Endogenous Labor Force Participation, Involuntary Unemployment and Monetary Policy

Yuelin Liu

This paper can be downloaded without charge from The Social Science Research Network Electronic Paper Collection:
http://ssrn.com/abstract=2546625
Endogenous Labor Force Participation, Involuntary Unemployment and Monetary Policy

Yuelin Liu*
University of New South Wales
December 17, 2014

Abstract
This paper develops a New Keynesian model with search frictions in which generated frictional unemployment is consistent with the time series of involuntary unemployment collected by the U.S. Bureau of Labor Statistics. Thus, it can shed light on the relevant impact of labor market frictions and policy interventions on the observed unemployment about which policy makers and the public are concerned. The data-consistent unemployment is achieved in the model via introduction of partial consumption insurance and an endogenous labor force participation channel. In particular, I find that allowing for endogenous labor force participation greatly improves the model fit for U.S. data. It appears that the price markup shock and matching efficiency shock are the two key driving forces of unemployment fluctuations. Monetary policy that stabilizes the participation gap can be welfare improving.

JEL Classification: C11, E24, E31, E32.

Keywords: New Keynesian DSGE, Involuntary unemployment, Endogenous labor force participation, Search and matching, Bayesian inference.

*Email address: yuelin.liu@unsw.edu.au. I thank Jamie Hall, Mariano Kulish, Gary Hansen, Yunjong Eo, Aarti Singh, Punnoose Jacob, Benjamin Wong, Gunes Kamber, Fang Yao, Chengsi Wang and my discussant Alexandre Dmitriev for helpful suggestions or comments and especially my advisor James Morley for his guidance, encouragement and constant support in various ways. I also benefit from conference and seminar participants at the Inter-University (UNSW-USYD) Postgraduate Research Seminar Series, 2014 Asian Meeting of the Econometric Society, 2014 China Meeting of Econometric Society, the Reserve Bank of New Zealand and the 27th PhD Conference in Economics and Business. All remaining errors are my own.
1 Introduction

As one of the key measures of economic conditions and social welfare, the time series of unemployment (rate) collected by government agencies is at the center of policy discussions. In addition, the private sector manages business activities by assessing implications from the observed economy-wide unemployment as well. To understand the impact of policy interventions on unemployment fluctuations and how its performance interacts with other macro aggregates, a theoretical model used for structural analysis must inherently generate a concept of unemployment that is consistent with the unemployment data to which agents in the economy actually respond. Developing a monetary dynamic stochastic general equilibrium (DSGE) model, which fulfills the aforementioned rationale, with an explicitly characterized labor market is my aim in this paper.

The Diamond-Mortensen-Pissarides (DMP) framework, see Diamond (1982) and Mortensen and Pissarides (1994), with search and matching frictions is the prevalent approach to embedding unemployment into monetary DSGE models. Frictional unemployment generated from this framework is an equilibrium outcome. Examples include Blanchard and Galí (2010), Gertler et al. (2008), Krause et al. (2008) and Christoffel et al. (2009), among others. Nevertheless, as full consumption insurance within the representative family and negative utility from work are typically assumed in this class of models, it implies that unemployment derived from these models is in fact voluntary. Under full consumption insurance, the employed households are allocated with the same amount of final consumption goods as the unemployed households. But the employed households are subject to disutility from work while the unemployed households can enjoy leisure. Therefore, within this class of models, the unemployed households enjoy a higher level of utility than the employed households. The unemployed households will voluntarily stay unemployed based on utility criteria. This feature indicates that labor force participation in a model setup of this sort must be collectively decided by the representative family, or there will not be any labor force participants from individual household’s perspective. More importantly, it contradicts the concept of unemployment defined by the U.S. Bureau of Labor Statistics (BLS) who collects the labor market data. As per the official definition of (involuntary) unemployment, to be defined as an unemployed, a person who currently does not have a job must prefer to work when offered one. This is clearly not the case in the above models. Thus, it is questionable whether implications with respect to unemployment fluctuations derived from this class of models are relevant to what we can observe from the data.

This paper still characterizes a frictional labor market explicitly under the search and matching paradigm. However, in contrast to the standard New Keynesian Diamond-Mortensen-
Pissarides (NK-DMP) model with voluntary unemployment à la Blanchard and Galí (2010), I introduce data-consistent involuntary unemployment into the monetary model in this paper following in the spirit of Christiano et al. (2012) via partial consumption insurance and an endogenous labor force participation channel within the representative family. To the best of my knowledge, this paper is the first in the literature to introduce data-consistent unemployment into the NK-DMP framework. In particular, the employed households are allocated with a higher level of final consumption goods compared to those households which are unemployed or non-participants. Thus, absent the collective labor force participation decision by the family, individual households are incentivized to participate in the labor force by paying a common participation cost.¹ In sum, the unemployed would have been better off if they had been matched with a job and they are willing to work once offered a job. This feature of the model is attractive in that the concept of unemployment generated from the model is identical to the time series of the unemployment rate that the policy makers actually target and, therefore, can shed light on the relevant impact of policy interventions on unemployment.

The search and matching framework is a conventional and useful approach to modeling frictional labor market. It aggregates job vacancies posted by firms and job seekers from families in the labor market, so the market-wide transition rates between labor status are corrected for externality generated from an additional job seeker or job vacancy on the market-wide job-finding or job-filling rates. Ignoring impact of this inherent externality in the labor market can be very misleading. Another implication from frictional labor market is the existence of a shared surplus derived from an established job-worker pair, which is supposed to be split between the family and the firm conditional on their relative bargaining power. That is to say, the wage rate under a frictional labor market should be given by the bargaining solution. In addition, the standard matching function used in the DMP framework leaves room for structural unemployment which is driven by exogenous shocks to the matching technology that shifts the Beveridge curve. It enables me to disentangle the extent to which variation in unemployment is structural or cyclical. However, to derive the aggregate utility function at the family level from individual household’s welfare, I have to abandon the standard random matching in the DMP framework and impose simplifying assumptions for the sake of tractability to track down the history of labor status of each household. Then the model is closed with a standard production sector à la Blanchard and Galí (2010), market clearing conditions, a Taylor-type interest rule and six exogenous shocks.

¹The participation cost is referred to as search effort to the unemployed and work effort to the employed, respectively. A search cost is required for the job seekers to be officially defined as an unemployed, as explained in Christiano et al. (2012). In addition, this participation cost excludes uninteresting trivial case of full participation.
Bayesian inferences based on U.S. data between 1985Q1 and 2008Q4 deliver three main findings. First, the model in this paper fits the data very well. Specifically, it greatly improves the model fit compared to the standard monetary model _à la_ Blanchard and Galí (2010) with unitary labor force participation and full consumption insurance. This is not surprising given the rich dynamics of the labor force participation rate in the sample period. Second, impulse response analysis and forecast error variance decomposition suggest that price markup and matching efficiency shocks are the two key driving forces of unemployment fluctuations. This is consistent with the finding by Krause et al. (2008) in which the matching efficiency shock explains 37 percent of the variation in unemployment. My study also shows that the matching efficiency shock that captures variation in structural factors accounts for 86% of the variation in the natural rate of unemployment (NRU). Third, I show that stabilizing the participation gap, which is the percent deviation of labor force participation from its natural level, in a Taylor-type rule can be welfare improving because it stabilizes inflation more efficiently than a simple rule that only responds to inflation and output gap. This channel is unavailable in a large branch of monetary DSGE literature from which labor force participation margin is abstracted.

The rest of this paper proceeds as follows. Section 2 discusses the related literature. Section 3 describes the structure of the economy and lays out the model. Section 4 presents the estimation results. Model fit is discussed in Section 5. Section 6 investigates the driving forces of unemployment fluctuations. Section 7 discusses implications of labor force participation margin for monetary policy design. Section 8 concludes.

## 2 Related Literature

This paper contributes to the literature by introducing data-consistent (involuntary) unemployment, following in the spirit of Christiano et al. (2012), into an otherwise standard New Keynesian model with search frictions in which the labor market is explicitly modeled and showing that allowing for endogenous labor force participation greatly improves the model fit and has important implications for monetary policy design.

The concept of (involuntary) unemployment developed in this paper is different from (voluntary) unemployment generated in standard NK-DMP models, see Blanchard and Galí (2010), Gertler et al. (2008), Krause et al. (2008), Christoffel et al. (2009) and Campolmi and Gnocchi (2014), among others. In standard NK-DMP models, a combination of assumptions of full consumption insurance and disutility from work within the representative family
leads to a conclusion that an unemployed household prefers to stay unemployed based on individual household’s preferences, which contradicts the official unemployment data collection that policy authorities actually target. Thus, policy implications from standard NK-DMP models are ambiguous, whereas the model in this paper is able to assess relevant impact of policy interventions on the labor market and, in particular, unemployment.

My approach differs from Christiano et al. (2012), which develops a welfare-based concept of involuntary unemployment that this paper builds on, in three important ways. First, I model the labor market explicitly and take into account the externality of an extra job or an additional worker, whereas Christiano et al. (2012) model the job-finding rate with a simple linear rule increasing in search efforts, thus ignoring the externality of search effort of one worker on another’s job-finding probability. Second, with uncertainty in the job searching process, an established job-worker pair generates a surplus shared by the worker and the firm. Thus, it is more appropriate to bargain over the wage rate bilaterally. However, Christiano et al. (2012) derive the wage rate à la Calvo-pricing in their medium-scale DSGE model because of monopoly power of the family. Instead, the real wage in this paper is subject to the efficient Nash bargaining and real wage rigidity. Lastly, as a Cobb-Douglas matching function is included in this paper, it enables me to investigate the extent to which unemployment is structural by looking into the contribution of matching efficiency shock that is a measure of overall variation in structural factors. The framework of Christiano et al. (2012) is silent on this topic.

In terms of the literature on generating involuntary unemployment in a New Keynesian framework, Galí et al. (2011) is another example. The unemployment in that framework results from monopoly power of the union. The union sets the wage rate above the market clearing wage relying on its monopoly power leading to excess supply of labor. The generated unemployment is involuntary in the sense that the unemployed are willing to participate in the labor force to search for a job conditional on the wage rate is higher than their marginal rate of substitution.

This paper also relates to the strand of literature on studying the tie between endogenous labor force participation and labor market variables or monetary policy design, see Ebell (2008), Krusell et al. (2012), Campolmi and Gnocchi (2014) and Erceg and Levin (2013), among others. For example, Krusell et al. (2012) find that an active labor supply channel is operative in explaining the labor market data. In addition, evidence from Erceg and Levin (2013) suggests that the labor force participation margin can be crucial to monetary policy design, especially in deep recessions. Campolmi and Gnocchi (2014) also show that evaluation of alternative policies may be misleading if labor force participation is treated constant. However, again the concept of unemployment involved in their models is not
consistent with the official data collection.

3 Model

The economy comprises three types of agents: families, wholesale firms and retail firms. The economy is populated with a large number of identical families in which households band together to insure themselves against idiosyncratic labor market outcomes. The representative family consumes, provides labor to wholesale firms, and saves by purchasing bonds. The wholesale firms hire households from the labor market and use labor as the only input to produce a homogeneous intermediate good under perfect competition. The retail firms purchase the intermediate good from wholesale firms at the marginal cost, transform them into differentiated final consumption goods and sell them to the families with a markup over the marginal cost under monopolistic competition.

In particular, the labor market is subject to search frictions à la Diamond (1982) and Mortensen and Pissarides (1994) and frictional unemployment exists as an equilibrium outcome. Each household is allowed to make its own labor force participation decision based on its (limited) information set. They can be either employed, unemployed or non-participating in each period of time. Thus, in order to incentivize labor force participation, the representative family allocates a larger bundle of final consumption goods to the employed and a smaller bundle to households in the other two categories of labor status. In this way, labor force participation is endogenized and the unemployment generated from this model is involuntary in terms of welfare comparisons.

3.1 Families

The representative family consists of a continuum of households indexed by \( j \in [0, 1] \). The \( j \)th household bears a disutility of \( \chi_t j^\varphi \ (\chi_t > 0, \varphi > 0) \) from work when it is employed and zero when it is not employed.\(^2\) Each household only knows its own degree of work aversion and can not verify that of any others. Instead, the representative family has full information about households’ work aversion. Each household either provides one unit of labor or none at all (indivisible labor). This assumption reflects the stylized fact that total hours worked over the business cycle mostly vary in the number of employed workers (extensive margin) rather than hours per capita (intensive margin). Departing from the large literature of full consumption insurance within the representative family since Merz (1995) and Andolfatto (1996), I only pursue partial insurance for consumption in this model so as to make unemployed households

\(^2\chi_t\) is an exogenous labor supply shock, or it can be interpreted as a preference shock.
in the labor force better off when they find a job. Therefore, the unemployment defined in this paper is involuntary, as discussed in Christiano et al. (2012). All earnings from wages, profits, dividends, etc. are pooled together. Then, the employed households are allocated with $C^h_t$ of a bundle of final (differentiated) consumption goods, while a non-employed (unemployed or non-participating) household only receives a bundle of final consumption goods $C^l_t$, where $C^h_t > C^l_t$ and $C^h_t/C^l_t$ is referred to as the consumption premium. Thus, households are incentivized to participate in the labor force. Meanwhile, as long as a household is in the labor force, no matter whether they are employed or not, it has to pay a common cost $e(F_t) \equiv \tau F_t^\Gamma$, where $\tau > 0$, $\Gamma \in (0,1)$ and $F_t$ is the job-finding rate that is endogenously determined within the search and matching process in the labor market.\(^3\)

**Labor Flows and Timing** Each household can be labeled with a unique labor status: employed, unemployed, or non-participant. Let $L_t$ and $N_t$ be the labor force and the pool of employed workers, respectively, in time period $t$. At the beginning of each time period, a constant fraction $\delta$, $\delta \in (0, 1)$, of the established job-worker pairs in the previous time period are discontinued. Searching workers, denoted by $U_t$ in time period $t$ including the newly separated workers, continuously unemployed workers since the last period, and new entrants to the labor force, are allowed to receive a job with a probability $F_t$ at the beginning of each time period after separation. When a searching worker is matched with a job, it becomes productive instantaneously in that period. Also, the aggregate new hires $H_t$ is given by

$$H_t = F_t U_t = F_t [L_t - (1 - \delta) N_{t-1}].$$

The level of unemployment $U_t$ in period $t$ is given by

$$U_t = L_t - N_t.$$

Likewise, the unemployment rate $u_t$ is defined by

$$u_t = \frac{U_t}{L_t} = 1 - \frac{N_t}{L_t}. \quad (1)$$

\(^3\)Recall the definition of unemployment by the BLS, the unemployed must somehow pay a cost to search for jobs that is reflected by $e(F_t)$ which is a concave function increasing in the job-finding rate implying a procyclical search intensity as discussed in Christiano et al. (2012). For employed households, I also impose the same cost that can be interpreted as procyclical working efforts, which is consistent with procyclical hours per worker during 1984-2007 as documented by Galí and van Rens (2010). $\Gamma$ restricted in $(0,1)$ is crucial in generating a procyclical labor supply. I will discuss this in more detail in the subsection on labor force participation.
The aggregate employment \( N_t \) evolves as

\[
N_t = (1 - \delta)(1 - F_t)N_{t-1} + F_tL_t. \tag{2}
\]

**Labor Force Participation** The labor force participation decision is made individually by each household, which is in contrast to a large literature in which labor force participation decisions are collectively decided by the family, for example, see Merz (1995) and Galí et al. (2011). Suppose at the beginning of time period \( t \), after exogenous separations have concluded, the representative family announces the job-finding rate \( F_t \), high consumption bundle \( C^h_t \), and low consumption bundle \( C^l_t \). Thus, each household’s information set at time \( t \) includes the above public information released by the family, its own work aversion \( \chi_tj^\varphi \), and the participation cost \( e(F_t) \). Thus, individual household’s decision for labor force participation is intratemporal.\(^4\) The unemployed or non-participating \( j^{th} \) household makes its participation decision by comparing the expected utility gain from participation in period \( t \) of

\[
F_t \left[ \log C^h_t - \chi_tj^\varphi - e(F_t) \right] + (1 - F_t) \left[ \log C^l_t - e(F_t) \right]
\]

with the deterministic utility gain in the case of non-participation of

\[
\log C^l_t.
\]

The unemployed or non-participating \( j^{th} \) household will participate in the labor force if

\[
F_t \left[ \log C^h_t - \chi_tj^\varphi - e(F_t) \right] + (1 - F_t) \left[ \log C^l_t - e(F_t) \right] \geq \log C^l_t.
\]

i.e.

\[
\log \frac{C^h_t}{C^l_t} \geq \bar{F}_t^{-1} \Gamma - 1 + \chi_tj^\varphi.
\]

As to continuously employed households surviving from the job separation, they naturally keep their jobs and stay in the labor force.

For the sake of feasibility of tracking down the history of each household’s labor status, which is indispensable for derivation of the family aggregate utility function, I assume

\(^4\)One may prefer to allow households to look far into the future, as under the search and matching framework, the current labor status has an impact on its future path. However, intertemporal labor force participation decision will substantially complicate the algebra without significantly sharpening the economic intuition behind the interactions among the variation in job-finding rate \( F_t \), consumption premium \( C^h_t/C^l_t \) and labor force participation \( L_t \). More importantly, without the simplifying assumption of intratemporal labor force participation decision, there is no a simple rule to generate procyclical labor supply as in Equation (3).
1) given the aggregate labor market conditions and full information about households’ work aversion, the representative family collects established job offers from the labor market and takes charge of the job distribution and separation (a constant fraction of matched job-worker pairs) within the family;

2) in each period $t$, the representative family assigns job offers to $N_t$ households with the lowest work aversion;

3) at the beginning of period $t$, a constant fraction $\delta$ of the employment pool $N_{t-1}$ in period $t-1$ with the highest work aversion are separated;

4) $\log C_t^h - \chi_t[(1 - \delta)N_{t-1}]^\omega - \frac{e(F_t)}{F_t} \geq \log C_t^l$.

Assumption 1) builds the foundation of Assumptions 2) and 3). Assumption 2) implies that households indexed by $j \in [0, N_t]$ are employed in period $t$, and Assumption 3) implies that households indexed by $j \in [0, (1 - \delta)N_{t-1}]$ survives the job separation at the beginning of period $t$. Assumptions 1), 2) and 3) are for the sake of two convenient thresholds in index $j$ which determine scales of the labor force and employment in order to help track down the history of labor status of each household. Under the standard search and matching framework, workers and vacancies are randomly matched and discontinued. Nevertheless, households are heterogeneous in work aversion in this paper, the random matching and separation mechanism makes it impossible to track down the history of each household’s labor status via which corresponding work aversion of the employed households are included in the aggregate utility function at the family level. Thus, with random matching and separation, it is intractable to derive the aggregate utility function at the family level from individual household’s welfare. Instead, Assumptions 1), 2) and 3) are not only efficient in that they minimize the disutility from work at the family level given any level of employment, but they also help to uncover the history of labor status of each household. Assumptions 1), 2) and 3) generate two cut-offs in index $j$ that conveniently deliver the size of labor force and the level of employment, respectively. Therefore, given the labor force $L_t$ and employment $N_t$, the labor status of each household can be identified by its index $j$. That is to say, households $j \in [0, N_t]$ are employed, $j \in (N_t, L_t]$ are unemployed, and $j \in (L_t, 1]$ are non-participants in period $t$. Moreover, Assumption 4) excludes the ill-defined case that $L_t < (1 - \delta)N_{t-1}$.

In sum, the labor force participation mechanism is summarized through (I) - (III):
(I). the size of labor force $L_t$ in period $t$ is determined via:\(^5\)

$$
\log \frac{C^h_t}{C^d_t} = e^{F_t^{\Gamma-1} + \chi_t L_t^\varphi};
$$

(II). households indexed by $j \in [0, L_t]$ represent the labor force;

(III). households indexed by $j \in [0, N_t]$ represent the employment pool.

**Representative Family’s Problem** Denote the log of consumption premium $\log \frac{C^h_t}{C^d_t}$ in Equation (3) as a function of job-finding rate $F_t$ and labor force $L_t$:

$$
g(F_t, L_t) = e^{F_t^{\Gamma-1} + \chi_t L_t^\varphi}.
$$

Recall the resource constraint in each period:

$$
N_t C^h_t + (1 - N_t) C^d_t = C_t,
$$

where $C_t \equiv \left( \int_0^1 C_t(i) \frac{\epsilon_{pt}^{-1}}{\epsilon_{pt}^{\epsilon_{pt}^{-1}}} di \right)^{\epsilon_{pt}^{-1}}$ defines the aggregate consumption, $C_t(i)$ is the demand for final good $i$ and $\epsilon_{pt}$ is the stochastic elasticity of substitution between final goods which can be interpreted as the exogenous price markup shock. Combined with Equation (3), we have

$$
C^h_t = \frac{1}{N_t (e^{g(F_t, L_t)} - 1) + 1} C_t
$$

and

$$
C^d_t = \frac{1}{N_t (e^{g(F_t, L_t)} - 1) + 1} C_t.
$$

---

\(^5\)The restriction $\Gamma \in (0, 1)$ is consistent with the empirically procyclical labor supply. As studied in Christiano et al. (2012), the consumption premium is procyclical. The job-finding rate is procyclical as well, see Hall (2006). Hence, if $\Gamma$ is restricted in $(0, 1)$, then the model in this paper will generate a procyclical labor supply. For example, following an expansionary monetary policy shock, both the consumption premium and the job-finding rate increase. That is, in Equation (3), the left hand side increases and the first term of the right hand side declines instead. Given an exogenous labor supply shock $\chi_t$, the labor supply increases which proves to be procyclical.
It follows that the period aggregate utility function at the family level can be written as

\[
U(C_t, N_t) = \int_0^{N_t} \left[ \log C^h_t - \chi_t j^\varphi - e(F_t) \right] dj + \int_{N_t}^{L_t} \left[ \log C^d_t - e(F_t) \right] dj + \int_0^{L_t} \log C^l_t dj
\]

utility from the employed

utility from the unemployed

utility from non-participants

\[= \log C_t + h(F_t, N_t, \chi_t),\]

where

\[
h(F_t, N_t, \chi_t) = N_t g(F_t, L_t) - \log \left[ N_t \left( e^{g(F_t, L_t)} - 1 \right) + 1 \right] - L_t e(F_t) - \chi_t \frac{N_t^{1+\varphi}}{1+\varphi},
\]

and \(L_t\) is a function of \(N_{t-1}\), \(N_t\) and \(F_t\) according to Equation (2).

Thus, the aggregate utility at the family level is a reduced form objective function which is consistent with individual household’s preferences. By varying \(C^h_t\) and \(C^d_t\), the representative family is able to choose the aggregate consumption \(C_t\) and employment \(N_t\). In addition, because consumption is heterogeneous among households, their saving behaviour may be different if individual households can access the bond market. For simplicity, I assume the representative family chooses savings on behalf of households in order to maximize the aggregate utility at the family level.

Note also the optimal demand for final good \(i\) is given by

\[
C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\epsilon_{pi}} C_t,
\]

where \(P_t \equiv \left( \int_0^1 P_t(i)^{1-\epsilon_{pi}} di \right)^{1-\frac{1}{\epsilon_{pi}}} \) denotes the aggregate price. The above optimal demand equation implies that the consumption expenditure holds with an identity

\[
\int_0^1 P_t(i)C_t(i)di = P_tC_t.
\]

The representative family treats the job-finding rate \(F_t\) and real wage \(W_t\) as given, and solves the optimization problem:

\[Equation (2) tells us, given the job-finding rate, \(L_t\) and \(N_t\) are chosen simultaneously.\]
\[
\max \{ C_t, N_t, B_t \} \left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}
\]

s.t.
\[
U(C_t, N_t) = \log C_t + h(F_t, N_t, \chi_t);
\]
\[
P_tC_t + \frac{B_t}{R_t \epsilon_{bt}} \leq P_t W_t N_t + B_{t-1} + \Lambda_t;
\]
\[
N_t = (1 - \delta)(1 - F_t)N_{t-1} + F_t L_t.
\]

Here \( \beta \in (0, 1) \) is the discount factor, \( R_t \) gross interest rate, \( B_t \) quantity of one-period bonds, \( \epsilon_{bt} \) risk-premium shock, \( W_t \) real wage and \( \Lambda_t \) the profits and lump-sum transfers.\(^7\) The inclusion of the risk-premium shock is to implicitly capture the non-modeled financial sector.

The FOCs are derived as follows:
\[
1 = \beta \mathbb{E}_t \left\{ \frac{\tilde{C}_t}{1 - F_t} \frac{R_t \epsilon_{bt}}{\Pi_{t+1} a_{t+1}} \right\}, \tag{5}
\]

where \( \Pi_t \equiv \frac{P_t}{P_{t-1}} \) is the gross inflation rate, \( \tilde{C}_t \equiv \frac{C_t}{A_t} \) is the stationarized consumption\(^8\) and
\[
\frac{1}{F_t} O_t = \tilde{W}_t - J_t + \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\tilde{C}_t}{C_{t+1}} \frac{1 - F_{t+1}}{F_{t+1}} O_{t+1} \right\}, \tag{6}
\]

where
\[
\tilde{W}_t \equiv \frac{W_t}{A_t},
\]
\[
O_t \equiv -\tilde{C}_t \left[ \phi N_t \chi_t L_t^{\phi-1} - \bar{e} F_t^\Gamma \right] - \frac{\phi N_t \chi_t e^{g(F_t, L_t)} L_t^{\phi-1}}{N_t (e^{g(F_t, L_t)} - 1) + 1},
\]
\[
J_t \equiv -\tilde{C}_t \left[ g(F_t, L_t) - \chi_t N_t^{\phi} - \frac{e^{g(F_t, L_t)} - 1}{N_t (e^{g(F_t, L_t)} - 1) + 1} \right].
\]

Equation (5) is the conventional intertemporal Euler equation. Equation (6) can be interpreted as the optimal labor force participation rule. \( O_t \) is the stationarized cost of transferring a marginal non-participant into the unemployment pool in terms of consumption goods.

---

\(^7\)In principle, the wage rates may be dispersed among wholesale firms. But the firms are facing an identical optimization problem of hiring and wage setting, therefore, the wage rate (in terms of a bundle of final consumption goods) and employment at each wholesale firm are the same. Thus, the firm-specific subscripts are omitted and the overall real wages earned at the family level is given by \( W_t N_t \).

\(^8\)Consumption is stationarized to be consistent with the balanced growth path and to guarantee the existence of steady states.
Likewise, $J_t$ denotes the stationarized cost of allocating a marginal searching worker to the employment pool. Hence, corrected for the search and matching frictions, the labor force participation rule implies that the cost of marginal participation equals the sum of current net payoffs $\tilde{W}_t - J_t$ and discounted expected continuation values in the last term. Note that under the assumptions of full consumption insurance, no matching frictions ($F_t = 1$) and no participation costs needed ($e(F_t) = 0$), Equation (6) collapses to the well-known labor supply curve presented in a standard New Keynesian model with a Walrasian labor market.

### 3.2 Search and Matching Process

The matching technology is characterized by

$$H_t = m_t U^\zeta_t V_t^{1-\zeta}, \quad (7)$$

where $U_t$ is the pool of searching workers, $V_t$ is the amount of vacancies, $\zeta \in (0, 1)$ is the elasticity to unemployment and $m_t$ is the stochastic exogenous matching efficiency at the beginning of period $t$. Within the time period $t$, the representative family is offered $H_t$ new labor contracts from the labor market and understands $H_t$ searching workers with lowest work aversion will move into the employment pool. The job-finding rate is given by

$$F_t = \frac{H_t}{U_t}. \quad (8)$$

Likewise, a vacancy can be filled with a probability

$$Q_t = \frac{H_t}{V_t}. \quad (9)$$

The labor market tightness $x_t$ is defined as

$$x_t = \frac{V_t}{U_t}. \quad (9)$$

### 3.3 Wholesale Firms

The homogenous intermediate good is produced by a continuum of identical and perfectly competitive wholesale firms with unit mass, indexed by $k \in [0, 1]$. The intermediate good

---

9In this model, as jobs are distributed to households with lowest work aversion instead of random matching, from the family’s point of view, the job-finding rate is interpreted as the fraction of new hires in the searching pool. As an individual job-seeking household, it treats the job-finding rate as the probability that it can find a job.
firm $k$ employs labor as the only input and produces $Y_t^I(k)$ intermediate good following the production function:

$$Y_t^I(k) = A_t N_t(k),$$

where $A_t$ is the technology applied to all intermediate good firms and $N_t(k)$ is the employment at firm $k$. I assume $A_t$ is non-stationary and the unconditional mean of its gross growth rate is $\bar{a}$. Let $a_t = \frac{A_t}{A_{t-1}}$, I assume $a_t$ follows a stationary AR(1) process:

$$\log a_t - \log \bar{a} = \rho a (\log a_{t-1} - \log \bar{a}) + \varepsilon_{at},$$

(10)

where $\bar{a} > 1$, $0 < \rho_a < 1$ and $\varepsilon_{at} \sim iid N(0, \sigma^2_a)$. Hence, technology $A_t$ has a positive unconditional growth rate of 100($\bar{a} - 1$) in percentage points.

The evolution of employment $N_t(k)$ at firm $k$ reads

$$N_t(k) = (1 - \delta)N_{t-1}(k) + H_t(k),$$

where $H_t(k)$ represents the new hires at firm $k$ during period $t$. Once matched with a job, a new hire becomes productive instantaneously. This assumption is consistent with the business cycle literature, though at odds with the standard timing in the search and matching framework.

In the model, hiring is costly. The intermediate good firm has to pay a hiring cost $G_t$ for each new hire. The hiring cost is common to all intermediate good firms and is defined by

$$G_t \equiv A_t G m_{t-\delta} F_t^{1-\delta},$$

(11)

where $\bar{G} > 0$ is a scaling parameter. To be consistent with the balanced growth path, the hiring cost $G_t$ is proportional to the non-stationary technology shock.\textsuperscript{11}

The wholesale firms face the same optimization problem and the identical optimal hiring rule reads

$$MC_t = \bar{W}_t + \tilde{G}_t - \beta(1 - \delta)E_t \left\{ \frac{\tilde{C}_t}{C_{t+1}} \tilde{G}_{t+1} \right\},$$

(12)

where $MC_t \equiv \frac{P_t'}{P_t}$ denotes the real marginal cost, and $\tilde{G}_t \equiv \frac{G_t}{A_t}$ is the stationarized hiring cost. The marginal cost of the intermediate good firm includes the (stationarized) real wage

\textsuperscript{10}Thus, $H_t \equiv \int_0^1 H_t(k) dk$ and $N_t \equiv \int_0^1 N_t(k) dk$ are the aggregate new hires and aggregate employment in time period $t$.

\textsuperscript{11}I assume a vacancy will be immediately filled if the firm pays the hiring cost. In the search and matching literature with a matching function specified in Equation (7), the (pre-hiring) job-posting cost is constant for each vacancy. Thus, the hiring cost is proportional to the expected vacancy duration, i.e. the inverse of job-filling rate $Q_t = \frac{H_t}{N_t}$. Rewritten in terms of job-finding rate $F_t$, it gives Equation (11).
paid to a marginal unit of labor and the amount of (stationarized) hiring cost for filling a vacancy. In addition, the last term on the right hand side of Equation (12) implies that a filled vacancy in the current period saves the firm an amount of hiring cost for filling this vacancy in the next period if it survives the separation at the beginning of the next period.

3.4 Retail Firms

There exists a continuum of monopolistically competitive retail firms indexed by $i \in [0, 1]$, each producing a differentiated final good. Firm $i$ produces according to an identical technology

$$Y_t(i) = X_t(i),$$

where $X_t(i)$ is the quantity of the intermediate good used by firm $i$ as the only input. The final goods firm $i$ purchases intermediate good at the competitive price $P^I_t$ and sells the differentiated final good $i$ to the family with a markup at price $P_t(i)$.

As the final goods are differentiated, this gives market power to the final goods firms, entitling them to set the prices with a markup over the marginal cost. Following the standard Calvo-pricing paradigm, see Calvo (1983), it is assumed that each firm can only reoptimize their prices occasionally, making aggregate price sticky. Specifically, each firm is able to adjust its price with probability $1 - \theta$ ($0 \leq \theta \leq 1$) in each period, i.e. a fraction $1 - \theta$ of the firms can optimally reset their prices in any given period of time. The remaining fraction $\theta$ of the firms keep the prices inherited from the previous period unchanged. Thus, $\theta$ serves as an index of price rigidities. All firms that are able to adjust their prices will choose the same price denoted by $P^*_t$, because they confront an identical optimization problem.

The reoptimized price $P^*_t$ is set via the FOC:

$$E_t \sum_{k=0}^{\infty} \beta^k \beta_{t,t+k} Y_{t+k|t} \left( \frac{P^*_t}{P_{t-1}} - M_{t+k} MC_{t+k} \Pi_{t-1,t+k} \right) = 0,$$

where $\beta_{t,t+k} \equiv \beta^k \frac{C_{t+k}}{C_{t+k}^*} \frac{P_{t+k}}{P_{t+k}^*}$ is the stochastic discount factor, $Y_{t+k|t}$ is the level of output in period $t + k$ for a firm resetting its price in period $t$, $M_t \equiv \frac{\epsilon_{opt}}{\epsilon_{opt}-1}$ is the desired level of markup, and $\Pi_{t-1,t+k} \equiv \frac{P_{t+k}}{P_{t-1}}$ is the gross inflation rate between periods $t-1$ and $t+k$.

Log-linearization of the above FOC in the zero inflation steady state delivers the inflation equation:

$$\pi_t = \beta E_t \pi_{t+1} - \lambda \left[ \log \frac{1}{MC_t} - \mu_t \right],$$

where $\lambda \equiv \frac{(1-\theta)(1-\theta^\theta)}{\theta}$, $\mu_t \equiv \log M_t$, $\pi_t \equiv p_t - p_{t-1}$ is the price inflation rate and $\log \frac{1}{MC_t} - \mu_t$ denotes the deviation of log average price markup from its desired level.
3.5 Wage Setting

Under search frictions, there is a shared surplus generated from an established job-worker pair to be split between the firm and the worker. Therefore, in an environment like ours, the wages are determined by bargaining solutions. In a standard real business cycle (RBC) model with search frictions, the wage rate is determined under efficient Nash bargaining. Shimer (2005) criticizes the standard Diamond-Mortensen-Pissarides (DMP) model, see Diamond (1982) and Mortensen and Pissarides (1994), for its inability to introduce reasonable volatility in unemployment as the period-by-period Nash bargained wage is perfectly flexible that offsets the changes in labor market conditions without substantially varying the levels of unemployment. Shimer (2005) and Hall (2005) argue that introduction of real wage rigidity can solve the problem.

In the New Keynesian literature, the New Keynesian Phillips Curve (NKPC) is problematic in explaining highly persistent inflation, while the Beveridge curve, i.e., a downward sloping curve indicating a negative correlation between vacancy and unemployment in the data, is usually absent due to lack of rigid real marginal cost. In the context of the New Keynesian framework, a sticky real wage is one possible source of real rigidity. Krause and Lubik (2007) find that a rigid real wage leads to a downward sloping Beveridge curve. Nonetheless, it only weakly affects the persistence of inflation due to the presence of volatile cost of hiring derived from the long-run employment relationship. In this paper, with procyclical endogenous labor force participation, the labor market tightness is less volatile compared to the case with inelastic labor force participation. The reason lies in that the firm is keen to create jobs in a boom leading to higher tightness in the labor market, which raises the cost of hiring new workers resulting in less new jobs than enough. However, during economic expansion, more households pour into the labor force because their expected return is higher, which loosens the labor market tightness. Thus, the effects from labor demand and supply offset each other to some extent and reduce the volatility of the labor market tightness. In order to determine if it is still necessary to include real wage rigidity to account for the downward sloping Beveridge curve and examine its impact on inflation dynamics under this framework, I also introduce real wage rigidity in the wage setting.

Under a flexible-wage framework, wages are bargained on a period-by-period basis after the labor force participation and search/matching process have concluded. The stationarized surplus $S^F_t$ in terms of a bundle of final consumption goods derived from a filled job

---

12 The wage rate is common across wholesale firms, as wholesale firms are identical. Therefore, the firm specific subscripts are abstracted.
accruing to the wholesale firm is given by

\[ S^F_t = \tilde{G}_t. \]  \hfill (14)

It is intuitive to see that the surplus accruing to the firm is equal to the hiring cost, as any incumbent worker can be replaced by an unemployed immediately when the hiring cost is paid. That is to say, the hiring cost derived from a new hire “eats up” the surplus. In addition, the stationarized surplus \( S^H_t \) accruing to the family derived from a marginal worker satisfies the following recursive equation:

\[ S^H_t = \tilde{W}_t - J_t + \beta(1 - \delta)\mathbb{E}_t \left\{ (1 - F_{t+1})\frac{\tilde{C}_t}{\tilde{C}_{t+1}} S^H_{t+1} \right\}. \]  \hfill (15)

For detailed derivations, please refer to Appendix A.

Under the period-by-period efficient Nash bargaining, the firm and its workers split the shared surplus to determine the stationarized real wage \( \tilde{W}_t^N \) by solving the problem

\[
\max_{\{\tilde{W}_t^N\}} \quad \left[ S^F_t \right]^\xi \left[ S^H_t \right]^{1-\xi}
\]
subject to Equations (14) and (15) and \( \xi \in (0, 1) \) is the bargaining power of the firm.

The optimization problem implies the following surplus-sharing rule:

\[ \xi S^H_t = (1 - \xi) S^F_t. \]

Together with Equations (14) and (15), the stationarized Nash bargained real wage \( \tilde{W}_t^N \) is given by

\[
\tilde{W}_t^N = J_t + \frac{1 - \xi}{\xi} \tilde{G}_t - \beta(1 - \delta)\mathbb{E}_t \left\{ (1 - F_{t+1})\frac{\tilde{C}_t}{\tilde{C}_{t+1}} \frac{1 - \xi}{\xi} \tilde{G}_{t+1} \right\}. \]  \hfill (16)

Equation (16) demonstrates that under the efficient Nash bargaining, the flexible wage rate covers the cost \( J_t \) of transferring a marginal worker from unemployment pool to employment and the hiring cost \( G_t \) corrected for the relative bargaining power of workers \( \frac{1 - \xi}{\xi} \), less the discounted value of future hiring costs \( G_{t+1} \) in the next period from an extra unit of employment in the current period.

Finally, following the “adaptive wage” discussed in Hall (2005), I introduce real wage rigidity by assuming that the actual stationarized real wage is determined by the law of motion:

\[ \tilde{W}_t = \gamma \tilde{W}_{t-1} + (1 - \gamma)\tilde{W}_t^N, \]  \hfill (17)
where \( \gamma \in [0, 1] \) defines the degree of real wage rigidity. The rigid wage has two sources: one is inherited from the wage rate in the previous period that makes the actual wage sticky; another derives from the hypothetical flexible wage based on period-by-period Nash bargaining. I assume that the shocks are small enough to maintain the actual wage lying well in the bargaining set \([W_t, \overline{W}_t]\), where \(W_t\) and \(\overline{W}_t\) are reservation wages of the workers and firms, respectively. This assumption assures that the actual wage is Pareto efficient.

### 3.6 Market Clearing Conditions

Aggregate resource constraint:
\[
\tilde{Y}_t = \tilde{C}_t + \tilde{G}_t H_t,
\]
(18)
where \(\tilde{Y}_t \equiv Y_t/A_t\).

The aggregate production function\(^{13}\)
\[
Y_t = A_t N_t.
\]
(19)
Rewritten in stationarized form
\[
\tilde{Y}_t = N_t.
\]
(20)

### 3.7 Monetary Policy Rule and Exogenous Stochastic Shocks

To close the model, I specify a monetary policy rule and characterize exogenous shocks.

#### 3.7.1 Monetary Policy Rule

The benchmark monetary policy follows a standard Taylor-type rule with the following specification:
\[
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[ e^{\phi_\pi \pi_t} \left( \frac{Y_t}{Y^n_t} \right)^{\phi_Y} \right]^{1-\rho_R} e^{\epsilon_{rt}},
\]
(21)
where \(\bar{R}\) is the steady state of gross nominal interest rates, \(Y^n_t\) denotes the natural rate of output under flexible prices and wages and \(\log \frac{Y_t}{Y^n_t}\) is referred to as the output gap.\(^{14}\) The monetary policy shock \(\epsilon_{rt}\) is governed by
\[
\epsilon_{rt} = \rho_r \epsilon_{r,t-1} + \tilde{\epsilon}_{rt},
\]
(22)
\(^{13}\)It can be shown that in zero inflation steady state, the aggregate production function is given by Equation (19) to the first order approximation.

\(^{14}\)In this paper, the natural level of a variable refers to its prevailing value under flexible prices and wages.
where $\rho_R \in (0, 1), \varepsilon_{rt} \sim iidN(0, \sigma^2_r)$.

### 3.7.2 Other Exogenous Shocks

- **Risk-premium shock $\epsilon_{bt}$**:
  \[
  \log \epsilon_{bt} = \rho_b \log \epsilon_{b,t-1} + \varepsilon_{bt},
  \]  
  where $\rho_b \in (0, 1), \varepsilon_{bt} \sim iidN(0, \sigma^2_b)$.

- **Labor supply shock $\chi_t$**:
  \[
  \log \chi_t - \log \bar{\chi} = \rho_\chi (\log \chi_{t-1} - \log \bar{\chi}) + \varepsilon_{\chi t},
  \]  
  where $\rho_\chi \in (0, 1), \varepsilon_{\chi t} \sim iidN(0, \sigma^2_\chi)$.

- **Price markup shock $\epsilon_{pt}$**:
  \[
  \log \epsilon_{pt} - \log \bar{\tau} = \rho_p (\log \epsilon_{p,t-1} - \log \bar{\tau}) + \varepsilon_{pt},
  \]  
  where $\rho_p \in (0, 1), \varepsilon_{pt} \sim iidN(0, \sigma^2_p)$.

- **Matching efficiency shock $m_t$**:
  \[
  \log m_t - \log \bar{m} = \rho_m (\log m_{t-1} - \log \bar{m}) + \varepsilon_{mt},
  \]  
  where $\rho_m \in (0, 1), \varepsilon_{mt} \sim iidN(0, \sigma^2_m)$.

Hereafter, I refer to the model discussed in Section 3 as the baseline model.

### 4 Model Estimation

#### 4.1 Data and Estimation Method

I estimate the model using the full-information Bayesian approach. The full-information method allows for a full characterization of the joint distribution of the data generating process (DGP), instead of matching certain moments of a single observable or cross correlations within the data set. The Bayesian approach is attractive as it combines one’s prior belief built from economic theory with the likelihood function based on data to form posterior distributions of the targets that not only takes advantage of reliable economic theory to assign smaller weight to uninteresting parameter space, but also usually smooths the irregular likelihood function in practice to facilitate numerical implementations. Another advantage of
the Bayesian approach is that the Markov Chain Monte Carlo (MCMC) sampler can divide the whole group of parameters into subgroups and simulate from each block iteratively until each block converges to its ergodic mean. This is very useful especially when the dimension of parameter space is so large that it is infeasible to find the mode of likelihood relying on numerical optimization.

The time period in the model is one quarter. With six exogenous shocks, to avoid stochastic singularity, the model is estimated with six quarterly data series running from 1985Q1 to 2008Q4: (i) annualized growth rate of real GDP per capita; (ii) annualized growth rate of real wage (compensation per hour in the non-farm business sector); (iii) growth rate of vacancy (Help-Wanted Index as proxy); (iv) annualized effective federal funds rate (FFR); (v) demeaned annualized inflation rate measured by GDP deflator; (vi) civilian unemployment rate (16 years old and over).\footnote{The Help-Wanted Index used in the estimation is the Composite Help-Wanted Index constructed by Regis Barnichon (2010). Please refer to that paper for details. I would also like to thank Kenneth Goldstein for providing print Help-Wanted data.} The sample period is chosen to align with a relatively uniform monetary policy regime between the onset of Great Moderation and the implementation of unconventional Quantitative Easing. Except for the Help-Wanted Index, all of the time series were downloaded from the FRED database maintained by the St. Louis Fed at http://research.stlouisfed.org/fred2/. The real variables are calculated by dividing their nominal counterparts by the GDP deflator. To be consistent with the model, I measure population by Civilian Noninstitutional Population aged over 16 years old. Thus, the per capita quantity is evaluated from total amount divided by the population. Note also that the model assumes zero steady state for inflation rate so that the original inflation data series is demeaned. I implement the estimation within Dynare (a free preprocessor for estimation of DSGE models with random walk Metropolis-Hastings algorithm), running for 3,000,000 replications (the 1st 1,500,000 draws discarded) and targeting the acceptance rate at around 26\%.\footnote{See http://www.dynare.org and Adjemian et al. (2011).}

The convergence diagnostics are satisfactory. For further details on the algorithm, please refer to the excellent review by An and Schorfheide (2007).

4.2 Calibrated Parameters

Before turning to estimation, I calibrate 6 out of 17 structural parameters in the model with independent evidence. The discount factor $\beta$ is set to 0.992 to match the 5% sample mean of the observed FFR. The steady state of technology gross growth rate $\alpha$ takes 1.004 to match the unconditional mean quarterly growth rate of real GDP per capita at 0.392%. I choose the steady state matching efficiency $m = 1$ as suggested by Michaillat (2012). In the
Table 1: Calibrated parameters, baseline model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.992</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.004</td>
</tr>
<tr>
<td>$\pi^m$</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>6</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Baseline model refers to the model discussed in Section 3.

Table 2: Priors and posteriors of structural parameters, baseline model

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Scale of search effort</td>
<td>$\tau$</td>
<td>Uniform(0, 2)</td>
<td>0.0439</td>
</tr>
<tr>
<td>Power in search effort</td>
<td>$\Gamma$</td>
<td>Beta(0.5, 0.2)</td>
<td>0.3954</td>
</tr>
<tr>
<td>Steady-state labor supply shifter</td>
<td>$\chi$</td>
<td>Gamma(1.7, 0.1)</td>
<td>1.5771</td>
</tr>
<tr>
<td>Power in labor disutility</td>
<td>$\varphi$</td>
<td>Uniform(0, 20)</td>
<td>5.2168</td>
</tr>
<tr>
<td>Scale of hiring cost</td>
<td>$\bar{G}$</td>
<td>Gamma(0.3, 0.15)</td>
<td>0.3944</td>
</tr>
<tr>
<td>Bargaining power of the firm</td>
<td>$\xi$</td>
<td>Beta(0.3, 0.02)</td>
<td>0.2499</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\theta$</td>
<td>Beta(0.75, 0.05)</td>
<td>0.7783</td>
</tr>
<tr>
<td>Real wage rigidity</td>
<td>$\gamma$</td>
<td>Beta(0.5, 0.2)</td>
<td>0.9774</td>
</tr>
<tr>
<td>Taylor rule: smoothing</td>
<td>$\rho_R$</td>
<td>Beta(0.75, 0.1)</td>
<td>0.8599</td>
</tr>
<tr>
<td>Taylor rule: response to inflation</td>
<td>$\phi_\pi$</td>
<td>Gamma(1.5, 0.2)</td>
<td>1.4791</td>
</tr>
<tr>
<td>Taylor rule: response to output gap</td>
<td>$\phi_Y$</td>
<td>Gamma(0.2, 0.1)</td>
<td>0.5347</td>
</tr>
</tbody>
</table>

The two arguments in the uniform distributions specify the lower and upper bounds, whereas the two arguments denotes the mean and standard deviation of Beta or Gamma distribution.

Baseline model refers to the model discussed in Section 3.

literature, the constant separation rate $\delta$ varies from 0.05 in Krause et al. (2008) to 0.15 in Andolfatto (1996). I set $\delta$ equal to the upper bound 0.15 in the literature to characterize the U.S. labor market as fluid as possible. Following Blanchard and Galí (2010), $\tau_p = 6$ implying a steady state net price markup at 20%. The elasticity of unemployment in the matching function, $\zeta$, is set to 0.65 according to the estimation of the Cobb-Douglas matching function as conducted in Barnichon and Figura (2014). The calibrated parameters are listed in Table 1. Note that we are more interested in the implications of endogenous labor force participation decision within the representative family, therefore, the structural parameters with respect to the family are left for estimation.

4.3 Estimation Results

Table 2 reports the priors and posterior distributions of structural parameters, while Table 3 presents the priors and posterior distributions of exogenous shocks. In both tables, posterior means along with 5% and 95% percentiles are displayed.

There is no direct evidence on the search effort of individual household, share of the
Table 3: Priors and posteriors of shock processes, baseline model

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter</th>
<th>Prior</th>
<th></th>
<th>Posterior</th>
<th></th>
<th></th>
<th></th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part A: Autoregressive coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$</td>
<td>Beta(0.5, 0.2)</td>
<td></td>
<td>0.1136</td>
<td>0.0285</td>
<td>0.1917</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price markup</td>
<td>$\rho_p$</td>
<td>Beta(0.6, 0.1)</td>
<td></td>
<td>0.9099</td>
<td>0.8733</td>
<td>0.9476</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>$\rho_m$</td>
<td>Beta(0.6, 0.1)</td>
<td></td>
<td>0.5896</td>
<td>0.4946</td>
<td>0.6830</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor supply</td>
<td>$\rho_\chi$</td>
<td>Beta(0.6, 0.1)</td>
<td></td>
<td>0.8956</td>
<td>0.8525</td>
<td>0.9415</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-premium</td>
<td>$\rho_b$</td>
<td>Beta(0.6, 0.1)</td>
<td></td>
<td>0.8412</td>
<td>0.7867</td>
<td>0.8975</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\rho_r$</td>
<td>Beta(0.6, 0.1)</td>
<td></td>
<td>0.4746</td>
<td>0.3584</td>
<td>0.5897</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part B: Standard deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$100\sigma_a$</td>
<td>IGamma(0.01, $\infty$)</td>
<td></td>
<td>0.6570</td>
<td>0.5777</td>
<td>0.7349</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price markup</td>
<td>$100\sigma_p$</td>
<td>IGamma(2, $\infty$)</td>
<td></td>
<td>5.5109</td>
<td>4.1646</td>
<td>6.8476</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>$100\sigma_m$</td>
<td>IGamma(0.01, $\infty$)</td>
<td></td>
<td>3.6568</td>
<td>3.1849</td>
<td>4.1268</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor supply</td>
<td>$100\sigma_\chi$</td>
<td>IGamma(0.01, $\infty$)</td>
<td></td>
<td>1.1540</td>
<td>0.8762</td>
<td>1.4180</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-premium</td>
<td>$100\sigma_b$</td>
<td>IGamma(0.1, $\infty$)</td>
<td></td>
<td>0.3231</td>
<td>0.2225</td>
<td>0.4177</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$100\sigma_r$</td>
<td>IGamma(0.1, $\infty$)</td>
<td></td>
<td>0.1106</td>
<td>0.0926</td>
<td>0.1284</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The two arguments in the prior distributions specify the mean and standard deviation. Baseline model refers to the model discussed in Section 3.

hiring cost in the output or the degree of real wage rigidity. Hence, to formulate the priors for these parameters, I remain agnostic about their “reasonable” values and impose rather noninformative priors on $\bar{\tau}$, $\varphi$, $\Gamma$, $\bar{G}$ and $\gamma$. The prior on the steady-state labor supply shifter $\bar{\chi}$ is relatively tight compared to other family parameters, as it is suggested by Blanchard and Gali (2010) that $\bar{\chi}$ is slightly over 1 for the United States. The bargaining power $\xi$ of the firm ranges from 0.1 to 0.4 as suggested by Gertler et al. (2008) and Krause et al. (2008) with similar environments to this paper. Hence, the prior of $\xi$ is set to reconcile the evidence. There is consensus in the literature that the average duration of prices is about 3-4 quarters in US which leads to a tight prior on $\theta$ centering at 0.75. In addition, priors on the parameters in the Taylor-type policy feedback rule are relatively informative based on common findings in the literature.

Generally, posterior estimates of the structural parameters conform with these priors. Three features of the posterior estimates deserve special attention. First, the posterior estimate of $\varphi$ tells us that the individual disutility from work is convex in $j$, a feature that curbs the increasing labor force participation in a boom but encourages households dropping out of the labor force in a recession. Second, the estimation implies that the hiring cost is about 3% of the output in steady state which seems a plausible value. Third, somewhat surprisingly, the posterior estimate of the degree of real wage rigidity is as high as 0.9774.
As discussed in Section 3.5, the endogenous labor force participation margin is able to partially stabilize the labor market tightness, leading to a rigid component in real wage that is driven by the labor market tightness so as to introduce real rigidity in the New Keynesian framework. This real rigidity helps generate persistence in price inflation and downward sloping Beveridge curve. Nevertheless, the data still favor an extremely high degree of *ad hoc* real wage rigidity. This suggests that the endogenous labor force participation mechanism formulated in this model may not be able to tell the whole story about the endogenous behavior of price stickiness and labor market flows.

Table 4 lists the steady states of selected key labor market variables. The steady states of unemployment rate, employment-population ratio and labor force participation rate track their sample counterparts well. The steady state of job-finding rate is also consistent with the evidence in the literature. In particular, the steady state of consumption premium indicates that the consumption level of the employed is about three times the amount of the non-employed in steady state.\(^\text{17}\)

### 5 Model Fit and Comparison

Although two new features, i.e. an endogenous extensive margin of labor force participation and partial consumption insurance within the representative family, have been introduced into an otherwise standard New Keynesian model with search frictions, the reliability of implications from this model depends on model fit, especially compared to a model with constant labor force participation. To this end, I also estimate a model with unitary (inelastic) labor force and full consumption insurance *à la* Blanchard and Galí (2010) incorporating the same exogenous shocks and data as described above, see Appendix B for the equilibrium equations. Hereafter, I refer to the model *à la* Blanchard and Galí (2010) as the BG-type model. The parameter and shock estimation results are shown in Table 5 and Table 6.

\(^\text{17}\)Chéron and Langot (2004) find that the steady-state ratio of the unemployed consumption to the employed one is about 16% based on micro data from the Consumer Expenditure Survey (CEX) in 1990.
respectively.

Parameters associated with the search cost in the baseline model $\bar{c}$ and $\Gamma$ are absent from the BG-type model. The same calibration in Table 1 applies to the BG-type model. In addition, for the parameters shared between the baseline model and the BG-type model, the same priors are imposed. As shown in Table 5, most structural parameters vary a lot across two models except for $\chi$ and $\xi$. The power in the individual disutility function from work $\varphi$, which now has an interpretation of inverse of the Frisch elasticity of labor supply with respect to the extensive margin, is substantially higher than the value in the baseline model, which suggests rather inelastic labor supply. The scaling parameter $\overline{G}$ is much smaller at the value of 0.0775 implying the share of hiring costs in the output in steady state is merely 0.6%. The degree of real wage rigidity $\gamma$ declines a bit, although it still stays at an obviously high level of about 0.8. Thus, in the BG-type model, the data seem to favor a combination of high real wage rigidity and inelastic labor supply along the extensive margin, which is also the finding by Gertler et al. (2008) under their full model. Another important deviation appears in the policy rule. The interest rate inertia is substantially lower in the BG-type model, however, the central bank responds to deviations in inflation and output gaps much more aggressively. Table 6 reports on the estimates of shock parameters and standard deviations. The persistence of technology and labor supply shocks is stable across models, while the AR(1) coefficients for the other shocks vary a lot. Furthermore, there is generally a rise in the standard deviations of the shocks in the BG-type model estimation, especially in the price markup shock. This is consistent with the policy rule. As the exogenous shocks become more important for driving the economy, policy makers have to adjust the interest rate more flexibly and respond to the shocks even more aggressively to stabilize the economy.

However, which model is preferred depends on their fit to the data. Table 7 compares the marginal likelihoods and summary statistics of key variables derived from the baseline model and the BG-type model.\footnote{The marginal likelihood is computed by Dynare using the modified harmonic mean method.} Panel A presents the marginal likelihoods, and Panels B-D report standard deviations, 1st-order autocorrelations and contemporaneous cross correlations of key variables. One of the advantages of Bayesian estimation lies in that it enables us to compare model fit of non-nested models, for example in this case the baseline and BG-type models, relying on marginal likelihoods. It can be seen from Table 7 that the baseline model outperforms the BG-type model, with a great improvement of about 114 log points in the marginal likelihood. Performance of two competitive models can also be assessed by looking at summary statistics of key variables. Standard deviations of inflation and unemployment rates generated from the baseline model are similar to their sample counterparts. However, the baseline model exaggerates the volatility of inflation and unemployment rates
Table 5: Priors and posteriors of structural parameters, BG-type model

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter</th>
<th>Prior</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state labor supply shifter</td>
<td>$\chi$</td>
<td>Gamma(1.7, 0.1)</td>
<td>1.7141</td>
<td>1.5492</td>
<td>1.8759</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\varphi$</td>
<td>Uniform(0, 20)</td>
<td>12.8424</td>
<td>10.3255</td>
<td>15.2121</td>
</tr>
<tr>
<td>Scale of hiring cost</td>
<td>$G$</td>
<td>Gamma(0.3, 0.15)</td>
<td>0.0775</td>
<td>0.0234</td>
<td>0.1325</td>
</tr>
<tr>
<td>Bargaining power of the firm</td>
<td>$\xi$</td>
<td>Beta(0.3, 0.02)</td>
<td>0.2977</td>
<td>0.2657</td>
<td>0.3307</td>
</tr>
<tr>
<td>Price rigidity</td>
<td>$\theta$</td>
<td>Beta(0.75, 0.05)</td>
<td>0.8875</td>
<td>0.8684</td>
<td>0.9059</td>
</tr>
<tr>
<td>Real wage rigidity</td>
<td>$\gamma$</td>
<td>Beta(0.5, 0.2)</td>
<td>0.8139</td>
<td>0.7594</td>
<td>0.8724</td>
</tr>
<tr>
<td>Taylor rule: Interest smoothing</td>
<td>$\rho_R$</td>
<td>Beta(0.75, 0.1)</td>
<td>0.3412</td>
<td>0.2332</td>
<td>0.4498</td>
</tr>
<tr>
<td>Taylor rule: response to inflation</td>
<td>$\phi_\pi$</td>
<td>Gamma(1.5, 0.2)</td>
<td>2.1834</td>
<td>1.7578</td>
<td>2.5958</td>
</tr>
<tr>
<td>Taylor rule: response to output gap</td>
<td>$\phi_Y$</td>
<td>Gamma(0.2, 0.1)</td>
<td>1.3618</td>
<td>1.1230</td>
<td>1.5849</td>
</tr>
</tbody>
</table>

The two arguments in the uniform distributions specify the lower and upper bounds, whereas the two arguments denotes the mean and standard deviation of Beta or Gamma distribution. The BG-type model refers to a model with unitary (inelastic) labor force and full consumption insurance à la Blanchard and Galí (2010).

Table 6: Priors and posteriors of shock processes, BG-type model

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Parameter</th>
<th>Prior</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part A: Autoregressive coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>$\rho_a$</td>
<td>Beta(0.5, 0.2)</td>
<td>0.1627</td>
<td>0.0457</td>
<td>0.2731</td>
</tr>
<tr>
<td>Price markup</td>
<td>$\rho_p$</td>
<td>Beta(0.6, 0.1)</td>
<td>0.5793</td>
<td>0.4679</td>
<td>0.6886</td>
</tr>
<tr>
<td>Matching efficiency</td>
<td>$\rho_m$</td>
<td>Beta(0.6, 0.1)</td>
<td>0.8469</td>
<td>0.7922</td>
<td>0.9017</td>
</tr>
<tr>
<td>Labor supply</td>
<td>$\rho_\chi$</td>
<td>Beta(0.6, 0.1)</td>
<td>0.8520</td>
<td>0.7778</td>
<td>0.9292</td>
</tr>
<tr>
<td>Risk-premium</td>
<td>$\rho_b$</td>
<td>Beta(0.6, 0.1)</td>
<td>0.3376</td>
<td>0.2341</td>
<td>0.4382</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\rho_r$</td>
<td>Beta(0.6, 0.1)</td>
<td>0.5770</td>
<td>0.4330</td>
<td>0.7195</td>
</tr>
</tbody>
</table>

| Part B: Standard deviations      |           |                     |       |       |        |
| Technology                       | $100\sigma_a$ | IGamma(0.01, $\infty$) | 0.53 | 0.47 | 0.59 |
| Price markup                     | $100\sigma_p$ | IGamma(2, $\infty$)    | 37.34| 28.13| 46.39 |
| Matching efficiency              | $100\sigma_m$ | IGamma(0.01, $\infty$) | 1.44 | 1.26 | 1.61 |
| Labor supply                     | $100\sigma_\chi$ | IGamma(0.01, $\infty$) | 8.10 | 5.55 | 10.51 |
| Risk-premium                     | $100\sigma_b$ | IGamma(0.1, $\infty$)  | 1.21 | 1.18 | 1.25 |
| Monetary policy                  | $100\sigma_r$ | IGamma(0.1, $\infty$)  | 1.21 | 1.18 | 1.26 |

The two arguments in the prior distributions specify the mean and standard deviation. The BG-type model refers to a model with unitary (inelastic) labor force and full consumption insurance à la Blanchard and Galí (2010).
Table 7: Marginal likelihoods and summary statistics

<table>
<thead>
<tr>
<th>Part A: Marginal likelihoods</th>
<th>Baseline</th>
<th>BG-type</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal likelihood</td>
<td>-762.69</td>
<td>-876.76</td>
<td>—</td>
</tr>
</tbody>
</table>

| Part B: Standard deviations          |          |         |      |
| inflation rate                       | 0.0031   | 0.0049  | 0.0024 |
| unemployment rate                    | 0.0111   | 0.0142  | 0.0098 |

| Part C: Contemporaneous cross correlations |          |         |      |
| corr(inflation rate, unemployment rate)| 0.1514   | -0.3770 | 0.0553 |
| corr(unemployment rate, vacancy rate)  | -0.2142  | -0.8446 | -0.2341 |

| Part D: 1\textsuperscript{st}-order autocorrelations |          |         |      |
| inflation rate                           | 0.6721   | 0.7881  | 0.5320 |
| unemployment rate                        | 0.7665   | 0.5741  | 0.9440 |

remarkably. Specially, it drastically amplifies inflation fluctuations to twice the amount of the sample value. Likewise, the baseline model produces much better autocorrelations of inflation and unemployment rates and cross correlations among inflation, unemployment and vacancy rates. All of the evidence suggests that the introduction of endogenous labor force participation margin and partial consumption insurance is able to account for the data better than a model with unitary inelastic labor force and full consumption insurance à la Blanchard and Galí (2010).

6 Sources of Unemployment Fluctuations

The branch of literature studying the importance of reallocation shocks, or more generally mismatch shocks, dates back to Lilien (1982). Early empirical studies by Abraham and Katz (1986) and Blanchard and Diamond (1989) suggest that demand shocks are dominant in producing unemployment fluctuations, whereas reallocation shocks are almost irrelevant at business cycle frequencies. Recently, under the framework of dynamic stochastic general equilibrium (DSGE) modeling, Krause et al. (2008), Furlanetto and Groshenny (2012) and Justiniano and Michelacci (2011) examine the impact of matching efficiency shock, which implicitly captures all structure factors in the labor market, on unemployment. Justiniano and Michelacci (2011) estimate an RBC model for six countries including the United States and find that matching efficiency shock explains only 11 percent of the variance of unemployment. By contrast, working with a monetary New Keynesian model, Krause et al. (2008) ascribe 37 percent of the variation in unemployment to the matching efficiency shock.
Other than looking into variance decomposition, Furlanetto and Groshenny (2012) study the transmission mechanism of matching efficiency shock. They conclude that the propagation of the shock hinges on specific forms of hiring costs.

Different from Justiniano and Michelacci (2011), this paper studies a monetary New Keynesian model with demand shocks. Additionally, an endogenous labor force participation margin, partial consumption insurance within the representative family, and the concept of involuntary unemployment that satisfies the official definition of unemployment by the BLS are introduced, all of which are absent from Krause et al. (2008) and Furlanetto and Groshenny (2012). In particular, as the unemployment defined in this paper is consistent with the data collection for unemployment in the United States, it is more closely related to countercyclical policy interventions than other frameworks in which unemployment is not in accordance to the data that policy makers are targeting. Hence, under the framework in this paper, the role of matching efficiency shock in shaping the key macro variables at business cycle frequencies can be reassessed.

Figures 1-2 plot the impulse responses of selected model variables to a 1 standard deviation positive shock to matching efficiency. A positive matching efficiency shock increases the job finding rate instantaneously, other things equal. Because the expected return from participation is higher, more households pour into the labor force. The positive comovement between the consumption premium and labor force participation, as characterized in Equation (3), is also justified by an increase in the consumption premium as a result of a positive matching efficiency shock. The matching efficiency shock has negative effects on both unemployment rate and vacancy rate (not reported) consistent with the finding based on pre-match hiring cost in Furlanetto and Groshenny (2012). However, the decline in the vacancy rate is larger, making the labor market slacker and bringing down the real wage, real marginal cost, and the price level. Following a decline in the real marginal cost, the intermediate good firm is willing to hire additional workers until the equality condition in Equation (12) is satisfied. This boosts employment and, equivalently, output. As for the natural rates of selected model variables under flexible prices and wages, the responses have the same signs as that of the actual model variables. However, the magnitude of responses is larger for natural rates of output and labor force, while the response of natural rate of unemployment is close to that of the actual model unemployment rate. In particular, a one standard deviation shock to matching efficiency decreases the output and participation gaps by roughly 0.7 percentage points and increases the participation gap by 0.8 percentage points at peak.\(^{19}\)

---

\(^{19}\)The participation gap is defined as the percent deviation of the actual labor force from its natural level, whereas the unemployment gap is defined as the percentage point deviation of unemployment rate from its natural level.
Figure 1: Impulse responses to 1 standard deviation shock to matching efficiency. All of the responses are in percent deviation from the steady state except for the unemployment rate which is in percentage point deviation. The solid circled lines are posterior means of responses and dashed lines denote 90% credible intervals.
Figure 2: Impulse responses to 1 standard deviation shock to matching efficiency. Responses of natural rates are in percent deviation from the steady state except for the natural rate of unemployment which is in percentage point deviation. Responses of gap variables are in percent deviation from the natural rate. The solid circled lines are posterior means of responses and dashed lines denote 90% credible intervals.
The impulse responses presented in Figures 1-2 illustrate the basic propagation mechanism of exogenous shocks. To study the relative importance of driving forces of the model variables, we need to look at forecast error variance decomposition. Forecast error variance decomposition at the horizon of 10 quarters ahead are summarized in Table 8.\textsuperscript{20} As evident from the table, the transitory shock to technology growth rate plays a trivial role in explaining the key variables listed. The other supply shock, i.e. the price markup shock, is a key driving force for many variables. It explains roughly 35-40% of the variation in output, unemployment rate, employment, consumption premium, vacancy rate, and especially 56% of the unemployment gap. Thus, the supply shocks seem to relate more to real activities.

Turning to the demand side, I consider the risk-premium and monetary policy shocks. The monetary policy shock itself plays a minor role in explaining any selected variable in accordance to the consensus in the literature that the monetary policy shock is not crucial to shape the business cycle. By contrast, the other demand shock i.e., the risk-premium shock plays a non-trivial role in driving both real and nominal variables. The risk-premium shock accounts for a large fraction of the variation in inflation, labor force and vacancy rate. In addition, the risk-premium shock is the dominant driving force of fluctuations of the interest rate, the output gap and the participation gap. Lastly, the labor market shocks comprise of the labor supply shock and matching efficiency shock. The labor supply shock dominates other shocks in driving labor force participation. As for the matching efficiency shock, though it is basically irrelevant to most of the selected variables, it drives three crucial variables characterizing the business cycle: the unemployment rate and the natural rate of unemployment. Specifically, the matching efficiency shock accounts for around 42% of the variation in the unemployment rate. Meanwhile, the matching efficiency dominates the fluctuations in the natural rate of unemployment for which 86% of the variation originates from the matching efficiency shock.\textsuperscript{21}

In summary, the price markup shock and the matching efficiency shock dominate other shocks in generating variation in the unemployment rate, the natural rate of unemployment and the unemployment gap. The matching efficiency shock, as a catch-all shock, implicitly integrates all of the structural factors and has non-trivial impact on shaping the business cycle within the estimated dynamic stochastic general equilibrium model considered in this paper.

\textsuperscript{20}Results related to other horizons are quantitatively similar.

\textsuperscript{21}A strand of rapidly growing literature argues that treating matching efficiency exogenously can overestimate its importance. Efforts are made to endogenize matching efficiency, see Barnichon and Figura (2014), Davis et al. (2013) and Sedláček (2014) among others. Thus, the evidence presented here should be taken with a grain of salt. Endogenizing matching efficiency under the framework in this paper is left for my future research.
Table 8: Forecast error variance decomposition based on posterior mean of model parameters, in percent, horizon = 10 quarters

<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Risk-premium</th>
<th>Labor supply</th>
<th>Price markup</th>
<th>Matching efficiency</th>
<th>Monetary policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>0.14</td>
<td>22.86</td>
<td>18.71</td>
<td>37.44</td>
<td>12.21</td>
<td>8.63</td>
</tr>
<tr>
<td>inflation</td>
<td>0.01</td>
<td>35.20</td>
<td>5.80</td>
<td>28.74</td>
<td>9.25</td>
<td>21.00</td>
</tr>
<tr>
<td>interest rate</td>
<td>0.03</td>
<td>67.48</td>
<td>7.14</td>
<td>5.37</td>
<td>16.04</td>
<td>3.95</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>0.07</td>
<td>12.66</td>
<td>4.97</td>
<td>34.71</td>
<td>42.39</td>
<td>5.19</td>
</tr>
<tr>
<td>labor force</td>
<td>0.32</td>
<td>28.10</td>
<td>35.74</td>
<td>25.39</td>
<td>1.10</td>
<td>9.35</td>
</tr>
<tr>
<td>labor market tightness</td>
<td>0.08</td>
<td>25.06</td>
<td>1.36</td>
<td>58.95</td>
<td>4.06</td>
<td>10.49</td>
</tr>
<tr>
<td>consumption premium</td>
<td>0.38</td>
<td>20.67</td>
<td>6.45</td>
<td>24.91</td>
<td>41.00</td>
<td>6.59</td>
</tr>
<tr>
<td>natural rate of unemployment</td>
<td>0.00</td>
<td>0.00</td>
<td>5.13</td>
<td>8.22</td>
<td>86.65</td>
<td>0.00</td>
</tr>
<tr>
<td>output gap</td>
<td>0.31</td>
<td>50.03</td>
<td>2.33</td>
<td>0.41</td>
<td>28.03</td>
<td>18.88</td>
</tr>
<tr>
<td>unemployment gap</td>
<td>0.14</td>
<td>25.81</td>
<td>1.37</td>
<td>56.79</td>
<td>5.31</td>
<td>10.58</td>
</tr>
<tr>
<td>participation gap</td>
<td>0.47</td>
<td>41.80</td>
<td>1.31</td>
<td>13.44</td>
<td>29.07</td>
<td>13.91</td>
</tr>
</tbody>
</table>

Due to rounding errors, the sum of percentages may not be exactly 100%.

7 Labor Force Participation and Monetary Policy

Inflation is a source of welfare loss in a New Keynesian framework with sticky prices. The presence of sticky prices hinders the optimal adjustments of prices for differentiated final consumption goods, which leads to relative price distortions. This distortion results in inefficiency by violating optimal symmetric equilibrium conditions for differentiated final consumption goods. Hence, stabilizing inflation fluctuations around its natural level is desirable for improving social welfare in that relative price distortions will be minimized under this strategy. Another inefficiency comes from the monopolistic competition among final goods firms which delivers inefficient output and employment levels. In this paper, stationarized output is identical to employment. To minimize the welfare loss from market power of final goods firms, it is beneficial to smooth the employment gap (equivalently the output gap) in the presence of stochastic price markup shocks.

It is clear that both of the above objectives closely relate to the labor force participation margin in this paper. First, the marginal cost, which feeds into price inflation via the inflation equation, is driven by the labor market tightness as shown in Equations (12), (16) and (17). As the marginal cost is the key determinant of inflation fluctuations, smoothing the marginal cost by managing the labor market tightness is promising for stabilizing price inflation. Second, the level of employment is an outcome of aggregate labor market conditions. The outside option is clearly influential in generating this outcome when an adjustment along the extensive margin is allowed for. In this paper, the labor force participation margin is a fundamental channel that defines labor flows and aggregate labor market conditions including
the labor market tightness, which is no longer pinned down solely by the employment as in a model with unitary labor supply. Thus, a simple interest rate rule stabilizing the labor force participation gap can be welfare improving.\footnote{See Erceg and Levin (2013) for discussions on implications of the participation gap for optimal monetary policy design during the Great Recession.}

In this section, to understand implications for welfare gains by taking into account the participation gap in the monetary policy design, I consider an extended Taylor-type rule as

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ e^{\phi_{\pi} \pi_t} \left( \frac{Y_t}{Y_{t^*}} \right)^{\phi_Y} \left( \frac{L_t}{L_{t^*}} \right)^{\phi_L} \right]^{1-\rho_R} e^{\epsilon_{rt}}, \hspace{1cm} (27)$$

where $L_t$ is the natural rate of labor supply and $\phi_L > 0$ in order to stabilize the participation gap.

The expected lifetime utility of the representative family at time zero under policy regime $pr$, denoted by $V_{0}^{pr}$, is used as the measure of welfare. Specifically, $V_{0}^{pr}$ is given by

$$V_{0}^{pr} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, N_t \right). \hspace{1cm} (28)$$

I evaluate the conditional welfare along the lines of Schmitt-Grohé and Uribe (2007) by setting initial values of all of the state variables at their respective deterministic steady states.\footnote{The welfare and policy functions are approximated by second-order perturbation methods. The expected lifetime conditional welfare at time zero is based on 50,000 simulations.} Because the deterministic steady states are the same across policy regimes, this guarantees the economy under alternative policy regimes starts from the same initial conditions. The welfare cost is measured following in the spirit of Lucas (1987) by computing the fraction $\Delta$ (in percentage point) of steady-state consumption that the representative family is willing to give up to be as well off under the steady state as under a specific policy regime $pr$, i.e.

$$V_{0}^{pr} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left( \left( 1 - \frac{\Delta}{100} \right) C_t, N_t \right), \hspace{1cm} (29)$$

where variables without time subscripts represent their corresponding steady states. Note that a higher value of $\Delta$ implies lower welfare.

Standard deviations for selected variables and welfare costs under alternative monetary policy regimes are reported in Table 9. The fluctuations of inflation, marginal cost, employment (output), labor force and labor market tightness are mitigated when the Taylor-type simple rule responds to the participation gap. In particular, under the policy regime with $\phi_L = 0.5$, the standard deviation of inflation is reduced by about 30% compared to the
Table 9: Welfares under alternative monetary policy regimes in the baseline model

<table>
<thead>
<tr>
<th>Policy regime</th>
<th>φ_L = 0</th>
<th>φ_L = 0.1</th>
<th>φ_L = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part A: Standard deviations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inflation</td>
<td>0.0037</td>
<td>0.0034</td>
<td>0.0026</td>
</tr>
<tr>
<td>marginal cost</td>
<td>0.0306</td>
<td>0.0300</td>
<td>0.0288</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>0.0107</td>
<td>0.0107</td>
<td>0.0108</td>
</tr>
<tr>
<td>employment</td>
<td>0.0188</td>
<td>0.0184</td>
<td>0.0179</td>
</tr>
<tr>
<td>labor force</td>
<td>0.0143</td>
<td>0.0139</td>
<td>0.0132</td>
</tr>
<tr>
<td>labor market tightness</td>
<td>0.1688</td>
<td>0.1663</td>
<td>0.1621</td>
</tr>
<tr>
<td>Part B: Welfare costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ (in percent)</td>
<td>0.0172</td>
<td>0.0170</td>
<td>0.0161</td>
</tr>
</tbody>
</table>

inflation fluctuations under the original Taylor-type rule in Equation (21) with φ_L = 0. Not surprisingly, welfare under policy regimes stabilizing the participation gap exceeds that under the original Taylor-type rule. As shown in Table 9, though unemployment fluctuations may be slightly amplified under policy regimes stabilizing the participation gap, the gain from stabilizing inflation outweighs that loss and improves the overall welfare. Therefore, the interest rate rule taking into account stabilization of participation gap can be beneficial in terms of welfare comparisons.

8 Conclusion

In this paper, I introduced endogenous labor force participation, partial consumption insurance, and involuntary unemployment into an otherwise standard New Keynesian with search frictions. Following in the spirit of Christiano et al. (2012), the unemployed pay costs to search for a job and they are better off if matched with one. Hence, the resulting unemployment is involuntary and satisfies the U.S. official definition of unemployment. Taking the model to U.S. data, I find that the introduction of endogenous labor force participation greatly improves the model fit compared to a model with unitary inelastic labor supply and full consumption insurance à la Blanchard and Galí (2010). The model estimation prefers an extremely high degree of real wage rigidity to account for the labor market data. Through the lens of impulse response analysis and forecast error variance decomposition, the price markup shock and matching efficiency shock appear to be the two key driving forces behind unemployment fluctuations. In addition, 86% of the variation in the natural rate of unemployment can be ascribed to the matching efficiency shock. I also show that labor force
participation margin is essential to monetary policy design. Stabilizing the participation gap in a simple Taylor-type rule can be welfare improving.

References


Appendix A. Derivations of the Surpluses Accruing to the Firm and Family in the Efficient Nash Wage Bargaining

The surpluses accruing to the firm and family derived from a marginal job-worker pair are evaluated as follows:

I. Surplus to the firm

$\mathcal{J}^F_t$: value of a filled job accruing to the wholesale firm, which is given by

$$
\mathcal{J}^F_t = \frac{P_t}{P_t} MPL_t - W_t + (1 - \delta) \mathbb{E}_t \{ \beta_{t+1} \mathcal{J}^F_{t+1} \},
$$

where the stochastic discount factor $\beta_{t+s} \equiv \beta^s C_t / C_{t+s}$ and $MPL_t \equiv A_t$ is the marginal product of labor.

$\mathcal{J}^V_t$: value of a vacancy accruing to the wholesale firm. The firm will create an additional vacancy until the value of a vacancy equals zero. According to the free entry condition:

$$
\mathcal{J}^V_t = 0.
$$

Combined with Equation (12), we have

$$
S^F_t = \tilde{G}_t,
$$

where $S^F_t \equiv \frac{\mathcal{J}^F_t - \mathcal{J}^V_t}{A_t}$ denotes the stationarized surplus from a filled job accruing to the firm. It is intuitive to see that the surplus accruing to the firm is simply given by the hiring cost, as any incumbent worker can be replaced by an unemployed immediately when the hiring cost is paid. That is to say, the hiring cost derived from a new hire “eats up” the surplus.

II. Surplus to the representative family

Rewrite the family utility function in a recursive way:

$$
U_t = \log C_t + N_t g(F_t, L_t) - L_t c(F_t) - \chi_t \frac{N_t^{1+\varphi}}{1+\varphi} - \log \left[ N_t \left( e^{g(F_t, L_t)} - 1 \right) + 1 \right] + \beta \mathbb{E}_t U_{t+1}.
$$

Because the bargaining occurs after the labor force participation and hiring decisions have been made, once $L_t$ is fixed, $\frac{\partial U_t}{\partial N_t}$ defines the surplus in terms of utility gain accruing to the family from an established employment relationship in period $t$.

Combining Equation (A.2) with the binding budget constraint of the representative family

$$
P_t C_t + \frac{B_t}{R_t \epsilon_{kt}} = P_t W_t N_t + B_{t-1} + \Lambda_t
$$
and the law of motion for employment
\[ N_t = (1 - \delta)(1 - F_t)N_{t-1} + F_t L_t, \]
it follows that
\[
\frac{\partial U_t}{\partial N_t} = \frac{W_t}{C_t} + g(F_t, L_t) - \chi_t N_t^p - \frac{e^{g(F_t, L_t)} - 1}{N_t (e^{g(F_t, L_t)} - 1)} + \beta \mathbb{E}_t \left\{ \frac{\partial U_{t+1}}{\partial N_t} \right\}.
\] (A.3)

Note that given the job-finding rate and labor force participation in period \( t + 1 \),
\[
\frac{\partial U_{t+1}}{\partial N_t} \frac{\partial N_{t+1}}{\partial N_{t+1}} = (1 - \delta)(1 - F_{t+1}) \frac{\partial U_{t+1}}{\partial N_{t+1}}.
\]

Recall the definition of \( J_t \), Equation (A.3) is simplified to a recursive representation
\[
\frac{\partial U_t}{\partial N_t} = \frac{W_t}{C_t} - A_t J_t + \beta (1 - \delta) \mathbb{E}_t \left\{ (1 - F_{t+1}) \frac{\partial U_{t+1}}{\partial N_{t+1}} \right\}.
\]

The stationarized surplus in terms of consumption goods \( S^H_t \) accruing to the family from an established employment relationship is given by
\[
S^H_t \equiv \frac{1}{A_t} \frac{\partial U_t}{\partial N_t} / U_{c,t} = \tilde{C}_t \frac{\partial U_t}{\partial N_t}.
\]

Thus, the stationarized surplus accruing to the family in terms of consumption goods satisfies the following recursive equation:
\[
S^H_t = \tilde{W}_t - J_t + \beta (1 - \delta) \mathbb{E}_t \left\{ (1 - F_{t+1}) \frac{\tilde{C}_t}{C_{t+1}} S^H_{t+1} \right\}.
\] (A.4)
Appendix B. Equilibrium Conditions of the Model à la Blanchard and Galí (2010)

1. Euler equation

\[ 1 = \beta \mathbb{E}_t \left\{ \frac{\bar{C}_t}{\bar{C}_{t+1}} \frac{R_t \epsilon_{t+1}}{e^{\pi_{t+1} a_{t+1}}} \right\} \]  

(B.1)

2. Hiring rule

\[ MC_t = \bar{W}_t + \bar{G}_t - \beta(1 - \delta) \mathbb{E}_t \left\{ \frac{\bar{C}_t}{\bar{C}_{t+1}} \bar{G}_{t+1} \right\} \]  

(B.2)

3. Hiring cost

\[ \bar{G}_t = \bar{G} m_t \frac{1}{1 - \zeta} F_t^{\frac{\zeta}{1 - \zeta}} \]  

(B.3)

4. Inflation equation

\[ \pi_t = \beta \mathbb{E}_t \pi_{t+1} - \lambda \left[ \log \frac{1}{MC_t} - \mu_t \right] \]  

(B.4)

5. Desired markup

\[ \mu_t = \log \left( \frac{\epsilon_{pt}}{\epsilon_{pt} - 1} \right) \]  

(B.5)

6. Employment stock

\[ N_t = (1 - \delta) N_{t-1} + H_t \]  

(B.6)

7. Unemployment rate

\[ u_t = 1 - N_t \]  

(B.7)

8. Matching function

\[ H_t = m_t U_t^{\zeta} V_t^{1-\zeta} \]  

(B.8)

9. Job-finding rate

\[ F_t = \frac{H_t}{U_t} \]  

(B.9)

10. Labor market tightness

\[ x_t = \frac{V_t}{U_t} \]  

(B.10)
11. Searching workers
\[ U_t = 1 - (1 - \delta)N_{t-1} \tag{B.11} \]

12. Vacancy rate
\[ vr_t = V_t \tag{B.12} \]

13. Flexible wage under Nash bargaining
\[ \tilde{W}_t^N = \tilde{MRS}_t + \frac{1 - \xi}{\xi} \tilde{G}_t - \beta(1 - \delta)E_t \left\{ (1 - F_{t+1}) \frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}} \frac{1 - \xi}{\xi} \tilde{G}_{t+1} \right\} \tag{B.13} \]

14. Marginal rate of substitution
\[ \tilde{MRS}_t = \tilde{C}_t \chi_t N_t^\phi \tag{B.14} \]

15. Real wage rigidity
\[ \tilde{W}_t = \gamma \tilde{W}_{t-1} + (1 - \gamma)\tilde{W}_t^N \tag{B.15} \]

16. Monetary policy
\[ \frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \phi_y \pi_t \left( \frac{Y_t}{Y_t^n} \right) \phi_y \right]^{-\rho_R} e^{\epsilon_{rt}} \tag{B.16} \]

17. Market clearing
\[ \tilde{Y}_t = \tilde{C}_t + \tilde{G}_t H_t \tag{B.17} \]

18. Aggregate production function
\[ \tilde{Y}_t = N_t \tag{B.18} \]

Specifically, variables with tildes are detrended by technology. The natural rate of output \( Y_t^n \) is determined under flexible prices and wages. In addition, six exogenous shocks are characterized as in the baseline model.
Appendix C. Comparisons between Priors and Posteriors of Structural Parameters, Baseline Model

Figure 3: baseline Model: Priors (blue) and Posteriors (red) of Structural Parameters.
Appendix D. History of Smoothed Shocks, Baseline Model

Figure 4: History of Smoothed Shocks, 1985:Q2-2008:Q4.