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Abstract

Motivated by the use of forward guidance, we propose a method to estimate DSGE models in which the central bank holds the policy rate fixed for an extended period. Private agents' beliefs about how long the fixed-rate regime will last influences, among other observable variables, current output, inflation and interest rates of longer maturities. We estimate the shadow policy rate and construct counterfactual scenarios to quantify the severity of the zero lower bound constraint. Using the Smets and Wouters (2007) model, we find that the expected duration of the zero interest rate policy has been around 2 years, that the shadow rate has been around -3 per cent and that the zero lower bound has imposed a significant output loss.

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1 Introduction

To combat the recent financial crisis and the resulting economic downturn, the Federal Reserve and many other central banks in advanced economies pushed their policy interest rates close to the zero lower bound and turned, among other policies, to forward guidance. Forward guidance refers to announcements about the future path of the policy rate. This communications policy has received increasing attention in the press and the academic literature. In particular, while some central banks have previously given guidance about the direction or timing of future policy rates, these recent announcements have been interpreted as an explicit attempt to influence expectations so as to increase the current degree of monetary policy accommodation.[1]

There is a good argument in theory why forward guidance can alleviate the contractionary impact of the zero lower bound. In forward-looking models the current stance of monetary policy depends on the expected path of the nominal interest rate, and therefore forward guidance can, in principle, stimulate aggregate demand to the extent it lowers private agents' forecasts of future nominal interest rates. So, a credible commitment to maintain interest rates at zero for longer than would have otherwise been implied by the zero bound itself represents an additional channel of monetary stimulus. Eggertsson and Woodford (2003), Jung et al. (2005) and more recently Werning (2012) all make this point: monetary policy can stimulate the economy by creating the right kind of expectations about the way the policy rate will be used once the constraint ceases to bind.[2]

Since December 2008, the Federal Reserve has made use of forward guidance. As is evident from FOMC statements, its forward guidance evolved over time and it is likely that the public's interpretation has changed as well. From early 2009 to mid 2011 the statements were somewhat vague, as is the case for example in the December 2009

[1]See Woodford (2012).

[2]Krugman (1998) was the first to recast the liquidity trap as an expectations-driven phenomenon.

statement which reads:

> "*The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions, [...], are likely to warrant exceptionally low levels of the federal funds rate for an extended period.*"

Then, from August 2011 to October 2012 the statements gave more precision about the "extended period" and the language changed, as the October 2012 statement shows:

> "*In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.*"

But then, starting in December of 2012, the FOMC statements provided clearer state-contingent conditions linking the path of interest rates to the state of the economy, as is the case of the June 2013 statement which reads:

> "*[...] the Committee currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, [...]*".

The existing empirical literature on forward guidance is sparse. Swanson and Williams (2014), for example, use high-frequency data to study the effects of the zero lower bound on interest rates of longer maturities and find that market participants often expected the zero bound to constrain policy for only a few quarters. Bauer and Rudebusch (2014) use a shadow rate affine dynamic term structure model that accounts for the zero lower bound to infer expected future policy and estimate the future lift-off date. They find that the expected duration of the zero interest rate policy was quite short prior to mid 2011, when it noticeably increased. Campbell et al. (2012) study the response of asset prices and private macroeconomic forecasts to FOMC forward guidance before and after the crisis and conclude that the zero lower bound has not prevented the Federal Reserve from communicating future policy intentions. Aruoba et al. (2013) estimate a small-scale model in which sunspot shocks move the economy between an intended steady state an a deflationary steady state.

Our approach here is different. In our setup, fundamental shocks drive the economy to the zero lower bound. We build on Kulish and Pagan (2012) and construct the likelihood function for the case in which monetary policy switches at the zero lower bound from following a standard Taylor-type rule to forward guidance. We use the Smets and Wouters (2007) model and incorporate information from the yield curve, and therefore use a larger set of observable variables.

Using Bayesian methods we estimate, for the period 1983Q1-2014Q2, both the structural parameters and the expected duration of the zero interest rate policy in each quarter since the beginning of 2009. The joint estimation of structural parameters and expected duration has not been done in the context of DSGE estimation and this constitutes a key methodological contribution of our paper. By now, the United States economy has had over 20 quarters for which the Federal Funds rate has been zero. This raises a challenge in estimation that will persist into the future: even if the zero lower bound were to never bind again, future samples of macroeconomic data will have a long spell of zero interest rates. We suggest a feasible way in which DSGE models can be estimated accounting for the zero lower bound.

In measuring the impact of communications on expectations of future interest rates and the economy, it is useful to distinguish between two kinds of forward guidance: a *threshold-based* forward guidance in which the central bank publicly states its forecasts and anticipated policy actions based on its own objectives and a *calendar-based* forward guidance in which the central bank publicly commits to a particular course of action. In our analysis, we let the data speak through the lens of a model in which forward guidance is 'calendar-based'. During the zero lower bound regime, agents base current expectations on the assumption that the central bank will unconditionally keep to its zero interest rate policy for a certain number of quarters in the future, after which it will revert back to its temporarily abandoned Taylor rule.

We achieve identification because variation in the expected duration gives rise to distinct dynamics of the observable variables, because the sub-sample prior to the zero lower bound helps identify competing sources of exogenous variation and because there are no unanticipated monetary policy shocks at the zero lower bound. It is well known that in models with rational expectations, forward guidance is powerful in the sense that it can generate very large responses of aggregate variables, a phenomenon del Negro et al. (2012) call the 'forward guidance puzzle'.[3] For the purposes of econometric identification, however, the sensitivity of aggregate variables to forward guidance turns out to be quite useful in pinning down the expected durations in estimation. An absence of 'unreasonably large' fluctuations in the data, however, does not necessarily imply short expected durations because forward guidance potentially can offset the impact of large shocks.

We find that including the zero lower bound regime in our sample and estimating the expected durations of the zero lower bound produces structural parameter estimates that are in line with those found in the literature on the pre-crisis data only. The mean expected duration for the zero interest rate policy is estimated to vary between 8 and 9 quarters over the zero lower bound regime.

Our estimation approach produces other results that are likely to be of interest to policymakers. In particular, for the zero lower bound regime, we compute shadow policy rates – i.e., the policy rate implied by the monetary policy rule not subject to the zero constraint. Shadow rates allow us to assess the extent to which the constraint is binding. We find persistently negative shadow rates of around -3% after the first quarter of 2009,

[3] Carlstrom et al. (2012) show that an announcement to keep the interest rate at zero for more than 8 quarters delivers 'unreasonably large' responses in perfect foresight simulations of the Smets and Wouters (2007) model. This need not be the case, however, in stochastic simulations.

which suggests that the zero lower bound has placed a significant constraint on monetary policy in the United States. We use the estimated structural shocks for a counterfactual analysis and find that the 22 quarters of zero interest rates cost the US economy a cumulative output loss of 22 per cent.

The rest of the paper is structured as follows. Section 2 presents the benchmark model, then in Section 3 we discuss how we solve for equilibria under forward guidance. Section 4 discusses the estimation methodology. Section 5 contains the main results, and section 6 presents the sensitivity analysis.

2 The Model

In our empirical analysis, we focus on the Smets and Wouters (2007) model and incorporate yield curve data in a manner similar to Graeve et al. (2009). Because the model of Smets and Wouters (2007) is well-known we briefly sketch its key properties and present the equations for the sticky-price economy in Appendix C.

The model is log-linearized around a balanced-growth path with deterministic labour-augmenting technology. The nominal frictions include both sticky intermediate goods prices and wages introduced via Calvo-pricing, with partial indexation for those firms who do not have the opportunity to re-optimise their prices. The real rigidities include investment adjustment costs (which depend on the growth rate of investment), external habits in consumption, variable capacity utilisation and fixed costs in production. Monetary policy is set according to a Taylor rule and Government spending is exogenous. There are seven exogenous processes: total factor productivity; a risk-premium, which drives a wedge between the policy rate the central bank sets and the one that households face; investment-specific technology; government spending; price and wage mark-ups and monetary policy. These are all modelled as first-order autoregressive processes with normal $i.i.d$ innovations, except for the price and wage mark-up shocks, which are assumed to follow a first order autoregressive moving-average process.[4]

As the aim of forward guidance is to influence expectations about the future path of the policy rate, long bond yields may contain useful information. The Smets and Wouters (2007) model, however, only explicitly includes one period bonds; we describe how we incorporate the yield curve drawing on Graeve et al. (2009).

According to the expectations hypothesis long yields are the average of the expected path of short rates:

$$\hat{r}_{j,t} = \frac{1}{j}\mathbb{E}_t\left(\hat{r}_t + \hat{r}_{t+1} + \cdots + \hat{r}_{t+j-1}\right) \text{ for } j = 2, 3, ..., m, \tag{1}$$

where $\hat{r}_{j,t}$ denotes the log deviation from steady-state. Consequently, longer yields may be incorporated by augmenting the model with Equation 1 for various maturities.

It is well-known that the version of the expectations hypothesis in Equation 1 does not perform well empirically (e.g. Campbell and Shiller (1991)), but even if it did, for estimation it is necessary to add additional shocks. In particular, we relate the model-implied yields, $\hat{r}_{j,t}$, to observed ones, $r_{j,t}$, as follows:

$$r_{j,t} = \hat{r}_{j,t} + r + c_j + \eta_t + \varepsilon_{j,t} \text{ for } j = 2, 3, ..., m \tag{2}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_{\eta,t}, \tag{3}$$

where r is the steady state of the one-period nominal interest rate. There are two components to the term premia. First, c_j which is a maturity-specific time-invariant component, and consequently $r + c_j$ is the steady state of the observed yield with j periods to maturity. Second, there is a time-varying component; this is composed of a persistent shock, η_t, common to all observed maturities, and an idiosyncratic, *i.i.d.* maturity-specific shock, $\varepsilon_{j,t}$.[5]

[4] The government spending shock also responds contemporaneously to the innovation to the technology shock.

[5] Graeve et al. (2009) allow for correlation between the idiosyncratic measurement errors (i.e. across

3 Solution with Forward Guidance

To solve for equilibria under forward guidance we use a special case of the solution developed by Kulish and Pagan (2012) for forward-looking models in the presence of possibly anticipated structural change. That solution provides an econometric representation of Cagliarini and Kulish (2013) and has more general application than the context we are considering here, so we provide a simplified discussion to highlight some important features.[6]

To discuss the solution, we introduce notation. Below we take a sample of data of size T to estimate the model. For presenting the solution under forward guidance, however, it is useful to take the start of the zero lower bound regime to be at $t = 1$. In Figure 1, the zero lower bound starts in period 1 and lasts for d periods and conventional policy is assumed to resume out of sample.

In the form of Binder and Pesaran (1995), the system of linearised equations can be written as

$$y_t = Ay_{t-1} + B\mathbb{E}_t y_{t+1} + D\varepsilon_t, \tag{4}$$

where y_t is an $n \times 1$ vector of state and jump variables and with no loss of generality ε_t is a $l \times 1$ vector of white noise shocks; the matrices A, B and D are of conformable dimensions.

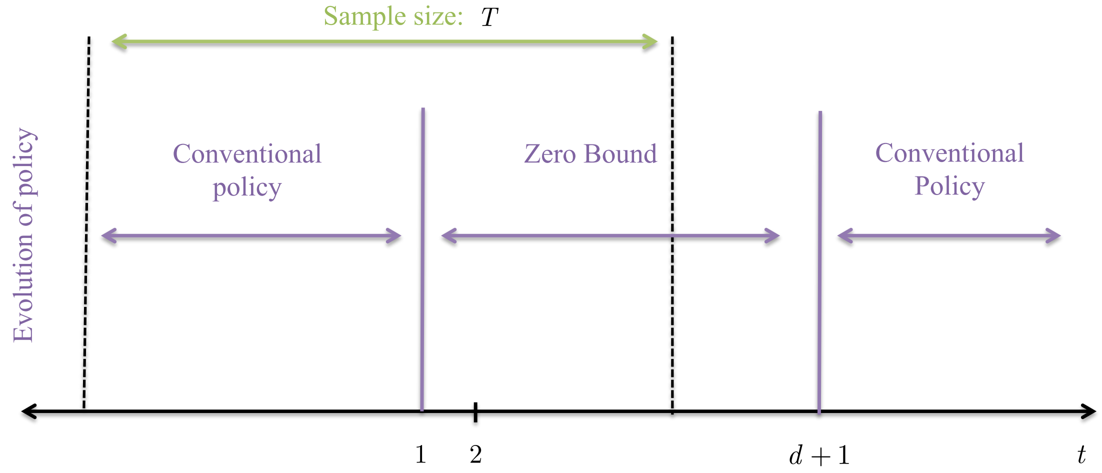
Prior to the zero lower bound, the economy follows equation (4) and the standard rational expectations solution applies. If the solution exists and is unique, then y_t follows the VAR process

$$y_t = Qy_{t-1} + G\varepsilon_t. \tag{5}$$

maturities) but do not allow for persistence in the measurement equation as we do with η_t.

[6] del Negro et al. (Forthcoming) also use this approach to solve for equilibria under forward guidance.

Figure 1: Timing of Events



Assume now that at $t = 1$ the implementation of the central bank's policy rule implied that it set a negative interest rate; in other words, $\hat{r}_t < -r$, where r is the steady-state value. As this is not possible, monetary policy sets its policy rate to its lower bound, $\hat{r}_t = -r$, and communicates its intentions to revert back to conventional policy at a later time, $t = d^e + 1$.[7] This later date may or may not coincide with the realised duration of the zero lower bound regime. If agents, in fact, expect conventional policy to resume at that time, then the expected duration of the zero lower bound regime in period $t = 1$ is given by d^e. During the zero lower bound regime, the structural equations are given by

$$y_t = \bar{A}y_{t-1} + \bar{B}\mathbb{E}_t y_{t+1} + \bar{D}\varepsilon_t, \tag{6}$$

and monetary policy now follows $\hat{r}_t = -r$. A monetary policy rule that fixes the nominal interest rate would give rise to indeterminacy if agents indeed expected this rule to be implemented indefinitely. Alternatively, if monetary policy is expected to adopt a rule consistent with a unique equilibrium in the future, then, as shown in Cagliarini and Kulish (2013), a rule like $\hat{r}_t = -r$ can be temporarily consistent with a unique equilibrium as well. Suppose then that monetary policy will indeed revert back to conventional policy

[7] In our empirical application, we assume that this conventional policy which is reverted back to includes an inflation target of 2 per cent per annum.

by $t = d^e + 1$, so we assume $d^e = d$. For periods $t = 1, 2, ..., d$ the solution for y_t becomes a time-varying coefficient VAR process

$$y_t = Q_t y_{t-1} + G_t \varepsilon_t, \tag{7}$$

which implies that

$$\mathbb{E}_t y_{t+1} = Q_{t+1} y_t. \tag{8}$$

Using equations (8) and (6), it is possible to establish via undetermined coefficients that

$$\begin{aligned} (I - \bar{B} Q_{t+1})^{-1} \bar{A} &= Q_t & (9) \\ (I - \bar{B} Q_{t+1})^{-1} \bar{D} &= G_t. & (10) \end{aligned}$$

Starting from the solution to the final structure, $Q_{d+1} = Q$, equation (9) determines via backward recursion the sequence $\{Q_t\}_{t=1}^{d}$. With the sequence $\{Q_t\}_{t=1}^{d}$ in hand, equation (10) yields the sequence $\{G_t\}_{t=1}^{d}$.

The sequence of time-varying reduced form matrices, $\{Q_t\}_{t=1}^{d}$, provide the solution for the case in which the expected duration of the zero lower bound follows $d, d-1, d-2, ..., 1$. In the sequence $\{Q_1, Q_2, ..., Q_d\}$, the matrix associated with an expected duration of d quarters is Q_1, the matrix associated with an expected duration of $d - 1$ quarters is Q_2 and so on. This sequence, however, can be thought of in two ways: as one announcement made in $t = 1$ and carried out as announced or as a sequence of announcements with expected durations $d, d-1, d-2, ..., 1$. This implies, for instance, that if in every period monetary policy were to announce, or alternatively, if agents were to expect zero interest rates to last for d^e periods then the resulting sequence of reduced form matrices would simply be $\{Q_1, Q_1, ..., Q_1\}$. The point is that a sequence of expected durations maps uniquely into a sequence of reduced-form matrices.

In work that is contemporaneous and independent to this analysis, Guerrieri and Iacoviello (2015) propose a solution for models with occasionally binding constraints. Their solution may be viewed as an iterative application of the solution of Cagliarini and Kulish (2013) where the iterations are used to find the expected duration which is consistent with the binding constraint. Once this expected duration is found, the reduced form matrix associated with that duration is the same as we described above. Thus, the occasionally binding approach places a restriction on the sequence of expected durations, a restriction that may or may not hold in the data. Our estimation approach is unconstrained in this respect, so it is free to choose the expected duration that best fits the data. We allow – but do not require – the expected duration of the zero interest rate policy to co-exist with a non-binding constraint. It is important to allow for this possibility in estimation given the optimal policy prescription of prolonging the duration of the zero interest rate policy once the constraint ceases to bind.

4 Estimation

We use Bayesian methods, as is common in the estimated DSGE model literature.[8] Our case, however, is non standard in a few ways. First, forward guidance implies a form of regime change as we have described above. Second, it is necessary to adjust the Kalman filter to handle missing observations. As the Federal Funds rate has no variance at the zero lower bound, it must be removed as an observable to prevent the variance-covariance matrix of the one-step ahead predictions of the observable variables from becoming singular.[9] Third, we jointly estimate two sets of distinct parameters: the structural parameters of the model, θ, that have continuous support and the sequence

[8]See An and Schorfheide (2007)).

[9]See Appendix A, which is available at https://sites.google.com/site/marianokulish/home/research, for a description of how this is implemented. One could alternatively allow for measurement error in the observation equation of the Federal Funds rate. Although there is little variation of the Federal Funds rate throughout the zero lower bound regime, we do not consider this variation to be a form of measurement error.

of expected durations, $\{d_t^e\}$, that have discrete support. In other words, the sequence of expected durations can take on only integer values and have to be treated differently. For notational convenience we will denote a sequence of expected durations hereafter simply by $\mathbf{d}$.

Next, we describe how we construct the joint posterior density of θ and $\mathbf{d}$:

$$p(\theta, \mathbf{d}|Z) \propto \mathcal{L}(Z|\theta, \mathbf{d})p(\theta, \mathbf{d}), \tag{11}$$

where $Z \equiv \{z_t\}_{t=1}^T$ is the data and z_t is a $n_z \times 1$ vector of observable variables. The likelihood is given by $\mathcal{L}(Z|\theta, \mathbf{d})$, the priors for the structural parameters and the sequence of expected durations are assumed to be independent, so that $p(\theta, \mathbf{d}) = p(\theta)p(\mathbf{d})$. Our baseline results are based on a flat prior for $\mathbf{d}$ such that $p(\mathbf{d}) \propto 1$, which is proper given its discrete support.

4.1 The Likelihood with Forward Guidance

The sample runs from 1983q1 to 2014q2 and has two distinct sub-samples, one before and one after the zero lower bound. Before the zero lower bound, from 1983q1 to 2008q4, we postulate a constant regime so that the reduced-form solution that governs the system for those periods is $y_t = Qy_{t-1} + G\varepsilon_t$. Once the zero lower bound regime is in place, for 2009q1 onwards, the reduced-form solution follows Equation (7), that is $y_t = Q_t y_{t-1} + G_t\varepsilon_t$, where the sequence of reduced-form matrices is a function of the expected duration that prevails at each quarter.

Before the zero lower bound, the model variables, y_t, are related to the observable variables, z_t, via the measurement equation

$$z_t = Hy_t + v_t. \tag{12}$$

For the zero lower bound regime, we define a new vector of observables, $\bar{z}_t \equiv W z_t$, where W is an $(n_z - 1) \times n_z$ matrix that selects a subset of the observable variables in z_t. In our case, we remove the Federal Funds rate from the set of observable variables. Defining $\bar{H} \equiv WH$ and $\bar{v}_t \equiv W v_t$, the model variables relate to the subset of the observables during the zero lower bound regime by

$$\bar{z}_t = \bar{H} y_t + \bar{v}_t. \tag{13}$$

Equations (5) and (7) from the previous section summarize the evolution of the state and Equations (12) and (13) the evolution of the measurement equations. Together they form a state space model to which the Kalman filter can be applied to construct the likelihood, $\mathcal{L}(Z|\theta, \mathbf{d})$, as described in Appendix A.

4.2 The Prior

As mentioned above, the joint prior for the parameters is split into two independent priors, one for the structural parameters, $p(\theta)$, and one for the sequence of expected durations. We discuss each in turn.

The joint prior for the structural parameters is factorized into independent priors for each structural parameter, which are set following Smets and Wouters (2007). The priors, together with the posterior estimates, are given in Table 2. We use an uninformative prior for the expected durations.[10]

4.3 The Posterior Sampler

To simulate from the joint posterior of the structural parameters and the expected durations of the zero lower bound, $p(\theta, \mathbf{d}|Z)$, we use the Metropolis-Hastings algorithm. As we have two distinct sets of parameters we consider a slight modification to the standard

[10] In a previous version of the paper, Kulish et al. (2014), we demonstrated one possible informative prior that alternatively could be used.

setup for estimating DSGE models. We separate the parameters into two natural blocks: the expected durations of the zero lower bound policy and the structural parameters. To be clear, though, our sampler delivers draws from the joint posterior of both sets of parameters.

The first block of the sampler is for the expected duration of the zero lower bound, $\mathbf{d}$. It is possible to update the entire sequence of expected durations at each iteration of the sampler. However, preliminary estimation with simulated data suggested that more accurate results were obtained when only a subset of the parameters were possibly updated at each iteration. The approach we use is to randomize both the *number* of expected durations in $\mathbf{d}$ to be updated, and *which* particular expected durations in $\mathbf{d}$ to update. Our approach is motivated by the randomized blocking scheme developed for DSGE models in Chib and Ramamurthy (2010). For this block, we use a uniform proposal density.

To be specific, the algorithm for drawing from the expected durations block is given as follows: Initial values of the expected durations, $\mathbf{d}_0$, and the structural parameters, θ_0, are set. Then, for the j^{th} iteration, we proceed as follows:

1. randomly sample the *number* of quarters to update in the proposal from a discrete uniform distribution $[1, d^*]$

2. randomly sample without replacement *which* quarters to update in the proposal from a discrete uniform distribution $[1, L]$

3. randomly sample the corresponding elements of the proposed sequence of durations, $\mathbf{d}_j^{'}$, from a discrete uniform distribution $[1, d^*]$ and set the remaining elements to their values in $\mathbf{d}_{j-1}$

4. calculate the acceptance ratio $\alpha_j^{\mathbf{d}} \equiv \frac{p(\theta_{j-1}, \mathbf{d}_j^{'}|Z)}{p(\theta_{j-1}, \mathbf{d}_{j-1}|Z)}$

5. accept the proposal with probability $\min\{\alpha_j^{\mathbf{d}}, 1\}$, setting $\mathbf{d}_j = \mathbf{d}_j^{'}$, or $\mathbf{d}_{j-1}$ otherwise.

The second block of the sampler is for the n_s structural parameters.[11] It follows a similar strategy to the expected-durations-block described above - we randomize over the number and which parameters to possibly update at each iteration. One difference, however, is that the proposal density is a multivariate Student's $t-$distribution.[12] Once again, for the j^{th} iteration we proceed as follows:

1. randomly sample the *number* of parameters to update from a discrete uniform distribution $[1, n_s]$

2. randomly sample without replacement *which* parameters to update from a discrete uniform distribution $[1, n_s]$

3. construct the proposed $\theta_j^{'}$ by drawing the parameters to update from a multivariate Student's $t-$ distribution with location set at the corresponding elements of θ_{j-1}, scale matrix based on the corresponding elements of the negative inverse Hessian at the posterior mode multiplied by a tuning parameter $\kappa = 0.2$, and degree of freedom parameter $\nu = 12$.

4. calculate the acceptance ratio $\alpha_j^\theta \equiv \frac{p(\theta_j^{'}, \mathbf{d}_j|Z)}{p(\theta_{j-1}, \mathbf{d}_j|Z)}$ or set $\alpha_j^\theta = 0$ if the proposed $\theta_j^{'}$ includes inadmissible values (e.g. a proposed negative value for the standard deviation of a shock) preventing calculation of $p(\theta_j^{'}, \mathbf{d}_j|Z)$

5. accept the proposal with probability $\min\{\alpha_j^\theta, 1\}$, setting $\theta_j = \theta_j^{'}$, or θ_{j-1} otherwise.

We use this multi-block algorithm to construct a chain of 575,000 draws from the joint posterior, $p(\theta, \mathbf{d}|Z)$, discarding the first 25 per cent as burn in (approximately 140,000 draws). The chain exhibits some persistence, although this in part reflects that blocking of the parameters, and simple trace plots suggest that the estimates of the structural parameters mix well.

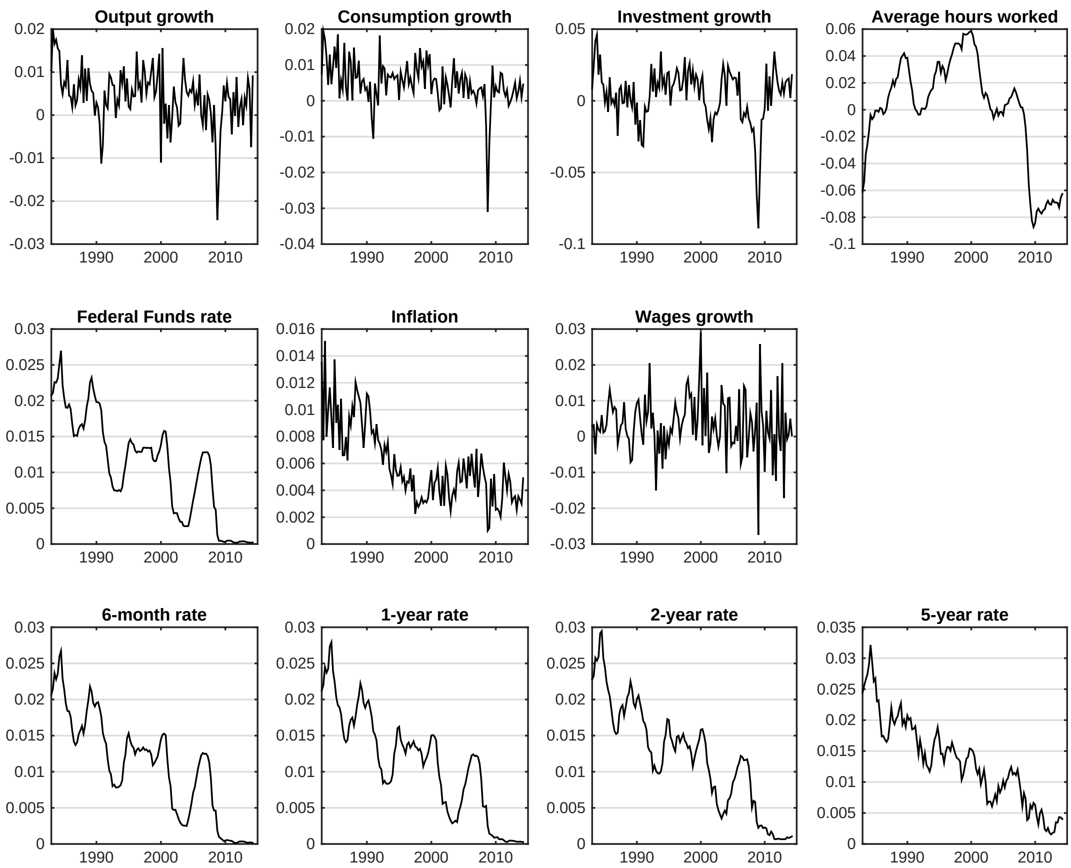
[11] In our benchmark application $n_s = 12$.

[12] For computational efficiency, the hessian of the proposal density is computed at the mode of the structural parameters rather than at each iteration as in Chib and Ramamurthy (2010).

4.4 The Data

The observable data used in estimation follow Smets and Wouters (2007), namely: consumption, investment and output per capita growth, average hours worked, the Federal Funds rate, real wages and inflation.[13] To these we add the yield curve data, namely yields at maturities of six months, a year and two and five years. The data were mostly obtained from the FRED database of the Federal Reserve of St Louis, and are shown in Figure 2. The details of their construction is provided in Appendix C.

Figure 2: Observable variables



[13]The sample is the period after the Volcker disinflation, 1983q1 to 2014q2, during which the objectives of US monetary policy are likely to be relatively stable, and is shorter than was used by Smets and Wouters (2007), namely 1957q1 to 2004q4.

5 Results

Our econometric methodology produces estimates of the structural parameters and of the sequence of expected durations of the zero lower bound. The estimation has implications for the stance of monetary policy and allows us to quantify the severity of the zero lower bound. We discuss these first and then present the estimates of the structural parameters of the model.

5.1 Expected Durations

Table 1 shows summary statistics of the posterior distribution of the expected durations. The mean estimate of the expected duration that the zero interest rate policy will continue is always 7 quarters or longer, and is particularly long until mid 2010.

Figure 3 shows marginal distributions of the expected durations for the quarters of 2011.[14] In August of 2011 the FOMC adopted calendar-based forward guidance. Our estimates suggest little change in the mean expected duration but as Figure 3 shows there is a shift in the distribution placing relatively more mass at longer durations. These distributions show that the expected durations are well-identified and change from quarter to quarter. There is also a significant change in the distribution in 2009: at the onset of the zero interest rate policy in the March quarter of 2009 there is substantial mass at short horizons, i.e. less than 5 quarters, and gradually throughout 2009 this shifts to longer expected durations. Generally the distributions are highly non-normal, and have long right-hand tails. Bauer and Rudebusch (2014) find similar shapes from both their shadow-rate yield curve model and when it is augmented with macroeconomic factors, and note that "*...the distribution is very strongly skewed to the right - even very distant horizons for policy liftoff are not uncommon*" (p. 15).

[14]The marginal distributions of every quarter of the zero lower bound are presented in Appendix B.

Table 1: Estimated Expected Duration of the Zero Lower Bound

	Expected Duration (quarters)	
Quarter	Mean	Std. Dev.
2009 Q1	8.70	4.88
2009 Q2	9.70	4.83
2009 Q3	10.57	4.90
2009 Q4	9.60	4.68
2010 Q1	9.29	4.61
2010 Q2	9.99	5.09
2010 Q3	7.32	4.53
2010 Q4	9.37	4.88
2011 Q1	8.29	4.52
2011 Q2	8.48	4.74
2011 Q3	8.37	5.02
2011 Q4	7.79	5.00
2012 Q1	7.88	4.82
2012 Q2	7.55	4.80
2012 Q3	7.25	4.79
2012 Q4	7.90	5.08
2013 Q1	7.49	4.88
2013 Q2	8.23	4.74
2013 Q3	8.57	4.84
2013 Q4	9.10	4.96
2014 Q1	8.24	4.61
2014 Q2	8.75	5.04

We estimate a posterior density for the sequence of expected durations. Each sequence has one duration for each of the 22 quarters of the zero interest rate regime. Figure 4 show the posterior distributions of the mean and standard deviation of this sequence. The average expected duration is around 10 quarters, and there is little mass below 5 quarters. The sequence also display considerable volatility: the mean standard deviation of expected durations is a little less than 5 quarters. Together, these figures imply that a sequence where the expected durations are very short, say fluctuating anywhere between 1 and 5 quarters, is improbable. In fact, we estimate a posterior probability of 72 per cent that the expected duration was greater than 16 quarters at least once during the zero lower bound regime and a posterior probability of 18 per cent that the expected duration reached 20 quarters at least once.

Figure 3: Distribution of Expected Durations: 2011

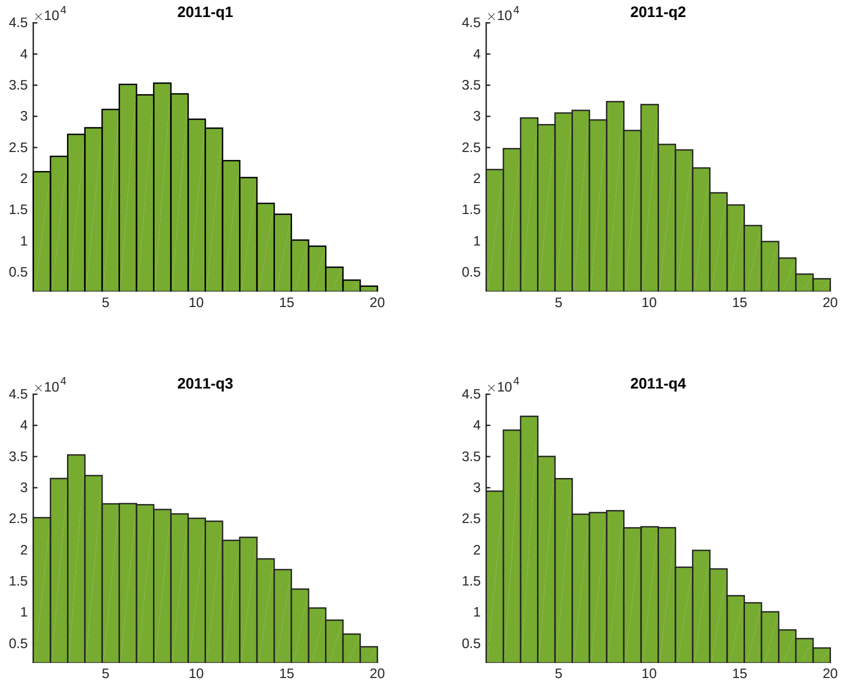
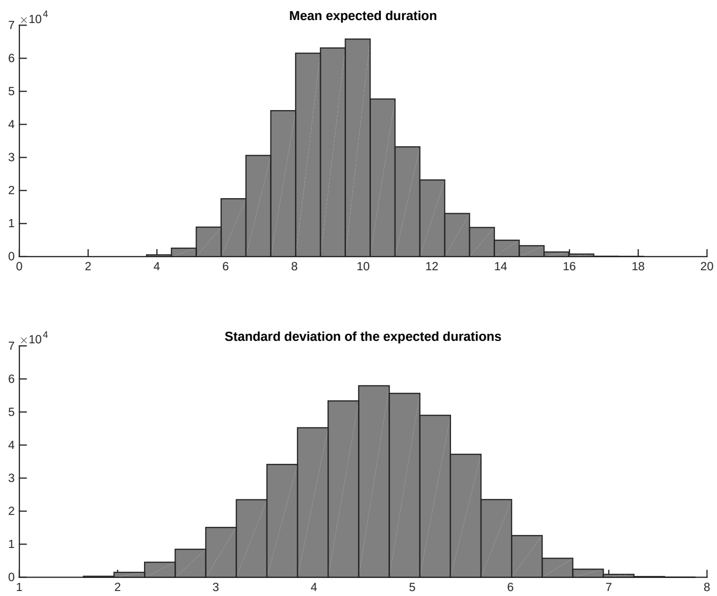


Figure 4: Posterior Distributions



Other authors have used different models and data to estimate the expected durations. For example, Swanson and Williams (2014) analyse the Blue Chip survey of forecasters and find that for the period after the Fed commenced its"mid-2013" guidance at the August 2011 FOMC meeting the expected duration is 6 quarters or longer.[15] For the period prior, however, they find a shorter expected duration - typically between 3 and 4 quarters.[16] The median estimates from the Federal Reserve Bank of New York's Primary Dealer Survey, presented in Bauer and Rudebusch (2014), display a similar pattern. Swanson and Williams (2014) construct estimates of the probability of the Federal Funds rate being less than 50 basis points in five quarters time from options data, and from August 2011 onwards these fluctuate around 0.85 per cent. Prior to this the probability is substantially lower, although throughout much of 2010 it is around 0.6 and begins to increase from the June quarter of 2011. Krippner (2014a) extracts the expected duration from a two-state variable shadow yield curve model and finds that from July 2011 until the end of April 2013 it exceeded 7 quarters, averaging around 8.5 quarters, close to our estimates.[17] Broadly similar estimates for this period, albeit a little shorter, are also obtained from a shadow yield curve model and one augmented with macro factors by Bauer and Rudebusch (2014).

5.2 Shadow Rates

The expected duration that the zero lower bound policy is expected to be maintained is related to the shadow rate, which we define as the interest rate that the Federal Reserve would have set following its Taylor rule if it were possible. This shadow rate does not feedback into the rest of the model; the interest rate that matters for agents in the

[15] See Figure 4 in Swanson and Williams (2014). The data is top-coded at 7 quarters.

[16] In a previous version of this paper (Kulish et al. (2014)) we estimated a standard New Keynesian model and made inferences of the expected durations from output growth and inflation data alone. Those estimates suggest an average duration of 3 to 4 quarters and reveal an increase in the duration in the second quarter of 2011.

[17] Outside this period, however, Krippner's estimated expected durations are shorter. His methodology is summarized in Krippner (2015).

model is that which is constrained at zero. The shadow rate as we have defined it, gives an indication of the extent to which the contraint is binding. During this period, however, the Federal Reserve obviously not only used forward guidance, but engaged in quantitative easing, considerably increasing its balance sheet. In the Smets and Wouters (2007) model there is no direct expansionary impact of such policy; however, quantitative easing can be thought of as a means of forward guidance, along the lines suggested by Bernanke and Reinhart (2004), as it demonstrates a commitment to holding rates low for an extended period.

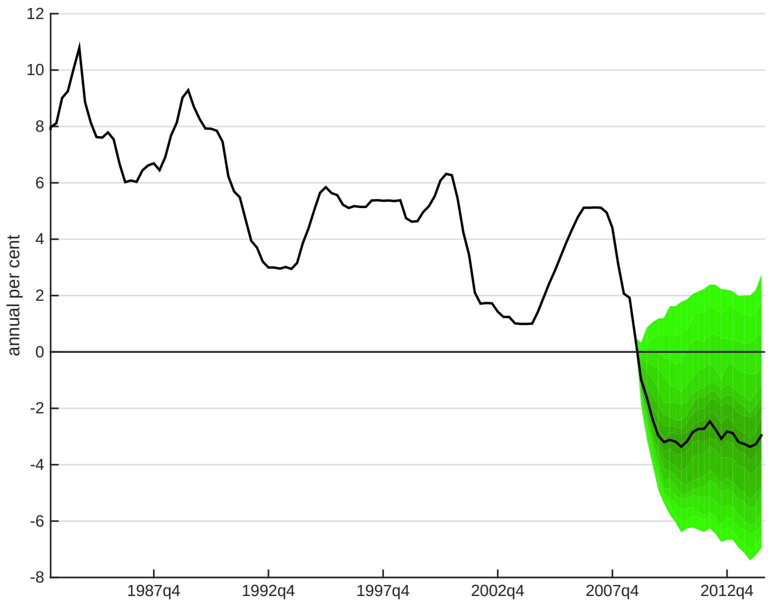
It is possible for the shadow rate to be positive during the zero interest rate regime. In this instance the Federal Reserve is pursuing more expansionary policy than it would have, on average, according to history. An increase in the expected duration of the zero interest rate policy when the shadow rate is positive can be identified as forward guidance announcements prolonging the duration of the zero interest rate regime. Alternatively, when the shadow rate is negative an increase in the expected duration of the zero lower bound policy could instead reflect the constraint binding, in a sense, to a greater extent, due to negative shocks hitting the economy. In this case, the direction of change of the shadow rate matters to interpret changes in the expected duration.

Figure 5 shows that the posterior distribution of the shadow rate from the Smets and Wouters (2007) model. The distribution reflects uncertainty stemming both from the shocks and parameters in the model.[18] The mean of the posterior becomes sharply negative with the onset of the zero interest policy, reaching around minus 3 per cent in 2010, and fluctuates broadly around that level thereafter. There is some mass given to the shadow rate being positive at any point over the entire zero lower bound period. And although the shadow rate does modestly increase in the second half of 2011, coinciding the with the shift to calendar-based guidance, we do not take this to be strong evidence in favour of calendar-based guidance because the increase is small and find no corresponding

[18] The fan ranges from the 10th quantile to the 90th quantile.

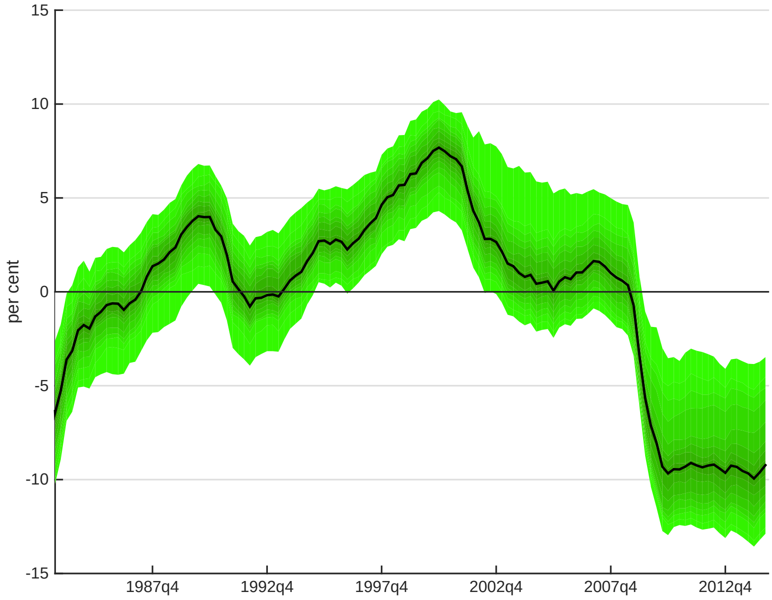
increase in the mean expected duration at that time.

Figure 5: Shadow Federal Funds Rate



It is perhaps surprising that our estimates of the shadow rate are so persistently negative over the entire zero lower bound period. To reiterate, this shadow rate represents what the Federal Reserve would have set following its Taylor rule if it were possible, given the state of the economy, that is the deviation of inflation from target and the output gap. The state of the economy is determined by the actual interest rate and not by the shadow rate. The output gap in the Smets and Wouters (2007) model is defined as the deviation of the level of output from the flex-price level, that is, that which would exist in the absence of nominal rigidities. The output gap has been persistently negative since the onset of the zero interest rate policy, and is a key variable determining the evolution of our shadow rate estimates (Figure 6). That the output gap has been persistently negative probably reflects the failure of average hours worked to recover - an observed variable in estimation - despite falls in the unemployment rate (see Figure 2).

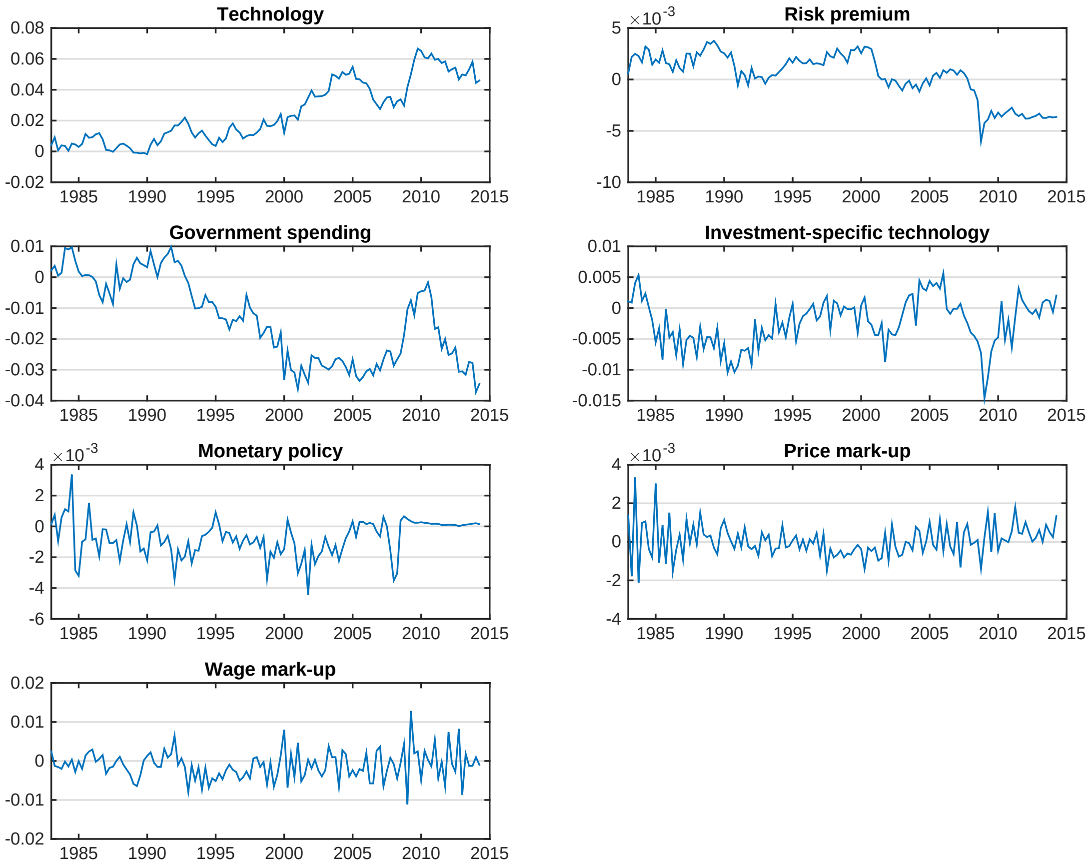
Figure 6: Output Gap From the Smets Wouters (2007) Model



Next, we look at what structural shocks explain the persistently negative shadow rate. We find that the risk-premium shock became sharply negative with the onset of the financial crisis (Figure 7).[19] The risk-premium shock drives a wedge between the Federal Funds rate and the interest rate that households receive; the negative value corresponds to this spread increasing, which lowers current consumption.[20] This shock also has the impact of lowering Tobin's Q and therefore investment. As, Smets and Wouters (2007) argue the interpretation of this shock is akin to a net worth shock in financial accelerator models (following Bernanke et al. (1999)). Essentially, it is a negative demand shock that can be thought to stem from the financial system. We find this estimate of the structural shock to be intuitively plausible given the financial crisis. It is also accompanied by a sharp decline in the investment technology shock, which has the impact of contemporaneously reducing both investment and the capital stock. Some offset to the negative impact of these shocks on output is provided by the government expenditure shock.

[19]This figure shows the posterior means of the autoregressive processes, not the innovations to them.
[20]The sign of this shock accords with Figure 4 in Smets and Wouters (2007) and their code available from the *American Economic Review* web-site (also see Appendix C), not with their equation 4.

Figure 7: Structural Shocks From the Smets Wouters (2007) Model



Several authors have constructed shadow rates for the US, although using a different methodology to that we propose here, namely a shadow yield curve model, e.g. Krippner (2011)), Wu and Xia (2014) and Bauer and Rudebusch (2014). Intuitively, this approach models the option value of holding cash when the interest rate is zero, which can be subtracted from the observed yields.[21] This method and the associated concept of the shadow rate is different from ours in many ways. First, it is intended to measure the stance of monetary policy, it is intended to capture the impact that unconventional policies have on the economy when the policy rate is constrained by the zero lower bound. A negative shadow rate in this case is an 'as-if-' measure of the monetary policy stance: a set of unconventional policies is 'as if' the Federal Reserve would have been

[21] For summaries of Krippner's methodology see Krippner (2015) or Bullard (2012).

able to set a negative interest rate. Our definition of shadow rate reflects the extent to which the constraint is binding; as emphasised above, it is the rate that would have been set according to the estimated Taylor rule which captures the Federal Reserve's pre zero lower bound behaviour. Second, our approach relies on a structural model estimated using yield curve and macroeconomic variables.[22]

The estimates presented in Figure 5 depend on the structural model. If instead of using the Smets and Wouters (2007) model we use a version of the canonical 3-equation New-Keynesian model based on Ireland (2004) and a monetary policy rule that responds to inflation and output growth, the resulting shadow rate would be *positive* for most of the sample. This is to be expected: different models yield different estimates, but the advantage of a structural model is that one can understand the differences. There also different estimates using the yield curve methodology, but these differences are harder to reconcile without the aid of a structural model. The estimates of Krippner (2013) and Wu and Xia (2014) differ, on average, by more than 1.5 percentage points and show different trends, reflecting differences in the methodology used.[23] For instance, Bauer and Rudebusch (2014) argue that

> ...model-implied shadow rates, which have been advocated as measures of the policy stance near the ZLB in some academic and policy circles (Bullard (2012); Krippner (2013); Wu and Xia (2014)), are largely uninformative, highly sensitive to model specification, and depend on the exact data at the short end of the yield curve. Their lack of robustness is striking and it raises a warning flag about using shadow short rates as a measure of monetary policy. (p.2.)

Krippner (2014b) also acknowledges the sensitivity of shadow yield curve based estimates of the shadow rate, and advocates alternatively the construction of an "economic stimulus

[22] Bauer and Rudebusch (2014) do incorporate macroeconomic factors in some of their shadow rate models, but only the unemployment gap and year-ended inflation.

[23] These were obtained from Krippner (2014a) and Federal Reserve Bank of Atlanta (2014).

measure".

Finally, it is worth noting that both our estimates of the expected durations of the zero interest rate policy and the shadow rate are influenced by the inclusion of the yield curve. In particular, if it is excluded the average of the posterior distributions of the expected durations in all quarters are considerably shorter, typically between 4.75 and 6.5 quarters over the zero interest rate period. The shadow rate remains negative, but typically is around 1 percentage point higher, and increases in a more pronounced way in 2011.[24]

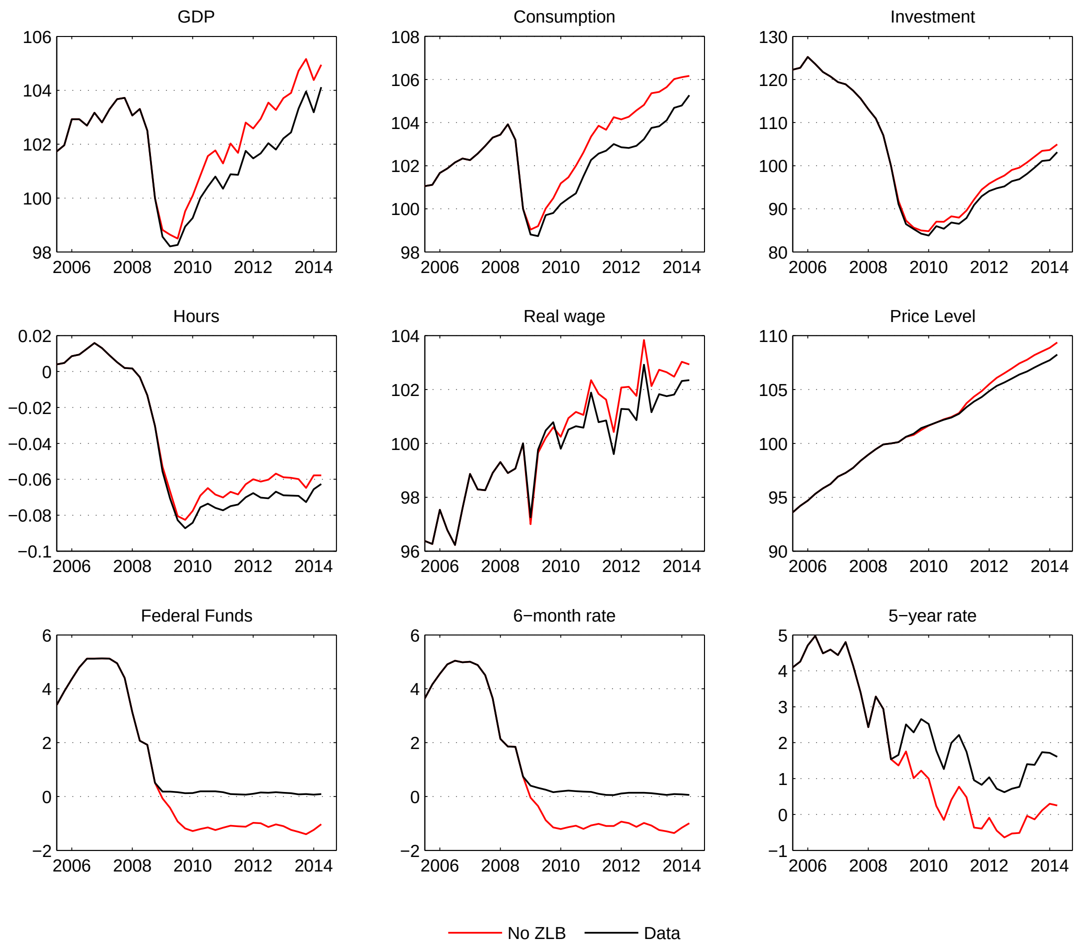
5.3 Quantifying the Severity of the Zero Lower Bound

The above analysis showed that, if possible, given the state of the economy the Federal Reserve would have set a negative interest rate of around minus 3 per cent during much of the zero interest rate period. This is a sizable difference, suggesting that the zero lower bound has been an important constraint. However, we want to evaluate more directly its importance for the macroeconomy, in particular, to answer the question, how would the economy evolved, given the estimated shocks, absent the zero lower bound?

To construct these counterfactuals we proceed as follows. We sample draws from the posterior distributions of the structural parameters and the expected durations.[25] For each draw we obtain the filtered estimates of the structural shocks using the Kalman filter. We then compute the difference between simulated time-paths for the observed variables, using these shocks, from models imposing the zero lower bound constraint and not. The average difference is displayed in Figure 8.

[24]These results are presented in Appendix B.
[25]750 draws were used.

Figure 8: Counterfactual



The differences in the growth rates of many of the variables are often small, but nevertheless imply significant and persistent differences in the evolution of the levels, which we show in Figure 8. With the exception of the interest rates, the levels are normalised to 100 in 2008q4. Our estimates suggest that, as one would expect, had monetary policy been able to set lower interest rates, output, consumption, investment, employment, real wages and the price level would have been higher and longer maturity yields lower. We find that the 22 quarters of zero interest rates imply a cumulative loss of output of 21.8 per cent of its pre-crisis level and slightly more for consumption. The effect on yields also is substantial; the 5-year rate, for example, is around 1.5 percentage points lower. The difference in inflation is modest as evidenced by the evolution of the price level. Our result that relaxing the zero lower bound has a larger impact on output than it does on inflation is consistent with the analysis of del Negro et al. (Forthcoming) who show that a version of the Smets Wouters model with financial frictions predicts a sharp contraction in economic activity along with a protracted but relatively small decline in inflation. In other words, the impact of relaxing the constraint is not independent of which shocks are driving fluctuations.

It is apparent that the negative Federal Funds rate in this counterfactual is higher than our estimates of the shadow rate in Figure 5. This is to be expected. The shadow rate above was the level according to the Taylor rule, given the output gap and inflation. But the output gap and inflation were determined by the rate set by the Federal Reserve - the zero interest rate. In the counterfactual, however, the negative interest rate in Figure 8 is the interest rate that determines inflation, the output gap, etc. - the zero lower bound constraint is not imposed.

5.4 Structural Parameters

Finally, we discuss estimates of the structural parameters. Relative to previous studies, we estimate the structural parameters jointly with the expected durations and extend the sample to include observations from the zero lower regime. Because of these two reasons, it is worth comparing the estimates of the structural parameters from those previously found. The priors used for the shock variances and the structural parameters are the same as in Smets and Wouters (2007).[26]

The estimated parameters include: φ governs investment adjustment costs; σ_c governs the intertemporal elasticity of substitution; h is the degree of external habits; σ_l the elasticity of labour supply; ξ_w and ξ_p are the Calvo parameters and ι_w and ι_p the indexation parameters for wages and goods prices; $(\phi_p - 1)$ is the mark up for goods prices, α determines the share of capital services in production; ψ governs the elasticity

[26]In Table 2 we report the parameters of the marginal priors, whereas Smets and Wouters (2007) alternatively report their mean and standard deviation.

of capacity utilisation with respect to the rental rate of capital; ρ_R and ψ_i, $i \in [1, \ldots 3]$, are parameters in the Taylor rule; γ the rate of deterministic technology growth.

The notation for the shocks is as follows: a_t technology; b_t risk premium; g_t government expenditure; μ_t investment-specific technology; ε_t^r monetary policy; λ_t^p price mark-up; λ_t^w wage mark-up. The standard deviations of the innovations to the shocks are denoted with σ; the autoregressive terms ρ; the response of government expenditure to the technology shock ρ_{ga}.[27] Finally, the constants in the measurement equations for labour, $\bar{l}$ and inflation $\bar{\pi}$, were also estimated.

Table 2 shows that the posterior estimates of the structural parameters are in line with previous findings, although there are some differences in the estimates of the shock processes. Monetary policy appears to respond less aggressively to inflation than was found by Smets and Wouters (2007); while the posterior mean of the interest rate smoothing parameter is the same, the inflation parameter, ψ_1, has fallen from 2.04 to 1.61. Monetary policy is now estimated to respond more to the level of the output gap than to its change, as the posterior mean of ψ_2 and ψ_3 are 0.17 and 0.12, whereas Smets and Wouters (2007) found the opposite (0.08 and 0.22).

The posterior mean of the elasticity of the cost of adjusting capital, φ, is 8.36, higher than the prior and the estimate by Smets and Wouters (2007) (5.74), which has the effect of reducing the responsiveness of investment to fluctuations in Tobin's Q. The cost of adjusting the level of capacity utilisation, ψ, is also greater (0.72 versus 0.54). Both prices and wages appear to be stickier, with their Calvo parameters increasing, but particularly for goods prices (0.89 versus 0.66), which are also estimated to be less forward looking, as the posterior mean of the indexation parameter has increased from 0.24 to to 0.52.

[27] ρ_m denotes the autocorrelation in the monetary policy shock.

Table 2: Posterior Estimates of the Smets Wouters (2007) Model

Parameter	Posterior Mode	Posterior Mean	90 per cent C. I.		Prior Distribution	Prior Mean
Standard Deviations of the Innovations to the Shocks†						
σ_a	0.65	0.69	0.60	0.79	Inv. Gamma (2.0025,0.10025)	0.1
σ_b	0.23	0.31	0.21	0.47	Inv. Gamma (2.0025,0.10025)	0.1
σ_g	0.64	0.66	0.58	0.79	Inv. Gamma (2.0025,0.10025)	0.1
σ_q	0.54	0.57	0.43	0.71	Inv. Gamma (2.0025,0.10025)	0.1
σ_m	0.34	0.39	0.31	0.53	Inv. Gamma (2.0025,0.10025)	0.1
σ^p	0.30	0.32	0.24	0.40	Inv. Gamma (2.0025,0.10025)	0.1
σ^w	0.65	0.69	0.56	0.83	Inv. Gamma (2.0025,0.10025)	0.1
σ_η	0.21	0.23	0.18	0.31	Inv. Gamma (13,1)	0.083
σ_2	0.19	0.22	0.16	0.32	Inv. Gamma (13,1)	0.083
σ_4	0.16	0.17	0.13	0.21	Inv. Gamma (13,1)	0.083
σ_8	0.23	0.26	0.19	0.36	Inv. Gamma (13,1)	0.083
σ_{20}	0.53	0.62	0.45	0.91	Inv. Gamma (13,1)	0.083
Parameters of the Shock Processes						
ρ_a	0.99	0.95	0.82	1.00	Beta (2.625, 2.625)	0.5
ρ_b	0.95	0.81	0.38	0.97	Beta (2.625, 2.625)	0.5
ρ_g	0.98	0.93	0.78	0.99	Beta (2.625, 2.625)	0.5
ρ_q	0.77	0.73	0.48	0.94	Beta (2.625, 2.625)	0.5
ρ_m	0.53	0.58	0.38	0.84	Beta (2.625, 2.625)	0.5
ρ_p	0.88	0.52	0.07	0.93	Beta (2.625, 2.625)	0.5
ρ_w	0.65	0.54	0.13	0.96	Beta (2.625, 2.625)	0.5
μ_p	0.76	0.55	0.12	0.92	Beta (2.625, 2.625)	0.5
μ_w	0.84	0.54	0.12	0.92	Beta (2.625, 2.625)	0.5
ρ_{ga}	0.43	0.41	0.04	0.77	Normal (0.5, 0.25)	0.5
Structural Parameters						
φ	7.14	8.36	4.61	13.42	Normal (4, 1.5)	4
σ_c	0.90	1.22	0.71	2.19	Normal (1.5, 0.375)	1.5
h	0.67	0.65	0.44	0.84	Beta (14, 6)	0.7
ξ_w	0.88	0.80	0.43	0.94	Beta (12, 12)	0.5
σ_l	2.14	2.12	0.04	4.54	Normal (2, 0.75)	2
ξ_p	0.92	0.89	0.74	0.96	Beta (12, 12)	0.5
ι_w	0.40	0.48	0.13	0.86	Beta (5.05556, 5.05556)	0.5
ι_p	0.80	0.52	0.09	0.92	Beta (5.05556, 5.05556)	0.5
ψ	0.83	0.72	0.37	0.94	Beta (5.05556, 5.05556)	0.5
ϕ_p	1.42	1.49	1.21	1.80	Normal (1.25, 0.125)	1.25
ψ_1	1.49	1.61	0.98	2.42	Normal (1.5, 0.25)	1.5
ρ_r	0.84	0.82	0.70	0.92	Beta (13.3125, 4.4375)	0.75
ψ_2	0.15	0.17	-0.002	0.33	Normal (0.125, 0.05)	0.125
ψ_3	0.10	0.12	-0.009	0.30	Normal (0.125, 0.05)	0.125
$\bar{\pi}$	0.58	0.61	0.35	0.88	Gamma (39.0625, 0.016)	0.625
$\beta^{-1}-1$	0.13	0.24	0.05	0.60	Gamma (6.25, 0.04)	0.25
$\bar{l}$	1.10	1.59	-2.49	5.91	Normal (0, 2)	0
$\gamma-1$	0.47	0.38	0.16	0.51	Normal (0.4, 0.1)	0.4
α	0.15	0.16	0.10	0.25	Normal (0.3, 0.05)	0.3
c_2	-0.32	-0.36	-2.03	1.00	Normal (0.1, 1)	0.1
c_4	0.11	0.03	-1.62	1.35	Normal (0.15, 1)	0.15
c_8	0.88	0.90	-0.77	2.24	Normal (0.2, 1)	0.2
c_{20}	2.34	2.23	0.50	3.68	Normal (0.5, 1)	0.5
ρ_s	0.82	0.80	0.59	0.96	Beta (5, 2)	0.71

† Moments multiplied by 10^{-2}.

Turning to the shocks, we find difference in persistence. In particular, the risk premium has become more persistent (the posterior mean of ρ_b is 0.81, compared with 0.22 found by Smets and Wouters (2007)), as have monetary policy shocks. Alternatively, the estimated persistence of both the price and wage mark-up shocks has fallen (the posterior mean of ρ_p is 0.52, whereas Smets and Wouters (2007) found 0.89). We also find larger posterior distributions of the standard deviations of all shocks, with the largest change for the wages mark-up shock. The standard deviations of both the monetary policy and technology shocks have also risen.

6 Conclusions

The Great Recession has had important ramifications for the implementation of monetary policy. Facing the zero lower bound on nominal interest rates, many central banks, including the Federal Reserve, have engaged in other policies such as forward guidance in an attempt to stimulate the economy. The contribution of this paper is to suggest a way in which DSGE models can be estimated for an economy in which the central bank holds the interest rate fixed for an extended period and conducts forward guidance.

Our approach builds on Kulish and Pagan (2012), which developed a method for solving DSGE models with structural change that yields a solution as a time-varying VAR process, a form which is convenient for estimation. The solution of Kulish and Pagan (2012) also accommodates private agents' expectations about structural changes to differ from a policymaker's announcement. In this paper, we cast this approach in a Bayesian framework and estimate the expected duration of the zero interest rate policy and the structural parameters of the model jointly.

We apply our approach to the workhorse Smets and Wouters (2007) model of the United States economy augmented with a yield curve, drawing upon Graeve et al. (2009). Our estimates suggests that the expected duration of the zero interest rate policy typically has been around 8 quarters. This is broadly in line with yield-curve model based estimates over the second half of the zero interest rate sample, but are longer in the early period. We find that the posterior distributions of the expected durations at all points in time place considerable mass on long durations, which was also found by Bauer and Rudebusch (2014). The posterior distributions of the expected durations shift over time, but not in a manner that can be closely related to changes in the conduct of forward guidance.

The shadow rate we propose is the rate the Federal Reserve would set if negative interest rates were possible, given the estimated policy rule and the state of the economy. The posterior mean of the shadow rate is negative throughout the zero interest rate period, fluctuating around -3 per cent. This suggests that the zero lower bound has significantly constrained monetary policy in the United States. Using the structural shocks we estimate a cumulative loss of output stemming from the zero lower bound on interest rates of 22 per cent of the pre-crisis level of GDP.

In summary, we propose a method of estimating DSGE models when the policy rate is fixed (possibly at zero) during part of the sample. Our method takes the zero lower bound explicitly into account, allows inferences to be made about the expected duration that such a policy will be maintained.

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