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UNSW Business School Research Paper No. 2016 ECON 01

# Age, Time, Vintage, and Price Indexes: Measuring the Depreciation Pattern of Houses

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February 2016

**Abstract:** Age, time and vintage are key determinants of house prices, yet they cannot be included together linearly or as dichotomous variables in hedonic regressions as *construction time + age of house = sale time*. We introduce a method where the estimates of the age, time and vintage effects on prices are obtained in a flexible manner, without requiring us to specify a pre-determined functional form for either of these variables. Applying our method to Dutch data, we find that the estimated depreciation pattern over the life of houses does not follow the functional forms typically specified for the age of houses in hedonic regressions.

**Keywords:** Age-price profile, capital formation, hedonic regressions, GEKS index

**JEL Classification Codes:** C43, E01, E31, R31

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The authors thank Ana Aizcorbe, Erwin Diewert, Kevin Fox, Alicia Rambaldi, Michael Scholz, Nigel Stapledon, Tsutomu Watanabe and Emma Wise for helpful comments, and the Australian Research Council (LP140101020) for providing financial support for this research. The corresponding author is Iqbal Syed.

# 1 Introduction

It is well known that time of sale, product age and cohort—three temporal variables—cannot all be included linearly or as dichotomous variables in hedonic regressions because of the identity: *age + cohort = sale time*. This is problematic in the housing context because these variables are regarded as important determinants of house prices, and the exclusion of one of the variables potentially biases the estimates of the other variables (Bailey, Muth and Nourse 1963; Goodman and Thibodeau 1995; Knight and Sirmans 1996; Hill, Knight and Sirmans 1997; Englund, Quigley and Redfearn 1998; Chau, Wong and Yiu 2005; Harding, Rosenthal and Sirmans 2007).

Perhaps two of the most important measures in the housing context are housing inflation and age-price profile of houses. In hedonic regressions, it is customary to include the time of sale as dummy variables to capture the movement of house prices. The age variable, on the other hand, is included in a non-linear form, such as *log(age)*, *squared-age*, or *squared- and cubic-age*, in order to obtain estimates of depreciation rates. This approach averts the perfect collinearity problem and provides an algebraic solution to the ordinary least squares problem. However, forcing the measure of the age effect to follow a pre-defined functional form may produce biased estimates of the depreciation pattern of houses (Malpezzi, Ozanne and Thibodeau 1987; Coulson and McMillen 2008). Malpezzi et al., surveying the empirical literature of housing depreciation, found a large variability in the estimates of depreciation rates, ranging from 0.5% to 2.5% per year. They make the following observation:

> "One shortcoming of the first two methods [the observed age and perpetual inventory methods] and of most hedonic studies, which may be a source of variability of results, is that they restrict functional form in a manner which arbitrarily imposes a particular depreciation pattern". (p. 373)

In this paper, we introduce a method that provides estimates of age, cohort and time effects emerging directly from the variation in the data without requiring us to specify a pre-defined functional form for any of these variables. The method follows the logic of the *hedonic imputation* approach and uses state-of-the-art index number formulae in the housing context. The advantages of the hedonic imputation method in the context of measuring inflation have been discussed in the literature (e.g. Triplett 1996; Pakes 2003; Silver and Heravi 2007; Hill and Melser 2008; Diewert, Heravi and Silver 2009; de Haan 2010; Syed 2010; Erickson and

Pakes 2011; Hill 2013; Rambaldi and Fletcher 2014); these include the method's flexibility in terms of its treatment towards the regression parameters of the models and, when the double imputation is applied, its ability to reduce the omitted variable bias in estimated price indexes. However, the use of the hedonic imputation method in disentangling the age, cohort and time effects, and measuring the depreciation pattern of houses remains unexplored. We show how the method can address one of the challenging econometric tasks of separating the effects of these three highly correlated temporal variables in product prices.

Applying our method to data for a city in the Netherlands, we find that the functional forms typically used in the literature to represent the depreciation pattern do not provide reasonable approximations of the actual pattern of depreciation over the life of a house. We also find that omitting the cohort effect in hedonic regressions significantly overestimates the ageing effect on house prices. Although we apply our model to the housing market, its application extends to many other durable products, such as furniture, used cars, electronic items, machineries and non-residential structures.

In the next section, we briefly discuss the hedonic method typically used in the literature for measuring depreciation rates, including the approaches pursued so far to disentangle the age, time and vintage effects in house prices. Section 3 provides a detailed description of our method. In section 4, we apply our method to Dutch data and compare our measures with those obtained from standard approaches. Section 5 concludes the paper.

# 2 Hedonic Approaches to Measuring the Depreciation of Houses

The economic depreciation of assets is typically defined as the decline in asset prices due to the ageing of assets (Hotelling 1925; Fieldstein and Rothschild 1974; Hall 1968; Hulten and Wykoff 1981), and the measurement of economic depreciation centers around establishing an empirical relationship between price and age of assets (Jorgenson 1996; Clapp and Giaccotto 1998).[1] The most widely used method to obtain a measure of economic depreciation is the

[1]While the empirical literature of depreciation focusses on finding the relationship between price and age, some question whether the ageing effect actually identifies the economic depreciation. For example, Rubin (1993) argues that the negative age effect is the consequence of age premiums for new houses rather than the consequence of depreciation of old houses. Hulten and Wykoff (1996) provide an excellent review of the debate surrounding the theory and measurement of economic depreciation of capital goods (see also Goodman and Thibodeau 1995; Clapp and Giaccotto 1998; Redfearn 2009).

hedonic method where data are pooled across time, and the natural log of prices for houses is hypothesised as a function of time dummies; characteristics of houses, including their age; and a random error term ($\epsilon_i$), as follows:[2]

$$ln\, p_i = \sum_{t=1}^{T} \delta_t d_{t,i} + \sum_{c=1}^{C} \beta_c z_{c,i} + \gamma f(a_i) + \epsilon_i, \qquad i = 1, \ldots, I; \tag{1}$$

where, $ln\, p_i$ refers to the natural log of prices of house $i$, for $i = 1, \ldots, I$. $d_{t,i}$ refers to time dummies taking the value of 1 if the period of sale of house $i$ is $t$, and 0 otherwise, for $t = 1, \ldots, T$, with the exponentials of the coefficients, $\delta_t$, providing a quality-adjusted price index. The contribution of each characteristic of a house to its log(price) is given by the coefficient $\beta_c$, where $z_{c,i}$ is a measure of characteristic $c$ for house $i$, $c = 1, \ldots, C$. A non-linear function of age, $f(a_i)$, is included as one of the characteristics in hedonic regressions, and $\gamma$ provides a measure of the economic depreciation of houses. This measure provides a measure of the net depreciation rather than the gross depreciation rate as the effect of maintenance expenses on prices is not removed from $\gamma$ (see Knight and Sirmans 1996; Harding et al. 2007).[3]

Different non-linear specifications of $f(a_i)$ have been used in the literature, which might have influenced the estimates of depreciation rates obtained in different studies. For example, Malpezzi et al. (1987) specify a cubic age function and a dummy variable if the dwelling belongs to the oldest cohort, while Lee, Chung and Kim (2005) specify cubic and log age functions. Smith (2004) specifies a squared age function and interaction terms of age with location and period of sale, and Fletcher, Gallimore and Mangan (2000) and Fisher et al. (2005) use a quadratic age function. Knight and Sirmans (1996) and Wilhelmsson (2008) specify a quadratic age function and interaction variables of age with maintenance, and Clapp and Giaccotto (1998) apply a non-linear weight to the age variable and allow the age coefficient to change with shifts in supply and demand over time. Ong, Ho and Lim (2003) and Harding et al. (2007) specify a log age function, the latter within a repeat-sales regression framework.[4]

[2]Goodman and Thibodeau (1995) argue that $\epsilon_i$ can be heteroscedastic if dwelling improvements are not adequately captured in hedonic regressions, with larger variations of the errors occurring for older houses rather than for newer houses.

[3]To our best knowledge, Harding et al. (2007) is the only paper that provides estimates of gross depreciation rates of houses which they obtain after controlling for maintenance expenses of the sample houses in the American Housing Survey data.

[4]The collinearity problem between time and age would not arise if data for a fixed period of time is

While the typical approach in specifying hedonic regressions in the housing context does not properly account for age and cohort effects (Sirmans et al. 2006), the problem that this may cause has been noted at least since Baily et al. (1963). Case and Quigley (1991), Quigley (1995), Hill et al. (1997) and Englund et al. (1998) approached the perfect collinearity issue by jointly estimating hedonic and repeat-sales models. A common feature of these models, the hybrid models, is that the single and repeat prices are combined in a single regression through some explicit assumptions about the error structures of the single and repeat prices, where the details of the error structures vary across the models. Intuitively, these models exploit the cross sectional variation of price levels to obtain estimates of the ageing effect which are then augmented in the repeat-sales prices to obtain estimates of the inflationary effects (see also Yiu 2009). The findings of these papers indicate that both age and time effects should be accounted for in the regression of housing values.

In a more recent work, and following a different approach, Coulson and McMillen (2008) find that the age and cohort effects on the house prices in Chicago are different and argue for "treating cohort and age effects separately and more flexibly than is possible in a standard hedonic [model]" (p. 148). Following McKenzie (2006), they assume that each price consists of additive components of age, time and cohort effects. They take the second-difference of house prices in a particular order to nullify two temporal effects and, with some normalization assumptions, identify the third temporal effect on the prices (see also Yiu 2009, and Karato, Movshuk and Shimizu 2015). Recently, Karato et al. (2015) have constructed bivariate splines of age and cohort effects on house prices in the framework of the generalized additive model and find that the depreciation patterns vary across different cohorts of houses in the city of Tokyo.

used (see e.g. Shilling, Sirmans and Dombrow 1991; Lee et al. 2005; Wilhelmsson 2008). Some researchers have, however, argued that the results obtained from a single point in time may not represent the actual depreciation patterns of houses (Clapp and Giaccotto 1998; Smith 2004; Coulson and McMillen 2008). Furthermore, the difference in the estimated depreciation rates between the periods would be due to a mix of quality change and inflationary effects (Dixon, Crosby and Law 1999).

# 3 Disentangling the Age, Cohort and Time Effects in House Prices

We apply the logic of the hedonic imputation method in conjunction with the GEKS index number formulae in a new context. Our objective is to obtain separate measures of age, cohort and time effects, which are not convoluted by the other effects on houses prices. For example, we obtain measures of the depreciation rate which are controlled for (1) the inflationary effect as indicated by the time of sale, (2) the cohort effect as indicated by the period of construction of houses and (3) the quality change effect on prices, net of the maintenance effect, as indicated by the differences in the characteristics of houses.

In order to achieve this, we run hedonic regressions separately for each age-cohort pair of houses. Although the age and cohort of the houses are fixed in each regression, the time of sale of these houses would differ. This is because a cohort typically covers a long period of time. For instance, if a unit of age is 1–2 years and a cohort is 10–20 years, there would be sample houses in each age-cohort pair which are sold in different periods, where each period corresponds to a month or quarter. These houses would be of different quality as indicated by their locations and physical characteristics. Each of the age-cohort hedonic regressions includes time dummies to control for the difference in the time of sale, and the characteristics of houses to control for the difference in the quality of houses. There are no restrictions on how these variables and their interactions are included in the hedonic regressions, and whether the hedonic models of *log(price)* or *price level* are specified.[5]

Once the hedonic models have been estimated for each age-cohort pair of houses, the price of a house of a given cohort sold at one age in a particular period is imputed if the same house in the same period was sold at another age. Suppose a house was sold in 2010, 10 years after its construction. We would impute the price of this house sold in the same period but when it was 9 and 11 years old. The imputation process provides us with estimates of price ratios, referred to as price relatives in the index number literature, which compare the prices of houses sold at two different ages, holding the time of sale, cohort and characteristics

[5] Note that this set up is not the same as running a single hedonic regression on pooled data, as shown in equation (1), with cohort, age and time of sale defined linearly but in different intervals. This is because in a single equation, these three temporal variables would be highly collinear leading to unstable and imprecise estimates, the degree of which would depend on the differences in the time intervals for these temporal variables. Our method separates these three variables at the regression stage, and then pull together the individual level estimates using indexes to obtain average measures of these temporal effects.

constant. These price relatives are aggregated using index number formulae in order to obtain price indexes measuring the price changes due to the ageing of the houses. In a similar way, we can measure the cohort effect by estimating the price relatives, comparing the prices of houses belonging to different cohorts, holding the age, time and characteristics constant. The inflationary effects, on the other hand, are obtained directly from the estimated time dummy coefficients in the regressions.

## 3.1 Hedonic Imputations

For the purpose of illustration, let us consider two ages, $j$ and $k$, and two cohorts, $l$ and $m$, of houses. This gives us houses belonging to four age-cohort pairs: $(j,l),(k,l),(j,m)$ and $(k,m)$. We estimate hedonic regressions of the following form for each age-cohort pair of houses:

$$ln\, p_i^{a,v} = \sum_{t=1}^{T} \delta_t^{a,v} d_{t,i}^{a,v} + \sum_{c=1}^{C} \beta_c^{a,v} z_{c,i}^{a,v} + u_i^{a,v}, \qquad (j,k) \in a, \quad (l,m) \in v, \quad i = 1,\ldots,I^{a,v}; \quad (2)$$

where $ln\, p_i^{a,v}$ denotes the price of house $i$ belonging to cohort $v$ and sold at age $a$, for $i = 1,\ldots,I^{a,v}$, where $I^{a,v}$ is the number of houses of age $a$ and cohort $v$ in the entire sample, with $(j,k) \in a$ and $(l,m) \in v$. $d_{t,i}^{a,v}$ takes the value of 1 if house $i$ is sold in period $t$ and 0 otherwise. $z_{c,i}^{a,v}$ refers to the value of characteristic $c = 1,\ldots,C$ of house $i$, and the $u_i^{a,v}$ are *i.i.d.* error terms. The periods $t = 1,\ldots,T$ are those in which transactions took place. The exponential of $\delta_t^{a,v}$ provides a measure of price change of the houses of cohort $v$ sold at age $a$ in period $t$ holding other characteristics constant, and $\beta_c^{a,v}$ refers to the implicit value of the characteristic $c$ of the $(a,v)$ pair houses.

Consider a house, $h$, which belongs to cohort $l$ and was sold when it was at age $j$. Then the price of house $h$ reaching age $k$ (i.e. when house $h$ becomes older) can be imputed from equation (2) as follows, for $x_h^{j,l} = (d_{1,h}^{j,l},\ldots,d_{T,h}^{j,l}, z_{1,h}^{j,l},\ldots,z_{C,h}^{j,l})$, and where a hat denotes an estimated parameter:[6]

$$\widehat{p}_s^{k,l}(x_h^{j,l}) = exp\left(\sum_{t=1}^{T} \widehat{\delta}_t^{k,l} d_{t,h}^{j,l} + \sum_{c=1}^{C} \widehat{\beta}_c^{k,l} z_{c,h}^{j,l}\right). \qquad (3)$$

[6]This requires making an assumption that $E(exp(u_i^{a,v})) = 1$, which is not the case because we are taking the expectation of non-linear transformation of a random variable. One can carry out a correction (Kennedy 1981), but these corrections are typically small enough that they can be ignored.

Hence, in order to obtain the imputed price of house $h$ of cohort $l$ as it reaches age $k$ (originally sold at age $j$): (1) we insert the characteristics of house $h$ into the corresponding estimated implicit values obtained from the hedonic regression for the houses of the $(k, l)$ pair, and (2) we stick the period of the sale of house $h$ with the estimated coefficient for the same period obtained from the hedonic regression for $(k, l)$ houses. Therefore, $\widehat{p}_h^{k,l}(x_h^{j,l})$ provides the imputed price of house $h$ if this house was sold in period $t$, but instead of being sold at age $j$, the house was sold at age $k$.

In a similar way, using the estimated coefficients obtained from the $(j, m)$ hedonic regression, we can impute the price of a house sold at age $j$, originally belonging to cohort $l$, had this house belonged to cohort $m$:

$$\widehat{p}_h^{j,m}(x_h^{j,l}) = exp\left(\sum_{t=1}^{T} \widehat{\delta}_t^{j,m} d_{t,h}^{j,l} + \sum_{c=1}^{C} \widehat{\beta}_c^{j,m} z_{c,h}^{j,l}\right). \tag{4}$$

We can use the estimated hedonic regression of the $(j, l)$ pair houses to impute a price for the same house sold at age $j$ and cohort $l$:

$$\widehat{p}_h^{j,l}(x_h^{j,l}) = exp\left(\sum_{t=1}^{T} \widehat{\delta}_t^{j,l} d_{t,h}^{j,l} + \sum_{c=1}^{C} \widehat{\beta}_c^{j,l} z_{c,h}^{j,l}\right). \tag{5}$$

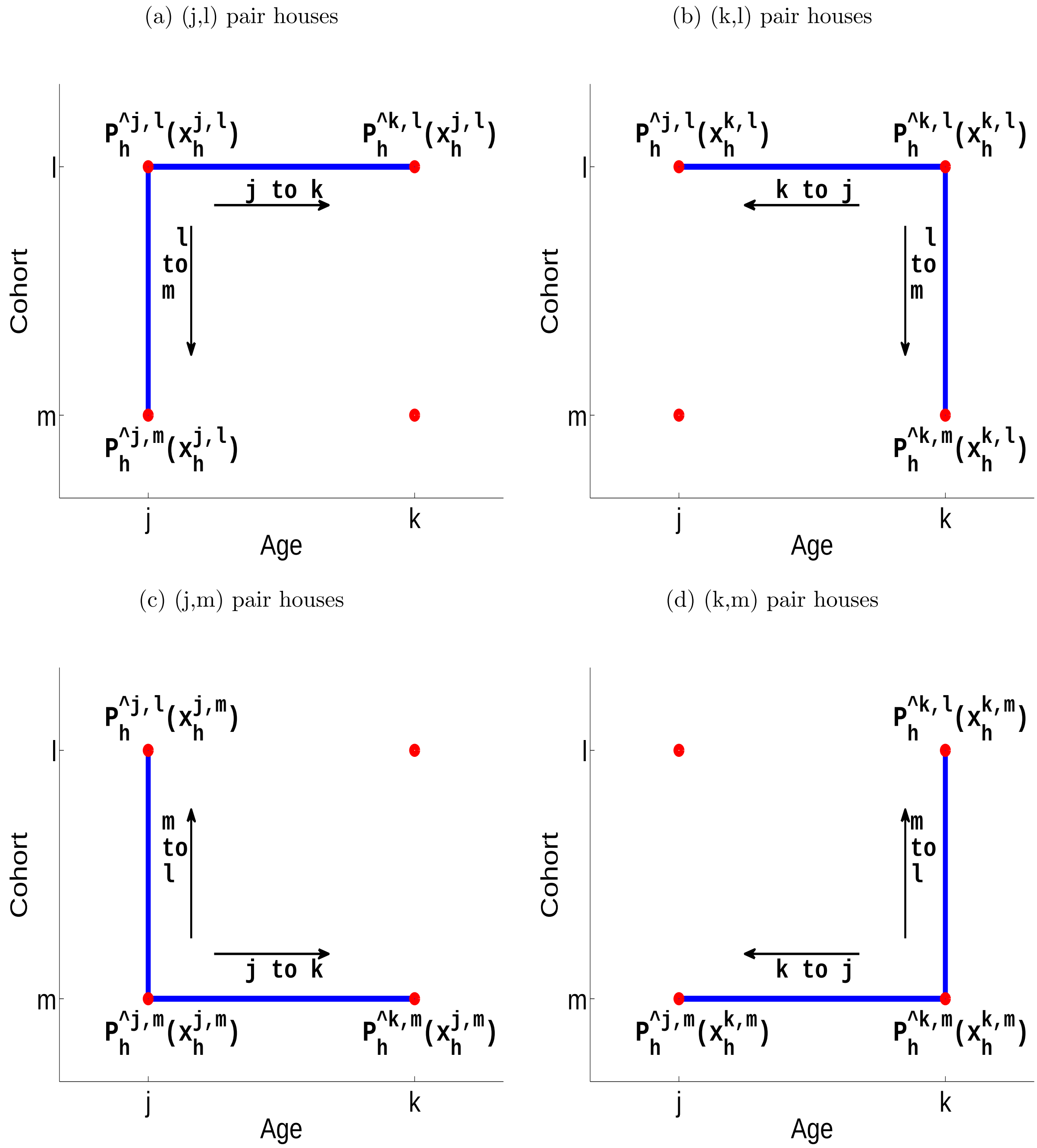
Hence, the imputation process provides us with three imputed prices for each house in the sample. This is shown in Figure 1. For example, if a house is in the $(j, l)$ age-cohort pair, as in Figure 1(a), then the three imputed prices are $\widehat{p}_s^{k,l}(x_h^{j,l})$, $\widehat{p}_h^{j,m}(x_h^{j,l})$ and $\widehat{p}_h^{j,l}(x_h^{j,l})$; if a house belongs to the $(k, l)$ age-cohort pair, as in Figure 1(b), then the three imputed prices are $\widehat{p}_s^{j,l}(x_h^{k,l})$, $\widehat{p}_h^{k,m}(x_h^{k,l})$ and $\widehat{p}_h^{k,l}(x_h^{k,l})$; and so on.[7]

A price relative measuring the change in the price of house $h$ as it ages from $j$ to $k$ — holding the cohort, time of sale and the characteristics constant — is obtained from dividing the imputed price of the house at age $k$, shown in equation (3), by its actual price:

$$\wp_h^{(k,l)/(j,l)}(SI) = \frac{\widehat{p}_h^{k,l}(x_h^{j,l})}{p_h^{j,l}}. \tag{6}$$

[7]This imputation process essentially generates numbers for the matrix specified in equation (1) of Hulten and Wykoff (1981a) although in our case each matrix belongs to a single construction vintage of assets.

Figure 1: Imputed Price of Each House in our Four Age-Cohort Pair Houses




Equation (6), referred to as single imputation (SI) price relative, uses the imputed price only when the price is unobserved. An alternative is to use the double imputation (DI) method where we replace the observed price in equation (6) with the imputed price obtained from equation (5). This provides the following measure of price change between age $j$ and $k$:

$$\wp_h^{(k,l)/(j,l)}(DI) = \frac{\widehat{p}_h^{k,l}(x_h^{j,l})}{\widehat{p}_h^{j,l}(x_h^{j,l})}. \tag{7}$$

We can likewise obtain two price relatives measuring the change in price of house $h$ if the cohort changes from $l$ to $m$, while the age, time of sale and other characteristics remain the same. The SI price relative divides the imputed price in equation (4) by its actual price to obtain the following measure of cohort effect:

$$\wp_h^{(j,m)/(j,l)}(SI) = \frac{\widehat{p}_h^{j,m}(x_h^{j,l})}{p_h^{j,l}}. \tag{8}$$

The corresponding DI price relative, which replaces the actual price with the imputed price shown in equation (5), is the following:

$$\wp_h^{(j,m)/(j,l)}(DI) = \frac{\widehat{p}_h^{j,m}(x_h^{j,l})}{\widehat{p}_h^{j,m}(x_h^{j,l})}. \tag{9}$$

Similarly, we can obtain the ageing and cohort effects for each house in the sample belonging to $(j, l)$, $(j, m)$, $(k, l)$ and $(k, m)$ pairs of houses. Table 1 shows the imputed price relatives measuring these effects for the houses belonging to our 4 age-cohort pairs. For example, suppose now that house $h$ belongs to the $(k, m)$ pair. The effect of the change in the cohort from $m$ to $l$ on the price of house $h$ can be obtained from the price relative, $\wp_h^{(k,l)/(k,m)}(DI)=\widehat{p}_h^{k,l}(x_h^{k,m})/\widehat{p}_h^{k,m}(x_h^{k,m})$, where $\widehat{p}_h^{k,m}(x_h^{k,m}) = exp\left(\sum_{t=1}^{T} \widehat{\delta}_t^{k,m} d_{t,h}^{k,m} + \sum_{c=1}^{C} \widehat{\beta}_c^{k,m} z_{c,h}^{k,m}\right)$ is the imputed price of the house at its original cohort $m$ and $\widehat{p}_h^{k,l}(x_h^{k,m})$ $= exp\left(\sum_{t=1}^{T} \widehat{\delta}_t^{k,l} d_{t,h}^{k,m} + \sum_{c=1}^{C} \widehat{\beta}_c^{k,l} z_{c,h}^{k,m}\right)$ is the imputed price of house $h$ if the house had belonged to cohort $l$.[8]

## 3.2 Constructing Price Indexes

In order to obtain aggregated measures of the ageing and cohort effects of houses, we use the resulting price relatives as inputs in index number formulae. We calculate the Fisher Ideal index because it falls in the class of "superlative" index numbers used for measuring price changes (Diewert 1976) and also satisfies the largest number of desirable axiomatic properties of index numbers (see Balk 1995). It is argued that, if data permits, superlative

[8]The SI method uses less imputation and therefore minimizes the estimation variance (Triplett 1996). The DI method potentially reduces the omitted variables bias (Silver and Heravi 2001; Hill and Melser 2008; Syed 2010). We estimate our price indexes using both methods, and find that the difference is very small and does not make any qualitative difference in the result. Therefore, in the subsequent sections we limit our discussions mainly to the DI indexes while the SI results are obtained for robustness checks.

Table 1: Price Relatives Measuring the Ageing and Cohort Effect of Houses

(a) Ageing effect of a house due to change in age from $j$ to $k$

| Age-Cohort $(l,m)\in v$ | Imputation Method | Price Relative for a Single House† |
|---|---|---|
| $(j,v)$ | Single | $\wp_h^{(k,v)/(j,v)}(SI)=\frac{\widehat{p}_h^{k,v}(x_h^{j,v})}{p_h^{j,v}}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{k,v}d_{t,h}^{j,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{k,v}z_{c,h}^{j,v}\right)}{p_h^{j,v}}$ |
| $(j,v)$ | Double | $\wp_h^{(k,v)/(j,v)}(DI)=\frac{\widehat{p}_h^{k,v}(x_h^{j,v})}{\widehat{p}_h^{j,v}(x_h^{j,v})}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{k,v}d_{t,h}^{j,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{k,v}z_{c,h}^{j,v}\right)}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{j,v}d_{t,h}^{j,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{j,v}z_{c,h}^{j,v}\right)}$ |
| $(k,v)$ | Single | $\wp_h^{(k,v)/(j,v)}(SI)=\frac{p_h^{k,v}}{\widehat{p}_h^{j,v}(x_h^{k,v})}=\frac{p_h^{k,v}}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{j,v}d_{t,h}^{k,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{j,v}z_{c,h}^{k,v}\right)}$ |
| $(k,v)$ | Double | $\wp_h^{(k,v)/(j,v)}(DI)=\frac{\widehat{p}_h^{k,v}(x_h^{k,v})}{\widehat{p}_h^{j,v}(x_h^{k,v})}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{k,v}d_{t,h}^{k,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{k,v}z_{c,h}^{k,v}\right)}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{j,v}d_{t,h}^{k,v}+\sum_{c=1}^{C}\widehat{\beta}_c^{j,v}z_{c,h}^{k,v}\right)}$ |

† $x_h^{a,v}=(d_{1,h}^{a,v},\ldots,d_{T,h}^{a,v},z_{1,h}^{a,v},\ldots,z_{C,h}^{a,v})$, $a\in(j,k)$, $v\in(l.m)$ and $t=1,\ldots,T$.

(b) Cohort Effect of a house due to change in cohort from $l$ to $m$

| Age-Cohort, $(j,k)\in a$ | Imputation Method | Price Relative for a Single House |
|---|---|---|
| $(a,l)$ | Single | $\wp_h^{(a,m)/(a,l)}(SI)=\frac{\widehat{p}_h^{a,m}(x_h^{a,l})}{p_h^{a,l}}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,m}d_{t,h}^{a,l}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,m}z_{c,h}^{a,l}\right)}{p_h^{a,l}}$ |
| $(a,l)$ | Double | $\wp_h^{(a,m)/(a,l)}(DI)=\frac{\widehat{p}_h^{a,m}(x_h^{a,l})}{\widehat{p}_h^{a,l}(x_h^{a,l})}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,m}d_{t,h}^{a,l}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,m}z_{c,h}^{a,l}\right)}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,l}d_{t,h}^{a,l}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,l}z_{c,h}^{a,l}\right)}$ |
| $(a,m)$ | Single | $\wp_h^{(a,m)/(a,l)}(SI)=\frac{p_h^{a,m}}{\widehat{p}_h^{a,l}(x_h^{a,m})}=\frac{p_h^{a,m}}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,l}d_{t,h}^{a,m}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,l}z_{c,h}^{a,m}\right)}$ |
| $(a,m)$ | Double | $\wp_h^{(a,m)/(a,l)}(DI)=\frac{\widehat{p}_h^{a,m}(x_h^{a,m})}{\widehat{p}_h^{a,l}(x_h^{a,m})}=\frac{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,m}d_{t,h}^{a,m}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,m}z_{c,h}^{a,m}\right)}{exp\left(\sum_{t=1}^{T}\widehat{\delta}_t^{a,l}d_{t,h}^{a,m}+\sum_{c=1}^{C}\widehat{\beta}_c^{a,l}z_{c,h}^{a,m}\right)}$ |

indexes should be used in order to measure price changes (see e.g. Triplett 1996; Hill 2006a).[9]

The Fisher index can be obtained by taking the geometric mean of the Laspeyres and Paasche indexes. The latter two indexes are probably the two best known price index formulae, whose inception dates back to the nineteenth century. A Laspeyres index based on the houses in the $(j,v)$ pair with $(l,m)\in v$ and measuring the price changes of houses as

[9]Diewert (1976) shows that the superlative indexes are free of substitution bias and have the advantage of being able to approximate the underlying cost of living to the second order. Hill (2006b), however, notes that Diewert's result breaks down for the quadratic mean of order $r$ indexes as $r$ becomes too large in magnitude. However, Diewert's results hold for the standard superlative index such as the one we are using in this paper.

they age from $j$ to $k$ is as follows:

$$P_{\underline{Las}}^{(j,k),v} = \sum_{i=1}^{I^{j,v}} w_i^{j,v} \left[ \frac{\widehat{p}_i^{k,v}(x_i^{j,v})}{\widehat{p}_i^{j,v}(x_i^{j,v})} \right], \qquad where \quad w_i^{j,v} = \frac{p_i^{j,v} q_i^{j,v}}{\sum_{i=1}^{I^{j,v}} p_i^{j,v} q_i^{j,v}}, \quad (l,m) \in v. \tag{10}$$

Here the price relative is obtained from equation (7); $q_i^{j,v}$ refers to the quantity of house $i$ belonging to cohort $v$ sold at age $j$, and $w_i^{j,v}$ reflects the corresponding expenditure share. In constructing consumer price indexes, the weighting of items according to their expenditure shares in the basket of goods and services consumed provides the best representation of the average price movements faced by households. However, the case of housing is different to the regular type of goods and services consumed by households. This is because each house is different and, therefore, irrespective of the price, only one house of a particular type is bought. This means that, in equation (10), $q_i^{j,v} = 1$ and $w_i^{j,v} = p_i^{j,v} / \sum_{i=1}^{I^{j,v}} p_i^{j,v}$. This implies that $w_i^{j,v}$ gives more weight to more expensive houses in the construction of price indexes, which is not the same as giving more weight to items which account for larger expenditure shares in household consumption. Since each house is somewhat different from other houses, it is more reasonable to give equal weight to each house in the sample, $w_i^{j,v} = q_i^{j,v} / \sum_{i=1}^{I^j} q_i^{j,v} = 1/I^{j,v}$ (Hill 2013). Hence, our Laspeyres-type index that measures the price change due to houses ageing from $j$ to $k$, with holding cohort, time of sale and other characteristics constant, is as follows:

$$P_{Las}^{(j,k),v} = \frac{1}{I^{j,v}} \sum_{i=1}^{I^{j,v}} \left[ \frac{\widehat{p}_i^{k,v}(x_i^{j,v})}{\widehat{p}_i^{j,v}(x_i^{j,v})} \right], \qquad (l,m) \in v, \tag{11}$$

which is the arithmetic mean of the imputed price relatives corresponding to the houses sold in the $(j,v)$ pair. A Paasche index based on the houses in the $(k,v)$ pair measuring the price changes due to houses ageing from $j$ to $k$ is as follows:

$$P_{\underline{Pas}}^{(j,k),v} = \left\{ \sum_{i=1}^{I^{k,v}} w_i^{k,v} \left[ \frac{\widehat{p}_i^{k,v}(x_i^{k,v})}{\widehat{p}_i^{j,v}(x_i^{k,v})} \right]^{-1} \right\}^{-1}, \qquad where \quad w_i^{k,v} = \frac{p_i^{k,v} q_i^{k,v}}{\sum_{i=1}^{I^{k,v}} p_i^{k,v} q_i^{k,v}}, \quad (l,m) \in v. \tag{12}$$

Here $q_i^{k,v}$ refers to the quantity of house $i$ of cohort $v$ sold at age $k$. Following the argument as in the Laspeyres case above, we set $w_i^{k,v} = 1/I^{k,v}$. Hence, our Paasche-type index that measures the price change due to houses ageing from $j$ to $k$ is as follows:

$$P_{Pas}^{(j,k),v} = \left\{ \frac{1}{I^{k,v}} \sum_{i=1}^{I^{k,v}} \left[ \frac{\widehat{p}_i^{k,v}(x_i^{k,v})}{\widehat{p}_i^{j,v}(x_i^{k,v})} \right]^{-1} \right\}^{-1}, \qquad (l,m) \in v, \tag{13}$$

which is the harmonic mean of the imputed price relatives corresponding to the houses in the $(k, v)$. We take the geometric mean of the Laspeyres- and Paasche-type indexes to obtain the Fisher-type index as follows:

$$P_F^{(j,k),v} = \sqrt{P_{Las}^{(j,k),v} \times P_{Pas}^{(j,k),v}}, \qquad (l,m) \in v. \tag{14}$$

In a similar way, we construct the price indexes measuring the cohort effect of the houses in $(a, l)$ and $(a, m)$ pairs for $a \in (j, k)$. A Laspeyres index based on the houses in the $(a, l)$ pair and measuring the price change if the cohort had changed from $l$ to $m$ is the following:

$$P_{Las}^{a,(l,m)} = \frac{1}{I^{a,l}} \sum_{i=1}^{I^{a,l}} \left[ \frac{\widehat{p}_i^{a,m}(x_i^{a,l})}{\widehat{p}_i^{a,l}(x_i^{a,l})} \right], \qquad (j,k) \in a. \tag{15}$$

The corresponding Paasche index based on the houses in the $(a, m)$ pair is the following:

$$P_{Pas}^{a,(l,m)} = \left\{ \frac{1}{I^{a,m}} \sum_{i=1}^{I^{a,m}} \left[ \frac{\widehat{p}_i^{a,m}(x_i^{a,m})}{\widehat{p}_i^{a,l}(x_i^{a,m})} \right]^{-1} \right\}^{-1}, \qquad (j,k) \in a. \tag{16}$$

The corresponding Fisher index is the following:

$$P_{Fis}^{a,(l,m)} = \sqrt{P_{Las}^{a,(l,m)} \times P_{Pas}^{a,(l,m)}}, \qquad (j,k) \in a. \tag{17}$$

An alternative to the Fisher index is the Törnqvist index which is also a widely used superlative index. The Törnqvist-type index for our purpose can be calculated by taking the geometric mean of the geometric analogues of the Laspeyres-type and Paasche-type indexes. Diewert (1978) shows that superlative indexes approximate each other to the second order, and thus empirically, it should not matter which one is used. In fact, conforming to Diewert's theory we find that our measures of Fisher- and Törnqvist-type indexes are very similar (see Table 4).[10]

[10]See Hill and Melser (2008) for alternative ways of constructing price indexes using imputed prices, and

The above illustration is shown for two ages and two cohorts, whereas in most cases we would construct indexes covering many ages, $a = 1, \ldots, A$, and, at least, a few cohorts, $v = 1, \ldots, V$. This extension can be attained by constructing direct and chained indexes based on the constructed superlative indexes. In constructing the direct index measuring the ageing effect, one particular age is taken as the base age, and the prices corresponding to other ages are compared with the prices of the base age. Suppose the base age is 1, then in equation (11), (13) and (14), we set $j = 1$, and $k = 2$ for obtaining the price indexes between age 1 and 2, $k = 3$ for obtaining indexes between age 1 and 3, and so on. The direct index measuring the ageing effect between age 1 and any arbitrary age $\alpha$, and for $(l, m) \in v$, is the following:

$$P_{FD}^{(1,\alpha),v} = \sqrt{P_{Las}^{(1,\alpha),v} \times P_{Pas}^{(1,\alpha),v}} = \sqrt{\frac{1}{I^{1,v}} \sum_{i=1}^{I^{1,v}} \left[ \frac{\widehat{p}_i^{\alpha,v}(x_i^{1,v})}{\widehat{p}_i^{1,v}(x_i^{1,v})} \right] \times \left\{ \frac{1}{I^{\alpha,v}} \sum_{i=1}^{I^{\alpha,v}} \left[ \frac{\widehat{p}_i^{\alpha,v}(x_i^{\alpha,v})}{\widehat{p}_i^{1,v}(x_i^{\alpha,v})} \right]^{-1} \right\}^{-1}}. \tag{18}$$

Similarly, a direct index measuring the cohort effect between cohort 1 and any arbitrary cohort $\kappa$, and for $(j, k) \in a$, can be obtained by setting $l = 1$ and $m = \kappa$ in equations (15), (16) and (17) as follows:

$$P_{FD}^{a,(1,\kappa)} = \sqrt{P_{Las}^{a,(1,\kappa)} \times P_{Pas}^{a,(1,\kappa)}} = \sqrt{\frac{1}{I^{a,1}} \sum_{i=1}^{I^{a,1}} \left[ \frac{\widehat{p}_i^{a,\kappa}(x_i^{a,1})}{\widehat{p}_i^{a,1}(x_i^{a,1})} \right] \times \left\{ \frac{1}{I^{a,\kappa}} \sum_{i=1}^{I^{a,\kappa}} \left[ \frac{\widehat{p}_i^{a,\kappa}(x_i^{a,\kappa})}{\widehat{p}_i^{a,1}(x_i^{a,\kappa})} \right]^{-1} \right\}^{-1}}. \tag{19}$$

A problem with the direct index is that it makes the price comparison dependent on the choice of base age or cohort. Furthermore, as the distance between the comparison ages and, similarly, the comparison cohorts get larger, the price comparisons may become less reliable. For example, houses constructed in two adjacent periods would probably entail more 'like with like' comparisons than the houses constructed in longer periods apart. An alternative to constructing the direct indexes is to chain the bilateral indexes for two adjacent ages and, similarly, for two adjacent cohorts. The chained index measuring the ageing effect between age 1 and $\alpha$ for $(l, m) \in v$ is as follows:

$$P_{FC}^{(1,\alpha),v} = P_F^{(1,2),v} \times P_F^{(2,3),v} \times \ldots \times P_F^{(\alpha-2,\alpha-1),v} \times P_F^{(\alpha-1,\alpha),v}, \tag{20}$$

Rambaldi and Fletcher (2014) for attaining stability in the price indexes using Kalman filters.

where the indexes on the right hand side are the Fisher indexes shown in equation (14). In a similar way, the chained index measuring the cohort effect between cohort 1 and $\kappa$, using the Fisher index in equation (17) and for $(j,k) \in a$, is the following:

$$P_{FC}^{a,(1,\kappa)} = P_F^{a,(1,2)} \times P_F^{a,(2,3)} \times \ldots \times P_F^{a,(\kappa-2,\kappa-1)} \times P_F^{a,(\kappa-1,\kappa)}. \tag{21}$$

Although the chained indexes circumvent the comparability problem inherent in the direct indexes, and may reduce the Paasche-Laspeyres spread (see Hill 2006a), the chained indexes have a shortcoming in that they are not transitive, i.e. $P_F^{1,3} \neq P_F^{1,2} \times P_F^{2,3}$, even when they are based on superlative index number formulae. The chaining may introduce a drift in the price comparison causing the chained index to deviate from the direct index counterpart (Ivancic, Diewert and Fox 2011; de Haan and van der Grient 2011). This would make the measurement of price changes dependent on the selection of base age or cohort. A solution to this problem is to apply the GEKS formula (Gini 1931; Eltetö and Köves 1964; Szulc 1964) on the Fisher indexes in equation (14) and (17). The GEKS index is the geometric mean of the ratios of the Fisher indexes between a number of entities, where each entity is taken as the base. Let $P_F^{(j,\alpha),v}$ and $P_F^{(k,\alpha),v}$ be the indexes shown in equation (14) measuring the ageing effect between $j$ and $\alpha$, and $k$ and $\alpha$, respectively, where $a = 1,\ldots,A$. The GEKS index measuring the ageing effect between $j$ and $k$ is the following:

$$P_{FGEKS}^{(j,k),v} = \prod_{a=1}^{A} \left( \frac{P_F^{(j,a),v}}{P_F^{(k,a),v}} \right)^{1/A} = \prod_{a=1}^{A} \left( P_F^{(j,a),v} \times P_F^{(a,k),v} \right)^{1/A}, \tag{22}$$

where the second expression holds because the bilateral Fisher index satisfies the entity reversal property of indexes, so that $P_F^{(k,\alpha),v} = 1/P_F^{(\alpha,k),v}$. Unlike the Fisher chained indexes, it can be shown that the GEKS indexes are transitive, i.e. $P_{GEKS}^{1,3} = P_{GEKS}^{1,2} \times P_{GEKS}^{2,3}$, making the price comparison independent of the choice of the base.[11] In a similar way, we can obtain the GEKS index measuring the cohort effect between $l$ and $m$ using the Fisher index specified in equation (17) in the following way, where $v = 1,\ldots,V$:

$$P_{FGEKS}^{a,(l,m)} = \prod_{v=1}^{V} \left( \frac{P_F^{a,(l,v)}}{P_F^{a,(m,v)}} \right)^{1/V} = \prod_{v=1}^{V} \left( P_F^{a,(l,v)} \times P_F^{a,(v,m)} \right)^{1/V}. \tag{23}$$

[11]GEKS indexes suffer from the loss of characteristicity because they make a price comparison between two entities dependent on the price comparisons between other entities. This would be a problem in the cases where revisions of the measurement of price changes are discouraged.

The price indexes $P_Z^{(j,k),v}$ for $Z \in (FD, FC, FGEKS)$ shown in equations (18), (20) and (22), respectively, measure the ageing effect of the houses belonging to a particular cohort, $v$, as these houses age from $j$ to $k$. In order to obtain an overall measure of the ageing effect for all houses in the sample, we aggregate $P_Z^{(j,k),v}$ across all cohorts, $v = 1, \ldots, V$, as follows:

$$P_Z^{(j,k)} = \prod_{v=1}^{V} \left[ P_Z^{(j,k),v} \right]^{0.50(S^{j,v}+S^{k,v})}, \tag{24}$$

where $S^{j,v}$ and $S^{k,v}$ are the proportion of houses in the $(j,v)$ and $(k,v)$ pairs in the sample. The overall measure of the cohort effect is obtained by aggregating $P_Z^{(l,m),a}$ indexes shown in equations (19), (21) and (23) across all ages as follows:

$$P_Z^{(l,m)} = \prod_{a=1}^{A} \left[ P_Z^{a,(l,m)} \right]^{0.50(S^{a,l}+S^{a,m})}, \tag{25}$$

where $S^{a,l}$ and $S^{a,m}$ are the share of houses in the $(a,l)$ and $(a,m)$ pairs in the sample. Similarly, we construct the Törnqvist based GEKS index measuring the ageing and cohort effects of houses.[12]

The time effects are obtained from the estimated coefficients of the time dummies in equation (2). Let $exp(\hat{\delta}^{a,v}_{t-1,t})$ provide the estimate of the price change from period $t-1$ to $t$ for a particular age-cohort pair of houses. Aggregating these estimates across all age-cohort pair of houses provides the measurement of housing inflation as follows:

$$P_{TD}^{t-1,t} = \prod_{a=1}^{A} \left[ \prod_{v=1}^{V} \left( exp(\hat{\delta}^{a,v}_{t-1,t}) \right)^{0.5(S^{a,v}_{t-1}+S^{a,v}_{t})} \right]^{0.5(S^{a}_{t-1}+S^{a}_{t})}, \tag{26}$$

where $S^{a,v}_{t-1}$ and $S^{a,v}_{t}$ are the shares of houses in the $(a,v)$ pairs in period $t-1$ and $t$, respectively. In the second stage of aggregation, $S^{a}_{t-1}$ and $S^{a}_{t}$ refer to the shares of houses sold at age $a$ in period $t-1$ and $t$, respectively.

It should be noted that in the above framework, the ageing and cohort effects are estimated using the hedonic imputation method and the inflationary effects are estimated using the time dummy method. Both methods allow these effects to emerge directly from the variation in the data rather than being forced to take a pre-determined functional form.

[12] The Fisher based GEKS index is generally referred to as the GEKS index. In order to avoid any terminological confusion, and following de Haan and Krsinich (2014), we refer to Fisher based GEKS as the Fisher-GEKS and Törnqvist based GEKS as the Törnqvist-GEKS index.

While de Haan (2010) derives the conditions required for the hedonic imputation and time dummy price indexes to be equivalent, the hedonic imputation method outperforms the time dummy method in a number of different aspects. Evidence shows that the implicit values of the characteristics vary across different parts of a data set belonging to a market (Hill et al. 1997; Berndt and Rappaport 2001; Pakes 2003; Hill and Melser 2008; Erickson and Pakes 2011; Hill 2013). That is, in our context, the estimated hedonic coefficients of the, for example, physical attributes are expected to vary across different ages and cohorts of houses. The hedonic imputation method would allow this variation to take place.[13]

Silver and Heravi (2007) show, through formal algebraic exposition, that two factors lead to the difference between the hedonic imputation and time dummy indexes; the parameter instability and the changes in the value of characteristics. Diewert, Heravi and Silver (2009), Eurostat (2013) and Rambaldi and Fletcher (2014) argue that the parameter flexibility is a significant advantage of the hedonic imputation method and favor using hedonic imputation price indexes unless degrees of freedom are very limited. This implies that if the interest of a study is, for example, to measure the age-price profile of houses, one should set the framework so that the ageing effect is one of the two temporal effects estimated through hedonic imputations while the control for the third temporal effect is attained through the dummy variables in the regressions.[14]

Another advantage of the hedonic imputation method lies in the flexibility in its application. This arises from the fact that the regression and compilation stages are separate in the hedonic imputation method. The method provides separate estimates of price relatives for each observation, which essentially adds new columns to the data. Once the price relatives are obtained, one is free to compile these using index number formulae in order to obtain different aggregated measures of choice, such as for different sections of the market and periods in the sample. The double imputation method has an added advantage because

[13]Perhaps the main criticism of the hedonic imputation method is related to the loss of efficiency of the estimates due to not exploiting the cross-equation correlations and to lower degrees of freedom for each regression (Eurostat 2013; Hill 2013).

[14]One could run regressions separately for each cohort-time pair of houses with age-dummy variables included in the regressions and obtain the ageing effect for that particular cohort-time houses from the estimated age-dummy coefficients. The flexibility in the ageing parameters, similar to what prevails in hedonic imputations, can be attained by adding interaction terms to the age-dummies, such as age-dummies $\times$ house types, age-dummies $\times$ ln(lotsize), age-dummies $\times$ room-dummies, and so on. However, it would then attenuate the parsimony of the time dummy type hedonic model (here, the age-dummy model) which is probably its greatest advantage (de Haan 2010; Hill 2013). Furthermore, it would not be straightforward to obtain an aggregated measure of price changes from compiling all the estimated direct and interactive age coefficients.

it has the potential for correcting for omitted variable bias incurred in the estimated hedonic regressions (Silver and Heravi 2001; Hill and Melser 2008; Syed 2010; Hill 2013). Hill argues that since houses are so heterogeneous, there is likely to be serious omitted variable bias problems in hedonic regressions of house prices. It can be shown that under certain plausible assumptions the omitted variable bias in the numerator and denominator of the double imputation price relative tend to cancel each other out.[15]

In addition, our method provides an in-built mechanism to correct for sample selection bias occurring when depreciation is estimated only on the surviving houses. The issue of potential sample selection bias has been raised, among others, by Hulten and Wykoff (1981a), Dixon, Crosby and Law (1999) and Coulson and McMillen (2008). The problem is that the houses which have retired early might have depreciated faster than the average. Hence, if this attrition is not accounted for it would lead to an underestimation of the measure of depreciation rates. In our method, the prices of houses are compared between two adjacent ages; between age 1 and 2, 2 and 3, and so on. Therefore, in each comparison the houses would have the same or a similar survival rate. This implies that the sample selection bias incurred in each regression would tend to cancel each other out while constructing the indexes between two consecutive ages. We obtain the estimates of the depreciation rates between ages of larger gaps by chaining the price indexes obtained for consecutive ages, implying that the depreciation pattern covering the life of houses would also tend to be free of sample selection bias.

# 4 Empirical Results

We apply our framework to housing data consisting of 6,348 observations of the quarterly sales of detached houses for a city, "Assen", in the Netherlands, covering the period between 1998:1 and 2008:2.[16] Assen is a small city with a population of around 60,000. Each observation in the data contains information on the address, sale price, quarterly period of sale, lot size, floor space, number of rooms, number of toilets, construction period, house type, maintenance indicator, and whether the house has a garage, balcony, dormer and roof

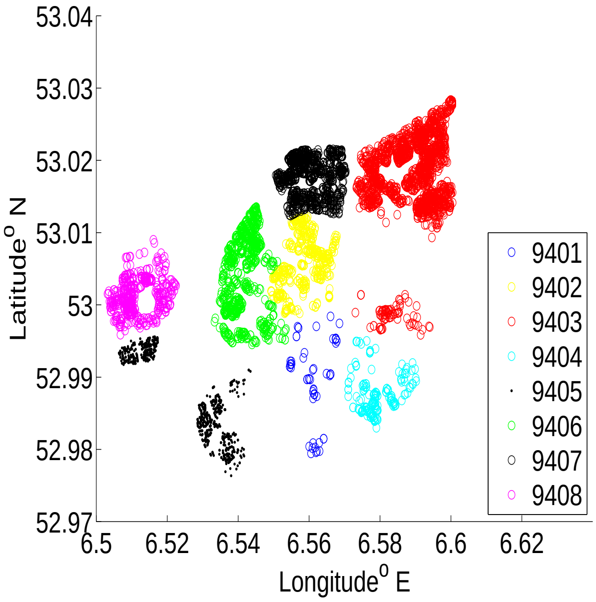
[15] A simple derivation showing how the omitted variable bias is reduced in the double imputation price relatives is provided in the supplementary materials of this paper.

[16] This data set has been used as a research database in the Eurostat *Handbook on Residential Property Prices Indices (RPPIs)*, 2013 edition (Eurostat 2013), and in Diewert, de Haan and Hendriks (2015).

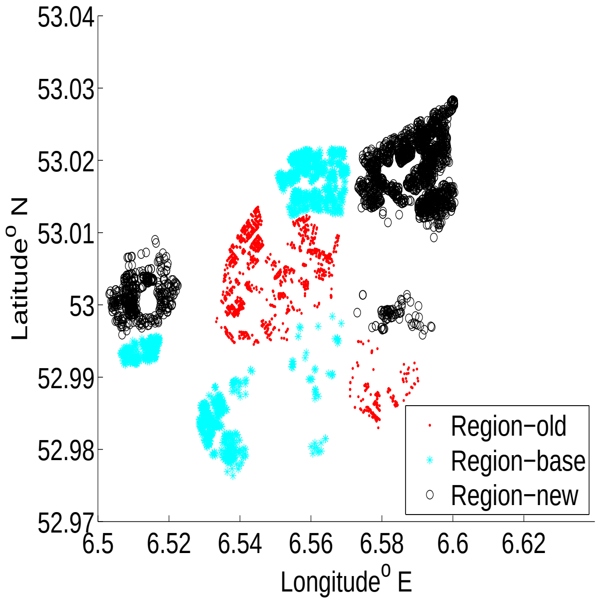
terrace. For the determination of the age variable, we use the available information on the construction period of the houses. These are available in tens of years as follows: 1960–1970; 1971–1980; 1981–1990; 1991–2000; and 2001–2008. The houses built during 2001–2008 are assigned to Age0, 1991–2000 to Age1, 1981–1990 to Age2, 1971–1980 to Age3, and 1960–1970 to Age4. Figure 2 provides the location plots of the sample houses and Table 2 provides some summary information about the data.

Figure 2: Assen City, its Postcodes and the Location of the Sample Houses

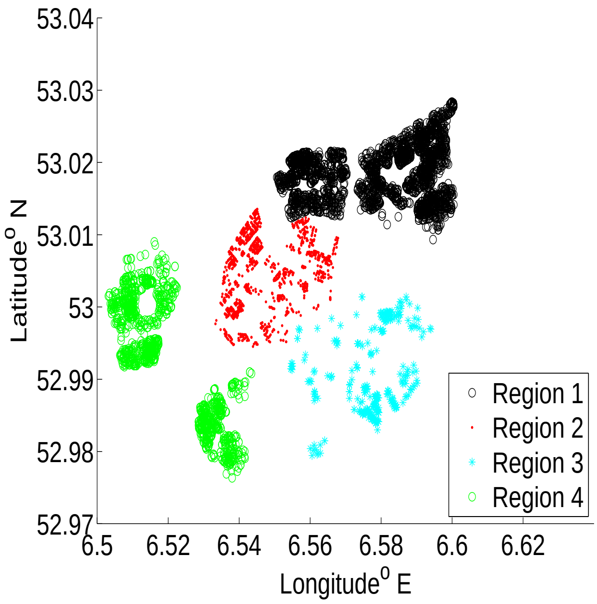
(a) Postcodes and sample houses




(b) Sample houses in cohort specific regions




(c) Sample houses in postcode specific regions



The coverage of the data is 1998-2008 and the last identified construction period is 2001-2008. The fact that these two periods nearly overlap makes the age and cohort of houses indistinguishable from each other. This would not, however, be the case in data sets where the actual construction time is known (e.g., in years) and sample covers a longer period of time (e.g., for 20 years).[17] However, the data set corresponds to a small and homogenous town which works as a natural control to location specific heterogeneity of prices. This lets us use the locational information to construct a cohort specific variable, which somewhat counteracts the above shortcoming. The cohort or construction vintage effect measures the separate impact attributed to the period of construction of a house. When a new area is opened up for residential development, the houses in the area are built around the same period. Therefore, these houses tend to share similar features in terms of style, and use of technology and materials which are specific to the construction period, making region and cohort effects correlated with each other.

We divide the city into 3 regions, shown in Figure 2(b), based on whether the postcodes are old or new as indicated by the construction period of the sample houses. We do the division by ordering the postcodes in ascending order in terms of the average construction period and then combining the adjacent postcodes in the order to create old, mid-age and new regions. These three regions comprise of 1,619 (3 postcodes), 2,000 (3 postcodes) and 2,729 observations (2 postcodes), respectively, and their average construction periods are 1972, 1983, and 1994, respectively (the average construction period for all houses is 1985). The dummy variables corresponding to these construction period specific regions are used as proxies to control for the cohort effect in our hedonic regressions.[18]

[17] Many data sets used in the housing literature contain information on actual construction year and cover a long period of time. Some examples include the data sets used by Coulson and McMillen 2008, Ferreira and Gyourko 2015, and the American Housing Survey data used by Harding et al. 2007. Additionally, historical transaction data sets from the county offices in the US are becoming increasingly available. We believe that the application of the method in relation to the availability of the data would not be a problem.

[18] Because region dummies represent cohorts we include them directly as regressors in our hedonic regressions. In order to conduct sensitivity analysis, we also consider two other scenarios. In one of the scenarios, we do not consider any geographical division of the city (hence, there are no region dummies in regressions) and in the other. we consider four regions in the city where regions are constructed by combining adjacent postcodes in terms of their geographical proximity (see Figure 2(c)).

Table 2: Data Description

| | Age0 | Age1 | Age2 | Age3 | Age4 | All |
|---|---|---|---|---|---|---|
| No. of Observations | 432 | 2169 | 1529 | 1418 | 800 | 6348 |
| 1998 | - | 154 | 164 | 165 | 66 | 549 |
| 1999 | - | 176 | 181 | 123 | 72 | 552 |
| 2000 | - | 218 | 153 | 125 | 68 | 564 |
| 2001 | 4 | 227 | 140 | 145 | 64 | 580 |
| 2002 | 12 | 230 | 144 | 135 | 77 | 598 |
| 2003 | 36 | 198 | 146 | 143 | 79 | 602 |
| 2004 | 51 | 235 | 117 | 128 | 77 | 608 |
| 2005 | 75 | 217 | 148 | 121 | 61 | 622 |
| 2006 | 83 | 217 | 136 | 140 | 77 | 653 |
| 2007 | 108 | 193 | 134 | 139 | 108 | 682 |
| 2008 | 63 | 104 | 66 | 54 | 51 | 338 |
| Median Price('000 Euros) | 230.13 | 159.28 | 127.06 | 130.00 | 122.00 | 142.94 |
| Mean Price('000 Euros) | 249.96 | 178.14 | 136.23 | 140.44 | 137.88 | 159.44 |
| Median lot size ($M^2$) | 274.00 | 244.00 | 189.00 | 190.00 | 215.00 | 209.50 |
| Mean lot size ($M^2$) | 314.58 | 263.26 | 226.63 | 243.79 | 291.63 | 257.16 |
| Median floor space ($M^2$) | 140.00 | 120.00 | 115.00 | 125.00 | 105.00 | 120.00 |
| Mean floor space ($M^2$) | 148.57 | 129.55 | 119.34 | 128.02 | 113.28 | 125.99 |
| Median no. of rooms | 5.00 | 5.00 | 5.00 | 5.00 | 4.00 | 5.00 |
| Mean no. of rooms | 4.75 | 4.61 | 4.67 | 4.79 | 4.59 | 4.67 |
| Median no. of toilet-baths | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| Mean no. of toilet-baths | 1.92 | 2.00 | 1.97 | 1.97 | 1.82 | 1.96 |

Following equation (2), we estimate hedonic models of log prices separately for each age($a$) of houses as follows:

$$ln(price)_i^a = \sum_{t=1}^{11} \delta_t^a d_{t,i}^a + \sum_{r=1}^{R-1} \phi_r^a d_{r,i}^a + \beta_1^a ln(lotsize)_i^a + \beta_2^a ln(flrspace)_i^a + \sum_{c=1}^{C-1} \eta_c^a d_{c,i}^a + \epsilon_i^a$$
$$i = 1, \ldots, I^a; \quad a = 0, \ldots, 4. \tag{27}$$

In equation (27), $d_{t,i}^a$ refers to a dummy variable taking the value of 1 if the period of sale of house $i$ is $t$, and 0 otherwise, for all $a = 0, \ldots, 4$. $t$, running from 1 to 11, indexes the year of sale, 1998,…,2008. $\delta_t^a$ is the time dummy coefficient corresponding to period $t$ and age $a$, and $exp(\delta_{t+1}^a - \delta_t^a)$ provides a measure of price change between period $t+1$ and $t$. $d_{r,i}^a$ refers to a dummy variable taking the value of 1 if house $i$ is located in region $r$, and 0 otherwise, where $r$ denotes old, mid-age and new regions in the city. The dummy variables—hereafter, called the cohort-region dummies—are constructed for the old and new regions, with the mid-age region being treated as the base. $ln(lotsize)$ and $ln(flrspace)$ refer to the natural log of lot size and floor space measured in square meters, respectively. $d_{c,i}^a$ refers to a dummy variable taking the value of 1 if the physical attribute $c$ is present in house $i$ of age $a$, and 0 otherwise. $\epsilon_i^a$ is assumed to be $i.i.d.$ error terms with a zero mean and constant variances.

Table 3 provides the results of the hedonic regressions specified in equation (27). The adjusted-$R^2$ ranges from 0.90 to 0.94, with the average being 0.92. The estimated time dummy and, in most cases, the region dummy coefficients are found to be significant at the 5% level. The estimated coefficients corresponding to the physical attributes have the expected sign in most cases, particularly for the ones which are significant. While the estimates are quite stable across the regressions for different ages of the houses, their differences are significant at the 5% level in many cases. An interesting result corresponds to the estimated coefficients of $ln(lotsize)$ and $ln(flrspace)$. The average of the sum of these elasticity measures is around 0.5, indicating that a 1 percent increase in the land and floor area leads to a 0.5 percent increase in the value of the sample houses.[19]

Before we move to our measures of depreciation, we construct price indexes measuring inflation using the estimated time dummy coefficients provided in Table 3. Taking 2001 as the base year, we obtain a measure of price change between $t$ and 2001 from $exp(\hat{\delta}_t^a - \hat{\delta}_{2001}^a)$, for $a = 0, \ldots, 4$. Figure 3 shows the plots of these price indexes measuring the inflation of houses, which are aggregated using equation (26) to obtain an overall measure of inflation of the sample houses.

Table 4 shows the median and double imputation GEKS indexes measuring the age-price profiles for our houses. The median index shows a much larger depreciation rate than the quality-adjusted indexes. This is because the old houses are in general of lower quality than the new houses and, therefore, the quality adjustments in the construction of the GEKS indexes reduce the ageing effect on house prices. The Fisher-GEKS and Törnqvist-GEKS indexes are, as expected, very similar, prompting us to focus on the former in the subsequent discussion. From the resulting age-price profile, we find that the houses in our sample depreciate by 20.7% over 40 years of life. They depreciate by 16.1% in the first 20 years and by 4.6% in the next 20 years of life. The estimated depreciation pattern implies that the simple annual average depreciation rate is 0.52% per year, given that in the calculation we take the average age of Age0 as 5 years and Age4 as 45 years, giving us a total of 40 years

[19]The implementation of hedonic imputation method requires that the regression models of two consecutive ages are specified on the same set of characteristics. In order to match with the characteristics present in the Age0 regression, a separate Age1 hedonic regression was estimated which excluded observations from 1998, 1999, and 2000, and observations with medium maintenance indicator of houses. This estimated regression for Age1 was used while constructing the price relatives comparing prices between Age0 and Age1.

Table 3: Hedonic Regression Results for Each Age of Dwellings

| Variables†,* | Age0 | | Age1 | | Age2 | | Age3 | | Age4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std |
| 1998 dummy | - | - | 1.648 | 0.081 | 2.108 | 0.114 | 2.027 | 0.107 | 1.442 | 0.131 |
| 1999 dummy | - | - | 1.818 | 0.081 | 2.265 | 0.114 | 2.164 | 0.107 | 1.592 | 0.130 |
| 2000 dummy | - | - | 1.934 | 0.081 | 2.403 | 0.114 | 2.318 | 0.107 | 1.662 | 0.132 |
| 2001 dummy | 2.240 | 0.155 | 2.048 | 0.081 | 2.523 | 0.115 | 2.411 | 0.107 | 1.792 | 0.129 |
| 2002 dummy | 2.243 | 0.156 | 2.072 | 0.081 | 2.585 | 0.114 | 2.493 | 0.107 | 1.886 | 0.129 |
| 2003 dummy | 2.258 | 0.155 | 2.114 | 0.081 | 2.608 | 0.114 | 2.535 | 0.106 | 1.933 | 0.130 |
| 2004 dummy | 2.328 | 0.154 | 2.158 | 0.081 | 2.661 | 0.114 | 2.547 | 0.107 | 1.953 | 0.130 |
| 2005 dummy | 2.342 | 0.153 | 2.198 | 0.081 | 2.713 | 0.114 | 2.621 | 0.106 | 2.035 | 0.130 |
| 2006 dummy | 2.402 | 0.154 | 2.239 | 0.081 | 2.726 | 0.114 | 2.621 | 0.107 | 2.020 | 0.129 |
| 2007 dummy | 2.427 | 0.153 | 2.279 | 0.081 | 2.763 | 0.114 | 2.661 | 0.106 | 2.053 | 0.130 |
| 2008 dummy | 2.448 | 0.154 | 2.291 | 0.081 | 2.779 | 0.115 | 2.681 | 0.107 | 2.061 | 0.131 |
| region-old | 0.067 | 0.039 | -0.085 | 0.017 | 0.083 | 0.015 | -0.107 | 0.007 | -0.093 | 0.023 |
| region-new | -0.052 | 0.036 | -0.014 | 0.006 | 0.097 | 0.006 | -0.032 | 0.021 | -0.023 | 0.034 |
| semi-detached | 0.260 | 0.037 | 0.156 | 0.020 | 0.034 | 0.015 | 0.103 | 0.012 | 0.229 | 0.056 |
| corner-house | 0.029 | 0.015 | 0.010 | 0.009 | 0.038 | 0.008 | 0.031 | 0.009 | 0.004 | 0.015 |
| oneside-duplex | 0.195 | 0.017 | 0.152 | 0.010 | 0.214 | 0.013 | 0.181 | 0.013 | 0.114 | 0.018 |
| detached | 0.383 | 0.027 | 0.326 | 0.017 | 0.472 | 0.022 | 0.449 | 0.023 | 0.317 | 0.027 |
| log(lotsize) | 0.222 | 0.019 | 0.258 | 0.012 | 0.156 | 0.014 | 0.218 | 0.012 | 0.253 | 0.017 |
| log(flrspace) | 0.354 | 0.031 | 0.268 | 0.016 | 0.266 | 0.023 | 0.215 | 0.024 | 0.306 | 0.027 |
| maintain-med | - | - | 0.032 | 0.008 | 0.013 | 0.005 | 0.012 | 0.004 | 0.018 | 0.003 |
| maintain-high | 0.011 | 0.002 | 0.041 | 0.008 | 0.026 | 0.005 | 0.021 | 0.004 | 0.037 | 0.005 |
| room4 dummy | -0.075 | 0.035 | 0.044 | 0.009 | 0.017 | 0.016 | 0.088 | 0.015 | 0.021 | 0.020 |
| room5 dummy | -0.059 | 0.035 | 0.057 | 0.009 | 0.036 | 0.016 | 0.100 | 0.015 | 0.054 | 0.021 |
| room6 dummy | -0.025 | 0.037 | 0.096 | 0.011 | 0.079 | 0.019 | 0.145 | 0.018 | 0.077 | 0.026 |
| toiletbath2 | -0.022 | 0.011 | 0.019 | 0.007 | -0.011 | 0.010 | 0.019 | 0.010 | 0.036 | 0.011 |
| toiletbath3 | 0.010 | 0.016 | 0.018 | 0.010 | -0.001 | 0.014 | 0.018 | 0.014 | 0.118 | 0.022 |
| balcony-yes | 0.007 | 0.013 | 0.041 | 0.009 | 0.071 | 0.020 | 0.049 | 0.015 | -0.014 | 0.012 |
| garage-yes | 0.021 | 0.011 | 0.053 | 0.006 | 0.055 | 0.010 | 0.077 | 0.009 | 0.051 | 0.013 |
| dormer-yes | 0.029 | 0.017 | 0.001 | 0.011 | 0.021 | 0.013 | 0.010 | 0.014 | 0.033 | 0.017 |
| roofter-yes | 0.016 | 0.015 | 0.057 | 0.008 | 0.191 | 0.039 | 0.066 | 0.022 | 0.030 | 0.026 |
| Adjusted $R^2$ | 0.940 | | 0.931 | | 0.906 | | 0.897 | | 0.912 | |
| Deg. freedom | 406 | | 2139 | | 1499 | | 1388 | | 770 | |

Note: Dependent variable is the natural log of prices where the prices are expressed in '000 euros.
† The base case for the dummy variables corresponding to the physical attributes are the following: terraced house for the type of houses, low maintenance for the maintenance indicator, number of toilets and bathrooms is 1 for the toiletbath dummies and number of rooms is 3 and less for the room dummies. The room6 dummy indicates houses with the number of rooms 6 and above. The dummy variables corresponding to whether a house has a balcony, garage, dormer and roof top terrace take the value 1 for 'yes' and 0 for 'no'. lotsize and flrspace are in square meters.
* Most of the estimated coefficients are significant at the 5% level (128 out of 147 coefficients). Therefore, we have not separately indicated whether an estimated coefficient is significant or not.

of life of the houses.

The finding that the depreciation rate is greater in the early years of life than in the later years is in concordance with the expectation of the general shape of the depreciation pattern of houses (Grether and Mieszkowski 1974; Hulten and Wykoff 1981a, 1996; Jorgenson 1996).

Figure 3: Time-dummy Price Indexes Measuring Inflation Constructed from Different Ages of Houses

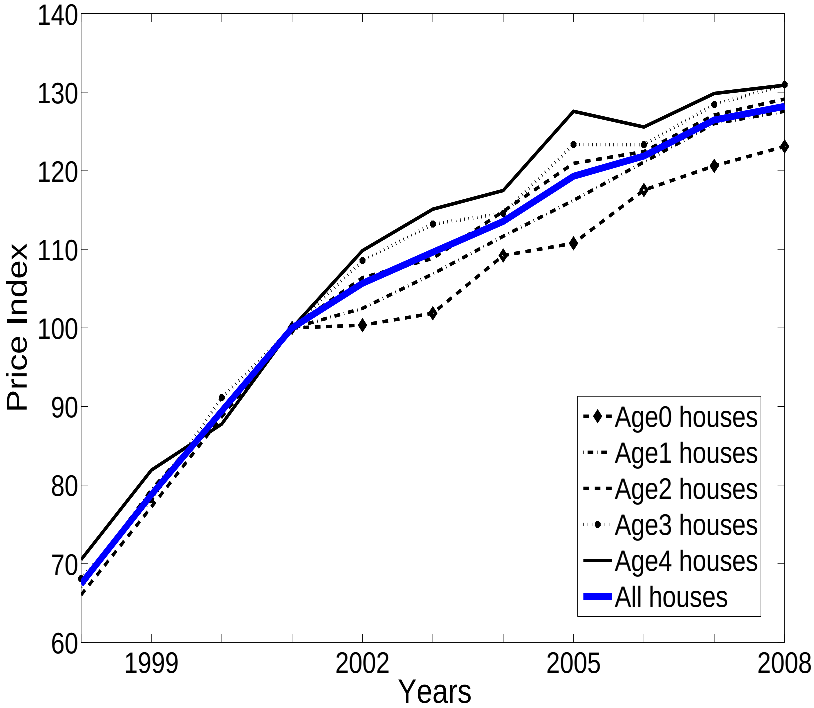



Table 4: Age-Price Profiles: Median and GEKS Indexes

| | Median Index | Fisher-GEKS Index | Törnqvist-GEKS Index |
|---|---|---|---|
| Age0 | 100.00 | 100.00 | 100.00 |
| Age1 | 69.21 | 92.46 | 92.45 |
| Age2 | 55.21 | 83.94 | 83.93 |
| Age3 | 56.49 | 81.25 | 81.32 |
| Age4 | 53.02 | 79.32 | 79.31 |
| Depreciation rates (%): | | | |
| Annual geometric* | 1.57 | 0.58 | 0.58 |
| Annual average† | 1.17 | 0.52 | 0.52 |

* Example: Let $d_r$ be the geometric depreciation rate, then $79.32 = 100(1 - d_r)^{40} \Longrightarrow d_r = 0.58\%$.

† Calculated by dividing the cumulative depreciation by 40.

For example, Hulten and Wykoff (1981a), using the Box-Cox power transformation, find that the value of used buildings approximately follows a geometric pattern of depreciation rather than a linear or one-horse shay pattern. However, the depreciation pattern in our data does not follow a geometric rate of decline over the life of the houses. If our finding of 20.7% depreciation over 40 years was incurred through an annual geometric rate, then it would imply a decline in value of the houses by 0.58% per year. This annual geometric rate would imply a depreciation of 11.0% in the first 20 years of life and 9.7% in the next 20 years of life. This is much lower than our measure of depreciation in the first 20 years (which is 16.1%) and higher than our measure of depreciation in the next 20 years of life of the houses (which is 4.6%). Hence, our measure of depreciation pattern—though indicating a decline in the rate of decay of the houses as they age—deviates for the depreciation pattern implied by the geometric rate of decline of houses.

Our measures of annual depreciation rate, 0.52–0.58% falls within the range reported by many authors. Malpezzi et al. (1987) find that the annual depreciation rate in 59 metropolitan areas in the U.S. ranges from 0.43-0.93%. Cannaday and Sunderman (1986) estimate the depreciation rate of single-family homes in Champaign, Illinois, ranging from 0.38 to 0.75%. Wilhelmsson (2008) estimates a depreciation rate of 0.77% per year for well-maintained properties in a municipality of Stockholm, Sweden (see also Chinloy 1979; Fletcher et al. 2000; Smith 2004; and Chau et al. 2005).

We compare our measures of the depreciation pattern with the estimates obtained from equation (1). In equation (1), data is pooled across all ages of houses and a non-linear function of age is included in the regression. The regression includes all variables, including the cohort specific regional dummies, that were included in the regressions corresponding to equation (27). We consider four different non-linear specifications of the age function in equation (1) by setting $\gamma f(a)$ equal to (1) $\gamma_1 ln(a)$, (2) $\gamma_1 a^2$, (3) $\gamma_1 a^2 + \gamma_2 a^3$, and (4) $\gamma_1 e^{-a}$ ('$a$' denotes age). The regression results are provided in Table 5, and they show that different age functions do not impact the overall performance of the models; the adjusted-$R^2$s are around 0.92 for the four regressions. The estimated coefficients, including the age coefficients, have the expected signs and are significant at the 5% level (with the exception of one coefficient). The estimated coefficients of the physical attributes are around the same across regressions. We find that the price indexes constructed from the estimated time dummy coefficients from the regressions are virtually identical.

The only estimated coefficients that are showing some difference across the regressions correspond to the cohort-region dummies, though the difference is small. The positive estimated coefficients for the new region and negative estimated coefficients for the old region (with the mid-age region as the base) indicate that newer houses are in general perceived to be of higher quality. This may be because houses built later in the period embody technical progress which make them more valuable (Hulten and Wykoff 1981b, 1996; Englund et al. 1998).[20] However, the regression results corresponding to different age specifications do not provide any indication that would let us select one from the four regressions. The question is: do they imply a similar depreciation pattern of houses?

The age-price profiles obtained from the estimates of the age functions are shown in Figure 4.[21] The solid blue line shows the Fisher-GEKS index obtained from our method. The $ln(a)$ and $e^{-a}$ specifications show that the houses depreciate by 23.5% and 24.2% over 40 years, respectively, which exceed our estimates. The depreciation patterns of the $ln(a)$ and $e^{-a}$ specifications deviate from our estimates from the very beginning of the life of the houses. The $a^2$ specification, on the other hand, underestimates the depreciation rate, and the estimated depreciation pattern exhibits an increase in the depreciation rate as the houses age, which is exactly the opposite of what our estimates exhibit. The $\gamma_1 a^2 + \gamma_2 a^3$ specification also provides a poor approximation to our measure of depreciation pattern of the houses, estimating a depreciation of 18.8% over 40 years, with only 10.9% in the first 20 years (lower than our estimate of 16.1%) and 7.9% in the next 20 years (higher than our estimate of 4.6%). These results provide strong evidence that (1) the choice of functional form for the age variable has a significant impact on the estimation of depreciation rates and patterns of houses, and (2) the functional forms which are typically used do not provide a reasonable approximation of the actual pattern of depreciation over the life of houses.

We also obtain depreciation patterns with two modifications to the specification of the regression models. First, we exclude the cohort-region dummies without any replacements. The idea is that since cohort and age variables are highly correlated, if the cohort effect is not controlled for in hedonic regressions, it would affect our measure of the ageing effect of house prices. Second, we replace the cohort-region dummies with the regional dummies constructed from combining adjacent postcodes (postcode-region dummies) as shown in Figure 2(c). This

[20] While we acknowledge that our empirical application defines cohort through regional amalgamation rather than construction period, the finding that a cohort exhibits a distinctive effect on house prices is not new in the literature (see also Hill et al. 1997; Karato et al. 2015). The method we described in Section 3 provides a more direct and flexible way of accounting for cohort effects in house prices.

[21] The age-price profiles are calculated using $P_{a+1} = P_a + dP/da$, where we set the initial price $P_1$ at 100 and obtain $dP/da$ from the estimated coefficient of $f(a)$ in equation (1). For example, in the case of $f(a) = log(a)$, $dP/da = -0.123 \times (P/a)$, hence, $P_2 = 100 - 0.123 \times (100/1) = 87.70$, $P_3 = 87.70 - 0.123 \times (87.70/2) = 82.306$, and so on. Note that if we could include the linear specification $f(a) = a$, given that our hedonic regressions are $ln(price)$ models, we would have obtained an estimate of geometric depreciation rates.

Table 5: Time-dummy Hedonic Regressions with Non-linear Specifications of Age

| Variables[†,*] | $f(age) = log(age)$ | | $f(age) = age^2$ | | $f(age) = age^2, age^{3‡}$ | | $f(age) = e^{-age}$ | |
|---|---|---|---|---|---|---|---|---|
| | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std |
| $f(age)$ | -0.123 | 0.005 | -0.007 | 0.001 | -0.029 | 0.002 | 0.454 | 0.022 |
| | | | | | 0.004 | 0.001 | | |
| 1998 dummy | 1.823 | 0.050 | 1.700 | 0.050 | 1.802 | 0.050 | 1.665 | 0.049 |
| 1999 dummy | 1.978 | 0.050 | 1.856 | 0.049 | 1.956 | 0.049 | 1.820 | 0.049 |
| 2000 dummy | 2.110 | 0.050 | 1.989 | 0.050 | 2.088 | 0.049 | 1.953 | 0.049 |
| 2001 dummy | 2.223 | 0.050 | 2.103 | 0.049 | 2.201 | 0.049 | 2.066 | 0.049 |
| 2002 dummy | 2.279 | 0.049 | 2.161 | 0.049 | 2.257 | 0.049 | 2.122 | 0.049 |
| 2003 dummy | 2.316 | 0.050 | 2.200 | 0.049 | 2.296 | 0.049 | 2.158 | 0.049 |
| 2004 dummy | 2.354 | 0.050 | 2.239 | 0.049 | 2.332 | 0.049 | 2.200 | 0.049 |
| 2005 dummy | 2.403 | 0.050 | 2.291 | 0.049 | 2.383 | 0.049 | 2.245 | 0.049 |
| 2006 dummy | 2.426 | 0.049 | 2.314 | 0.049 | 2.406 | 0.049 | 2.268 | 0.049 |
| 2007 dummy | 2.461 | 0.049 | 2.352 | 0.049 | 2.442 | 0.049 | 2.302 | 0.049 |
| 2008 dummy | 2.471 | 0.050 | 2.363 | 0.050 | 2.452 | 0.049 | 2.312 | 0.049 |
| region-old | -0.043 | 0.004 | -0.025 | 0.005 | -0.042 | 0.005 | -0.066 | 0.004 |
| region-new | 0.047 | 0.004 | 0.061 | 0.004 | 0.037 | 0.004 | 0.057 | 0.004 |
| semi-detached | 0.121 | 0.008 | 0.120 | 0.008 | 0.129 | 0.008 | 0.115 | 0.008 |
| corner house | 0.024 | 0.005 | 0.022 | 0.005 | 0.027 | 0.005 | 0.024 | 0.005 |
| oneside-duplex | 0.176 | 0.006 | 0.173 | 0.006 | 0.179 | 0.006 | 0.179 | 0.006 |
| detached | 0.386 | 0.010 | 0.383 | 0.010 | 0.389 | 0.010 | 0.388 | 0.010 |
| ln(lotsize) | 0.226 | 0.006 | 0.231 | 0.010 | 0.221 | 0.006 | 0.222 | 0.006 |
| ln(flrspace) | 0.283 | 0.010 | 0.289 | 0.010 | 0.296 | 0.010 | 0.286 | 0.010 |
| maintain-med | 0.017 | 0.002 | 0.016 | 0.002 | 0.017 | 0.002 | 0.018 | 0.002 |
| maintain-high | 0.028 | 0.002 | 0.029 | 0.002 | 0.028 | 0.002 | 0.030 | 0.002 |
| room4 dummy | 0.029 | 0.007 | 0.033 | 0.007 | 0.027 | 0.007 | 0.027 | 0.007 |
| room5 dummy | 0.040 | 0.007 | 0.038 | 0.007 | 0.039 | 0.007 | 0.038 | 0.007 |
| room6 dummy | 0.080 | 0.008 | 0.077 | 0.008 | 0.079 | 0.008 | 0.077 | 0.008 |
| toiletbath2 | 0.010 | 0.005 | 0.007 | 0.005 | 0.010 | 0.005 | 0.012 | 0.005 |
| toiletbath3 | 0.036 | 0.007 | 0.033 | 0.007 | 0.037 | 0.007 | 0.037 | 0.007 |
| balcony-yes | 0.028 | 0.006 | 0.037 | 0.006 | 0.025 | 0.006 | 0.022 | 0.006 |
| garage-yes | 0.059 | 0.004 | 0.060 | 0.004 | 0.058 | 0.004 | 0.059 | 0.004 |
| dormer-yes | 0.022 | 0.007 | 0.234 | 0.007 | 0.024 | 0.007 | 0.020 | 0.007 |
| roofter-yes | 0.065 | 0.007 | 0.072 | 0.007 | 0.062 | 0.007 | 0.066 | 0.007 |
| Adjusted $R^2$ | 0.921 | | 0.919 | | 0.921 | | 0.919 | |

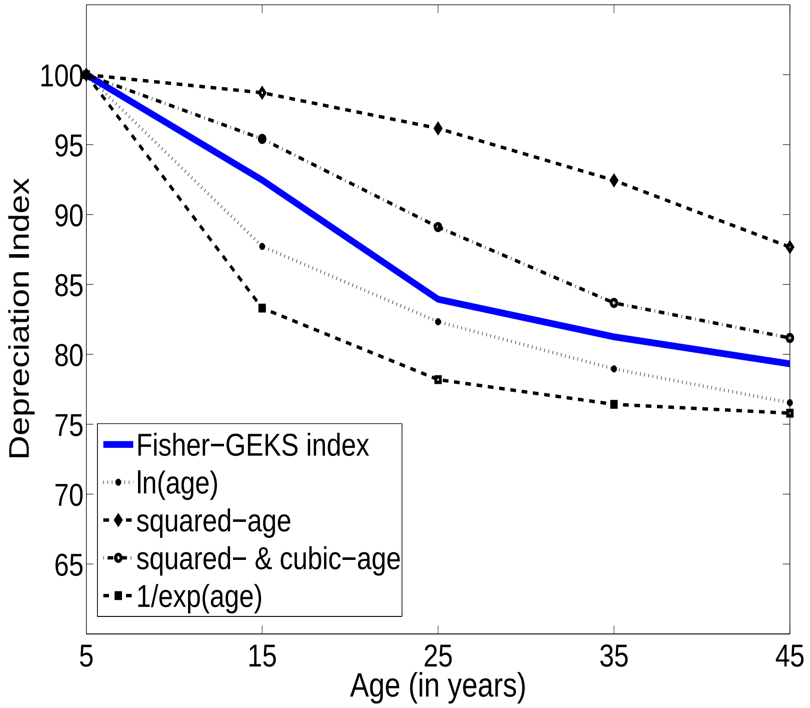
† Dependent variable is the natural log of prices where the prices are expressed in thousand euros. No. of observations: 6348

‡ The numbers corresponding to row $f(age)$ are for $age^2$, and the row below $f(age)$ are for $age^3$.

* With the exception of the estimated coefficient corresponding to toiletbath2 in the $f(age) = age^2$ regression, all estimated coefficients are significant at the 5% level.

is probably typical for how regional heterogeneity is accounted for in house prices. However, as discussed earlier, since houses in one area tend to be built around the same period, the postcode-region dummies are expected to, at least, partially control for construction vintage effects. Hence, our base case and two additional scenarios differ in terms of the extent to which the cohort effect is controlled for in hedonic regressions of houses prices. Our interest

Figure 4: GEKS Age-Price Index versus Alternative Age Functions



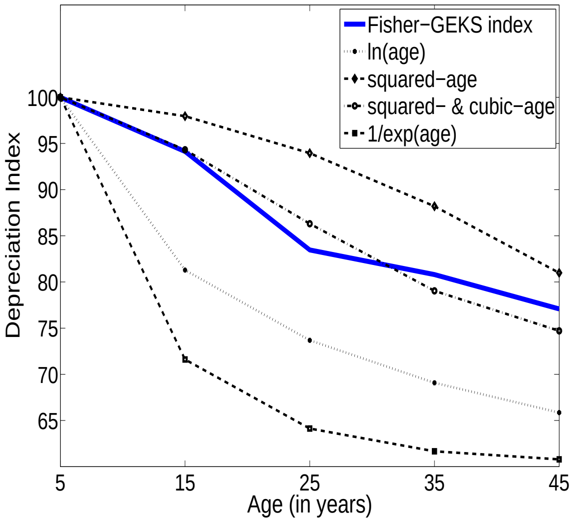

is to see how this variation affects the measured depreciation pattern.[22]

The estimated depreciation patterns are shown in Figure 5. The omission of cohort-region dummies without any replacement swings the depreciation index downward in all our models and, hence, increases the ageing effect on the price of houses (Figure 5(a)). This larger ageing effect is expected because the estimated cohort-region dummy coefficients shown in Table 5 imply that older houses on average are valued less, *ceteris paribus*, in the market. However, the extent to which the changes take place differs across models. For example, the impact is the largest for the $e^{-a}$ specification, with the average decline of house prices increasing from 0.61% to 0.98% per year (by 0.37% points). In the log(age) specification, the annual average decline increases from 0.59% to 0.85% (by 0.26% points). The impact of this

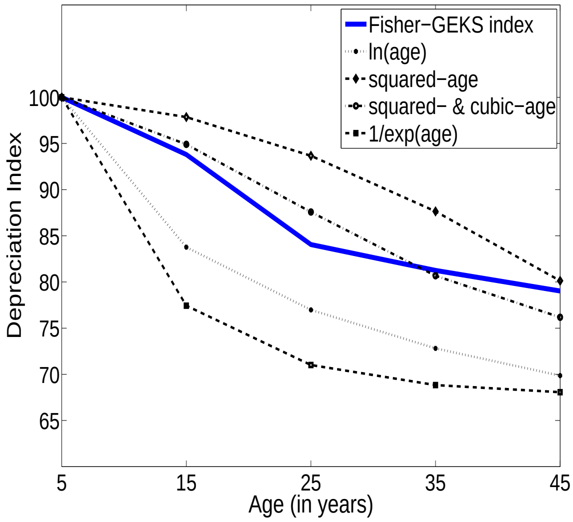
[22]We should mention that we do not delve into the debate of whether the cohort effect should or should not be separated from the ageing effect and the consequent complications it creates with regard to the interpretation of the ageing effect as a measure of depreciation of houses (see discussion in Hulten and Wykoff 1996; Englund et al. 1998; Coulson and McMillen 2008). We conduct this sensitivity analysis in order to check irrespective of whether we control for cohort effect or not, whether any of the commonly used non-linear age functions provides a good approximation of the depreciation pattern of houses. If they do not, it would make our case even stronger for obtaining the estimate of the age-price profile of houses in a flexible manner. We show the plots of the estimated depreciation patterns under the two scenarios in the paper while the detailed regression results are provided in the supplementary materials of the paper.

Figure 5: GEKS Age-Price Index versus Alternative Age Functions, without Directly Controlling for the Cohort Effect

(a) Without any region dummy variables




(b) With postcode-region dummy variables




omission is the lowest in the Fisher-GEKS index where the average price fall increases from 0.52% to 0.57% (by 0.05% points). The lower impact in the Fisher-GEKS index might be because of the inherent correction to the omitted variable bias that takes place in the hedonic imputation methods. The depreciation indexes similarly exhibit a larger fall in prices when cohort-region dummies are replaced with postcode-region dummies in the models (Figure 5(b)). However, in line with our expectation, the extent to which the changes take place in each model is lower than the corresponding model when no region dummies are included.[23]

Figure 5 shows that despite the changes in the depreciation patterns obtained from different models due to differing levels at which the construction vintage is controlled for, none of the depreciation patterns obtained from the non-linear age functions provide a reasonably good approximation to the depreciation pattern exhibited by the Fisher-GEKS index. Referring to Figure 4 and 5, the closest approximation is probably provided by the squared- and cubic-age function ($\gamma_1 a^2 + \gamma_2 a^3$), yet this approximation is inconsistent; while in Figure 4, $\gamma_1 a^2 + \gamma_2 a^3$ underestimates the overall depreciation rates, in Figure 5 it overestimates

[23]See also Appendix Figure A-1 for comparisons of the estimated depreciation patterns obtained under different cohort scenarios separately for each model. We also assess the relative impact of omitting temporal variables vis-à-vis omitting other physical attributes of houses on the measurement of depreciation rates of houses. The results provided in Appendix Table A-1 clearly show that omitting the time dummies and cohort-region dummies has the largest impact on the estimated ageing effect of houses. This is in most part expected because both the time of sale and cohort are potentially highly correlated with the age of houses.

the depreciation rates. All in all, our empirical results provide a strong support in favor of estimating the depreciation pattern of houses in a flexible manner, such as the way we have described in Section 3.

# 5 Conclusion

Age, cohort and time cannot be included in a flexible manner in hedonic regressions because of the identity, *age + cohort = sale time*. The typical procedure in hedonic regressions is to include time-dummies in order to measure inflation flexibly and a non-linear age function in order to obtain a measure of depreciation rate, while not accounting for the potential cohort effect on the price of houses. In support of the concerns raised previously about this approach in the literature, we find that this procedure distorts the measurement of depreciation pattern of houses. We find robust evidence that the commonly used functional forms provide poor approximation of the actual depreciation patterns of houses.

We introduce a method where each of these temporal effects on house prices can be accounted for within a hedonic regression framework in a flexible manner. Our method uses hedonic imputation approach and state-of-the-art index number formulae. We show how the method provides an estimate of the ageing effect, controlled for time and cohort effects, in the form of a separate price relative for each dwelling in the sample, which are then compiled using superlative-GEKS index number formulae in order to obtain an aggregated measure of the depreciation pattern of houses. Our method includes an in-built mechanism that reduces the omitted variable bias and corrects for the sample selection bias.

Applying our method to a housing data set for a town in the Netherlands, we find that the houses depreciate by around 20.7% over the life of 40 years, with 16.1% depreciation in the first 20 years and by 4.6% over the next 20 years (0.52% per year). If the cohort effect is not controlled for, the depreciation index swings downward exhibiting a larger annual depreciation rate for our houses. In either case, the estimated depreciation pattern does not follow the pattern implied by the commonly assumed geometric rate of decline in housing value. We also find that the ln(age) and exp(-age) specifications overestimate the depreciation rates, and the squared-age specification underestimates the depreciation rates of our houses. The results obtained from the cubic-age function are also found to be unreliable, providing an incorrect pattern of the age-price profile and sometimes overestimating and at other times underestimating depreciation rates.

While we apply our method to obtain the age-price profile of houses, it would be interesting to undertake similar exercise for a broad category of asset classes (such as those listed in Hulten and Wykoff 1981b; Jorgenson 1996) and see the implications the results would have on national accounts and productivity measurements.

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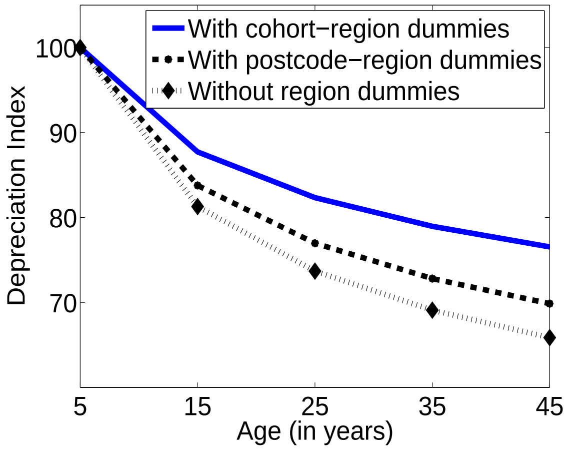
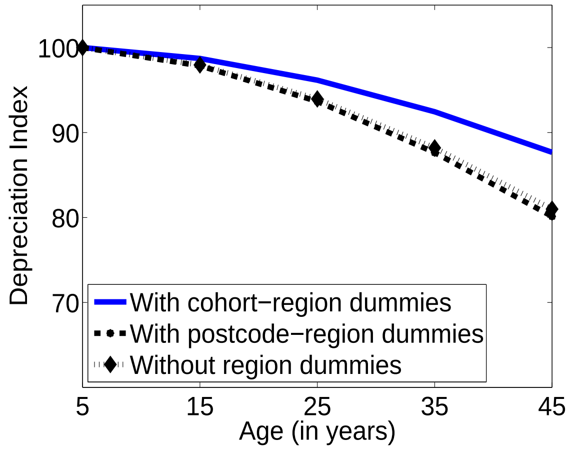
# Appendix

Figure A-1: Depreciation patterns, with and without controlling for cohort effect

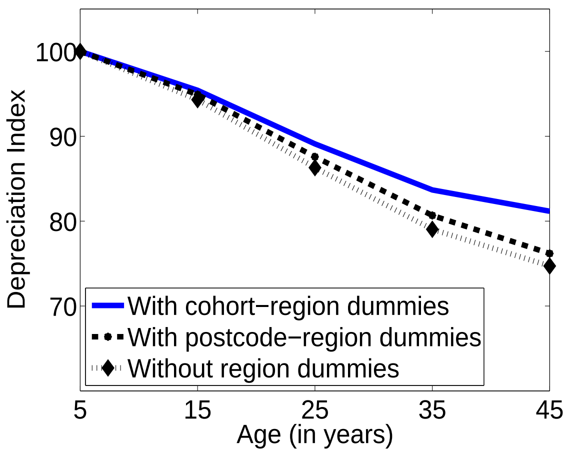
(a) Log(age) specification



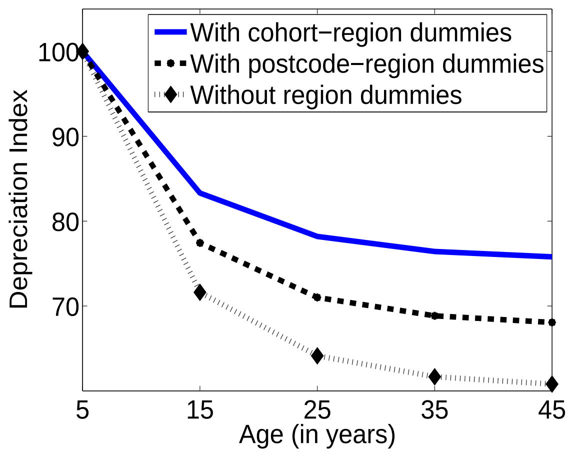

(b) Squared-age specification




(c) Squared- and cubic-age specification




(d) Inverse of exponential-age specification




(e) Hedonic imputation method

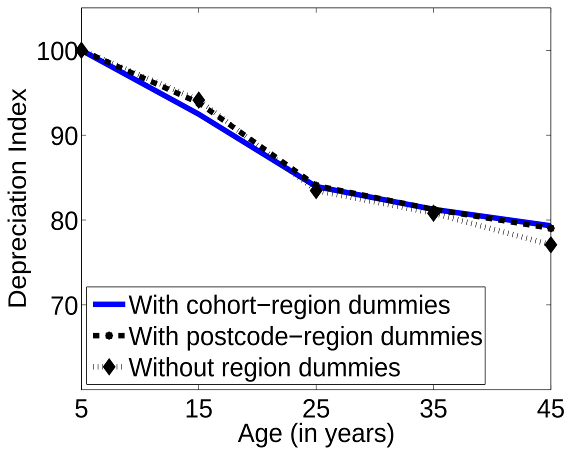


Table A-1: The Impact of Omitting Temporal and Physical Attributes on the Estimated Age Coefficient of Hedonic Regressions of House Prices

| Excluded Variables*,† | Replaced with: | $f(age) = log(age)$ | $f(age) = age^2$ | $f(age) = age^2, age^3$‡ | | $f(age) = e^{-age}$ |
|---|---|---|---|---|---|---|
| None: | n.a. | -0.123 | -0.007 | -0.029 | 0.004 | 0.454 |
| Base case | | (0.005) | (0.001) | (0.002) | (0.001) | (0.022) |
| | | {0.921} | {0.919} | {0.921} | | {0.919} |
| | | [0.587] | [0.308] | [0.471] | | [0.605] |
| Cohort- | None | -0.187 | -0.010 | -0.036 | 0.005 | 0.772 |
| region | | (0.004) | (0.001) | (0.002) | (0.001) | (0.019) |
| dummies | | {0.917} | {0.915} | {0.919} | | {0.910} |
| | | [0.854] | [0.475] | [0.632] | | [0.980] |
| Cohort- | Postcode- | -0.162 | -0.011 | -0.032 | 0.004 | 0.613 |
| region | region | (0.005) | (0.001) | (0.002) | (0.001) | (0.021) |
| dummies | dummies | {0.920} | {0.919} | {0.922} | | {0.916} |
| | | [0.754] | [0.497] | [0.596] | | [0.798] |
| Time | n.a. | -0.218 | -0.009 | -0.063 | 0.010 | 1.086 |
| (yearly) | | (0.011) | (0.001) | (0.003) | (0.001) | (0.043) |
| dummies | | {0.662} | {0.649} | {0.662} | | {0.672} |
| | | [0.973] | [0.439] | [0.778] | | [1.287] |
| House | n.a. | -0.125 | -0.007 | -0.027 | 0.004 | 0.454 |
| type | | (0.006) | (0.001) | (0.002) | (0.001) | (0.025) |
| dummies | | {0.894} | {0.892} | {0.894} | | {0.893} |
| | | [0.598] | [0.325] | [0.469] | | [0.606] |
| Lot size | n.a. | -0.134 | -0.006 | -0.034 | 0.005 | 0.527 |
| & | | (0.006) | (0.001) | (0.002) | (0.001) | (0.026) |
| floor space | | {0.887} | {0.884} | {0.894} | | {0.887} |
| | | [0.635] | [0.304] | [0.502] | | [0.695] |
| Maintenance | n.a. | -0.146 | -0.008 | -0.035 | 0.005 | 0.547 |
| indicators | | (0.005) | (0.003) | (0.002) | (0.001) | (0.022) |
| | | {0.917} | {0.914} | {0.894} | | {0.916} |
| | | [0.685] | [0.369] | [0.547] | | [0.719] |
| Room | n.a. | -0.117 | -0.006 | -0.028 | 0.004 | 0.428 |
| dummies | | (0.005) | (0.001) | (0.002) | (0.001) | (0.022) |
| | | {0.919} | {0.917} | {0.919} | | {0.918} |
| | | [0.559] | [0.296] | [0.448] | | [0.573] |
| Toilet & | n.a. | -0.122 | -0.006 | -0.029 | 0.004 | 0.451 |
| bathroom | | (0.005) | (0.001) | (0.002) | (0.001) | (0.022) |
| dummies | | {0.920} | {0.918} | {0.921} | | {0.919} |
| | | [0.584] | [0.307] | [0.468] | | [0.602] |
| Balcony, dormer, | n.a. | -0.127 | -0.006 | -0.033 | 0.005 | 0.477 |
| garage, roof | | (0.006) | (0.001) | (0.002) | (0.001) | (0.023) |
| terrace dummies | | {0.916} | {0.914} | {0.917} | | {0.915} |
| | | [0.605] | [0.302] | [0.494] | | [0.634] |

* Indicates the set of variables excluded from the hedonic regression specified in equation (1).
† The numbers at the top of each result are the estimated age coefficients ($\hat{\gamma}$), and the numbers in the first bracket are their standard errors. All age coefficients are significant at the 5% level. The numbers inside the second (curly) bracket are the adjusted-$R^2$s of the regressions and the numbers inside the third bracket are the annual average depreciation rates of houses.
‡ The numbers on the left are for $age^2$, and the ones on the right are for $age^3$.

# Supplementary Materials

## Omitted Variable Bias Correction in the Double Imputation Price Relatives

Let the following be the data generating process for the price of houses sold at age 0 and 1, respectively:

$$\ln\, p_i^0 \;= \beta_0 + \beta_1 z_{1,i}^0 + \beta_2 z_{2,i}^0 + \epsilon_i^0, \qquad \forall i = 1, \ldots, I \tag{S.1}$$

$$\ln\, p_j^1 \;= \delta_0 + \delta_1 z_{1,j}^1 + \delta_2 z_{2,j}^1 + \epsilon_j^1, \qquad \forall j = 1, \ldots, J \tag{S.2}$$

where ln $p_i^0$ denotes the natural log of price of house $i$ sold at age 0 and ln $p_j^1$ denotes the natural log of price of house $j$ sold at age 1. $z_1$ and $z_2$ denote the characteristics of houses. $\epsilon_i^0$ and $\epsilon_j^1$ are *i.i.d.* error terms with a zero mean and constant variance.

Let us consider two models: Model-A and Model-B. Using the OLS, Model-A is estimated on an intercept, $z_1$ and $z_2$, and Model-B on an intercept and $z_1$. The price relative obtained from Model-A is free of omitted variable bias and is used as the benchmark to compare with the price relative obtained from Model-B. Here $z_2$ takes the role of an omitted characteristic.

Model-A: Let $\widehat{\ln\, p_h^0}$ and $\widehat{\ln\, p_h^1}$ be the imputed price from Model-A of house $h$ sold at age 0 and 1, respectively. The probability limits of the imputed prices are as follows:

$$\begin{aligned} plim\,\widehat{\ln\, p_h^0} \;&= \beta_0 + \beta_1 z_{1,h} + \beta_2 z_{2,h} = \Theta_h^0 \\ plim\,\widehat{\ln\, p_h^1} \;&= \delta_0 + \delta_1 z_{1,h} + \delta_2 z_{2,h} = \Theta_h^1 \end{aligned}$$

Hence, the probability limit of the log of the double imputation price relative of house $h$ obtained from Model-A is:

$$plim\,\widehat{ln\left(\frac{p^1}{p^0}\right)_h} = \Theta_h^1 - \Theta_h^0$$

Model-B: Let $\widetilde{\ln\, p_h^0}$ and $\widetilde{\ln\, p_h^1}$ be the imputed price of house $h$ sold at age 0 and 1 obtained from Model-B, respectively. The corresponding probability limits are as follows:

$$\begin{aligned} plim\,\widetilde{\ln\, p_h^0} \;&=\; \left(\beta_0 + \phi_{02}^0 \beta_2\right) + \left(\beta_1 + \phi_{12}^0 \beta_2\right) z_{1,h} \\ &=\; \Theta_h^0 + \beta_2 \left(\phi_{02}^0 + \phi_{12}^0 z_{1,h} - z_{2,h}\right) \\ plim\,\widetilde{\ln\, p_h^1} \;&=\; \Theta_h^1 + \delta_2 \left(\phi_{02}^1 + \phi_{12}^1 z_{1,h} - z_{2,h}\right) \end{aligned}$$

From the above two equations, the probability limit of the log of the double imputation price relative obtained from Model-B is:

$$\begin{aligned} plim \ln \widetilde{\left(\frac{p^1}{p^0}\right)_h} &= \left(\Theta_h^1 - \Theta_h^0\right) + \delta_2 \left(\phi_{02}^1 + \phi_{12}^1 z_{1,h} - z_{2,h}\right) \\ &- \beta_2 \left(\phi_{02}^0 + \phi_{12}^0 z_{1,h} - z_{2,h}\right). \end{aligned}$$

The above probability limits provide the following measure of omitted variable bias for the predicted price of house $h$ sold at age 0 and 1, respectively:

$$plim \left[\widetilde{\ln p_h^0} - \widehat{\ln p_h^0}\right] = \beta_2 \left(\phi_{02}^0 + \phi_{12}^0 z_{1,h} - z_{2,h}\right) \quad \text{(S.3)}$$

$$plim \left[\widetilde{\ln p_h^1} - \widehat{\ln p_h^1}\right] = \delta_2 \left(\phi_{02}^1 + \phi_{12}^1 z_{1,h} - z_{2,h}\right). \quad \text{(S.4)}$$

The omitted variable bias of the corresponding double imputation price relative is:

$$plim \left[\ln \widetilde{\left(\frac{p^1}{p^0}\right)_h} - \ln \widehat{\left(\frac{p^1}{p^0}\right)_h}\right] = \delta_2 \left(\phi_{02}^1 + \phi_{12}^1 z_{1,h} - z_{2,h}\right) - \beta_2 \left(\phi_{02}^0 + \phi_{12}^0 z_{1,h} - z_{2,h}\right) \quad \text{(S.5)}$$

Hence, the bias in $\widetilde{ln\, p_h^0}$ depends on $\beta_2$, $\phi_{02}^0$ and $\phi_{12}^0$, and the bias in $\widetilde{ln\, p_h^1}$ depends on $\delta_2$, $\phi_{02}^1$ and $\phi_{12}^1$. The bias in $ln\widetilde{(p^1/p^0)}_h$, on the other hand, depends on the differences between the above set of parameters, $\delta_2 - \beta_2$, $\phi_{02}^1 - \phi_{02}^0$ and $\phi_{12}^1 - \phi_{12}^0$.

Suppose, for the houses at age 0 and 1, $z_2$ is a desirable characteristic, then $\beta_2, \delta_2 > 0$; its average unit takes a positive value, implying $\phi_{02}^0, \phi_{02}^1 > 0$; and it is positively correlated with $z_1$, implying $\phi_{12}^0, \phi_{12}^1 > 0$. An example of $z_2$ is the number of bedrooms in a house.

We now consider two *stability* conditions related to the regression models of two consecutive ages: (1) the signs of the above parameters remain the same in the two regressions and (2) the differences in the magnitude of the parameters are small in comparison to the parameter values themselves. If these two conditions hold, then the bias in equation (S.5) would be smaller than the biases in equations (S.3) and (S.4).

An indication of whether such stability conditions would hold in our regressions can be observed from the results shown in Table 3. We compare the estimated regression coefficients, $\hat{\beta}$ and $\hat{\delta}$, obtained from the regressions of two consecutive ages, i.e. Age0 and Age1, Age1 and Age2, and so on. The table shows that the sign of the estimated regression coefficients remain the same in 87.9% of such comparisons. The average difference of the estimated coefficients is 27.7% of the estimated values themselves. If a similar level of stability holds with respect to the omitted characteristics, their average values and how they are configured with the included characteristics, then the omitted variable bias in each estimated hedonic regression would tend to cancel each other out in the double imputation price relatives.

Table S-1: Hedonic Regression Results for Each Age of Dwellings with Postcode Specific Regional Dummies included in the Regressions†

| Variables‡,§ | Age0 | | Age1 | | Age2 | | Age3 | | Age4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std |
| 1998 dummy | - | - | 1.676 | 0.080 | 2.041 | 0.123 | 1.948 | 0.107 | 1.564 | 0.175 |
| 1999 dummy | - | - | 1.844 | 0.080 | 2.194 | 0.122 | 2.088 | 0.107 | 1.703 | 0.173 |
| 2000 dummy | - | - | 1.961 | 0.079 | 2.343 | 0.123 | 2.242 | 0.107 | 1.782 | 0.175 |
| 2001 dummy | 2.089 | 0.156 | 2.072 | 0.079 | 2.456 | 0.123 | 2.333 | 0.107 | 1.909 | 0.173 |
| 2002 dummy | 2.079 | 0.154 | 2.097 | 0.079 | 2.522 | 0.122 | 2.419 | 0.107 | 2.000 | 0.173 |
| 2003 dummy | 2.100 | 0.153 | 2.135 | 0.080 | 2.538 | 0.122 | 2.459 | 0.106 | 2.046 | 0.174 |
| 2004 dummy | 2.164 | 0.153 | 2.178 | 0.079 | 2.604 | 0.123 | 2.470 | 0.107 | 2.069 | 0.174 |
| 2005 dummy | 2.180 | 0.151 | 2.220 | 0.079 | 2.641 | 0.123 | 2.545 | 0.106 | 2.155 | 0.174 |
| 2006 dummy | 2.236 | 0.152 | 2.258 | 0.079 | 2.655 | 0.123 | 2.545 | 0.107 | 2.142 | 0.173 |
| 2007 dummy | 2.261 | 0.151 | 2.301 | 0.079 | 2.703 | 0.123 | 2.585 | 0.106 | 2.173 | 0.173 |
| 2008 dummy | 2.280 | 0.152 | 2.310 | 0.079 | 2.711 | 0.123 | 2.604 | 0.107 | 2.178 | 0.174 |
| region2 | 0.243 | 0.041 | -0.059 | 0.017 | 0.056 | 0.016 | -0.068 | 0.013 | -0.198 | 0.119 |
| region3 | 0.237 | 0.042 | 0.170 | 0.020 | 0.107 | 0.031 | -0.016 | 0.018 | -0.143 | 0.119 |
| region4 | 0.120 | 0.038 | 0.027 | 0.005 | 0.017 | 0.084 | 0.045 | 0.014 | -0.093 | 0.125 |
| semi-detached | 0.260 | 0.036 | 0.160 | 0.035 | 0.016 | 0.015 | 0.097 | 0.012 | 0.247 | 0.055 |
| corner-house | 0.033 | 0.014 | 0.012 | 0.008 | 0.036 | 0.009 | 0.027 | 0.009 | 0.008 | 0.015 |
| oneside-duplex | 0.203 | 0.017 | 0.156 | 0.010 | 0.202 | 0.014 | 0.177 | 0.013 | 0.129 | 0.017 |
| detached | 0.390 | 0.025 | 0.328 | 0.017 | 0.473 | 0.024 | 0.438 | 0.023 | 0.319 | 0.026 |
| log(lotsize) | 0.216 | 0.019 | 0.256 | 0.012 | 0.157 | 0.015 | 0.221 | 0.012 | 0.257 | 0.017 |
| log(flrspace) | 0.359 | 0.029 | 0.263 | 0.016 | 0.278 | 0.025 | 0.220 | 0.024 | 0.298 | 0.027 |
| maintain-med | - | - | 0.031 | 0.007 | 0.017 | 0.006 | 0.019 | 0.004 | 0.018 | 0.003 |
| maintain-high | 0.010 | 0.002 | 0.039 | 0.008 | 0.030 | 0.006 | 0.021 | 0.004 | 0.037 | 0.005 |
| room4 dummy | -0.074 | 0.033 | 0.042 | 0.009 | 0.026 | 0.017 | 0.086 | 0.015 | 0.015 | 0.020 |
| room5 dummy | -0.057 | 0.033 | 0.056 | 0.009 | 0.045 | 0.017 | 0.099 | 0.015 | 0.040 | 0.021 |
| room6 dummy | -0.022 | 0.035 | 0.096 | 0.011 | 0.089 | 0.019 | 0.144 | 0.018 | 0.068 | 0.026 |
| toiletbath2 | -0.021 | 0.010 | 0.019 | 0.007 | -0.011 | 0.010 | 0.021 | 0.010 | 0.035 | 0.011 |
| toiletbath3 | 0.009 | 0.015 | 0.012 | 0.010 | 0.001 | 0.015 | 0.016 | 0.014 | 0.104 | 0.022 |
| balcony-yes | 0.014 | 0.013 | 0.038 | 0.009 | 0.064 | 0.021 | 0.051 | 0.015 | -0.007 | 0.012 |
| garage-yes | 0.013 | 0.010 | 0.052 | 0.006 | 0.082 | 0.010 | 0.075 | 0.009 | 0.044 | 0.013 |
| dormer-yes | 0.028 | 0.016 | -0.005 | 0.011 | 0.030 | 0.013 | 0.010 | 0.014 | 0.024 | 0.017 |
| roofter-yes | 0.015 | 0.014 | 0.060 | 0.008 | 0.189 | 0.042 | 0.082 | 0.022 | 0.037 | 0.026 |
| Adjusted $R^2$ | 0.945 | | 0.934 | | 0.893 | | 0.896 | | 0.913 | |
| Deg. freedom | 405 | | 2138 | | 1498 | | 1387 | | 769 | |

† The dependent variable is natural log of prices, where the prices are entered in thousand euros.

‡ Region1 combines postcodes 9403 and 9407, region2 combines postcodes 9402 and 9406, region3 combines postcodes 9401, 9404 and some adjacent houses from postcode 9403, and region4 combines postcodes 9405 and 9408 (see Figure 2). The dummy variables are constructed with region1 as the base.

§ With the exception of a few estimated coefficients, all are significant at the 5% level. Therefore, we have not separately indicated whether an estimated coefficient is significant or not.

Table S-2: Hedonic Regression Results for Each Age of Dwellings without any Regional Dummy Variables

| Variables | Age0 | | Age1 | | Age2 | | Age3 | | Age4 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std |
| 1998 dummy | - | - | 1.634 | 0.081 | 2.106 | 0.123 | 1.795 | 0.114 | 1.310 | 0.130 |
| 1999 dummy | - | - | 1.802 | 0.081 | 2.257 | 0.122 | 1.927 | 0.114 | 1.459 | 0.129 |
| 2000 dummy | - | - | 1.920 | 0.081 | 2.408 | 0.123 | 2.077 | 0.114 | 1.527 | 0.130 |
| 2001 dummy | 2.393 | 0.156 | 2.035 | 0.081 | 2.521 | 0.123 | 2.170 | 0.114 | 1.652 | 0.127 |
| 2002 dummy | 2.363 | 0.155 | 2.059 | 0.081 | 2.584 | 0.122 | 2.256 | 0.114 | 1.749 | 0.128 |
| 2003 dummy | 2.380 | 0.154 | 2.101 | 0.081 | 2.601 | 0.122 | 2.297 | 0.113 | 1.796 | 0.128 |
| 2004 dummy | 2.451 | 0.154 | 2.145 | 0.081 | 2.669 | 0.123 | 2.307 | 0.114 | 1.818 | 0.128 |
| 2005 dummy | 2.471 | 0.152 | 2.185 | 0.081 | 2.706 | 0.123 | 2.387 | 0.113 | 1.900 | 0.129 |
| 2006 dummy | 2.525 | 0.153 | 2.227 | 0.081 | 2.719 | 0.123 | 2.379 | 0.114 | 1.884 | 0.128 |
| 2007 dummy | 2.551 | 0.152 | 2.267 | 0.081 | 2.767 | 0.123 | 2.424 | 0.113 | 1.921 | 0.128 |
| 2008 dummy | 2.571 | 0.153 | 2.278 | 0.081 | 2.777 | 0.123 | 2.432 | 0.114 | 1.930 | 0.129 |
| semi-detached | 0.271 | 0.039 | 0.158 | 0.020 | 0.047 | 0.016 | 0.155 | 0.012 | 0.245 | 0.056 |
| corner-house | 0.029 | 0.016 | 0.010 | 0.009 | 0.041 | 0.009 | 0.054 | 0.010 | 0.005 | 0.015 |
| oneside-duplex | 0.214 | 0.018 | 0.150 | 0.010 | 0.206 | 0.014 | 0.186 | 0.014 | 0.127 | 0.018 |
| detached | 0.397 | 0.028 | 0.326 | 0.017 | 0.485 | 0.023 | 0.460 | 0.025 | 0.335 | 0.027 |
| log(lotsize) | 0.225 | 0.020 | 0.257 | 0.012 | 0.157 | 0.015 | 0.221 | 0.013 | 0.256 | 0.017 |
| log(flrspace) | 0.324 | 0.032 | 0.269 | 0.016 | 0.266 | 0.024 | 0.247 | 0.026 | 0.310 | 0.027 |
| maintain-med | - | - | 0.032 | 0.008 | 0.013 | 0.017 | 0.013 | 0.004 | 0.018 | 0.003 |
| maintain-high | 0.011 | 0.002 | 0.041 | 0.008 | 0.030 | 0.021 | 0.005 | 0.004 | 0.038 | 0.005 |
| room4 dummy | -0.116 | 0.036 | 0.043 | 0.009 | 0.023 | 0.017 | 0.098 | 0.016 | 0.023 | 0.020 |
| room5 dummy | -0.102 | 0.036 | 0.057 | 0.009 | 0.040 | 0.017 | 0.088 | 0.016 | 0.054 | 0.022 |
| room6 dummy | -0.071 | 0.038 | 0.095 | 0.011 | 0.085 | 0.020 | 0.123 | 0.019 | 0.079 | 0.026 |
| toiletbath2 | -0.027 | 0.011 | 0.018 | 0.007 | -0.011 | 0.010 | 0.018 | 0.011 | 0.039 | 0.012 |
| toiletbath3 | 0.016 | 0.017 | 0.019 | 0.010 | 0.004 | 0.015 | 0.044 | 0.015 | 0.123 | 0.022 |
| balcony-yes | 0.004 | 0.014 | 0.043 | 0.009 | 0.075 | 0.021 | 0.053 | 0.016 | -0.015 | 0.012 |
| garage-yes | 0.014 | 0.011 | 0.053 | 0.006 | 0.081 | 0.010 | 0.064 | 0.010 | 0.049 | 0.013 |
| dormer-yes | 0.056 | 0.017 | -0.001 | 0.012 | 0.028 | 0.014 | 0.021 | 0.015 | 0.034 | 0.018 |
| roofter-yes | 0.016 | 0.016 | 0.060 | 0.008 | 0.219 | 0.042 | 0.065 | 0.024 | 0.026 | 0.027 |
| Adjusted $R^2$ | 0.933 | | 0.931 | | 0.891 | | 0.880 | | 0.909 | |
| Deg. freedom | 408 | | 2141 | | 1501 | | 1390 | | 772 | |

Note: The dependent variable is natural log of prices, where the prices are in thousand euros.

Table S-3: Impact of Different Regional Divisions on the Time-dummy Hedonic Regression with Non-linear Specification of Age

| Variables† | $f(age) = log(age)$ | | | | $f(age) = age^2$ | | | | $f(age) = age^2, age^3$ ‡ | | | | $f(age) = e^{-age}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Postcode-regions | | No region | | Postcode-regions | | No region | | Postcode-regions | | No region | | Postcode-regions | | No region | |
| | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std | Coefs | Std |
| $f(age)$ | -0.162 | 0.005 | -0.187 | 0.004 | -0.011 | 0.001 | -0.010 | 0.001 | -0.032 | 0.002 | -0.036 | 0.002 | 0.613 | 0.021 | 0.772 | 0.019 |
| | - | - | - | - | - | - | - | - | 0.004 | 0.001 | 0.005 | 0.001 | - | - | - | - |
| 1998 dummy | 1.912 | 0.050 | 1.896 | 0.051 | 1.832 | 0.050 | 1.753 | 0.051 | 1.890 | 0.049 | 1.843 | 0.050 | 1.684 | 0.050 | 1.607 | 0.052 |
| 1999 dummy | 2.065 | 0.050 | 2.051 | 0.050 | 1.987 | 0.049 | 1.910 | 0.050 | 2.042 | 0.049 | 1.997 | 0.050 | 1.838 | 0.050 | 1.764 | 0.051 |
| 2000 dummy | 2.200 | 0.050 | 2.183 | 0.050 | 2.122 | 0.049 | 2.044 | 0.050 | 2.176 | 0.049 | 2.128 | 0.050 | 1.975 | 0.050 | 1.898 | 0.052 |
| 2001 dummy | 2.309 | 0.050 | 2.293 | 0.050 | 2.230 | 0.049 | 2.155 | 0.050 | 2.284 | 0.049 | 2.239 | 0.050 | 2.085 | 0.050 | 2.007 | 0.051 |
| 2002 dummy | 2.366 | 0.049 | 2.350 | 0.050 | 2.289 | 0.049 | 2.214 | 0.050 | 2.342 | 0.049 | 2.297 | 0.049 | 2.141 | 0.050 | 2.063 | 0.051 |
| 2003 dummy | 2.399 | 0.049 | 2.385 | 0.050 | 2.322 | 0.049 | 2.251 | 0.050 | 2.376 | 0.049 | 2.333 | 0.049 | 2.174 | 0.050 | 2.095 | 0.051 |
| 2004 dummy | 2.436 | 0.049 | 2.421 | 0.050 | 2.359 | 0.049 | 2.292 | 0.050 | 2.411 | 0.049 | 2.371 | 0.049 | 2.213 | 0.050 | 2.133 | 0.051 |
| 2005 dummy | 2.484 | 0.049 | 2.469 | 0.050 | 2.411 | 0.049 | 2.342 | 0.050 | 2.461 | 0.049 | 2.420 | 0.049 | 2.261 | 0.050 | 2.181 | 0.051 |
| 2006 dummy | 2.506 | 0.049 | 2.489 | 0.050 | 2.432 | 0.049 | 2.363 | 0.050 | 2.482 | 0.049 | 2.441 | 0.049 | 2.283 | 0.050 | 2.200 | 0.051 |
| 2007 dummy | 2.542 | 0.049 | 2.527 | 0.050 | 2.473 | 0.049 | 2.406 | 0.050 | 2.521 | 0.049 | 2.480 | 0.049 | 2.317 | 0.050 | 2.235 | 0.051 |
| 2008 dummy | 2.551 | 0.050 | 2.536 | 0.050 | 2.482 | 0.049 | 2.418 | 0.050 | 2.529 | 0.049 | 2.489 | 0.050 | 2.328 | 0.050 | 2.245 | 0.052 |
| region2 | -0.045 | 0.004 | - | - | -0.001 | 0.005 | - | - | -0.019 | 0.005 | - | - | -0.086 | 0.004 | - | - |
| region3 | 0.028 | 0.007 | - | - | 0.080 | 0.008 | - | - | 0.052 | 0.008 | - | - | -0.017 | 0.007 | - | - |
| region4 | 0.023 | 0.004 | - | - | 0.056 | 0.004 | - | - | 0.043 | 0.004 | - | - | 0.003 | 0.004 | - | - |
| semi-detached | 0.108 | 0.008 | 0.126 | 0.008 | 0.095 | 0.008 | 0.115 | 0.008 | 0.112 | 0.008 | 0.135 | 0.008 | 0.104 | 0.008 | 0.120 | 0.008 |
| corner-house | 0.021 | 0.005 | 0.026 | 0.005 | 0.015 | 0.005 | 0.020 | 0.005 | 0.023 | 0.005 | 0.029 | 0.005 | 0.022 | 0.005 | 0.028 | 0.005 |
| oneside-duplex | 0.173 | 0.006 | 0.175 | 0.006 | 0.165 | 0.006 | 0.168 | 0.006 | 0.173 | 0.006 | 0.176 | 0.006 | 0.178 | 0.006 | 0.181 | 0.006 |
| detached | 0.380 | 0.010 | 0.389 | 0.010 | 0.370 | 0.010 | 0.379 | 0.010 | 0.380 | 0.010 | 0.389 | 0.010 | 0.384 | 0.010 | 0.397 | 0.010 |
| log(lotsize) | 0.229 | 0.006 | 0.225 | 0.007 | 0.242 | 0.006 | 0.237 | 0.007 | 0.229 | 0.006 | 0.223 | 0.007 | 0.222 | 0.007 | 0.212 | 0.007 |
| log(flrspace) | 0.273 | 0.010 | 0.283 | 0.010 | 0.260 | 0.010 | 0.283 | 0.010 | 0.276 | 0.010 | 0.295 | 0.010 | 0.283 | 0.010 | 0.298 | 0.011 |
| maintain-med | 0.018 | 0.002 | 0.018 | 0.002 | 0.016 | 0.002 | 0.016 | 0.002 | 0.017 | 0.002 | 0.017 | 0.002 | 0.019 | 0.002 | 0.022 | 0.002 |
| maintain-high | 0.028 | 0.002 | 0.028 | 0.002 | 0.028 | 0.002 | 0.029 | 0.002 | 0.027 | 0.002 | 0.027 | 0.002 | 0.031 | 0.002 | 0.034 | 0.002 |
| room4 dummy | 0.028 | 0.007 | 0.032 | 0.007 | 0.035 | 0.007 | 0.037 | 0.007 | 0.029 | 0.007 | 0.031 | 0.007 | 0.025 | 0.007 | 0.031 | 0.007 |
| room5 dummy | 0.039 | 0.007 | 0.042 | 0.007 | 0.041 | 0.007 | 0.039 | 0.007 | 0.041 | 0.007 | 0.041 | 0.007 | 0.036 | 0.007 | 0.039 | 0.007 |
| room6 dummy | 0.083 | 0.008 | 0.085 | 0.008 | 0.085 | 0.008 | 0.081 | 0.008 | 0.085 | 0.008 | 0.083 | 0.008 | 0.078 | 0.008 | 0.079 | 0.009 |
| toiletbath2 | 0.013 | 0.005 | 0.013 | 0.005 | 0.009 | 0.005 | 0.006 | 0.005 | 0.012 | 0.004 | 0.011 | 0.005 | 0.015 | 0.005 | 0.020 | 0.005 |
| toiletbath3 | 0.032 | 0.007 | 0.040 | 0.007 | 0.025 | 0.007 | 0.031 | 0.007 | 0.031 | 0.007 | 0.038 | 0.007 | 0.033 | 0.007 | 0.045 | 0.007 |
| balcony-yes | 0.026 | 0.006 | 0.022 | 0.006 | 0.039 | 0.006 | 0.040 | 0.006 | 0.027 | 0.006 | 0.025 | 0.006 | 0.020 | 0.006 | 0.005 | 0.006 |
| garage-yes | 0.062 | 0.004 | 0.061 | 0.004 | 0.065 | 0.004 | 0.066 | 0.004 | 0.060 | 0.004 | 0.060 | 0.004 | 0.064 | 0.004 | 0.062 | 0.004 |
| dormer-yes | 0.019 | 0.007 | 0.021 | 0.007 | 0.021 | 0.007 | 0.024 | 0.007 | 0.022 | 0.007 | 0.024 | 0.007 | 0.016 | 0.007 | 0.016 | 0.007 |
| roofter-yes | 0.063 | 0.007 | 0.062 | 0.008 | 0.069 | 0.007 | 0.074 | 0.008 | 0.059 | 0.007 | 0.060 | 0.007 | 0.067 | 0.008 | 0.063 | 0.008 |
| Adjusted $R^2$ | 0.920 | | 0.917 | | 0.919 | | 0.915 | | 0.922 | | 0.919 | | 0.916 | | 0.910 | |
| Deg. freedom | 6316 | | 6319 | | 6316 | | 6319 | | 6315 | | 6318 | | 6316 | | 6319 | |

† The dependent variable is natural log of prices, where the prices are in thousand euros.
‡ The numbers corresponding to row $f(age)$ are for $age^2$, and the row below $f(age)$ are for $age^3$.

Table S-4: Age-Price Profiles: Direct and Chained Fisher and Törnqvist Indexes

| Indexes* | Direct Index (Base=Age0) | | | Chained Index | | |
|---|---|---|---|---|---|---|
| | Fisher SI Index | Fisher DI Index | Törnqvist DI Index | Fisher SI Index | Fisher DI Index | Törnqvist DI Index |
| Age0 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| Age1 | 94.39 | 94.44 | 94.38 | 94.39 | 94.44 | 94.38 |
| Age2 | 85.63 | 85.86 | 85.87 | 85.17 | 85.36 | 85.33 |
| Age3 | 79.19 | 79.30 | 79.40 | 82.60 | 82.73 | 82.87 |
| Age4 | 77.70 | 77.79 | 77.77 | 81.43 | 81.56 | 81.66 |
| Depreciation rates (%): | | | | | | |
| Annual geometric† | 0.63 | 0.63 | 0.63 | 0.51 | 0.51 | 0.51 |
| Annual average‡ | 0.56 | 0.56 | 0.56 | 0.46 | 0.46 | 0.46 |

* SI refers to single imputation and DI refers to double imputation methods.

† Example: Let $d_r$ be the geometric depreciation rate, then $77.77 = 100(1 - d_r)^{40}$ $\Longrightarrow d_r = 0.63\%$.

‡ Calculated by dividing the cumulative depreciation by 40.