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Testing for Symmetry in Weakly Dependent Time Series

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Abstract

I propose a test of symmetry for a stationary time series based on the difference between the dispersion above the central tendency of the series with that below it. The test has many attractive features: it is applicable to dependent processes, it has a familiar form, it can be implemented using regression, and it has a standard Gaussian limiting distribution under the null hypothesis of symmetry. The finite sample properties of the test are examined via Monte Carlo simulation and suggest that it is more powerful than competing tests in the literature for the DGPs considered. I apply the test to investigate business cycle asymmetry in sectoral data and confirm previous findings that asymmetry is more often detected in goods producing sectors than service producing sectors.

Keywords: Symmetry; Weak dependence; Hypothesis testing; Monte Carlo simulation; Business cycle asymmetry.

JEL Classification: C12; C15; C22; C52; E32.

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1 Introduction

Understanding whether a variate originates from a symmetric or asymmetric distribution has been a topic of interest in statistics since Pearson (1895). In economics, the business cycle and the question about its symmetry, or lack there of, has been of interest for nearly as long a period (see Mitchell 1927 and Keynes 1936). In finance, symmetry is of interest because it is often an underlying assumption in asset pricing models. Meanwhile, the concept of symmetry of a distribution is also important during the initial step of a model selection procedure. For instance, if a time series process were found to be asymmetric, then a standard ARMA-type model with the assumption of a symmetric innovation term would be misspecified and some other, potentially nonlinear, model could provide a better fit to the data.

Within economics, Ramsey and Rothman (1996) characterise asymmetry into two different forms: longitudinal or transversal. Longitudinal asymmetry is related to differences in the duration and slope of upswings of a time series that are not mirrored by downswings in the same series. Transversal asymmetry is asymmetry related to differences in the vertical displacement of a (stationary) time series from its mean. It relates to the 'deepness' form of asymmetry proposed by Sichel (1993) and is most similar to the original description of symmetry found in Pearson (1895).

Many of the existing symmetry tests in the statistics and economics literature make use of the sample skewness coefficient (standardised third moment) which can be very sensitive to outliers (Kim and White 2004), or are only applicable to *iid* data such as the triples test of Randles et al. (1980). Only a relatively small number of tests are also applicable to weakly dependent data (see Psaradakis and Vávra 2015). However, simulation evidence indicates that the power of these symmetry tests can be very low when considering processes with sample sizes of around 100–200 observations and moderate amounts of dependence as is usually encountered in macroeconomics.

In this paper I propose a new test which is suitable for weakly dependent time series processes. The test uses the original Pearson (1895) definition by comparing the difference in the dispersion of the process above its central tendency to that below it. If the difference is zero, then the process is deemed to be symmetric. If the difference is non zero, then the process is considered asymmetric. The test has the appearance of the familiar 'paired difference' t-test and has a standard Gaussian limiting distribution under the null hypothesis of symmetry.

One benefit of my test compared to other tests which use the sample skewness coefficient, is that it only requires finite fourth moments. This makes my test is less restrictive than the skewness-base alternatives which require finite sixth moments. Importantly, the test can be implemented easily using standard regression techniques combined with Heteroskedasticity and Autocorrelation Consistent (HAC) standard error estimates.

In a range of Monte Carlo simulations the symmetry test is shown to have good power properties relative to the competing tests of Bai and Ng (2005) and Psaradakis and Vávra (2015) for the Data Generating Processes (DGPs) investigated. As an empirical application I use the test developed in this paper to investigate the prevalence of asymmetry in monthly U.S. nonfarm payroll data by sector. My findings confirm previous results that asymmetry is detected more often in durable goods manufacturing-related sectors than service-related sectors.

The remainder of the paper is organised as follows: Section 2 introduces the symmetry test and its limiting distribution under the null hypothesis of symmetry. Section 3 provides simulation evidence on the sampling properties of the test statistic as well as its empirical size and power for a range of DGPs. An empirical application is discussed in Section 4, before concluding in Section 5. Auxiliary information and results are provided in the Appendix.

2 Testing for Symmetry

2.1 The Test Statistic

In this Section and throughout the remainder of the paper I will use the following definition to describe a stationary time series which is symmetric:

Definition 1. *A stationary time series* $\{X_t, t \in \mathbb{Z}\}$ *with central tendency given by* ζ*, is symmetric if* $\mathbb{V}ar\left[X_t \mid X_t > \zeta\right] = \mathbb{V}ar\left[X_t \mid X_t < \zeta\right]$.

Definition 1 states that a time series is symmetric if the dispersion of the subsequence of values strictly greater than its central tendency is equal to the dispersion of the subsequence of values strictly less than its central tendency. This definition follows from the original description provided by Pearson (1895). It is also similar to the descriptions provided by DeLong and Summers (1984), as well as Sichel (1993) in the context of describing the property 'deepness' as it relates to asymmetry and Ramsey and Rothman (1996) in relation to transversal asymmetry. However, I have specified the variance as the measure of 'deviation' and the more general term, 'central tendency' in place of any reference to the mean of the time series. As a result, central tendency can also include other statistics such as the median which is equivalent to the mean for a symmetric and stationary time series.

Definition 1 could form the basis of a statistical test for equality in the variance of the subsequence of X_t above the central tendency to the subsequence of X_t below the central tendency. This would involve using a test for equal variances such as Levene (1960) or one of the alternative versions proposed by Brown and Forsythe (1974). One problem with this approach is that the two subsequences might not be independent from each other. The potential lack of independence can be avoided by constructing a new sequence, $\{\hat{\delta}_n, n \in \mathbb{Z}\}$, representing the difference between squared deviations from the two subsequences. More formally, let $\{X_{1,t_i}\}$ be the subsequence of $\{X_t\}$ defined by the indices $\{t_i \in \mathbb{Z} : X_t > \hat{\theta}\}$, and $\{X_{2,t_j}\}$ be the second subsequence of $\{X_t\}$ defined by the indices $\{t_j \in \mathbb{Z} : X_t < \hat{\theta}\}$ and let $\hat{\theta}$ be the sample median of $\{X_t\}$. Then $\{\hat{\delta}_n\}$ can be defined as:

$$\hat{\delta}_n = \left(X_{1,t_i} - \hat{\mu}_{X_1}\right)^2 - \left(X_{2,t_j} - \hat{\mu}_{X_2}\right)^2 \qquad (1)$$

Where $\hat{\mu}_{X_k}$ $k \in \{1, 2\}$ is an estimate of the mean for that particular subsequence. Defining the central tendency of a time series in terms of the median has an important implication: if the median is unique, which it is for a continuous random variable, then the two subsequences X_1 and X_2 will be equal in length (exactly half the length of the original sequence X_t), thereby allowing us to compute the differences between the two subsequence square deviations.

Using this sequence of differences, we can state the following proposition:

Proposition 2. *If $\{X_t\}$ is a stationary and symmetric time series with a continuous distribution function, then* $\mathbb{E}\left[\hat{\delta}_n\right] = 0$.

The proof follows immediately from Definition 1 and the linearity of the expectation operator. Using Proposition 2 we can state the following theorem:

Theorem 3. *Assuming $\{\hat{\delta}_n\}$, is a sequence of N mixing random scalars such that either $\phi(m)$ or $\alpha(m) = \mathcal{O}\left(m^{-\lambda}\right)$ for $\lambda > -r/2(r-1)$, or $\lambda > -r/(r-1)$, $r \geq 2$, and $\mathbb{E}\left|\hat{\delta}_n\right|^r < \infty$ and where $\phi(m)$ and $\alpha(m)$ are the uniform or strong mixing coefficients as defined in Definition 3.45 of White (2000). Then by application of Theorem 5.20 of White (2000):*

$$\sqrt{N}\bar{\delta} \xrightarrow{d} \mathcal{N}\left(0, \sigma_{\delta}^2\right) \tag{2}$$

where $\bar{\delta} = N^{-1}\sum_{n=1}^{N}\hat{\delta}_n$ and $\sigma_{\delta}^2 = \lim_{N\longrightarrow\infty} N\mathbb{V}ar\left[\hat{\delta}_n\right]$ are the sample mean and long run variance of the difference sequence, $\hat{\delta}_n$.

From Equation (2), I define the symmetry test statistic $\mathcal{S}$ as follows:

Theorem 4. *Assuming the previous assumptions all hold and let $\hat{\sigma}_{\delta}^2$ be a consistent estimate of σ_{δ}^2, then under the null hypothesis of symmetry:*

$$\mathcal{S} = \frac{\sqrt{N}\bar{\delta}}{\hat{\sigma}_{\delta}} \xrightarrow{d} \mathcal{N}(0,1) \tag{3}$$

This result leads directly from Theorem 3 and Theorem 5.20 of White (2000). Serial dependence in $\hat{\delta}_n$ is explicitly taken into account through σ_{δ}^2. As such, Theorem 4 permits testing for symmetry in weakly dependent time series. The test statistic $\mathcal{S}$ is conceptually just a standard 'paired difference' t-test statistic (see Casella and Berger 2002) and it could be computed using linear regression. This would involve regressing the estimated difference sequence, $\hat{\delta}_n$, on a constant and then compute the HAC-corrected standard error for this term. The estimate of the constant is identical to $\bar{\delta}$ and the standard error is a consistent estimator for σ_{δ} even in the presence of heteroskedasticity and/or serial dependence (see

Newey and West 1987 and den Haan and Levin 1997). The test would reject the null hypothesis of symmetry at a given significance level α if $|\mathcal{S}| > Z_{\alpha/2}$ where Z_i is the upper i^{th} quantile of the standard Gaussian distribution.

The method of computing the test statistic $\mathcal{S}$ in this way is not new and is the basis for alternative symmetry-type tests such as those proposed by Sichel (1993) using the sample skewness coefficient, Ramsey and Rothman (1996) and their test of time irreversibility, as well other tests such as the Diebold and Mariano (1995) test for comparing predictive accuracy.

Remarks:

1. The pairing of $\{X_{1,t_i}\}$ and $\{X_{2,t_j}\}$ in Equation (1) is arbitrary and the computation of the test statistic $\mathcal{S}$ is invariant to permutations in the ordering of either subsequence.

2. The sign of the test statistic $\mathcal{S}$ provides information on the form of symmetry. A negative value indicates left skewness, while a positive value indicates right skewness.

3. The symmetry test only requires $\{\hat{\delta}_n\}$ have finite second moments and by implication that $\{X_t\}$ have finite fourth moments. This requirement is less restrictive than tests based on the sample skewness coefficient which requires finite six moments (for example the skewness tests of DeLong and Summers 1984, Sichel 1993, Bai and Ng 2001, or Bai and Ng 2005).

4. To implement the symmetry test the sample median must be unique otherwise the test cannot be computed since we are unable to construct the sequences of differences.[1]

5. The sequence $\{X_t\}$ need not have mean equal to zero to use the symmetry test.

6. The symmetry test can be applied to the residuals from an OLS regression using Theorem 5 of Bai and Ng (2005).

[1] In the Monte Carlo simulations to be discussed in the Section 3, this was never a problem (with 1240 difference experiments); however, in the empirical application in Section 4, there was one series for which the test could not be computed. This issue is most likely to occur with floating point numbers which have only a small number of significant figures after the decimal place and periods of no change.

3 Monte Carlo Simulation Analysis

3.1 Simulation Design

Two Monte Carlo experiments were undertaken to investigate the finite sample properties of the symmetry test $\mathcal{S}$. The first one is to ascertain the sampling distribution of the $\mathcal{S}$ under the null hypothesis of symmetry based on variates generated from a Gaussian distribution and using sample sizes $T = 100$ to represent a small sample and $T = 1000$ to represent a large sample. This is also helpful to judge how quickly the sample distribution of the test statistic approaches that of the limiting Gaussian distribution as $T \longrightarrow \infty$. The second experiment is to assess the empirical size and power properties of the proposed symmetry test applying various well known and frequently used distributions in the symmetry testing literature including: Gaussian and Student-t as examples of symmetric distributions as well as Chi-squared to represent an asymmetric distribution. Additionally, the Generalised Lambda Distribution (GLD) of Ramberg and Schmeiser (1974) is also considered. This is a distribution which can be used to generate both symmetric and asymmetric variates depending on the values chosen for the four parameters which describe it. The inverse function of the GLD is defined as:

$$F^{-1}(u) = \lambda_1 + \lambda_2^{-1}\left[u^{\lambda_3} - (1-u)^{\lambda_4}\right], \qquad 0 \leq u \leq 1 \tag{4}$$

The skewness and peakedness of the GLD are determined by the two parameters λ_3 and λ_4. If $\lambda_3 = \lambda_4$ the the GLD will be symmetric and it will be asymmetric when $\lambda_3 \neq \lambda_4$. Given these two values, the variance is determined by λ_2, while the mean can be shifted to any value by the appropriate choice of λ_1. Note, if the GLD is asymmetric, then its mean will not be equal to λ_1 which is the case when it is symmetric.

The data used in both experiments are generated as an AR(1) model: $y_t = \phi y_{t-1} + \epsilon_t$ were ϵ is generated as *iid* from one of a total of 12 different distributions with six representing symmetry and the remaining six representing asymmetry. In all cases the initial observation is drawn from the process's stationary distribution. I adopt, with some modifications to be explained, the distribution parameter values from Bai and Ng (2005). These parameter values were also used by Psaradakis and Vávra (2015).

To help with comparison across the different distributions examined, each process is constructed so that its mean is zero and its variance is unity. For the Student-t and Chi-squared distributions this was achieved as follows: Let ν represent the degrees of freedom parameter, then for the Student-t variates, which already have a zero mean, I divided the variates by $\nu/(\nu-2)$. With the Chi-squared variates it was necessary to also demean the variates by subtracting ν before dividing them by 2ν to get a process with zero mean and unit variance.

GLD-generated variates are not likely to have zero mean and unit variance for all parameter values investigated in Bai and Ng (2005). As was mentioned previously, if $\lambda_3 \neq \lambda_4$ then the mean of the process will not be λ_1. This might have implications when constructing the various DGPs and could impact the statistical properties of the symmetry test. To avoid this potential issue, I deviate from both Bai and Ng (2005) and Psaradakis and Vávra (2015) and instead follow the guidance of Ramberg and Schmeiser (1974). to ensure the GLD-generated variates do have the desired mean and variance they recommend the following: To set the variance of the GLD to be equal to σ let:

$$\lambda_2 = \pm\Bigg(\Big[1/(2\lambda_3+1) - 2\beta(\lambda_3+1,\lambda_4+1) + 1/(2\lambda_4+1)\Big] - \Big[1/(\lambda_3+1) - 1/(\lambda_4+1)\Big]^2\Bigg)^{1/2}\sigma^{-1} \quad (5)$$

Where the sign of λ_2 depends on the sign of both λ_3 and λ_4 and β is the beta function. Then, given this value of λ_2, the mean of the GLD can be set to be equal to μ by letting:

$$\lambda_1 = \mu - \lambda_2^{-1}\Big[1/(\lambda_3+1) - 1/(\lambda_4+1)\Big] \quad (6)$$

Table A1 in the Appendix lists the set of parameter values used for each distribution employed.

In the two experiments ten evenly spaced values from 0.0 to 0.9 were used for the AR parameter, with results presented for three cases: the best situation of *iid* data, (i.e., $\phi = 0.0$), the worst case of very strong dependence (i.e., $\phi = 0.9$), and somewhere in between with a modest amount of dependence (i.e., $\phi = 0.5$). In the second experiment a grid of ten evenly spaced sample sizes from 100 to 1000 were used (additional results are available from the author upon request).

In both experiments, the symmetry test was computed using the previously described DGPs and for the first experiment a sequence of symmetry test statistics were estimated and saved, while for the second experiment the number of rejections of the null hypothesis of symmetry using a nominal size of $\alpha = 0.05$ was recorded. When estimating the standard error in the regression used to compute the test statistic I adopt the Quadratic Spectral density kernel and the automatic bandwidth method described by Andrews (1991). Furthermore, I apply the OLS residual adjustment of Hartigan (2016) to improve the finite sample properties of the test statistic. In each case the number of repetitions was set to 10000. All experiments were undertaken in C++ using the Armadillo Template Linear Algebra library (Sanderson and Curtin 2016) and the Mersenne twister pseudo random number engine (`std::mt19937_64`) from the C++ standard library. The default seed (5489) was used in each experiment to aid in comparisons.

3.2 Simulation Results

Figure 1 plots the finite sampling distribution for the symmetry test statistic for an AR(1) process and Gaussian innovation term with $T = 100$ and $T = 1000$. The top two panels show the estimated density of the test statistic overlaid with a Gaussian probability density function (PDF) for comparison. The bottom two panels show the empirical cumulative distribution function (CDF) for the test statistic compared to the true Gaussian distribution function. Additional plots for $\phi = 0.0$ and $\phi = 0.9$ are presented in Figure A1 and Figure A2 in the Appendix.

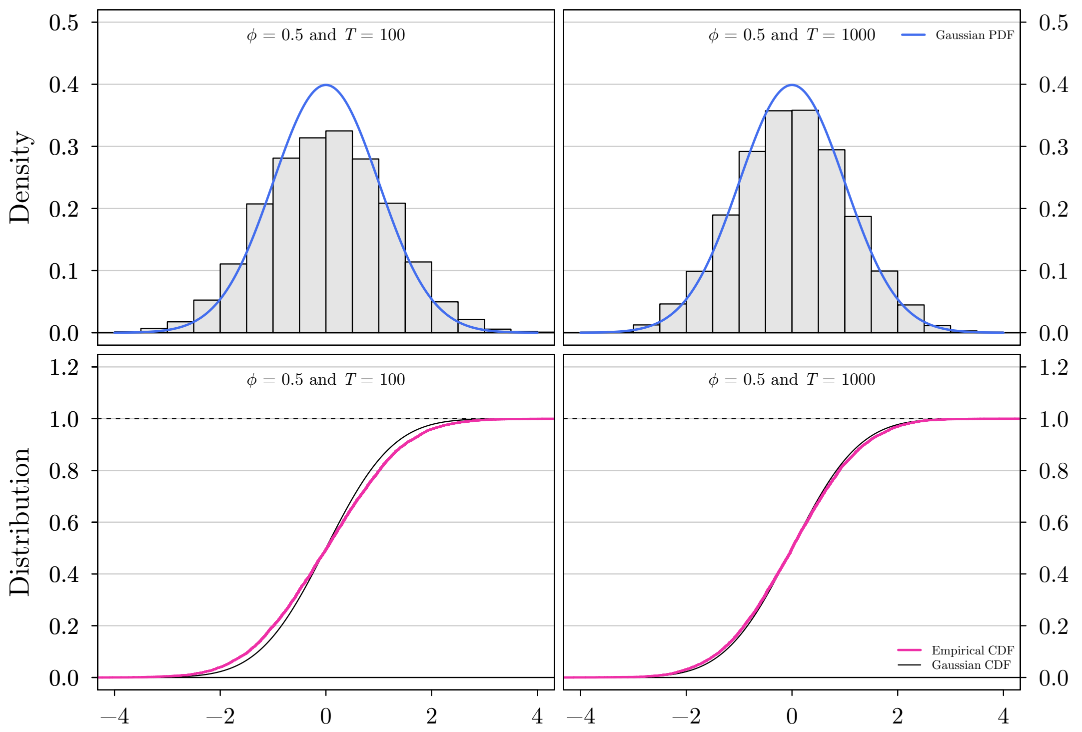

Figure 1: Symmetry Test Sampling Distribution – AR(1) Process: $\phi = 0.5$

For both the *iid* and moderate dependence cases, the symmetry test statistic displays a reasonable approximation to the limiting Gaussian distribution at the smaller sample size of $T = 100$. In the strong dependence case; however, the statistic's sampling distribution is clearly different to the standard Gaussian distribution with much more mass located in the tails than is the case with a Gaussian distributed variate. Nonetheless, in the second example with $T = 1000$ all three cases show a much closer approximation to the limiting Gaussian distribution, although the very persistent AR(1) process with $\phi = 0.9$ still shows a slight difference which is more evident in the empirical CDF plot.

Empirical size for the six different symmetric DGPs are displayed in Figure 2. Generally, the symmetry test statistic is modestly over-sized, especially for the Gaussian process (S1) and the smaller sample sizes . In contrast, the test shows more correctly sized test statistics for the more heavy-tailed distributions such as the Student-t distributed variates with degrees of freedom equal to 5 (S2). This finding is interesting because previous research has highlighted how leptokutic (i.e., heavy-tailed) distributions can adversely impact measures of symmetry based on the use of the skewness coefficient (see for example Kim and White 2004 and

Premaratne and Bera 2005). The performance of the symmetry test deteriorates when testing the more highly persistent processes and the test statistic is over-sized for all six DGPs. Although, as the number of observations grows, the performance of the symmetry test improves as would be expected.

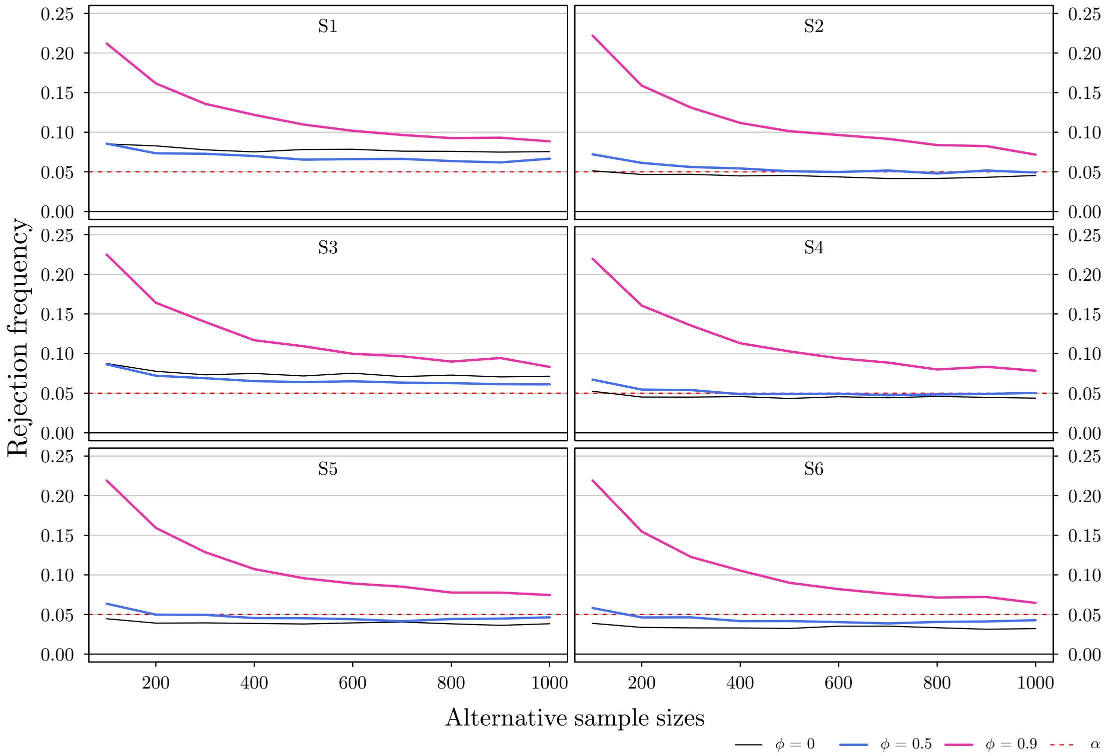

Figure 2: Symmetry Test Empirical Size

Figure 3 illustrates the empirical power for the six asymmetric DGPs. Generally, the test statistic shows good power properties, predominately for *iid* and moderately dependent data. However, the test's power deteriorate's significantly for the highly persistent DGPs, but this does improve marginally as T increases with the notable exception of DGP A2 which shows no real improvement at all as the sample size increases.

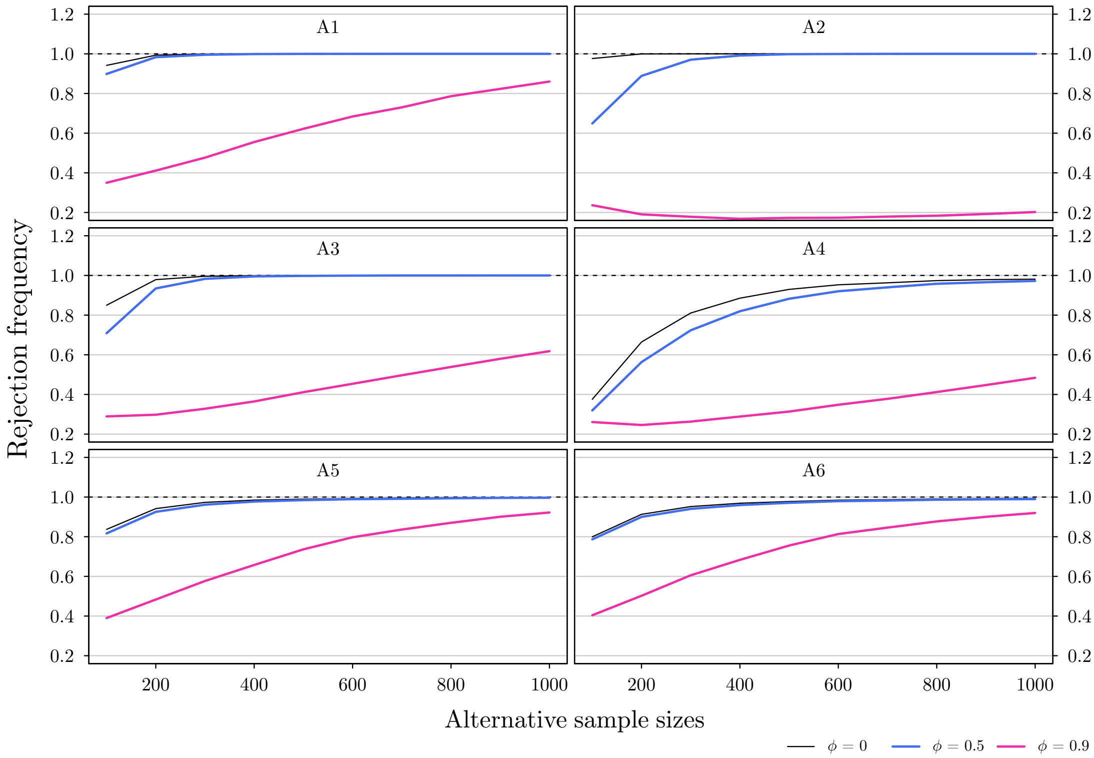

Figure 3: Symmetry Test Empirical Power

The notable loss in power of the symmetry test for very persistent processes is something which has been previously highlighted in the literature (see Kim et al. 1996, Bai and Ng 2001, and Bai and Ng 2005). Indeed, Bai and Ng (2001) noted that with an ARMA model even if the innovation term ϵ_t is asymmetric, it does not necessarily imply that the process y_t will also be asymmetric as well.

This is supported by Figure 4 which plots $T = 250$ *iid* standardised Chi-squared variates with 2 degrees of freedom (top left) as well as an AR(1) process with $\phi = 0.9$ generated using the same 250 standardised Chi-squared variates (top right). Below each of these are histograms for the two processes. As can be seen, the histogram for the very dependent process is much more symmetrical relative to the *iid* variates. Furthermore, when tested using the symmetry test, the test statistic for the *iid* variates is 3.69 with a p–value of 0.0002, rejecting the null of symmetry at the $\alpha = 0.05$ significance level. Whereas, the test statistic for the highly dependent process is 1.08 with a p–value of 0.28, indicating a lack of evidence against the null of symmetry.

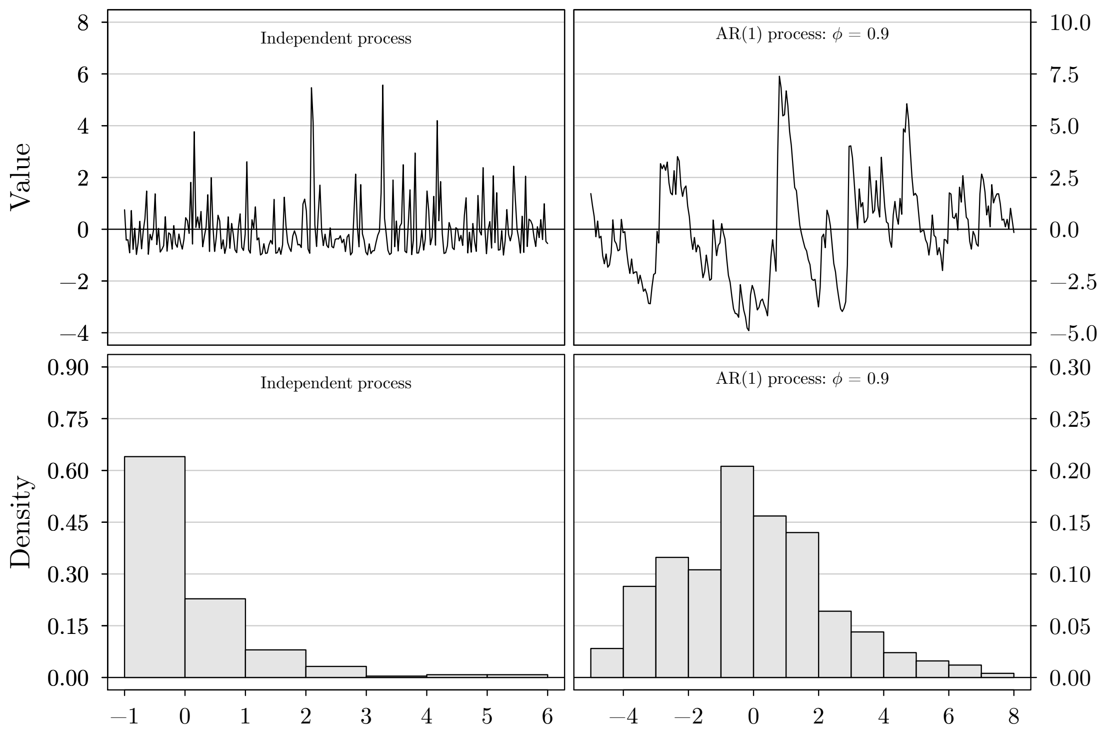

Figure 4: Asymmetry & Dependence – Standardised $\chi^2_{(2)}$ Process

Finally, it is important to compare the finite sample performance of the symmetry test proposed in this paper with alternative tests in the literature. The tests I use for comparison are the two-sided skewness test of Bai and Ng (2005) and the quantile-based symmetry test of Psaradakis and Vávra (2015). These two tests are relatively recent developments in the symmetry testing literature and, most importantly, are applicable to weakly dependent time series data. I take the published results from each paper and compare them with those of the symmetry test statistic $\mathcal{S}$ using the same DGPs for comparison. The comparisons for empirical size are in Table A2, while the comparisons for empirical power are in Table A3, both of which are in the Appendix.

From the size comparisons, the symmetry test $\mathcal{S}$ shows a tendency to reject more frequently than the other two tests, especially at the smaller sample sizes and more persistent DGPs. However, as the sample size increases, the differences between the three tests become smaller. When considering a sample size of $T = 500$ and a DGP from an AR(1) process with $\phi = 0.5$, then the test of Bai and Ng (2005) shows the best empirical size properties, followed closely by $\mathcal{S}$ and the test of Psaradakis and Vávra (2015).

However, when considering empirical power for these three tests, the results are very much in favour of $\mathcal{S}$. The $\mathcal{S}$ test statistic displays relatively good power, at the smallest sample size considered for the three different AR(1) processes examined. The results for the most dependent process are particularly favourable for $\mathcal{S}$ with the lowest empirical power of 0.22 for DGP A2, while the highest figure for the test of Bai and Ng (2005) is 0.06 for the same DGP. When considering a sample size of $T = 500$ and a DGP from an AR(1) process with $\phi = 0.5$, then $\mathcal{S}$ shows the best empirical power properties followed closely by the test of Psaradakis and Vávra (2015) and then the test of Bai and Ng (2005).

4 Sectoral Evidence of Business Cycle Asymmetry

In this section I employ the previously developed symmetry test in an empirical application to investigate business cycle asymmetry using sectoral data. The existence of business cycle asymmetry has been an area of active research since the seminal work of Neftçi (1984). The idea has it origins in the works of Mitchell (1927) and Keynes (1936) who both characterised downturns in activity as being distinctly different to upturns in activity and therefore are likely to result in time series which are not symmetric. Other notable studies using both U.S. and international data include: DeLong and Summers (1984), Falk (1986), Rothman (1991), Sichel (1993), Ramsey and Rothman (1996), Verbrugge (1997) Bidarkota (1998), Razzak (2001), and Korenok et al. (2009). Generally, these papers have tended to use data related to activity such as growth rates of GDP/GNP or industrial production or the unemployment rate as a measure of the business cycle.

The main findings have been that there is some evidence for both longitudinal and transversal asymmetry in activity-based output measures of the U.S. economy. This evidence is stronger for industrial production than it is for GDP/GNP. Meanwhile, the results for the unemployment rate are mixed. Neftçi (1984) found support for asymmetry in the U.S. unemployment rate; however, Sichel (1989) identified an error in Neftçi (1984)'s calculations which when corrected weakened the original result. Rothman (1991) did find evidence for asymmetry when using a corrected version of Neftçi (1984)'s method, but in a more recent study and with a longer data set, Rothman (2008) found less compelling evidence and this depended on the form of differencing employed and whether asymptotic or estimated p–values were used. Some authors have found asymmetry when looking at subsectors of the economy using disaggregated data. In the cases when asymmetry is detected, it appears to originate more from sectors connected to durable goods manufacturing compared to the more service-based sectors.

4.1 Testing for Asymmetry in U.S. Sectoral Nonfarm Payroll Data

To investigate this hypothesis I applied the symmetry test to the Bureau of Labor Statistics's monthly nonfarm payrolls by industry sector time series data set which has not previously been used in the asymmetric business cycle testing literature. Another reason for using this particular data set is that is a relatively long data set by macroeconomic standards with a broad coverage of many different sectors of the U.S. economy. The time period examined is 1960M1–2016:M8, providing 680 observations for each of the 18 different series considered. This sample size should give the symmetry test good power properties given the findings of the simulation experiments in Section 3.

Before computing the test statistic, I convert the data to growth rates by taking the first difference of the logarithm of the data. In addition, I also compute the sample skewness coefficient as well as the Bowley coefficient (BC) for the data set. The BC is a nonparametric measure of skewness bounded between ± 1 that has been shown to be robust to outliers (Groeneveld and Meeden 1984) and Kim and White (2004) recommended that robust measures such as it should be routinely reported. The BC can be defined as:

$$BC = \frac{Q_{(0.75)} + Q_{(0.25)} - 2Q_{(0.5)}}{Q_{(0.75)} - Q_{(0.25)}} \tag{7}$$

where $Q_{(i)}$ is the i^{th} quantile of the series examined. A value of -1 indicates extreme left skewness, a value of 1 indicates extreme right skewness, while a value near 0 indicates a lack of skewness. The results for the different nonfarm payroll sectors are presented in Table 1. The method for computing the HAC-corrected standard error in the regression used to estimate the test statistic is the same as that used in the Monte Carlo simulations in Section 3.

Table 1: Selected U.S. Nonfarm Payroll Data

Sector	Skewness	*BC*	Symmetry Test $\mathcal{S}$	p–value	Bai and Ng (2005) Tests $\hat{\pi}_3$	p–value	$\hat{\mu}_{35}$	p–value
Goods producing	-1.11	-0.11	-2.51	0.01	2.18	0.03	5.40	0.07
Mining & logging (A%)	-0.03	0.01	-0.69	0.49	-0.08	0.94	2.53	0.28
Construction	0.06	-0.19	-0.35	0.73	0.16	0.88	0.05	0.98
Manufacturing	-1.30	-0.01	-2.51	0.01	-2.33	0.02	7.01	0.03
Durable goods	-1.22	-0.05	-2.43	0.02	-2.01	0.04	7.09	0.03
Nondurable goods	-1.03	-0.11	-1.64	0.10	-1.57	0.12	4.42	0.11
Service producing	-0.14	-0.01	-0.94	0.35	-0.35	0.73	3.22	0.20
Trade, transport & utilities	-0.34	-0.07	-2.12	0.03	-1.40	0.16	3.78	0.15
Wholesale trade	-0.52	-0.10	-1.99	0.05	-1.54	0.12	5.62	0.06
Retail trade	-0.32	0.02	-1.21	0.23	-1.13	0.26	1.99	0.37
Information	7.40	-0.15	0.41	0.68	0.93	0.35	1.10	0.58
Financial services	-0.56	-0.03	-1.19	0.24	-1.17	0.24	1.38	0.50
Professional & business services	-1.24	-0.14	-2.06	0.04	-1.70	0.09	5.59	0.06
Education & health services	0.30	0.10	1.58	0.11	1.63	0.10	2.68	0.26
Leisure & hospitality	0.04	-0.14	-0.80	0.42	0.14	0.89	4.41	0.11
Other services	-0.13	0.02	-0.74	0.46	-0.58	0.56	0.35	0.84
Government	1.00	0.14	1.38	0.17	1.48	0.14	1.86	0.40
Total nonfarm	-0.48	-0.08	-1.97	0.05	-1.48	0.14	4.84	0.09

Note: The time series are sourced from the Bureau of Labor Statistics, Table B-1a Employees on nonfarm payrolls by industry sector and selected industry detail, seasonally adjusted (in thousands). Each series is converted to monthly compounding growth rates, defined as $\ln(y_t/y_{t-1})$. The sample size is 680 observations for each series covering the time period: 1960:M1 – 2016:M8. 'A%' denotes that series was converted to annual compounding growth rates, defined as $\ln(y_t/y_{t-12})$. The skewness statistic is computed using a bias correction for the standard deviation (i.e., $T-1$). '*BC*' is the Bowley coefficient measure of skewness. Values close to -1 indicate extreme left skewness, values close to 1 indicate extreme right skewness, and values close to 0 indicate an absence of skewness. '$\hat{\pi}_3$' and '$\hat{\mu}_{35}$' are the two-sided skewness test and joint test of the third and fifth moments which are distributed as $\mathcal{N}(0,1)$ and $\chi^2_{(2)}$ respectively and were developed by Bai and Ng (2005).

Note, one series, 'Mining & logging' did not have a uniquely defined median for the time period considered when converted to monthly compounding growth rates. As a result, the series cannot be tested using the symmetry test. However, the problem could be overcome by using an alternative detrending mechanism. As long as the resultant series is stationary the test is still applicable. Hence, I use the test on the Mining & logging series converted to annual compounding growth rates (denoted by 'A%' in Table 1).

For comparison purposes I also include the results from using the two alternative skewness tests developed in Bai and Ng (2005). As was shown in Table A2, Bai and Ng (2005)'s test based on the two-sided skewness test ($\hat{\pi}_3$) had the overall best empirical size properties of the three symmetry tests compared, but it also had the worst power (Table A3). As a robustness measure, I also include the results using their second test, a joint test that the third and fifth moments are jointly zero ($\hat{\mu}_{35}$). Bai and Ng (2005) state that this test should have better power properties relative to the skewness-based test.[2]

Using $\mathcal{S}$ the null hypothesis of symmetry is rejected at the standard $\alpha = 0.05$ level for five of the 18 different series tested. The sectors were: Goods producing, Manufacturing, Durable goods, Trade, transport & utilities, and Professional & business services. Furthermore, the results are borderline for Total nonfarm as well as Wholesale trade. In comparison, the two tests of Bai and Ng (2005) only detect asymmetry in the Goods producing (only $\hat{\pi}_3$), Manufacturing, and Durable goods sectors. Finally, the conclusions reached for the Mining & logging series using annual compounding growth rates confirm those obtained via the two tests of Bai and Ng (2005).

This result is in accordance with the simulation results from Section 3 which indicated that the symmetry test $\mathcal{S}$ was likely to be more powerful relative to Bai and Ng (2005)'s two-sided skewness test at least. Generally, the results confirm findings of previous studies (see Rothman 1991, Sichel 1993, and Korenok et al. 2009) and indicate business cycle asymmetry is more likely to be concentrated in goods producing sectors than service producing sectors.

Furthermore, the test results also indicate that transversal (or 'depth') asymmetry is not as prevalent in the U.S. economy as claimed by Verbrugge (1997), at least when using the BLS nonfarm payroll data set, and when using first differences as the detrending mechanism. This conclusion is supported by both the symmetry test proposed in this paper as well as the two tests proposed in Bai and Ng (2005).

Finally, it is worth comparing the test statistic $\mathcal{S}$, which provides and indicator of the direction of skewness, to both the sample skewness coefficient and the BC. Both $\mathcal{S}$ and BC agree on all but two cases on the direction of skewness (Information and Education & health services), while there are five disagreements between the skewness statistic and the BC, indicating some possible distortion coming from the presence of outliers. This is especially

[2]Both tests were estimated using the GAUSS codes obtained from Serena Ng's web site: `http://www.columbia.edu/~sn2294/`. I used the Quadratic Spectral density and the Andrews (1991) automatic bandwidth in estimation.

evident for the Information category. Visual inspection of this series in first differences reveals two very large outliers (1983:M8 -25%, 1983:M9 +35%) while being otherwise relatively symmetric. The effect of these two outliers is apparent in the sample skewness statistic having a relatively large value of 7.4, while having no real impact on the BC or $\mathcal{S}$.

5 Conclusion

A test for symmetry suitable for weakly dependent time series is developed. It is based on comparing the dispersion of a process above its central tendency to that below it; equality between the two measures indicates symmetry while any difference suggests asymmetry.

The test statistic is similar to the familiar t-test for paired differences and has a standard Gaussian limiting distribution under the null hypothesis of symmetry. In simulation experiments it was shown to converge to this limiting distribution relatively quickly and with processes containing different amounts of dependence. In comparisons with competing symmetry tests in the literature applicable for dependent time series, it was shown to be slightly over-sized at smaller sample sizes, but provided a noticeable improvement in power relative to those alternative tests for the DGPs considered.

An empirical application to monthly U.S. nonfarm payroll data by sector, suggests that when asymmetry is present it appears to be detected more often in durable goods manufacturing-related sectors than service-related sectors.

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Appendix

Data Generating Processes

Table A1: Distributions

Label	Distribution	Parameter Values
		Symmetric Processes
S1	Gaussian	$\mu = 0.00$, $\sigma = 1.00$
S2	Student-t	$\nu = 5.00$
S3	Generalised Lambda Distribution	$\lambda_1 = 0.00000$, $\lambda_2 = 0.19745$, $\lambda_3 = 0.13492$, $\lambda_4 = 0.13492$
S4	Generalised Lambda Distribution	$\lambda_1 = 0.00000$, $\lambda_2 = -0.16809$, $\lambda_3 = -0.08000$, $\lambda_4 = -0.08000$
S5	Generalised Lambda Distribution	$\lambda_1 = 0.00000$, $\lambda_2 = -0.39791$, $\lambda_3 = -0.16000$, $\lambda_4 = -0.16000$
S6	Generalised Lambda Distribution	$\lambda_1 = 0.00000$, $\lambda_2 = -0.72959$, $\lambda_3 = -0.24000$, $\lambda_4 = -0.24000$
		Asymmetric Processes
A1	Chi-squared	$\nu = 2.00$
A2	Generalised Lambda Distribution	$\lambda_1 = 0.83503$, $\lambda_2 = 0.45906$, $\lambda_3 = 1.40000$, $\lambda_4 = 0.25000$
A3	Generalised Lambda Distribution	$\lambda_1 = -0.62900$, $\lambda_2 = -0.03716$, $\lambda_3 = -0.00750$, $\lambda_4 = -0.03000$
A4	Generalised Lambda Distribution	$\lambda_1 = -0.30656$, $\lambda_2 = -0.03536$, $\lambda_3 = -0.10000$, $\lambda_4 = -0.18000$
A5	Generalised Lambda Distribution	$\lambda_1 = -0.85162$, $\lambda_2 = -0.17429$, $\lambda_3 = -0.00100$, $\lambda_4 = -0.13000$
A6	Generalised Lambda Distribution	$\lambda_1 = -0.81183$, $\lambda_2 = -0.25217$, $\lambda_3 = -0.00010$, $\lambda_4 = -0.17000$

Note: Each process, x, is generated so that $\mathbb{E}[x] = 0$ and $\mathbb{V}ar[x] = 1$.

Additional Symmetry Test Sampling Distribution Plots

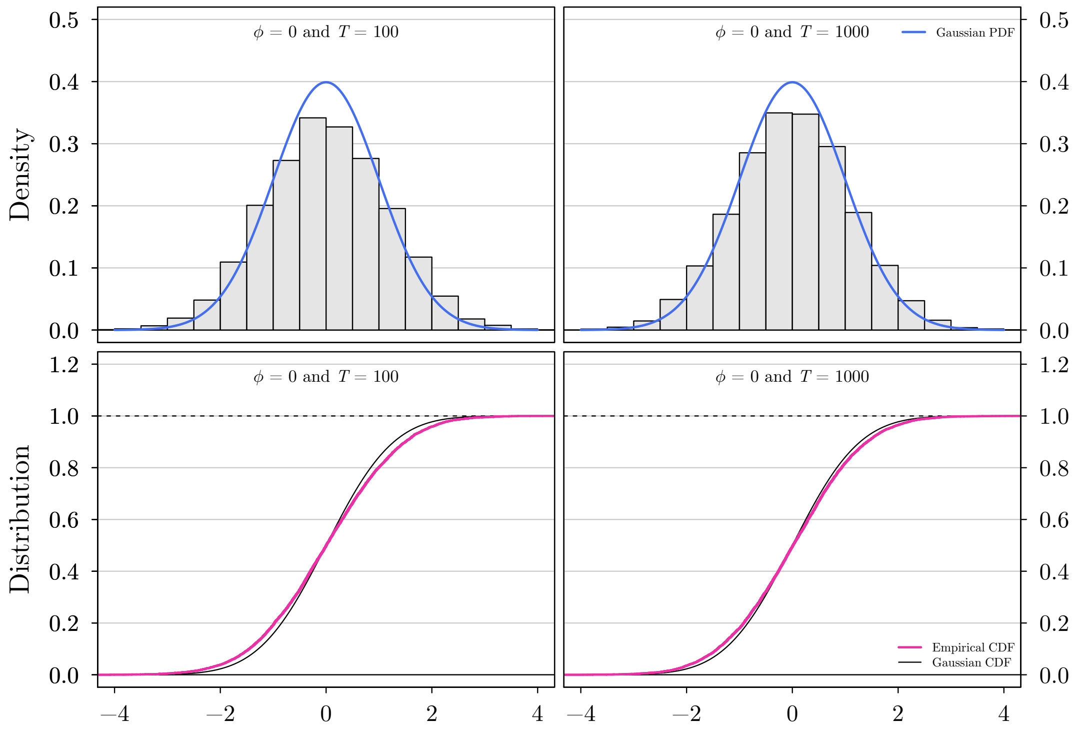

Figure A1: Symmetry Test Sampling Distribution – *iid* Process

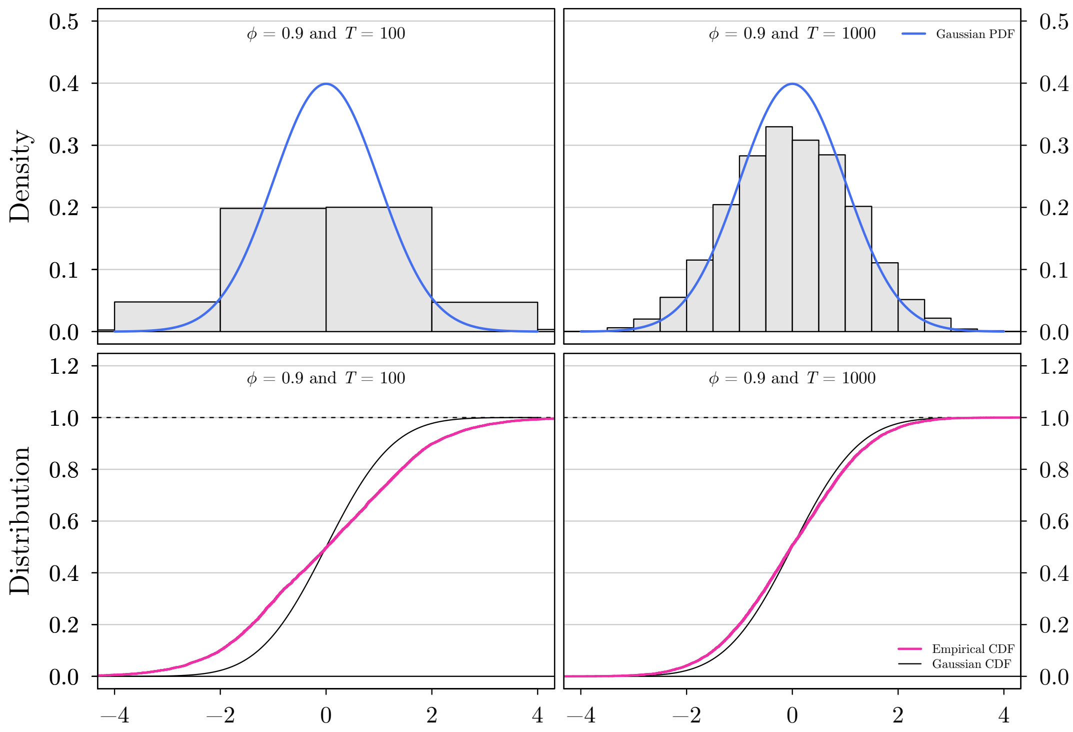

Figure A2: Symmetry Test Sampling Distribution – AR(1) Process: $\phi = 0.9$

Symmetry Test Comparisons

Table A2: Symmetry Test Empirical Size Comparison

	$T = 100$			$T = 200$			$T = 500$		
DGP	$\mathcal{S}$	BN	PV	$\mathcal{S}$	BN	PV	$\mathcal{S}$	BN	PV
AR(1) Process: $\phi = 0.0$									
S1	0.09	0.04	n.a.	0.08	0.05	n.a.	0.08	0.06	n.a.
S2	0.05	0.03	n.a.	0.05	0.04	n.a.	0.05	0.03	n.a.
S3	0.09	0.03	n.a.	0.08	0.04	n.a.	0.07	0.05	n.a.
S4	0.05	0.02	n.a.	0.05	0.04	0.07	0.04	0.04	0.05
S5	0.04	0.02	n.a.	0.04	0.04	0.05	0.04	0.03	0.07
S6	0.04	0.03	n.a.	0.03	0.02	0.06	0.03	0.03	0.07
AR(1) Process: $\phi = 0.5$									
S1	0.09	0.03	n.a.	0.07	0.04	n.a.	0.07	0.05	n.a.
S2	0.07	0.03	n.a.	0.06	0.04	n.a.	0.05	0.04	n.a.
S3	0.09	0.03	n.a.	0.07	0.04	n.a.	0.06	0.05	n.a.
S4	0.07	0.03	n.a.	0.05	0.04	0.07	0.05	0.05	0.05
S5	0.06	0.03	n.a.	0.05	0.04	0.05	0.05	0.04	0.05
S6	0.06	0.04	n.a.	0.05	0.02	0.06	0.04	0.03	0.07
AR(1) Process: $\phi = 0.8$									
S1	0.14	0.02	n.a.	0.12	0.04	n.a.	0.09	0.03	n.a.
S2	0.14	0.04	n.a.	0.10	0.04	n.a.	0.07	0.04	n.a.
S3	0.15	0.02	n.a.	0.11	0.04	n.a.	0.08	0.05	n.a.
S4	0.14	0.02	n.a.	0.10	0.03	n.a.	0.07	0.04	n.a.
S5	0.13	0.04	n.a.	0.10	0.03	n.a.	0.07	0.06	n.a.
S6	0.13	0.04	n.a.	0.09	0.04	n.a.	0.06	0.04	n.a.

Note: 'BN' is the two-sided skewness test $\hat{\pi}_3$ from Table 1 in Bai and Ng (2005). 'PV' refers is the $QS_k\,(A)$ test statistic from Table 2 in Psaradakis and Vávra (2015). 'n.a.' means that particular author did not consider that DGP in their Monte Carlo simulation experiment.

Table A3: Symmetry Test Empirical Power Comparison

	$T = 100$			$T = 200$			$T = 500$		
DGP	$\mathcal{S}$	BN	PV	$\mathcal{S}$	BN	PV	$\mathcal{S}$	BN	PV
				AR(1) Process: $\phi = 0.0$					
A1	0.94	0.74	n.a.	0.99	0.90	n.a.	1.00	0.98	n.a.
A2	0.98	0.85	n.a.	1.00	1.00	n.a.	1.00	1.00	n.a.
A3	0.85	0.67	n.a.	0.98	0.85	0.94	1.00	0.98	1.00
A4	0.38	0.23	n.a.	0.66	0.43	0.62	0.93	0.72	0.95
A5	0.84	0.56	n.a.	0.94	0.73	1.00	0.99	0.87	1.00
A6	0.80	0.53	n.a.	0.91	0.67	n.a.	0.98	0.83	n.a.
				AR(1) Process: $\phi = 0.5$					
A1	0.90	0.62	n.a.	0.98	0.87	n.a.	1.00	0.97	n.a.
A2	0.65	0.30	n.a.	0.89	0.70	n.a.	1.00	0.99	n.a.
A3	0.71	0.44	n.a.	0.93	0.76	0.71	1.00	0.96	0.96
A4	0.32	0.19	n.a.	0.56	0.37	0.40	0.88	0.69	0.76
A5	0.82	0.48	n.a.	0.93	0.67	0.98	0.98	0.86	1.00
A6	0.79	0.46	n.a.	0.90	0.64	n.a.	0.97	0.82	n.a.
				AR(1) Process: $\phi = 0.8$					
A1	0.53	0.01	n.a.	0.75	0.13	n.a.	0.97	0.73	n.a.
A2	0.22	0.06	n.a.	0.24	0.12	n.a.	0.36	0.32	n.a.
A3	0.36	0.04	n.a.	0.50	0.20	n.a.	0.83	0.64	n.a.
A4	0.25	0.06	n.a.	0.32	0.15	n.a.	0.56	0.45	n.a.
A5	0.57	0.02	n.a.	0.77	0.18	n.a.	0.95	0.72	n.a.
A6	0.58	0.04	n.a.	0.77	0.22	n.a.	0.93	0.72	n.a.

Note: 'BN' is the two-sided skewness test $\hat{\pi}_3$ from Table 1 in Bai and Ng (2005). 'PV' refers is the $QS_k(A)$ test statistic from Table 2 in Psaradakis and Vávra (2015). 'n.a.' means that particular author did not consider that DGP in their Monte Carlo simulation experiment.