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Abstract

In this paper, the role of matching efficiency (equivalently, mismatch) at the aggregate level in driving unemployment fluctuations is estimated using a TVP-SVAR model. Modelling mismatch at the aggregate level sidesteps the problematic implicit assumption of orthogonality of sources of mismatch at disaggregated levels (industrial, occupational, geographical, etc.) and is not sensitive to the level of disaggregation by construction. Observing that estimated aggregate matching efficiency lags business cycles, I identify a structural shock to aggregate matching efficiency using standard timing restrictions. Based on impulse response analysis and forecast error variance decompositions, I conclude that the matching efficiency shock explains no more than 20% of the variation in unemployment in the United States between 1967-2013, whereas aggregate shocks explain well above 70% of unemployment fluctuations. Related, the rise in the unemployment rate during the Great Recession is dominated by a slump in aggregate demand rather than driven by structural factors.

JEL Classification: C11, E24, E32, E37.

Keywords: Aggregate matching efficiency, Mismatch, Structural unemployment, Time-varying parameter vector autoregression (TVP-VAR).

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1 Introduction

In the aftermath of the recent global financial crisis, the United States is suffering from a frustratingly slow recovery in the labor market. This painful experience has resurrected an old debate about the source of the slow recovery in employment among both economists and policymakers. Among others, Delong (2010) argues that it is a shortfall in aggregate demand rather than structural factors that raised the unemployment rate drastically during the Great Recession. In contrast, Kocherlakota (2010) favors the opposite hypothesis that mismatch should be blamed for the sluggish recovery, as quoted from his speech that1

“Monetary stimulus has provided conditions so that manufacturing plants want to hire new workers. But the Fed does not have a means to transform construction workers into manufacturing workers.”

Debate about the role of mismatch in shaping unemployment fluctuations can be traced back to Lilien (1982), Abraham and Katz (1986), and Davis (1987). Lilien (1982) finds that the unemployment rate is positively correlated to the dispersion in the employment growth rates across sectors, with this dispersion heightened in recessions. Thus, he attributes a large part of the rise in unemployment to sectoral reallocations. However, Abraham and Katz (1986) argue that the positive correlation between the unemployment rate and the dispersion in employment growth rates is not necessarily a result from sectoral reallocations. As long as elasticities of different sectors to the aggregate demand shock are diverse enough, I would expect a positive correlation of this sort. Subsequently, Davis (1987) provides evidence for the existence of reallocations. Specifically, the sectoral employment growth rates tend to be highly autocorrelated so that they can not reverse themselves easily. Otherwise, they would if purely driven by diverse sensitivities to business cycles. More recent empirical studie on gauging the importance of mismatch in determining unemployment fluctuations using disaggregated data include Sahin et al. (2012), Barnichon and Figura (2010, 2014), Herz and Van Rens (2011) and Davis et al. (2013), among others.

To shed light on this issue, it is important to construct a proper measure of mismatch (no matter at the aggregate or disaggregated level) and understand its transmission effect to unemployment. As one of the contributions of this paper, I develop an original two-stage approach relying on aggregate data only to understand mismatch at the aggregate level and its transmission mechanism. In the first stage, I define the residuals from the standard Cobb-Douglas matching function as aggregate matching efficiency which measures the aggregate

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1Some authors argue that the matching efficiency has slumped during the Great Recession that ends up with much less new hires than what would be predicted absent heightened mismatch in the labor market, for example, see Barlevy (2011) and Veracierto (2011).
level of mismatch implicitly “averaged” over every source of mismatch. In the second stage, aggregate matching efficiency is endogenized in the time-varying parameter vector autoregression (TVP-VAR) model. The intrinsic matching efficiency shock is identified with the standard recursive scheme by ordering aggregate matching efficiency first in the TVP-SVAR model. There is evidence showing that aggregate matching efficiency displays cyclical fluctuations and typically lags business cycles. Thus, I argue it is necessary to endogenize aggregate matching efficiency to properly assess the impact of mismatch at the aggregate level and that my identification of matching efficiency shock with timing restrictions is reasonable.

The advantages of my approach are three-fold. First, because I am modeling matching efficiency at the aggregate level, the results are not sensitive to the level of disaggregation, which would be a serious problem for modeling disaggregated data as noted by Sahin et al. (2012) and Barnichon and Figura (2014). Second, looking into the transmission mechanism of one specific source of mismatch at a time implicitly assumes that disaggregated sources of mismatch are orthogonal. This orthogonality assumption is questionable. For example, a (severe) local mismatch at industry level facilitates the incentives of worker mobility. However, the migration can be hindered by “house-locking” effects due to a (drastic) decline in average family income that leads to a slump in house prices, see Valletta (2013). In return, the average unemployment spell may last longer and result in a large shortfall in aggregate demand that imposes upward pressure on industrial mismatch, and so forth. Instead, the transmission mechanism of aggregate matching efficiency shock does not rely on an orthogonality assumption for different sources of mismatch. Third, a TVP-VAR model is a natural way to endogenize aggregate matching efficiency without the need to explicitly model disaggregated sources of mismatch. With a justifiable identification scheme, the underlying structural VAR provides us standard and powerful tools of impulse response analysis, forecast error variance decomposition, and counterfactual analysis for the sake of understanding the importance of mismatch at the aggregate level in shaping unemployment fluctuations. In addition, under the framework of a TVP-VAR model, I can capture changes in the transmission mechanism of structural shocks, if any, which is not done in the aforementioned empirical literature.

Based on the framework developed above, I analyze unemployment fluctuations in the United States between 1967:Q3 and 2013:Q2, especially focusing on the Great Recession. The evidence suggests that mismatch (evaluated by the aggregate matching efficiency shock) plays a minor role in determining the dynamics of unemployment. Instead, aggregate shocks explain well above 80% of the variation in the unemployment rate over the sample periods. My finding in this paper is consistent with the finding by Sahin et al. (2012) that mismatch at the industry and occupation level at most accounts for one third of the total increase

\[^2\text{A vivid example is the rise and fall of Detroit in United States over the past century.}\]
in the unemployment rate, while geographical mismatch plays a trivial role. Additionally, Barnichon and Figura (2010) and Herz and Van Rens (2011) also conclude that mismatch contributes little to movements in unemployment.

To address the controversy between Delong (2010) and Kocherlakota (2010), counterfactual analyses are conducted for the Great Recession era by setting the structural factors to their conditional means, i.e. set both the matching efficiency shock and labor supply shock in the Great Recession to zero. The simulations do not support Kocherlakota’s hypothesis that mismatch should be blamed for the sluggish recovery in the labor market as the simulated paths of unemployment rate track closely the actual rate. My counterfactuals are a sharp contrast to the findings by Sahin et al. (2012) and Barnichon and Figura (2010). In particular, Sahin et al. (2012) argue that mismatch across industries and occupations adds 0.75 percentage points and 1.5 percentage points in the unemployment rate during 2006-2009, respectively. Meanwhile, Barnichon and Figura (2010) suggest that mismatch adds in 1.5 percentage points to the unemployment rate in 2009. The striking differences may lie in the nature of aggregate matching efficiency treated in the models. Sahin et al. (2012) and Barnichon and Figura (2010) treat aggregate matching efficiency as an exogenous shock, while I endogenize aggregate matching efficiency and allow it to respond to aggregate shocks. As evident from my findings, I argue that, by treating aggregate matching efficiency exogenously, the analyses in Sahin et al. (2012) and Barnichon and Figura (2010) mistakenly attribute part of the contribution of aggregate shocks to mismatch, which exaggerates the importance of mismatch in determining the unemployment fluctuations.

In relation to the previous literature, my study implements an original approach that involves modeling at the aggregate level to gauge the role of mismatch, departing from the other approaches that rely on disaggregated data. Sahin et al. (2012) investigate the importance of mismatch, which is defined as the deviation of actual unemployment from the optimal unemployment derived from the social planner’s problem under costless between-sector mobility, using disaggregated data at the industry, occupation, and metropolitan levels. Barnichon and Figura (2010) decompose the movements in the Beveridge curve into three parts driven by labor demand, labor supply, and matching efficiency in order to understand the determinants of unemployment rate, whereas Herz and Van Rens (2011) develop a simple model of a segmented labor market with search frictions within segments. In terms of endogenizing aggregate matching efficiency, my approach to endogenization using a TVP-VAR model is related to empirical studies by Barnichon and Figura (2014) and Davis et al. (2013), who generalize the standard matching function along different lines. Barnichon and Figura (2014) focus on the roles of composition of the unemployment pool and dispersion in labor market conditions, while Davis et al. (2013) incorporate a measure of recruiting
intensity (per vacancy). My work is also related to Garin et al. (2013) and Sedlacek (2014), both of which explore in general equilibrium models. Garin et al. (2013) develop a two-island general equilibrium model of labor reallocation, in the spirit of Lucas and Prescott (1974), with each island subjected to a common aggregate productivity shock and an island-specific productivity shock that triggers reallocation of workers between islands and affects the level of mismatch in the economy. Sedlacek (2014) endogenizes the matching efficiency via firing costs and endogenous separation driven by firms’ hiring standards in a general equilibrium model.

The rest of the paper is organized as follows. Section 2 provides the motivation and details of my proposed empirical method for assessing the role of aggregate matching efficiency (mismatch at aggregate level) in driving unemployment. Section 3 describes the data source, priors, and Markov Chain Monte Carlo estimation. Section 4 discusses my empirical findings in detail. Section 5 concludes.

2 Modeling Strategy

In this section, I describe in detail the empirical methods employed in this paper to understand movements in aggregate matching efficiency and its impact on other macroeconomic variables, especially the unemployment rate. In the first step, the standard matching function (in logs), as surveyed in Petrongolo and Pissarides (2001), is estimated using two-stage least squares. Aggregate matching efficiency is defined by the residuals from the regression related to the matching function that reflects the fundamental ability of matching an existing vacancy with a searching worker. This ability, i.e. aggregate matching efficiency, is not necessarily exogenous. Endogeneity of aggregate matching efficiency may stem from cyclical movements in search intensity and reallocations of workers driven by unequal elasticities of sectoral productivities to aggregate shocks in the spirit of Abraham and Katz (1986).

In the second step, I endogenize the matching efficiency by considering a time-varying parameter vector autoregression (TVP-VAR) model with stochastic volatility. Identifying the fundamental matching efficiency shock that is orthogonal to aggregate shocks via timing restrictions allows me to investigate the importance of mismatch in determining the fluctuations in unemployment over time. The two steps are discussed in more detail in the following subsections.
2.1 Construction of Aggregate Matching Efficiency

The matching function is built to associate the new hires with the vacancies posted and the stock of searching workers.\(^3\) The advantage of the matching function lies in its simplicity in accommodating frictions from various sources without modeling them explicitly. The flow of new hires \(H_t\) is modeled with a standard Cobb-Douglas matching function with constant returns to scale:

\[
H_t = e^{m + \nu_t} U_t^\alpha V_t^{1-\alpha},
\]

where \(m\) is a constant capturing the average aggregate ability of matching a vacancy with a searching worker, \(\alpha\) the constant elasticity of unemployment, \(U_t\) the number of searching workers, \(V_t\) the stock of vacancies, and \(\nu_t\) is a shifter that moves the aggregate ability of matching and is allowed to be driven by aggregate structural shocks. I define \(\nu_t\) as aggregate matching efficiency capturing variation in the flow of new hires that can not be explained by the labor market tightness, i.e. the vacancy-unemployment ratio \(V_t/U_t\), alone. Then, I rewrite Equation (1) as

\[
\ln f_t = m + (1 - \alpha) \ln \frac{v_t}{u_t} + \nu_t,
\]

where \(f_t = \frac{H_t}{U_t}\) is the job-finding rate, \(v_t = \frac{V_t}{L_t}\) the vacancy rate, \(u_t = \frac{U_t}{L_t}\) the unemployment rate, and \(L_t\) defines the level of the labor force.

The time series of job-finding rate allowing for elastic labor participation is constructed following Shimer (2012) using the Current Population Survey (CPS) Basic Monthly Data at the National Bureau of Economic Research (NBER).\(^4\) The Composite Help-Wanted Index constructed by Regis Barnichon (2010) serves as the proxy for vacancies posted. The level of the labor force and the unemployment rate are downloaded from the FRED database maintained by the St. Louis Fed at http://research.stlouisfed.org/fred2/. All of the data are quarterly and run from 1967:Q3 to 2013:Q2.

To obtain aggregate matching efficiency \(\nu_t\), I regress \(\ln f_t\) on \(\ln \frac{v_t}{u_t}\) and a constant. Then the residuals record the history of \(\nu_t\). Before estimation, I conduct the Hausman test for \(\ln \frac{v_t}{u_t}\) using the lagged vacancy rate and unemployment rate as instruments. The test suggests that \(\ln \frac{v_t}{u_t}\) is endogenous implying that aggregate matching efficiency \(\nu_t\) is correlated with the labor market tightness, which is affected by exogenous disturbances over the business cycle. This evidence suggests the importance of endogenizing aggregate matching efficiency for assessing the impact of mismatch properly, as movements in aggregate matching efficiency

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\(^3\)Since I allow nonparticipation in this paper, searching workers are referred to the people who are able and willing to work but fail to be matched with a vacancy. Meanwhile, the universe of non-employed agents consists of searching workers and individuals who are outside the labor force.

\(^4\)The data from June 1967 to December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley.
can originate from its intrinsic shock and aggregate structural shocks that shift demand and
supply as well. Hence, I estimate the regression with two-stage least squares using the lagged
vacancy rate and unemployment rate as instruments.

Figure 1 plots the history of aggregate matching efficiency between 1967:Q3 and 2013:Q2. Aggregate
matching efficiency clearly displays cyclical fluctuations and typically lags business cycles, which
is consistent with the finding by Barnichon and Figura (2014). Specifically, aggregate matching
efficiency starts to decline at the end of recessions or the beginning of expansions, while it rebounds in
the later stage of expansions or shortly before the onset of recessions. Meanwhile, the pattern of
aggregate matching efficiency during the Great Recession and its aftermath, except its deeper and faster
slump at the onset of the recession, is not dramatically different from previous recessions. At first sight,
the lagged responses of aggregate matching efficiency may look puzzling. Nevertheless, given the aggregate
numbers of vacancies and searching workers, the redistribution of workers from low productivity
industries to high productivity industries or moving from a slack local labor market to a tight
one, as discussed in Garin et al. (2013) and Sahin et al. (2012), generally takes time to reach
the new equilibrium because of the training costs and moving costs involved. During the
process of adjustment, aggregate matching efficiency first falls and then recovers thereafter.
Another possible source of the lagged behavior is the variation in search intensity over busi-
ness cycles. Normally, searching workers are losing confidence in succeeding to get a job and
their search intensity may decline gradually as an unemployment spell lasts longer which
is standard in a recession. By contrast, they are confident that they will be matched with
a vacancy and search harder in a boom. Aggregate matching efficiency can stay at a high
level even in the early stage of recessions and keep low until the middle of expansions, as
long as the typical responses of search intensity lag aggregate shocks. Figure 2 displays the
time series of the level of discouraged workers between 1994:Q1 and 2013:Q2 obtained from
FRED with Series ID: LNU05026645. Although the data only cover the most recent two
recessions, the number of discouraged workers generally climbs substantially during the re-
cessions and the early stage of expansions and drops drastically only when a boom is clearly
confirmed. It is arguably convincing that an apparent increase (decrease) in the number of
discouraged workers signals a general decline (rise) in search intensity. At the same time,
other things equal, search intensity positively comoves with aggregate matching efficiency.
Therefore, the observed pattern of discouraged workers implies that movements in search
intensity lag aggregate shocks, which at least partially results in the mildly procyclical but
lagged behaviors of aggregate matching efficiency.

As evident from the Hausman test on the labor market tightness and cyclical fluctuations
of the estimated aggregate matching efficiency, it is crucial to endogenize aggregate matching
efficiency for the sake of properly assessing the role of mismatch in shaping the dynamics of unemployment. Furthermore, observing the predeterminacy of aggregate matching efficiency, it is natural to identify the intrinsic matching efficiency shock using a standard timing restrictions. Both of the exercises are illustrated in Section 2.2.

2.2 Understanding the Endogeneity of Aggregate Matching Efficiency and Its Role

2.2.1 Motivation and Background

A rapidly growing literature studies the source of mismatch, equivalently matching efficiency, and its implications for the labor market fluctuations. Among others, Barnichon and Figura (2014) and Davis et al. (2013) generalize the standard matching function along different lines. Barnichon and Figura (2014) take into account composition of the unemployment pool and dispersion in labor market conditions as determinants of matching efficiency, although their importance is sensitive to the level of disaggregation. Davis et al. (2013) find job-filling rates vary with industries and can be accounted for by a generalized matching function incorporating recruiting intensity (per vacancy). As to research under the general equilibrium framework, see Garin et al. (2013) and Sedlacek (2014). Garin et al. (2013) develop a two-island general equilibrium model of labor reallocation, in the spirit of Lucas and Prescott.
Sedlacek (2014) endogenizes the matching efficiency via firing costs and endogenous separation driven by firms’ hiring standards, which captures almost half of the variation of the estimated matching efficiency in his paper.

My empirical analysis adopts the framework of vector autoregression (VAR) with aggregate data that departs significantly from the existing approaches that rely on disaggregated data in the empirical literature on matching efficiency. The works of Barnichon and Figura (2014), Davis et al. (2013), and many others investigate mismatch and its influences by dividing the labor market into numerous sectors according to certain characteristics - industries, geography and occupations, etc.. Relying on corresponding disaggregated data, they can only evaluate the impact of mismatch for one specific reason at a time. Furthermore, their conclusions are sensitive to the level of disaggregation. If the sources of mismatch are truly orthogonal, the one-at-a-time exercise delivers the correct effects of each type of mismatch. However, such orthogonality is questionable. For example, if a geographical location is dominated by one industry, a persistent shrinkage in that industry clearly generates enormous local long-term unemployment, thereby inducing a slump in local matching efficiency due to mismatch of skills and lower search intensity. This sectoral shrinkage also lowers average household incomes that brings down local house prices drastically. The unemployed
houseowners may be unwilling to sell their houses at a nominal loss and move to another place with greater job opportunities, a phenomenon referred to as the “house-locking” effect, see Valletta (2013). The “house-locking” effect reduces the geographic mobility and amplifies mismatch at the industry level. Thus, the one-at-a-time strategy can be misleading on measuring impact of any specific type of mismatch. By contrast, considering matching efficiency at the aggregate level allows me to take the interactions of different types of mismatch into account and is immune to the sensitivity of disaggregation by construction.

In addition, a vector autoregressive model is a natural statistical device for endogenizing aggregate matching efficiency. In a VAR model, aggregate matching efficiency is allowed to respond to its own intrinsic structural shock as well as aggregate structural shocks suggested by its cyclical behaviors in Figure 1 and the hypothesis in the spirit of Abraham and Katz (1986) that mismatch can rise from sectoral unequal sensitivities to the common aggregate shocks. With the time series of unemployment rate and an estimated aggregate matching efficiency in the VAR model, it can shed light on the contribution of mismatch to unemployment fluctuations under an appropriate identification scheme via the standard impulse response analysis, forecast error variance decomposition, and counterfactual analysis.  

Therefore, a careful and appropriate extraction of the intrinsic (aggregate) matching efficiency shock is the key to our empirical analysis.

For my analysis, the estimated aggregate matching efficiency (AME) $\nu_t$, the labor force participation rate (LFPR) $l_t$, the unemployment rate $u_t$, and the inflation rate $\pi_t$ are included in the VAR model. The inclusion of unemployment and inflation captures aggregate shocks, whereas the labor force participation rate captures a shock to labor supply. The labor force participation, unemployment, and inflation, all of which follow aggregate shocks at no lag or at least a shorter lag than aggregate matching efficiency, ought to adjust ahead of aggregate matching efficiency. Hence, I order the estimated aggregate matching efficiency first in the VAR model and identify the matching efficiency shock by the standard recursive scheme. Based on the identified structural VAR, I am able to determine the competing roles of different structural shocks in shaping the dynamics of unemployment.

Last but not least, the VAR model allows for stochastic volatility and time-varying parameters for which stationarity is imposed in each period of time. It is well known that the United States suffered from highly volatile unemployment and inflation in the 1970s and

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5One may worry about the measurement error derived from the estimation of aggregate matching efficiency. However, parameters in the structural Equation (2) are quite precisely estimated. For example, the elasticity of unemployment in the matching function $\alpha$ is estimated at 0.42 with a very small standard deviation of 0.03. Under the definition of aggregate matching efficiency as the residuals from the standard aggregate matching function, measurement error should not be an issue then.

6This modeling strategy has been widely applied to monetary VAR models, see Cogley et al. (2010) and Primiceri (2005) among others.
early 1980s, and enjoyed stabilized macroeconomic aggregates since the middle of 1980s at
least to the onset of the recent credit crisis in 2007. This stabilization is dubbed the “Great
Moderation” and motivates the inclusion of stochastic volatility in my model to allow for
stochastic volatility in the endogenous variables. Furthermore, the transmission mechanism
of the structural shocks may have changed, for example it is widely noted that recoveries in
the labor market after the three most recent recessions have been slower compared to previ-
ous recessions, a phenomenon known as a “jobless recovery”. Changes in the transmission
mechanism, if present, are easily accounted for by time-varying parameters in the model.
Finally, stationarity imposed on the parameters in each period of time stems from the stance
that one can never expect explosive matching efficiency, labor participation, unemployment
or inflation given a well-defined structure of the economy.

2.2.2 The Time-Varying Parameter Vector Autoregressive (TVP-VAR) Model

The reduced-form TVP-VAR model of order $p$ can be cast in the following form:

$$
y_t = X'_t \theta_t + \mu_t, \quad \mu_t \sim iid. N(0, \Omega_t)
$$

$$
X'_t = I_n \otimes [1, y'_{t-1}, \cdots, y'_{t-p}],
$$

where “$\otimes$” denotes the Kronecker product, $y_t$ is an $n \times 1$ vector including the current
observations of endogenous variables, $X_t$ is an $m \times n$ matrix including intercepts and lagged
variables, $\theta_t$ stacks time-varying reduced-form VAR coefficients and $\Omega_t$ is the time-varying
variance-covariance matrix of the error term $\mu_t$, for $t = 1, 2, \cdots, T$. In this paper, $y_t$ contains
four endogenous variables, so $n = 4$ and $m = 36$ because I set $p = 2$ in order to keep the
dimension of parameter space manageable.

As discussed in the previous subsection on the identification of the matching efficiency
shock, in my benchmark model, $y_t = [\nu_t, l_t, u_t, \pi_t]'$ where $\nu_t$ denotes the (aggregate) matching
efficiency estimated in Section 2.1, $l_t$ the labor force participation rate, $u_t$ the unemployment
rate, and $\pi_t$ is the inflation rate. The order of endogenous variables reflects my identifica-
tion assumption that the matching efficiency only responds to the labor supply shock and aggregate
shocks with at least one-period lag. In addition, the LFPR adjusts to the matching
efficiency shock and the labor supply shock instantaneously, but responds to aggregate
shocks with at least one-period lag.

It is well understood that the order of endogenous variables matters for structural inter-
pretations. However, my results are remarkably robust to the orders of $[\nu_t, l_t, \pi_t, u_t]'$,
$[\nu_t, u_t, \pi_t, l_t]'$, and $[\nu_t, \pi_t, u_t, l_t]'$, i.e. orderings by interchanging unemployment and inflation
within the block of aggregate shocks and/or swapping the LFPR and the block of aggre-
gate shocks. Furthermore, although I do not intend to identify aggregate shocks explicitly under the recursive identification scheme, the structural shocks to unemployment and inflation resemble the aggregate demand and supply shocks, respectively, under the above four specifications. Their behaviors can be seen from the impulse response analysis conducted in Section 4, which shows that the structural shock to unemployment pushes unemployment and inflation in opposite directions, whereas responses of unemployment and inflation to the structural shock to inflation have the same sign.\footnote{Trivariate models without the LFPR are also studied in the early stage of this project. However, the trivariate models convey a “matching efficiency puzzle”, i.e. a positive matching efficiency shock rises the unemployment rate instead of suppressing it. This counterintuitive implication may indicate an omitted variable problem. Suppose a positive labor supply shock hits the economy, this implies more household members participating in the labor force and signals an ongoing higher general search intensity. Absent the LFPR from the model, the positive labor supply shock would be translated into an increase in the matching efficiency. If the new entrants brought about by the positive labor supply shock are just marginal workers who are low-skilled, they may not be absorbed by the economy even in a boom, thereby remaining unemployed and increasing the unemployment rate. Thus, a positive matching efficiency shock may generate a puzzling response of an increase in unemployment rate in the trivariate models that do not include the LFPR.}

Technically, structural shocks are recovered via a Cholesky decomposition of the variance-covariance matrix of the reduced-form error terms as follows:

\[ A_t \Omega_t A_t' = \Sigma_t \Sigma_t', \quad A_t^{-1} \varepsilon_t = \mu_t, \quad \varepsilon_t = \Sigma_t \epsilon_t, \quad \epsilon_t \sim iid. N(0, I_4), \]

\[
A_t = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & a_{21,t} & 1 & 0 \\
0 & a_{31,t} & a_{32,t} & 1 \\
0 & a_{41,t} & a_{42,t} & a_{43,t} & 1
\end{bmatrix}_{4 \times 4}, \quad \Sigma_t = \begin{bmatrix}
\sigma_{11,t} & 0 & 0 & 0 \\
0 & \sigma_{22,t} & 0 & 0 \\
0 & 0 & \sigma_{33,t} & 0 \\
0 & 0 & 0 & \sigma_{44,t}
\end{bmatrix}_{4 \times 4},
\]

where \( \varepsilon_t = [\varepsilon_{\nu t}, \varepsilon_{lt}, \varepsilon_{ut}, \varepsilon_{\pi t}]' \), with the four elements representing the matching efficiency shock, the labor supply shock, the structural shock to unemployment and the structural shock to inflation, respectively. Then, the reduced-form time-varying VAR model can be rewritten as

\[ y_t = X'_t \theta_t + A_t^{-1} \varepsilon_t, \quad (3) \]

\[ X'_t = I_4 \otimes [1, y'_{t-1}, y'_{t-2}], \quad \varepsilon_t \sim iid. N(0, \Sigma_t \Sigma_t'). \]

The time-varying parameters \( \theta_t \) follows a parsimonious driftless random walk as

\[ \theta_t = \theta_{t-1} + \xi_t, \quad \xi_t \sim iid. N(0, Q), \quad (4) \]

where \( Q \) is positive definite.

In terms of the variance-covariance matrix for the VAR errors, let \( \alpha_t \) be a vector collecting
the non-diagonal and non-zero elements in $A_t$ and $\sigma_t$ be a vector collecting the diagonal elements in $\Sigma_t$. Then the evolution of elements in $\alpha_t$ and $\sigma_t$ is as follows:

$$\alpha_t = \alpha_{t-1} + \eta_t, \quad \eta_t \sim iid. N(0, S),$$  \hspace{1cm} (5)

$$\ln \sigma_t = \ln \sigma_{t-1} + \zeta_t, \quad \zeta_t \sim iid. N(0, W),$$  \hspace{1cm} (6)

where $S, W$ are positive definite and $S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$ is block diagonal with each block corresponding to parameters in different equations.

I assume that all of the innovation blocks in the dynamic system are uncorrelated contemporaneously and at all lags and leads—i.e., they are jointly normally distributed with the following variance-covariance matrix $V$:

$$V = \text{Var} \begin{pmatrix} \epsilon_t \\ \xi_t \\ \eta_t \\ \zeta_t \end{pmatrix} = \begin{bmatrix} I_4 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}.$$

## 3 Model Estimation

As discussed above, I study the role of mismatch in the United States by using a two-stage modeling strategy. In the first stage, aggregate matching efficiency is derived as in Section 2.1. This section discusses the data source and how the TVP-VAR model can be estimated in the second stage.

In particular, the TVP-VAR model incorporates the estimated aggregate matching efficiency $\nu_t$, the LFPR $l_t$, the unemployment rate $u_t$, and the inflation rate $\pi_t$. The LFPR $l_t$ is the seasonally adjusted Civilian Labor Force Participation Rate (Series ID: CIVPART), and $u_t$ denotes seasonally adjusted civilian unemployment rate of all workers over 16 (Series ID: UNRATE). The inflation rate is measured by the GDP deflator (continuously compounded annual rate of change, Series ID: CIVPART). All of the data series were downloaded from FRED managed by Federal Reserve Bank of St. Louis at http://research.stlouisfed.org/fred2/. The series are quarterly (averaged from monthly data) and the sample period runs from 1967:Q3 to 2013:Q2.

Estimation of parameters for this model are based on Markov Chain Monte Carlo (MCMC) methods and Bayesian inference. Priors for state vectors and hyperparameters are calibrated from a training sample which consists of the first six years of the sample periods (26 quar-
Specifically, a time-invariant VAR model is estimated to produce point estimates, \( \hat{\theta}_0 \), for the conditional mean parameters and their corresponding variances, \( V(\hat{\theta}_0) \). Estimates, \( \hat{\Omega}_0 \), of the variance-covariance matrix for the VAR errors are obtained as well and \( \hat{\alpha}_0, \hat{\sigma}_0 \) are derived from decomposing \( \hat{\Omega}_0 \). The variance, \( V(\hat{\alpha}_0) \), of \( \hat{\alpha}_0 \) is obtained by simulation from a Wishart distribution with scatter matrix \( \hat{\Omega}_0 \) and degree of freedom set to 24. I set the variance of \( \ln(\hat{\sigma}_0) \) to \( 10I_3 \) which is large in log-scale, implying a small weight is put on the prior. As for the hyperparameters \( Q, S, W \), the priors are inverse-Wishart distributions. In order to put as little weight as possible on prior beliefs, the degree of freedom corresponding to each inverse-Wishart distribution is set to the minimum plausible value \( \text{dim}(Q) + 1 = 37 \), \( \text{dim}(S_1) + 1 = 2 \), \( \text{dim}(S_2) + 1 = 3 \), \( \text{dim}(S_3) = 4 \) and \( \text{dim}(W) + 1 = 5 \), respectively.

In summary, the priors are as follows:

\[
\begin{align*}
\theta_0 &\sim N(\hat{\theta}_0, 4V(\hat{\theta}_0)), \\
\alpha_0 &\sim N(\hat{\alpha}_0, 4V(\hat{\alpha}_0)), \\
\ln \sigma_0 &\sim N(\ln \hat{\sigma}_0, 10I_3), \\
Q &\sim \mathbb{IW}(24k_Q^2V(\hat{\theta}_0), 37), \\
S_1 &\sim \mathbb{IW}(2k_S^2V(\hat{\alpha}_1,0), 2), \\
S_2 &\sim \mathbb{IW}(3k_S^2V(\hat{\alpha}_2,0), 3), \\
S_3 &\sim \mathbb{IW}(4k_S^2V(\hat{\alpha}_3,0), 4), \\
W &\sim \mathbb{IW}(5k_W^2I_3, 5),
\end{align*}
\]

where \( k_Q = k_W = 0.01 \), \( k_S = 0.1 \), and \( \hat{\alpha}_{1,0}, \hat{\alpha}_{2,0}, \hat{\alpha}_{3,0} \) correspond to each block of \( \hat{\alpha}_0 \).

Posteriors of the parameters and hyperparameters are obtained via a Metropolis-within-Gibbs sampler. Except for the rejection sampling procedure for imposing stationarity on \( \theta_t \), the sampler follows Primiceri (2005) and its recent correction by Del Negro and Primiceri (2013) closely in that it adapts methods in Carter and Kohn (1994) and Kim et al. (1998) to draw state vectors \( \theta_t, \alpha_t \) and \( \ln \sigma_t \) from three Gaussian linear state-space systems separately. In terms of the rejection sampling, once the unrestricted \( \theta^T = (\theta_1, \theta_2, \cdots, \theta_T) \) are drawn from the unconditional posterior, I evaluate the vector autoregressive roots related to \( \theta^T \) in each period of time and reject the whole draw of \( \theta^T \) if there is any root lying inside the unit circle, as proposed by Cogley and Sargent (2005). \(^{10}\)

\(^8\)The training sample is chosen to accord with the typical durations of business cycles and sidestep the abnormal era of stagflation since the late 1970s.

\(^9\)See Primiceri (2005) for a full discussion of the reasons behind these values of \( k_Q, k_S, k_W \).

\(^{10}\)The acceptance rate of posterior draws is about 10%.
I consider 70,000 draws from the posterior sampler, discarding the first 10,000 draws known as the “burn-in” period, to allow for convergence to the ergodic distribution. Every 20th draw is saved from the remaining 60,000 draws to economize the storage space. Therefore, Bayesian inferences are carried out based on 3,000 draws from the posterior distribution. Convergence diagnostics are conducted by inspecting sample ACFs for parameter draws. See the detailed MCMC algorithm and the convergence diagnostics in the technical appendix.

4 Empirical Results

Lilien (1982) and Abramham and Katz (1986) sparked a long-lasting debate on the importance of mismatch in the labor market in shaping the unemployment fluctuations in the United States, especially when the unemployment is at an unusually high level and reverses sluggishly to lower values. Understanding the determinants of unemployment is crucial for designing appropriate policy response to the slack labor market. If mismatch resulting from any reason dominates the unemployment fluctuations, it would not help even if the monetary policy becomes much more accommodative because the unemployment would be structural in the sense that it needs to be fixed by longer-term reallocation of workers. By contrast, if aggregate shocks explain the bulk of variation in unemployment, the central bank’s countercyclical policy may be able to address the worst effects of a recession on the unemployment rate.

The TVP-SVAR model developed in this paper provides a powerful tool for analyzing the relative importance of structural shocks, i.e. the impulse response analysis, forecast error variance decomposition and counterfactual analysis. The above three exercises in my analysis are carried out by assuming that VAR parameters remain constant at their current values as time goes forward—i.e., in each period of time, a time-invariant VAR model is assumed based on the time-varying parameter estimates for that period. Evidence obtained from the above three tools are reported in the following Sections 4.1-4.3.

4.1 Impulse Response Analysis

The impulse response analysis indicates the magnitude and persistence of responses of endogenous variables to each structural shock. Hence, it gives us a clear visual impression of how the (aggregate) matching efficiency shock affects key macro aggregates, something

\footnote{This “local-to-date” assumption is common in the literature on bounded rationality and learning, see the “anticipated-utility” model in Kreps (1998), and is implemented in Cogley and Sargent (2005) and Cogley et al. (2010) as well, among many other studies using time-varying parameter models.}
that cannot be properly done in the current literature using disaggregated data. Figures 3 and 4 plot the responses of unemployment and inflation to the unitary structural shocks at selected dates, respectively. Our framework allows the transmission mechanism of structural shocks to change with the time-varying parameters. Therefore, it is interesting to find out how the transmission mechanism evolves over time and if the evolution is nonlinear to recessions and normal times. To do this, I select 1981:Q3, 1990:Q4, 2001:Q3 and 2008:Q4 as the representative dates of the four most recent recessions and also take down representative dates—1984:Q2, 1996:Q1, 2006:Q1 and 2010:Q1 for their corresponding follow-up expansions.\footnote{The results are qualitatively and quantitatively robust to parameters at single dates and parameters averaged over an internal of dates in recessions and expansions.} Then, impulse responses of unemployment and inflation to four structural shocks are evaluated for these selected dates.

Three findings stand out. First, it can be seen from Figures 3 and 4 that the structural shock to unemployment pushes unemployment and inflation in opposite directions, while responses of unemployment and inflation to the structural shock to inflation have the same sign. Hence, the structural shock to unemployment behaves like a demand shock, whereas the structural shock to inflation resembles a supply shock. Second, the impulse responses of unemployment and inflation track each other well over time and across different phases of business cycles, except for the transmissions of the matching efficiency shock and the labor supply shock to unemployment fluctuations, as depicted in Figure 3. This evidence suggests that changes in the transmission mechanism happen in a highly structured way.\footnote{Liu and Morley (2014) study a trivariate monetary VAR under the mixture innovation framework and find that the transmission of monetary policy shock to unemployment is stable over time. However, the response of inflation to monetary policy shock has weakened a lot since the Great Moderation. Figures 3 and 4 do not contradict these findings. Note that the monetary policy shock can be interpreted as one source of a demand shock. In Figures 3 and 4, the responses of unemployment to the structural shock to unemployment over time are almost identical, while the response of inflation to the same shock does decline a bit (about 0.1 percentage points) since the 2000s. Structured changes in time-varying parameters are also documented in Cogley and Sargent (2005).} Generally speaking, except for the counterintuitive positive response of unemployment to a unitary increase in the matching efficiency shock in the early 1980s, the positive effect of the matching efficiency shock has strengthened since the 1990s. Likewise, the sign of the response of unemployment to a positive labor supply shock has been reversed from positive to negative since the 1990s. The negative response becomes sharper since the 1990s and has somewhat weakened following the Great Recession in the late 2007. Finally, the evidence of nonlinearity in the impulse responses with respect to the phase of business cycles is weak. As discussed in the above, there is basically no difference in the responses of inflation to any structural shock over time. The only possible sources of a nonlinear transmission mechanism lie in the impact of the matching efficiency shock and the labor
(a) Matching efficiency shock (AME equation)  
(b) Labor supply shock (LFPR equation)  
(c) Demand shock (unemployment equation)  
(d) Supply shock (inflation equation)

supply shock on the unemployment. However, the evidence here is mixed. For example, the negative responses of unemployment to a positive unitary matching efficiency shock in the 1990 and 2001 recessions are smaller than their counterparts in the follow-up expansions. Nevertheless, the response of unemployment in 2008 recession is somewhat larger than its counterpart in the 2010 expansion. Similarly, in terms of the transmission of labor supply shock, the response of unemployment in the 1990s with respect to business cycle phases is clearly at odds with that of the 2000s. Thus, the evolution of the impulse responses reflects the consequences of variation in the underlined structural parameters over time rather than nonlinearity with respect to phases of business cycles.

4.2 Forecast Error Variance Decomposition

In order to understand the determinants of unemployment fluctuations, it is not enough to merely look at the impulse responses. Besides the transmission mechanism, the variances of structural shocks also play a key role in shaping the dynamics of unemployment. Taking the matching efficiency shock as an example, a unitary increase in log points in the matching efficiency shock is huge and far above the standard error of matching efficiency of 0.089 in log points. However, the matching efficiency shock only leads to a peak decline in unemployment of 0.3 percentage points in 2008:Q4. Hence, the impact of matching efficiency shock may not be as large as it may seem at first sight.

The forecast error variance decomposition results complement the impulse response analysis. Following the “local-to-state” assumption on the VAR parameters, I evaluate the forecast error decomposition in each period of time. Figures 5 and 6 report the time-varying contributions of each structural shock at the horizons of 4 and 20 quarters ahead, respectively. Figure 5 reveals that for a one-year-ahead forecast of the unemployment rate, the structural shock to unemployment almost explains almost all of the forecast error. Conversely, it implies that the other three structural shocks, including the matching efficiency shock, are trivial. At the longer horizon of 20 quarters ahead, in Figure 6, the structural shock to unemployment still dominates in the unemployment fluctuations. However, its contribution decreases from well above 90% to around 75% of the variation in the unemployment rate observed from the posterior medians. Contributions of the other three structural shocks roughly preserve constant patterns no matter whether the horizon is one year ahead or 5 years ahead. The matching efficiency shock generally makes a much larger contribution to variation in the unemployment rate before the 2000s than after. Its contribution peaks in the 1980s at about 14% and 17% and reduces to approximately 1% and 5% at the horizons

14The forecast error variance decomposition at the infinite horizon ahead is similar to the results at the horizon of 20 quarters ahead.
Figure 5: Forecast error variance decomposition of unemployment (in percentage points) at the horizon of 4 quarters ahead between 1967:Q3 and 2013:Q2. Posterior median is in red solid lines. The dashed lines indicate 68% equal-tailed credible intervals.
Figure 6: Forecast error variance decomposition of unemployment (in percentage points) at the horizon of 20 quarters ahead between 1967:Q3 and 2013:Q2. Posterior medians are in red solid lines. The dashed lines indicate 68% equal-tailed credible intervals.
of 4 and 20 quarters, respectively. Meanwhile, the contributions of labor supply shock peak at about 8% of the forecast error in the unemployment rate one year and 5 years ahead. In terms of the structural shock to inflation, its contribution peaks in the late 1970s and starts to decline with the Great Moderation. In particular, for the 5 years ahead forecasting, the structural shock to inflation can explain 10% of the variation in the forecasted unemployment rate before 1982 and around 7% of the fluctuations afterwards.

In summary, aggregate shocks dominate in terms of driving fluctuations in unemployment between 1967:Q3 and 2013:Q2 for the United States. That is to say, unemployment of the United States in this sample period is mainly driven by aggregate demand and supply rather than mismatch in any sense. The evidence provides possible grounds for countercyclical policy interventions to smooth the pain of a recession. The evidence also provides some hints about how to fight the sluggish recovery in the labor market after the Great Recession.

4.3 Counterfactual Analysis: the Great Recession

My findings in terms of the forecast error variance decomposition of unemployment is consistent with the vast empirical literature on mismatch during the Great Recession using disaggregated data. Sahin et al. (2012) argue that mismatch at the industry and occupation level at most accounts for one third of the total increase in the unemployment rate, while geographical mismatch plays a trivial role. In addition, mismatch across industries and occupations adds 0.75 percentage points and 1.5 percentage points in the unemployment rate during 2006-2009, respectively. Barnichon and Figura (2010) draw the conclusion that mismatch contributes little to unemployment. In fact, labor demand dominates the determination of cyclical unemployment, whereas labor supply explains all of the secular trend in unemployment since 1976. They find that mismatch adds in 1.5 percentage points to the unemployment rate in 2009. Herz and Van Rens (2011) develop a simple model of a segmented labor market with search frictions within segments and also find no support for dominance of mismatch in unemployment fluctuations.

However, as I have discussed above, one must be very careful when dealing with the disaggregated data since the sources of mismatch may not be orthogonal. Additionally, the matching efficiency in Sahin et al. (2012), Barnichon and Figura (2010), Herz and Van Rens (2011), and many others is exogenous. If the variation in matching efficiency is actually partly attributed to aggregate shocks, the impact of mismatch would be exaggerated. Observing the cyclical behavior of the estimated aggregate matching efficiency in Figure 1, there is a clear signal of endogeneity for matching efficiency. This can be also seen in Figure 7, which reveals the determinants of aggregate matching efficiency according to my model. Generally,
the aggregate shocks explain 20-40% of the variation in aggregate matching efficiency over time. This evidence casts serious doubts on counterfactual analysis conducted by simply setting matching efficiency to its pre-crisis level in order to assess its implications for the Great Recession in which matching efficiency is treated exogenous.

Figure 8 plots the histories of structural shocks. It is clear that the economy suffers from negative shocks in aggregate matching efficiency, labor supply, and aggregate demand since the onset of the credit crisis in 2007. The controversy between Delong (2010) and Kocherlakota (2010) lies in that whether structural factors, i.e. the matching efficiency shock and labor supply shock in my model, account for the bulk of unemployment fluctuations during the Great Recession. To address this issue, I conduct three counterfactual exercises: I simulate paths of the unemployment rate from 2008:Q2 to 2013:Q2 given “local-to-date” parameters and other structural shocks by (1) setting the matching efficiency shock to zero; (2) setting the labor supply shock to zero; (3) setting both the matching efficiency shock and labor supply shock to zero. Hence, the simulated paths of unemployment rate reflect the hypothetical scenarios in which the structural factors return to their conditional means in the absence of negative shocks to matching efficiency and labor supply.

Figure 9 displays the simulated unemployment rates. The simulated paths of unemployment rate in all of the three exercises track the actual rate closely. This clearly rejects the hypothesis that it was exacerbated structural factors driving the slump in unemployment during the Great Recession. Instead, I can draw a conclusion that aggregate shocks account for the slump and recovery in the labor market, supporting Delong’s argument. My finding is in sharp contrast to the evidence suggested by Sahin et al. (2012) and Barnichon and Figura (2010). They find significantly larger add-ins to the unemployment rate during the Great Recession. This can be ascribed to the exogeneity of matching efficiency in their models, where the variation in matching efficiency driven by other structural shocks are mistakenly attributed to matching efficiency shock. To echo the discussion in Section 2, this counterfactual analysis provides evidence appealing to endogenizing the matching efficiency for the sake of properly measuring the impact of variation in matching efficiency.

5 Conclusion

In this paper, I developed an original two-stage approach to assessing the impact of matching efficiency (equivalently, mismatch) at the aggregate level on unemployment fluctuations. Specifically, in the first stage, aggregate matching efficiency is estimated from the standard Cobb-Douglas matching function. In the second stage, aggregate matching efficiency is incorporated in a TVP-VAR model with the labor force participation rate, the unemployment
Figure 7: Forecast error variance decomposition of matching efficiency (in percentage points) at the horizon of 20 quarters ahead between 1967:Q3 and 2013:Q2. Posterior medians are in red solid lines. The dashed lines indicate 68% equal-tailed credible intervals.
Figure 8: Posterior medians of the histories of structural shocks between 1967:Q3 and 2013:Q2.

(a) Matching efficiency shock (AME equation)
(b) Labor supply shock (LFPR equation)
(c) Demand shock (unemployment equation)
(d) Supply shock (inflation equation)
Figure 9: Simulated paths of unemployment rate between 2008:Q2 and 2013:Q2. (a) Matching efficiency shock set to zero; (b) Labor supply shock set to zero; (c) Both matching efficiency shock and labor supply shock set to zero. Actual rate is in green. Simulated rates are in red. The posterior median in red is reported with the 68% equal-tailed credible intervals in dashed lines.
rate, and inflation. In contrast to the existing literature studying disaggregated data, dealing with mismatch at the aggregate level sidesteps the problematic implicit assumption of orthogonality of sources of mismatch at disaggregated levels (industrial, occupational, geographical, etc.), which is a questionable assumption. Observing that the estimated aggregate matching efficiency lags business cycles, I identify the (aggregate) matching efficiency shock using timing restrictions, i.e. the standard recursive scheme, by ordering aggregate matching efficiency first in the TVP-SVAR model.

Under the identified structural VAR model, with the help of impulse response analysis and forecast error variance decomposition, I find that aggregate shocks rather than aggregate matching efficiency shock dominate unemployment fluctuations in the United States from 1967:Q3 to 2013:Q2. Based on the counterfactual analysis during the Great Recession, the conclusion can be drawn that the surge in the unemployment rate as consequences of the recent credit crisis in 2007-2009 was not structural in the sense that the slump in the labor market appears to have been mainly driven by a shortfall in the aggregate demand rather than by structural factors.

References


Appendix A. Simulating $p(\theta^T, \alpha^T, \sigma^T, Q, S, W|y^T)$

0. State-space models

Model One:

\[ y_t = X_t\theta_t + A_t^{-1}\varepsilon_t, \quad (A.1) \]

\[ \theta_t = \theta_{t-1} + \xi_t. \quad (A.2) \]

Model Two:

\[ \hat{y}_t = D_t\alpha_t + \Sigma_t\varepsilon_t, \quad (A.3) \]

\[ \alpha_t = \alpha_{t-1} + \eta_t, \quad (A.4) \]

where $D_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -\hat{y}_{1,t} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\hat{y}_{1,t} & -\hat{y}_{2,t} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\hat{y}_{1,t} & -\hat{y}_{2,t} & -\hat{y}_{3,t} \end{bmatrix}$.

Model Three:

\[ y^{**}_t = 2h_t + e_t, \quad (A.5) \]

\[ h_t = h_{t-1} + \zeta_t, \quad (A.6) \]

where $y^{**}_t = [y^{**}_1, y^{**}_2, y^{**}_3, y^{**}_4]'$, $y^{**}_t = \ln[(y^*_t)^2 + c]$, $y^*_t = A_t(y_t - X_t\theta_t)$, $c = 0.001$ and $e_t = [e_{1t}, e_{2t}, e_{3t}, e_{4t}]'$ in which $e_{jt}, j = 1, 2, 3, 4$ are log-chi-square distributed.

1. Drawing unrestricted reduced VAR parameters $\theta^T$

Conditional on $y^T, \alpha^T, \sigma^T, Q, S, W$, the state vector $\theta^T$ can be drawn from the state-space model A.1 and A.2 by Gibbs sampling developed in Carter and Kohn (1994). Denote the unrestricted posterior density of $\theta^T$ by $p_U(\theta^T|\alpha^T, \sigma^T, Q, S, W, y^T)$, then

\[
p_U(\theta^T|\alpha^T, \sigma^T, Q, S, W, y^T)
= p_U(\theta^T|\alpha^T, \sigma^T, Q, y^T)
= p_U(\theta_T|\alpha^T, \sigma^T, Q, y_T) \prod_{t=1}^{T-1} p_U(\theta_t|\theta_{t+1}, y_t, \alpha^T, \sigma^T, Q),
\]
where

\[ \theta_t \mid \theta_{t+1}, y^t, \alpha^T, \sigma^T, Q \sim N(\theta_{t|t+1}, P_{t|t+1}), \]
\[ \theta_{t|t+1} = E(\theta_t | \theta_{t+1}, y^t, \alpha^T, \sigma^T, Q), \]
\[ P_{t|t+1} = Var(\theta_t | \theta_{t+1}, y^t, \alpha^T, \sigma^T, Q). \]

The last recursion of forward Kalman filter gives \( \theta_T \) and \( P_T \) from which \( \theta_T \) can be simulated. Then \( \theta_{t|t+1} \) and \( P_{t|t+1}, t = 1, 2, \cdots, T-1 \), are obtained by backward recursions from \( \theta_T \) and \( P_T \). From \( N(\theta_{t|t+1}, P_{t|t+1}) \), I am able to simulate the smoothed estimates of \( \theta_t, t = 1, 2, \cdots, T-1 \). Please see the details of Gibbs sampling in Appendix B.

2. Rejection sampling: drawing stationary reduced VAR parameters \( \theta^T \)

To sample from the target posterior density, \( p(\theta^T | \alpha^T, \sigma^T, Q, S, W, y^T) \), of stationary \( \theta^T \), I evaluate roots of the VAR polynomials associated with the unrestricted \( \theta^T \) simulated from \( p_U(\theta^T | \alpha^T, \sigma^T, Q, S, W, y^T) \) in each period of time and discard the whole draw \( \theta^T \) if there is any root lying inside the unit circle at any date.

3. Drawing hyperparameter \( Q \)

Since I assume the prior of \( Q \) is the inverse-Wishart distribution \( \mathbb{IW}(Q, \nu_Q) \), hence \( Q^{-1} \) is governed by Wishart distribution as:

\[ Q^{-1} \sim \mathbb{W}(Q^{-1}, \nu_Q). \]

Then, the posterior for \( Q^{-1} \) conditional on other blocks is Wishart as well:

\[ Q^{-1}|y^T, \theta^T, \alpha^T, \sigma^T, S, W \sim \mathbb{W}(\overline{Q}^{-1}, \overline{\nu}_Q), \]

where

\[ \overline{Q}^{-1} = \left[ Q^{-1} + \sum_{t=1}^{T} (\theta_{t+1} - \theta_t)(\theta_{t+1} - \theta_t)' \right]^{-1}, \quad \text{and} \quad \overline{\nu}_Q = \nu_Q + T. \]

4. Drawing covariances \( \alpha^T \)

Reconsider the Gaussian linear state-space model A.3 and A.4 under the assumption of block-diagonal \( S \). Since \( \hat{y}_{1,t} \) is determined by exogenous identity shock \( \epsilon_{1t} \) and \( \sigma_{11,t} \), thus conditional on other blocks, \( \hat{y}_{1,t} \) is predetermined in \( \hat{y}_{2,t} \)'s equation. So are \( \hat{y}_{1,t} \) and \( \hat{y}_{2,t} \) predetermined in
\( \tilde{y}_{3,t} \)'s equation. Likewise, \( \tilde{y}_{1,t}, \tilde{y}_{2,t} \) and \( \tilde{y}_{3,t} \) are predetermined in \( \tilde{y}_{4,t} \)'s equation. Therefore, \( \alpha_t \) can be obtained by applying Kalman filter and the backward recursion equation by equation. Let \( \alpha_t = [\alpha_{1,t}, \alpha_{2,t}, \alpha_{3,t}]' \), where \( \alpha_{1,t} = \alpha_{21,t}, \alpha_{2,t} = [\alpha_{31,t}, \alpha_{32,t}]' \) and \( \alpha_{3,t} = [\alpha_{41,t}, \alpha_{42,t}, \alpha_{43,t}]' \) are corresponding to different blocks in \( S \), then the smoothed estimate of \( \alpha_t \) is derived from

\[
\begin{align*}
\alpha_{i,t} | \alpha_{i,t+1}, y^t, \theta^T, S_i, \sigma^T & \sim N(\alpha_{i,t+1}, A_{i,t+1}), \\
\alpha_{i,t+1} & = E(\alpha_{i,t} | \alpha_{i,t+1}, y^t, \theta^T, S_i, \sigma^T), \\
A_{i,t+1} & = Var(\alpha_{i,t} | \alpha_{i,t+1}, y^t, \theta^T, S_i, \sigma^T), \quad i = 1, 2, 3.
\end{align*}
\]

5. Drawing hyperparameter \( S \)

Recall that I separate \( S \) into three blocks \( S_1, S_2 \) and \( S_3 \) each governed by inverse-Wishart distribution \( \mathcal{IW}(S_j, \nu_{S_j}) \), \( j = 1, 2, 3 \). Equivalently, \( S_j^{-1} \sim \mathcal{IW}(S_j^{-1}, \nu_{S_j}) \), \( j = 1, 2, 3 \). Thus, the conditional posterior for \( S_j, j = 1, 2, 3 \), are as follows:

\[
S_j^{-1} | y^T, \theta^T, \alpha^T, \sigma^T, Q, W \sim \mathcal{IW}(S_j^{-1}, \nu_{S_j}),
\]

where

\[
\overline{S}_j^{-1} = \left[ S_j^{-1} + \sum_{t=1}^{T} (\alpha_{j,t+1} - \alpha_{j,t})(\alpha_{j,t+1} - \alpha_{j,t})' \right]^{-1} \quad \text{and} \quad \nu_{S_j} = \nu_{S_j} + T.
\]

6. Drawing stochastic volatility \( \sigma^T \)

The stochastic volatility \( \sigma^T \) are drawn from the non-Gaussian linear state-space model A.5 and A.6 based on a mixture of seven normals approximation \( \text{a la} \) Kim et al. (1998) with component probabilities \( q_t \), means \( m_l - 1.2704 \) and variances \( v_l^2, l = 1, 2, \cdots, 7 \). Please see the constants \( \{q_l, m_l, v_l^2\} \) chosen for matching a number of moments of the \( \log(\chi^2(1)) \) distribution in Kim et al. (1998). Note that \( y_{it}^* \) and \( y_{jt}^* \) are independent of one another for \( i \neq j \), hence, \( e_{it} \) is independent of \( e_{jt} \) as well. Thus, it allows us to employ the same mixture of normals to approximate any element in \( e_t \).

Define the state-indicator matrix \( s^T = [s_1, s_2, \cdots, s_T]' \), \( s_t = [s_{1t}, s_{2t}, s_{3t}]' \), \( s_{jt} \in \{1, 2, \cdots, 7\}, \ j = 1, 2, 3 \) and \( t = 1, 2, \cdots, T \), showing in each period of time which member of the mixture of normals is used for each element of \( e_t \). Then, \( s^T \) can be updated as in Kim et al. (1998) for each \( s_{jt} \) independently from the discrete density

\[
Pr(s_{jt} = l | y_{jt}^*, h_{jt}) \propto q_l f_N(y_{jt}^* | 2h_{jt} + m_l - 1.2704, v_l^2), \quad j = 1, 2, 3, \ l = 1, 2, \cdots, 7,
\]

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where $f_N(\cdot)$ stands for the normal density.

Conditional on other blocks, after determining the members of the mixture of normals used for approximation for $e_t$, the system obtained is a Gaussian linear state-space model in which $h_t$ can be easily drawn based on the standard Kalman filtering and backward recursions as in previous steps. Specifically, smoothed estimates of $h_t$ can be drawn recursively from

\[
\begin{align*}
    h_t \mid h_{t+1}, y^t, \theta^T, \alpha^T, W, s^T & \sim N(h_{t|t+1}, H_{t|t+1}), \\
    h_{t|t+1} & = E(h_t \mid h_{t+1}, y^t, \theta^T, \alpha^T, W, s^T), \\
    H_{t|t+1} & = Var(h_t \mid h_{t+1}, y^t, \theta^T, \alpha^T, W, s^T).
\end{align*}
\]

Finally, the smoothed estimate of $\sigma_t$ can be recovered by transform $\sigma_t = \exp\{0.5h_t\}$.

\section*{7. Drawing hyperparameter $W$}

Note that $W \sim \mathbb{IW}(W, \nu_W)$, i.e. $W^{-1} \sim \mathbb{W}(W^{-1}, \nu_W)$, where $\mathbb{W}(\cdot, \cdot)$ and $\mathbb{IW}(\cdot, \cdot)$ stand for Wishart distribution and inverse-Wishart distribution, respectively. Hence, the posterior for $W^{-1}$ conditional on other blocks reads:

\[
W^{-1} \mid y^T, \theta^T, \alpha^T, \sigma^T, Q, S \sim \mathbb{W}(\overline{W}^{-1}, \overline{\nu}_W),
\]

where

\[
\overline{W}^{-1} = \left[ W^{-1} + \sum_{t=1}^T (h_{t+1} - h_t)(h_{t+1} - h_t)^T \right]^{-1} \quad \text{and} \quad \overline{\nu}_W = \nu_W + T.
\]
Appendix B. Gibbs Sampling for State-Space Models

I cast the Gaussian linear state-space models considered in this paper into the following state-space form:

Measurement equation:
\[ y_t = F_t \beta_t + u_t, \]

State equation:
\[ \beta_t = \beta_{t-1} + v_t, \]

where
\[
\begin{bmatrix} u_t \\ v_t \end{bmatrix} \sim iid. \ N \left( \begin{bmatrix} u_t \\ v_t \end{bmatrix}, \begin{bmatrix} R_t & 0 \\ 0 & Q \end{bmatrix} \right).
\]

Define
\[
\beta_{t|s} = E(\beta_t|y^s, F^s, R^s, Q),
\]
\[
P_{t|s} = Var(\beta_t|y^s, F^s, R^s, Q).
\]

Given the mean and variance of the initial state, \( \beta_{0|0} \) and \( P_{0|0} \), the forward Kalman filter yields:

\[
\beta_{t|t-1} = \beta_{t-1|t-1},
\]
\[
P_{t|t-1} = P_{t-1|t-1} + Q,
\]
\[
\kappa_t = P_{t|t-1}F_t'\left(F_tP_{t|t-1}F_t' + R_t\right)^{-1},
\]
\[
\beta_{t|t} = \beta_{t|t-1} + \kappa_t(y_t - F_t\beta_{t|t-1}),
\]
\[
P_{t|t} = P_{t|t-1} - \kappa_tF_tP_{t|t-1}.
\]

After obtaining \( \beta_{T|T} \) and \( P_{T|T} \), I draw \( \beta_T \) from \( N(\beta_{T|T}, P_{T|T}) \). Then the draw of \( \beta_T \) and the output derived from the above forward Kalman filter are used for backward recursion as follows:

\[
\beta_{t|t+1} = \beta_{t|t} + P_{t|t}P_{t+1|t}^{-1}(\beta_{t+1} - \beta_{t|t}),
\]
\[
P_{t|t+1} = P_{t|t} - P_{t|t}P_{t+1|t}^{-1}P_{t|t},
\]

which provide \( \beta_{T-1|T} \) and \( P_{T-1|T} \) that are used to generate \( \beta_{T-1} \). Likewise, \( \beta_{T-2}, \beta_{T-3}, \ldots, \beta_1 \) are drawn from \( N(\beta_{T-2|T-1}, P_{T-2|T-1}) \), \( N(\beta_{T-3|T-2}, P_{T-3|T-2}) \), \ldots, \( N(\beta_1|2, P_1|2) \), respectively.
Appendix C. Convergence Diagnostics

Figure 10: 20th-Order Autocorrelations for Parameter Draws. From left to right, conditional mean parameters $\theta_t$ from 1-5688, covariances $\alpha_t$ from 5689-6636, variances $\sigma_t$ from 6637-7268 and hyper-parameters $Q, S, W$ from 7269-8616, where $t = 1, 2, \cdots, T$ and $j = 1, 2, \cdots, 6$. 