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Abstract

Using the new Bureau of Economic Analysis (BEA) Integrated Macroeconomic Accounts as well as other BEA data, we construct productivity accounts for two key sectors of the US economy: the Corporate Nonfinancial Sector (Sector 1) and the Noncorporate Nonfinancial Sector (Sector 2). Calculating user costs of capital based on, alternatively, ex post and predicted asset price inflation rates, we provide alternative estimates for capital services and Total Factor Productivity growth for the two sectors. Rates of return on assets employed are also reported for both sectors. In addition, we compare rates of return on assets employed and TFP growth rates when the land and inventory components are withdrawn from the asset base. Finally, implications for labour and capital shares from using alternative income concepts are explored.

Key Words

User cost of capital, Total Factor Productivity, rate of return on assets, Integrated Macroeconomic Accounts, Bureau of Economic Analysis, ex post and ex ante asset inflation rates, US Nonfinancial Sector, Austrian model of production, balancing rates of return, inequality.

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1. Introduction

The US Bureau of Economic Analysis (BEA), in conjunction with the Bureau of Labor Statistics (BLS) and the Board of Governors of the Federal Reserve, have developed a new set of production accounts, the Integrated Macroeconomic Accounts, for two major private sectors of the US economy: the Corporate Nonfinancial Sector (which we will call Sector 1) and the Noncorporate Nonfinancial Sector (which we will call Sector 2). For both sectors we work out the rate of return on assets employed back to 1960 and compute estimates of Total Factor Productivity (TFP) growth. In addition to comparing results across the sectors, we are particularly interested in determining whether rates of return and TFP growth have declined in recent years compared to the long run trends.

Another contribution is to document what can happen to user costs when ex post asset inflation rates are used in the user cost formula. Dale Jorgenson and his coworkers have advocated the use of ex post inflation rates in a user cost formula and so we call the resulting user costs “Jorgensonian”. We show that for many assets, Jorgensonian user costs can be quite volatile and even negative at times which means that they cannot be used in many contexts. We advocate the use of predicted asset inflation rates in the user cost formula and we suggest a very simple moving average method for forming these predicted asset inflation rates, which we implement and compare with their Jorgensonian counterparts. We use Jorgensonian and predicted user costs to construct alternative measures of capital services and TFP growth for our two sectors of the US economy and, somewhat surprisingly, we find that there was little difference in the resulting trend measures of TFP growth, even though there are very large differences in the two sets of user costs.

An additional contribution is the examination of what happens to ex post rates of return on assets employed and on TFP growth as we withdraw assets from the asset base. This research has relevance for existing estimates of rates of return and TFP growth since many productivity studies exclude land and inventories from their asset base. We find that excluding these assets leads to exaggerated estimated rates of return on the remaining assets (as could be expected) but the effects on estimates of TFP growth are more variable. For our Sector 1, we found that excluding land and inventories had little effect on measured TFP growth but in Sector 2, the exclusion of land dramatically lowered measured TFP growth.

Finally, we use our data set to provide evidence on the debate regarding growing inequality due to a falling labour share in income. We find that moving from value added shares to (Hayekian) income shares provides stronger evidence of falling labour shares, indicative of growing inequality, for both our sectors.

Our accounting framework is laid out in the following section and the empirical results for the above measurement exercises follow in the subsequent sections.[2]

[2] The Appendix in Diewert and Fox (2016) explains in detail how we used the Integrated Macroeconomic Accounts to construct our data set for the two sectors of the US economy.

2. The Accounting Framework, User Costs and Rates of Return on Assets

Following Jorgenson and Griliches (1967), the Total Factor Productivity growth of a firm or industry of a sector is generally measured as an output index divided by an input index. The basic ingredients that go into an index number formula are two price vectors and two quantity vectors that list the output quantities and their prices (or the input quantities and their prices) produced or used for the production unit for the two observations being compared. Compiling prices and quantities for outputs and nondurable inputs for each period or observation is generally straightforward, but determining the flow price for a durable input is not straightforward. In order to accomplish the latter task, we will use a model of production that is due to the economist Hicks (1961) and the accountants Edwards and Bell (1961).[3] In each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock and inventory components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period). The model could be viewed as an Austrian model of production in honour of the Austrian economist Böhm-Bawerk (1891) who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.[4] The beauty of this model is that a complex intertemporal production model with many periods can be reduced to a sequence of single period models.

Using this one period framework, we can now explain how user costs arise. Consider a production unit which produces quantities q_O of a single output, uses q_I units of an intermediate input, uses q_L units of labour services during say period t and purchases q_K units of a capital stock at the beginning of the period. After using the services of the capital input during period t, the production unit will have q_K^u units of used (depreciated) capital on hand at the end of period t. We suppose that the production unit faces the positive prices P_O^t, P_I^t, P_L^t for its output and variable inputs during period t and it faces the beginning of period t price for units of the capital input equal to P_K^t and the price P_K^{t+1u} for (used) units of the depreciated capital good at the end of period t. Finally, we assume that the production unit has a one period financial opportunity cost of capital at the beginning of period t (i.e., a beginning of the period nominal interest rate) equal to r^t. We also assume that the period t production possibilities set for this production unit is the set S^t. Using all of these assumptions, the production unit's (competitive) one period profit maximization problem is the following constrained optimization problem:

$$(1)\quad \max_{q_O,q_I,q_L,q_K,q_K^*} \{P_O^t q_O - P_I^t q_I - P_L^t q_L - P_K^t q_K + (1+r^t)^{-1} P_K^{t+1u} q_K^* : (q_O,q_I,q_L,q_K,q_K^u) \in S^t\}.$$

Note that (1) assumes that all outputs and all variable inputs are paid for at the beginning of period t, as is the payment for the initial capital stock, which is an input. The depreciated

[3] This model can be traced back in part to Walras (1954; 267-269) and Böhm-Bawerk (1891; 342) and more explicitly to von Neumann (1945; 2).

[4] For more on this Austrian model of production and additional references to the literature, see Diewert (1977; 108-111) (1980; 473) (2010) (2014).

capital stock $q_K{}^u$ is an output that is "produced" at the end of period t and its end of period t market value, $P_K{}^{t+1u}q_K{}^u$, is discounted by $(1+r^t)$ to account for the opportunity cost of tying up financial capital in the asset over period t.

We make some additional assumptions at this point in order to further simplify the constrained optimization problem defined by (1). First we assume that the capital input depreciates at the constant geometric rate δ per period. The geometric model of depreciation has been advocated by Jorgenson (1989) and his coworkers and it is currently used by the BEA to construct US business sector capital stocks. The geometric model of depreciation implies that the depreciated quantity of end of period capital, $q_K{}^u$, is related to the corresponding beginning of the period capital stock, q_K, by the following equation:[5]

(2) $q_K{}^u = (1-\delta)q_K$

where δ is the geometric rate of depreciation and satisfies the inequalities $0 \leq \delta < 1$. Let $P_K{}^{t+1}$ be the end of period t price of a unit of the capital stock that has the same quality as the beginning of the period unit of the capital stock. Define the *constant quality asset inflation rate* over period t, i^t, by the following equation:

(3) $1+ i^t \equiv P_K{}^{t+1}/P_K{}^t$.

Thus i^t is the constant quality inflation rate for the capital stock component from the beginning of period t to the end of period t. We assume that the anticipated end of period t price for the used beginning of the period capital stock is equal to the end of period price for a constant quality unit of the capital stock, i.e., we assume that $P_K{}^{t+1u} = P_K{}^{t+1}$ and thus we have the following equation:

(4) $P_K{}^{t+1u} = (1+i^t)P_K{}^t$.

Our final additional assumption is that all revenues and variable input costs are received and paid for at the end of period t instead of the beginning of period t. With these changes, the producer's constrained optimization problem becomes:

(5) $\max_{q_O, q_I, q_L, q_K} \{(1+r^t)^{-1}[P_O{}^tq_O - P_I{}^tq_I - P_L{}^tq_L + (1+i^t)(1-\delta)P_K{}^tq_K] - P_K{}^tq_K : (q_O,q_I,q_L,q_K) \in S^{t*}\}$.

The terms involving q_K (the beginning of the period capital stock) in the objective function of (5) simplify to $-f_K{}^t \equiv -(1+r^t)^{-1}[1+r^t - (1+i^t)(1-\delta)]P_K{}^t$. Thus $f_K{}^t$ is the *discounted to the beginning of period t user cost of capital* using the geometric model of depreciation.[6] However, instead of discounting end of period cash flows to the beginning of period t, we

[5] The assumption of equation (2) allows us to replace the initial production possibilities set S^t with a new set S^{t*} which is the feasible set of $(q_O{}^t,q_I,q_L,q_K)$.

[6] This simple discrete time derivation of a user cost (as the net cost of purchasing the durable good at the beginning of the period and selling the depreciated good at an interest rate discounted price at the end of the accounting period) was developed by Diewert (1974; 504), (1980; 472-473), (1992; 194). Simplified user cost formulae (the relationship between the rental price of a durable input to its stock price) date back to Babbage (1835; 287) and to Walras (1954; 268-269). The original version of Walras in French was published in 1874. The early industrial engineer, Church (1901; 907-909) also developed a simplified user cost formula.

could anti-discount or appreciate beginning of the period cash flows to the end of period t.[7] This can be accomplished by multiplying the objective function in (5) by $(1+r^t)$. If we do this, we obtain the following one period profit maximization problem:

$$(6)\quad \max_{q_O,q_I,q_L,q_K} \{[P_O{}^t q_O - P_I{}^t q_I - P_L{}^t q_L - u_K{}^t q_K : (q_O,q_I,q_L,q_K) \in S^{t*}\}$$

where the *end of period user cost of capital* $u_K{}^t$ is defined as follows:

$$(7)\ u_K{}^t \equiv [1+r^t - (1+i^t)(1-\delta)]P_K{}^t = [r^t - i^t + (1+i^t)\delta]P_K{}^t.$$

This formula for the user cost of capital was obtained by Christensen and Jorgenson (1969; 302) for the geometric model of depreciation. It plays a fundamental role in our analysis.[8]

There are two versions of the user cost formula $u_K{}^t$ defined by (7) that we will use in this paper: (i) An *ex post version* that uses the actual beginning and end of period constant quality asset prices, $P_K{}^t$ and $P_K{}^{t+1}$, in order to define the asset inflation rate as $i^t \equiv (P_K{}^{t+1}/P_K{}^t) - 1$; and (ii) an *ex ante version* that uses the actual beginning of period t constant quality asset price, $P_K{}^t$, and an anticipated price for the asset at the end of period t, $P_K{}^{t+1*}$, in order to define an *anticipated asset inflation rate* as $i^{t*} \equiv (P_K{}^{t+1*}/P_K{}^t) - 1$.

Jorgenson (1995) (1996) and his coworkers[9] have endorsed the use of ex post user costs, arguing that producers can perfectly anticipate future asset prices, and so we refer to the user costs defined by (7) when ex post asset inflation rates are used in the formula as *Jorgensonian user costs*. On the other hand, Diewert (1980; 476) (2005a; 492-493) and Hill and Hill (2003) endorsed the ex ante version for most purposes, since these ex ante user costs will tend to be smoother than their ex post counterparts and they will generally be closer to a rental or leasing price for the asset.[10] We will use our sectoral data on the US corporate and noncorporate financial sector to compute capital services aggregates and the resulting rates of TFP growth using both Jorgensonian and smoothed user costs that use predicted asset inflation rates.

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[7] Assuming that all of the flow transactions within the accounting period are realized at the end of each period is consistent with traditional accounting treatments of assets at the beginning and end of the accounting period and the cash flows that occur during the period; see Peasnell (1981; 56). The idea of anti-discounting to the end of the period to form *end of period user costs* $u_K{}^t$ (as opposed to the usual discounted to the *beginning of period user costs* $f_K{}^t$) was explicitly suggested by Diewert (2005; 485). Anti-discounting is implicit in the derivation of the user cost of an asset using the geometric model of depreciation that was made by Christensen and Jorgenson (1969; 302).

[8] We have ignored tax complications in deriving (6). Any specific capital taxes (such as property taxes on real estate assets) should be added to the user cost formula for the relevant assets. In our empirical work, we were not able to obtain a breakdown of property taxes into land and structure components and so property tax rates are missing in our user costs that we construct in the following sections of this study. Business income taxes that fall on the gross return to the asset base can be absorbed into the cost of capital, r^t, so that r^t can be interpreted as the before income tax gross return to the asset base used by the production unit. For material on the construction of user costs for more complex systems of business income taxation, see Diewert (1992) and Jorgenson (1996).

[9] See in particular Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969).

[10] Of course, the problem with using ex ante user costs is that there are many methods that could be used to predict asset inflation rates and these different methods could generate very different user costs. For empirical evidence on this point, see Harper, Berndt and Wood (1989), Diewert (2005a) and Schreyer (2012).

We now discuss the issues surrounding the choice for the cost of capital, r^t, in the user cost formula. There are many methods for choosing r^t that have been suggested in the literature but the methods break down into two classes: those that choose exogenous estimates for r^t and those that choose r^t endogenously as the rate of return which will just make the value of inputs used during the period (including capital services) equal to the value of outputs produced during the accounting period. We will use endogenous estimates for the cost of capital in this study.[11]

In order to explain how the cost of capital is determined endogenously, we need to consider the case where the production unit uses N types of capital. Let P_{Kn}^t and P_{Kn}^{t+1} be the beginning and end of period t prices for a new asset of type n, let $0 \leq \delta_n < 1$ be the associated geometric depreciation rate, let $i_n^t \equiv (P_{Kn}^{t+1}/P_{Kn}^t) - 1$ be the associated period t ex post asset n inflation rate over period t and let r^t be the endogenously determined period t ex post rate of return on the asset base for the production unit. The *ex post end of period t user cost for asset n* is defined as:

$$(8)\ u_{Kn}^t \equiv [1+r^t - (1+i_n^t)(1-\delta_n)]P_{Kn}^t = [r^t - i_n^t + (1+i_n^t)\delta_n]P_{Kn}^t; \qquad n = 1,...,N.$$

The period t technology set for the production unit is now the set of feasible production vectors $(q_O,q_I,q_L,q_{K1},q_{K2},...,q_{KN})$ that belong to a period t production possibilities set S^{t*}. Let q_O^t,q_I^t,q_L^t denote the period t output produced, intermediate input used and labour used for the production unit and let $(q_{K1}^t,q_{K2}^t,...,q_{KN}^t)$ denote the vector of beginning of period t capital stocks used by the production unit during the period. The *ex post rate of return on the period t asset base*, r^t, is defined as the solution to the following (linear) equation which sets the value of period t outputs equal to the value of period t inputs where capital inputs are valued at their ex post user costs:

$$(9)\ 0 = P_O^tq_O^t - P_I^tq_I^t - P_L^tq_L^t - \Sigma_{n=1}^N u_{Kn}^tq_{Kn}^t$$
$$= P_O^tq_O^t - P_I^tq_I^t - P_L^tq_L^t - \Sigma_{n=1}^N [1+r^t - (1+i_n^t)(1-\delta_n)]P_{Kn}^tq_{Kn}^t.$$

The ex post cost of capital method for determining the opportunity cost of capital that is based on solving equation (9) for r^t is due to Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969). This method has been used frequently in the regulatory context. The method can be applied to both a single enterprise as well as to the economy as a whole. National statistical agencies that have programs that measure the productivity of market sector industries generally use this method.[12] From a national income accounting perspective, this method has the great advantage for statistical agencies that it preserves the structure of the *System of National Accounts 1993* (SNA 1993); i.e., the resulting user cost values just sum to the Gross Operating Surplus that was already in SNA 1993. Thus this method can be viewed as a straightforward elaboration of the present system of accounts which does not change its

[11] The problem with the exogenous method is that it is difficult to determine exactly the appropriate external cost of financial capital. In particular, it is difficult to estimate the risk premium that is associated with investing in a production unit that generates variable ex post rates of return on its asset base over time. Nevertheless, the exogenous method is probably the preferred method from a theoretical point of view. These issues are discussed more fully in Schreyer, Diewert and Harrison (2005) and Schreyer (2009) (2012).

[12] The Bureau of Labor Statistics in the U.S. was the first to introduce an official program to measure Multifactor Productivity or Total Factor Productivity in 1983; see Dean and Harper (2001). Other countries with TFP programs now include Canada, Australia, the UK and New Zealand.

basic structure; it only provides a decomposition of Gross Operating Surplus or Cash Flow into more basic components.[13]

In the following sections of this study, we will calculate these ex post rates of return on assets for our Sectors 1 and 2 and also use the Jorgensonian user costs defined by (8) when we calculate TFP growth rates for our two sectors.

The major disadvantage of using Jorgensonian user costs is their volatility and their tendency to become negative for at least some periods when asset inflation rates for particular assets (such as land) are high. These volatile and sometimes negative user costs do not approximate corresponding asset rental prices (when they exist), which do not exhibit the same volatility. Moreover, if these bouncing user costs are used in production function studies where the underlying technology is estimated using derived supply and demand functions, the resulting estimated parameters are unlikely to be reliable. Finally, if statistical agencies report these volatile user costs in their system of productivity accounts, users are likely to be skeptical of these estimates. Thus there is a need to produce smoother user costs for a variety of reasons.

Our approach to producing smoother user costs will be to use *predicted asset inflation rates*, say i_n^{t*}, in the user cost formula instead of the actual ex post asset inflation rates, i_n^t. The method for calculating these predicted asset inflation rates will be explained more fully in subsequent sections but the predicted rates are basically simple long run geometric averages of past ex post inflation rates. Once the smoothed or ex ante asset inflation rate for asset n in period t, i_n^{t*}, has been defined for n = 1,...,N, the *ex ante or smoothed end of period t user cost for asset n in period t*, u_{Kn}^{t*}, is defined as:

$$(10)\ u_{Kn}^{t*} \equiv [1+r^{t*} - (1+i_n^{t*})(1-\delta_n)]P_{Kn}^t = [r^{t*} - i_n^{t*} + (1+i_n^{t*})\delta_n]P_{Kn}^t; \qquad n = 1,...,N$$

where *the smoothed balancing rate of return for period t*, r^{t*}, is defined as the solution to the following equation (which is linear in r^{t*}):

$$\begin{aligned}(11)\ 0 &= P_O^t q_O^t - P_I^t q_I^t - P_L^t q_L^t - \Sigma_{n=1}^N u_{Kn}^{t*} q_{Kn}^t \\ &= P_O^t q_O^t - P_I^t q_I^t - P_L^t q_L^t - \Sigma_{n=1}^N [1+r^{t*} - (1+i_n^{t*})(1-\delta_n)]P_{Kn}^t q_{Kn}^t.\end{aligned}$$

The smoothed rate of return r^{t*} can be viewed as a planned rate of return on assets that is expected on the beginning of the period value of the capital stock used by the production unit, provided expected asset inflation rates, the i_n^{t*}, are realized.[14] The smoothed user costs defined by (10) will also provide a decomposition of Gross Operating Surplus into meaningful components. As we shall see, the ex ante user costs are considerably smoother

[13] This method for decomposing Gross Operating Surplus into explanatory factors (that are useful when measuring TFP growth), was endorsed in the *System of National Accounts, 2008*; see Schreyer, Diewert and Harrison (2005) for a discussion of the issues.

[14] Period t predicted prices for output, intermediate input and labour, say P_O^{t*}, P_I^{t*} and P_L^{t*}, should be used in equation (11) in order to calculate the period t predicted rate of return, r^{t*}, instead of the actual ex post prices for output, intermediate input and labour, P_O^t, P_I^t and P_L^t. However, it is the usual convention in production theory to assume that actual ex post unit value prices for variable outputs and inputs are equal to their predicted counterparts.

than their Jorgensonian counterparts.[15] Note that both of our user cost models use endogenous rates of return. One of the main purposes of this study is to determine whether the choice of user cost formula affects our estimates of TFP growth.

We conclude this section by discussing some of the problems associated with the valuation of investments made by the production unit during period t and with the sales of assets that might have occurred during period t. We discuss these issues in the context of equation (9) but a similar discussion holds for the accounting framework defined by equation (11).

Consider the second equation in (9). Upon noting that $(1+i_n{}^t)P_{Kn}{}^t$ is equal to the end of period t price of a new unit of the nth capital stock component, $P_{Kn}{}^{t+1}$, (9) can be rewritten as follows:

$$(12)\ 0 = P_O{}^t q_O{}^t - P_I{}^t q_I{}^t - P_L{}^t q_L{}^t - \Sigma_{n=1}{}^N (1+r^t)P_{Kn}{}^t q_{Kn}{}^t + \Sigma_{n=1}{}^N P_{Kn}{}^{t+1}(1-\delta_n)q_{Kn}{}^t.$$

Recall our Austrian one period model of production where the beginning of period t capital stocks are regarded as inputs and the end of period capital stocks are regarded as outputs. The initial value of the capital stock, $\Sigma_{n=1}{}^N P_{Kn}{}^t q_{Kn}{}^t$, is appreciated to end of period values by multiplying this initial capital stock value by $(1+r^t)$ so that the anti-discounted price for input asset n is $(1+r^t)P_{Kn}{}^t$. Looking at (12), we see that the term $-\Sigma_{n=1}{}^N (1+r^t)P_{Kn}{}^t q_{Kn}{}^t$ is (minus) the cost of the beginning of period t capital stock at end of period prices. The other prices on the right hand side of (12) are also expressed in end of period t prices. The first three terms on the right hand side of (12) correspond to the value of outputs produced during period t, less the value of intermediate and labour inputs used during the period. The final set of terms, $\Sigma_{n=1}{}^N P_{Kn}{}^{t+1}(1-\delta_n)q_{Kn}{}^t$, is the end of period t value of the depreciated beginning of the period capital stock. Thus $(1-\delta_n)q_{Kn}{}^t$ is the depreciated quantity of the beginning of the period capital stock for asset n that is left over at the end of period t. But this quantity is not the entire end of period t capital stock for asset n: during period t, there may have been investments in asset n. Suppose $q_{GIn}{}^t$ is the *gross investment* in asset n during period t (and the average price that the statistical agency assigns to this investment is $P_{GIn}{}^t$) for n = 1,...,N. Thus the actual end of period t quantity of asset n that the production unit has at its disposal is $q_{Kn}{}^{t+1} \equiv q_{GIn}{}^t + (1-\delta_n)q_{Kn}{}^t$ and according to our accounting conventions, it should be valued at the end of period t asset price $P_{Kn}{}^{t+1}$. Hence the terms $\Sigma_{n=1}{}^N P_{Kn}{}^{t+1}q_{GIn}{}^t$ seem to be missing from the right hand side of (12). There is an explanation for this apparent puzzle.

Suppose asset n is a reproducible capital stock; i.e., an asset which is produced internally by the production unit or purchased from another producer. In this case, the value of the gross investment in asset n during period t, $P_{GIn}{}^t q_{GIn}{}^t$, will be part of the period t value of output for

[15] There is a problem with interpreting these smoothed user costs as rental prices that might be anticipated at the beginning of the accounting period. When there is a severe recession in the economy in say period t, both r^t defined by solving (9) and r^{t*} defined by solving (11) will become unusually low (or even negative) and it is unlikely that the resulting low (or negative) user costs defined by (10) could be anticipated in practice. This limitation of our analysis should be kept in mind, particularly when looking at the user costs for 2008. This suggests that exogenous estimates for the cost of capital may be a more appropriate strategy for forming user costs that more closely approximate rental prices. If an exogenous r^{t*} is used, then equation (11) will not hold in general and it will be necessary to include pure profits (or losses) as a balancing item in the SNA. However, we do not pursue this line of inquiry in the present study.

the production unit; i.e., it should be included as part of $P_O^t q_O^t$. This resolves the puzzle for reproducible capital stock components.[16]

Now suppose asset n is an inventory stock. External purchases of the inventory stock will be part of intermediate input purchases, $P_I^t q_I^t$. Sales of the inventory item will be reflected in the value of gross output, $P_O^t q_O^t$. But at the end of period t, there will be a net change in inventory stocks equal to $q_{Kn}^{t+1} - q_{Kn}^t$. Hence it appears that the term $P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ is missing on the right hand side of (12). Note that since asset n is an inventory item, we assume $\delta_n \equiv 0$ and so the term $P_{Kn}^{t+1}(1-\delta_n)q_{Kn}^t = P_{Kn}^{t+1}q_{Kn}^t$ is present on the right hand side of (12) and adding $P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ to this term gives us the end of period value of inventory stocks, $P_{Kn}^{t+1}q_{Kn}^{t+1}$, which is the right answer from the perspective of the Austrian approach to production theory. But statistical agencies treat inventory change over a period as part of sectoral output and so the missing term $P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ should be included as a part of the value of gross output, $P_O^t q_O^t$.[17] This resolves the puzzle for inventory components of the capital stock.

Suppose asset n is a type of land asset. As was the case for inventory items, we assume that the land depreciation rate is $\delta_n = 0$ and again, we find that the term $P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ is missing on the right hand side of (12). This term now represents the value of net purchases of land of type n over period t, $q_{Kn}^{t+1} - q_{Kn}^t$, valued at end of period t price for this type of land, P_{Kn}^{t+1}. Statistical agencies typically do not treat land as an output or an intermediate input so in this case, the net quantity of land purchases over period t, valued at end of period land prices, will not appear as part of the gross output (if land was sold during period t) or intermediate input of the sector (if land was purchased during period t). Thus we need to treat these net purchases as an input cost item, so $-P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ should be added to the right hand side of (12), but this net cost value is offset by the increase in the value of land holdings at the end of the period, so $+P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ should be added to the right hand side of (12). These two entries cancel and so this resolves the puzzle for the land components of the capital stock.[18]

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[16] However, to make the accounting precisely consistent with the Austrian model of production, we require that the price used to value gross investments in asset n during period, P_{GIn}^t, be equal to the end of period t imputed value for a unit of the nth capital stock. Setting $P_{Kn}^{t+1} = P_{GIn}^t$ will ensure consistency. In our empirical work, we used the BEA end of period price for reproducible units of the capital stock which may be slightly different from the corresponding investment price for the asset.

[17] The BEA in particular *does* include the value of inventory change as part of the gross output of an industry. However, they may not value the change in inventories at end of period prices of the inventory item and so again there may be a slight inconsistency in our empirical work due to this pricing difference. For a more complete treatment of the accounting problems associated with the treatment of inventories in the Austrian model of production, see Diewert (2005b).

[18] Suppose some land is purchased during period t at the price P_{Kn}^{t*} where this purchase price is not equal to the end of period price of land, P_{Kn}^{t+1}. The quantity of new land purchased will be equal to $q_{Kn}^{t+1} - q_{Kn}^t$. Then the term $-P_{Kn}^{t*}(q_{Kn}^{t+1} - q_{Kn}^t)$ should be added to the right hand side of (12) as a purchase of a primary input (a cost item) and at the same time, we should add the term $P_{Kn}^{t+1}(q_{Kn}^{t+1} - q_{Kn}^t)$ to the right hand side of (12) to value this land purchase at the end of period t price of this type of land (a revenue item). Thus in principle, we should add the term $(P_{Kn}^{t+1} - P_{Kn}^{t*})(q_{Kn}^{t+1} - q_{Kn}^t)$ to the right hand side of (12). If some land is sold during the period at the price P_{Kn}^{t*}, then $q_{Kn}^{t+1} - q_{Kn}^t$ is negative and is equal to minus the quantity sold. In this case, we should still add the term $(P_{Kn}^{t+1} - P_{Kn}^{t*})(q_{Kn}^{t+1} - q_{Kn}^t)$ to the right hand side of (12) to make the accounting consistent with our Austrian model of production. In our empirical work, we did not make these adjustments to the accounting identity given by (12); we simply assumed that P_{Kn}^{t*} is equal to our end of period price for the asset, P_{Kn}^{t+1}.

Real monetary balances are not regarded as productive inputs by national income accountants. However, we treat real monetary balances as being necessary for production.[19] Our accounting treatment of real balances is entirely analogous to our treatment of land and, as was the case with land, the accounting decomposition given by (9) or (12) is consistent with our Austrian theory of production.

Equations (9) and (12) provided an accounting treatment of production using ex post asset prices. As mentioned above, it is possible to build a similar accounting treatment of production using ex ante asset prices; i.e., instead of using equation (9) as our starting point for our accounting decomposition, we could have used equation (11). The consistency of equation (11) with the Austrian view of production is similar to our analysis of the consistency of equations (9) and (12) with the Austrian approach to production theory.

We conclude this section with an important observation. Although we do not think that the Jorgensonian ex post user costs are useful in all contexts, we do think that *they are the right user costs to use in the context of finding the ex post rate of return on assets for a production unit*. Ex post rates of return are extremely important indicators of economic efficiency (along with TFP growth rates) and it is important to measure these rates of return accurately to guide the allocation of resources between sectors.[20]

Before we use the data that are described in the Appendix to construct ex post rates of return on assets and TFP growth rates, in the following section we describe the use of our data base to construct estimates for real wages and labour productivity.

3. Real Wages and Labour Productivity Growth in Sectors 1 and 2

In this section, we draw on our data base in order to calculate real wages and labour productivity for the two sectors.[21] We start with the data for Sector 1, the Nonfinancial Corporate Sector of the US economy.

Value Added of Sector 1 in year t, $V_{VA1}{}^{t}$ (in billions of dollars), and the corresponding year t price index, $P_{VA1}{}^{t}$ are listed in Table 1. Define the year t real value added of Sector 1 as $Q_{VA1}{}^{t} \equiv V_{VA1}{}^{t}/P_{VA1}{}^{t}$ for t = 1960,...,2014. The price and quantity of employee labour in Sector 1, are $P_{L1}{}^{t}$ and $Q_{L1}{}^{t}$ and define the value of labour input in Sector 1 for year t as $V_{L1}{}^{t} \equiv P_{L1}{}^{t}Q_{L1}{}^{t}$. The labour series $V_{L1}{}^{t}$ and $P_{L1}{}^{t}$ are also listed in Table 1. The value of capital services in Sector 1 for year t, $V_{KS1}{}^{t}$, can be defined residually by subtracting the value of labour input from value added; i.e., $V_{KS1}{}^{t} \equiv V_{VA1}{}^{t} - V_{L1}{}^{t}$. The shares of labour and capital services in value added are defined as $s_{L1}{}^{t} \equiv V_{L1}{}^{t}/V_{VA1}{}^{t}$ and $s_{KS1}{}^{t} \equiv V_{KS1}{}^{t}/V_{VA1}{}^{t}$. These Sector 1 value added shares along with the value of capital services are also listed in Table 1.

[19] This is consistent with the cash-in-advance, or vending machine model of the demand for money consider by Fischer (1974). For a more extensive discussion of the issues surrounding money in the production function, see Diewert and Fox (2015).
[20] See Harberger (1998) on the importance of the rate of return on assets.
[21] This data base is described in more detail in the Appendix of Diewert and Fox (2016).

A beginning of year t price index for personal consumption expenditures, P_C^t, for t = 1960-2015, is converted to a centered consumer price index for year t, P_C^{t*}, by averaging P_C^t and P_C^{t+1}; i.e., define $P_C^{t*} \equiv (1/2)(P_C^t + P_C^{t+1})$ for t = 1960,...,2014.[22] This series, along with the wage rate index P_{L1}^t, was used to define the *Sector 1 real wage for year t*, defined as follows:

(13) $RW_1^t \equiv P_{L1}^t/P_C^{t*}$; t = 1960,...,2014.

Finally, *Sector 1 Labour Productivity in year t* (relative to the level in 1960), $ProdL_1^t$, is defined as follows (and is listed in Table 1):

(14) $ProdL_1^t \equiv [Q_{VA1}^t/Q_{L1}^t]/[Q_{VA1}^{1960}/Q_{L1}^{1960}]$; t = 1960,...,2014.

The price of (value added) output in Sector 1 grew 4.56 fold over the sample period while employee wages grew 13.95 fold. The geometric rates of growth were 3.61% per year for output and 5.00% per year for wages. Real wages grew 2.25 fold over the sample period while labour productivity grew 3.41 fold (the corresponding geometric rates of growth were 1.51% and 2.30% per year). The sample average labour and capital services shares were 68.6% and 31.4% respectively. The upward trend in the capital services share is noticeable in Figure 1 which plots the series s_{L1}^t, s_{KS1}^t, P_{VA1}^t, RW_1^t and $ProdL_1^t$. Note that the capital services share finishes up at 36.7%, well above its long term average of 31.4%. It can be seen that real wages have grown very slowly since 2007. Note also that real wage growth was fairly similar to labour productivity growth until 1982 and then labour productivity grew substantially faster than real wages. Finally, it can be seen that labour productivity in Sector 1 is still growing fairly steadily since 2006 at the geometric average rate of 1.20% per year but this rate is lower than the historical average rate of 1.51% per year.

We turn our attention to Sector 2, the Nonfinancial Noncorporate Sector of the US economy. Value Added of Sector 2 in year t, V_{VA2}^t (in billions of dollars) and the corresponding year t price index, P_{VA2}^t are listed in Table 2. Define the year t real value added of Sector 2 as $Q_{VA2}^t \equiv V_{VA2}^t/P_{VA2}^t$ for t = 1960,...,2014. The value and price of labour in Sector 2, V_{L2}^t and P_{L2}^t are listed in Table 2. The value of capital services in Sector 2 for year t, V_{KS2}^t, can be defined residually by subtracting the value of labour input from value added; i.e., $V_{KS2}^t \equiv V_{VA2}^t - V_{L2}^t$. The shares of labour and capital services in value added for Sector 2 are defined as $s_{L2}^t \equiv V_{L2}^t/V_{VA2}^t$ and $s_{KS2}^t \equiv V_{KS2}^t/V_{VA2}^t$. These Sector 2 value added shares along with the value of capital services are also listed in Table 2.

Again, we use the consumption price series P_C^{t*} along with the Sector 2 wage rate index P_{L2}^t to define the *Sector 2 real wage for year t*, $RW_2^t \equiv P_{L2}^t/P_C^{t*}$ for t = 1960,...,2014. This series also appears in Table 2.

Finally, *Sector 2 Labour Productivity of Sector 1 in year t* (relative to the level in 1960), $ProdL_2^t$, is defined as follows (and listed in Table 2):

(15) $ProdL_2^t \equiv [Q_{VA2}^t/Q_{L2}^t]/[Q_{VA2}^{1960}/Q_{L2}^{1960}]$; t = 1960,...,2014.

[22] This series was normalized to equal 1 in 1960. Note that the Sector 1 wage rate series P_{L1}^t is also normalized to equal 1 in 1960.

Table 1: Sector 1 Value Added V_{VA1}^t, Value of Labour Input V_{L1}^t, Value of Capital Services V_{KS1}^t, Value Added Shares of Labour and Capital Services, s_{L1}^t and s_{KS1}^t. Price of Labour P_{L1}^t, Real Wage RW_1^t and Labour Productivity $ProdL_1^t$ for Year t

Year	V_{VA1}^t	V_{L1}^t	V_{KS1}^t	s_{L1}^t	s_{KS1}^t	P_{VA1}^t	P_{L1}^t	RW_1^t	$ProdL_1^t$
1960	255.9	180.4	75.5	0.7050	0.2950	1.0000	1.0000	1.0000	1.0000
1961	262.8	184.5	78.3	0.7021	0.2979	1.0030	1.0290	1.0181	1.0301
1962	286.9	199.3	87.6	0.6947	0.3053	1.0095	1.0725	1.0505	1.0781
1963	306.1	210.1	96.0	0.6864	0.3136	1.0145	1.1094	1.0725	1.1231
1964	330.6	225.7	104.9	0.6827	0.3173	1.0239	1.1640	1.1104	1.1739
1965	364.7	245.4	119.3	0.6729	0.3271	1.0419	1.2085	1.1364	1.2152
1966	403.1	272.9	130.2	0.6770	0.3230	1.0723	1.2792	1.1753	1.2422
1967	423.9	291.1	132.8	0.6867	0.3133	1.0962	1.3501	1.2058	1.2643
1968	465.4	320.9	144.5	0.6895	0.3105	1.1302	1.4519	1.2537	1.3134
1969	504.4	356.1	148.3	0.7060	0.2940	1.1779	1.5545	1.2842	1.3178
1970	518.6	374.5	144.1	0.7221	0.2779	1.2216	1.6690	1.3173	1.3337
1971	558.5	396.2	162.3	0.7094	0.2906	1.2657	1.7739	1.3441	1.3927
1972	622.2	439.9	182.3	0.7070	0.2930	1.3108	1.8849	1.3791	1.4339
1973	698.7	495.1	203.6	0.7086	0.2914	1.3876	2.0157	1.4004	1.4452
1974	755.7	542.9	212.8	0.7184	0.2816	1.5239	2.2140	1.4058	1.4256
1975	818.1	569.0	249.1	0.6955	0.3045	1.6735	2.4284	1.4139	1.4708
1976	928.1	640.0	288.1	0.6896	0.3104	1.7549	2.6233	1.4415	1.5282
1977	1052.9	723.3	329.6	0.6870	0.3130	1.8543	2.8320	1.4697	1.5673
1978	1201.4	829.5	371.9	0.6904	0.3096	1.9868	3.0722	1.4886	1.5788
1979	1341.6	942.4	399.2	0.7024	0.2976	2.1497	3.3668	1.5002	1.5718
1980	1452.7	1030.7	422.0	0.7095	0.2905	2.3508	3.7310	1.5079	1.5770
1981	1641.7	1139.8	501.9	0.6943	0.3057	2.5530	4.0746	1.5100	1.6206
1982	1701.4	1183.3	518.1	0.6955	0.3045	2.7049	4.3920	1.5317	1.6459
1983	1817.5	1250.1	567.4	0.6878	0.3122	2.7546	4.5656	1.5253	1.6988
1984	2040.5	1388.2	652.3	0.6803	0.3197	2.8397	4.7838	1.5419	1.7456
1985	2172.9	1490.1	682.8	0.6858	0.3142	2.8900	5.0489	1.5719	1.7959
1986	2260.7	1578.2	682.5	0.6981	0.3019	2.9303	5.3247	1.6154	1.8349
1987	2425.7	1685.5	740.2	0.6949	0.3051	2.9859	5.5326	1.6337	1.8799
1988	2640.7	1825.3	815.4	0.6912	0.3088	3.0624	5.8270	1.6549	1.9406
1989	2772.6	1934.8	837.8	0.6978	0.3022	3.1554	6.0175	1.6431	1.9265
1990	2897.7	2037.5	860.2	0.7031	0.2969	3.2507	6.3362	1.6568	1.9542
1991	2946.1	2071.1	875.0	0.7030	0.2970	3.3222	6.6568	1.6788	2.0093
1992	3074.6	2188.7	885.9	0.7119	0.2881	3.3645	6.9976	1.7209	2.0597
1993	3216.0	2271.0	945.0	0.7062	0.2938	3.4346	7.1064	1.6915	2.0656
1994	3465.8	2398.7	1067.1	0.6921	0.3079	3.4868	7.2530	1.6773	2.1188
1995	3682.7	2524.6	1158.1	0.6855	0.3145	3.5344	7.3930	1.6759	2.1510
1996	3924.4	2667.7	1256.7	0.6798	0.3202	3.5578	7.6890	1.7111	2.2413
1997	4219.5	2862.6	1356.9	0.6784	0.3216	3.5860	7.9603	1.7545	2.3067
1998	4470.8	3093.8	1377.0	0.6920	0.3080	3.5955	8.4875	1.8610	2.4048
1999	4745.3	3310.0	1435.3	0.6975	0.3025	3.6193	8.8505	1.9151	2.4714
2000	5063.1	3597.3	1465.8	0.7105	0.2895	3.6610	9.4524	2.0011	2.5618
2001	5026.2	3584.6	1441.6	0.7132	0.2868	3.7132	9.8280	2.0421	2.6163
2002	5066.0	3542.0	1524.0	0.6992	0.3008	3.7109	10.0373	2.0525	2.7272
2003	5228.7	3595.7	1633.0	0.6877	0.3123	3.7486	10.4012	2.0878	2.8444
2004	5577.0	3762.8	1814.2	0.6747	0.3253	3.8262	10.8273	2.1235	2.9567
2005	5958.9	3930.3	2028.6	0.6596	0.3404	3.9577	11.2147	2.1357	3.0287
2006	6377.9	4129.3	2248.6	0.6474	0.3526	4.0789	11.6165	2.1598	3.1010
2007	6571.4	4305.3	2266.1	0.6552	0.3448	4.1610	12.0778	2.1894	3.1233
2008	6624.1	4358.0	2266.1	0.6579	0.3421	4.2492	12.4226	2.1993	3.1326
2009	6253.9	4088.4	2165.5	0.6537	0.3463	4.3182	12.6545	2.2109	3.1601
2010	6605.7	4158.7	2447.0	0.6296	0.3704	4.3216	12.8807	2.2228	3.3375
2011	6921.7	4363.4	2558.3	0.6304	0.3696	4.4176	13.1669	2.2282	3.3331
2012	7321.5	4593.3	2728.2	0.6274	0.3726	4.4916	13.4816	2.2322	3.3727
2013	7591.9	4747.4	2844.5	0.6253	0.3747	4.5205	13.6248	2.2228	3.3978
2014	7895.8	4995.8	2900.0	0.6327	0.3673	4.5568	13.9548	2.2503	3.4121

Note: All values are in billions of dollars.

Table 2: Sector 2 Value Added V_{VA2}^t, Value of Labour Input V_{L2}^t, Value of Capital Services V_{KS2}^t, Value Added Shares of Labour and Capital Services, s_{L2}^t and s_{KS2}^t. Price of Labour P_{L2}^t, Real Wage RW_2^t and Labour Productivity $ProdL_2^t$ for Year t

Year	V_{VA2}^t	V_{L2}^t	V_{KS2}^t	s_{L2}^t	s_{KS2}^t	P_{VA2}^t	P_{L2}^t	RW_2^t	$ProdL_2^t$
1960	107.4	76.6	30.8	0.7135	0.2865	1.0000	1.0000	1.0000	1.0000
1961	110.2	76.8	33.4	0.6967	0.3033	1.0161	1.0297	1.0187	1.0378
1962	114.2	78.4	35.8	0.6864	0.3136	1.0306	1.0720	1.0500	1.0811
1963	117.1	79.2	37.9	0.6767	0.3233	1.0424	1.1087	1.0719	1.1216
1964	123.0	82.9	40.1	0.6737	0.3263	1.0599	1.1635	1.1100	1.1626
1965	130.0	84.7	45.3	0.6512	0.3488	1.0810	1.2053	1.1334	1.2216
1966	138.5	87.5	51.0	0.6318	0.3682	1.1142	1.2675	1.1646	1.2848
1967	142.1	89.4	52.7	0.6291	0.3709	1.1494	1.3281	1.1862	1.3104
1968	149.8	93.5	56.3	0.6241	0.3759	1.2014	1.4232	1.2289	1.3543
1969	157.8	99.1	58.7	0.6279	0.3721	1.2540	1.5223	1.2577	1.3795
1970	163.4	102.8	60.6	0.6294	0.3706	1.3032	1.6246	1.2823	1.4131
1971	173.1	106.8	66.3	0.6172	0.3828	1.3646	1.7248	1.3069	1.4613
1972	191.2	113.2	78.0	0.5919	0.4081	1.4302	1.8305	1.3393	1.5429
1973	223.5	124.1	99.4	0.5553	0.4447	1.4928	1.9487	1.3539	1.6773
1974	235.2	135.6	99.6	0.5763	0.4237	1.6334	2.1193	1.3456	1.6064
1975	252.0	144.1	107.9	0.5718	0.4282	1.8018	2.3053	1.3423	1.5966
1976	275.6	155.4	120.2	0.5638	0.4362	1.9586	2.4811	1.3634	1.6031
1977	300.7	170.1	130.6	0.5655	0.4345	2.1231	2.6649	1.3830	1.5835
1978	340.7	189.5	151.2	0.5562	0.4438	2.2748	2.8772	1.3941	1.6224
1979	380.3	211.6	168.7	0.5565	0.4435	2.5432	3.1382	1.3983	1.5821
1980	399.8	233.2	166.6	0.5832	0.4168	2.7131	3.4595	1.3982	1.5599
1981	435.5	253.4	182.1	0.5819	0.4181	2.9761	3.7703	1.3972	1.5534
1982	454.5	272.6	181.9	0.5998	0.4002	3.1681	4.0618	1.4166	1.5252
1983	480.7	290.0	190.7	0.6032	0.3968	3.4070	4.2399	1.4165	1.4720
1984	556.3	314.0	242.3	0.5644	0.4356	3.4506	4.4382	1.4305	1.6259
1985	600.4	330.0	270.4	0.5497	0.4503	3.6182	4.6619	1.4514	1.6725
1986	636.3	346.1	290.2	0.5439	0.4561	3.6821	4.8851	1.4821	1.7404
1987	667.5	365.3	302.2	0.5472	0.4528	3.8627	5.0870	1.5021	1.7171
1988	727.1	390.9	336.2	0.5376	0.4624	4.0499	5.3541	1.5206	1.7546
1989	774.1	413.1	361.0	0.5337	0.4663	4.2668	5.5205	1.5074	1.7298
1990	807.5	437.7	369.8	0.5420	0.4580	4.4449	5.7961	1.5155	1.7165
1991	815.0	458.3	356.7	0.5623	0.4377	4.6295	6.0487	1.5254	1.6579
1992	869.8	471.9	397.9	0.5425	0.4575	4.7022	6.3684	1.5661	1.7813
1993	903.5	501.4	402.1	0.5550	0.4450	4.8129	6.4767	1.5417	1.7301
1994	951.2	522.6	428.6	0.5494	0.4506	4.8685	6.6155	1.5299	1.7648
1995	992.0	541.2	450.8	0.5456	0.4544	5.0990	6.7770	1.5363	1.7381
1996	1069.5	566.7	502.8	0.5299	0.4701	5.3611	7.0498	1.5688	1.7707
1997	1136.9	601.8	535.1	0.5294	0.4706	5.5402	7.3528	1.6206	1.7889
1998	1229.4	637.3	592.1	0.5184	0.4816	5.6619	7.7929	1.7087	1.8943
1999	1312.3	662.7	649.6	0.5050	0.4950	5.7664	8.1347	1.7602	1.9933
2000	1420.7	712.9	707.8	0.5018	0.4982	6.0680	8.6602	1.8334	2.0293
2001	1637.1	830.6	806.5	0.5073	0.4927	6.3365	8.9251	1.8545	1.9809
2002	1707.0	861.7	845.3	0.5048	0.4952	6.3650	9.0834	1.8575	2.0170
2003	1800.5	934.1	866.4	0.5188	0.4812	6.4184	9.3894	1.8847	2.0118
2004	1953.7	1023.3	930.4	0.5238	0.4762	6.5464	9.7942	1.9209	2.0381
2005	2088.6	1102.8	985.8	0.5280	0.4720	6.6842	10.1199	1.9272	2.0460
2006	2293.1	1210.0	1083.1	0.5277	0.4723	6.8051	10.5245	1.9567	2.0912
2007	2356.3	1303.0	1053.3	0.5530	0.4470	6.9719	10.9629	1.9872	2.0289
2008	2474.5	1315.8	1158.7	0.5317	0.4683	6.9994	11.2585	1.9932	2.1583
2009	2321.0	1271.1	1049.9	0.5477	0.4523	6.8638	11.4066	1.9929	2.1651
2010	2395.5	1291.9	1103.6	0.5393	0.4607	7.0960	11.6811	2.0158	2.1779
2011	2592.9	1324.2	1268.7	0.5107	0.4893	7.2228	11.9747	2.0264	2.3162
2012	2742.3	1386.7	1355.6	0.5057	0.4943	7.3719	12.2744	2.0323	2.3493
2013	2839.8	1410.8	1429.0	0.4968	0.5032	7.5334	12.3977	2.0226	2.3635
2014	2966.3	1470.3	1496.0	0.4957	0.5043	7.7110	12.6666	2.0426	2.3645

Note: All values are in billions of dollars.

Figure 1: Sector 1 Labour and Capital Shares of Value Added, Output Price, Real Wage and Labour Productivity

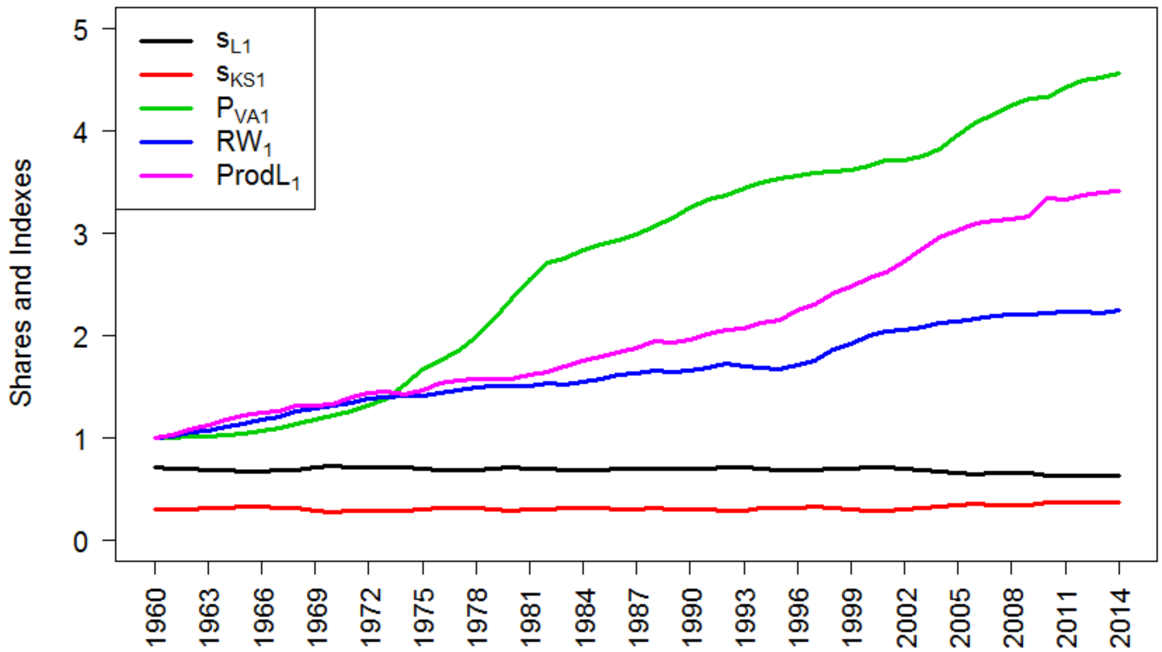
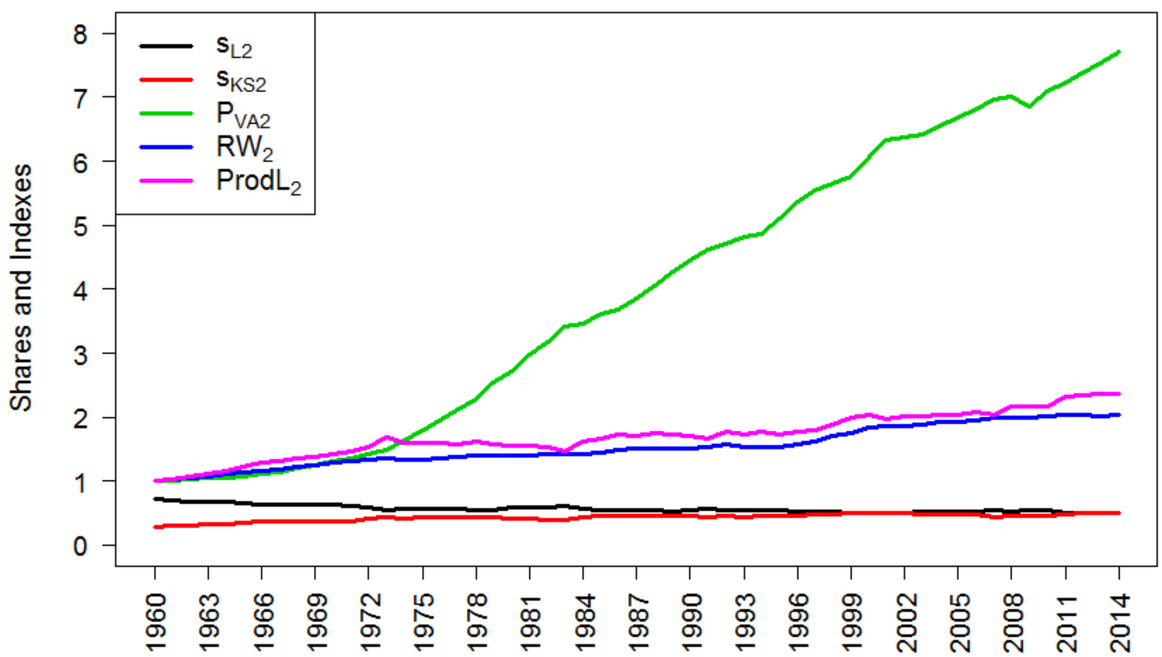


Figure 2: Sector 2 Labour and Capital Shares of Value Added, Output Price, Real Wage and Labour Productivity



The price of (value added) output in Sector 2 grew 7.71 fold over the sample period (much higher than the Sector 1 price growth of 4.56 fold) while wages grew 12.67 fold. The geometric rates of growth were 3.61% per year for real value added, 3.86% per year for the value added deflator and 4.81% per year for wages. Real wages grew 2.04 fold over the sample period while labour productivity grew 2.36 fold, much lower than the 3.41 fold of labour productivity in Sector 1. The long run average geometric rates of growth of real wages and labour productivity for Sector 2 were 1.33% and 1.61% per year while the corresponding growth rates for Sector 1 were 1.51% and 2.30% per year. Thus real wage growth and labour productivity growth in Sector 2 were substantially below their Sector 1 counterparts. The sample average labour and capital services shares in Sector 2 were 56.7% and 43.3% (68.6% and 31.4% in Sector 1). It can be seen that Sector 2 is much more capital intensive than Sector 1. The upward trend in the capital services share is very noticeable in Figure 2, which plots the series s_{L2}^t, s_{KS2}^t, P_{VA2}^t, RW_2^t and $ProdL_2^t$. Note that the capital services share finishes up at 50.4%, well above its long term average of 43.3%. It can be seen that real wages have grown very slowly since 2001. Note also that real wage growth stagnated after 2007 while labour productivity continued to grow. *It can be seen that the structure of production is entirely different in the noncorporate nonfinancial sector as compared to the corporate nonfinancial sector*.

In the following section, we will calculate price and quantity indexes for the capital stocks used in both sectors as well as the corresponding real and nominal capital output ratios for the two sectors.

4. Capital stocks and Capital Output Ratios for Sectors 1 and 2

We constructed chained Fisher capital stock price and quantity indexes for Sector 1 using price and quantity information for each of the nine assets that are used as inputs, which are as follows: 1 = Equipment; 2 = Intellectual property products; 3 = Nonresidential structures; 4 = Residential structures; 5 = Residential land; 6 = Farm land; 7 = Commercial land; 8 = Beginning of year inventory stocks, and 9 = Beginning of the year real holdings of currency and deposits.

Denote the resulting period t price and quantity indexes as P_{K1}^t and Q_{K1}^t for t = 1960,...,2015. Define the Sector 1 capital stock value at the beginning of year t as $V_{K1}^t \equiv P_{K1}^t Q_{K1}^t$. Now define the year t nominal and real capital output ratios as $V_{K/O,1}^t \equiv V_{K1}^t/V_{VA1}^t$ and $Q_{K/O,1}^t \equiv Q_{K1}^t/Q_{VA1}^t$. V_{K1}^t, Q_{K1}^t, P_{K1}^t, $V_{K/O,1}^t$ and $Q_{K/O,1}^t$ are listed in Table 3.

It can be seen that the Sector 1 aggregate capital stock price P_{K1}^t increased 7.36 fold over the sample period. The average geometric growth rates for the price and quantity of the Sector 1 capital stock were 3.70% per year and 2.74% per year respectively. The real capital output ratio, $Q_{K/O,1}^t$, declined more or less steadily from 2.47 in 1960 to 1.59 in 2014. The nominal capital output ratio, $V_{K/O,1}^t$, did not decline nearly as much due to increasing land prices.[23]

[23] We constructed chained Fisher land price and quantity indexes for Sector 1 and then compared the value of land to value added and the quantity of land to the quantity of output. The nominal land to output ratio went from 36.7% in 1960 to a peak of 51.2% in 2006, declined to 22.0% in 2012 and finished up in 2014 at 30.4%. The corresponding real land to output ratio declined steadily from 36.7% in 1960 to 9.8% in 2014. The inclusion or exclusion of land from the productive asset base does make a significant difference to capital output ratios.

The nominal capital output ratio started at 2.47 and ended up at 2.52 with many fluctuations in between ($V_{K/O,1}{}^t$ had a low of 2.00 in 1966 and a high of 2.80 in 2009).

We similarly constructed chained Fisher capital stock price and quantity indexes for Sector 2 using the price and quantity information for each of the fourteen assets that are used as inputs,[24] which are as follows: 1 = Equipment held by sole proprietors; 2 = Equipment held by partners; 3 = Equipment held by cooperatives; 4 = Intellectual property products held by sole proprietors; 5 = Intellectual property products held by partners; 6 = Nonresidential structures held by sole proprietors; 7 = Nonresidential structures held by partners; 8 = Nonresidential structures held by cooperatives; 9 = Residential structures held by the noncorporate nonfinancial sector; 10 = Residential land held by the noncorporate nonfinancial sector; 11 = Farm land held by the noncorporate nonfinancial sector; 12 = Commercial land held by noncorporate nonfinancial sector; 13 = Beginning of the year inventories held by the noncorporate nonfinancial sector, and 14 = Beginning of the year real holdings of currency and deposits by noncorporate nonfinancial sector.

Denote the resulting beginning of period t price and quantity indexes as $P_{K2}{}^t$ and $Q_{K2}{}^t$ for t = 1960,...,2015. Define the Sector 2 capital stock value at the beginning of year t as $V_{K2}{}^t \equiv P_{K2}{}^t Q_{K2}{}^t$. Now define the year t nominal and real capital output ratios for Sector 2 as $V_{K/O,2}{}^t \equiv V_{K2}{}^t/V_{VA2}{}^t$ and $Q_{K/O,2}{}^t \equiv Q_{K2}{}^t/Q_{VA2}{}^t$. $V_{K2}{}^t$, $Q_{K2}{}^t$, $P_{K2}{}^t$, $V_{K/O,2}{}^t$ and $Q_{K/O,2}{}^t$ are listed in Table 3.

It can be seen that the Sector 2 aggregate capital stock price $P_{K2}{}^t$ increased 14.76 fold over the sample period whereas the Sector 1 capital stock price increased only 7.36 fold. The average geometric growth rates for the price and quantity of the Sector 2 capital stock were 5.02% per year (3.70% per year for Sector 1) and 1.44% per year (2.74% per year for Sector 1) respectively. This large difference in growth rates between sectors is explained by the relatively very large land component in the Sector 2 capital stock.[25] The price of land tends to grow more rapidly and the quantity less rapidly than other assets. The real capital output ratio for Sector 2, $Q_{K/O,2}{}^t$, increased (erratically) from 3.43 in 1960 to 4.01 in 1983 and then declined to 2.07 in 2014. The corresponding nominal capital output ratio, $V_{K/O,2}{}^t$, did not decline nearly as much, due to increasing land prices. The nominal capital output ratio started at 3.43 and remained roughly constant until 1969 and then increased rapidly to hit a peak of 5.83 in 1982 and then fell to 3.48 in 2012 and increased a little to end up at 3.80 in 2014. It can be seen that the real and nominal capital output ratios are in general, much larger in Sector 2 than in Sector 1.

The nominal and real capital output ratios for Sectors 1 and 2 are plotted in Figure 3, where the overall decline in the real capital output ratios from 1983 is visible. The much higher capital output ratios for Sector 2 over Sector 1 are also apparent.

[24] The BEA Fixed Asset Tables are organized somewhat differently for the Nonfinancial Noncorporate Sector as compared to Sector 1, with a decomposition of Sector 2 into subsectors. This led us to organize the capital stock data for Sector 2 into fourteen rather than nine components.

[25] The average share of residential land in Sector 2 value of the capital stock is 28.2%, farm land is 16.4% and commercial (nonresidential and nonfarm) land is 7.1%. Thus the overall average land share in the total value of Sector 2 assets is 51.6% and for reproducible assets is 48.4%. The average land share of asset value in Sector 1 is only 14.4% and the corresponding reproducible asset share is 85.6%.

Table 3: Capital Stock Values, Prices and Quantities and Nominal and Real Capital Output Ratios for Sectors 1 and 2

Year	V_{K1}^t	Q_{K1}^t	P_{K1}^t	$V_{K/O,1}^t$	$Q_{K/O,1}^t$	V_{K2}^t	Q_{K2}^t	P_{K2}^t	$V_{K/O,2}^t$	$Q_{K/O,2}^t$
1960	633.4	633.4	1.0000	2.4752	2.4752	368.9	368.9	1.0000	3.4345	3.4345
1961	641.2	650.0	0.9865	2.4399	2.4807	373.3	371.4	1.0049	3.3872	3.4248
1962	662.5	668.8	0.9906	2.3092	2.3533	385.6	376.4	1.0244	3.3762	3.3969
1963	689.9	691.8	0.9972	2.2539	2.2930	402.2	382.0	1.0530	3.4348	3.4002
1964	712.8	713.3	0.9993	2.1561	2.2090	411.5	384.3	1.0707	3.3455	3.3116
1965	750.5	739.3	1.0151	2.0579	2.1122	422.5	389.6	1.0844	3.2499	3.2397
1966	804.3	773.4	1.0400	1.9953	2.0572	445.9	399.5	1.1162	3.2197	3.2139
1967	874.7	816.8	1.0709	2.0634	2.1124	472.1	407.0	1.1601	3.3222	3.2916
1968	944.1	858.8	1.0994	2.0286	2.0855	500.5	412.4	1.2136	3.3409	3.3074
1969	1028.6	895.4	1.1487	2.0393	2.0909	548.1	422.7	1.2967	3.4733	3.3591
1970	1131.9	931.5	1.2152	2.1826	2.1942	614.3	436.1	1.4086	3.7596	3.4784
1971	1233.1	961.7	1.2823	2.2079	2.1794	675.1	445.3	1.5161	3.8998	3.5101
1972	1342.0	989.3	1.3565	2.1569	2.0841	746.7	459.5	1.6249	3.9052	3.4373
1973	1462.6	1020.1	1.4338	2.0933	2.0259	838.5	476.2	1.7609	3.7515	3.1803
1974	1663.0	1056.7	1.5738	2.2006	2.1309	1005.7	495.0	2.0320	4.2761	3.4374
1975	2016.4	1109.2	1.8180	2.4647	2.2689	1183.2	500.6	2.3634	4.6952	3.5795
1976	2216.9	1130.3	1.9613	2.3886	2.1373	1342.6	506.9	2.6486	4.8716	3.6025
1977	2449.5	1166.5	2.0999	2.3264	2.0544	1497.6	509.0	2.9422	4.9803	3.5939
1978	2725.9	1206.9	2.2587	2.2689	1.9958	1705.4	515.1	3.3106	5.0057	3.4396
1979	3098.1	1249.0	2.4806	2.3093	2.0013	2007.9	527.2	3.8085	5.2798	3.5257
1980	3569.1	1296.1	2.7537	2.4569	2.0974	2296.6	538.1	4.2683	5.7445	3.6514
1981	4058.3	1331.1	3.0488	2.4720	2.0700	2488.8	544.4	4.5719	5.7149	3.7201
1982	4528.4	1377.0	3.2886	2.6616	2.1892	2647.9	554.8	4.7728	5.8260	3.8671
1983	4779.9	1405.0	3.4020	2.6299	2.1295	2736.8	565.4	4.8408	5.6934	4.0070
1984	4966.4	1436.6	3.4570	2.4339	1.9993	2866.2	570.4	5.0250	5.1523	3.5380
1985	5280.1	1493.8	3.5346	2.4300	1.9868	2962.8	582.3	5.0879	4.9347	3.5092
1986	5540.5	1546.5	3.5826	2.4508	2.0046	3134.5	597.4	5.2473	4.9262	3.4568
1987	5742.8	1584.8	3.6237	2.3675	1.9508	3314.2	603.7	5.4899	4.9651	3.4934
1988	6041.1	1614.4	3.7420	2.2877	1.8722	3522.6	609.2	5.7828	4.8448	3.3930
1989	6434.3	1643.3	3.9156	2.3207	1.8701	3777.3	616.6	6.1259	4.8796	3.3988
1990	6749.5	1674.7	4.0303	2.3293	1.8787	4005.2	622.3	6.4360	4.9600	3.4255
1991	7057.4	1707.1	4.1343	2.3955	1.9250	4108.2	627.4	6.5480	5.0407	3.5639
1992	7229.2	1732.9	4.1716	2.3513	1.8963	4206.9	626.7	6.7131	4.8367	3.3879
1993	7421.0	1754.3	4.2302	2.3075	1.8735	4256.2	625.6	6.8033	4.7108	3.3327
1994	7775.5	1793.9	4.3345	2.2435	1.8047	4388.0	626.2	7.0077	4.6132	3.2049
1995	8173.3	1842.8	4.4353	2.2194	1.7686	4487.7	629.3	7.1314	4.5239	3.2346
1996	8648.2	1895.7	4.5619	2.2037	1.7187	4688.9	635.2	7.3817	4.3842	3.1841
1997	9059.0	1956.6	4.6299	2.1469	1.6629	4897.0	647.3	7.5658	4.3073	3.1541
1998	9577.5	2032.2	4.7129	2.1422	1.6343	5191.2	658.6	7.8816	4.2225	3.0334
1999	10131.9	2110.4	4.8009	2.1352	1.6096	5607.1	669.6	8.3735	4.2727	2.9424
2000	10861.3	2200.2	4.9366	2.1452	1.5909	6082.2	682.9	8.9060	4.2811	2.9169
2001	11758.9	2291.5	5.1316	2.3395	1.6929	6733.2	701.0	9.6050	4.1129	2.7133
2002	12238.7	2324.1	5.2661	2.4159	1.7024	7256.3	709.3	10.230	4.2509	2.6448
2003	12832.2	2348.3	5.4645	2.4542	1.6836	7847.9	710.4	11.048	4.3588	2.5323
2004	13575.4	2393.6	5.6715	2.4342	1.6422	8495.9	715.8	11.869	4.3486	2.3986
2005	14835.3	2432.0	6.1000	2.4896	1.6153	9558.0	726.9	13.149	4.5763	2.3262
2006	16327.0	2484.1	6.5727	2.5599	1.5887	10742.6	737.0	14.577	4.6847	2.1870
2007	17248.2	2519.9	6.8447	2.6247	1.5956	11334.8	750.9	15.094	4.8104	2.2219
2008	17652.5	2538.6	6.9537	2.6649	1.6284	11138.3	764.9	14.561	4.5012	2.1637
2009	17527.8	2561.5	6.8429	2.8027	1.7687	10275.4	775.7	13.247	4.4272	2.2938
2010	17154.9	2583.4	6.6405	2.5970	1.6901	9937.9	772.2	12.869	4.1486	2.2876
2011	17460.2	2620.2	6.6637	2.5225	1.6723	9464.8	773.8	12.232	3.6503	2.1555
2012	18064.0	2648.8	6.8197	2.4673	1.6250	9539.9	779.4	12.240	3.4788	2.0953
2013	18919.5	2688.7	7.0367	2.4921	1.6010	10491.4	788.3	13.309	3.6944	2.0912
2014	19908.8	2754.4	7.2281	2.5215	1.5896	11284.8	797.0	14.160	3.8043	2.0717
2015	20661.5	2806.7	7.3615			11960.3	810.2	14.762		

Note: All values are in billions of dollars and the quantities are in billions of 1960 dollars.

Figure 3: Nominal and Real Capital Output Ratios for Sectors 1 and 2

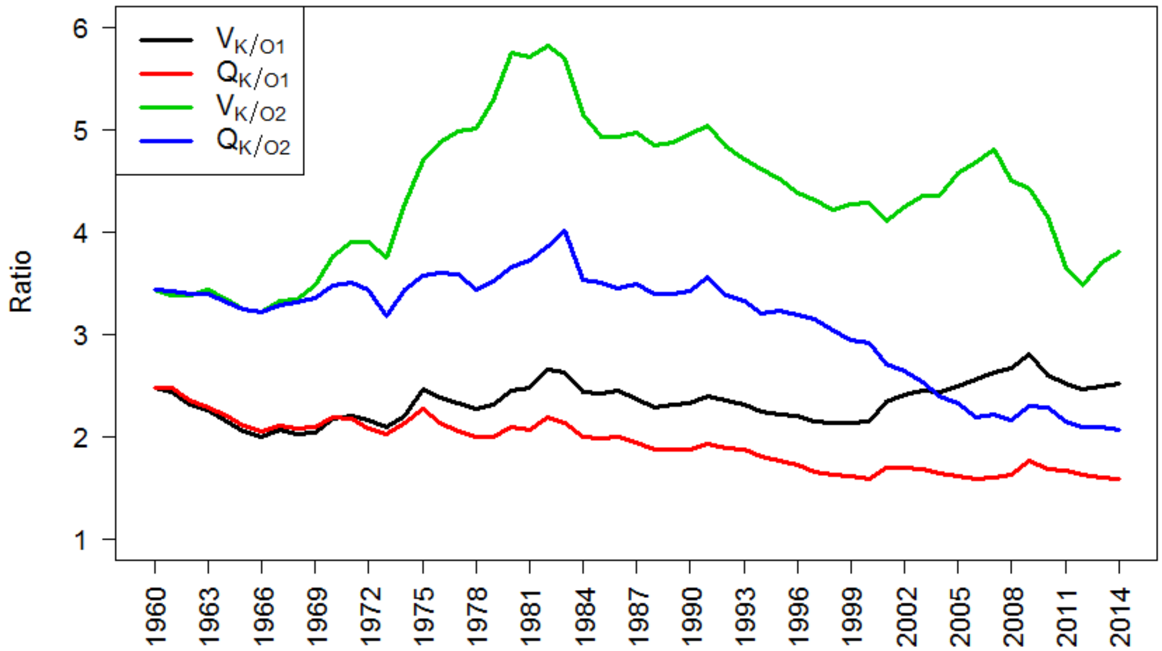


Figure 4: Normalized and Real Value Added, Labour Input and Capital Stocks for Sectors 1 and 2

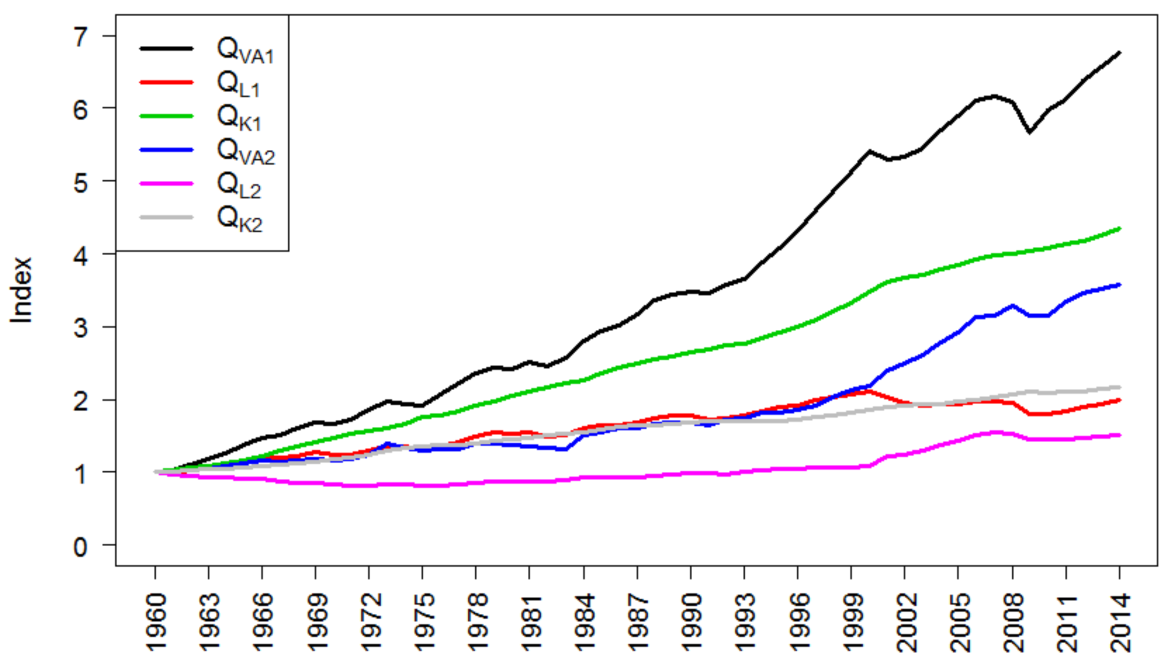


Figure 4 plots real value added, labour input and beginning of the period capital stocks for Sectors 1 and 2, except that each series is divided by its starting 1960 value. Thus real value added in Sectors 1 and 2 grew 6.77 fold and 3.58 fold respectively, labour input grew 1.98 fold in Sector 1 and 1.51 fold in Sector 2 and capital stocks grew 4.35 fold in Sector 1 and only 2.16 fold for Sector 2. Note that labour input in Sector 2 did not recover to its starting value in 1960 until 1993 after which it grew fairly rapidly until 2007 when it levelled off.

In the following section, we turn our attention to deriving the alternative balancing rates of return on assets, and the resulting user costs, for our two sectors that were discussed in Section 2.

5. Balancing Rates of Return and Alternative User Costs for Sectors 1 and 2

Denote the beginning of the year t asset prices for Sector 1 by $P_{K1,n}{}^{t}$ for n = 1,...,9. The year t inflation rate for asset n, $i_{1,n}{}^{t}$, is defined as follows:

$$(16)\ i_{1,n}{}^{t} \equiv (P_{K1,n}{}^{t+1}/P_{K1,n}{}^{t}) - 1\ ; \qquad n = 1,...,9\ ;\ t = 1960,...,2014.$$

Denote the depreciation rate for asset n in year t used in Sector 1 by $\delta_{1,n}{}^{t}$. Define the depreciation rates for assets n = 5,...,9 to be 0 for all years t.[26]

Recall equation (9) in Section 2 which defined the ex post rate of return on assets for year t, r^t. For Sector 1, we will use the following counterpart to (9) to define the year t *ex post rate of return on assets for Sector 1*, $r_1{}^{t}$:

$$(17)\ V_{VA1}{}^{t} - V_{L1}{}^{t} - \Sigma_{n=1}{}^{9}\ [1+r_1{}^{t} - (1+i_{1,n}{}^{t})(1-\delta_{1,n}{}^{t})]P_{K1,n}{}^{t}Q_{K1,n}{}^{t} = 0\ ; \qquad t = 1960,...,2014,$$

where Sector 1 value added and the value of labour input in year t, $V_{VA1}{}^{t}$ and $V_{L1}{}^{t}$, are listed in Table 1. The Sector 1 ex post rates of return on assets (the $r_1{}^{t}$ which solve (17) for year t data) are plotted in Figure 5.

Recall that the personal consumption deflator for the beginning of year t was defined in Section 3 as $P_C{}^{t}$ for t = 1960,...,2014. Define the corresponding year t *consumption inflation rate*, $i_C{}^{t}$, by (18) and the corresponding year t *ex post real rate of return on assets* for Sector 1, $R_1{}^{t}$, by (19):

$$(18)\ i_C{}^{t} \equiv (P_C{}^{t+1}/P_C{}^{t}) - 1\ ; \qquad t = 1960\ ...,2014;$$
$$(19)\ R_1{}^{t} \equiv [(1+r_1{}^{t})/(1+i_C{}^{t})] - 1\ ; \qquad t = 1960\ ...,2014.$$

The personal consumption deflator inflation rates $i_C{}^{t}$ and the Sector 1 ex post real rates of return $R_1{}^{t}$ are also plotted in Figure 5.

We also calculated a balancing rate of return for Sector 1 for each year t, $r_1{}^{t*}$, using a modification of equation (11) in section 3. In order to calculate this alternative rate of return

[26] The nonzero depreciation rates for assets n = 1,2,3,4 used in Sector 1 are listed in Table A10 in the Appendix of Diewert and Fox (2016).

on assets, we need to form *expected or predicted asset inflation rates*, $i_{1,n}^{t*}$, for each asset n. For the first six years in our sample, we used the actual geometric average growth rate of the asset prices, starting at the beginning of 1960 and ending at the beginning of 1965. Thus we defined $i_{1,n}^{t*}$ as follows for the first six years in our sample:

$$(20)\ i_{1,n}^{t*} \equiv (P_{K1,n}^{1965}/P_{K1,n}^{1960})^{1/5} - 1\ ; \qquad n = 1,...,9;\ t = 1960,...,1965.$$

For the years 1966-1985 we defined the $i_{1,n}^{t*}$ as geometric average growth rates of the asset price from the beginning of 1960 to the beginning of year t as follows for n = 1,...,9:

$$(21)\ i_{1,n}^{1966*} \equiv (P_{K1,n}^{1966}/P_{K1,n}^{1960})^{1/6} - 1,$$
$$i_{1,n}^{1967*} \equiv (P_{K1,n}^{1967}/P_{K1,n}^{1960})^{1/7} - 1,$$
$$...$$
$$i_{1,n}^{1985*} \equiv (P_{K1,n}^{1985}/P_{K1,n}^{1960})^{1/25} - 1.$$

For t greater than 1985, we simply used the geometric average growth rate of the asset price over the 25 years prior to year t; i.e., define $i_{1,n}^{t*}$ for t ≥ 1985 as follows:[27]

$$(22)\ i_{1,n}^{t*} \equiv (P_{K1,n}^{t}/P_{K1,n}^{t-25})^{1/25} - 1; \qquad n = 1,...,9;\ t = 1985,...,2014.$$

Recall equation (11) in Section 2 which decomposed value added into labour and capital service components using predicted asset inflation rates, which we now denote by $i_{1,n}^{t*}$, and a predicted or expected balancing nominal rate of return on assets for year t, which we now denote by r_1^{t*}. For Sector 1, we will use the following counterpart to (11) to define the year t *predicted balancing rate of return on assets for Sector 1*, r_1^{t*}:

$$(23)\ V_{VA1}^{t} - V_{L1}^{t} - \Sigma_{n=1}^{9}\,[1+r_1^{t*} - (1+i_{1,n}^{t*})(1-\delta_{1,n}^{t})]P_{K1,n}^{t}Q_{K1,n}^{t} = 0\ ; \qquad t = 1960,...,2014$$

where Sector 1 value added and the value of labour input in year t. The Sector 1 predicted rates of return on assets (the r_1^{t*} which solve (23) for year t data) are plotted in Figure 5.[28] The corresponding year t *predicted real rate of return on assets* for Sector 1, R_1^{t*}, is defined by (24) and also plotted in Figure 5:

$$(24)\ R_1^{t*} \equiv [(1+r_1^{t*})/(1+i_C^{t})] - 1\ ; \qquad t = 1960\ ...,2014.$$

The mean nominal rate of return r_1^{t} over the sample period in Sector 1 was 11.25% (minimum rate was 3.21% in 2009 and the maximum was 21.97% in 1974) while the mean real ex post rate of return on assets R_1^{t} was 7.57% (minimum was 1.99% in 2009; maximum was 11.83% in 1965). These ex post real rates have been above average for the last three years at 9.73%, 9.82% and 8.68%. The mean nominal predicted rate of return r_1^{t*} over the sample period in Sector 1 was 10.04% (minimum rate was 6.96% in 2001 and the maximum was 12.56% in

[27] It may be that the length of our moving average process is too long or that better methods for predicting asset prices one year hence could be devised. However, our goal is to obtain user costs that could approximate one year rental prices for assets used in production (when they exist). Since observed rental prices are relatively smooth, our suggested method for generating predicted asset prices does lead to relatively smooth user costs as will be seen later.

[28] Tabulated data for the series in this and following figures are available in Diewert and Fox (2016).

1978) while the mean expected real rate of return on assets R_1^{t*} was 6.44% (minimum was -0.94% in 1974; maximum was 9.77% in 1965).[29]

The most important series is R_1^t, the before income tax realized real rate of return on assets used in the Corporate Nonfinancial Sector.[30] This real rate has remained above 5% except for the 10 years 1960, 1982-83, 1985, 1990-93 and 2008-09, and has remained below 11% except for the 3 years 1965 and 2004-05. There is no indication of a real rate of return slowdown that shows up in our data. However, the 2008 financial crisis certainly drove down ex post realized rates of return temporarily in 2008 and 2009.

We turn our attention to Sector 2. Denote the beginning of the year t asset prices for Sector 2 by $P_{K2,n}^t$ for n = 1,...,14. The year t inflation rate for asset n in Sector 2, $i_{2,n}^t$, is defined as follows:

$$(25)\ i_{2,n}^t \equiv (P_{K2,n}^{t+1}/P_{K2,n}^t) - 1\ ; \qquad n = 1,...,14\ ;\ t = 1960,...,2014.$$

Denote the depreciation rate for asset n in year t used in Sector 2 by $\delta_{2,n}^t$. Define the depreciation rates for assets n = 10,...,14 to be 0 for all years t.[31] Again recall equation (9) in Section 2 which defined the ex post rate of return on assets for year t, r^t. For Sector 2, we will use the following counterpart to equation (9) to define the year t *ex post rate of return on assets for Sector 2*, r_2^t:

$$(26)\ V_{VA2}^t - V_{L2}^t - \Sigma_{n=1}^{14} [1+r_2^t - (1+i_{2,n}^t)(1-\delta_{2,n}^t)]P_{K2,n}^tQ_{K2,n}^t = 0\ ; \qquad t = 1960,...,2014$$

where Sector 2 value added and the value of labour input in year t, V_{VA2}^t and V_{L2}^t, are listed in Table 2. The Sector 2 ex post rates of return on assets (the r_2^t which solve (26) for year t data) are plotted in Figure 6. The year t *ex post real rate of return on assets* for Sector 2, R_2^t, is defined by (27):

$$(27)\ R_2^t \equiv [(1+r_2^t)/(1+i_C^t)] - 1\ ; \qquad t = 1960\ ...,2014.$$

We also calculated a balancing rate of return for Sector 2 for each year t, r_2^{t*}, using a modification of equation (11) in Section 3. In order to calculate this alternative rate of return on assets, we need to form *expected or predicted asset inflation rates*, $i_{2,n}^{t*}$, for each asset n. We formed Sector 2 predicted asset inflation rates using exactly the same method that we used to form Sector 1 predicted inflation rates.

Recall equation (11) in Section 2 which decomposed value added into labour and capital service components using predicted asset inflation rates, which we now denote by $i_{2,n}^{t*}$, and a predicted or expected balancing nominal rate of return on assets for year t, which we now

[29] Note that our expected real rate of return on Sector 1 assets has been fairly stable over the period 1982-2014. R_1^{t*} ranged between 4.62% (1990) and 9.33% (1997) over this period.
[30] The average corporate income tax paid by the nonfinancial corporate sector on assets during our sample period as a percentage of the asset base is 1.98% per year; see the series V_{TI1}^t in Appendix Table A3 of Diewert and Fox (2016).
[31] The nonzero depreciation rates for assets n = 1,...,9 used in Sector 2 are listed in Table A11 in the Appendix of Diewert and Fox (2016).

denote by r_2^{t*}. For Sector 2, we will use the following counterpart to (11) to define the year t *predicted balancing rate of return on assets for Sector 2*, r_2^{t*}:

(28) $V_{VA2}^t - V_{L2}^t - \Sigma_{n=1}^{14} [1+r_2^{t*} - (1+i_{2,n}^{t*})(1-\delta_{2,n}^t)]P_{K2,n}^t Q_{K2,n}^t = 0$; t = 1960,...,2014

where Sector 2 value added and the value of labour input in year t, V_{VA2}^t and V_{L2}^t, are listed in Table 2. The Sector 2 predicted rates of return on assets (the r_2^{t*} which solve (28) for year t data) are plotted in Figure 6, along with the corresponding year t *predicted real rate of return on assets* for Sector 2, R_2^{t*}, as defined by (29):

(29) $R_2^{t*} \equiv [(1+r_2^{t*})/(1+i_C^t)] - 1$; t = 1960 ...,2014.

The mean nominal ex post rate of return r_2^t over the sample period in Sector 2 was 12.76% (minimum rate was −0.70% in 2008 and the maximum was 24.60% in 1973) while the mean real ex post rate of return on assets R_2^t was 9.03% (minimum was −2.14% in 2008; maximum was 18.29% in 2012). Note that the average real rate of return in Sector 2 was a very high 9.03% per year which is considerably above the average real rate of return on assets used in Sector 1, which was 7.57% per year. This result was somewhat surprising. The Sector 2 ex post real rates have been above average for the last three years at 18.29%, 16.13% and 13.86%. These are very high real rates of return. The corresponding Sector 1 ex post real rates were only 9.73%, 9.82% and 8.68%.[32] The mean *nominal predicted* rate of return r_1^{t*} over the sample period in Sector 2 was 11.35% (minimum rate was 7.12% in 1960 and the maximum was 13.72% in 1979) while the mean expected *real predicted* rate of return on assets R_1^{t*} was 7.73% (minimum was 0.92% in 1974; maximum was 11.91% in 2013).

The most important series is R_2^t, the before income tax realized real rate of return on assets used in the Noncorporate Nonfinancial Sector.[33] This series has fluctuated considerably during the sample period, driven by large fluctuations in the price of land. There does not appear to be a long run decline in the real rate of return on assets in Sector 2. The predicted nominal rate of return series r_2^{t*} is much smoother than the corresponding realized return series r_2^t and so the use of the r_2^{t*} series in our user costs will lead to much smoother user costs for this sector.

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[32] The reason why nominal and real ex post rates of return on assets are much higher in Sector 2 compared to Sector 1 can be explained by the fact that production in Sector 2 is highly land intensive and land inflation rates are much higher than inflation rates for other assets.

[33] The average business income tax paid by the nonfinancial noncorporate sector on assets during our sample period as a percentage of the asset base is only 0.15% per year; see the series V_{TI2}^t in Appendix Table A3 of Diewert and Fox (2016). This income tax rate for Sector 2 seems to be too low to be true!

Figure 5: Sector 1 Nominal and Real Rates of Return, Predicted Nominal and Real Rates of Return, and Personal Consumption Deflator Inflation Rate

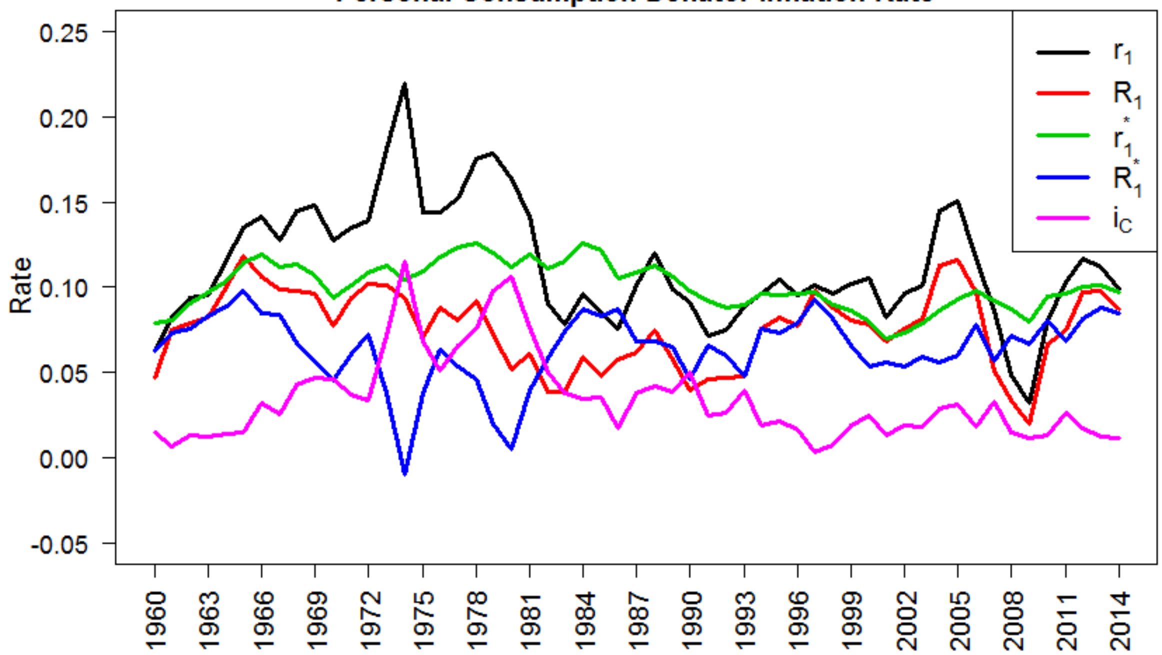
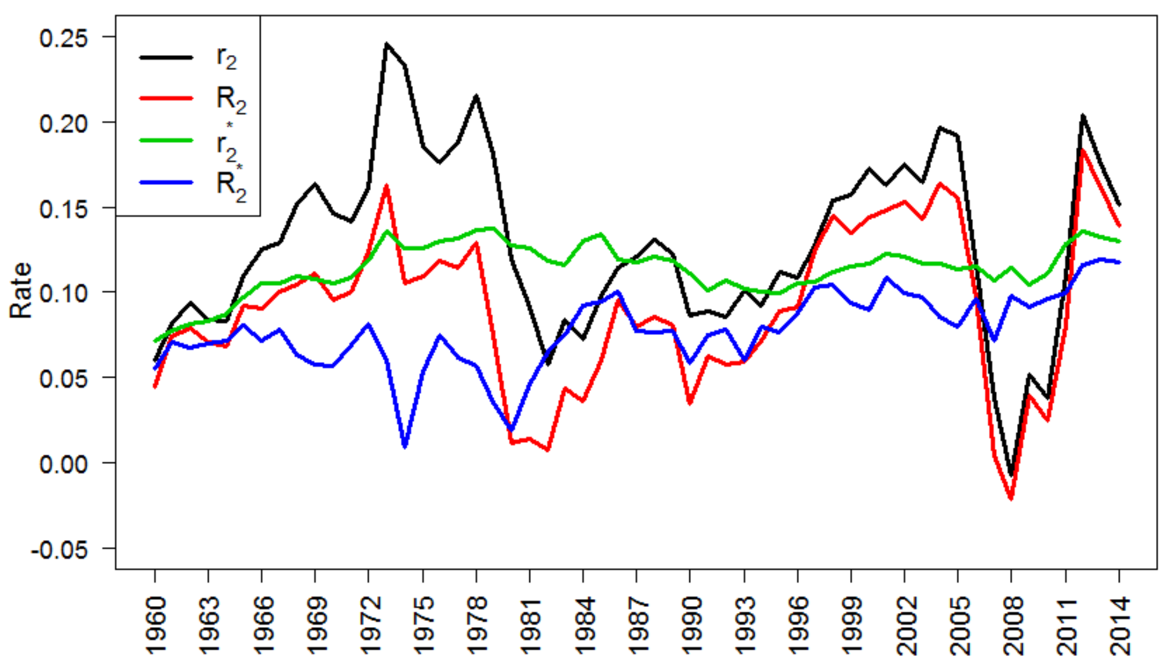


Figure 6: Sector 2 Nominal and Real Rates of Return, Predicted Nominal and Real Rates of Return



We turn our attention to the calculation of user costs for Sector 1. Recall equations (16) and (17). The *year t Jorgensonian user cost for asset n used in Sector 1*, $u_{1,n}{}^t$, is defined as follows:

$$(30)\ u_{1,n}{}^t \equiv [1+r_1{}^t - (1+i_{1,n}{}^t)(1-\delta_{1,n}{}^t)]P_{K1,n}{}^t\ ; \qquad n = 1,...,9;\ t = 1960,...,2014$$

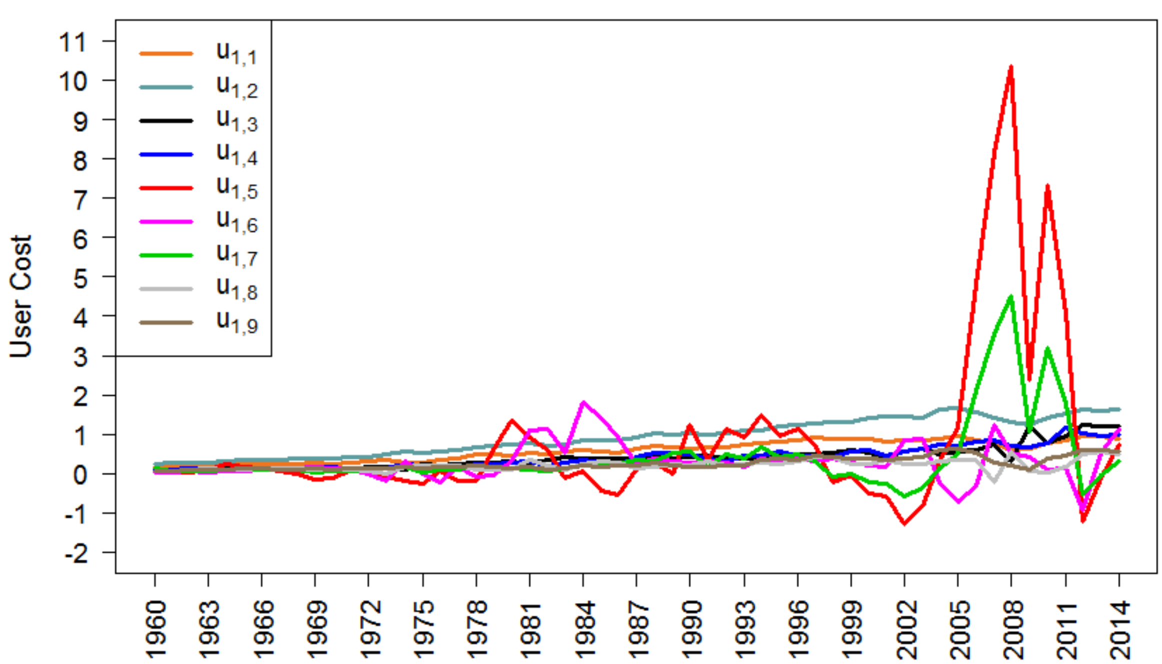
where the $i_{1,n}{}^t$ are the ex post asset inflation rates defined by (16) and the $r_1{}^t$ are the Sector 1 balancing nominal rates of return defined by equations (17). These Jorgensonian user costs are plotted in Figure 7. It can be seen that there are numerous negative Jorgensonian user costs for assets 5-8 (residential land, farm land, commercial land and inventory stocks). It can also be seen that these user costs are in general quite volatile. Thus while Jorgensonian user costs are the "right" user costs to use when computing ex post rates of return on assets, they are not good approximations to rental prices for these assets.[34]

Recall equations (20)-(23). The *year t predicted user cost for asset n used in Sector 1*, $u_{1,n}{}^{t*}$, is defined as follows:

$$(31)\ u_{1,n}{}^{t*} \equiv [1+r_1{}^{t*} - (1+i_{1,n}{}^{t*})(1-\delta_{1,n}{}^t)]P_{K1,n}{}^t\ ; \qquad n = 1,...,9;\ t = 1960,...,2014$$

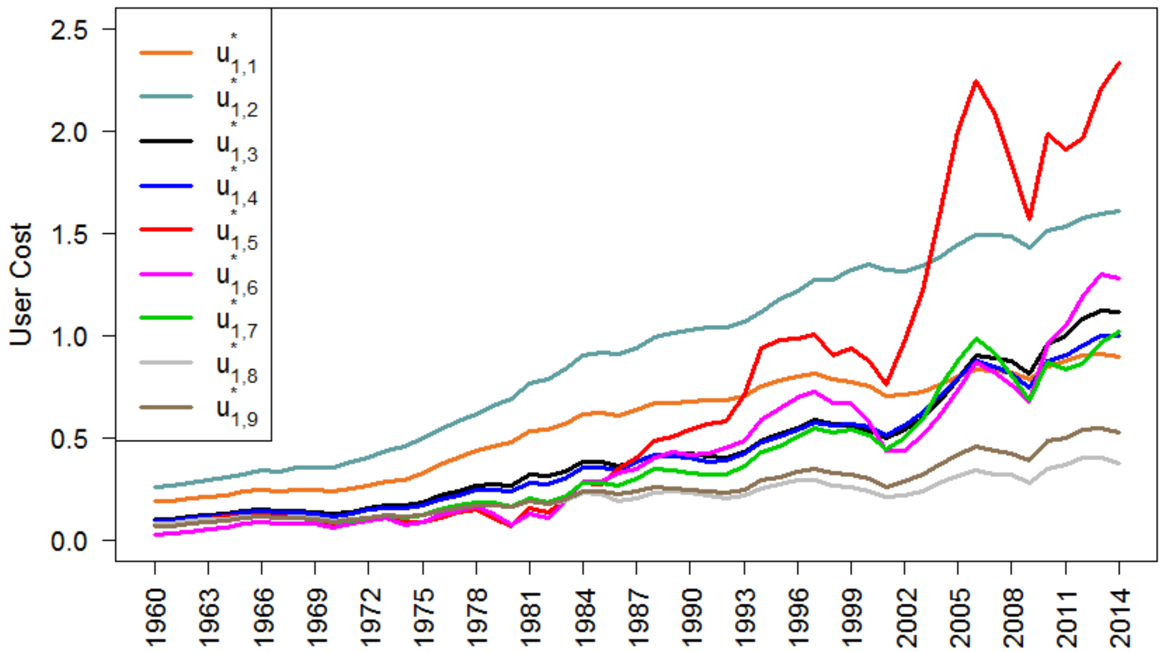
where the $i_{1,n}{}^{t*}$ are the predicted asset inflation rates defined by (20)-(22) and the $r_1{}^{t*}$ are the predicted Sector 1 balancing nominal rates of return defined by equations (23). These predicted user costs are plotted in Figure 8.

Figure 7: Jorgensonian User Costs for Sector 1



[34] Thus the use of Jorgensonian user costs is not recommended in econometric studies where cost functions are estimated or where production functions are estimated using inverse factor demand equations as additional estimating equations.

Figure 8: Predicted User Costs for Sector 1



The predicted user costs are much smoother than the Jorgensonian user costs and the negative user costs have been eliminated. Thus in what follows, we will sometimes refer to these predicted user costs as *smoothed user costs*. These user costs are suitable for production or cost function econometric studies. They are also more suitable for statistical agencies to use when computing capital services aggregates for publication. It can be seen that the user costs for residential, farm and commercial land ($u_{1,5}^{t*}$, $u_{1,6}^{t*}$ and $u_{1,7}^{t*}$) have been quite volatile for the last 20 years in our sample period but the remaining user cost series are fairly smooth.

We turn our attention to the calculation of user costs for Sector 2. Recall equation (26). The *year t Jorgensonian user cost for asset n used in Sector 2*, $u_{2,n}^{t}$, is defined as follows:

(32) $u_{2,n}^{t} \equiv [1+r_2^{t} - (1+i_{2,n}^{t})(1-\delta_{2,n}^{t})]P_{K2,n}^{t}$; $n = 1,...,14;\ t = 1960,...,2014$

where the $i_{2,n}^{t}$ are the ex post asset inflation rates defined by (25) and the r_2^{t} are the Sector 2 balancing nominal rates of return defined by equations (26). These Jorgensonian user costs are plotted in Figure 9. Again, these user costs are volatile and there are numerous negative user costs in assets, 6-7 (nonresidential structures held by proprietors, partners and cooperatives) and 10-14 (residential land, farm land, commercial land, inventory stocks and monetary stocks). It can be seen at a glance that these user costs are not suitable approximations to asset rental prices.

Recall equation (28). The *year t predicted user cost for asset n used in Sector 2*, $u_{2,n}^{t*}$, is defined as follows:

(33) $u_{2,n}^{t*} \equiv [1+r_2^{t*} - (1+i_{2,n}^{t*})(1-\delta_{2,n}^{t})]P_{K2,n}^{t}$;   n = 1,...,14; t = 1960,...,2014

where the $i_{2,n}^{t*}$ are the predicted asset inflation rates for Sector 2 defined by counterparts to definitions (20)-(22) and the r_2^{t*} are the predicted Sector 2 balancing nominal rates of return defined by equations (28). These predicted user costs are plotted in Figure 10. It can be seen that these predicted user costs are all positive, and that all of the series have fairly smooth trends, with the exception of assets 10, 11 and 12 (residential land, farm land and commercial land).

We conclude that our rather simple method for forming predicted asset inflation rates does lead to relatively smooth (and reasonable) user costs that could be published by statistical agencies for general use by economic analysts as well as for the construction of capital services aggregates. In the following section, we will compute capital services aggregates (and the resulting measures of Total Factor Productivity) using both Jorgensonian and predicted user costs to determine if the alternative user costs affect aggregate capital services growth for our two sectors.

Figure 9: Jorgensonian User Costs for Sector 2

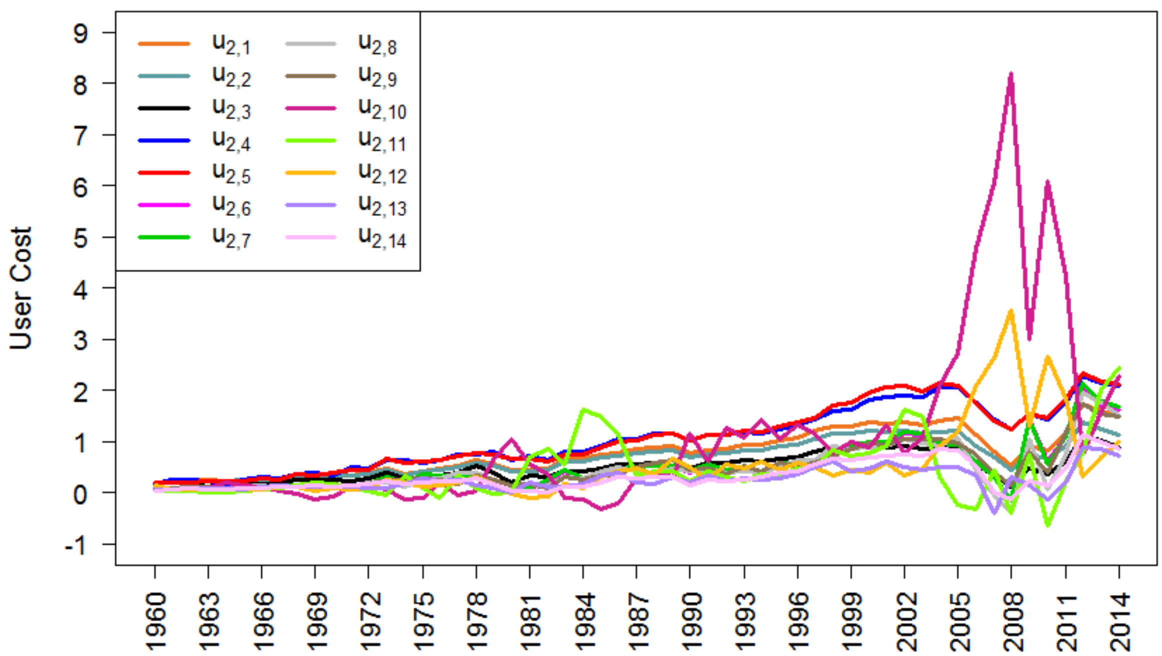
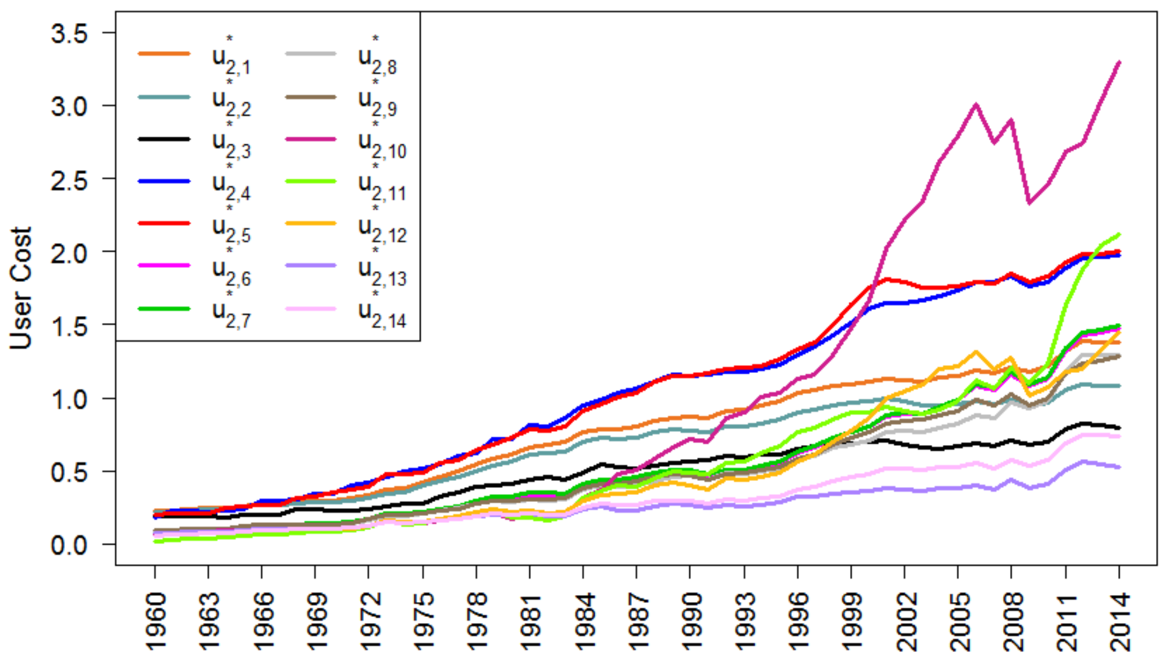


Figure 10: Predicted User Costs for Sector 2



6. Jorgensonian and Predicted Measures of Capital Services and Total Factor Productivity Growth

We use the Törnqvist formula to aggregate capital services and to aggregate all inputs, including labour services.[35] Our methodology for measuring Total Factor Productivity growth follows the methodology proposed by Diewert and Morrison (1986) and Kohli (1990). This methodology measures TFP growth over two periods as an implicit Törnqvist quantity index defined over gross outputs and intermediate inputs divided by a direct Törnqvist quantity index of primary inputs.[36] Since we have only one value added output in our BEA data base for each sector, our output index going from year t to year t+1 is simply $Q_{VA1}{}^{t+1}/Q_{VA1}{}^{t}$ for Sector 1 and $Q_{VA2}{}^{t+1}/Q_{VA2}{}^{t}$ for Sector 2. However, we will use the Törnqvist quantity index to aggregate inputs.

Let $p^t \equiv [p_1{}^t,...,p_N{}^t]$ and $q^t \equiv [q_1{}^t,...,q_N{}^t]$ denote a generic price and quantity vector for year t. Then the logarithm of the *Törnqvist chain link quantity index* Q_T going from year t to t+1 is defined as follows:

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[35] This formula was attributed to Törnqvist (1936) by Jorgenson and Griliches (1972; 83) as a discrete time approximation to the continuous time Divisia indexes that Jorgenson and Griliches (1967) (1972) advocated for aggregating inputs and outputs in productivity studies. The formula does not explicitly appear in Törnqvist (1936) but it is explicit in a follow up paper co-authored by Törnqvist; see Törnqvist and Törnqvist (1937). The formula was derived in an instructive manner by Theil (1967; 136-137) and so it is also known as the Törnqvist-Theil formula. Jorgenson and Nishimizu (1982) called the index the translog index. Diewert (1976; 118-129), Diewert and Morrison (1986) and Kohli (1990) related Törnqvist price and quantity indexes to various translog functional forms for cost, revenue and production functions.

[36] See Diewert (2014b) for a detailed explanation of the methodology and an application to US data. The land data used in this earlier study was of lower quality than the land data used in the current study.

(34) $\ln Q_T(p^t,p^{t+1},q^t,q^{t+1}) \equiv \Sigma_{n=1}^{N} (1/2)(s_n^t + s_n^{t+1})\ln (q_n^{t+1}/q_n^t)$

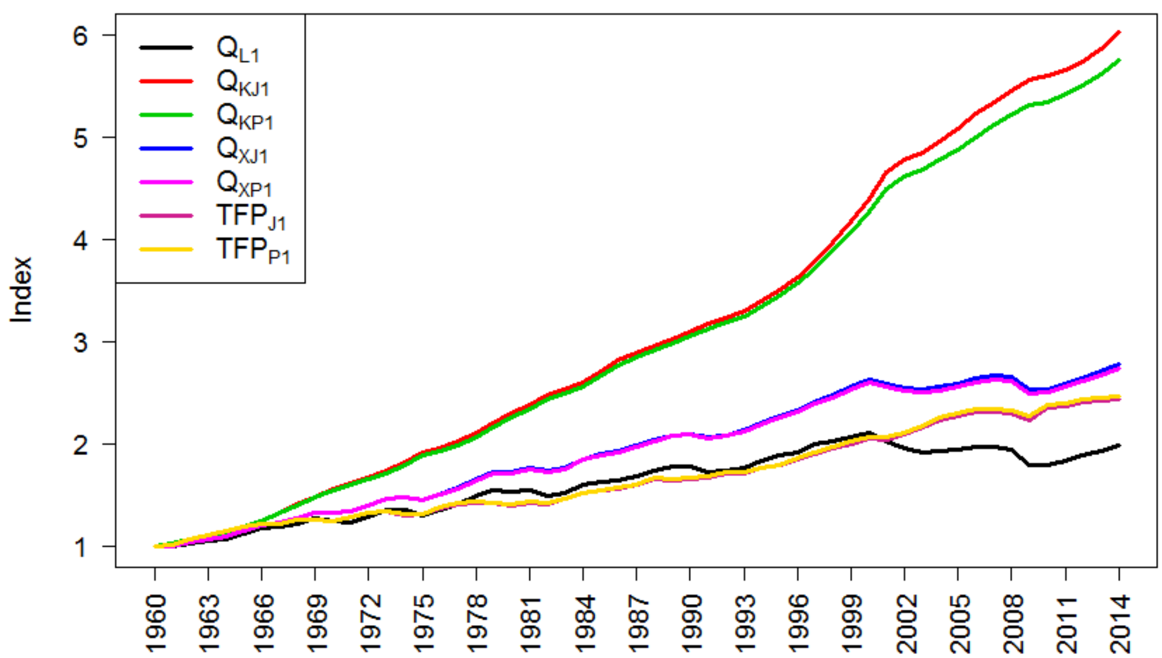
where the cost share of input n in year t is defined as $s_n^t \equiv p_n^t q_n^t/p^t \cdot q^t$ for n = 1,...,N. Note that this index can be used to aggregate quantities as long as they are all positive even though some prices may be negative.

The Törnqvist quantity index was used to aggregate the nine types of capital services used by Sector 1. Denote the aggregate *chained Törnqvist quantity index of Jorgensonian capital services* and of *predicted capital services* for Sector 1 for year t by Q_{KJ1}^t and Q_{KP1}^t respectively.[37] The Törnqvist quantity index was also used to aggregate the nine types of capital services and the one type of labour used by Sector 1. Denote the chained index for year t using Jorgensonian and predicted user costs by Q_{XJ1}^t and Q_{XP1}^t respectively.[38] Finally, the *year t levels of Jorgensonian and Predicted TFP* are defined as follows:

(35) $TFP_{J1}^t \equiv [Q_{VA1}^t/Q_{VA1}^{1960}]/ Q_{XJ1}^t/Q_{XJ1}^{1960}]$; t = 1960,...,2014;
(36) $TFP_{P1}^t \equiv [Q_{VA1}^t/Q_{VA1}^{1960}]/ Q_{XP1}^t/Q_{XP1}^{1960}]$; t = 1960,...,2014.

The quantity and TFP series are plotted in Figure 11, along with the labour input series Q_{L1}^t (normalized to equal 1 in 1960).

Figure 11: Sector 1 Indexes of Labour Quantity, and Alternative Capital Services, Aggregate Input and TFP Estimates



[37] These series are normalized to equal one in 1960.
[38] These series were also normalized to equal one in 1960. The price and value of labour input for Sector 1 in year t, P_{L1}^t and V_{L1}^t, are listed in Table 1. Define the quantity of labour used in Sector 1 in year t as $Q_{L1}^t \equiv V_{L1}^t/P_{L1}^t$. Thus we added P_{L1}^t and Q_{L1}^t to our user costs and capital stock quantities to form the overall chained Törnqvist input quantity indexes.

It can be seen that labour input into the Corporate Nonfinancial Sector grew fairly steadily to a 2.11 fold increase in 2000 but then growth levelled off and fell to a 1.79 fold increase over 1960 in 2009 and 2010. Labour input has since increased to finish off at a 1.98 fold increase over 1960 in 2014. We note that the price of labour has increased steadily (even through the Great Recession period) to end up increasing 13.95 fold over the sample period. The geometric average rate of growth of Q_{L1}^t was 1.28% per year and the geometric average rate of growth of P_{L1}^t over the sample period was 5.00% per year.

The quantity of Jorgensonian capital services increased 6.02 fold over the sample period while the quantity of predicted capital services increased only 5.75 fold. The geometric average rates of growth for these two measures of capital services were 3.38% and 3.29% per year. This difference is surprisingly small considering how different the two sets of user costs were. The price index of Jorgensonian capital services increased 6.37 fold over the sample period while the price index of predicted capital services increased 6.68 fold. The geometric average rates of growth for these two measures of capital services prices were 3.49% and 3.58% per year. One reason why there is so little difference between the two measures of capital services is that land as a share of total capital services in Sector 1 is relatively small.[39]

Sector 1 Jorgensonian input Q_{XJ1}^t increased 2.78 fold over the sample period while the quantity of predicted capital services Q_{XP1}^t increased 2.74 fold.[40] The geometric average rates of growth for these two input measures were 1.91% and 1.89% per year. This is a very small difference in growth rates. Sector 1 real value added Q_{VA1}^t grew 6.77 fold over the sample period (geometric average rate of growth was 3.61% per year). Jorgensonian TFP in Sector 1, TFP_{J1}^t, grew 2.43 fold over the sample period while predicted TFP, TFP_{P1}^t, grew 2.47 fold. The geometric average rates of growth for these two measures of Total Factor Productivity were 1.66% and 1.69% per year, a surprisingly small difference.

Another surprise is the rather high overall rate of TFP growth that the Corporate Nonfinancial Sector has been able to achieve over the 55 years in our sample. To see if there has been a TFP slowdown over the past fifteen years, we computed decade by decade geometric average rates of TFP growth.[41] Using Jorgensonian estimates for input growth, the resulting decade by decade averages were as follows: 2.57% (1960s), 1.22% (1970s), 1.51% (1980s), 1.99% (1990s), 1.09% (2000s) and 1.71% (2010s) per year.[42] There is little evidence of a productivity slowdown in Sector 1 using these sub-periods; the average TFP growth rate over the last five years in our sample is 1.71% per year, which is slightly higher than long run Jorgensonian average of 1.66% per year. However, if we consider the sub-period 2005-2014, the geometric average was 0.88%, which is substantially lower than the long run average.[43]

We turn our attention to developing alternative measures of capital services and productivity growth for Sector 2, the Noncorporate Nonfinancial Sector of the US private sector.

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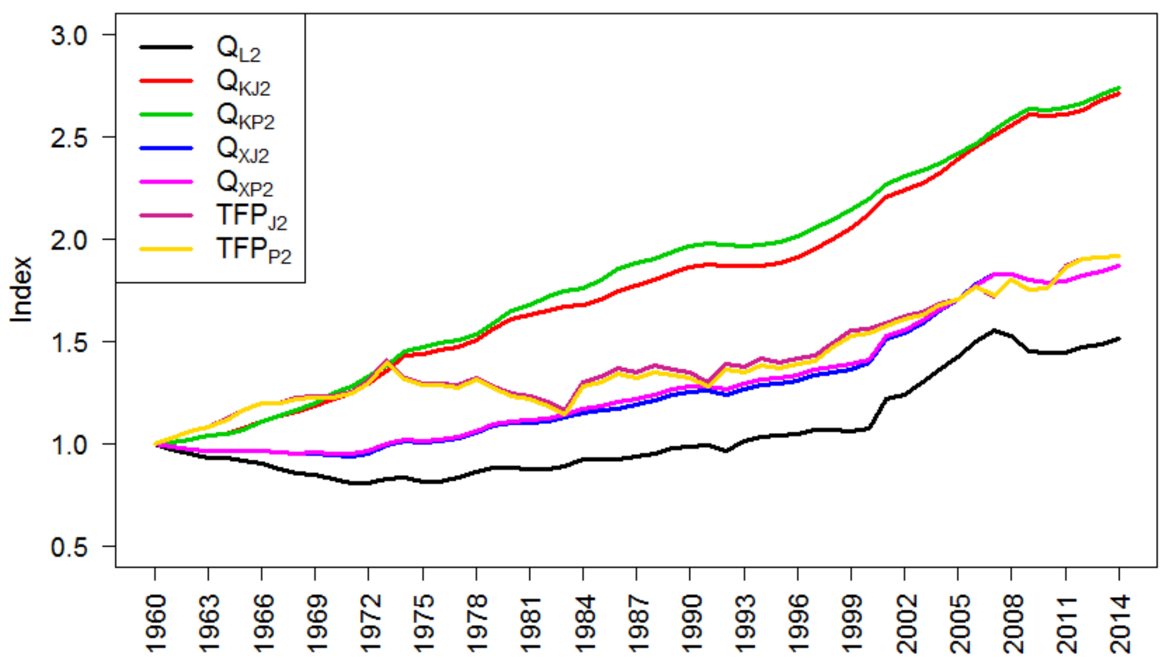
[39] Using Jorgensonian user costs, we find that the sample average input cost shares of labour, land services and reproducible capital stock services in Sector 1 were 68.6%, 2.1% and 29.3%. The sample average cost shares of residential, farm and commercial land (assets 5, 6 and 7) were only 0.05%, 0.17% and 1.85%.
[40] Note that Q_{XJ1}^t and Q_{XP1}^t (and TFP_{J1}^t and TFP_{P1}^t) cannot be distinguished in Figure 11.
[41] The last "decade" covers only the years 2010-2014.
[42] Using predicted user costs, the corresponding decade by decade geometric average rates of TFP growth in Sector 1 were as follows: 2.59%, 1.26%, 1.49%, 2.02%, 1.16% and 1.72% per year.
[43] See Diewert and Fox (2017) on potential sources of the productivity slowdown.

Again, the Törnqvist quantity index was used to aggregate the fourteen types of capital services used by Sector 2. Denote the aggregate *chained Törnqvist quantity index of Jorgensonian capital services* and of *predicted capital services* for Sector 2 for year t by $Q_{KJ2}{}^{t}$ and $Q_{KP2}{}^{t}$ respectively.[44] The Törnqvist quantity index was also used to aggregate the fourteen types of capital services and the one type of labour used by Sector 2. Denote the chained index for year t using Jorgensonian and predicted user costs by $Q_{XJ2}{}^{t}$ and $Q_{XP2}{}^{t}$ respectively.[45] Finally, the *year t levels of Jorgensonian and Predicted TFP* are defined as follows:

$$(37)\ TFP_{J2}{}^{t} \equiv [Q_{VA2}{}^{t}/Q_{VA2}{}^{1960}]/\ Q_{XJ2}{}^{t}/Q_{XJ2}{}^{1960}]\ ; \qquad t = 1960,...,2014;$$
$$(38)\ TFP_{P2}{}^{t} \equiv [Q_{VA2}{}^{t}/Q_{VA2}{}^{1960}]/\ Q_{XP2}{}^{t}/Q_{XP2}{}^{1960}]\ ; \qquad t = 1960,...,2014.$$

These quantity and TFP series, along with the labour input series $Q_{L2}{}^{t}$ (normalized to equal 1 in 1960), are plotted in Figure 12. The rates of input, output and productivity growth in Sector 2 are quite different from the corresponding rates in Sector 1 as can be seen by comparing figures 11 and 12.

Figure 12: Sector 2 Indexes of Labour Quantity, and Alternative Capital Services, Aggregate Input and TFP Estimates



Labour input into the Noncorporate Nonfinancial Sector fell to 80.7% of its initial 1960 level in 1972 but then grew fairly steadily to a 1.07 fold increase in 2000 over its initial level. Then

[44] These series are normalized to equal one in 1960 when they are listed in Table 13. The input price and quantity series used in the index number formula for $Q_{KJ2}{}^{t}$ and $Q_{KP2}{}^{t}$ are the $u_{2,n}{}^{t}$ and $u_{2,n}{}^{t*}$ listed in Tables 9 and 11 respectively and the corresponding quantity series $Q_{K2,n}{}^{t}$ are described in Table 5.

[45] These series were also normalized to equal one in 1960. The price and value of labour input for Sector 1 in year t, $P_{L2}{}^{t}$ and $V_{L2}{}^{t}$, are listed in Table 2. Define the quantity of labour used in Sector 2 in year t as $Q_{L2}{}^{t} \equiv V_{L2}{}^{t}/P_{L2}{}^{t}$. Thus we added $P_{L2}{}^{t}$ and $Q_{L2}{}^{t}$ to our user costs and capital stock quantities to form the overall chained Törnqvist input quantity indexes.

labour input growth grew rapidly to a 1.55 fold increase in 2007, fell to 1.44 in 2010 and then slowly increased to finish up with a 1.51 fold increase over its initial level. We note that the price of labour P_{L2}^t has increased steadily to end up increasing 12.67 fold over the sample period. The geometric average rate of growth of Q_{L2}^t was only 0.77% per year (compared to a 1.28% geometric rate of increase for Q_{L1}^t) and the geometric average rate of growth of P_{L2}^t over the sample period was 4.81% per year, which is close to the rate of increase for P_{L1}^t (5.00% per year).

The quantity of Jorgensonian capital services increased 2.71 fold in Sector 2 over the sample period (the Sector 1 increase was 6.02 fold) while the quantity of Sector 2 predicted capital services increased 2.74 fold. The geometric average rates of growth for these two measures of capital services were 1.86% and 1.88% per year (compared to 3.38% and 3.29% per year for Sector 1). Again, this difference in average rates of capital services growth is surprisingly small considering how different the two sets of user costs were.[46] The price index for Jorgensonian capital services increased 17.94 fold over the sample period while the price index of predicted capital services increased 17.75 fold (only 6.37 fold and 6.68 fold increases for Sector 1 capital service prices). The geometric average rates of growth for these two measures of Sector 2 capital services prices were 5.49% and 5.47% per year (the corresponding rates for Sector 1 were 3.49% and 3.58% per year). Thus since land is a much more important input in Sector 2 compared to Sector 1, the overall rate of growth in the price of capital services in Sector 2 is much greater than in Sector 1.[47] Note that these rates of service price increase for Sector 2 are higher than the rate of increase in wages for Sector 2, which was only 4.81% per year.[48] Looking at Figure 12, it can be seen that the level of predicted capital services, Q_{KP2}^t, bulged above the corresponding level of Jorgensonian capital services, Q_{KJ2}^t, over the middle of the sample period but the two series were quite close near the endpoints of our sample period.

Sector 2 Jorgensonian input Q_{XJ2}^t and predicted input Q_{XP2}^t increased 1.87 fold over the sample period (the Sector 1 counterparts were 2.78 fold and 2.74 fold increases).[49] The geometric average rates of growth for these two input measures were both 1.17% per year (1.91% and 1.89% for Sector 1). Sector 2 real value added Q_{VA2}^t grew 3.58 fold (6.77 fold for Sector 1) over the sample period and the geometric average rate of growth was 2.39% per year (3.61% for Sector 1). Jorgensonian TFP and predicted TFP in Sector 2, TFP_{J2}^t and TFP_{P2}^t, both grew 1.91 fold over the sample period (2.43 and 2.47 for Sector 1). The geometric

[46] However, the predicted asset price inflation rates are on average quite close to the average ex post asset price inflation rates. Thus on average, the two sets of user costs are similar, giving rise to similar trends in the two sets of capital service prices.

[47] Using Jorgensonian and predicted user costs, we find that the sample average input cost shares of labour and capital services were 56.7% and 43.3%. Using Jorgensonian user costs, the sample average cost shares of residential, farm and commercial land services (assets 10, 11 and 12) were 7.51%, 4.44% and 2.43%. Using predicted user costs, the sample average input cost shares for assets 10, 11 and 12 were 8.04%, 4.05% and 2.44%. These input cost shares for land are low compared to the share of land assets in total asset value: the average overall land share of total asset value was 51.6% while reproducible assets contributed 48.4% of total asset value. The average shares of the three types of land in total asset value were 28.2%, 16.4% and 7.1%. The user cost shares of capital services for land are lower than their corresponding asset value shares because the high land price inflation terms dramatically reduce land user costs relative to their asset prices.

[48] These trends in the prices and quantities of labour and capital input into Sector 2 indicate the presence of labour saving technical progress in this sector.

[49] Note that Q_{XJ2}^t and Q_{XP2}^t (and TFP_{J2}^t and TFP_{P2}^t) can hardly be distinguished in Figure 12.

average rates of growth for the two Sector 2 measures of Total Factor Productivity were both 1.21% per year (1.66% and 1.69% per year for Sector 1).

Another surprise is the rather high overall rate of TFP growth that the Noncorporate Nonfinancial Sector has been able to achieve over the 55 years in our sample. To see if there has been a TFP slowdown over the past fifteen years, we computed decade by decade geometric average rates of TFP growth.[50] Using Jorgensonian estimates for input growth, the resulting decade by decade TFP_{J2}^{t} averages were as follows: 2.32% (1960s), 0.41% (1970s), 0.64% (1980s), 1.29% (1990s), 1.22% (2000s) and 1.81% (2010s) per year.[51] Thus there is little evidence of a productivity slowdown in Sector 2 using these sub-periods; the average Jorgensonian TFP growth rate for Sector 2 over the last five years in our sample is 1.81% per year, which is slightly higher than the corresponding Jorgensonian rate of 1.71% for Sector 1 over the past five years and higher than the long run Jorgensonian average of 1.66% per year for Sector 1. However, if we consider the sub-period 2005-2014, the geometric average was 1.27%, which is substantially lower than the long run average.

Finally, we note that for the period 2000-2009, Jorgensonian TFP growth averaged 1.22% per year while the corresponding predicted TFP growth averaged 1.37% per year. This is a substantial difference. Thus, although for the most part Jorgensonian TFP growth rates based on the use of ex post asset inflation rates are close to our preferred TFP growth rates based on the use of predicted asset inflation rates, it can be seen that it is not always the case that these rates are close.

In the following section, we look at what happens to the rate of return on assets and on Jorgensonian TFP growth rates when we drop assets from the asset boundary.

7. Rates of Return and TFP Growth in Sector 1 with Alternative Asset Bases

Many national and international productivity data bases do not include money, inventories or land in their asset base.[52] Thus it is of interest to see what happens to rates of return on assets and on TFP growth when these assets are dropped from the list of productive inputs.

Recall equations (17) and (19) in Section 5 which defined the year t nominal and real rate of return on all nine assets used in Sector 1, r_1^{t} and R_1^{t} respectively. Modify equation (17) by dropping asset 9 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without monetary services, which we denote by $r_{1,M}^{t}$ and $R_{1,M}^{t}$ respectively. Now modify equation (17) by dropping assets 8 and 9 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without inventory and monetary services, which we denote by $r_{1,IM}^{t}$ and $R_{1,IM}^{t}$ respectively. Finally modify equation (17) by dropping assets 5-9 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without land, inventory and monetary services,

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[50] Again, the last "decade" covers only the years 2010-2014.

[51] Using predicted user costs, the corresponding decade by decade geometric average rates of predicted TFP growth, TFP_{P2}^{t}, were as follows, with the corresponding Jorgensonian rates of growth in brackets: 2.28% (2.32), 0.39% (0.41), 0.49% (0.64), 1.35% (1.29), 1.37% (1.22) and 1.80% (1.81) per year. Note that the difference is particularly large for the 2000s.

[52] See the EUKLEMS and World KLEMS data bases on line; European Commission (2011) and Jorgenson and Timmer (2016).

which we denote by $r_{1,LIM}^t$ and $R_{1,LIM}^t$ respectively. The alternative nominal rates can be found in Figure 13 and the alternative real rates of return can be found in Figure 14.

For each asset base, the value of capital services adds up to value added less the value of labour input. Thus as we decrease the number of assets in the asset base, the nominal and real rate of return on the remaining assets must increase and this fact is reflected in Figures 13 and 14. With all assets in the asset base, the average nominal rate of return on assets is 11.25%. Dropping monetary holdings from the asset base increases the average rate of return to 11.60% and then dropping inventory stocks further increases the average rate of return to 12.97%. Finally dropping residential, farm and commercial land from the asset base further increases the average rate of return on the remaining assets to 14.59% per year. Similarly, decreasing the asset base causes the average real rate of return on the remaining assets to go from 7.57% per year when all assets are included to 10.80% per year when money, inventories and land are dropped from the asset base. Our conclusion here is that dropping assets can substantially distort the estimated return on assets.

Recall that the year t Jorgensonian user costs $u_{1,n}^t$ for the nine assets used by Sector 1 were defined by equations (30) in Section 5. These user costs involved the nominal rates of return on assets for Sector 1, the r_1^t. The user costs $u_{1,n}^t$ were used to form the Sector 1 Jorgensonian year t capital services aggregate, Q_{KJ1}^t, and the overall Sector 1 year t input aggregate, Q_{XJ1}^t. These input aggregates along with the Sector 1 output aggregates, Q_{VA1}^t, were used to form the year t Total Factor Productivity levels, TFP_{J1}^t, for Sector 1; see equations (35). When we drop monetary assets from the list of assets, we obtain the new year t Jorgensonian balancing nominal rate of return for year t, $r_{1,M}^t$, and this new rate of return can be inserted into equations (30) for n = 1,...,8 in order to obtain new year t Jorgensonian user costs for Sector 1, which we define as $u_{1M,n}^t$. These new user costs can be used to form new year t capital services aggregates, Q_{K1M}^t, and new year t aggregate input indexes, Q_{X1M}^t, for Sector 1. In a similar fashion, when we drop both monetary assets and inventory stocks, we obtain the year t capital services aggregates, Q_{K1IM}^t, and the year t aggregate input indexes, Q_{X1IM}^t, for Sector 1. Finally, when we drop monetary assets, inventory and land stocks from the list of productive assets, we obtain the year t capital services aggregates, Q_{K1LIM}^t, and the year t aggregate input indexes, Q_{X1LIM}^t, for Sector 1. These alternative measures of aggregate capital services are used to form the alternative TFP levels, TFP_{1M}^t, TFP_{1IM}^t and TFP_{1LIM}^t. These alternative measures of (normalized) Jorgensonian capital services and TFP are plotted in Figure 15 along with the (normalized) measure of labour input for Sector 1, Q_{L1}^t.[53]

It can be seen that there are some small differences in the growth of Jorgensonian capital services for Sector 1 as we drop assets. With all assets included, capital services grew 6.026 fold; dropping money led to a 5.959 fold increase; dropping money and inventories led to a 5.706 fold increase and dropping money, inventories and land led to a 6.184 fold increase (see the highest line on Figure 15). These small differences in the rates of growth of capital services as we decrease the number of assets led to even smaller differences in the rates of TFP growth. With all assets included, Jorgensonian TFP increased 2.433 fold and as we dropped assets, there were 2.444, 2.478 and 2.416 fold increases in TFP over the sample

[53] To recover the un-normalized Q_{L1}^t, multiply the listed Q_{L1}^t series by the value of labour input in Sector 1 for 1960, which is 180.4. To recover the four un-normalized capital services series, multiply Q_{KJ1}^t, Q_{K1M}^t, Q_{K1IM}^t and Q_{K1LIM}^t by the Gross Operating Surplus for Sector 1 for 1960, which is 75.5.

period for Sector 1. The corresponding geometric rates of growth were 1.661%, 1.668%, 1.695% and 1.647% per year so that all of these annual average TFP growth rates were within 0.05 of a percentage point. These differences are too small to show up in Figure 15.

Figure 13: Sector 1 Nominal Rates of Return on Alternative Asset Bases

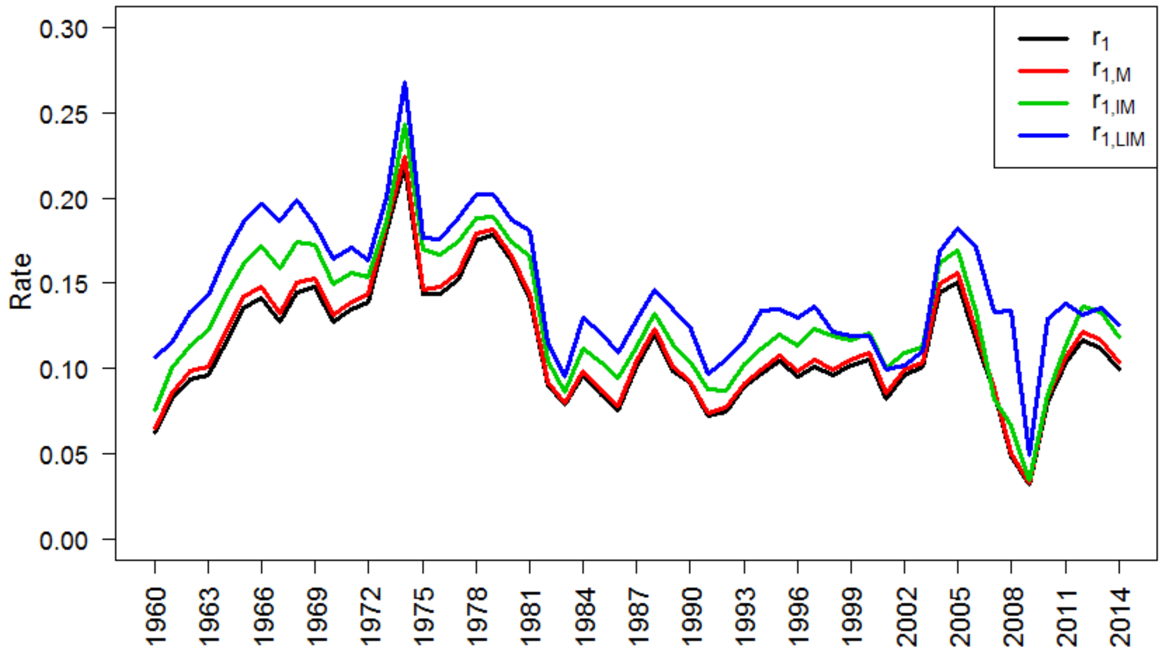


Figure 14: Sector 1 Real Rates of Return on Alternative Asset Bases

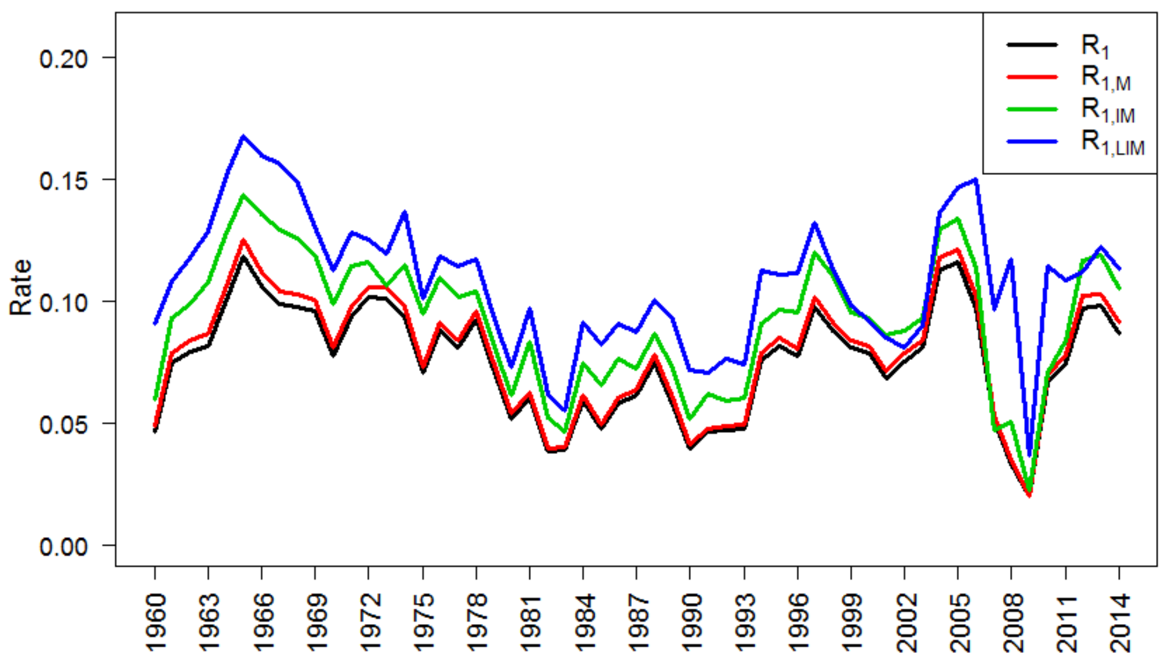
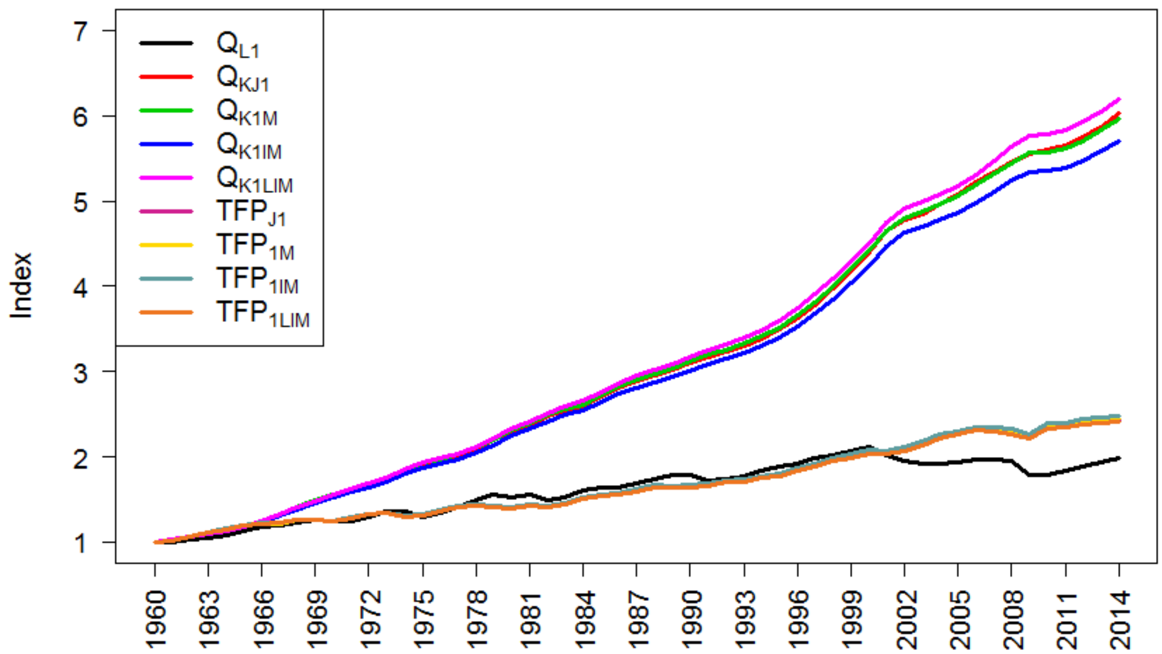


Figure 15: Sector 1 Labour and Measures of Capital Services and TFP with Alternative Asset Bases



Dropping nonreproducible assets (or zero depreciation assets) from the asset base had a significant effect on ex post rates of return on assets employed in the US Corporate Nonfinancial Sector. However, dropping zero depreciation assets had a negligible effect on overall rates of TFP growth for Sector 1. In the following section, we will see if the same conclusions hold for the US Noncorporate Nonfinancial Sector.

8. Rates of Return and TFP Growth in Sector 2 with Alternative Asset Bases

Recall equations (26) and (27) in Section 5 which defined the year t nominal and real rate of return on all fourteen assets used in Sector 2, r_2^t and R_2^t respectively. Modify equation (26) by dropping asset 14 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without monetary services, which we denote by $r_{2,M}^t$ and $R_{2,M}^t$ respectively. Further modify equation (26) by dropping assets 13 and 14 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without inventory and monetary services, which we denote by $r_{2,IM}^t$ and $R_{2,IM}^t$ respectively. Finally modify equation (26) by dropping assets 10-14 from the asset base, which gives rise to a new nominal and real rate of return on the new asset base without land, inventory and monetary services, which we denote by $r_{2,LIM}^t$ and $R_{2,LIM}^t$ respectively. The alternative nominal rates can be found in Figure16 and the alternative real rates of return can be found in Figure 17.

It can be seen that dropping assets leads to significant increases in the measured rates of return on the asset base. With all assets included, the Sector 2 average real rate of return was 9.03%; dropping money leads to a 9.49% rate of return, further dropping inventory stocks leads to a 10.03% rate of return and further dropping land leads to a huge 19.68% average rate of return on the remaining assets. Again, our conclusion here is that dropping assets can substantially distort the estimated return on assets.

Figure 16: Sector 2 Nominal Rates of Return on Alternative Asset Bases

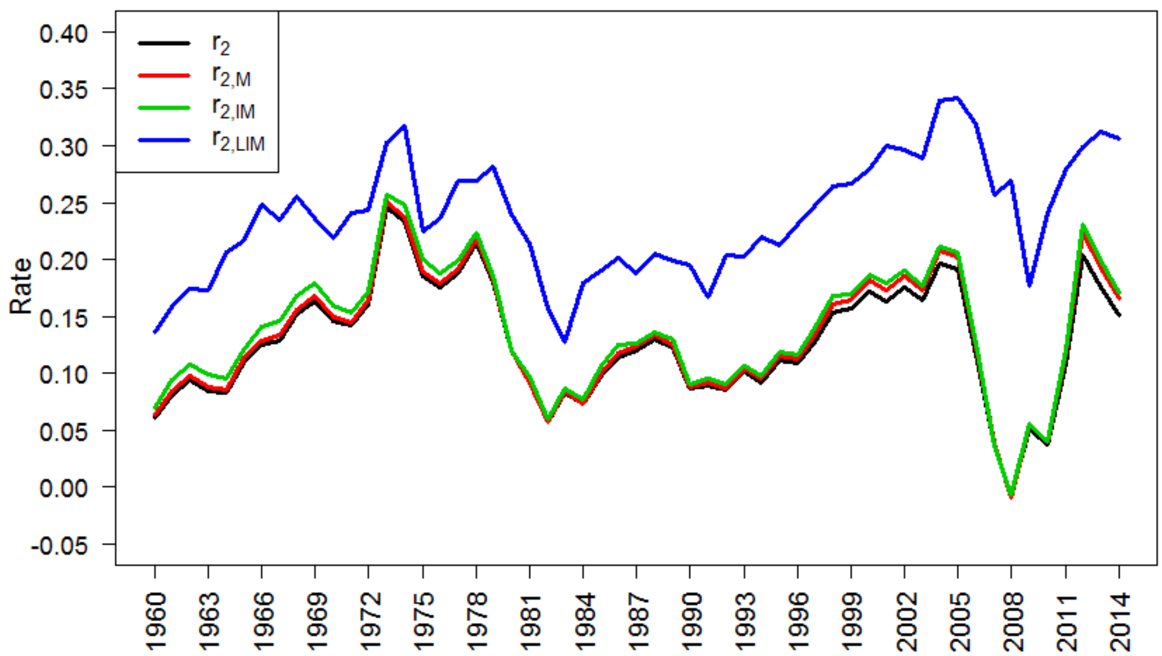
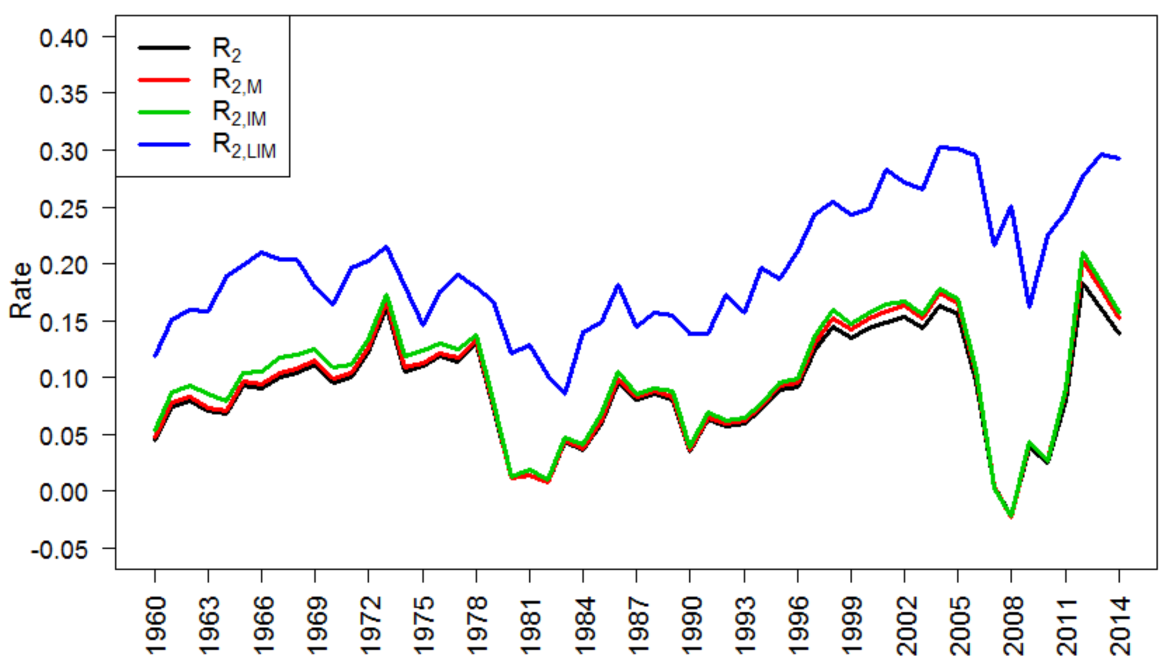
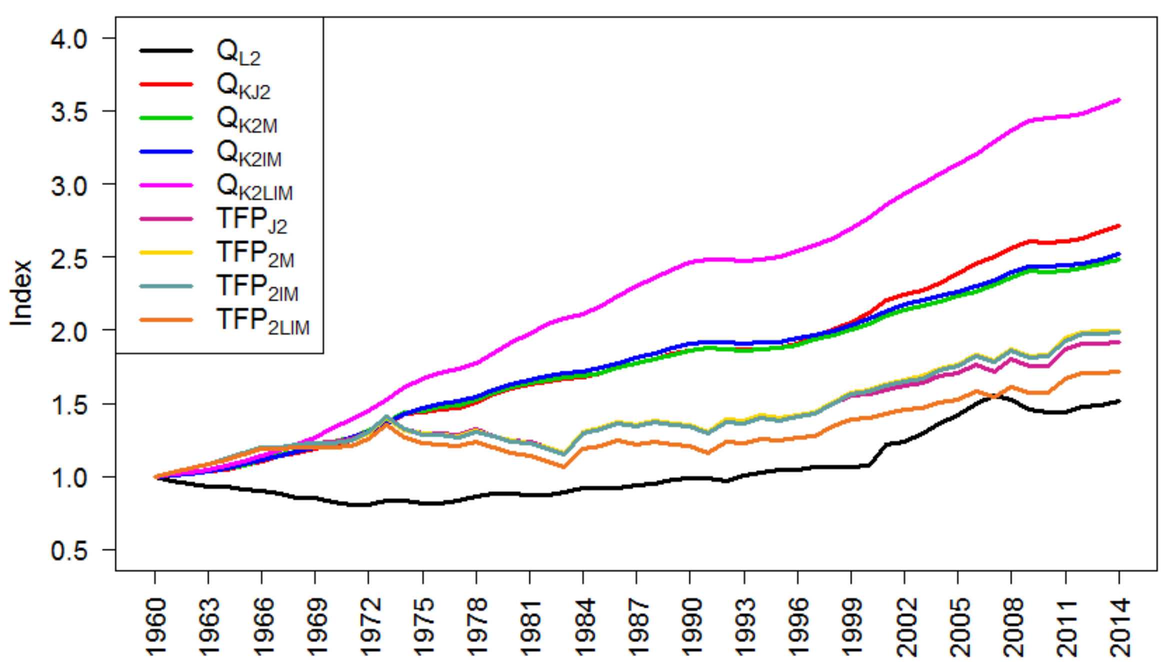


Figure 17: Sector 2 Real Rates of Return on Alternative Asset Bases



Recall that the year t Jorgensonian user costs $u_{2,n}^t$ for the fourteen assets used by Sector 2 were defined by equations (32) in Section 5. These user costs involved the nominal rates of return on assets for Sector 2, the r_2^t defined by equations (26). The user costs $u_{2,n}^t$ were used to form the Sector 2 Jorgensonian year t capital services aggregate, Q_{KJ2}^t, and the overall Sector 2 year t input aggregate, Q_{XJ2}^t. These input aggregates along with the Sector 2 output aggregates, Q_{VA2}^t, were used to form the year t Total Factor Productivity levels, TFP_{J2}^t, for Sector 2; see equations (37). When we drop monetary assets from the list of assets, we obtain the new year t Jorgensonian balancing nominal rate of return for year t, $r_{2,M}^t$, and this new rate of return can be inserted into equations (32) for n = 1,...,13 in order to obtain new year t Jorgensonian user costs for Sector 2, which we define as $u_{2M,n}^t$. These new user costs can be used to form new year t capital services aggregates, Q_{K2M}^t, and new year t aggregate input indexes, Q_{X2M}^t, for Sector 2. In a similar fashion, when we drop both monetary assets and inventory stocks, we obtain the year t capital services aggregates, Q_{K2IM}^t, and the year t aggregate input indexes, Q_{X2IM}^t, for Sector 2. Finally, when we drop monetary assets, inventory and land stocks from the list of productive assets, we obtain the year t capital services aggregates, Q_{K2LIM}^t, and the year t aggregate input indexes, Q_{X2LIM}^t, for Sector 2. These alternative measures of aggregate capital services are used to form the alternative TFP levels, TFP_{2M}^t, TFP_{2IM}^t and TFP_{2LIM}^t. These alternative measures of (normalized) Jorgensonian capital services and TFP are plotted in Figure 18 along with the (normalized) measure of labour input for Sector 2, Q_{L2}^t.[54]

Figure 18: Sector 2 Labour and Measures of Capital Services and TFP with Alternative Asset Bases



[54] To recover the un-normalized Q_{L2}^t, multiply the listed Q_{L2}^t series by the value of labour input in Sector 2 for 1960, which is 76.6. To recover the four un-normalized capital services series, multiply Q_{KJ2}^t, Q_{K2M}^t, Q_{K2IM}^t and Q_{K2LIM}^t by the Gross Operating Surplus for Sector 2 for 1960, which is 30.8.

It can be seen that there are some large differences in the growth of Jorgensonian capital services for Sector 2 as we drop assets. With all assets included, capital services grew 2.71 fold; dropping real monetary balances (which increased more rapidly than other assets, particularly in recent years) led to a 2.49 fold increase in the remaining capital services; dropping money and inventories led to a 2.52 fold increase and dropping money, inventories and land led to a 3.57 fold increase in the remaining capital services (see the highest line on Figure 17). Since land stocks grow more slowly than other capital stocks and since land is a very large component of the Sector 2 capital stock, these results are not unexpected. These large differences in the rates of growth of capital services as we decrease the number of assets led to significant differences in the rates of TFP growth. With all assets included, Jorgensonian TFP increased 1.91 fold and as we dropped assets, there were 2.00, 1.99 and 1.71 fold increases in TFP over the sample period for Sector 2. The corresponding geometric average rates of TFP growth for Sector 2 were 1.21%, 1.29%, 1.28% and 1.00% per year. Thus dropping land from the list of in scope assets significantly reduced the measured rate of Jorgensonian TFP growth. Excluding money from the list of assets also had a significant (but smaller) effect.

Our conclusion is that dropping zero depreciation assets will in general significantly increase measured rates of return on assets. On the other hand, dropping zero depreciation assets will not *always* significantly affect long run average rates of TFP growth for a sector but for land intensive sectors, it is likely to significantly decrease measured long run average rates of TFP growth.

9. Changing Shares and Inequality

There has been significant recent interest in the measured fall in the labour share of income across many industrialised economies; the implication is that there has been a change in the distribution of income as households have hetereogeneous assets, and skills which are not equally substitutable with capital.[55] In this section we examine the issue of relative labour and capital shares using our two sector data set. Specifically, we consider how the shares change if we draw a distinction between value added and (net) income.

Our approach is based on that of Hayek (1941). Recall the expression of Jorgensonian user cost for asset n from either (30) or (32): $u_{m,n}{}^t \equiv [1+r_m{}^t - (1+i_{m,n}{}^t)(1-\delta_{m,n}{}^t)]P_{Km,n}{}^t$, for sectors m = 1, 2. It is convenient for current purposes to express the *user cost value*, $UCV_{m,n}{}^t$, for asset n in sector m in the following form:

$$(39)\ UCV_{m,n}{}^t \equiv [r_m{}^t - i_{m,n}{}^t + (1+ i_{m,n}{}^t)\delta_{m,n}{}^t)]P_{Km,n}{}^tQ_{Km,n}.$$

Thus, for each asset n the user cost value, or capital services of asset n, can be decomposed into the sum of the following terms: financing cost, or waiting services,[56] $r_m{}^t\ P_{Km,n}{}^tQ_{Km,n}$, asset revaluation, $-i_{m,n}{}^t\ P_{Km,n}{}^tQ_{Km,n}$, and depreciation, $\delta_{m,n}{}^tP_{Km,n}{}^{t+1}Q_{Km,n}$. Pigou (1941) argued that an appropriate measure of income is valued added less depreciation; this accounts for the physical deterioration of assets used in producing consumption goods. It is hence an income concept that emphasizes the maintenance of physical capital. Hayek (1941), however, argued

[55] See, for example, Karabarbounis and Neiman (2014), Bridgman (2014) and Cho, Hwang and Schreyer (2017).
[56] See Rymes (1968)(1983) on the concept of waiting services.

that this would overstate income due to not taking into account the revaluation of assets from, for example, foreseen obsolescence. Thus, Hayek's is an income concept that emphasizes the real financial maintenance of capital.

Bridgman (2014) and Cho, Hwang and Schreyer (2017) have examined the impact on relative labour and capital shares of changing from a value added measure of income to a Pigou-type of income by subtracting depreciation from value added. Here we highlight the Hayekian concept of income, and thus also subtract asset revaluation to form our income measure.

That is, income is equal to the wage bill plus the capital stock times the ex post nominal rate of return on this stock, or $r_m^t P_{Km,n}^t Q_{Km,n}$ rather than the full user cost value of (39). Hence the difference between value added and this income measure is the value of depreciation and asset revaluation.

A comparison of nominal value added with Hayekian and Pigouvian nominal income is provided in Figure 19 for Sector 1.[57] It can be seen that nominal value added is generally higher than nominal income, especially since 2007. Comparing the Hayekian and Pigouvian income measures, it can be seen that the Hayekian measure is typically larger, due to positive asset revaluations in most years, and more volatile. With depreciation rates evolving relatively smoothly,[58] changes in prices of residential and commercial land in particular appear to drive much of this difference in volatility, especially around 2008.[59]

The share of capital services in value added is the user cost value of (39) summed over all assets and divided by nominal value added. These shares are plotted in Figure 20.[60] The greater volatility of nominal Hayekian income seen in Figure 19 is reflected in the capial income shares in Figure 20.[61] The generally lower capital shares in either Hayekian of Pigouvian income indicate less inequality than implied by the corresponding value added shares. In terms of long-term trends, the share of capital services in value added goes from 0.295 in 1960 to 0.367 in 2014 (a 24% increase), while our preferred share of capital in total Hayekian income goes from 0.179 in 1960 to 0.283 in 2014 (a 58% increase). Thus, while all capital shares have grown, the Hayekian income share has grown more, although with much higher year-on-year volatility. This somewhat strengthens the view of long-term increasing inequality through a shift in the relative distribution of income from labour to capital.

The corresponding results for Sector 2 are shown in figures 21 and 22.[62] From Figure 21 we again see the increased deviation between value added and income from 2007. As for Sector 1, the shares in Figure 22 indicate less inequality using income shares compared to value added shares. In terms of long-term trends, the share of capital services in value added goes from 0.287 in 1960 to 0.504 in 2014 (a 76% increase), while the share of capital income in total Hayekian income goes from 0.209 in 1960 to 0.472 in 2014 (a 125% increase). Hence, while all capital income shares have grown, the Hayekian income share has grown more. Thus,

[57] Jorgensonian ex post rates of return are appropriate in this context; see Section 5.
[58] See Appendix A8 of Diewert and Fox (2016).
[59] See tables A1 and A9 of Diewert and Fox (2016).
[60] The value added shares are the same as those in Table 1.
[61] Labour shares are of course a mirror image of these capital shares.
[62] The value added shares are the same as those in Table 2.

evidence for Sector 2 also strengthens the view of increasing inequality through a shift in the relative distribution of income from labour to capital.

Figure 19: Sector 1 Nominal Value Added and Nominal Income

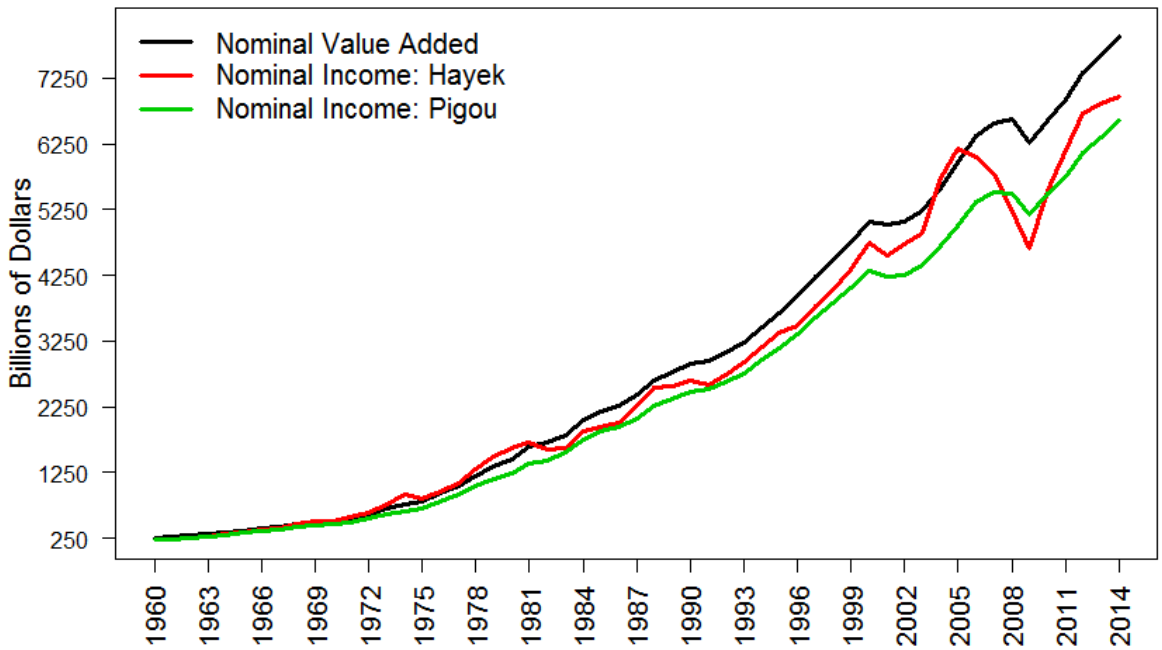


Figure 20: Sector 1 Capital Shares of Value Added and Income

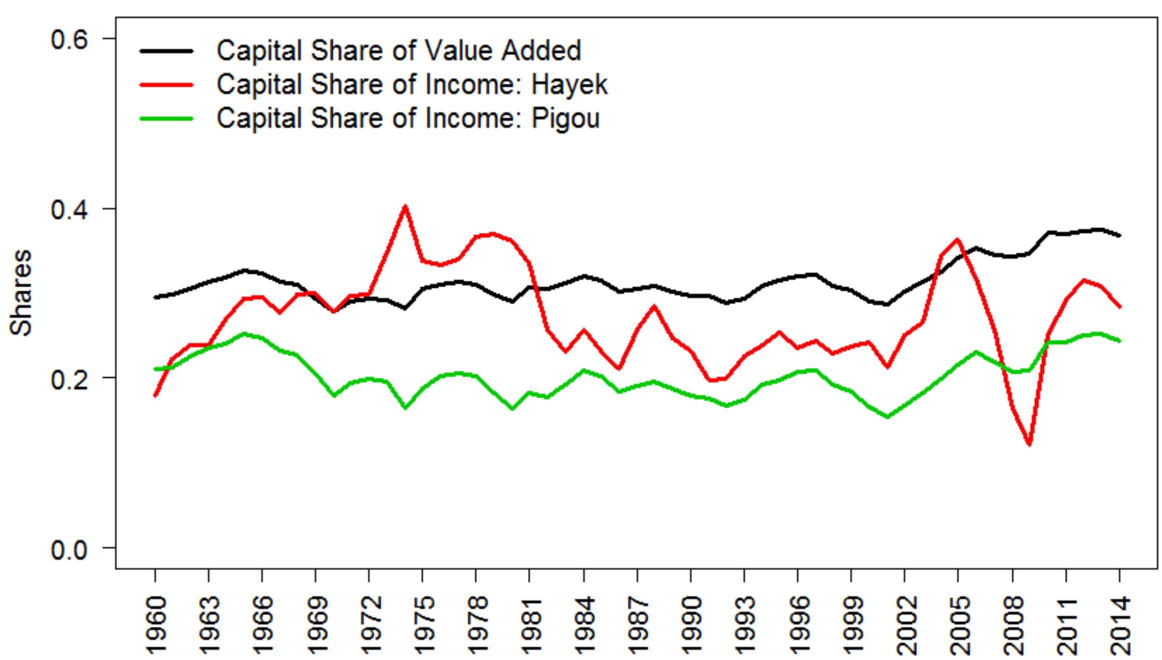


Figure 21: Sector 2 Nominal Value Added and Nominal Income

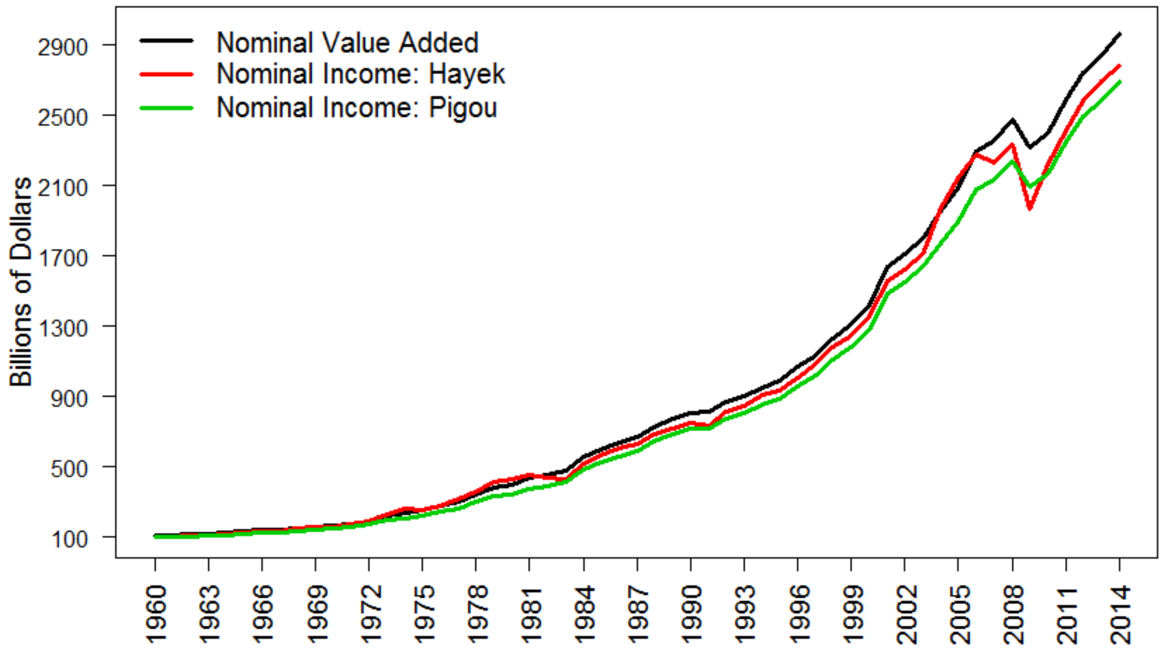
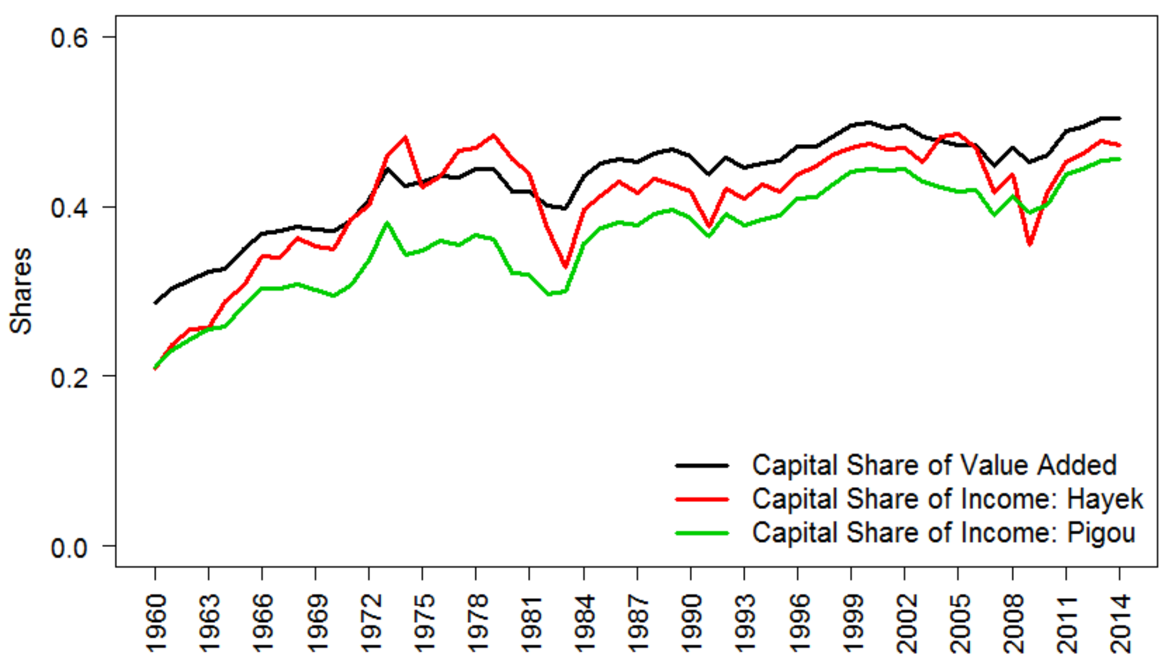


Figure 22: Sector 2 Capital Shares of Value Added and Income



10. Conclusions

A number of tentative conclusions can be drawn from the above analysis:

1. The technologies used in the Corporate Nonfinancial (Sector 1) and Noncorporate Nonfinancial (Sector 2) sectors are quite different. Sector 1 uses reproducible assets quite intensively while Sector 2 uses land and structure assets quite intensively.
2. Total Factor Productivity growth in our two sectors over the years 1960-2014 has been excellent: the TFP growth rate for Sector 1 averaged 1.66% per year using Jorgensonian user costs (1.69% per year using predicted user costs) and 1.21% per year for Sector 2 using both sets of user costs. These are very high average rates of TFP growth over such a long period, even though there has been a significant productivity slowdown over 2005-2014, especially for Sector 1.
3. Average real rates of return on productive assets employed have been quite high in both sectors. The average annual real rate of return was 7.6% per year in Sector 1 and 9.0% per year in Sector 2. There is no indication of a long run slowdown in these rates of return (but there have been massive short run fluctuations in these rates).
4. Jorgensonian user costs use actual ex post asset inflation rates in place of predicted asset inflation rates and as a result, Jorgensonian user costs are volatile and frequently negative if land assets are included in the asset boundary. These user costs are not suitable for many analytical purposes. Our predicted asset inflation rates generated relatively smooth user costs that could be used in production and cost function studies. However, Jorgensonian user costs are the right type of user cost to use when calculating ex post rates of return on assets employed.
5. Somewhat surprisingly, Jorgensonian and predicted user costs can give rise to rates of growth of capital services and Total Factor Productivity that are very close to each other. Thus for Sector 1, we found that the long run average geometric rate of capital services growth generated by the alternative user cost approaches were 3.38% and 3.29% per year for Sector 1 and 1.86% and 1.88% per year for Sector 2. The resulting alternative annual rates of TFP growth were 1.66% and 1.69% per year in Sector 1 and 1.21% per year using both Jorgensonian and predicted user costs for Sector 2. These differences are not large.
6. Dropping assets from the asset base can lead to very large biases in the measured rates of return on assets employed. Dropping land, inventory and monetary balances from the list of assets in scope increased the measured average ex post real rate of return on assets from 7.6% to 10.8% per year for Sector 1 and from 9.0% to 19.7% per year for Sector 2.
7. Dropping assets from the asset base can lead to little change in measured TFP growth rates or it can lead to significant changes. Thus the Jorgensonian average TFP growth rate for Sector 1 changed from 1.66% per year with all assets in the base to 1.65% per year, after land, inventories and real monetary balances were dropped from the list of assets. On the other hand, the Jorgensonian average TFP growth rate for Sector 2 changed from 1.21% per year with all assets in the base to 1.00% per year after land, inventories and real monetary balances were dropped from the list of assets. This is a significant change.
8. Our data are subject to a considerable degree of uncertainty. Hopefully, in future years, the BEA in cooperation with the BLS, the USDA and the Federal Reserve Board of

Governors will be able to improve the quality of the underlying data. In particular, we note that our land and labour data are weak and we are missing data on resource stocks.

9. More research is needed on choosing appropriate predicted asset inflation rates.
10. Using (net of depreciation and asset revaluation) income is more appropriate than value added for examining changes in the relative distribution of income between labour and capital, with potentially different results relating to the extent of changes in inequality.

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