{\partial \gamma^*}.$$

From Roy's identity,

$$\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} = f(p^T)(F_1(.) - C_1(.))\frac{\partial p^T}{\partial \gamma^*}.$$

Since an increase in γ confer comparative advantage in good 1,

$$\frac{\partial p^T(\gamma,\ \gamma^*)}{\partial \gamma^*} < 0.$$

Suppose $\gamma^* > \gamma$, which implies that the home country is an importer of good 1,

$$F_1 < C_1,\ \text{and}\ \frac{\partial U(\gamma,\ \gamma^*)}{\delta \gamma^*} > 0.$$

Similarly, $\gamma^* < \gamma$ implies $\frac{\partial U(\gamma,\ \gamma^*)}{\delta \gamma^*} < 0$. A similar proof works if an increase in γ confer comparative advantage in good 2. ■

Proof of Proposition 6. Given $\gamma_1^* < \gamma_2^* < \gamma_2$ and $\frac{\partial U^T(\gamma,\gamma^*)}{\partial \gamma^*} < 0$ for $\gamma > \gamma^*$, (by Lemma 5),

$$U^T(\gamma_2, \gamma_2^*) < U^T(\gamma_2, \gamma_1^*).$$

But,

$$U^T(\gamma_2, \gamma_1^*) < U^T(\gamma_1, \gamma_1^*)$$

since γ_1 is the best response to γ_1^*. Similarly, given $\gamma_2^* < \gamma_2 < \gamma_1$ and $\frac{\partial U^T(\gamma^*,\ \gamma)}{\partial \gamma} > 0$ for $\gamma^* < \gamma$,

$$U^T(\gamma_2^*, \gamma_2) < U^T(\gamma_2^*, \gamma_1).$$

But γ_1^* is the best response to γ_1, which implies

$$U^T(\gamma_2^*, \gamma_1) < U^T(\gamma_1^*, \gamma_1).$$

■

Proof of Proposition 7. I prove this proposition in three steps.

Step 1: Properties of Cooperative Welfare Maximization: *A symmetric strategy profile in which both countries choose the autarky optimum, $(\widetilde{\gamma}, \widetilde{\gamma})$, is the only candidate for a symmetric Pareto optimum. If $\widetilde{\gamma}$ lies in the interior of the policy space and $W(\gamma, \gamma^*)$ is quasiconvex in (γ, γ^*) at $(\widetilde{\gamma}, \widetilde{\gamma})$, all the Pareto optima are asymmetric.*

Proof. The result that $(\widetilde{\gamma}, \widetilde{\gamma})$ is the only candidate for a symmetric Pareto optimum follows directly from the symmetry of the setup which ensures $W(\gamma, \gamma) = 2U^A(\gamma)$, and optimality of $\widetilde{\gamma}$.

If $\widetilde{\gamma}$ is an interior autarky optimum, first order condition (FOC) of maximization must be satisfied at $\widetilde{\gamma}$. From the definition of $U^A(.)$,

$$\frac{dU^A(.)}{d\gamma} = \frac{\partial V}{\partial p}\frac{\partial p^A}{\partial \gamma} + \frac{\partial V}{\partial \gamma}.$$

From Roy's identity,

$$\frac{\partial V}{\partial p} = f(p^A)(F_1(.) - C_1(.)),$$

which equals zero from (4). Therefore,

$$\frac{dU^A(.)}{d\gamma} = \frac{\partial V}{\partial \gamma} = 0 \text{ at } \widetilde{\gamma} \text{ from the FOC.}$$

Similarly from the definition of $W(.)$ and using (5),

$$\frac{\partial W(.)}{\partial \gamma} = \frac{\partial V}{\partial \gamma}.$$

Therefore, the FOC of optimization of the world welfare is satisfied at $(\widetilde{\gamma},\ \widetilde{\gamma})$. If $W(\gamma, \gamma^*)$ is quasiconvex in $(\gamma,\ \gamma^*)$ at $(\widetilde{\gamma},\ \widetilde{\gamma})$, every Pareto optimum is asymmetric since $(\widetilde{\gamma},\ \widetilde{\gamma})$ is the unique candidate for symmetric Pareto optimum. ■

Step 2: Properties of Non-cooperative Welfare Maximization: *A symmetric strategy profile in which both countries choose the autarky optimum, $(\widetilde{\gamma},\ \widetilde{\gamma})$, is the only candidate for a symmetric PSNE. Suppose that $\widetilde{\gamma}$ is an interior optimum. If $W(\gamma, \gamma^*)$ is strictly quasiconcave, an asymmetric PSNE does not exist. If $U^T(\gamma,\ \gamma^*)$ is strictly quasiconvex in own strategy, there is a unique pair of asymmetric NE given by the extremes $(\underline{\gamma},\ \overline{\gamma})$.and $(\overline{\gamma},\ \underline{\gamma})$.*

Proof. Suppose that $(\widehat{\gamma},\ \widehat{\gamma})$, $\widehat{\gamma} \neq \widetilde{\gamma}$ is a PSNE. The home country attains a payoff of $U^T(\widetilde{\gamma},\ \widehat{\gamma})$ by deviating to $\widetilde{\gamma}$, given that the foreign country is choosing $\widehat{\gamma}$. The resulting change in payoff $(U^T(\widetilde{\gamma}, \widehat{\gamma}) - U^T(\widehat{\gamma}, \widehat{\gamma}))$ consists of a gain from trade component $((U^T(\widetilde{\gamma}, \widehat{\gamma}) - U^A(\widetilde{\gamma}))$ and an increase in autarky utility $(U^A(\widetilde{\gamma}) - U^A(\widehat{\gamma}))$. Hence, the autarky optimum is a profitable unilateral deviation from the strategy profile $(\widehat{\gamma},\ \widehat{\gamma})$. Thus $(\widehat{\gamma},\ \widehat{\gamma})$, $\widehat{\gamma} \neq \widetilde{\gamma}$ cannot be a PSNE.

From the proof of step 1, the strategy profile, $(\widetilde{\gamma},\ \widetilde{\gamma})$, satisfies the necessary condition for an interior maximum of $W(\gamma, \gamma^*)$. If $W(\gamma, \gamma^*)$ is strictly quasiconcave in $(\gamma,\ \gamma^*)$, there is a unique interior Pareto optimum at $(\widetilde{\gamma},\ \widetilde{\gamma})$. From the Proposition 6, an asymmetric PSNE, if it exists, is a Pareto improvement over the autarky optimum. Hence, if $W(\gamma, \gamma^*)$ is strictly quasiconcave, a Pareto improvement over $(\widetilde{\gamma},\ \widetilde{\gamma})$ is not possible. Hence, if $W(\gamma, \gamma^*)$ is strictly quasiconcave, an asymmetric PSNE does not exist.

From the definition of strict quasiconvexity,

$$U^T(\lambda\gamma^1 + (1-\lambda)\gamma^2) < \max\{U^T(\gamma^1),\ U^T(\gamma^2)\}, \forall \gamma^1,\ \gamma^2 \in [\underline{\gamma},\ \overline{\gamma}]^2.$$

Let $\gamma^1 = (\underline{\gamma},\ \gamma)$ and $\gamma^2 = (\overline{\gamma},\ \gamma)$. Any $\gamma' \in (\underline{\gamma},\ \overline{\gamma})$ yields a lower pay off than $\gamma' = \underline{\gamma}$ or $\gamma' = \overline{\gamma}$, for any given foreign policy $\gamma \in [\underline{\gamma},\ \overline{\gamma}]$. Hence, $\forall \gamma \in [\underline{\gamma},\ \overline{\gamma}]$, the best response is either $\underline{\gamma}$ or $\overline{\gamma}$. Neither $\underline{\gamma}$ nor $\overline{\gamma}$ is an autarky optimum. Therefore, $\underline{\gamma}$ $(\overline{\gamma})$ is not the best response to $\underline{\gamma}$ $(\overline{\gamma})$. Thus, the best response to $\underline{\gamma}$ is $\overline{\gamma}$ and vice versa. The only two PSNEs are $(\overline{\gamma},\ \underline{\gamma})$ and $(\underline{\gamma},\ \overline{\gamma})$. ■

Step 3: Relation between quasiconvexity of welfare function and social increasing retun : *If* $\frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2} = 0, W(\gamma, \gamma^*)$ is strictly quasiconcave. if $\frac{\partial^2 Y(p^T,\gamma)}{\partial \gamma^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ is positive and sufficiently large, $U^T(\gamma,\ \gamma^*)$ is quasiconvex at $(\widetilde{\gamma},\ \widetilde{\gamma})$ even though $U^A(\gamma)$ is quasiconcave at $\widetilde{\gamma}$.

Proof. The sufficient condition for equilibrium asymmetry in step 2 requires a quasiconvex $U^T(\gamma,$

γ^*), even though $\widetilde{\gamma}$ is an interior autarky optimum. Let us consider the difference between the curvatures of $U^T(\gamma,\, \gamma^*)$ at $(\widetilde{\gamma},\, \widetilde{\gamma})$ and $U^A(\gamma)$ at $\widetilde{\gamma}$. Note that the curvature of $U^A(\gamma)$ is given by,

$$\frac{d^2U^A(\gamma)}{d\gamma^2} = \frac{dp^A}{d\gamma}(2\frac{\partial^2 V(p^A,.)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^A,.)}{\partial p^2}\frac{dp^A}{d\gamma}) + \frac{\partial^2 V(p^A,.)}{\partial\gamma^2}. \tag{14}$$

First, we show that $\frac{\partial^2 V(p^A,.)}{\partial\gamma\partial p} > 0$.To prove $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \geqslant 0$, note that,

$$\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} = f'(p)\frac{\partial F_2}{\partial\gamma} + f(p)\frac{\partial F_1}{\partial\gamma}f_2(p), \text{ using (12) and (13).}$$

Indirect utility is decreasing in p given Y,

$$f'(p) < 0$$

Since γ gives absolute advantage in good 1 and absolute disadvantage in good 2,

$$\delta F_2/\delta\gamma < 0,\ \delta F_1/\delta\gamma > 0.$$

Hence, $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \geqslant 0$.

Now let us consider $\frac{\partial^2 V(p^A,.)}{\partial p^2}$. Note that,

$$\frac{\partial^2 V(p^A,.)}{\partial p^2} = f(p^A)(\underset{+}{\frac{\partial F_1}{\partial p}} - f_1'(p)Y) + \underset{-}{f'(p^A)}F_1.$$

Thus the sign of $\frac{\partial^2 V(p^A,.)}{\partial p^2}$, in general, is ambiguous and $\frac{\partial^2 V(p^A,.)}{\partial\gamma^2}$ is given by,

$$f(p^A)\frac{\partial^2 Y(p^A,\gamma)}{\partial\gamma^2} = f(p^A)[p^A\frac{\partial^2 F_1(p^A,\gamma)}{\partial\gamma^2} + \frac{\partial^2 F_2(p^A,.)}{\partial\gamma^2}].$$

The first principal minor of the Hessian of $U^T(\gamma,\, \gamma^*)$ has a similar expression as (14) given by,

$$\frac{\partial p^T}{\partial\gamma}(2\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial\gamma}) + \frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2} + \frac{\partial V(p^T,\gamma)}{\partial p}\frac{\partial^2 p^T}{\partial\gamma^2}. \tag{15}$$

At a point of symmetry, $\frac{\partial p^T}{\partial\gamma} = .5\frac{dp^A}{d\gamma}$, $p^T(\gamma,\gamma) = p^A(\gamma)$, and $\frac{\partial V(p^T,\gamma)}{\partial p} = 0$. If $|\frac{\partial^2 V(p^T,\gamma)}{\partial p^2}|_{(\widetilde{\gamma},\widetilde{\gamma})}$ is sufficiently low, $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} > 0$ and sufficiently high, it is possible that (14) is negative, while (15) is positive. The second principal minor of the Hessian of $U^T(\gamma,\, \gamma^*)$ at a point of symmetry is given by,

$$\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2}\left(2\frac{\partial p^T}{\partial\gamma}(2\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial\gamma}) + \frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2}\right)$$

Hence, sufficiently high values of $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}$ also ensures that the second principal minor of the Hessian of $U^T(\gamma,\, \gamma^*)$ is positive. Thus sufficient convexity in production with respect to the policy in question ensures that there are only asymmetric equilibria in the open economy, even though $\widetilde{\gamma}$ is an interior autarky optimum.

Convexity in production also plays a crucial role to satisfy the necessary condition for existence of an asymmetric equilibrium. To see this, suppose that production of goods is linear in policy, $\frac{\partial^2 F_1(p^T,\gamma)}{\partial\gamma^2} = \frac{\partial^2 F_2(p^T,\gamma)}{\partial\gamma^2} = \frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$. This ensures that the second principal minor of the Hessian of $U^T(\gamma,\, \gamma^*)$ is zero, and the first principal minor is negative. Hence, the Hessian of $U^T(\gamma,\, \gamma^*)$ is negative semidefinite, and $U^T(\gamma,\, \gamma^*)$ is strictly quasiconcave. Hence, if $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0$ there is no asymmetric equilibrium in the open economy. ■

Steps 1, 2 and 3 combined complete the proof of proposition 7. ■

Remark 11 *Every symmetric equilibrium* (γ, γ) *is stable.*

Proof. We need to show that $|\frac{d^2U}{d\gamma d\gamma^*}| < |\frac{d^2U}{d\gamma^2}|$ at any (γ, γ).This implies that the best response has absolute slope less than unity at any symmetric equilibrium. Note that, $\frac{d^2U}{d\gamma d\gamma^*}$ is given by,

$$\frac{\partial p^T}{\partial \gamma^*}(\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma\partial p} + \frac{\partial^2 V(p^T,\gamma)}{\partial p^2}\frac{\partial p^T}{\partial\gamma^*}) + \frac{\partial V(p^T,\gamma)}{\partial p}\frac{\partial^2 p^T}{\partial\gamma\partial\gamma^*}$$

and $\frac{d^2U}{d\gamma^2}$ is given by (15).Provided $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \gtrless 0$ and $\frac{\partial^2 V(p^T,\gamma)}{\partial\gamma^2} = sign(\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2}) \gtrless 0$, $|\frac{d^2U}{d\gamma d\gamma^*}| < |\frac{d^2U}{d\gamma^2}|$ at any (γ, γ). Proof of step 3 of proposition 7 shows that $\frac{\partial^2 V(.,.)}{\partial\gamma\partial p} \gtrless 0$. Thus, when $\frac{\partial^2 Y(p^T,\gamma)}{\partial\gamma^2} = 0, (\widetilde{\gamma}, \widetilde{\gamma})$ is the unique stable equilibrium. ■

6.2 Derivation and Proofs for Section 3

The aggregate indirect utility is a function of the equilibrium price and the aggregate income,

$$V(p,\gamma) = \; cp^{-\mu}Y(p,\gamma).$$

Aggregate income, $Y(p,\gamma)$, is given by $w_H(p)H^e(\gamma) + w_L(p)L^e(\gamma)$, where

$$w_L(p) = \; c_h p^{\eta_2/(\eta_2-\eta_1)}, \; w_H(p) = c_l p^{-(1-\eta_2)/(\eta_2-\eta_1)}.$$

Here c, c_h, c_l, and c_p denote constants that depend on the economic fundamentals.

Let us first derive (8). A more able agent with initial ability h chooses h^e by maximizing

$$p^{-\mu}w_H(p)h^e - \beta_h(\frac{h^e}{h})^{\epsilon},$$

where $w_H(p) = c_h p^{\frac{1-\eta_2}{\eta_1-\eta_2}}$. From the FOC, one can derive

$$h^e = (\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon\beta_h}h^{\epsilon})^{\frac{1}{\epsilon-1}}.$$

Multiplying both sides by n_H I arrive at (8). The second order condition of optimality requires $\epsilon > 1$. Similarly one can solve the optimal skill-choice problem of the low-skilled agents.

The optimal skill choice function is given by

$$\begin{aligned} H^e &= c_{h'}\Big(\frac{H^\epsilon p^{\frac{1-a}{\eta_1-\eta_2}}}{n_H\beta_H}\Big)^{\frac{1}{\epsilon-1}},\ c_{h'} = f(\eta_1,\eta_2,\mu,\epsilon),\ a = \mu\eta_1 + (1-\mu)\eta_2, \\ L^e &= c_{l'}\Big(\frac{L^\epsilon p^{\frac{-a}{\eta_1-\eta_2}}}{n_l\beta_L}\Big)^{\frac{1}{\epsilon-1}},\ c_{l'} = f(\eta_1,\eta_2,\mu,\epsilon),\ H = n_H h,\ L = n_l * l. \end{aligned} \tag{16}$$

Note that,

$$\frac{\partial H^e}{\partial \gamma} = -\Big(\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon n_H}h^\epsilon\Big)^{\frac{1}{\epsilon-1}}\Big(\frac{1}{\epsilon-1}\Big)\beta_h^{-\frac{1}{\epsilon-1}-1}\underset{-}{\frac{\partial \beta_h}{\partial \gamma}} > 0$$

and $\frac{\partial^2 H^e}{\partial \gamma^2}$ is equal to

$$\Big(\frac{c_h p^{\frac{1-a}{\eta_1-\eta_2}}}{\epsilon n_H}h^\epsilon\Big)^{\frac{1}{\epsilon-1}}\left(\Big(\frac{1}{\epsilon-1}\Big)\Big(\frac{1}{\epsilon-1}+1\Big)\beta_h^{-\frac{1}{\epsilon-1}-2}\Big(\underset{-}{\frac{\partial\beta_h}{\partial\gamma}}\Big)^2 - \Big(\frac{c_h p^{\frac{1-e}{\eta_1-\eta_2}}}{\epsilon}h^\epsilon\Big)^{\frac{1}{\epsilon-1}}\Big(\frac{1}{\epsilon-1}\Big)\beta_h^{-\frac{1}{\epsilon-1}-1}\frac{\partial^2\beta_h}{\partial\gamma^2}\right) > 0,$$

provided $\frac{\partial^2 \beta_h}{\partial \gamma^2} \leq 0$. A similar result hold for L^e.

We can rewrite

$$H^e = A_H g_H(\gamma).$$

An increase in the government spending increases the effective endowment, $g'_H(\gamma) > 0$. Similarly,

$$L^e = A_L g_L((1-\gamma))$$

where $g_L^{'}(1-\gamma) \geqslant 0$. A_H and A_L are constants that depend on all the parameters and price, p. Impose parameter restrictions such that $H^e \geqslant H$ and $L^e \geqslant L$ for any choice of policy.

In the competitive equilibrium of the closed economy only domestic policy determines the equilibrium price,

$$p^A = c_p\Big(\frac{H^e}{L^e}\Big)^{\eta_2-\eta_1},$$

and in the open economy both domestic and foreign policy determine the equilibrium terms-of-trade,

$$p^T = \Big(\frac{H^e + H^{e*}}{L^e + L^{e*}}\Big)^{\eta_2-\eta_1} c_p(\mu,\eta_1,\eta_2). \tag{17}$$

The equilibrium price in the open economy is given by,

$$p^T(\gamma,\gamma^*) = \{\frac{\beta_h^{\frac{1}{1-\epsilon}} + \beta_h^{*\frac{1}{1-\epsilon}}}{\beta_l^{\frac{1}{1-\epsilon}} + \beta_l^{*\frac{1}{1-\epsilon}}}\}^{\frac{(\eta_1-\eta_2)(1-\epsilon)}{\epsilon}} c_p'(\eta_1,\eta_2,\mu,H,L,\epsilon,n_H,n_l),$$

where c_p denotes a constant that does not depend on policy.

Welfare in the open economy is given by

$$U^T = gq^a(\frac{H^e + H^{e*}}{L^e + L^{e*}})^e(L^e + \frac{H^e}{qf(\mu,\eta_1,\eta_2)}(\frac{H^e + H^{e*}}{L^e + L^{e*}})^{-1}), \tag{18}$$

where $a = \eta_2 + \mu(\eta_1 - \eta_2)$, $qf = ((1-\mu)f_1 + \mu f_2)/(\mu f_2(\eta_1/(1-\eta_1)) + (1-\mu)f_1(\eta_2/(1-\eta_2))$, $f = (((1-\eta_1)/(1-\eta_2))(\eta_2/(1-\eta_2))^{-\eta_2}(\eta_1/(1-\eta_1))^{\eta_1})^{1/(\eta_2-\eta_1)}$, $f_2 = (\eta_2/(1-\eta_2))^{\eta_2} f^{\eta_2-1}/(\eta_1/(1-\eta_1) - \eta_2/(1-\eta_2))$, $f_1 = (\eta_1/(1-\eta_1))^{\eta_1} f^{\eta_1-1}/(\eta_1/(1-\eta_1) - \eta_2/(1-\eta_2))$, and $g = (1-\eta_1)(\eta_1/(1-\eta_1))^{\eta_1} f^{\eta_1}$ are constants depending on the production and demand parameters.

Proof of Lemma 9. Consider a pair of policies $(\gamma_1,\ \gamma_2)$ such that $\gamma_1 > \gamma_2$. Relative welfare of higher education, (10), is given by,

$$r^A(\gamma_1,\gamma_2) = \frac{(\frac{g_H(\gamma_1)}{g_L(1-\gamma_1)})^a g_L(1-\gamma_1)}{(\frac{g_H(\gamma_2)}{g_L(1-\gamma_2)})^a g_L(1-\gamma_2)}.$$

Evidently given $g_t' > 0$, (10) increases in the demand share of the skill intensive good (μ) and in the skill intensities of production,

$$\frac{dr^A(\gamma_1,\gamma_2)}{d\kappa} > 0 \text{ for } \kappa = \mu,\ \eta_1,\ \eta_2.$$

Given the rest of the parameters, consider an increase in μ' to μ''. Let the original autarky optimal policy be $\widetilde{\gamma}'$ and the new autarky optimal policy be $\widetilde{\gamma}''$. Since $\widetilde{\gamma}'$ is the original autarky optimal,

$$\frac{U^A(\widetilde{\gamma}')}{U^A(\gamma)}|_{(\mu=\mu')} > 1\ \nabla\gamma \neq \widetilde{\gamma}'.$$

With increase in μ, $r^A(\widetilde{\gamma}',\gamma)$ increases $\nabla\ \gamma < \widetilde{\gamma}'$. Thus,

$$U^A(\widetilde{\gamma}')/U^A(\gamma)|_{(\mu=\mu'')} \geqslant U^A(\widetilde{\gamma}')/U^A(\gamma)|_{(\mu=\mu')} > 1\ \nabla\gamma < \widetilde{\gamma}'.$$

Hence, $\widetilde{\gamma}'' \geqslant \widetilde{\gamma}'$. This implies that the autarky optimal policy ($\widetilde{\gamma}$) is increasing in μ, η_i.

The relative welfare of higher education in the open economy, (9), is given by,

$$\underbrace{\frac{(\frac{g_H(\gamma_1)+g_H(\gamma^*)}{g_L(1-\gamma_1)+g_L(1-\gamma^*)})^a}{(\frac{g_H(\gamma_2)+g_H(\gamma^*)}{g_L(1-\gamma_2)+g_L(1-\gamma^*)})^a}}_{\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}} \times$$

$$\underbrace{\frac{(g_L(1-\gamma_1)(1+\lambda)+(1-\lambda)\frac{g_H(\gamma_1)}{qf(\mu,\eta_1,\eta_2)}(\frac{g_H(\gamma_1)+g_H(\gamma^*)}{g_L(1-\gamma_1)+g_L(1-\gamma^*)})^{-1})}{(g_L(1-\gamma_2)(1+\lambda)+(1-\lambda)\frac{g_H(\gamma_2)}{qf(\mu,\eta_1,\eta_2)}(\frac{g_H(\gamma_2)+g_H(\gamma^*)}{g_L(1-\gamma_2)+g_L(1-\gamma^*)})^{-1})}}_{\frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}}$$

where U_L^T is the aggregate indirect utility of the low-skilled workers and S_L^T stands for $\frac{Y}{w_L}$. From $\frac{\partial a}{\partial \mu} > 0$ and $\frac{\partial qf}{\partial \mu} < 0$, I can prove that both $\frac{U_L^T(\gamma_1,\gamma^*)}{U_L^T(\gamma_2,\gamma^*)}$ and $\frac{S_L^T(\gamma_1,\gamma^*)}{S_L^T(\gamma_2,\gamma^*)}$ increases in μ, if $\gamma_1 > \gamma_2$ and $g_t' > 0$. A similar proof works for increase in η_1. Hence using a similar logic as before, $BR(\gamma^*)$ shifts up in μ and η_1.

Proof of Proposition 10. **Step 1**: If $g_t''(.)$ is sufficiently high, skill intensities of production are sufficiently different and consumers do not prefer either of the two goods too strongly, only asymmetric PSNEs and hence, only asymmetric Pareto optima exist. ■

Note that

$$\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2} = w_H(p)g_H''(\gamma) + w_L(p)g_L''(1-\gamma).$$

From the derivation in appendix 6.1 $U^T(\gamma,\gamma^*)$ is quasiconvex in γ, if $\frac{\partial^2 Y(.,\gamma)}{\partial \gamma^2}$ is sufficiently high. Hence, if $g_t''(.)$ is sufficiently high, $U^T(\gamma,\gamma^*)$ is quasiconvex in γ. Now there are two possible cases.

Case 1: If $g_l''(.)$ is such that both U^A and U^T is quasiconvex, only asymmetric equilibria exist if preference for higher γ in the competitive economy is not very high. Define $\widetilde{\mu}$ such that the autarky optimum is 0 for $\mu \leq \widetilde{\mu}$, and the autarky optimum is $\overline{\gamma}$ for $\mu \geqslant \widetilde{\mu}$. Also, define $\underline{\mu}$ such that $r^T(\overline{\gamma},\underline{\gamma},\underline{\gamma}) = 1$ at the $\underline{\mu}$. By the gains from trade property (7), $r^T(\gamma',\underline{\gamma},\underline{\gamma})$ lies above $r^A(\gamma',\underline{\gamma})$ for any μ. Hence, $\underline{\mu}$ is strictly less than $\widetilde{\mu}$. Thus, for $\mu \in [\underline{\mu},\widetilde{\mu}]$ there is only asymmetric PSNE. Similarly I can define $\overline{\mu}$ such that $r^T(\underline{\gamma},\overline{\gamma},\overline{\gamma}) = 1$ at the $\overline{\mu}$, and for $\mu \in [\,\widetilde{\mu},\overline{\mu}]$ there is only asymmetric PSNE. It is straightforward to show that, $\underline{\mu} < \overline{\mu}$. To have a non empty interval of $[\underline{\mu}\ ,\ \overline{\mu}] \sqsubseteq [0,1]$, we need to check rest of the parameters of the economy. The sufficient condition for $\underline{\mu} \geqslant 0$ is

$$(1+g_L(0))^{\eta_2} - ((1+g_L(0))/(1+(g_L(0))\eta_2)) < 0.$$

For a given $g_L(0) > 0$, this condition depends only on η_2 and is more likely to be satisfied for a fall in η_2. A fall in η_2 makes the low-skill intensive good even more intensive in lower skill and hence in primary education. This increase in low-skill intensity of production for good 2 makes it more likely that investing only in primary education is the symmetric equilibrium for non-negative values of μ. In a similar vein, $\overline{\mu} \leq 1$ essentially means that for some values of the preference parameter investing only in higher education is the symmetric equilibrium. For a given $g_H(0) > 0$, this sufficient condition depends only on η_1 and is more likely to be satisfied for an increase in η_1. Thus higher is the difference in the skill intensities of production of the two goods, more likely is the existence of a non empty subset of the parameter space for the preference parameter $[\underline{\mu}, \overline{\mu}] \subseteq [0, 1]$ for which an asymmetric PSNE exists.

Case 2: If $g_t''(.)$ is sufficiently high such that U^T is quasiconvex at $(\widetilde{\gamma}, \widetilde{\gamma})$, by Proposition 7 there is only asymmetric PSNEs at the extremes provided there is a interior $\widetilde{\gamma}$. Since (10) increases in μ, there is an interior $\widetilde{\gamma}$ for values of $\mu \in [\mu_0, \mu_1]$, where μ_0, μ_1 are respectively the maximum and the minimum values of μ for which $\widetilde{\gamma}$ is not interior. Since (10) increases in η_1 and η_2, I can show that μ_0 rises in η_2 and μ_1 falls in η_1. Hence, a rise in $(\eta_1 - \eta_2)$ ensure that $[\mu_0, \mu_1] \subseteq [0, 1]$.

Step 2: Note that a fall in ϵ increases the magnitude of $g_t''(.)$. I can show that for sufficiently small ϵ objective function of the government in the open economy is quasiconvex in $\dot{\gamma}$. Combining step 1 and 2 completes the proof. ■