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Disinflations in a model of imperfectly anchored expectations*

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Abstract

We study disinflations under imperfect credibility of the central bank. Imperfect credibility is modeled as the extent to which agents rely on adaptive learning to form expectations. Lower credibility increases the mean, variance, and skewness of the distribution of sacrifice ratios. When credibility is low, disinflationary policies become very costly for adverse realizations of the shocks. Even if the impact of an announcement decreases with lower credibility, pre-announcing a disinflation reduces the sacrifice ratio. Additionally, disinflationary policies implemented after a period of below trend inflation lead to lower sacrifice ratios. Opportunistic disinflations are desirable when credibility is low.

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1 Introduction

Advanced economies have succeeded in keeping inflation low after the high inflation rates of the 1970's, but other economies have not. In Argentina, for example, inflation is once again the main macroeconomic concern. The case for price stability is by now well-established and studies of disinflations, like Ascari and Ropele (2013) and Ireland (1995), make clear that the long-term welfare gains from low inflation exceed the short-term costs of reducing it. Argentina, therefore, must disinflate. But it must do so in a context of imperfect credibility as official price statistics have been seriously questioned for the past 8 years.[1] In addition, many central banks that have run up against the zero lower bound have engaged in large monetary expansions. Critics of these policies warn that outbreaks of high inflation are possible and that the cost of lowering inflation is high, particularly if credibility is damaged due to missed targets.[2]

The empirical evidence suggests that the fear of costly disinflations is justified. Estimates of the mean sacrifice ratio - the cumulative percentage of output growth lost for decreasing the annual inflation rate by one percent - range between one and four. And each mean estimate comes from a distribution with a high variance.[3] These estimates, however, may be relatively uninformative for policy makers wishing to disinflate because the empirical distribution of sacrifice ratios is unconditional in the sense that it includes intended and unintended disinflations from countries with varying degrees of credibility. Katayama et al. (2015) for example note that attempts to explain the determinants of the sacrifice ratio in the data over the last three decades have yielded few robust predictors.

Recent theoretical studies of the cost of disinflation such as Ascari and Ropele (2012,b,

[1] Cavallo (2013) shows that inflation in Argentina is known to have accelerated while official statistics of inflation have not reflected the change. The official inflation rate has stood at around 10% in annual terms, while estimates from the Billion Prices Project at the MIT have put it closer to 40%.

[2] A prominent example of critics of unconventional monetary policy is the open letter signed by a number of prominent economists in November 2010 asking Federal Chairman Ben Bernanke to reconsider the use of quantitative easing (http://blogs.wsj.com/economics/2010/11/15/open-letter-to-ben-bernanke/).

[3] The range of sacrifice ratios we refer to are those estimated by Ball (1994b), Cecchetti and Rich (2001), and Gonçalves and Carvalho (2009).

2013) show that medium-scale monetary models of the kind proposed by Christiano et al. (2005) yield perfect foresight predictions that are consistent with the estimated positive mean of the unconditional distribution of sacrifice ratios. These studies conclude that the models give reasonable estimates for the mean sacrifice ratio. But the sacrifice ratio implied by a nonstochastic perfect foresight simulation represents a single point in a distribution of stochastic sacrifice ratios. It corresponds to the realization for which all other shocks are zero. In practice, for a policymaker wishing to implement a disinflation program, the properties of the conditional distribution are important. The contribution of this paper is to study how credibility, policy design, and luck act on the conditional distribution of sacrifice ratios of disinflationary policies in estimated sticky price models.

To do so, we propose a framework to model imperfect credibility that links agents' forecasting functions to past data. Our framework speaks to the view of policymakers that the path of inflation over the medium term is an important determinant of the extent to which long-term expectations of inflation are anchored.[4] Low credibility may be thought of as an environment in which expectations are poorly anchored and as such depend more on the path of data than what is captured by the textbook rational expectations solution. To capture this data dependence, we define credibility as the degree to which agents rely on adaptive learning expectations to form expectations. In particular, we assume boundedly rational agents form expectations using as a linear combination of the textbook rational expectations solution and adaptive learning expectations estimated on observed data. These expectations are akin to a combined forecast where the textbook rational expectations solution represents the forecast under full credibility. Combining expectations in this way allows us to introduce anticipated state-contingent policy changes into the traditional adaptive learning framework of Evans and Honkapohja (2001) in a parsimonious way. The combination of backward and forward-looking information in expectations makes policy outcomes a function of the degree of credibility

[4]See the 2009 speech by Donald Kohn, former Vice Chairman of the Federal Reserve, available here.

as well as of the path of aggregate variables. As a result, shocks play a prominent role in realized disinflation outcomes.[5]

Imperfect credibility, as the literature identifies, can go some way to explain a positive sacrifice ratio. In the specifications of imperfect credibility of Ball (1995) and Huh and Lansing (2000) the central bank may renege on a disinflation plan with some known probability. In Goodfriend and King (2005) and Ascari and Ropele (2013) this probability is either fixed or exogenously increasing as the disinflation program is implemented. In our case, however, imperfect credibility is different because it is explicitly linked to agents' reliance on realized data. The degree to which forecasting functions depend on data increases the mean of the sacrifice ratio, like in the alternative specifications of imperfect credibility, but it has implications which have not been noted in the literature for higher moments of the distribution.

The design of a disinflation policy such as whether it is anticipated or unanticipated, cold turkey or gradual, influences the distribution of sacrifice ratios. Pre-announcements shift the distribution of sacrifice ratios to the left and reduce right skewness. Perhaps surprisingly, these benefits exist at low levels of credibility. This is because even with low credibility an announcement moves expectations and inflation towards the new inflation target. This movement feeds back on expectations through adaptive learning, which reduces the cost of disinflation. A gradual policy, where the inflation target is lowered slowly, also shift the distribution to the left and reduces right skewness, but implies a larger variance because a gradual disinflation lasts longer and is therefore exposed to more shocks.[6]

[5]Huh and Lansing (2000) also study disinflation under imperfect credibility with adaptive learning. However, they do not consider the distribution of sacrifice ratios or anticipated policies. Hommes and Lustenhouwer (2015) consider the case of heterogeneous expectations where a continuum of agents select between a fundamentals based forecasting rule and naive forecasting rule. The degree to which agents rely on the fundamentals based rule is interpreted as a measure credibility, which corresponds closely with our notion of credibility. Our work is also related to Erceg and Levin (2003) who model imperfect credibility as an information problem.

[6]Our findings are consistent with those of Ascari and Ropele (2013) who find that gradual and anticipated disinflation also reduce the sacrifice ratio under perfect credibility.

When there is imperfect credibility, expectations depend more on data which opens up the possibility of opportunistic disinflations, where policymakers take advantage of periods of below trend inflation to announce a new inflation target. There is evidence to suggest that many central banks followed this strategy in the early 1990's when inflation targeting regimes were first established.[7] We find that an opportunistic approach effectively lowers the mean and variance of the sacrifice ratio. We quantify the gains from this policy to be large enough that, for empirically plausible values, opportunism can trump credibility.

The rest of the paper is structured as follows. Section 2 extends the sample of disinflation episodes of Ball (1994b) and characterizes the unconditional distribution of sacrifice ratios. A regression analysis shows evidence which supports the quantitative findings of Section 5. Section 3 discusses the general framework for expectations and how we model imperfect credibility. Section 4 contains analytical results on the mechanics of disinflation with imperfect credibility. Section 5 studies the conditional distribution of sacrifice ratios in estimated standard sticky-price models. Section 6 concludes.

2 Sacrifice ratios in the data

We begin by constructing an up-to-date empirical distribution of sacrifice ratios with the objective to document two stylized facts that point to shocks playing an important role in determining individual disinflation outcomes:

1. The empirical distribution of sacrifice ratios is large.

2. There is a weak relationship between the sacrifice ratio and observable characteristics of disinflation episodes.[8]

[7]For example, the Reserve Bank of Australia formalized its inflation targeting framework after a significant fall in inflation in the early 1990's. Bomfim and Rudebusch (2000) note that opportunistic policies where discussed in Federal Reserve FOMC meetings in the late 1980's.

[8]The first point is implicit both in Ball (1994b) and Gonçalves and Carvalho (2009) who note that

We use an unbalanced panel data set containing quarterly CPI and real GDP growth from 1960 to 2014 for 42 countries. Our data set is assembled from data obtained from the OECD, the World Bank and the Federal Reserve Economic Database. We use annual GDP for countries with long histories of quarterly CPI data that did not have corresponding quarterly GDP data. Places where this substitution is made are clearly marked in our tables.[9]

Following Ball (1994b), a disinflation episode is defined as a two percentage point or greater decrease in trend CPI inflation from peak to trough, where trend inflation is calculated as a nine quarter moving average. Peaks and troughs are defined as points where trend inflation is higher or lower than the previous and following four quarters respectively. We identified 150 disinflation episodes, a considerably larger sample than the 25 episodes of Ball (1994b) and the 58 episodes of Gonçalves and Carvalho (2009).[10]

The sacrifice ratio, SR, of each disinflation episode is defined as

$$SR = -\frac{1}{\Delta\pi} \sum_{t=Peak}^{Trough} \left(\frac{Y_t - \bar{Y}_t}{\bar{Y}_t} \right), \tag{1}$$

where the denominator, $\Delta\pi$, is the change in trend inflation, the difference between inflation at a *peak* and at a *trough*. The numerator is the sum of output losses, measured as the deviation between log output, Y_t, and trend log output, $\bar{Y}_t$. Again following Ball (1994b), trend log output is approximated by the line that connects log output in the quarter that inflation peaks to the level of log output four quarters after the

the sacrifice ratio can vary from 1 to 4 and -3 to 21, respectively, depending on the countries and time periods considered. The second point is made recently by Katayama et al. (2015) who note that there is mixed evidence in the empirical literature over what determines the size sacrifice ratio. Taken together, these two facts suggest that shocks play a large role in individual disinflation outcomes. Therefore, the distribution of sacrifice ratios, and not just its mean, should be of interest to policymakers wishing to disinflate.

[9]Our dataset is available upon request.

[10]Ball (1994b) and Gonçalves and Carvalho (2009) do not consider disinflation episodes for which trend inflation exceeds 20%. This restriction, however, omits intended disinflation episodes such as Chile's disinflation in the 1990's. We not consider disinflation episodes for which trend inflation exceeds 50%.

trough.[11] Tables 5 and 6 in the appendix report the dates, duration, size, and sacrifice ratio estimates for our sample.

Figure 1 shows the unconditional distribution of the sacrifice ratio. The mean sacrifice ratio is 3.08, which accords well with the existing literature, with a standard deviation of 9.04.[12] The estimate is unconditional because it includes intended and unintended disinflations.

Next, we regress the sacrifice ratio on observable characteristics of a disinflation episode (the size of the disinflation, the peak level of trend inflation, and disinflation duration) and on measures of central bank governance to explore conditional sacrifice ratios.

Table 1 reports the regression results for the observable characteristics of the disinflation episodes. Consistent with recent findings in the literature (Katayama et al (2015)), the observables characteristics of the disinflations explain only a small fraction (R-squared = 0.129) of the observed variation in sacrifice ratios. As we show with simulations of estimated models, this is not surprising given that shocks and imperfect credibility imply large distributions of sacrifice ratios for any disinflation policy. Reading the point estimates for economic significance, the size of the disinflation and the initial level of inflation are negatively correlated with the sacrifice ratio. And longer disinflations have an ambiguous relationship with the sacrifice ratio as the point estimates change signs based on the inclusion of covariates.

Table 2 shows the results for OLS regressions of the sacrifice ratio on measures of central bank credibility and intent to disinflate. The measures are constructed using data from Dreher et al. (2008) and Dreher et al. (2010) on central bank governance. The data contain information on appointment and tenure of central bank governors for all central banks from 1972 to the present.

[11] With annual GDP data, we assume GDP grew equally in each quarter.

[12] Gonçalves and Carvalho (2009) finds a mean sacrifice ratio of 5.6, Ball (1994b) finds 1.4, and for the Volcker disinflation Blinder (1987) finds 4.2, and Sachs (1985) 2.9.

Our first indicator variable, 'New CBG', takes a value equal to one if a new central bank governor is appointed in the year of, or the year preceding the start of a disinflation episode, and zero otherwise. This measure proxies both for credibility and intent to disinflate because a change in leadership near a peak in trend inflation is likely correlated with a desire to change policy. The second indicator, 'Reappointed', takes a value equal to one if a central bank governor is reappointed to an additional term during the course of a disinflation and zero otherwise. This measure proxies for credibility since reappointment during a disinflation episode suggests that the policy environment was deemed satisfactory. Finally, the third indicator, 'Irregular appointment', takes a value equal to one if there is a new central bank governor appointed in the year preceding the start of a disinflation episode and that appointment was irregular, i.e. not due to a term limit, and zero otherwise. This variable proxies for intent and for anticipation of the disinflation episode since an irregular appointment of a central bank governor may be triggered by high inflation. For example, this variable takes a value of one for the Volker-disinflation in the US.

The intent, credibility, and anticipation measures explain little of the observed variation in sacrifice ratios. The new appointment of a central bank governor and the reappointment variables, which proxy for intent and credibility, have an ambiguous relationship with the sacrifice ratio. Neither variable is statistically significant and the sign of the point estimate changes based on the covariates included in the regression. The irregular appointment variable, which proxies for intent and anticipation of the disinflation episode, is negatively correlated with the sacrifice ratio. This suggests that anticipated intended disinflations are less costly.

The regressions demonstrate that key determinants of the sacrifice ratio identified in the theoretical literature such as credibility or gradualism (Ascari and Ropele (2013)) are not based on strong correlations in the data. Luck plays a large role in observed disinflation outcomes which implies, as we flagged in the introduction, that the distribution (and not just its mean) matters. The framework we propose in the next section allows us to study the distribution. It shows that, although different disinflation policies impact the distribution of sacrifice ratios, these changes are not always enough to give rise to statistically significant differences in *mean* sacrifice ratios, a fact consistent with the econometric evidence.

3 A framework for modeling imperfect credibility

This section introduces our framework of imperfect credibility in the context of a linearized structural model and discusses stability in relation to adaptive learning. The basic idea is similar to the way in which others have incorporated imperfect credibility in studies of disinflations. In these studies, imperfect credibility is introduced by modifying inflation expectations (Goodfriend and King (2005); Ascari and Ropele (2013)) so that expectations are partially linked to the old inflation target ($\bar{\pi}$) and given by $\hat{\mathbb{E}}_t \pi_{t+1} = \lambda \mathbb{E}_t \pi_{t+1} + (1-\lambda)\bar{\pi}$, where the weight on the old target is the probability agents assign to a policy reversal. We build on this idea but expand it to the full vector of endogenous and exogenous variables.

Consider the class of linearized structural models of n equations of the form

$$y_t = \mathbf{\Gamma} + \mathbf{A} y_{t-1} + \mathbf{B} \hat{\mathbb{E}}_t y_{t+1} + \mathbf{D} \varepsilon_t, \tag{2}$$

where y_t is a $n \times 1$ vector of state and jump variables and ε_t is a $l \times 1$ vector of exogenous variables. Without loss of generality, we take the latter to be white noise.[13] We assume $\hat{\mathbb{E}}_t y_{t+1}$ is a linear combination of the form

$$\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbb{E}_t y_{t+1} + (1-\lambda) \tilde{\mathbb{E}}_t y_{t+1} \tag{3}$$

[13]All matrices in Equation (2) conform to the specified dimensions. The method we develop here may be further generalized as in Binder and Pesaran (1995) to allow additional lags of y_t as well as expectations at different horizons and from earlier dates.

where $0 \leq \lambda \leq 1$ and $\mathbb{E}_t y_{t+1}$ is the forecasting function that is obtained under rational expectations and $\tilde{\mathbb{E}}_t y_{t+1}$ is the forecasting function under adaptive learning.

Rational expectations

If $\lambda = 1$, expectations coincide with the textbook rational expectations solution ($\hat{\mathbb{E}}_t y_{t+1} = \mathbb{E}_t y_{t+1}$). The solution to Equation (2) is a VAR of the form

$$y_t = \mathbf{C} + \mathbf{Q} y_{t-1} + \mathbf{G} \varepsilon_t, \tag{4}$$

which we take to be the McCallum (1983) minimum state variable (MSV) solution. We restrict attention to parameter values that yield uniquely bounded solutions as is typically done in the literature. Given that $\mathbb{E}_t \varepsilon_{t+1} = 0$, it follows from Equation (4) that the forecasting function is $\mathbb{E}_t y_{t+1} = \mathbf{C} + \mathbf{Q} y_t$.

Adaptive learning

If $\lambda = 0$, we follow Evans and Honkapohja (2001) and solve the model given by Equation (2) under adaptive learning by assuming that agents understand the reduced form structure of the economy but do not know how it is parameterized. Without loss of generality, agents are assumed to have a *perceived law of motion* (PLM) of the economy given by

$$y_t = \tilde{\mathbf{C}} + \tilde{\mathbf{Q}} y_{t-1} + \tilde{\mathbf{G}} \varepsilon_t, \tag{5}$$

consistent with the MSV solution.[14]

Agents parameterize the model by recursively estimating a VAR of the form of Equation (5) using observed past data. In particular, agents estimate the parameters using the recursive least squares algorithm with information up until time $t-1$. To coincide

[14]The form of the MSV may vary depending on information set assumptions and may exclude terms to prevent multicollinearity in real time learning. See Evans and Honkapohja (2001) for a discussion of these issues.

with the timing of the rational expectations solution, however, we assume agents use information through time t in their forecasts.

Agents estimate the parameters of the PLM, $\xi_t' = \begin{pmatrix} \tilde{\mathbf{C}}_t, & \tilde{\mathbf{Q}}_t, & \tilde{\mathbf{G}}_t \end{pmatrix}$, recursively

$$\xi_t' = \xi_{t-1}' + \gamma \mathbf{\Omega}_t^{-1} z_{t-1}(y_{t-1} - \xi_{t-1}' z_{t-1}) \tag{6}$$

$$\mathbf{\Omega}_t = \mathbf{\Omega}_{t-1} + \gamma(z_{t-1} z_{t-1}' - \mathbf{\Omega}_{t-1}) \tag{7}$$

where $z_t' = \begin{pmatrix} 1 & y_t' & \varepsilon_t' \end{pmatrix}$, $\mathbf{\Omega}_t$ is the variance covariance matrix of z_t, and γ is the gain parameter. As is common in literature, we restrict our attention to the case where γ is a constant $0 < \gamma < 1$. A constant gain is similar to assuming rolling window regressions, where more weight is placed on newer observations and older observations are down-weighted or dropped from the sample.

The forecasting functions under adaptive learning are time-varying and given by $\tilde{\mathbb{E}}_t y_{t+1} = \tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t$. Substituting this expression into Equation (2) and re-arranging terms gives rise to the *actual law of motion* (ALM) for the economy

$$y_t = (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}(\mathbf{\Gamma} + \mathbf{B}\tilde{\mathbf{C}}_t + \mathbf{A}y_{t-1} + \mathbf{D}\varepsilon_t). \tag{8}$$

which implies a T-mapping given by

$$\begin{aligned} \bar{\mathbf{C}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}(\mathbf{\Gamma} + \mathbf{B}\tilde{\mathbf{C}}_t) \\ \bar{\mathbf{Q}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}\mathbf{A} \\ \bar{\mathbf{G}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}\mathbf{D}, \end{aligned}$$

which maps adaptively formed expectations into the structure of the economy. The fixed point of this mapping is exactly the rational expectations equilibrium given by $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$ and is the limit of the recursive learning algorithm under well-known regularity conditions.

The solution for $0 < \lambda < 1$

In the standard adaptive learning framework expectations are backward-looking in the sense that forecasting functions respond only to past data. To add a forward-looking component to learning, we propose to partially anchor expectations to the forecasting function that would prevail under the textbook rational expectations solution. That is,

$$\begin{aligned}
\hat{\mathbb{E}}_t y_{t+1} &= \lambda \mathbb{E}_t y_{t+1} + (1-\lambda)\tilde{\mathbb{E}}_t y_{t+1} \\
&= \lambda(\mathbf{C} + \mathbf{Q} y_t) + (1-\lambda)(\tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t)
\end{aligned}$$

where $0 \leq \lambda \leq 1$. Since a fraction λ of the forecasting function does not respond to incoming data, λ can be interpreted as the extent to which expectations are anchored.

The reduced form solution when expectations are given by Equation (3) is now

$$y_t = \hat{\mathbf{C}}_t + \hat{\mathbf{Q}}_t y_{t-1} + \hat{\mathbf{G}}_t \varepsilon_t \tag{9}$$

where

$$\begin{aligned}
\hat{\mathbf{C}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}(\mathbf{\Gamma} + \lambda\mathbf{BC} + (1-\lambda)\mathbf{B}\tilde{\mathbf{C}}_t) \\
\hat{\mathbf{Q}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{A} \\
\hat{\mathbf{G}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{D}.
\end{aligned}$$

The fixed point of the mapping is again $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$.

The reduced-form solution nests rational expectations and adaptive learning expectations as special cases. If all weight is placed on rational expectations or if all the weight is placed on adaptive learning, then the standard solutions are obtained.

Announcements under perfect credibility

Next, we incorporate announcements as in Cagliarini and Kulish (2013) using the Kulish and Pagan (2012) VAR representation of this solution. Under perfect credibility λ is equal to 1. Assume at time T^a there is an announcement. Agents' views of how the structural equations will evolve are

$$\begin{aligned} y_t &= \mathbf{\Gamma} + \mathbf{A} y_{t-1} + \mathbf{B}\mathbb{E}_t y_{t+1} + \mathbf{D}\varepsilon_t \quad \text{for} \quad t = T^a, T^a+1, \ldots, T^* \\ y_t &= \mathbf{\Gamma}^* + \mathbf{A}^* y_{t-1} + \mathbf{B}^*\mathbb{E}_t y_{t+1} + \mathbf{D}^*\varepsilon_t \quad \text{for} \quad t = T^*+1, T^*+2, \ldots \end{aligned}$$

Kulish and Pagan (2012) show that from the time of the announcement until the time of implementation, the solution for y_t evolves as a time varying coefficient VAR of the form

$$y_t = \mathbf{C}_t + \mathbf{Q}_t y_{t-1} + \mathbf{G}_t \varepsilon_t. \tag{10}$$

Because the announcement is taken to be certain and non-stochastic, it follows that $\mathbb{E}_t y_{t+1} = \mathbf{C}_{t+1} + \mathbf{Q}_{t+1} y_t$ which imply the equivalences

$$\begin{aligned} \mathbf{C}_t &= (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}(\mathbf{\Gamma} + \mathbf{B}\mathbf{C}_{t+1}) & (11) \\ \mathbf{Q}_t &= (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}\mathbf{A} & (12) \\ \mathbf{G}_t &= (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}\mathbf{D}. & (13) \end{aligned}$$

To solve for the sequence of forecasting functions agents use to form expectations, we use a backward recursion. One starts from the solution of the final structure $\mathbf{Q}_{T^*+1} = \mathbf{Q}^*$, and chooses the sequence $\{\mathbf{Q}_t\}_{t=T^a}^{T^*}$ that satisfies Equation (12). Equation (11) can be written as

$$\mathbf{C}_t = \mathbf{\Lambda}_t + \mathbf{F}_t \mathbf{C}_{t+1}$$

where $\mathbf{\Lambda}_t = \left(\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1}\right)^{-1}\mathbf{\Gamma}$ and $\mathbf{F}_t = (\mathbf{I} - \mathbf{B}_t\mathbf{Q}_{t+1})^{-1}\mathbf{B}_t$. With the sequence for $\mathbf{Q}_t$ in hand it is possible to solve for the sequence for $\mathbf{C}_t$ through a forward recursion, giving $\mathbf{C}_t = \mathbf{\Lambda}_t + \mathbf{F}_t\mathbf{\Lambda}_{t+1} + \mathbf{F}_t\mathbf{F}_{t+1}\mathbf{\Lambda}_{t+2} + \ldots$, which provides the sequence of forecasting functions that prevails under perfect credibility.

Announcements under imperfect credibility

Expectations in the presence of an announcement when $0 < \lambda < 1$ are given by

$$\begin{aligned}\hat{\mathbb{E}}_t y_{t+1} &= \lambda \mathbb{E}_t y_{t+1} + (1-\lambda)\tilde{\mathbb{E}}_t y_{t+1} \\ &= \lambda(\mathbf{C}_{t+1} + \mathbf{Q}_{t+1} y_t) + (1-\lambda)(\tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t).\end{aligned}$$

Substituting this expression into the Equation (2), the reduced form solution is

$$y_t = \hat{\mathbf{C}}_t + \hat{\mathbf{Q}}_t y_{t-1} + \hat{\mathbf{G}}_t \varepsilon_t \tag{14}$$

where

$$\begin{aligned}\hat{\mathbf{C}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}(\mathbf{\Gamma} + \lambda \mathbf{B}\mathbf{C}_{t+1} + (1-\lambda)\mathbf{B}\tilde{\mathbf{C}}_t) \\ \hat{\mathbf{Q}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{A} \\ \hat{\mathbf{G}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{D}.\end{aligned}$$

The reduced form now includes both backward ($\tilde{\mathbf{Q}}_t$) and forward-looking ($\mathbf{Q}_{t+1}$) information. The backward-looking information comes from the estimated coefficients from learning expectations and the forward-looking information comes from the textbook rational expectations solution. In this context, λ serves as a natural measure of credibility because it governs the impact of an announcement. In the extreme when $\lambda = 0$, an announcement would have no impact on the reduced-form and forecasting functions would respond only to past data. As λ increases, forecasting functions respond more strongly to an announcement.

The linear combination in Equation (3) effectively combines forecasting functions in the same way a professional forecaster would. In econometrics, forecast combination is a well-established approach to pooling multiple forecasts when confronted with uncertainty over which forecast is best. Empirically, forecast combination is often the most robust way to forecast.[15] The combined forecasting approach has also been applied in recent work in the adaptive learning literature by Evans et al. (2013) and Gibbs (2015) as a method for modelling forecast uncertainty.[16]

For the study of disinflations, we take λ to be fixed. In reality, λ is likely to evolve as a function of the success of policymakers in achieving known objectives. The assumption that λ is fixed is no different from assuming that λ is only updated following the completion of an announced policy. Because we are interested in how a given level of credibility affects a single disinflation outcome, we do not attempt to formalize the process by which credibility may evolve over time.[17]

Evans et al. (2009) and Mitra et al. (2013) have also proposed ways to incorporate the anticipated path of a policy variable into an adaptive learning framework. They propose giving agents perfect foresight of the path of a policy instrument into the infinite future while assuming that agents form adaptive forecasts of all other variable. Our approach is more general because we allow announcements of not only a given path of a policy variable but also of *state-contingent* policy rules.[18]

[15]See Timmermann (2006) for a survey of the literature

[16]The use of best forecasting practices places the method in line with cognitive consistency principle proposed by Evans and Honkapohja (2013). The cognitive consistency principle essentially states that a way to judge the legitimacy of a bounded rationality assumption is to compare it to the actual forecasting behavior of economists. If a reasonable economists would form expectations in the proposed way, then the method is a disciplined departure from RE.

[17]It is a worthwhile task for future research to formalize the process by which λ would evolve over time. Endogenizing this process along lines considered by Marcet and Nicolini (2003) and Sargent et al. (2009) for the gain parameter in adaptive learning may also prove useful for explaining hyperinflations.

[18]Evans, Honkapohja, and Mitra use the infinite horizon learning setup introduced by Preston (2005), while we use the standard Euler equation learning framework. The principle advantage of this choice is that it allows for imperfect credibility to be studied in the broad class of models already in use. Extending our notion of credibility to the setup of Preston (2005) is a worthwhile avenue though for future research.

3.1 E-stability under imperfect credibility

It is common in the learning literature to study the convergence properties of the proposed learning algorithms. Our framework behaves as one would expect. Combining adaptive learning with a fixed forecasting function is stabilizing. The more anchored expectation are (i.e. the larger λ is) the more stable the MSV solution is under learning.

Proposition 1: *There exists* $0 < \lambda < 1$ *such that the MSV solution given by* $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$ *is E-stable.*

The proof of Proposition 1 is in the Appendix. Proposition 1 states that our framework of imperfectly anchored expectations allows any parameterization of the MSV solution to be E-stable if λ is significantly large. This is because as λ approaches one, the learning dynamics become less important for expectations and the forward-looking component dominates. Therefore, any equilibrium that is E-stable in the standard adaptive learning setup will also be E-stable in our framework.

4 The sacrifice ratio in the New Keynesian model

This section illustrates our expectations framework in a New Keynesian model simple enough to allow for analytical results. We use this model to highlight the source of output gains and losses in response to a disinflation and to show how our measure of imperfect credibility changes outcomes relative to rational expectations. The model we consider is described by an IS-curve, a Phillips curve, and a Taylor-type monetary policy rule:

$$
\begin{aligned}
x_t &= \hat{\mathbb{E}}_t x_{t+1} - (r_t - \hat{\mathbb{E}}_t \pi_{t+1}) \\
\pi_t &= \beta \hat{\mathbb{E}}_t \pi_{t+1} + \psi x_t \\
r_t &= \bar{\pi} + \alpha \left(\pi_t - \bar{\pi} \right).
\end{aligned}
$$

where x_t is the output gap, π_t is inflation, r_t is the nominal interest rate, and $\bar{\pi}$ is the inflation target. We set β to one so that in steady state $\pi_t = \bar{\pi}$, $r_t = \bar{\pi}$ and $x_t = 0$.

4.1 Origins of the sacrifice ratio

A disinflation policy is a permanent change in the inflation target, $\bar{\pi}$. The output cost depends on the path of inflation expectations. To see this, substitute the Phillips curve and monetary policy rule into the IS-curve and iterate forward:

$$
x_t = (\alpha - 1) \sum_{j=1}^{\infty} (1 + \alpha\psi)^{-j} (\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j}), \tag{15}
$$

where we have used the fact that in a stable equilibrium $\lim_{j\to\infty} \hat{\mathbb{E}}_t x_{t+j} = 0$. Equation (15) shows that the cost of disinflation depends on inflation expectations relative to the central bank's inflation target. To the extent that inflation expectations do not fall when the inflation target falls, that is $(\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j}) < 0$, there is an output cost. The aggressiveness of monetary policy as captured by α and the degree of price stickiness as captured by ψ act to scale the output cost. In particular, the more aggressive the central bank is in fighting deviations from its current inflation target, the larger the output cost.

The cost of a credible cold turkey disinflation under rational expectations is zero because $\mathbb{E}_t \pi_{t+j} = \bar{\pi}$ for all j. The only way to generate a non-zero sacrifice ratio under rational expectations is for the policy to be anticipated. As noted by Ball (1994a), if the policy is anticipated, then expectations move before the change in the target. This causes either a boom or a slump in output depending on the aggressiveness of monetary policy. Output responds to the anticipated change in the target because the central bank temporarily defends its old target. The expectation of a decrease in the target causes expectations to fall, and hence actual inflation to fall relative to the old target, which causes monetary policy to reduce the nominal interest rate in response. The reduction leads to a fall in real interest rates if $\alpha > 1$, which stimulates aggregate demand and increases output.

4.2 Disinflations with imperfect credibility

With imperfect credibility, a disinflation program generates output losses because expectations do not immediately adjust to the new inflation target. The central bank, therefore, must contract demand to move inflation and inflation expectations to the desired level. With imperfect credibility, a disinflation has two components: an initial impact which depends on the response of expectations to the announcement and a convergence towards the new target once the policy is implemented.

4.2.1 Initial impacts

Consider first an unanticipated cold turkey disinflation. The central bank lowers the inflation target from π^H to π^L. The inflation target is π^H until T^* when the central bank announces and implements π^L. Because the announcement and implementation dates coincide, the change is unanticipated.

Substituting the monetary policy rule into the output gap equation, the model can be reduced to the following two equations

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \frac{\alpha - 1}{1 + \alpha\psi} \begin{pmatrix} \bar{\pi} \\ \psi\bar{\pi} \end{pmatrix} + \frac{1}{1 + \alpha\psi} \begin{pmatrix} 1 & 1 - \alpha \\ \psi & 1 + \psi \end{pmatrix} \begin{pmatrix} \hat{\mathbb{E}}_t x_{t+1} \\ \hat{\mathbb{E}}_t \pi_{t+1} \end{pmatrix}$$

and written in the form of Equation (2) as

$$y_t = \mathbf{\Gamma} + \mathbf{B}\hat{\mathbb{E}}_t y_{t+1}, \tag{16}$$

where $\mathbf{A}$ and $\mathbf{D}$ are zero. In this case, the MSV solution, Equation (4), reduces to

$$y_t = \mathbf{C}$$

where $\mathbf{C} = (0, \bar{\pi})'$. Assuming the economy is at steady state prior to the disinflation, one can show that expectations at T^* are $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C}^L + (1-\lambda)\mathbf{C}^H$, where $\mathbf{C}^L = (0, \pi^L)'$ and $\mathbf{C}^H = (0, \pi^H)'$. The output gap and inflation in period T^* are given by the expressions below

$$\begin{aligned} x_{T^*} &= (1-\lambda)\frac{(\pi^L - \pi^H)(\alpha - 1)}{(1+\alpha\psi)} \\ \pi_{T^*} &= (1-\lambda)\frac{(1+\psi)}{(1+\alpha\psi)}\pi^H + \frac{(\alpha-1)\psi + \lambda(1+\psi)}{(1+\alpha\psi)}\pi^L \end{aligned}$$

When credibility is perfect, $\lambda = 1$, then $x_t = 0$ and $\pi_t = \pi^L$. The disinflation is achieved at time T^* with no loss in output. The sacrifice ratio is zero.

When $\lambda < 1$, agents rely on past data to update their expectations and do not fully adjust to the new target upon implementation. Inflation expectations at time T^* are

$$\hat{\mathbb{E}}_t \pi_{t+1} = \lambda \pi^L + (1-\lambda)\pi^H.$$

The fact that inflation expectations remain somewhat anchored at the old target results in an insufficient decrease of inflation relative to the new inflation target. If the Taylor principle is satisfied, $\alpha > 1$, monetary policy generates a demand-driven recession by increasing the real interest rate to bring inflation towards its new lower target. The expressions above show that with $(\alpha - 1) > 0$ and $\pi^L < \pi^H$, the disinflation leads to a loss of output, $x_t < 0$. The loss of output is larger the more aggressive is the response of monetary policy to inflation because this determines the response of the real interest rate.[19] The loss of output decreases with credibility, consistent with the alternative specifications of imperfect credibility proposed in the disinflation literature.[20]

An anticipated cold-turkey disinflation works much the same way. The central bank announces the policy before it implements the new target. For this example let $T^a = T^* - 1$, where T^a is the time the announcement is made, and T^* now stands for the time of implementation. Because this corresponds to an anticipated policy change we must use the recursions given by Equations (11) to (13). Equation (12) implies $\mathbf{Q}_t = 0$ because $\mathbf{A} = 0$ and Equation (13) implies $\mathbf{G}_t = 0$ as $\mathbf{D} = 0$ in this case. Solving Equation (11), $\mathbf{C}_t = \mathbf{\Gamma} + \mathbf{B}\mathbf{C}_{t+1}$, implies:

$$\mathbf{C}_{T^a} = \mathbf{\Gamma} + \mathbf{B}\mathbf{C}^L,$$

and with imperfectly anchored expectations, $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C}_{T^a} + (1 - \lambda)\mathbf{C}^H$.

The output gap and inflation at the time of the announcement are given by the expressions below:

$$x_{T^a} = \lambda \frac{(\pi^H - \pi^L)(\alpha - 1)}{1 + \alpha\psi}$$

$$\pi_{T^a} = \pi^H - \lambda \frac{(\pi^H - \pi^L)(1 + \psi)}{1 + \alpha\psi}$$

The anticipation effect depends on credibility, λ. In the extreme case in which $\lambda = 0$,

[19] One can verify that $\partial x_t / \partial \alpha < 0$ provided $\psi > -1$, which is satisfied by model's theoretical restrictions. Interestingly, if monetary policy responds one-for-one to inflation, then $x_t = 0$ for all λ.

[20] If the Taylor principle is not satisfied ($\alpha < 1$), the initial impact leads to a boom in output. Therefore, on impact, passive monetary policy leads to lower output losses.

the expressions above reveal that the announcement has no impact. If $\lambda > 0$, the announcement influences output and inflation. The change in the inflation target and the aggressiveness of monetary policy matter. In particular, an anticipated disinflation results in a boom ($x_t > 0$) if the Taylor principle is satisfied ($\alpha > 1$). The intuition is this: the news of an impending disinflation causes expectations of inflation and actual inflation to decrease. The fall in inflation, however, represents a deviation from the current inflation target, π^H, so the central bank reduces the nominal interest rate. If the Taylor Principle is satisfied, $\alpha > 1$, the decrease in the nominal interest rate causes a decrease in the real interest rate and a corresponding boom in output. When the Taylor Principle is not satisfied, output falls.

Perhaps surprisingly, even with imperfect credibility, a pre-announcement can be beneficial. This is because, even with some credibility, inflation moves in the direction of the new inflation target. This leads to a second round effect on the learning component of expectations. Expectations are closer to the new target upon implementation resulting in lower sacrifice ratios.

One may wonder if increasing the time between the announcement and the implementation always increases the boom in output. In general the answer is no. In fact, the farther apart the announcement and implementation dates, the smaller is the impact of the announcement on inflation and output. At a mechanical level this is because the recursion is just a repeated iteration of the mapping from expectations to the structural parameters and when the mapping is E-stable, beliefs converge to the old steady state and initial impacts vanish.[21] At an intuitive level, it is because announcements regarding policy changes that will take place far into the future have little contemporaneous impact. But a boom would still take place as the implementation date approaches.

[21] The mapping could be explosive when the final structure has a unique equilibrium but the current structure is not E-stable. An example of this kind of explosive behaviour obtains in the case of forward guidance regarding fixed interest rates. See del Negro et al. (2012).

4.2.2 Convergence

Provided $\lambda < 1$, inflation expectations and inflation would not be at the new inflation target when the policy is implemented. Convergence to the new steady state hinges on adaptive learning. The adaptive learning rule that corresponds to the MSV solution in this case is a recursively estimated mean of the form

$$\mathbf{C}_t = \mathbf{C}_{t-1} + \gamma(y_{t-1} - \mathbf{C}_{t-1}). \tag{17}$$

Expectations are $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C} + (1-\lambda)\mathbf{C}_t$ and the actual law of motion is

$$y_t = \mathbf{\Gamma} + \mathbf{B}(\lambda \mathbf{C} + (1-\lambda)\mathbf{C}_t). \tag{18}$$

Substituting Equation (18) into Equation (17) results in

$$\mathbf{C}_t = \gamma\left(\mathbf{\Gamma} + \mathbf{B}\lambda\mathbf{C}\right) + \left(I - \gamma\left((1-\lambda)\mathbf{B} - I\right)\right)\mathbf{C}_{t-1}, \tag{19}$$

which is a stationary VAR process around $\mathbf{C}$ if

$$\alpha > \frac{\psi - \psi\lambda - \lambda^2}{\psi}.$$

If monetary policy is sufficiently aggressive, inflation will converge to $\mathbf{C}^L$ following implementation. Notice that setting $\lambda = 0$ recovers the familiar E-stability condition for this model that $\alpha > 1$.

Figure 2 shows the global convergence properties for an unanticipated disinflation. The two large dots on the figure correspond to $(0, \pi^H)$, the high inflation steady state, and to $(0, \pi^L)$, the low inflation steady state. The solid line describes the paths of output and inflation taking as initial condition the high inflation steady state. We compute paths for $\lambda = 0$ (left) or for $\lambda = 1/2$ (right). With no credibility, inflation can only be lowered through a demand contraction. The path to steady state is long and overshoots once in the neighborhood of the new target. With some credibility, inflation and output jump upon the simultaneous announcement and implementation of the policy. The path is more direct because more credibility pulls expectations toward the new steady state. With full credibility, there are no convergence dynamics; beliefs jump from the high steady state to the low steady state without output moving.

Credibility has a significant effect on the dynamics of convergence. But the paths also depends on the monetary policy response. The larger α is, the more severe the counterclockwise swirl of Figure 2 and the higher the implied sacrifice ratio. As α decreases, the swirl diminishes. If the Taylor principle is not satisfied, $\alpha < 1$, the swirl becomes clockwise.

Figure 2 reveals how shocks affect the cost of disinflation and the time to disinflate. When credibility is low, a shock that reduces inflation lowers the sacrifice ratio. Shocks that increase inflation unambiguously place the economy on trajectories that imply longer times to converge and larger output losses. As credibility increases, the cost of bad shocks is reduced because paths to the low inflation steady state are more direct. The next section makes this point in the context of a model with many shocks.

5 Quantitative results

To understand the conditional distribution of sacrifice ratios of intended disinflations, we study a sticky price DSGE model estimated on recent aggregate data from the United States and Argentina.[22] The estimated parameters discipline the numerical analysis. We use the United States parameter values to study disinflationary policies with small shocks and use the parameter values implied by Argentinean data to study disinflationary policies with large shocks.

[22]We use Billion Price Project data in place of official inflation statistics for Argentina.

5.1 The model

We consider a version of the New Keynesian model of Ireland (2004). The linearize approximation of the model is given by the equations below.[23]

$$
\begin{aligned}
x_t &= (z - \ln \beta) - (r_t - \hat{\mathbb{E}}_t \pi_{t+1}) + \hat{\mathbb{E}}_t x_{t+1} + (1-\omega)(1-\rho_a) a_t && (20)\\
\pi_t &= (1-\beta)\pi + \beta \hat{\mathbb{E}}_t \pi_{t+1} + \psi x_t - e_t && (21)\\
r_t &= r + \rho_r (r_{t-1} - r) + \rho_\pi (\pi_t - \bar{\pi}_t) + \rho_g (g_t - z) + \rho_x x_t + \varepsilon_{r,t} && (22)\\
\bar{\pi}_t &= (1-\rho_\pi^*)\bar{\pi} + \rho_\pi^* \bar{\pi}_{t-1} + \mu_a \varepsilon_{a,t} + \mu_e \varepsilon_{e,t} + \mu_v \varepsilon_{v,t} + \mu_z \varepsilon_{z,t} + \mu_\pi \varepsilon_{\pi,t} && (23)\\
\omega_t &= m_t - m_{t-1} - \pi_t + z_t && (24)\\
m_t &= \hat{y}_t + a_t - \frac{1}{g\bar{\pi}\beta^{-1} - 1}(r_t - r) + v_t && (25)\\
x_t &= \hat{y}_t - \omega a_t && (26)\\
g_t &= \hat{y}_t - \hat{y}_{t-1} + z_t && (27)\\
a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} && (28)\\
e_t &= \rho_e e_{t-1} + \varepsilon_{e,t} && (29)\\
v_t &= \rho_v v_{t-1} + \varepsilon_{v,t} && (30)\\
z_t &= (1-\rho_z) z + \rho_z z_{t-1} + \varepsilon_{z,t}, && (31)
\end{aligned}
$$

where x_t is the output gap, defined as the deviation of output from a socially efficient level of output; π_t is the gross rate of inflation, that is $\ln(p_t/p_{t-1})$ and $\bar{\pi}_t$ is time-varying target; r_t is the log of the gross nominal interest rate; g_t is the growth rate of output; $\hat{y}_t$ is the percentage deviation from steady state of the log of the stochastically de-trended level of output, Y_t/Z_t; and Z_t is labour augmenting productivity, whose growth rate,

[23] The appendix includes an overview of the micro foundations underpinning the model. In the model the assumption of Rotemberg (1982) price adjustment costs holds. The linear approximation of the model under this assumption does not depend on the level of steady state inflation. This is not the case for models that assume Calvo-pricing as emphasized by Ascari (2004) and Ascari and Sbordone (2014). The invariance of the linear model to the level of trend inflation means that there are no explicit costs associated with one inflation rate compared to another. Disinflations are costly through transitions between steady states.

$z_t = \ln(Z_t/Z_{t-1})$ follows the autoregressive process in Equation (31); m_t is the percentage deviation of stochastically de-trended real money balances and ω_t is the percentage deviation of nominal money growth from steady state; a_t is a demand shock, e_t is a cost-push shock, and v_t is a money demand shock. The ε_i's are identically and independently distributed shocks with zero mean and standard deviation σ_i's respectively. In steady state, $\pi_t = \bar{\pi}$, $r_t = r$, $g_t = g$ and $r = \bar{\pi} + g - \ln\beta$.

Unlike the model of Ireland (2004), we assume that there is a short-run inflation target, $\bar{\pi}_t$, that is time-varying around the long-run inflation target, $\bar{\pi}$, and accommodates shocks as shown in Equation (23). A specification like this is required when taking the model to the recent Argentinean data where inflation has been trending upwards. But this specification is also useful because it provides a parsimonious way to model gradual disinflations. For a given change in the long-run inflation target, $\Delta\bar{\pi}$, the persistence parameter ρ^*_π governs the speed with which the new unconditional mean of inflation is implemented.[24] All the disinflation simulation exercises we consider refer to changes of the *long-run* inflation target, $\bar{\pi}$.

The model is estimated using Bayesian methods as is standard in the literature. Details of the data and estimation are given in the appendix. We estimate the model under rational expectations and use the estimated parameter values for the disinflation simulation exercises.[25] The estimated parameter values and shock variances that we use for our quantitative exercise are in Table 3. There are two points to note. The first is that the variances of the shocks are larger for Argentina. The standard deviation of inflation in Argentina is more than four times that of the United States. This means that, for Argentina, small disinflations are indistinguishable from other shocks. The second point

[24] ρ^*_π set to zero corresponds to a cold turkey disinflation as the inflation target falls immediately to its new intended value; monetary policy responds to deviations from the new long-run level of inflation as soon as it is implemented.

[25] Alternatively, one could think of estimating the model jointly with λ. We do not do this for two reasons. First, we are interested in the distributions of sacrifice ratios as a function of λ. We are interested in changing λ whatever its estimate may be. Second, once learning converges to rational expectations the solutions are observational equivalent and λ is not likely to be identified.

is that monetary policy is more aggressive in the United States. The response to inflation is twice as large in the United States. This implies, as we have seen in the analytical section, different output costs for similar sized disinflations.

5.2 Measuring the sacrifice ratio in stochastic simulations

With knowledge of the data generating process at hand, the sacrifice ratio of interest is

$$SR = \frac{1}{\bar{\pi}^H - \bar{\pi}^L} \sum_{t=T^a}^{N} x_t, \tag{32}$$

where the denominator $\bar{\pi}^H - \bar{\pi}^L$ corresponds to the disinflation intended by the authorities and the output gap is the model measure, x_t. T^a is the time the policy is announced, and N corresponds to the end of the disinflation episode. We take the end to be the first quarter in which inflation is close to its new target, that is $|\pi^L - \pi_t| < 2\%$. In non-stochastic simulations, a disinflation episode is defined as completed when the output gap returns to zero. In a stochastic simulation, though, this definition is undesirable because shocks may close the output gap before inflation reaches its new target. As we know the disinflation objective, we select a cutoff based on inflation being close to its new target.

5.3 Conditional distributions of the sacrifice ratio

We study how the following four dimensions shape the distribution of sacrifice ratios: i) credibility, ii) anticipation, the extent to which the policy is pre-announced, iii) gradualism, the speed with which the disinflation is implemented and iv) the size of the disinflation. We use Monte Carlo simulations of disinflations of 10 percentage points at the parameter values of Table 3.

5.3.1 Credibility

Figure 3 shows distributions of sacrifice ratios under rational expectations ($\lambda = 1$), imperfect credibility ($\lambda = 0.5$), and adaptive learning ($\lambda = 0$) for an unanticipated cold turkey disinflation. Consistent with the findings in the literature, the mean sacrifice ratio increases with lower credibility. Lower credibility though also increases the variance of the distribution and causes significant right skewness. This is true for all disinflation programs.

High credibility lowers the chance of costly disinflations. Low credibility does not rule out costless disinflations but makes these very unlikely. Luck is important when expectations are poorly anchored: even with no credibility some mass of the distribution remains near zero.

These results support the regression estimates for proxies of credibility in Table 2. Changes in credibility have larger effects on the variance of the distribution than on its mean. Therefore, even with large sample sizes, empirical measures of credibility should not be expected to account for much of the observed variation in the sacrifice ratio.

5.3.2 Anticipation

Figure 4 compares the distribution of sacrifice ratios for anticipated disinflations with those of unanticipated disinflations. In the anticipated case, we assume that the change in the inflation target is announced two quarters in advance. We set λ to 0.5.

Consistent with the intuition from the analytical results, when the disinflation is anticipated the distribution of sacrifice ratios shifts to the left. Anticipated disinflations generally result in lower sacrifice ratios. This is because announcements generate increases in output and result in faster convergence to the new target. The positive effect of the announcement vanishes as λ approaches zero. But pre-announcing a disinflation remains beneficial in shifting the distribution of the sacrifice ratio even for low levels of credibility.

Figure 4 makes an important point regarding the empirical determinants of the sacrifice ratio. Although the distribution of sacrifice ratios changes in a meaningful way with the design of policy, the difference in the mean sacrifice ratios would be hard to detect in data. With this in mind, the point estimates in Table 2 are consistent with our findings in Figure 4. The 'irregular appointment' variable implies that anticipated disinflation are less costly. The point estimate reported from the empirical investigation is negative and large, but statistically insignificant.

5.3.3 Gradualism

Figure 5 compares the distribution of sacrifice ratios for gradual and cold turkey disinflations. We set $\lambda = 0.5$. The gradual disinflation is implemented with $\rho_{\pi}^{*} = 0.8$, which implies the short-run inflation target transitions from 12% to 2% in just over 8 quarters. A gradual disinflation is inherently less costly because the central bank responds less aggressively. This is because the short-run inflation target is closer to actual inflation when the policy is announced. The trade off, though, is that gradual disinflations last longer.

5.3.4 Size of the disinflation

Figure 6 compares the distribution of sacrifice ratios for different sizes of disinflation for full credibility and imperfect credibility ($\lambda = 0.25$). With full credibility, the size of the disinflation has a little impact on the distribution of sacrifice ratios.

With imperfect credibility, the larger disinflation is more costly and the variance of the sacrifice ratio is larger. The increased cost is because the larger disinflation requires a larger demand contraction to move the adaptive component of expectations to the new target, while the increased variances reflects the increased duration of the disinflation. The regression evidence in Table 1 found a negative relationship between the sacrifice ratio and the size of disinflation. From the perspective of the simulations, the empirical relationship may reflect that larger disinflations are more credible.

5.4 The sacrifice ratio of opportunistic disinflations

The results show that shocks play a prominent role. We consider two experiments. The first explores how the distribution of sacrifice ratios changes for a given disinflation program as a function of the size of the shocks. The second studies the possibility that a policymaker capitalizes on a series of good luck shocks by announcing the disinflation when inflation is below trend. Bomfim and Rudebusch (2000) note that the Federal Reserve discussed opportunistic disinflation strategies in FOMC meetings in the late 1980's and it seems consistent with the approach of many central banks who took advantage of the low inflation rates of the 1990's to implement inflation targeting.[26]

5.4.1 The size of shocks

Figure 7 shows the distribution of sacrifice ratios for a disinflation of 30% and $\lambda = 0.5$ in the Argentinean calibration compared with the distribution of sacrifice ratios that would have obtained if Argentina had experienced shocks of equivalent size to those found in the US. The difference is between the size of the shocks relative to the size of the disinflation. Large shocks significantly increase the variance of the distribution and reduce right skewness. The distribution of sacrifice ratios generated with small shocks, however, is right skewed and has no mass on benefit ratios. The size of shocks relative to the size of the disinflation is not large enough to move expectations (by chance) significantly towards the new inflation target. The bulk of the movement in inflation and inflation expectations must come either through a demand contraction or through the impact that the credibility of the policy has on expectations. The mean sacrifice ratio, however, with small shocks is below that with large shocks. The small shocks eliminate benefit ratios but also eliminate very costly disinflations.

[26]See Rasche and Williams (2007).

5.4.2 Opportunistic disinflations

Periods where inflation is above or below target have persistent effects on expectations when credibility is low. This fact opens up the possibility that disinflation policies implemented after periods of below trend inflation may have lower sacrifice ratios. To investigate if there are quantitative gains to an opportunistic disinflation, we simulate 5,000 disinflation episodes and compute sacrifice ratios for cases where average inflation is either above or below target over the four quarters prior to the policy change.

Table 4 reports the mean, standard deviation, and skewness of the simulated distribution for disinflations of 10% and 30% for the US and Argentinean calibrations, respectively. Column 1 reports the results for average inflation 1% below target, column 2 reports the results for average inflation 1% above target, and column 3 reports the results from the 5,000 simulated disinflations. Columns 4, 5, and 6 report results with the Argentinean calibration. Because shocks are larger in this case, we consider 5% below or above the prevailing target.

Implementing a disinflation following a period of below target inflation lowers the sacrifice ratio at all levels of credibility. There are, however, more significant gains to an opportunistic policy when credibility is low. In particular, an opportunistic disinflation unambiguously lowers the variance and skewness of the distribution of sacrifice ratios, which significantly reduces the chance of a very costly disinflation. The reductions in the mean sacrifice ratio are large enough that opportunism often trumps credibility. For example, in the Argentinean calibration with $\lambda = 1/3$, the mean sacrifice ratio of the opportunistic disinflation is smaller than the mean sacrifice ratio obtained for the whole distribution with perfect credibility ($\lambda = 1$). These results provide a rationale for the approach taken in practice by some central banks of implementing inflation targeting after a period of below trend inflation, consistent with the evidence in Rasche and Williams (2007).[27] Therefore, a policymaker operating in an environment of low credibility should

[27] The comparison above is done at the same value of the gain parameter for both calibrations. In

take advantage of periods of below target inflation that occur by chance.

6 Conclusion

In this paper, we propose a model of imperfectly anchored expectations to study the distribution of sacrifice ratios of intended disinflationary policies. Our framework departs from the well-established rational expectations and adaptive learning frameworks but nest these two as special cases. Our framework captures elements of the behavior of expectations that policy makers consider relevant: the degree to which expectations are anchored affects the impact of announcements and the extent to which past data influence beliefs.

We show that linking expectations to the realization of data is particularly relevant in a stochastic environment. Luck matters more when credibility is low in determining the output cost of a disinflation and leads to distributions of sacrifice ratios with large variances. Our simulations shed light on empirical findings of the sacrifice ratio. Disinflation episodes depend significantly on the interplay between shocks and credibility but the distributions of different policies often overlap. Therefore, it is not surprising that empirical studies that regress policy characteristics on sacrifice ratios often find that the measures explain only a small amount of the variation and are found to be statistically insignificant.

This is not to say that the design of disinflation policy does not matter for the distribution of sacrifice ratios. Pre-announced disinflations and gradual disinflations shift a significant mass towards lower sacrifice ratios. This is true even with low credibility.

One advantage of characterizing the distribution of sacrifice ratios is that it allows us to quantify the benefits of an opportunistic approach to disinflation. As we show, implementing a disinflation following a period of below trend inflation is particularly

reality, it is reasonable to expect a higher gain in countries like Argentina, which are exposed to more frequent regime changes. Because a higher gain implies that the most recent observations carry more weight, a higher gain makes the opportunistic approach to disinflation even more desirable.

helpful when credibility is low. The mean sacrifice ratio in these cases is often smaller than what is obtained for perfectly credible disinflations.

Finally, a methodological contribution of our paper is to propose a parsimonious way of modeling imperfect credibility as the extent to which expectations are anchored. This way of modeling expectations can prove useful in other applications, such as forward guidance and fiscal consolidations.

References

Ascari, Guido, "Staggered prices and trend inflation: some nuisances," *Review of Economic dynamics*, 2004, *7* (3), 642–667.

— **and Argia M Sbordone**, "The macroeconomics of trend inflation," *Journal of Economic Literature*, 2014, *52* (3), 679–739.

— **and Tiziano Ropele**, "Sacrifice Ratio in a Medium-Scale New Keynesian Model," *Journal of Money, Credit and Banking*, 2012, *44* (2-3), 457–467.

— **and** — , "Disinflation in a DSGE perspective: Sacrifice ratio or welfare gain ratio?," *Journal of Economic Dynamics and Control*, 2012b, *36* (2), 169 – 182.

— **and** — , "Disinflation effects in a medium-scale new keynesian model: money supply rule versus interest rate rule," *European Economic Review*, 2013, *61*, 77–100.

Ball, Laurence, "Credible disinflation with staggered price-setting," *The American Economic Review*, 1994, pp. 282 – 289.

— , "What determines the sacrifice ratio?," in "Monetary policy," The University of Chicago Press, 1994, pp. 155–193.

— , "Disinflation with imperfect credibility," *Journal of Monetary Economics*, 1995, *35* (1), 5–23.

Binder, Michael and M Hashem Pesaran, "Multivariate Rational Expectations Models and Macroeconometric Modelling: A Review and Some New Results," in in M Hashem Pesaran and M Wickens, eds., *Handbook of Applied Econometrics: Macroeconomics*, Basil Blackwell Oxford 1995, pp. 139–187.

Blinder, Alan S, *Hard heads, soft hearts: Tough-minded economics for a just society*, Basic Books, 1987.

Bomfim, Antulio N and Glenn D Rudebusch, "Opportunistic and deliberate disinflation under imperfect credibility," *Journal of Money, Credit and Banking*, 2000, pp. 707–721.

Cagliarini, Adam and Mariano Kulish, "Solving Linear Rational Expectations Models with Predictable Structural Changes," *Review of Economics and Statistics*, 2013, *95*, 328–336.

Cavallo, Alberto, "Online and official price indexes: Measuring Argentina's inflation," *Journal of Monetary Economics*, 2013, *60* (2), 152–165.

Cecchetti, Stephen G and Robert W Rich, "Structural estimates of the US sacrifice ratio," *Journal of Business & Economic Statistics*, 2001, *19* (4), 416–427.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, 2005, *113* (1), 1–45.

del Negro, Marco, Marc Giannoni, and Christina Patterson, "The Forward Guidance Puzzle," Staff Reports 574, Federal Reserve Bank of New York 2012.

Dreher, Axel, Jan-Egbert Sturm, and Jakob De Haan, "Does high inflation cause central bankers to lose their job? Evidence based on a new data set," *European Journal of Political Economy*, 2008, *24* (4), 778–787.

_ , _ , and _ , "When is a central bank governor replaced? Evidence based on a new data set," *Journal of Macroeconomics*, 2010, *32* (3), 766–781.

Erceg, Christopher J and Andrew T Levin, "Imperfect credibility and inflation persistence," *Journal of monetary economics*, 2003, *50* (4), 915–944.

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton: Princeton University Press, 2001.

_ **and** _ , "Learning as a Rational Foundation for Macroeconomics and Finance," in Roman Frydman and Edmund S. Phelps, eds., *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton Univ Pr, 2013.

_ **,** _ **, and Kaushik Mitra**, "Anticipated fiscal policy and adaptive learning," *Journal of Monetary Economics*, 2009, *56* (7), 930 – 953.

Evans, G.W., S. Honkapohja, T.J. Sargent, and N. Williams, "Bayesian Model Averaging, Learning, and Model Selection," in T.J. Sargent and J. Vilmunen, eds., *Macroeconomics at the Service of Public Policy*, Oxford University Press, 2013.

Gibbs, Christopher G, "Forecast Combination, Non-linear Dynamics, and the Macroeconomy," *mimeo*, 2015.

Gonçalves, Carlos Eduardo S and Alexandre Carvalho, "Inflation targeting matters: evidence from OECD economies' sacrifice ratios," *Journal of Money, Credit and Banking*, 2009, *41* (1), 233–243.

Goodfriend, Marvin and Robert G King, "The incredible Volcker disinflation," *Journal of Monetary Economics*, 2005, *52* (5), 981–1015.

Hommes, Cars and Joep Lustenhouwer, "Inflation targeting and liquidity traps under endogenous credibility," *working paper*, 2015.

Huh, Chan G and Kevin J Lansing, "Expectations, credibility, and disinflation in a small macroeconomic model," *Journal of Economics and Business*, 2000, *52* (1), 51–86.

Ireland, Peter N, "Optimal disinflationary paths," *Journal of Economic Dynamics and Control*, 1995, *19* (8), 1429–1448.

_ , "Technology Shocks in the New Keynesian Model," *The Review of Economics and Statistics*, November 2004, *86* (4), 923–936.

Katayama, Hajime, Natalia Ponomareva, and Malvin Sharma, "What really determines the sacrifice ratio: the bayesian model averaging approach," *mimeo*, 2015.

Kulish, Mariano and Adrian Pagan, "Estimation and Solution of Models with Expectations and Structural Changes," Research Discussion Paper 2012-08, Reserve Bank of Australia 2012.

Marcet, Albert and Juan P Nicolini, "Recurrent hyperinflations and learning," *The American Economic Review*, 2003, *93* (5), 1476–1498.

McCallum, Bennett T, "On non-uniqueness in rational expectations models: An attempt at perspective," *Journal of monetary Economics*, 1983, *11* (2), 139–168.

Milani, Fabio, "Expectations, learning and macroeconomic persistence," *Journal of monetary Economics*, 2007, *54* (7), 2065–2082.

Mitra, Kaushik, George W. Evans, and Seppo Honkapohja, "Policy change and learning in the RBC model," *Journal of Economic Dynamics and Control*, 2013, *37* (10), 1947 – 1971.

Preston, Bruce, "Learning about Monetary Policy Rules when Long-Horizon Expectations Matter," *International Journal of Central Banking*, 2005.

Rasche, Robert H and Marcela M. Williams, "The Effectiveness of Monetary Policy," *Federal Reserve Bank of St. Louis Review*, 2007, *89* (5), 447–89.

Rotemberg, Julio J, "Sticky prices in the United States," *The Journal of Political Economy*, 1982, pp. 1187–1211.

Sachs, Jeffrey, "The dollar and the policy mix: 1985," Technical Report, National Bureau of Economic Research 1985.

Sargent, Thomas, Noah Williams, and TAO Zha, "The Conquest of South American Inflation," *Journal of political economy*, 2009, *117* (2), 211–256.

Timmermann, Allan, "Forecast combinations," *Handbook of economic forecasting*, 2006, *1*, 135–196.

7 Appendix

Proposition 1 Proof: The proof follows Proposition 10.3 of EH. The T-map implied by combining expectations is

$$T\begin{pmatrix} \tilde{\mathbf{C}} \\ \tilde{\mathbf{Q}} \\ \tilde{\mathbf{G}} \end{pmatrix} = \begin{pmatrix} \chi^{-1}(\mathbf{\Gamma} + \mathbf{B}\lambda\mathbf{C} + \mathbf{B}(1-\lambda)\tilde{\mathbf{C}}) \\ \chi^{-1}\mathbf{A} \\ \chi^{-1}\mathbf{D} \end{pmatrix},$$

where $\chi = \mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}})$. Stability is determined by evaluating the eigenvalues of the mapping linearized around a fixed point of interest. The linearized mapping with respect to $\tilde{\mathbf{C}}$, $\tilde{\mathbf{Q}}$, and $\tilde{\mathbf{G}}$ and evaluated at the REE can be computed as

$$\begin{aligned} \mathbf{DT_C}(\mathbf{C},\mathbf{Q}) &= (\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{B}(1-\lambda) \\ \mathbf{DT_Q}(\mathbf{Q}) &= [(\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{A}(1-\lambda)]' \otimes [(\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{B}]. \end{aligned}$$

The requirement for E-stability is that the eigenvalues of the matrices $\mathbf{DT_C}(\mathbf{C},\mathbf{Q})$ and $\mathbf{DT_Q}(\mathbf{Q})$ have real part less than one. Consider

$$(1-\lambda)\mathbf{A}x = (1-\lambda)\xi x \equiv \hat{\xi}x,$$

where $\mathbf{A}$ is equal to either $(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{B}$ or $[(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{A}]' \otimes [(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{B}]$, $\hat{\xi}$ is a vector of eigenvalues associated with A, $\hat{\xi}_i$ is the real part of an eigenvalue element of $\hat{\xi}$, and x is the corresponding right eigenvector. The requirement for stability is $\hat{\xi}_i < 1$ for all i.

Let ξ_i be the real part of the eigenvalue of the T-map in the case where $\lambda = 0$, which corresponds to the standard adaptive learning case. Assume $\xi_i > 1$, then it must be the case that $\xi_i \geq \hat{\xi}_i$ since $\hat{\xi}_i = (1-\lambda)\xi_i$. Pick λ such that $\lambda < 1 - \xi_i^{-1} < 1$. Now, since $\hat{\xi}_1 = (1-(1-\xi_1^{-1}))\xi_1 = 1$ and $1 - \xi_i^{-1} < 1$ for all finite ξ_i, there always exists a λ such that $\hat{\xi}_i < 1$ for all i.

Micro foundations of the Ireland model: We consider a version of the sticky price DSGE model proposed in Ireland (2004). The model consists of representative household that maximize the following expected utility function

$$E \sum_{t=0}^{\infty} \beta^t \left[a_t \ln(C_t) + \ln(M/P) - \eta^{-1} h_t^{\eta} \right] \tag{33}$$

by choosing consumption, money holdings, labour supply, and by taking into account a preference shock a_t.[28] The representative household is faced with the budget constraint

$$M_{t-1} + B_{t-1} + T_t + W_t h_t + D_t \geq P_t C_t + B_t / r_r + M_t, \tag{34}$$

where M is nominal money balances, B is nominal bond, T is transfers, W is the nominal wage, and D is nominal profits the household receives from ownership of firms.

Production in the economy is separated into two sectors: a perfectly competitive finished goods sector and a monopolistically competitive intermediates goods sector. The finished goods sector uses a continuum of intermediates goods of prices $P(i)$ to construct the finished good. The production function is the familiar CES constant returns to scale technology described by

$$\left(\int_0^1 Y_t(i)^{(\theta_t - 1)/\theta_t} di \right)^{\theta_t/(\theta_t - 1)} \geq Y_t, \tag{35}$$

where θ_t is stochastic process that functions as a markup or cost push shock.[29] The

[28] It is assumed that $0 < \beta < 1$, $\eta \geq 1$, and $\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_{a,t}$.

[29] It is assumed that $\theta_t = (1-\rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \epsilon_(\theta, t)$.

finished-good-producing firms maximize profits by choosing

$$Y_t(i) = (P_t(i)/P_t)^{-\theta_t} Y_t, \tag{36}$$

which leads to the finished goods price of

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta_t} di \right)^{1/(1-\theta_t)} \tag{37}$$

for all t.

The intermediate-goods-producing firms hires $h_t(i)$ units of labour to manufacture $Y_t(i)$ units of outputs using the following production function

$$Z_t h_t(i) \geq Y_t(i), \tag{38}$$

where Z_t is an aggregate technology shock.[30] Since the intermediate-goods-producing firms' products are imperfect substitutes, the firms have a degree of pricing power. To introduce price stickyness it is assumed that firms face an explicit cost to adjust nominal prices, measured in terms of finished goods

$$\frac{\phi}{2} \left(\frac{P_t(i)}{\bar{\pi} P_{t-1}(i)} - 1 \right)^2 Y_t. \tag{39}$$

The firm's problem, therefore, is to pick a sequence of $P_t(i)$ to maximize

$$E \sum_{t=0}^{\infty} \beta^t \frac{a_t}{C_t} \frac{D_t(i)}{P_t}, \tag{40}$$

where $D_t(i)$ is nominal profits. Finally, assuming that monetary policy follows a Taylor-type rule, solving for the first order conditions of the household and firm decisions, and allowing a slight abuse of notation, the model can be log-linearly approximated around the $\pi \geq 0$ inflation steady state as presented in Section 5.

[30]It is assumed that $\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \epsilon_{z,t}$.

Data and estimation: We do two estimations of the model. One with data from Argentina and one with the data from the United States. The model is estimated on output per capita growth, nominal money growth and inflation for Argentina for the period 2003q3 to 2014q1 using Bayesian methods. In preliminary attempts to estimate the model we used a 3-month nominal interest rate. The estimation routine in this case, however, consistently led to implausibly parameter values. The issue is that the unconditional mean of the interest rate, inflation and output and money growth do not satisfy the steady state relations. The model implies that the steady state level of the nominal interest rate satisfies $r = z\pi/\beta$. But in our sample, the mean of the nominal interest rate is 0.0198 and the sample means for inflation and output growth require $\beta > 1$. Because the mean growth rate of money growth, 0.0541, is closer to the sum of mean inflation, 0.0361, and mean output growth, 0.0137, the estimation on those three observables seem more reliable.

In the case of inflation we use the official series up until 2007q4 and then use the series implied by Billion Price Project. We choose the post-default period as it entails a period of relatively little structural change. During this period inflation accelerated, but this can be captured with our structure as persistent changes in π_t^*.

The results of the estimation are shown in the left panel of Table 3. For the numerical analysis we set parameter values at the mode.

The US estimation uses PCE inflation, real GDP growth, and the effective federal funds rates as observables from 1984Q1 through 2008Q1. Data is stopped at 2008Q1 to avoid the financial crisis and issues with incorporating the zero lower bound of nominal interest rates. Estimate results and priors are given in the right panel of Table 3. For the numerical analysis we set parameter values at the mode.

List of Figures

Figure 1: Empirical distribution of sacrifice ratios

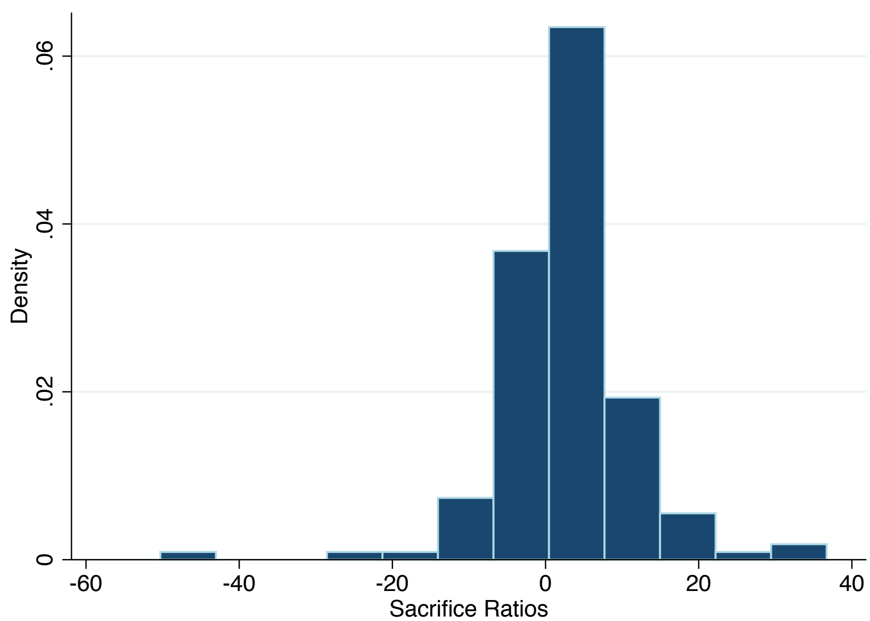
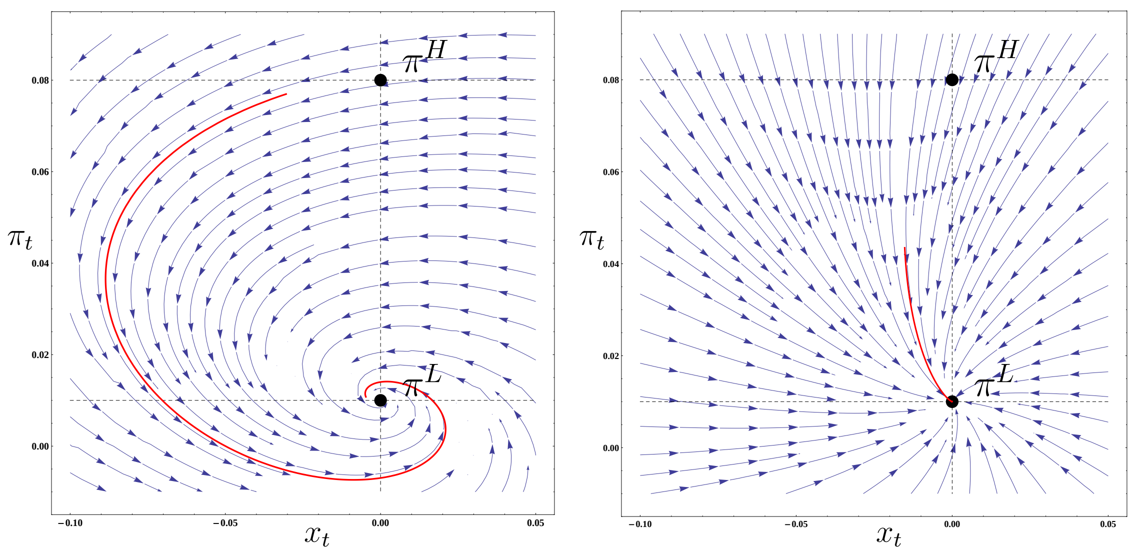
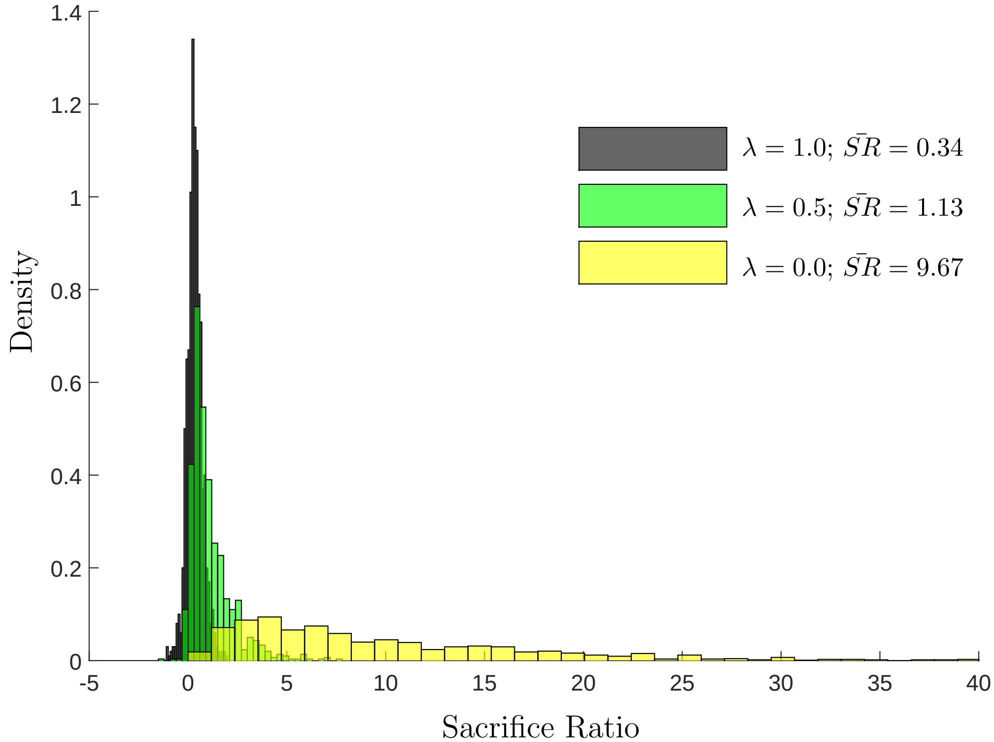


Figure 2: Global convergence



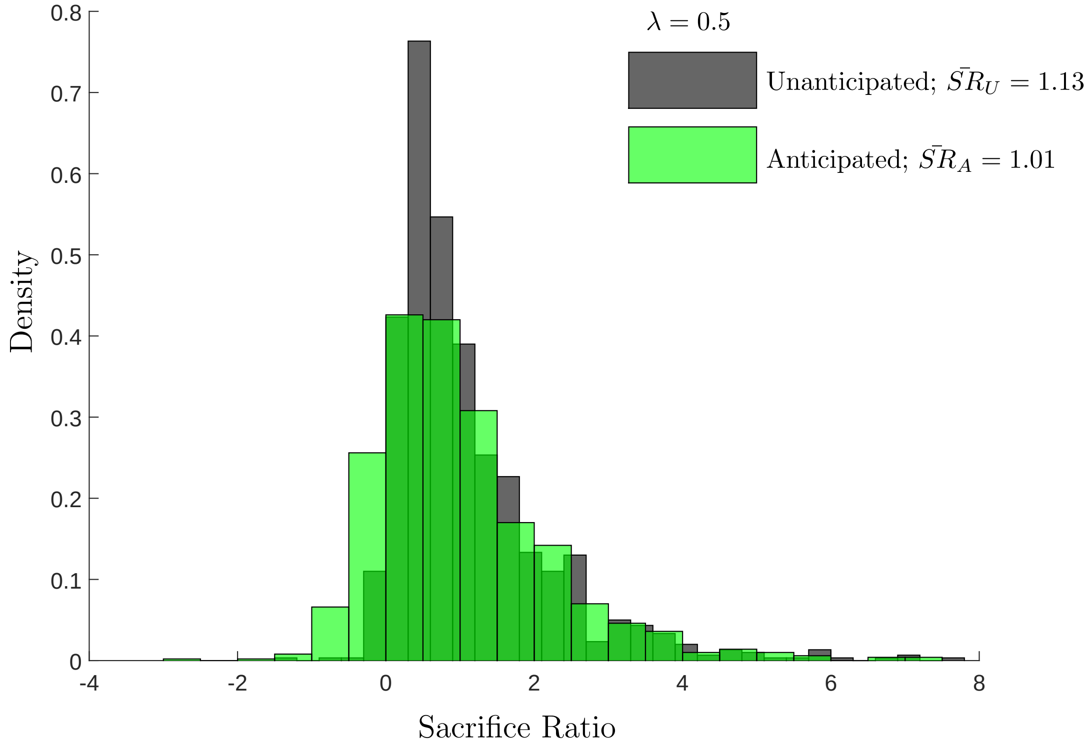
Notes: Stream plots illustrating convergence to new steady state with $\lambda = 0$ (left) and $\lambda = 0.5$ (right). The solid line indicates the non-stochastic path assuming inflation and output gap are at steady state upon implementation of the new policy.

Figure 3: Central bank credibility and the distribution of SRs



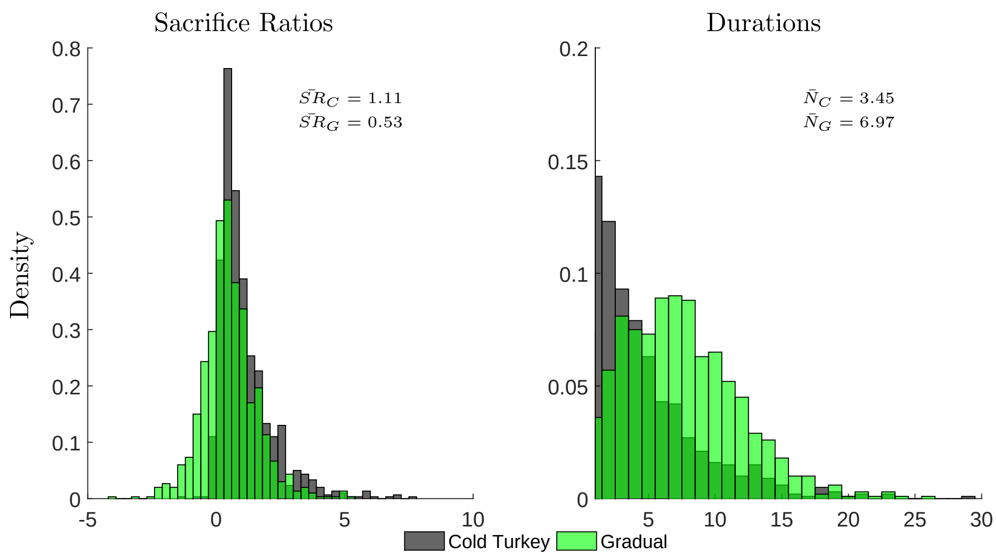
Notes: Distribution of sacrifice ratios for a 10% disinflation under the US calibration. $\bar{SR}$ is the mean sacrifice ratio.

Figure 4: Anticipation and the distribution of SRs



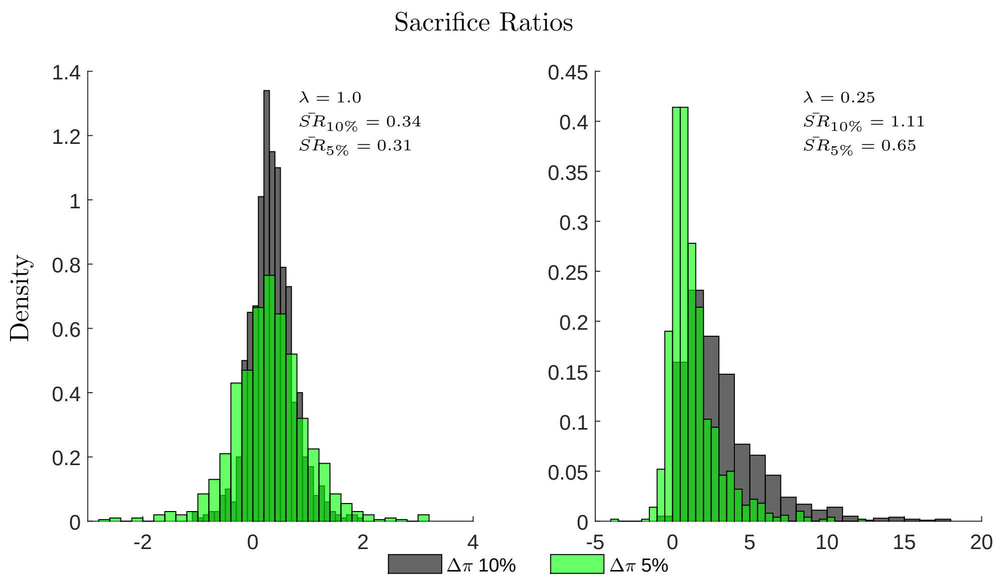
Notes: Distribution of sacrifice ratios for a 10% disinflation under the US calibration. The anticipated disinflation is announced two quarter prior to implementation. $\bar{SR}_U$ is the mean sacrifice ratio of unanticipated disinflations and $\bar{SR}_A$ is the mean sacrifice ratio of anticipated disinflations.

Figure 5: Gradual versus cold turkey disinflations



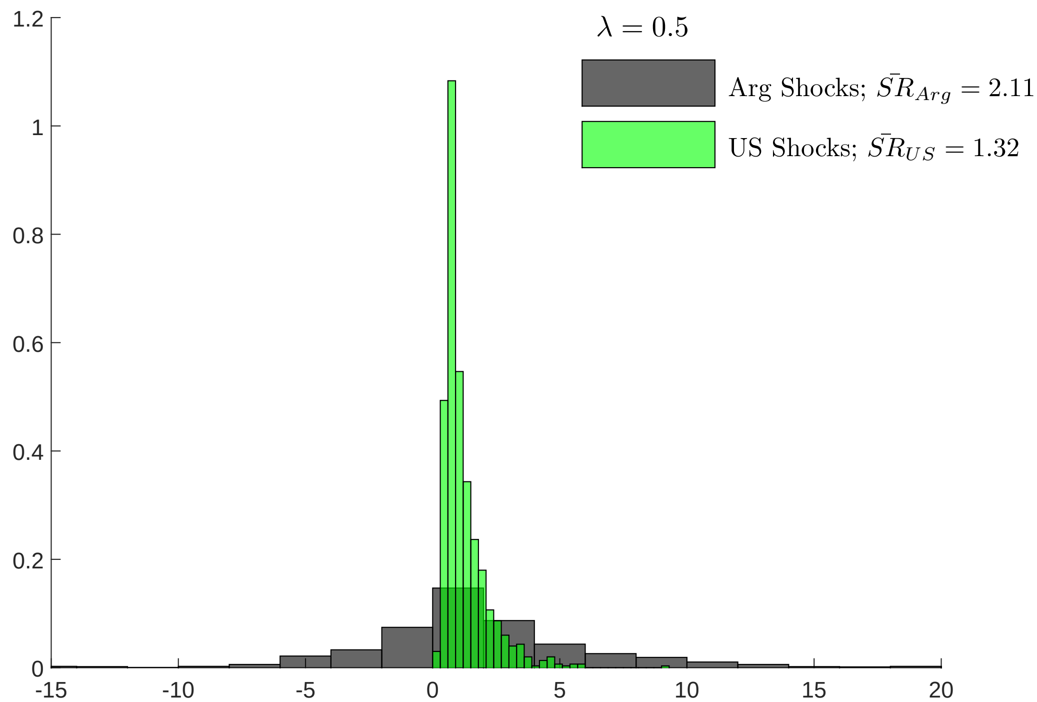
Notes: Distribution of sacrifice ratios and the distribution of durations of the disinflation episodes for a 10% disinflation under the US calibration for a gradual ($\rho_\pi^* = 0.8$) and a cold turkey disinflation. $\bar{SR}_C$ is the mean sacrifice ratio of cold turkey disinflations and $\bar{SR}_G$ is the mean sacrifice ratio of gradual disinflations. $\bar{N}_C$ is the mean duration in quarters of cold turkey disinflations and $\bar{N}_G$ is the mean duration in quarters of gradual disinflations.

Figure 6: Size of the disinflation



Notes: Distribution of sacrifice ratios and the distribution of durations of the disinflation episodes for a 10% and 5% disinflation under the US calibration. $\bar{SR}_{10\%}$ is the mean sacrifice ratio of 10% disinflations and $\bar{SR}_{5\%}$ is the mean sacrifice ratio of 5% disinflations.

Figure 7: The size of shocks and the distribution of the sacrifice ratio



Notes: Distribution of sacrifice ratios for 30% disinflation under the Argentinean calibrations for the policy and structural parameters. The shock parameters are set to the Argentinean values for the first distribution and the US values to construct the second distribution. $\bar{SR}_{Arg}$ is the mean sacrifice ratio of with large shocks and $\bar{SR}_{US}$ is the mean sacrifice ratio with small shocks.

List of Tables

Table 1: Determinants of the sacrifice ratio

	(1)	(2)	(3)	(4)
VARIABLES	SR	SR	SR	SR
$\Delta\pi$	-0.471***			-0.317
	(0.122)			(0.196)
	[0.000]			[0.108]
Peak Trend π		-0.356***		-0.217*
		(0.082)		(0.119)
		[0.000]		[0.071]
Duration			-0.024	0.088
			(0.081)	(0.091)
			[0.767]	[0.337]
Observations	150	150	150	150
R-squared	0.092	0.113	0.001	0.129

Notes: Hyperinflation episodes are excluded from the sample (π >50%). Estimated standard errors are reported in parentheses. *** significant at 1%; ** significant at 5%; * significant at 10% (with p-values in brackets).

Table 2: Determinants of the sacrifice ratio

VARIABLES	(1) SR	(2) SR	(3) SR	(4) SR	(5) SR
New CBG	-0.313			0.554	1.225
	(1.698)			(2.010)	(1.893)
	[0.854]			[0.783]	[0.519]
Reappointed		0.148		0.097	0.137
		(1.902)		(1.990)	(1.973)
		[0.938]		[0.961]	[0.945]
Irr. appointment			-2.210	-2.584	-2.004
			(2.478)	(2.845)	(2.681)
			[0.374]	[0.365]	[0.456]
Observations	126	126	126	126	126
R-squared	0.000	0.000	0.006	0.007	0.147
Observables	No	No	No	No	Yes

Notes: Hyperinflation episodes are excluded from the sample ($\pi > 50\%$). We only have data on central bank governance dating back to 1975, which is why the number of observations decreases in these regressions. The fifth regression includes the three observables of the disinflation episodes as controls. Estimated standard errors are reported in parentheses. *** significant at 1%; ** significant at 5%; * significant at 10% (with p-values in brackets).

Table 3: Estimation results

	Argentinean Data						US Data				
Parameter	Prior Mean	Mode	s.d.	Prior	Prior St. Dev	Parameter	Prior Mean	Mode	s.d.	Prior	Prior St. Dev
ρ_a	0.85	0.9179	0.0872	Beta	0.1	ρ_a	0.85	0.8592	0.0872	Beta	0.01
ρ_e	0.85	0.9606	0.0345	Beta	0.1	ρ_e	0.85	0.8620	0.0345	Beta	0.01
ρ_z	0.5	0.3755	0.2183	Beta	0.2	ρ_z	0.5	0.0715	0.2183	Beta	0.2
ρ_v	0.85	0.9178	0.0628	Beta	0.1	ρ_v	0.97	0.9632	0.0628	Beta	0.01
ρ_r	0.7	0.9674	0.0587	Beta	0.2	ρ_r	0.97	0.9736	0.0587	Beta	0.01
ρ_π	0.8	0.3749	0.0797	Normal	0.5	ρ_π	0.9	0.7901	0.0797	Normal	0.1
ρ_g	0.2	0.3228	0.0558	Normal	0.1	ρ_g	0.8	0.9662	0.0558	Normal	0.1
ρ_x	0.1	0.0075	0.0062	Normal	0.05	ρ_x	0.2	0.1846	0.0062	Normal	0.05
ρ_π^*	0.7	0.8044	0.0567	Beta	0.1	ρ_π^*	0.2	0.7299	0.0567	Beta	0.2
π	0.036	0.0454	0.0052	Normal	0.01	π	0.01	0.0081	0.0052	Normal	0.1
μ_a	-0.1	-0.1669	0.1642	Normal	0.2	μ_a	-0.3	-0.2999	0.1642	Normal	0.01
μ_e	-0.1	-0.0597	0.1969	Normal	0.2	μ_e	0.6	0.8573	0.1969	Normal	0.2
μ_z	0.1	0.1603	0.1918	Normal	0.2	μ_z	0.1	-0.0269	0.1918	Normal	0.2
μ_v	0.0	0.1776	0.0575	Normal	0.2	μ_v	0.0	0.0372	0.0575	Normal	0.2
Standard deviation of shocks						Standard deviation of shocks					
σ_a	0.02	0.0089	0.0034	Inv. Gamma	∞	σ_a	0.015	0.0089	0.0034	Inv. Gamma	∞
σ_e	0.01	0.0061	0.0008	Inv. Gamma	∞	σ_e	0.001	0.0139	0.0008	Inv. Gamma	∞
σ_z	0.012	0.0051	0.0018	Inv. Gamma	∞	σ_z	0.020	0.0008	0.0018	Inv. Gamma	∞
σ_r	0.02	0.0051	0.0009	Inv. Gamma	∞	σ_r	0.010	0.0090	0.0009	Inv. Gamma	∞
σ_π^*	0.01	0.0036	0.001	Inv. Gamma	∞	σ_π^*	0.010	0.0004	0.001	Inv. Gamma	∞
σ_v	0.07	0.0354	0.0129	Inv. Gamma	∞	σ_v	0.070	0.0799	0.0129	Inv. Gamma	∞

Notes: Parameters values used for the quantitative exercises. Values were obtained by estimating the model on US and Argentinean aggregate data. Details of the estimation are given in the appendix. The gain in the recursive least squares algorithm (γ) is set to 0.2 for all simulations, which is close to the value estimated by Milani (2007) on US data. The parameter ρ_π^* is set to zero for cold turkey disinflations.

Table 4: The sacrifice ratio and luck

	US Calibration: Small Shocks			Argentinean Calibration: Large Shocks		
	Inflation Below Target	Inflation Above Target	Full Distribution	Inflation Below Target	Inflation Above Target	Full Distribution
$\lambda = 1/3$						
Mean	0.92	2.59	2.35	0.10	7.54	3.59
St. Deviation	0.97	2.07	2.02	5.04	9.16	7.05
Skewness	1.21	1.61	1.67	0.51	2.85	2.62
N	1,200	1,211	5,000	1,123	1,110	5,000
$\lambda = 2/3$						
Mean	0.23	0.61	0.54	-1.30	4.24	1.59
St. Deviation	0.36	0.62	0.58	5.64	5.96	5.35
Skewness	1.35	2.54	2.68	-0.72	3.40	0.89
N	1,194	1,189	5,000	1,120	1,097	5,000
$\lambda = 1$						
Mean	0.12	0.37	0.34	-2.54	3.79	0.73
St. Deviation	0.53	0.38	0.40	7.31	5.94	6.77
Skewness	0.26	0.33	-0.05	-0.30	0.79	-0.83
N	1,178	1,173	5,000	1,108	1,094	5,000

Notes: The mean, standard deviation, and skewness refers to the distribution of sacrifice ratios for disinflation policies implemented after periods for which average inflation is below or above target. We define average inflation as being 'below or above target' if the deviation of $1/4 \sum_{t=T^*-4}^{T*-1} \pi_t$ is more than 1% with small shocks and more than 5% with the large shock.

Table 5: Disinflation episodes and sacrifice ratios

Country	Date of DE	Peak Trend Inflation	Δ Inflation	Duration (Q)	Sacrifice Ratio	Country	Date of DE	Peak Trend Inflation	Δ Inflation	Duration (Q)	Sacrifice Ratio
Argentina*	1962q4 : 1964q2	75.0	14.3	6	1.8	Denmark	1967q2 : 1969q4	8.9	4.1	10	-0.2
Argentina*	1965q4 : 1969q2	87.8	63.6	14	0.9	Denmark	1974q3 : 1976q4	12.9	3.8	9	3.7
Argentina*	1972q2 : 1973q4	151.2	42.2	6	0.1	Denmark	1980q4 : 1987q1	12.0	8.2	25	0.1
Argentina*	1976q2 : 1977q4	543.4	216.4	6	0.0	Denmark	1988q4 : 1994q1	4.6	3.0	21	7.4
Argentina*	1984q3 : 1986q4	820.1	581.5	9	0.1	Estonia	2001q2 : 2003q4	4.9	2.9	10	0.2
Argentina*	1989q2 : 1997q1	1987.2	1986.5	31	0.1	Estonia	2007q4 : 2010q2	8.0	6.1	10	9.0
Argentina	1997q3 : 2000q1	3.1	7.0	10	-4.2	Finland	1963q4 : 1966q3	7.5	3.1	11	-4.4
Argentina	2003q1 : 2004q1	54.2	38.7	4	-0.1	Finland	1967q4 : 1970q2	7.0	4.3	10	4.6
Argentina	2006q1 : 2008q4	31.3	16.4	11	-3.1	Finland	1975q1 : 1979q2	17.3	9.5	17	6.2
Argentina	2011q1 : 2012q2	19.2	2.2	5	-5.2	Finland	1981q2 : 1987q1	11.4	7.9	23	-0.1
Australia	1975q1 : 1979q1	15.1	6.4	16	-0.8	Finland	1989q4 : 1996q3	6.3	5.7	27	35.8
Australia	1982q3 : 1985q1	10.8	5.1	10	6.8	Finland	2001q1 : 2005q1	2.7	2.3	16	7.2
Australia	1989q2 : 1993q2	7.6	6.2	16	2.5	Finland	2007q4 : 2010q2	3.2	2.3	10	8.5
Australia	1995q3 : 1998q3	3.6	3.0	12	0.5	France	1963q1 : 1966q4	5.1	2.5	15	-3.4
Austria	1974q4 : 1979q1	8.8	5.1	17	5.3	France	1975q1 : 1978q1	12.3	2.9	12	2.6
Austria	1981q2 : 1987q3	6.6	5.0	25	-0.7	France	1981q2 : 1987q3	13.5	10.8	25	2.3
Austria	1993q1 : 1998q4	3.7	3.0	23	6.2	Germany	1973q4 : 1978q3	6.9	3.8	19	9.9
Belgium	1975q2 : 1979q2	12.6	7.9	16	-0.1	Germany	1981q1 : 1987q2	5.9	5.7	25	6.4
Belgium	1982q3 : 1987q4	8.3	7.1	21	5.9	Germany	1992q3 : 1996q3	4.9	3.3	16	3.6
Belgium	1990q3 : 1999q1	3.4	2.4	34	29.0	Greece	1966q2 : 1968q4	4.2	3.3	10	4.9
Brazil*	1985q2 : 1986q3	214.2	35.6	5	-0.1	Greece	1974q4 : 1978q1	21.2	8.9	13	-8.1
Brazil*	1989q4 : 1992q2	2453.8	1735.3	10	0.0	Greece	1980q4 : 1984q4	24.4	5.8	16	-2.5
Brazil*	1993q4 : 1998q4	2160.7	2156.8	20	0.0	Greece	1985q4 : 1989q1	20.8	7.0	13	3.3
Brazil	2002q4 : 2007q2	11.1	7.1	18	5.1	Greece	1991q1 : 2000q2	19.7	16.8	37	11.2
Canada	1975q1 : 1977q3	10.7	2.8	10	1.2	Hungary*	1991q1 : 1994q2	31.0	9.9	13	0.5
Canada	1981q3 : 1985q4	11.7	7.7	17	8.7	Hungary	1995q4 : 2003q2	25.3	20.2	30	3.1
Canada	1990q3 : 1994q2	5.3	4.3	15	13.2	Iceland	1964q1 : 1968q4	15.4	13.5	19	-10.7
Chile*	1974q4 : 1982q2	487.1	471.2	30	1.0	Iceland	1980q4 : 1981q4	55.6	4.5	4	-0.2
Chile*	1984q3 : 1988q4	25.7	9.2	17	20.7	Iceland	1983q1 : 1987q3	65.9	45.6	18	1.2
Chile	1990q3 : 2004q4	23.7	21.9	57	-24.6	Iceland	1988q4 : 1995q3	23.2	21.6	27	2.3
Chile	2008q1 : 2010q3	6.4	5.4	10	5.1	Iceland	2001q2 : 2003q4	6.2	3.6	10	-0.3
China*	1988q3 : 1991q2	17.6	14.1	11	-1.8	Iceland	2009q1 : 2012q1	11.8	7.4	12	9.7
China*	1994q3 : 1999q2	20.8	21.8	19	-11.8	India*	1967q1 : 1969q3	11.6	10.3	10	-0.3
China*	2007q4 : 2010q1	5.0	3.5	9	-11.2	India*	1974q2 : 1976q4	22.6	23.8	10	0.0
Colombia*	1974q3 : 1975q4	24.7	3.1	5	0.3	India*	1981q1 : 1985q3	12.0	5.3	18	-0.7
Colombia*	1977q1 : 1979q2	26.6	5.0	9	-3.3	India*	1991q3 : 1994q1	12.9	4.6	10	2.0
Colombia*	1980q4 : 1984q2	27.0	8.7	14	-10.0	India	1997q4 : 2000q4	10.1	6.8	12	-2.0
Colombia*	1991q2 : 2006q2	29.5	24.8	60	-50.3	India	2009q4 : 2012q1	11.3	2.2	9	-8.5
Colombia	2008q1 : 2010q4	6.3	3.6	11	7.0	Indonesia*	1974q1 : 1978q2	34.3	24.8	17	0.6
Czech Republic*	1992q4 : 1996q4	16.0	7.6	16	0.5	Indonesia*	1980q2 : 1982q4	17.7	7.5	10	-1.8
Czech Republic	1997q3 : 2000q2	9.7	6.5	11	4.2	Indonesia*	1983q4 : 1986q2	11.0	5.3	10	-0.9
Czech Republic	2001q1 : 2003q3	4.3	3.3	10	2.6	Indonesia	1998q3 : 2000q4	37.9	31.6	9	0.6
Czech Republic	2008q1 : 2010q3	4.4	3.1	10	5.3						

Notes: Sacrifice ratios calculated using the methodology proposed by Ball (1994b). *Denotes that annual GDP growth was used to calculate output losses.

Table 6: Disinflation episodes and sacrifice ratios II

Country	Date of DE	Peak Trend Inflation	Δ Inflation	Duration (Q)	Sacrifice Ratio	Country	Date of DE	Peak Trend Inflation	Δ Inflation	Duration (Q)	Sacrifice Ratio
Indonesia	2001q4 : 2004q3	11.4	4.8	11	0.5	Portugal	2007q1 : 2010q1	2.8	2.4	12	-2.7
Indonesia	2005q3 : 2010q4	11.3	6.5	21	0.7	Russia	1999q3 : 2007q1	58.0	48.4	30	0.6
Ireland	1981q2 : 1988q1	19.8	17.1	27	3.0	Slovak Republic	1994q1 : 1997q2	17.6	11.5	13	-0.7
Ireland	2001q2 : 2005q1	5.2	2.9	15	3.9	Slovak Republic	2000q3 : 2002q2	11.1	5.6	7	0.9
Ireland	2007q3 : 2010q2	4.7	6.8	11	1.1	Slovak Republic	2003q4 : 2006q4	7.5	4.0	12	4.8
Israel*	1974q4 : 1976q4	37.9	5.4	8	-1.5	Slovak Republic	2007q3 : 2010q2	3.7	2.2	11	-7.9
Israel*	1984q4 : 1988q3	325.8	307.9	15	0.0	Slovenia	2001q1 : 2006q2	8.6	6.1	21	7.7
Israel*	1990q1 : 1993q4	18.6	7.9	15	-1.0	Slovenia	2007q4 : 2010q3	4.4	3.0	11	3.1
Israel*	1994q1 : 2001q1	11.8	10.6	28	-15.5	South Africa	1981q4 : 1984q1	15.0	2.8	9	7.7
Israel	2002q2 : 2004q3	3.7	4.0	9	3.2	South Africa	1986q2 : 1989q2	17.6	3.7	12	-0.6
Italy	1963q4 : 1969q1	6.6	4.4	21	14.4	South Africa	1991q3 : 1997q3	14.9	7.4	24	8.4
Italy	1975q4 : 1978q4	17.7	4.3	12	-2.1	South Africa	1997q4 : 2000q4	7.9	2.9	12	6.0
Italy	1980q4 : 1988q1	19.3	14.5	29	5.6	South Africa	2002q2 : 2005q1	8.1	7.6	11	1.4
Italy	1990q2 : 1994q2	6.4	2.1	16	4.1	South Africa	2008q3 : 2011q1	8.5	3.9	10	5.4
Italy	1995q2 : 1998q4	4.7	2.9	14	0.0	Spain	1965q3 : 1969q3	10.3	6.9	16	0.9
Japan	1974q3 : 1979q1	17.8	13.6	18	2.0	Spain	1977q2 : 1988q2	22.1	16.9	44	12.3
Japan	1980q4 : 1987q4	6.2	6.0	28	5.4	Spain	1990q1 : 1998q3	6.7	4.8	34	12.8
Japan	1990q4 : 1995q4	3.1	3.1	20	4.9	Spain	2007q3 : 2010q1	3.5	2.5	10	-3.9
Korea	1970q3 : 1973q1	15.0	7.1	10	2.6	Sweden	1966q3 : 1969q1	5.7	3.2	10	5.0
Korea	1975q1 : 1977q4	24.4	12.8	11	0.0	Sweden	1977q2 : 1979q1	11.1	2.1	7	2.3
Korea	1980q3 : 1984q4	24.7	22.3	17	1.2	Sweden	1980q4 : 1987q2	12.5	8.2	26	9.2
Korea	1991q1 : 1996q4	8.8	4.2	23	-0.5	Sweden	1990q4 : 1998q1	9.5	9.4	29	13.2
Korea	1997q4 : 2000q2	5.9	4.1	10	14.7	Sweden	2007q4 : 2010q1	2.7	2.1	9	10.1
Latvia	2008q1 : 2010q4	12.4	11.3	11	6.2	Switzerland	1966q3 : 1969q3	4.4	2.1	12	17.7
Luxembourg	1975q3 : 1979q1	10.6	6.6	14	1.1	Switzerland	1974q2 : 1978q1	9.2	8.0	15	8.3
Luxembourg	1982q3 : 1987q3	9.0	9.0	20	3.9	Switzerland	1982q1 : 1987q2	5.9	4.8	21	9.4
Luxembourg	1992q3 : 1998q4	3.3	2.3	25	36.7	Switzerland	1991q1 : 1998q3	5.5	5.3	30	12.9
Mexico	1974q3 : 1975q4	19.5	3.5	5	-0.5	Turkey	1962q4 : 1965q1	5.3	2.1	9	-12.9
Mexico	1977q4 : 1979q2	23.5	5.1	6	-0.1	Turkey	1967q1 : 1969q3	10.8	5.2	10	0.6
Mexico	1983q3 : 1985q2	85.9	23.5	7	-0.3	Turkey	1974q4 : 1976q3	22.2	4.0	7	0.0
Mexico	1987q3 : 1994q1	127.0	118.1	26	0.2	Turkey	1979q4 : 1982q4	77.2	47.0	12	0.7
Mexico	1996q2 : 2007q1	34.6	30.9	43	-3.2	Turkey	1985q1 : 1986q4	45.8	8.9	7	1.3
Netherlands	1981q2 : 1987q3	6.7	7.0	25	8.9	Turkey	1989q2 : 1990q2	65.4	3.6	4	0.9
Netherlands	2001q4 : 2006q1	3.6	2.2	17	17.2	Turkey	1995q2 : 1996q3	96.6	16.3	5	-0.1
New Zealand *	1980q4 : 1984q2	16.3	8.8	14	0.3	Turkey	1997q3 : 2005q2	85.8	77.5	31	3.0
New Zealand *	1986q3 : 1993q1	15.4	14.2	26	4.8	Turkey	2007q4 : 2010q3	9.6	2.8	11	21.0
New Zealand	1995q4 : 1999q1	3.0	2.4	13	1.1	United Kingdom	1975q2 : 1978q4	20.7	9.7	14	3.4
Norway	1975q3 : 1979q2	10.6	4.0	15	-2.1	United Kingdom	1980q3 : 1984q2	15.7	10.8	15	3.2
Norway	1981q3 : 1985q3	12.6	6.7	16	8.6	United Kingdom	1991q1 : 2001q1	7.2	6.2	40	15.1
Norway	1987q3 : 1994q1	8.1	6.2	26	15.5	United States	1974q4 : 1977q2	9.9	3.7	10	4.7
Portugal	1977q4 : 1980q4	26.5	8.8	12	-0.4	United States	1980q3 : 1984q2	12.3	8.5	15	4.6
Portugal	1984q2 : 1988q1	26.2	16.1	15	2.6	United States	1990q2 : 1995q1	5.1	2.4	19	14.1
Portugal	1990q2 : 1999q2	13.2	10.8	36	7.3	United States	2007q3 : 2010q1	3.4	2.6	10	3.1

Notes: Sacrifice ratios calculated using the methodology proposed by Ball (1994b). *Denotes that annual GDP growth was used to calculate output losses.