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Sunk Costs and the Measurement of Commercial Property Depreciation

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Abstract

This paper develops a new framework for measuring prices and quantities of commercial properties. In particular, it addresses problems associated with obtaining separate estimates for the land and structure components of a property. A key contribution is to address the problem of estimating structure depreciation taking into account the fixity of the structure. We find that structure depreciation is determined primarily by the cash flows that the property generates rather than physical deterioration of the building.

JEL Classification Numbers: C2, C23, C43, D12, E31, R21.

Key Words: Property price indexes, net operating income, discounted cash flow, System of National Accounts, Balance Sheets, land and structure prices, goodwill amortization, intangible assets.

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1. Introduction

Besides their direct relevance for informing investment and policy decisions, price and quantity indexes are required for calculating stocks of commercial properties in the Balance Sheets of a country. Related price and quantity indexes are also required for the services of the land and structure components of a commercial property in the production accounts of the country. The main purpose of the paper is to provide a framework for commercial property capital measurement that would be suitable for national income accounting purposes.

In particular, the System of National Accounts (SNA) requires separate measures for the input contributions of a commercial property structure and the associated land plot. For the most part, this decomposition problem has been neglected in the commercial property academic literature, which has focused on the total investment return of a commercial property project; see for example Gatzlaff and Geltner (1998), Fisher, Geltner and Pollakowski (2007) and Bokhari and Geltner (2012) (2014).

The land and structure decomposition problem was recently studied in some detail by Diewert and Shimizu (2013). What is different in the present paper is that we take into account the *fixity* of the structure whereas Diewert and Shimizu simply used the traditional perpetual inventory method to measure the contributions of the structure and land components of a commercial property. A further contribution of the present paper is that it addresses the difficult problems associated with the amortization of property goodwill; i.e., a commercial property may generate profits that more than cover the project's cost of capital. The capitalized excess profits or goodwill need to be amortized over the lifetime of the project. The paper draws on the recent contributions of Cairns (2013) to address this problem of amortizing an intangible asset.

The rest of the paper is structured as follows. Section 2 introduces the notation and assumptions that define our simplified commercial property project. Section 3 studies the problem of determining when the structure should be demolished. Section 4 shows how

period-by-period asset values, user costs and depreciation schedules can be determined for the project as a whole. Section 5 decomposes these value aggregates into land and non-land components. The case where the project earns pure profits is analyzed in Section 6 where the aggregate values are decomposed into additive land, structure and goodwill components. Section 7 further decomposes the value components derived in section 6 into price and quantity components and section 8 concludes.

2. The Commercial Property Project

We assume that a group of investors has either purchased a commercial property building at the end of period 0 (or the beginning of period 1) or has constructed a new building which is just ready for occupancy at the end of period 0.¹ We assume that the total actual cost of the structure at the beginning of period 1 is known to the investor group and is $C_S^0 > 0$, and the opportunity cost value of the land plot at the beginning of period 1 is $V_L^0 > 0$. The *total initial cost of the commercial property*, C^0 , is then defined as

$$(1) C^0 \equiv C_S^0 + V_L^0.$$

Time is divided up into discrete periods, $t = 0, 1, 2, \dots$ and we assume that the *end of period t value of the land plot* is expected to be V_L^t for $t = 1, 2, \dots$. Thus the investors form definite expectations about the price movements of the land plot that the structure utilizes. It will be convenient to relate these expected land values to period-by-period *land price inflation rates* i_t ; i.e., we assume that the period t land prices V_L^t and land inflation rates i_t satisfy the following equations, with $1+i_t > 0$ for all t :

$$(2) V_L^t = (1+i_1)(1+i_2)\dots(1+i_t)V_L^0; \quad t = 1, 2, \dots$$

We assume that the *beginning of period t cost of capital* (or interest rate) that the investors face is $r_t > 0$ for $t = 1, 2, \dots$. Finally, we assume that the building is expected to

¹ The length of each period will typically be a quarter if our analysis is applied empirically because, usually, information on building cash flows will only be available on a quarterly (or annual) basis.

generate Net Operating Income (or cash flow) equal to $N^t \geq 0$, which following Peasnell (1981) and Diewert (2005; 485) we assume to be realized at the end of each period $t = 1, 2, \dots$. Thus the information set that we assume is known to the investors consists of the building cost C_S^0 , the sequence of end of period land values V_L^t (or equivalently V_L^0 and the sequence of land inflation rates i_t), the sequence of one period interest rates r_t and the sequence of cash flows N^t .

Using the above information set, we can define an expected discounted profit maximization problem for each choice of time period $t = 1, 2, \dots$. Problem t assumes that the firm demolishes the structure at the end of period t , at which time the structure has no value, but of course the land will have (expected) value V_L^t . The resulting *expected discounted profit* (Π^t) for the investor group will then be defined as follows:²

$$(3) \Pi^t \equiv -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \dots + \alpha_t N^t + \alpha_t \beta_t V_L^0; \quad t = 1, 2, \dots$$

where the α_t and β_t are defined recursively as follows:

$$(4) \alpha_1 \equiv (1+r_1)^{-1}; \quad \alpha_t \equiv (1+r_t)^{-1} \alpha_{t-1} \quad \text{for } t = 2, 3, \dots;$$

$$(5) \beta_1 \equiv (1+i_1); \quad \beta_t \equiv (1+i_t) \beta_{t-1} \quad \text{for } t = 2, 3, \dots$$

Thus Π^t is the sum of the discounted cash flows that the property is expected to generate over time periods 1 to t , $\alpha_1 N^1 + \alpha_2 N^2 + \dots + \alpha_t N^t$, plus the discounted expected land value of the property at the end of period t , $\alpha_t \beta_t V_L^0 = \alpha_t V_L^t = (1+r_1)^{-1} (1+r_2)^{-1} \dots (1+r_t)^{-1} V_L^t$, less the initial value of the structure at the beginning of period 1, C_S^0 , less the market value of the land at the beginning of period 1, V_L^0 .

² Our analysis utilizes the intertemporal production plan methodology that was pioneered by Hicks (1946).

3. Choosing the Length of Life of the Structure

We assume that the sequence of Π^t is maximized at t equal to $T \geq 1$.³ We also assume that Π^T is nonnegative:⁴

$$(6) \quad \Pi^T \equiv -C_S^0 - V_L^0 + \alpha_1 N^1 + \alpha_2 N^2 + \dots + \alpha_T N^T + \alpha_T \beta_T V_L^0 \geq 0.$$

Thus T is the *endogenously determined expected length of life for the structure*. Note that the determination of the length of life of the structure is not a simple matter of determining when the building will collapse due to the effects of aging and use: it is an *economic decision* that depends on all of the variables which were defined in the previous section.

In order to gain some insight into the nature of the Π^t , it is useful to consider the following special case of our general framework when the costs of capital r_t and the expected inflation rates for land i_t are constant over time:

$$(7) \quad r_t = r ; i_t = i ; \quad t = 1, 2, \dots$$

Substitute equations (7) into definitions (3) and calculate the following differences for $t \geq 1$:

$$(8) \quad \begin{aligned} \Pi^{t+1} - \Pi^t &= \alpha_{t+1} N^{t+1} + \alpha_{t+1} \beta_{t+1} V_L^0 - \alpha_t \beta_t V_L^0 \\ &= (1+r)^{-t} [(1+r)^{-1} N^{t+1} + (1+i)^{t+1} (1+r)^{-1} V_L^0 - (1+i)^t V_L^0]. \end{aligned}$$

Using (8), it can be seen that Π^{t+1} will be less than Π^t if the following inequality is satisfied:

³ If there is more than one maximizing t , choose the smallest one. Later we will look at conditions that ensure a finite maximizing T .

⁴ Our analysis largely follows that of Cairns (2013; 639) who noted that unless the inequality in (6) is satisfied, investors will not participate in the project: "This participation constraint provides that the cash flows of the project allow investors to recover their sunk investment as a stream of quasi-rents or user costs."

$$(9) N^{t+1}/V_L^0 < (1+i)^t(r-i).$$

As the structure becomes very old, the cash flows that can be generated by it will fall to zero. Thus for large t , N^t will equal zero and so will the left hand side of (9). We have assumed that $(1+i)$ is positive so the right hand side of (9) will be positive if the cost of capital r is greater than the expected land inflation rate i , an assumption that we now make.⁵ Thus for t large enough, we can safely assume that Π^{t+1} is less than Π^t , which ensures that the expected life of the building T is finite.

Return to the general expression for Π^t defined by (3) and evaluate $\Pi^{t+1} - \Pi^t$ using definitions (4) and (5):

$$\begin{aligned} (10) \Pi^{t+1} - \Pi^t &= \alpha_{t+1} N^{t+1} + \alpha_{t+1} \beta_{t+1} V_L^0 - \alpha_t \beta_t V_L^0 \\ &= \alpha_t [(1+r_{t+1})^{-1} N^{t+1} + (1+r_{t+1})^{-1} (1+i_{t+1}) \beta_t V_L^0 - \beta_t V_L^0] \\ &= \alpha_t (1+r_{t+1})^{-1} [N^{t+1} - (r_{t+1} - i_{t+1}) \beta_t V_L^0]. \end{aligned}$$

Thus $\Pi^{t+1} - \Pi^t$ will be negative if the following inequality is satisfied:

$$(11) N^{t+1}/V_L^0 < \beta_t (r_{t+1} - i_{t+1}) = (1+i_1)(1+i_2)\dots(1+i_t)(r_{t+1} - i_{t+1}).$$

It can be seen that if $1+i_t > 0$ for all t and $r_{t+1} > i_{t+1}$ and $N^t = 0$ for large t , then (11) will be satisfied for all large t . The inequalities (11) for large t imply that a finite T will exist where Π^t is maximized and thus the optimal length of life for the building will be well determined.

⁵ This assumption is a reasonable *long run* assumption since if the expected land inflation rate exceeded the cost of capital, it would pay investors to simply purchase land (and not build a structure) and make an infinite stream of period-by-period profits. If all investors held expectations such that $i > r$, the price of land would be bid up to eliminate these effortless profits. In the *short run*, land price bubbles tend to occur from time to time.

The above algebra shows that the eventual decline in the asset value of the property as the life of the building is extended depends entirely on the sequences of cash flows N^t , one period interest rates r_t and one period expected land inflation rates i_t . However, the initial land value V_L^0 and the initial structure cost C_S^0 do play a role once the optimal length of building life has been determined since we also require that the project be profitable; i.e., that $\Pi^T \geq 0$.

The above analysis shows that the decision to retire a commercial property structure is an endogenous one that is not determined exogenously by wear and tear physical deterioration of the building. The retirement decision depends crucially on the intertemporal pattern of cash flows generated by the building and on the movements in the price of the land plot over time.⁶ Thus our theory of structure retirement (and depreciation as will be seen below) is somewhat different from existing theories in the real estate literature about the retirement and depreciation of a commercial property structure.⁷

4. Period-by-Period Aggregate Asset Values, User Benefits and Depreciation

We assume that the optimal length of life of the structure T has been determined and that the nonnegative discounted profits constraint (6) holds. Our task in this section is to determine the sequence of project asset values and the changes in asset value over each time period.

⁶ Structures in locations with a higher land inflation rate will tend to have a shorter life than structures in locations with a lower land inflation rate.

⁷ Baum (1991; 59) and Dixon, Crosby and Law (1999; 162) distinguished *physical deterioration* and *obsolescence* of the structure as the primary causes of depreciation (decline in the value of the building over time). In our approach, it is increases in the price of land along with falls in cash flows that drives obsolescence. Dixon, Crosby and Law (1999; 168-170) also noted that *rental decline* (i.e., falls in net operating income as the building ages) contributed to building depreciation and of course, this effect is also part of our approach. Crosby, Devaney and Law (2012) investigate the rental decline phenomenon for UK commercial properties and they also take into account post construction capital expenditures on the properties. When depreciation rates for commercial properties are reported in the real estate literature, they are generally reported as fraction of property value (which includes the value of the land plot). Thus these reported property depreciation rates will understate depreciation rates on the structure by itself.

The sequence of expected *end-of-period t project asset values* A^t can be defined as follows:

$$\begin{aligned}
(12) \quad A^0 &\equiv \alpha_1 N^1 + \alpha_2 N^2 + \dots + \alpha_T N^T + \alpha_T V_L^T; \\
A^1 &\equiv (1+r_1)[\alpha_2 N^2 + \alpha_3 N^3 + \dots + \alpha_T N^T + \alpha_T V_L^T]; \\
A^2 &\equiv (1+r_1)(1+r_2)[\alpha_3 N^3 + \alpha_4 N^4 + \dots + \alpha_T N^T + \alpha_T V_L^T]; \\
&\dots \\
A^{T-1} &\equiv (1+r_1)(1+r_2)\dots(1+r_{T-1})[\alpha_{T-1} N^{T-1} + \alpha_T N^T + \alpha_T V_L^T] \\
&= N^{T-1} + (1+r_{T-1})^{-1}(N^T + V_L^T); \\
A^T &\equiv N^T + V_L^T.
\end{aligned}$$

Thus at the end of period t , the expected property asset value A^t is equal to the expected period t cash flow N^t plus the discounted to the end of period t cash flow for period $t+1$, $(1+r_{t+1})^{-1}N^{t+1}$, plus the discounted to the end of period t cash flow for period $t+2$, $(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}N^{t+2}$, ... , plus the discounted to the end of period t cash flow for period T , $(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}\dots(1+r_T)^{-1}N^T$, plus the discounted to the end of period t expected value of the land plot at the end of period T , $(1+r_{t+1})^{-1}(1+r_{t+2})^{-1}\dots(1+r_T)^{-1}V_L^T$.

Note that the last T equations in (12) can be rearranged to give us the following relationships between the end of period t asset values A^t and the period t cash flows N^t :

$$\begin{aligned}
(13) \quad N^t &= (1+r_t)A^{t-1} - A^t && t = 1, \dots, T \\
&= r_t A^{t-1} + (A^{t-1} - A^t) \\
&= r_t A^{t-1} + \Delta^t
\end{aligned}$$

where $r_t A^{t-1}$ reflects the opportunity costs of the capital that is tied up in the project at the beginning of period t , and Δ^t is the period t expected *asset value change* for the project defined by (14):

$$(14) \quad \Delta^t \equiv A^{t-1} - A^t; \quad t = 1, \dots, T.$$

Thus Δ^t is simply the anticipated decline in asset value of the project from the beginning of period t to the end of period t . In the zero discounted profits case where $\Pi^T = 0$, then Δ^t can be interpreted as *time series depreciation* for the project.⁸

It can be seen that the expressions on the right hand side of (13) are analogous to expressions for the traditional user cost of capital; see Jorgenson (1963)(1989).⁹ As our later discussion will show, the expression $r_t A^{t-1} + \Delta^t$ is not necessarily equal to a user cost if the project makes profits that are above and beyond the cost of capital for the project. In this latter case, $r_t A^{t-1} + \Delta^t$ can be interpreted as a *user benefit* expression rather than a user cost expression.

Diewert (2009; 3) noted that measuring depreciation for a *sunk cost asset* like a commercial structure is difficult since there are no second hand asset markets for a sunk cost asset that can provide period-by-period opportunity costs in order to value the structure asset as it ages. Sales of commercial properties can provide some information but are infrequent and the sale price is for the combined land and structure. It then seems difficult to obtain a sequence of objective measures of period-by-period depreciation or amortization amounts over the life of the building. Let $N^{t*} \geq 0$ be a *period t amortization amount* for the commercial property for $t = 1, 2, \dots, T$ where the N^{t*} satisfy the following equation:

$$(15) \alpha_1 N^{1*} + \alpha_2 N^{2*} + \dots + \alpha_T N^{T*} = C_S^0 + V_L^0.$$

N^{t*} can be interpreted as a payment made to the owners of the project at the end of period t for $t = 1, 2, \dots, T$. Equation (15) says that the initial project cost, $C_S^0 + V_L^0$, can be distributed across the T time periods before the building is demolished by the series of

⁸ The term *time series depreciation* is due to Hill (2000) but the concept dates back to Hotelling (1925; 341). We note that Δ^t incorporates both the effects of wear and tear depreciation and anticipated revaluation; see Hill (2000; 6), Hill and Hill (2003; 617), Diewert (2009; 9) and Cairns (2013; 640).

⁹ The expressions on the right hand side of equations (13) are analogous to end of period user costs; see Diewert (2005; 485) (2009; 8). Baumol, Panzar and Willig (1982; 384) identify $r_t A^{t-1} + \Delta^t$ as the period t *payment to capital*; see also Cairns (2013; 640).

period-by-period cost allocations N^{t*} where the discounted value of these cost allocations (to the beginning of period 1) is equal to the project cost. Note that the amortization schedules N^{t*} which satisfy (15) are largely arbitrary; the indeterminacy of amortization schedules for sunk cost assets was noticed by e.g. Peasnell (1981; 54), Schmalensee (1989; 295-296) and Diewert (2009; 9).

In the case where $\Pi^T = 0$, it can be shown that the following intertemporal cost allocations satisfy equation (15):

$$(16) N^{t*} \equiv N^t \text{ for } t = 1, 2, \dots, T-1 \text{ and } N^{T*} \equiv N^T + V_L^T.$$

Thus if the period t cash flow N^t is distributed back to the owners at the end of each period t and the end-of-period T market value of the land plot V_L^T is also distributed to the owners at the end of period T , then the present value of the resulting sequence of distributions will just be equal to the initial project cost. This distribution pattern is consistent with the sequence of end of period asset values A^t defined by (12) and the depreciation amounts Δ^t defined by (14). This intertemporal allocation of project cost is preferred to any other N^{t*} that satisfies (15) as it is useful for the property firm to value its assets at the end of each period at *market values*. As shown by Diewert (2009; 9-10) and Cairns (2013; 640-641), at the end of period t the market value of the firm's assets will be A^t defined by equation t in (12) (if anticipations are realized) and thus the project depreciation schedule defined by (14) will be uniquely determined.

The Δ^t defined by equations (14) can be interpreted as *aggregate period t time series depreciation allocations* for the project as a whole. In the $\Pi^T = 0$ case, the period t cash flow N^t can be interpreted as a period t *aggregate user cost of capital* value for the property.¹⁰ But for national income accounting purposes, it is necessary to decompose the aggregate asset values A^t into land and structure components, and for productivity accounts to similarly decompose the aggregate user cost values N^t .

¹⁰ If $\Pi^T > 0$, then the N^t are not equal to user *cost* allocations since they will contain a pure profit component.

5. The Decomposition of Asset Values into Land and Non-Land Components

We make use of the assumption that the firm forms expectations of the market value of the project land plot at the end of each period t , V_L^t for $t = 0, 1, 2, \dots, T$.¹¹ The availability of this information enables us to define the following end-of-period t expected *user cost for the use of the land* during period t , U_L^t , using the approach of Diewert (1974; 504):

$$\begin{aligned}
 (17) \quad U_L^t &\equiv (1+r_t)V_L^{t-1} - V_L^t && t = 1, \dots, T \\
 &= r_t V_L^{t-1} + \Delta_L^t \\
 &= (1+r_t)V_L^{t-1} - (1+i_t)V_L^{t-1} && \text{using (2)} \\
 &= (r_t - i_t)V_L^{t-1},
 \end{aligned}$$

where the period t *change in the asset value of land (land time series depreciation)* Δ_L^t is defined as:

$$(18) \quad \Delta_L^t \equiv V_L^{t-1} - V_L^t; \quad t = 1, \dots, T.$$

Definitions (17) and (18) could be used to provide estimates for the user cost and depreciation of land for national accounts purposes or for productivity studies for the commercial property sector. Note that the last equation in (17) shows that the user cost of land in period t will be negative if the anticipated period t land inflation rate i_t is greater than the period t cost of capital r_t , and Δ_L^t will also be negative. In the short run, this situation can occur but over long periods of time, we expect r_t to exceed i_t .

By making repeated use of the first set of equations in (17), it can be shown that the asset values for land, V_L^t , and the land user costs, U_L^t , satisfy the following discounted present value relationships:

¹¹ If the project is a Real Estate Investment Trust (REIT), then typically assessed values for the property will be available at the end of each reporting period. The assessed value will often have a decomposition into structure and land values. Assessed values for property taxation purposes always have a structure-land decomposition but these official assessed values may not be based on market values and they may not be up to date.

$$\begin{aligned}
(19) \quad V_L^0 &= \alpha_1 U_L^1 + \alpha_2 U_L^2 + \dots + \alpha_T U_L^T + \alpha_T V_L^T; \\
V_L^1 &= (1+r_1)[\alpha_2 U_L^2 + \alpha_3 U_L^3 + \dots + \alpha_T U_L^T + \alpha_T V_L^T]; \\
V_L^2 &= (1+r_1)(1+r_2)[\alpha_3 U_L^3 + \alpha_4 U_L^4 + \dots + \alpha_T U_L^T + \alpha_T V_L^T]; \\
&\dots \\
V_L^{T-1} &= (1+r_1)(1+r_2)\dots(1+r_{T-1})[\alpha_{T-1} U_L^{T-1} + \alpha_T U_L^T + \alpha_T V_L^T] \\
&= U_L^{T-1} + (1+r_{T-1})^{-1}(U_L^T + V_L^T); \\
V_L^T &= V_L^T.
\end{aligned}$$

Thus the period-by-period discounted present values of the user cost charges defined by (17) are consistent with the exogenously given sequence of expected land values.

We use the aggregate period t project asset values A^t of equations (12) along with the land asset values V_L^t in order to define the *end-of-period t non-land or residual asset value* V_R^t as follows:

$$(20) \quad V_R^t \equiv A^t - V_L^t; \quad t = 0, 1, 2, \dots, T.$$

Once the project residual asset values V_R^t have been defined by (20), period t *residual user benefit* U_R^t and *asset value change* Δ_R^t can be defined as follows:

$$\begin{aligned}
(21) \quad U_R^t &\equiv (1+r_t)V_R^{t-1} - V_R^t; & t = 1, 2, \dots, T \\
&= r_t V_R^{t-1} + \Delta_R^t,
\end{aligned}$$

where Δ_R^t is defined as

$$(22) \quad \Delta_R^t \equiv V_R^{t-1} - V_R^t; \quad t = 1, \dots, T.$$

It is straightforward to show that definitions (20)-(22) and our previous definitions provide us with additive decompositions of aggregate asset values A^t , user benefits N^t and asset value changes Δ^t ; i.e., the above definitions imply the following equations:

$$(23) A^t = V_L^t + V_R^t; \quad t = 0, 1, \dots, T;$$

$$(24) N^t = U_L^t + U_R^t; \quad t = 1, \dots, T;$$

$$(25) \Delta^t = \Delta_L^t + \Delta_R^t; \quad t = 1, \dots, T.$$

If expected discounted profits Π^T are equal to zero, then the end-of-period t non-land asset value V_R^t can be interpreted as the end-of-period t *structure asset value* $V_S^t \equiv V_R^t$, the period t non-land user benefit term U_R^t can be interpreted as the period t *user cost of the structure* and the period t non-land change in asset value Δ_R^t can be interpreted as period t *time series depreciation for the structure* $\Delta_S^t \equiv \Delta_R^t$. Thus in the case where $\Pi^T = 0$, we have decomposed property values, user costs and time series depreciation amounts into land and structure components.¹² In the following section, we tackle the more difficult case where Π^T is positive.

6. The Decomposition of Asset Values into Land, Structure and Goodwill Components

In this section, we assume that expected discounted project profits are positive; i.e., we assume that:

$$(26) \Pi^T = -C_S^0 - V_L^0 + A^0 > 0$$

where C_S^0 is the structure cost, V_L^0 is the end of period 0 land cost and A^0 is end of period 0 expected asset value (defined in equations (12)). Thus the end of period 0 value

¹² A property project has at least two sunk costs: a land component and a structure component. When there are two or more sunk cost assets in a project, Cairns (2013; 644) showed that asset values, user costs and depreciation schedules could not be uniquely determined for the *separate* assets. The reason why we are able to avoid the Cairns impossibility result for the case where $\Pi^T = 0$ is that we assumed extra information; that period-by-period exogenous asset values for one of our two assets were available.

of liabilities, $C_S^0 + V_L^0$, is less than the end of period 0 value of project asset, A^0 . In this situation, it is natural to define an *intangible goodwill asset*, V_G^0 , that is equal to the value of assets less tangible liabilities; i.e., define V_G^0 as follows:

$$(27) \quad V_G^0 \equiv A^0 - C_S^0 - V_L^0 \\ = \Pi^T.$$

Equation (23) for $t = 0$ implies that $A^0 = V_R^0 + V_L^0$ where V_R^0 is the non-land initial asset value for the project. Equation (27) implies that $A^0 = C_S^0 + V_G^0 + V_L^0$ and thus we deduce that the initial non-land asset value V_R^0 is equal to the sum of the initial goodwill asset V_G^0 and the structure cost C_S^0 :

$$(28) \quad V_R^0 = V_G^0 + C_S^0.$$

Define the *shares of the initial goodwill asset and structure cost* in initial non-land asset value V_R^0 , s_G^0 and s_S^0 , as follows:

$$(29) \quad s_G^0 \equiv V_G^0/V_R^0 ; s_S^0 \equiv C_S^0/V_R^0 = (1-s_G^0).$$

Recall that the non-land user benefits U_R^t were defined in the previous section by equations (21). We will use the shares defined by (29) above to decompose these user benefits into *goodwill and structure user cost components*, U_G^t and U_S^t , as follows:

$$(30) \quad U_G^t \equiv s_G^0 U_R^t ; U_S^t \equiv s_S^0 U_R^t = (1-s_G^0) U_R^t ; \quad t = 1, \dots, T.$$

The shares defined by (29) can also be used to decompose the non-land asset values V_R^t defined by (20) into end of period t *goodwill and structure asset values*, V_G^t and V_S^t :

$$(31) \quad V_G^t \equiv s_G^0 V_R^t ; V_S^t \equiv s_S^0 V_R^t ; \quad t = 0, 1, \dots, T.$$

It can be shown that (27)-(31) and the definitions in the previous section can be used to derive the following relationships between the user costs and the asset values defined by equations (30) and (31):

$$(32) U_G^t = (1+r_t)V_G^{t-1} - V_G^t = r_t V_G^{t-1} + \Delta_G^t; U_S^t = (1+r_t)V_S^{t-1} - V_S^t = r_t V_S^{t-1} + \Delta_S^t; \quad t = 1, \dots, T,$$

where period t *time series depreciation for goodwill and structures*, Δ_G^t and Δ_S^t are defined as follows:

$$(33) \Delta_G^t \equiv V_G^{t-1} - V_G^t; \Delta_S^t \equiv V_S^{t-1} - V_S^t; \quad t = 1, \dots, T.$$

We have labelled U_G^t and U_S^t as *user costs* (instead of user benefits) because we can show that these user costs provide for an intertemporal allocation of the initial goodwill asset value V_G^0 and for the initial structure cost C_S^0 ; i.e., the following equations are satisfied by the U_G^t and U_S^t :

$$(34) V_G^0 = (1+r_1)^{-1}U_G^1 + (1+r_1)^{-1}(1+r_2)^{-1}U_G^2 + \dots + (1+r_1)^{-1} \dots (1+r_T)^{-1}U_G^T;$$

$$(35) C_S^0 = (1+r_1)^{-1}U_S^1 + (1+r_1)^{-1}(1+r_2)^{-1}U_S^2 + \dots + (1+r_1)^{-1} \dots (1+r_T)^{-1}U_S^T.$$

It can be shown that the following additive decompositions for the period t aggregate commercial property project values hold:

$$(36) A^t = V_G^t + V_S^t + V_L^t; \quad t = 0, 1, \dots, T;$$

$$(37) N^t = U_G^t + U_S^t + U_L^t; \quad t = 1, \dots, T;$$

$$(38) \Delta^t = \Delta_G^t + \Delta_S^t + \Delta_L^t; \quad t = 1, \dots, T.$$

We have now succeeded in decomposing commercial property values into land, structure and goodwill components. In the case where the property project makes profits that more

than cover the cost of financial capital, we ended up with a depreciable goodwill asset that absorbs up these excess returns.¹³

To further complicate our discussion of the goodwill asset, it should be noted that Cairns (2013; 644) Impossibility Theorem applies to the residual asset;¹⁴ i.e., only the aggregate value of the joint goodwill and structure asset is uniquely determined under our assumptions. Thus instead of using the constant shares s_G^0 and s_S^0 defined by (29) in order to decompose the non-land user benefits U_R^t into goodwill and structure components by equations (30), we could use the following equations for the decomposition:

$$(39) U_G^t \equiv s_G^t U_R^t ; U_S^t \equiv (1-s_G^t) U_R^t ; \quad t = 1, \dots, T$$

where the period t goodwill shares of U_R^t , the s_G^t , satisfy the following restrictions:

$$(40) 0 \leq s_G^t \leq 1 ; \quad t = 1, \dots, T;$$

$$(41) V_G^0 = s_G^1(1+r_1)^{-1}U_R^1 + s_G^2(1+r_1)^{-1}(1+r_2)^{-1}U_R^2 + \dots + s_G^T(1+r_1)^{-1} \dots (1+r_T)^{-1}U_R^T.$$

We know V_G^0 and $V_S^0 (= C_S^0)$. Given V_G^0 , V_S^0 and the U_G^t and U_S^t defined by (39), use equations (32) to define the new end of period t asset values V_G^t and V_S^t iteratively for $t = 1, \dots, T$. Once the V_G^t and V_S^t have been determined, use equations (33) to define time series depreciation for period t , Δ_G^t and Δ_S^t , for the goodwill and structure assets. These newly defined user costs, asset values and time series depreciation amounts will also satisfy equations (32)-(38) and thus these new values also provide an additive decomposition of asset values, user costs and depreciation for the project. Thus the particular allocation that we initially generated in this section is not unique. However, it

¹³ There are no definitive guidelines on how this goodwill asset should be treated in the System of National Accounts. It could be treated as a separate intangible asset in the Balance Sheet Accounts or it could be absorbed into the structure component of the accounts or into the land component of the accounts. Any one of these three treatments could be justified.

¹⁴ "If there are two or more types of comprehensive capital (possibly including a form of intangible capital as a source of profit), economic rental schedules and their implied economic depreciation schedules are not unique. Such schedules apply to all forms of comprehensive capital. The sum of the rentals in any period is the cash flow or producer's surplus." R.D. Cairns (2013; 644).

does seem to be a sensible one. It simply allocates the period t free cash flow less the market determined user cost of land ($U_R^t = N^t - U_L^t$) to the goodwill and structure assets in a manner that is proportional to the initial asset values for goodwill and the structure (which can be observed).¹⁵ The proportional allocation method defined by (31) is consistent with the *matching principle* that has been suggested in accounting theory, where initial costs are distributed over time in proportion to future revenues that are generated by the initial investment.¹⁶

Up to this point, our analysis of a commercial property project has focused on values. In the following section, we turn our attention to the problems that are associated with splitting project values into price and quantity components.

7. The Decomposition of Values into Price and Quantity Components

The decomposition of land values into price and quantity components is straightforward since the quantity of land is constant over the life of the project. Let P_L^t and Q_L^t be the (asset) *price and quantity of project land* at the end of period t . Suitable definitions for these variables are the following ones:

$$(42) P_L^t \equiv V_L^t ; Q_L^t \equiv 1 ; \quad t = 0, 1, \dots, T.$$

Let p_L^t and q_L^t denote the period t *user cost price and quantity of project land*. Again, we can set the period t quantity of land equal to 1 and the corresponding price can be set equal to the user cost U_L^t defined by (17). Thus we have the following definitions:

$$(43) p_L^t \equiv (1+r_t)V_L^{t-1} - V_L^t ; q_L^t \equiv 1 ; \quad t = 1, \dots, T.$$

¹⁵ The proportional allocation of V_R^t to the goodwill and structure asset defined by (31) corresponds to Cairns' (2013; 645) first method for constructing a definite depreciation schedule for two sunk cost assets. He also noted that any standard depreciation model could be applied to the structure and then depreciation for the goodwill asset could be defined residually. Finally, Cairns noted that a *minimum economic payback allocation method* could be used where all available cash flows are allocated to tangible capital until their present value is equal to the initial investment value.

¹⁶ The matching principle in accounting theory can be traced back to Church (1917; 193); see also Paton and Littleton (1940; 123) and Diewert (2005; 533-540).

Goodwill does not have to be decomposed into price and quantity components if it is simply regarded as a repository for pure profits.¹⁷ The decomposition of structure values into price and quantity components is more complex. It would seem that we could treat the decomposition of end-of-period t structure asset value V_S^t in a manner that is analogous to our treatment of land value in (42); i.e., simply define the *end-of-period t asset price of structures* P_S^t as the corresponding value V_S^t and define the corresponding *asset quantity* Q_S^t as 1. However, the resulting prices do not give us the price of a constant quality amount of structure over time: the structure at the end of period $t+1$ is not the same as a structure at the end of period t , since its useful life has been reduced by one period. This changing quality problem does not apply to land and so the land prices defined by (42) and (43) can be regarded as constant quality prices.

Our suggested solution is to decompose structure asset value V_S^t at the end of period t into the price of a new structure of the same type at the end of period t times a corresponding quantity Q_S^t that is measured in equivalent units of new structure. Let P_S^{t*} be an appropriate (for the type of structure under consideration) *exogenous construction price index* for the end of period t and define the end of period *asset price and quantity* (in constant quality units of measurement) for the project structure as follows:¹⁸

$$(44) P_S^t \equiv P_S^{t*}; Q_S^t \equiv V_S^t / P_S^{t*}; \quad t = 0, 1, \dots, T.$$

We turn now to the problems associated with the decomposition of the *user cost value for structures* for period t , U_S^t , given by equations (32). Define the period t *structure inflation rate* i_S^t for the exogenous end of period t structure price index P_S^{t*} as follows:

$$(45) 1 + i_t^* \equiv P_S^{t*} / P_S^{t-1*}; \quad t = 1, \dots, T.$$

¹⁷ Other treatments of goodwill are possible but more discussion and research is needed in order to definitively decompose goodwill values into price and quantity components.

¹⁸ This simple method of quality adjustment is essentially the same method that was suggested by Diewert (2009; 16) in a different context.

Define the period t (constant quality) *structure depreciation rate* δ_t as follows:

$$(46) 1-\delta_t \equiv Q_S^t/Q_S^{t-1}; \quad t = 1, \dots, T.$$

Now use (44) to solve for $V_S^t = P_S^{t*} Q_S^t$ for $t = 0, 1, \dots, T$ and substitute these relationships into equations (32). We obtain the following expressions for the U_S^t :

$$(47) \begin{aligned} U_S^t &= (1+r_t)V_S^{t-1} - V_S^t && t = 1, \dots, T \\ &= (1+r_t) P_S^{t-1*} Q_S^{t-1} - P_S^{t*} Q_S^t \\ &= (1+r_t) P_S^{t-1*} Q_S^{t-1} - (1+i_t^*)(1-\delta_t)P_S^{t-1*} Q_S^{t-1} && \text{using (45) and (46)} \\ &= [r_t - i_t^* + (1+i_t^*)\delta_t]P_S^{t-1*} Q_S^{t-1}. \end{aligned}$$

Using the last equation in (47), the decomposition of the period t *user cost value for the structure*, U_S^t , into *price and quantity components*, p_S^t and q_S^t , is thus fairly simple:

$$(48) p_S^t \equiv [r_t - i_t^* + (1+i_t^*)\delta_t]P_S^{t-1*}; \quad q_S^t \equiv Q_S^{t-1}; \quad t = 1, \dots, T.$$

Note that p_S^t has the same form as the traditional user cost of capital. Equations (43) and (48) show that traditional capital measurement techniques can be adapted to measure the land and structure input contributions of a commercial property project.¹⁹

In order to implement our framework in a national accounting framework, the national statistician will require information by property on (i) projections of the market value of the land plot over time; (ii) the initial construction cost of the structure (or an estimate of its current value); (iii) the cost of capital faced by the commercial property firm; (iv) an exogenous construction cost index for the type of structure on the property; and current period cash flows (or net operating incomes) generated by the property and projections of

¹⁹ See Diewert and Shimizu (2013) for the description of a measurement framework that uses traditional methods. They used a one hoss shay model to describe structure depreciation in their framework. When the fixity of the structure is taken into account, it can be seen that the intertemporal pattern of project cash flows plays a decisive role in determining the time series depreciation of the structure. Thus actual structure depreciation is likely to be much more volatile than one hoss shay depreciation.

these cash flows. In addition, the national statistician will have to make assumptions on how any pure profits generated by the project will be amortized over time. This is a rather heavy load of data requirements which will certainly not be fulfilled in practice. Thus in order to obtain a practical measurement framework, additional assumptions will have to be made.

There are many additional problems associated with measuring capital input for a commercial property that we have not discussed, such as the treatment of capital expenditures, property taxes, and insurance payments; see Diewert and Shimizu (2013). These extensions of the above framework are left for further research.

8. Conclusion

The main purpose of the present paper has been to provide a framework for measuring capital input for a commercial property. Our suggested framework provides a decomposition of aggregate capital input into land, structure and goodwill components. What is new in the present paper is that the fixity of the structure and the endogeneity of the useful life of the structure are taken into account.

Three main practical conclusions emerge from the paper. First, taking the fixity of the structure into account does not lead to a dramatically different measurement framework as compared to more traditional approaches which ignore the fixity problem in the sense that we still obtain user costs for the structure that look familiar; see equations (47).

Second, the pattern of time series depreciation allocations for a commercial property structure is largely (but not exclusively) determined by the cash flows that the property generates over the lifetime of the structure. These cash flow patterns are likely to be very different over different property classes, leading to measurement challenges. In particular, traditional depreciation models for structures (such as the one hoss shay, geometric or straight line models) are unlikely to provide adequate descriptions of economic reality for the commercial property sector.

Finally, our theoretical measurement framework requires a great deal of data for implementation and these data are unlikely to be available. Thus further assumptions will have to be made in order to obtain a practical measurement framework. However, we believe that our framework does capture many realities of the commercial property market, and thereby significantly advances the capacity to understand this important market.

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