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Abstract

This paper explores the role of multilateral transfers in achieving *strict* Pareto improvements in welfare, focusing on identifying conditions under which their use is warranted when carbon prices differ internationally and there are impediments to international trade. Using a general equilibrium model of international trade with global emission externalities, it is shown that strict Pareto improvements in welfare may arise from multilateral income transfers when either trade or carbon taxes are constrained away from their Pareto optimal levels. The purpose of transfers is then to account for the impact on emissions of the trade distortions and inappropriate carbon pricing. Such transfers exist if and only if a *generalized normality condition* is violated. A numerical example illustrates the transfer mechanism.

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Keywords: Global emissions, environmental externalities, multilateral income transfers, Pareto-improving reforms.

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1 Introduction

In a world economy in which there are trade and pollution distortions, the latter arising from inefficient carbon pricing, the question arises as to whether there exist multilateral transfers of income (a key issue in climate change discussions and negotiations) that are strictly Pareto-improving in welfare. And, if they do exist, under what conditions? This is the theme of the paper: to explore conditions under which there exist international transfers of income that result in strict welfare gains to all participating countries, thereby partially ameliorating the negative welfare effects of carbon price distortions in the presence of global emissions externalities.

The key task in this paper is to elaborate on the conditions that are necessary and/or sufficient to hold in the initial equilibrium for multilateral transfers to general strict Pareto improvements in welfare. This is a deceptively simple question but, as it will be shown shortly below, with an answer that is surprisingly not simple. On a more practical note, the analysis highlights an issue that has been very prominent in current discussions in climate change negotiations, and in particular with the nature of the responsibilities and actions countries need to take in relation to financial transfers to compensate for enhanced climate action. There is, of course, another way to state the problem. If, for some reason, the global economy is constrained in setting the Pareto efficient carbon and trade tax instruments, can we nevertheless ensure that the use of multilateral transfers can generate welfare gains to all participating countries? This is the objective of this paper: to identify conditions under which this is, or is not, the case.

To this end, we construct a general equilibrium model of a trading world comprising many countries and goods in which production generates (carbon) emissions that result in global negative externalities on households. Within this framework, we consider three types of policy instruments – trade taxes (tariffs), carbon taxes and multilateral income transfers. Taking trade and carbon tax setting as given, attention is turned to the role of multilateral income transfers in yielding strict Pareto improvements in welfare.

To anticipate the results that follow, what emerges is that Pareto-improving income transfers exist only if there are initial trade and/or carbon tax distortions. In this case, international transfers of income can be used to generate a strict Pareto improvement in welfare, even though the policy instrument being used is different from the ones that are causing the distortion. The existence of strict Pareto-improving transfers is shown to depend on the violation of a generalization of a Hatta normality condition, the generalization taking into account the global externality created by carbon emissions and its general equilibrium impact on world prices and households. Strict Pareto improvements do not exist, unsurprisingly, if trade taxes and carbon taxes are set at their Pareto optimal levels. Nor do strict Pareto improvements exist if all goods are normal in all countries and the world substitution matrix (which accounts for the effect of global emissions on compensated demands in all countries) exhibits net substitutability.[1] Assuming that trade taxes are Pareto optimally set (at zero) while carbon prices are not optimally set, it is proved that multilateral transfers are able to yield strict Pareto improvements in welfare. This theoretical possibility is illustrated through a numerical example.

The analysis builds on two strands of literature – one that has discussed trade and pollution reforms and one that has discussed the possibility of immiserizing transfers arising when a recipient becomes worse off when the donor gives them resources (the latter taking place in a framework within which distortions from emissions are assumed away or, indeed, not part of the model).

The first strand of literature includes contributions in the theoretical literature that have addressed the linkages between climate (environmental, more generally) and trade policies. Some of these studies have focused on non-cooperative policy formation, characterizing nationally optimal trade and environmental policies and the interplay between them as in, for instance, Markusen (1975), Baumol and Oates (1988), Copeland (1996), Panagariya et al. (2004), Copeland (2011) and Ishikawa and Kiyono (2006). Others have focused on desirable directions of reform — whether for small or large economies — when one policy instrument, environment or trade, is for some reason constrained away from its optimal level as in, for example, Copeland (1994), Hoel (1996), Turunen-Red and Woodland (2004), Neary (2006), Vlassis (2013), Kotsogiannis and Woodland (2013), and Keen and Kotsogiannis (2014). This latter literature has, in particular, characterized Pareto-efficient allocations in which potentially three sets of policy instruments may be deployed: international lump-sum transfers, carbon pricing, and trade tariffs.[2] The first set of these policy instruments is naturally directed to equity concerns, moving the world around its utility possibly frontier; the second set is naturally targeted to controlling emissions; and the third set would have no role if the other two instruments were optimally deployed. Attention has thus focused on the implications of various constraints on these policy instruments for the setting of the other policy instruments to achieve constrained Pareto-efficient or Pareto-improving welfare outcomes. However, these analyses have been un-

[1] The perspective taken here reinforces, in some sense, a plausibly held belief that multilateral transfers might not deliver strict Pareto improvements — particularly under climate change conditions.

[2] See also the related work of Keen and Wildasin (2004), who characterize Pareto efficient taxation (commodity taxes and trade taxes), with and without lump sum transfers, in a world economy that does not incorporate environmental externalities.

dertaken without delving into the conditions required for international lump sum transfers amongst participating countries to deliver (or not) strict Pareto improvements in welfare.[3] That is the focus of the current paper.

The analysis here also relates to the international trade literature that has analyzed the transfer problem, as in Turunen-Red and Woodland (1988). They considered the age-old question of whether a transfer of income between two countries necessarily benefits the recipient at the expense of the donor. In a general equilibrium model with many countries, they showed that interesting paradoxes can occur and, in particular, that it may be possible for multilateral transfers of income to improve the welfare of every country in the world, provided that there are trade distortions in the initial equilibrium. The transfers thus exploit the trade tax distortions to generate strict Pareto improvements. Their model, however, did not (and did not need to) consider the possibility of environmental distortions. In the present paper, we explicitly model global environmental externalities and show that Strict Pareto-improving transfers may exist when there are carbon tax distortions but no trade tax distortions.

The plan of this paper is as follows. Section 2 sets out the model of a world economy comprising many countries and goods and embodying global carbon emission externalities. Section 3 provides a general characterization of the necessary and sufficient conditions required for strict Pareto-improving international lump sum transfers to exist, while Section 4 then characterizes conditions required for international lump sum transfers not to exist. Section 5 provides an example that further illustrates the mechanism at work. Finally, Section 6 provides some brief concluding remarks.

2 The structure of the model

The model is based upon those of Turunen-Red and Woodland (2004) and Keen and Kotsogiannis (2014). It is a standard perfectly competitive general equilibrium model of international trade in which there are J countries, indexed by the superscript j, that trade in N commodities the production of which generates pollution. The N-vector of international commodity prices is denoted by p. International trade is subject to trade taxes (or subsidies), the vector of which is denoted in country j by τ^j. If $\tau_i^j > 0$ ($\tau_i^j < 0$) and commodity i is being imported by country j, then τ_i^j is an import tariff (import subsidy); and if $\tau_i^j > 0$ ($\tau_i^j < 0$) and commodity i is being

[3] The point here is that many contributions in the literature (as the ones referred to above) are using implicitly the conditions identified here, but none has explicitly characterize the conditions required for international lump transfers to exist (or not).

exported by country j, then τ_i^j is an export subsidy (export tax). The domestic commodity price vector in country j is thus given by the N-vector $p^j = p + \tau^j$.[4,5]

The production of each commodity generates some pollutant with the N-vector z^j denoting emissions in country j.[6] This formulation allows for emissions to be distinguished by the industry (product) of origin. Total emissions in country j are thus given by $1_N^\intercal z^j$, where 1_N is the N-vector of 1s and the superscript $^\intercal$ indicates transposition. Global emissions, on which damage in each country depends, are thus given by the scalar

$$k = 1_N^\intercal \sum\nolimits_{j=1}^{J} z^j. \tag{1}$$

Pollution discharges in country j are subject to pollution taxes, given by the N-vector t^j. Such pollution taxes are, in general, permitted to be sector (product)-specific.

The production sector in country j is competitive and characterized by a revenue function (with the standard properties of homogeneity, convexity and differentiability), which takes the form

$$G^j(p^j, t^j, v^j) = \max_{y^j, z^j} \{ p^{j\intercal} y^j - t^{j\intercal} z^j \quad : \quad f^j(y^j, z^j) \leq 0 \}, \tag{2}$$

where $f^j(\cdot)$ is the implicit production possibility frontier in country j, with v^j being the vector of endowments and y^j the vector of net outputs of traded goods. Notice, following from (2) — and as an envelope property — that[7,8]

$$G_p^j(p^j, t^j) \equiv \frac{\partial G^j(p^j, t^j)}{\partial p^j} = y^j, \tag{3}$$

$$G_t^j(p^j, t^j) \equiv \frac{\partial G^j(p^j, t^j)}{\partial t^j} = -z^j. \tag{4}$$

[4] The framework is consistent with the most-favoured nation principle, in the sense that each country applies the same tariff rates to all other countries.

[5] Consumption taxes do not feature in the model as their inclusion does not offer any additional insights.

[6] This is, of course, a rather specific form of emissions (best suited to the concentration of greenhouse gasses in the global atmosphere). Generalizing this to include many types of pollutants (expressed, appropriately modified, by k being a vector) that may have differential impacts across countries (as in Turunen-Red and Woodland, 2004) is feasible at a cost of some additional notation. This generalization, however, is not pursued here as it is beyond the focus of the analysis.

[7] Throughout the dependence of function on the vector of (non-polluting) endowments for brevity (and being fixed) is suppressed. Any abatement technology that might be available to countries is implicit in the description of the production sector.

[8] Throughout, we use subscripts to denote derivatives as in the expressions below.

The consumption sector in country j is characterized by the (restricted) expenditure function

$$E^j(p^j, u^j, k) = \min_{x^j}\{p^{j\intercal}x^j : U^j(x^j, k) \geq u^j\}, \tag{5}$$

which is concave and linearly homogeneous in prices, increasing in utility u^j and increasing in global emissions k. The last property implies that higher global pollution requires greater expenditure on goods to maintain the level of utility (the utility function is decreasing in k). Shephard's lemma implies that the price gradient $E^j_p(p^j, u^j, k) \equiv \frac{\partial E^j(p^j,u^j,k)}{\partial p^j}$ gives the vector of compensated demands, whereas the scalar $E^j_k(p^j, u^j, k) \equiv \frac{\partial E^j(p^j,u^j,k)}{\partial k}$ is the compensation required for a marginal increase in global emissions. That is, it is the marginal willingness to pay for pollution reduction.

It will prove convenient to make use of the net revenue function, denoted by $S^j\left(p^j, t^j, u^j, k\right)$ and defined as the difference between national revenues $G^j\left(p^j, t^j\right)$ and expenditures $E^j\left(p^j, u^j, k\right)$. The net revenue function is

$$S^j\left(p^j, t^j, u^j, k\right) \equiv G^j\left(p^j, t^j\right) - E^j\left(p^j, u^j, k\right), \tag{6}$$

with the gradient vector with respect to product prices

$$S^j_p\left(p^j, t^j, u^j, k\right) = G^j_p\left(p^j, t^j\right) - E^j_p\left(p^j, u^j, k\right), \tag{7}$$

giving country j's compensated net-export vector and the gradient vector with respect to carbon taxes

$$S^j_t = G^j_t = -z^j, \tag{8}$$

giving the pollution N-vector in country j.[9] Since u^j and k only appear in the household expenditure function, the affects of marginal changes in these variables upon S^j are given by $S^j_u = -E^j_u < 0$ and $S^j_k = -E^j_k < 0$. We assume, without loss of generality, that $S^j_u = -E^j_u = -1$, and hence $-S^j_{pu} = E^j_{pu}$ may be interpreted as the income derivative of country j's Marshallian demand functions at the initial equilibrium.[10]

Assuming that countries impose arbitrary fixed tariffs on their net imports and carbon taxes

[9] For the properties of these functions, see Woodland (1982).

[10] This follows from the fact that $E^j_p\left(p^j, u^j\left(p^j, m^j\right), k\right) = x^j\left(p^j, m^j, k\right)$, which implies that $E^j_{pu}\left(p^j, u^j\left(p^j, m^j\right), k\right)\partial u^j/\partial m^j = x^j_m$. Since $\partial u^j/\partial m^j = E^j_u$. With $E^j_u = 1$, it then follows that $E^j_{pu}\left(p^j, u^j\left(p^j, m^j\right), k\right) = x^j_m$. Expression x^j_m is the income derivative of country j's Marshallian demand functions.

on pollution emissions, the world equilibrium is characterized by the system of equations

$$p^{\intercal} S_p^j(p^j, t^j, u^j, k) = b^j, \; j = 1, ..., J, \tag{9}$$

$$\sum\nolimits_{j=1}^{J} S_p^j \left(p^j, t^j, u^j, k\right) = 0_N, \tag{10}$$

$$\sum\nolimits_{j=1}^{J} b^j = 0, \tag{11}$$

$$-1_N^{\intercal} \sum\nolimits_{j=1}^{J} S_t^j \left(p^j, t^j\right) = k, \tag{12}$$

where, as noted earlier, $p^j = p + \tau^j$.

Equation (9) is the national budget constraint in every country j, stating that the value (at world prices) of net exports (the trade balance) must be equal to a constant b^j. If $b^j = 0$ then country j has a zero trade balance, while b^j different from zero implies a financial transfer from country j to the rest of the world.[11] Since world trade must balance in value, equation (11) must hold. The N equations in (10) are the world market equilibrium conditions for tradeable commodities, stating that the world excess supplies for these goods must be zero. Equation (12) indicates how aggregate world pollution depends on the national vectors of pollution, which depend on domestic prices for products and carbon taxes. Given the tariff vectors τ^j, $j = 1, ..., J$, the carbon tax vectors t^j, $j = 1, ..., J$, and the vector of multilateral transfers $b = (b^1, ..., b^J)^{\intercal}$ satisfying (11), the market equilibrium conditions (10), the national budget constraints (9) and the world pollution equation (12) determine the competitive equilibrium world price N-vector for tradeable commodities, p, the world pollution level, k, and the vector of country utilities $u = (u^1, ..., u^J)^{\intercal}$.[12]

Prior to undertaking a complete analysis of the comparative statics of the model, it is instructive to undertake a preliminary examination of the model. Perturbation of equation (9) for country j with respect to world prices, transfers and global emissions reveals that

$$p^{\intercal} S_{pu}^j du^j = db^j - S_p^{j\intercal} dp - \left(\tau^{j\intercal} S_{pp}^j + t^{j\intercal} G_{tp}^j\right) dp - p^{\intercal} S_{pk}^j dk. \tag{13}$$

This shows how utility for a country j is affected by an income transfer, a change in world

[11] Equation (9) can also be written as $E^j\left(p^j, u^j, k\right) = G^j\left(p^j, t^j\right) - b^j + R^j\left(p^j, t^j, u^j, k\right)$, $j \in J$, where the revenues (trade and carbon taxes) in country j are given (following (7) and (8)) by $R^j\left(p^j, t^j, u^j, k\right) = -\tau^{jT} S_p^j - t^{jT} S_t^j$. This implies that expenditure in country j (for given global emissions k) is equal to GDP less any financial transfer b^j to the rest of the world, plus any additional tax revenue R^j returned to the consumer in that country in a lump sum fashion.

[12] Some useful notation: If $x = (x_1, ..., x_N)^{\intercal}$ then $x \gg 0$ means $x_n > 0$ for all $n = 1, ..., N$; $x > 0_N$ means $x_n \geq 0$ for all $n = 1, ..., N$, and $x \neq 0$; and $x \geq 0$ means $x_n \geq 0$ for all $n = 1, ..., N$. The existence of a competitive equilibrium solution with $p \gg 0$ is assumed, and no restrictions are imposed on this equilibrium.

prices and a change in global emissions. Thus, there are several effects upon utility in country j. The first one, given by db^j, is the direct effect on utility in country j of an income transfer: if $p^\intercal S^j_{pu} < 0$, a reduction in the transfer b^j to the rest of the world (or an increase in the transfer received by j) confers a utility gain to this country. The second effect, given by $-S^{j\intercal}_p dp$, is the familiar terms-of-trade effect: if a change in international prices increases the terms of trade ($S^{j\intercal}_p dp > 0$) for country j, then its welfare increases. The third effect, given by $\left(\tau^{j\intercal} S^j_{pp} + t^{j\intercal} G^j_{tp}\right) dp$, gives the change in trade and carbon tax revenues — respectively, $\tau^{j\intercal} S^j_{pp}$ and $t^{j\intercal} G^j_{tp}$ — as the international price N-vector p changes, keeping utility, transfers and global emissions constant. If tariff/tax revenue increases as a result of the price change, then welfare increases. The final term, given by $p^\intercal S^j_{pk} dk$, indicates that utility is reduced by an increase in global emissions since $-p^\intercal S^j_{pk} > 0$.

Taking into account the fact that global emissions are a function of world prices, given tariffs and carbon taxes, as denoted by

$$\kappa(p) = -1^\intercal_N \sum\nolimits_{j=1}^{J} S^j_t, \tag{14}$$

with price derivative

$$\kappa^\intercal_p = -1^\intercal_N \sum\nolimits_{j=1}^{J} S^j_{tp}, \tag{15}$$

the expression (13) for the change in utility for a country j may be rewritten as

$$p^\intercal S^j_{pu} du^j = db^j - S^{j\intercal}_p dp - \left[\tau^{j\intercal}\left(S^j_{pp} + S^j_{pk}\kappa^\intercal_p\right) + t^{j\intercal} G^j_{tp}\right] dp. \tag{16}$$

The last term in this expression gives the complete change in tariff/tax revenue arising from a change in world prices, taking into account the impact of this change upon global emissions and the impact of this upon tariff/tax revenue. The term $S^j_{pp} + S^j_{pk}\kappa^\intercal_p$ is the pollution-augmented net substitution matrix in country j, which gives the responses in the net exports to changes in the terms of trade, when consumer compensated demands respond to the endogenous change in global pollution emissions.

Equation (16) is central in the analysis that follows as it identifies all welfare effects that are associated with the transfers and their summation that determines total welfare. What this points to is that, at least in principle, a multilateral transfer of purchasing power across countries (even in the absence of trade distortions) could reduce the welfare of every country. The first two effects will disappear if the objective is the maximization of total world utility.

To see this — making use of (10) and (11) — the sum of (13) over all J countries gives

$$\sum_{j=1}^{J} p^{\intercal} S_{pu}^{j} du^{j} = \sum_{j=1}^{J} \left[\tau^{j\intercal} \left(S_{pp}^{j} + S_{pk}^{j} \kappa_{p}^{\intercal} \right) + t^{j\intercal} G_{tp}^{j} \right] dp. \tag{17}$$

Clearly, (17) shows that if $p^{\intercal} S_{pu}^{j} < 0$, $j = 1, ..., J$, then a necessary condition for a strict Pareto improvement is for the right hand side of (17) to be negative. It is thus necessary that the world price vector alters so that the compensated world tariff revenue increases. If the right hand side is zero, such as when all countries have free trade and have zero carbon taxes, then a necessary (but not sufficient) for a strict Pareto improvement reform is that not all $p^{\intercal} S_{pu}^{j}$, $j = 1, ..., J$, terms be of the same sign. The implication of this is that some commodities must be inferior in some countries but normal in others.

We next turn to the formal characterization of necessary and sufficient conditions under which there exits a multilateral transfer of income that will raise the welfare in every country.

3 Existence of strict Pareto-improving multilateral transfers

The analysis that follows proceeds by characterizing the conditions under which there exists a multilateral transfer of income such that there is a Pareto improvement in welfare, assuming that all tariff and carbon tax rates are given but taking into account the general equilibrium impacts of the income transfer upon world prices, world pollution and national utility levels. Formally, the analysis proceeds by using Motzkin's Theorem of the Alternative to characterize the necessary and sufficient conditions under which a Pareto-improvement exists when multilateral transfers may be endogenously chosen.

The system (9)-(12) can be differentiated with respect to utility levels, world prices, world pollution and income transfers at the initial equilibrium. This yields the differential system

$$Adu + Bdp + Cdb + Ddk = 0_{J+N+2}, \tag{18}$$

where the matrices A, B, C and D are defined by[13]

$$A \equiv \begin{bmatrix} p^\intercal S^1_{pu} & 0 & \cdots & 0 \\ 0 & p^\intercal S^2_{pu} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & p^\intercal S^J_{pu} \\ S^1_{pu} & S^2_{pu} & \cdots & S^J_{pu} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; B \equiv \begin{bmatrix} p^\intercal S^1_{pp} + S^{1\intercal}_p \\ p^\intercal S^2_{pp} + S^{2\intercal}_p \\ \vdots \\ p^\intercal S^J_{pp} + S^{J\intercal}_p \\ S_{pp} \\ 0^\intercal \\ 1^\intercal_N S_{tp} \end{bmatrix},$$

$$C \equiv \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \\ 0_N & 0_N & \cdots & 0_N \\ 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; D \equiv \begin{bmatrix} p^\intercal S^1_{pk} \\ p^\intercal S^2_{pk} \\ \vdots \\ p^\intercal S^J_{pk} \\ S_{pk} \\ 0 \\ 1 \end{bmatrix}, \tag{19}$$

and the vectors of change are

$$du \equiv \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^J \end{bmatrix}, \; dp \equiv \begin{bmatrix} dp_1 \\ dp_2 \\ \vdots \\ dp_N \end{bmatrix}, \; db \equiv \begin{bmatrix} db^1 \\ db^2 \\ \vdots \\ db^J \end{bmatrix}. \tag{20}$$

In these expressions,

$$S_{pp} \equiv \sum\nolimits_{j=1}^J S^j_{pp} \tag{21}$$

is the world substitution matrix, which gives the aggregate world (compensated) substitution effects on net excess supply of changes in international prices, p, in the absence of environmental changes, and

$$S_{tp} \equiv \sum\nolimits_{j=1}^J S^j_{tp} \tag{22}$$

gives the negative of the change in global emissions due to changes in international prices, p, and

$$S_{pk} \equiv \sum\nolimits_{j=1}^J S^j_{pk} \tag{23}$$

[13] Matrix A is of dimension $(J+N+2)\times J$, B is of dimension $(J+N+2)\times N$, C is of dimension $(J+N+2)\times J$, and D is of dimension $(J+N+2)\times 1$.

gives the change in the compensated net supply vectors across all countries as a consequence of changes in global emissions, k.

Recalling from (14) that the impact of world prices on global emissions, given the trade taxes and carbon taxes, is given by the vector

$$\kappa_p^\intercal = -1_N^\intercal \sum\nolimits_{j=1}^{J} S_{tp}^j = -1_N^\intercal S_{tp}, \tag{24}$$

the matrix defined by

$$\tilde{S}_{pp} \equiv S_{pp} + S_{pk}\kappa_p^\intercal \tag{25}$$

is the pollution-augmented world net substitution matrix, which gives the responses in the world net exports to changes in the terms of trade, when consumer compensated demands respond to changes in pollution emissions arising from the world price change. According to (25), changes in world prices affect excess supply of products directly via S_{pp} and indirectly via the change in consumption plans resulting from changes in global emissions.[14] Notice also that this pollution-augmented world substitution matrix can be written in alternative forms as[15]

$$\tilde{S}_{pp} = \left[\begin{array}{cc} \tilde{S}_{p1} & \tilde{S}_{pq} \end{array}\right] = \left[\begin{array}{cc} \tilde{S}_{11} & \tilde{S}_{1q} \\ \tilde{S}_{q1} & \tilde{S}_{qq} \end{array}\right] = \left[\begin{array}{cc} S_{11} + S_{1k}\kappa_p^\intercal & S_{1q} + S_{1k}\kappa_p^\intercal \\ S_{q1} + S_{qk}\kappa_p^\intercal & S_{qq} + S_{qk}\kappa_p^\intercal \end{array}\right]. \tag{26}$$

It will be assumed that the sub-matrix corresponding to non-numeraire goods $j = 2, ..., N$ defined by

$$\tilde{S}_{qq} \equiv S_{qq} + S_{qk}\kappa_p^\intercal \tag{27}$$

is of full rank and therefore invertible.[16]

Next we establish conditions under which there exists a multilateral transfer of income that will raise the level of welfare in every country j. A strict differential Pareto-improving multilateral income transfer is defined as a transfer, db, such that (du, dp, db, dk) solves the differential system (19) with $du \gg 0$. That is, the income transfer, along with the general equilibrium changes in world prices and global emissions, yields an increase in the utility level for every

[14] To see this in a clear way first notice that $S_{pk}^j = -E_{pk}^j$ and hence that $S_{pk}^j\kappa_p^\intercal = -E_{pk}^j\kappa_p^\intercal$. The summation of this expression over all J countries gives $\sum_{j=1}^J S_{pk}^j\kappa_p^\intercal = -\sum_{j=1}^J E_{pk}^j\kappa_p^\intercal$, which implies that $S_{pk}\kappa_p^\intercal = -E_{pk}\kappa_p^\intercal$. This, as stated in the text above, is the reduction in world consumption of goods due to the change in global emissions caused by the change in world prices for goods.

[15] This follows from using (6) and (12), after using (8).

[16] $\tilde{S}_{qq}$ is an important matrix and will be central in the analysis that follows. Invertibility of this matrix is a regularity assumption that allows a solution for prices — it implies that there is sufficient substitutability so the function is smooth and can be solved for the prices.

country (du is strictly positive). The conditions for the existence of such a multilateral transfer of income are established using Motzkin's theorem of the alternative (Mangasarian, 1969, p.34).

We perform a conventional normalization by fixing the price of commodity 1, ignore the market equilibrium condition for that commodity, following Walras' Law, and assume without loss of generality that the numeraire good is freely traded. Accordingly, it follows that $\tau_1^j = 0$ and $p_1^j = 1$, $j = 1, ..., J$, and thus that $p^\intercal = (1, q^\intercal)$ where $q^\intercal = (p_2, \ldots, p_N)$. We have the following result.

Lemma 1 *A strict Pareto-improving multilateral transfer (with $p_1 = 1$) exists if and only if there is no vector $y = (y_2^\intercal, y_3)^\intercal \in R^N$ such that*

$$y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j \leq 0 \ , \ j = 1, \ldots, J, \tag{28}$$

$$y_2^\intercal \left(S_{qq} + S_{qk}\kappa_p^\intercal\right) + y_3 p^\intercal \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) = 0_{N-1}^\intercal, \tag{29}$$

with (28) holding with strict inequality for at least one country j.

Proof. The proof of the lemma makes use of Motzkin's theorem of the alternative and is provided in the Appendix. ■

While Lemma 1 provides the necessary and sufficient conditions for a strict Pareto-improving multilateral transfer to exist, we proceed to use this result to provide an alternative characterization that is more readily interpreted from an economics viewpoint. Equipped with Lemma 1, one can define the country-specific scalars

$$\beta^j \equiv p^\intercal S_{pu}^j - p^\intercal \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} S_{qu}^j, \ j = 1, \ldots J. \tag{30}$$

Using this definition, we can establish the following characterization of the existence conditions.

Proposition 1 *Let the pollution augmented world net substitution matrix $\tilde{S}_{qq} \equiv S_{qq} + S_{qk}\kappa_p^\intercal$ have full rank, and let $p_1 = 1$. Then, a strict Pareto-improving differential multilateral transfer exists if and only if there is no scalar $y_3 \in R$ such that*

$$y_3 \beta^j \leq 0 \ , \ j = 1, \ldots J, \tag{31}$$

and with strict inequality for at least one country j, where β^j is given by (30).

Proof. The proof of the proposition is provided in the Appendix. ■

Proposition 1 has a clear interpretation. It is evident from the inequalities in (31) that if all β^j have the same sign — either positive or negative — then a $y_3 \in R$ that solves (31) does exist (and it can be either positive or negative, but not zero). In these cases, there are no multilateral transfers that can generate strict Pareto improvements in welfare.[17]

On the other hand, strict Pareto-improving multilateral transfers exist when there does not exist a scalar y_3 that solves (31). This case occurs when at least two countries have non-zero β^j, $j = 1, \ldots J$, terms that differ in sign. What this implies in practice is that the existence of strict Pareto-improving multilateral transfers has been narrowed down to the sign structure of easily recognizable quantities defined by the country-specific scalars β^j, $j = 1, \ldots J$, in (30). This result may be formalized as in the following corollary to Proposition 1.

Corollary 1 *Under the assumptions of Proposition 1, a strict Pareto-improving transfer of income exists if, and only if, at least two of the scalars β^j, $j = 1, \ldots J$, differ in sign.*

We have demonstrated that a sufficient condition for Pareto-improving multilateral transfers to exist is that there exist at least two countries whose β^j scalars differ in sign. This condition has an intuitive interpretation, and one that relates to the well-known Hatta Normality Condition. To see this notice that for a small open economy j the Hatta Normality Condition is

$$p^{\intercal} S^j_{pu} < 0, \tag{32}$$

which implies that all income effects on net exports (weighted by the world N-vector of international commodity prices) are strictly negative.[18] In the present framework, however, things are different in the sense that our normality condition for country j is

$$\beta^j \equiv \hat{p}^{\intercal} S^j_{pu} < 0, \tag{33}$$

[17] The necessary and sufficient conditions in Proposition 1 involve a variable y_3 that can be thought of as the implicit social marginal value of income, evaluated at the Pareto-efficient allocation being characterized, common across countries. This interpretation follows from the formalities in the proof of Proposition 1 on noting that the conditions expressed there are equivalent to those of maximizing a social welfare function $W(u)$ with marginal weights $W_u^T = y^T Z_u$ (where Z is a matrix with elements from matrices A and D) with the typical elements being (after appropriate substitutions) $W_{u^j} = y_3 \beta^j$ (where β^j is given by (30)). This implies that $y_3 = W_{u^j}/\beta^j$.

[18] Notice that in this case (given the homogeneity property of the S^j function) $p^T S^j_{pu} = -E^j_u$. This implies that $y_3 = -W_{u^j}/E^j_u = -W_{u^j}$ (following from the fact that $E^j_u = 1$).

which is equivalent to the definition for β^j in (30), where

$$\hat{p}^\intercal \equiv p^\intercal - \left(0, p^\intercal \left(S_{pq} + S_{pk}\kappa_p^\intercal\right)\left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}\right). \tag{34}$$

As demonstrated in Appendix B, this last expression can be written alternatively as

$$\begin{aligned} \hat{p}^\intercal &= p^\intercal \left[I - \left(0, \left(S_{pq} + S_{pk}\kappa_p^\intercal\right)\left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}\right)\right] && (35)\\ &= \begin{bmatrix} p_1 & -p_1\left(S_{1q} + S_{1k}\kappa_p^\intercal\right)\left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{bmatrix}. && (36) \end{aligned}$$

We call (33) the Generalized Hatta Normality Emissions Condition (GHNEC) for country j. Here the income effects on net exports are weighted by a 'shadow price vector', $\hat{p}$, which accounts for the general equilibrium impacts of product prices on emissions and their subsequent effect on prices. These indirect general equilibrium effects operate through matrix κ_p, which indicates how price changes affect pollution, and S_{pk}, which gives the subsequent effect of the change in pollution on net exports.

The significance of Proposition 1 (and its corollary) is that it provides a straightforward condition that can be checked when evaluating the impact of international transfers. This requires knowledge of the matrix $S_{pq} + S_{pk}\kappa_p^\intercal = \sum_{j=1}^{J}\left[G_{pq}^j - E_{pq}^j - E_{pk}^j\left(1_N^\intercal \sum_{l=1}^{J} G_{tq}^l\right)\right]$ together with S_{pu}^j at the initial perfectly competitive equilibrium. In principle, these matrices are observable marginal responses to prices and emissions.

We turn now to the identification of conditions for the non-existence of strict Pareto-improving reforms.

4 Non-existence of strict Pareto-improving multilateral transfers

Proposition 1 established necessary and sufficient conditions for the existence of strict Pareto-improving multilateral transfers. Intuition suggests that if both policy instruments (tariffs and carbon taxes) are set at their Pareto efficient levels then no strict Pareto-improving multilateral transfers would exist. This intuition is confirmed by the following corollary to Proposition 1, in which we make use of the Pareto efficient characterization of Pareto efficient tariffs and carbon taxes, under the assumption that multilateral transfers are available, obtained by Keen and Kotsogiannis (2014, Proposition 2, p. 122).

Corollary 2 *Suppose that tariffs and carbon taxes in every country j, $j = 1, ..., J$, are set at their Pareto efficient levels (in the sense that $\tau^j = 0$ and $t^j = \sum_{i=1}^{J} E_k^i 1_N = t1_N$, where $t \equiv \sum_{i=1}^{J} E_k^i$). Then, if each good is normal in every country in the sense that $S_{pu}^j < 0$, $j = 1, ..., J$, a strict Pareto improvement in welfare from transfers does not exist.*

Proof. The proof of the corollory is provided in the Appendix. ■

This corollary shows that strict Pareto-improving welfare outcomes from a multilateral transfer of income can only exist if tariffs and/or carbon taxes are set at non-optimal levels. Besides this special case, the question that arises is: under what other conditions will a strict Pareto-improving transfer of income not exist? This is the question to which we now turn.

Following the analysis of the preceding section, if all countries satisfy the Generalized Hatta Normality Emissions Condition (*GHNEC)* then there is no multilateral transfer that can yield a strict Pareto improvement in welfare. Can we identify special cases of these conditions that have clear economic interpretations? In the following, we identify two such special cases.

The first special case is obtained by assuming that a change in global emissions does not affect the household's compensated demands for commodities at the margin and in the aggregate. If compensated demands (on the aggregate) are unresponsive to global emissions in the sense that

$$S_{pk} \equiv \sum\nolimits_{j=1}^{J} S_{pk}^j = 0, \tag{37}$$

then the shadow price vector $\hat{p}$ defined by (34) takes the special form

$$\hat{p}^\intercal \equiv p^\intercal - \left(0, p^\intercal S_{pq} S_{qq}^{-1}\right). \tag{38}$$

For this case, we arrive at the following result.

Proposition 2 *Assume that compensated net exports are unresponsive to global emissions in the sense that $S_{pk} = 0$, that the world substitution matrix S_{qq} is of full rank, that each good is normal in every country in the sense that $S_{pu}^j < 0$, $j = 1, ..., J$, and that all goods are world net-substitutes at the initial equilibrium. Then, a strict Pareto-improving transfer of income does not exist.*

Proof. The proof of the proposition is provided in the Appendix. ■

Proposition 2 re-confirms a result in Turunen-Red and Woodland (1988) in the present context: in the absence of pollution effects on compensated demands and if all goods are normal in all countries and there is sufficient substitutability in the world substitution matrix then income transfers across all countries do not generate a strict Pareto improvement.

Our second special case goes in a quite different direction. Suppose now, going to the opposite extreme to the circumstances of Proposition 2, that production and consumption, on the aggregate, are unresponsive to international prices. In this case, we have the following result.

Proposition 3 *Assume that there is no substitution in production or consumption in any of the J countries (and so, in the aggregate, $S_{pp} = 0$), that the matrix $S_{qk}\kappa_p^\intercal$ has full rank and exhibits net substitutability and that each good is normal in every country in the sense that $S_{pu}^j < 0$, $j = 1, ..., J$. Then, a strict Pareto-improving transfer of income does not exist.*

Proof. The proof of the proposition is provided in the Appendix. ■

Beyond its technical content, there is a practical element — related to the climate change discussions referred to at the outset — behind Proposition 3. In the presence of inefficiencies from carbon pricing, normality and sufficient substitutability rule out the possibility of strict Pareto-improving welfare gains from income transfers. Negative off-diagonal elements imply that the effects of a change in the price of l^{th} good, through a change in global emissions, on compensated demands for good i is negative. To give an example, suppose that the price of air-conditioning equipment increases (good l) and as a consequence more of this is being supplied. Assume further that pollution intensity (on the aggregate) increases. What Proposition 3 requires is that compensated demand for heating equipment (good i) — either for each country j of for the aggregate — decreases.[19]

Proposition 3 can be thought of as the generalization of Proposition 2 in Turunen-Red and Woodland (1988). Things, however, are somewhat more complicated here due to the presence of global pollution, which affects compensated demands.

5 An example

To illustrate the multilateral income transfer mechanism at work, this section presents a numerical example. The model has three commodities — with commodity 1 being taken as the numeraire with unit world price — and three countries engaged in perfectly competitive international trade. It is assumed that the countries all have free trade and have zero carbon taxes.

[19]Notice that this does not preclude the possibility that the technology is one of fixed emissions (per unit of output), in the sense that $G_t^j = -z^j = -Ay^j$, where A is a matrix with off-diagonal elements 0 and the diagonal elements α_i^j giving the emission of good i in country j. Since $y^j = G_p^j$ with $y_p^j = G_{pp}^j$ and so $G_{tp}^j = -AG_{pp}^j$.

If tariffs are set at their Pareto optimal levels (zero) but carbon taxes are set sub-optimally (zero), the conditions for the existence of a Pareto-improving set of multilateral transfers simplify somewhat.

Importantly, in this case multilateral transfers provide a clear and only mechanism for ameliorating the welfare-reducing distortionary affects of global emissions. The assumption of free trade implies that trade taxes are set at their Pareto optimal levels and so they do not constitute a reason why there might exist Pareto-improving transfers. However, the assumption that carbon taxes are zero implies that carbon taxes are not set at their Pareto optimal levels. Accordingly, non-optimality of carbon taxes is the only policy setting that may yield Pareto-improving international transfers. The following numerical example illustrates a case where such transfers exist.

The initial equilibrium is characterized by the equilibrium world prices and various matrices of derivatives. We assume that the initial world prices are all equal to unity. Furthermore, we assume that the positive semi-definite world substitution matrix, the world S_{pk} matrix and the matrix of national income effects $S_{pu} = \begin{bmatrix} S_{pu}^1 & S_{pu}^2 & S_{pu}^3 \end{bmatrix}$ are given by

$$\begin{aligned}
\begin{bmatrix} S_{pp} & S_{pt} \\ S_{tp} & S_{tt} \end{bmatrix} &= \begin{bmatrix} 4.7159 & -1.9277 & -2.7882 & 1.2121 & -0.2438 & 0.3413 \\ -1.9277 & 1.0000 & 0.9277 & -0.5141 & 0.0549 & -0.5347 \\ -2.7882 & 0.9277 & 1.8605 & -0.6979 & 0.1889 & 0.1934 \\ 1.2121 & -0.5141 & -0.6979 & 1.3132 & -0.1945 & -0.4895 \\ -0.2438 & 0.0549 & 0.1889 & -0.1945 & 1.0405 & -0.4018 \\ 0.3413 & -0.5347 & 0.1934 & -0.4895 & -0.4018 & 2.4390 \end{bmatrix} \\
S_{pk} &= \begin{bmatrix} -1.0000 \\ -1.0000 \\ -1.0000 \end{bmatrix} \\
S_{pu} &= \begin{bmatrix} -1.0000 & -1.0000 & -1.0000 \\ -4.0000 & -1.0000 & -1.0000 \\ -1.0000 & -1.0000 & -1.0000 \end{bmatrix}.
\end{aligned}$$

The matrices satisfy all the required conditions imposed by economic theory and are therefore valid for an example. The world net substitution matrix (aggregate of national net substitution matrices) indicates that the three goods are net substitutes for each other when the off-diagonal term is negative and net complements when the off-diagonals are positive. The equal and negative elements in S_{pk} indicate that global emissions have the same effects in each country and the same effects for each good (greater global emissions k negatively affect utility and so increase compensated demands and reduce compensated net exports). The S_{pu} matrix of national income effects has negative elements meaning that all goods are normal in consumption in every country; the large negative term of -4 shows that country 1 has a very high marginal propensity to consume the second good out of income.

Based on these assumptions about the initial equilibrium, the shadow price vector, $\widehat{p}$, and the vector $\widehat{\beta}$ are computed to be

$$\begin{aligned} \widehat{p}^{\intercal} &= \begin{bmatrix} 1.0000 & -1.3944 & 1.9328 \end{bmatrix} \\ \widehat{\beta}^{\intercal} &= \begin{bmatrix} 2.6447 & -1.5384 & -1.5384 \end{bmatrix}. \end{aligned}$$

Since the vector $\widehat{\beta}$ has elements of different sign, Corollary 1 implies that there exists a multilateral transfer of income that is strictly Pareto-improving in welfare for this world economy. The direction of transfers is from countries 2 and 3 to country 1. All countries gain in welfare from the income transfers.

Other such examples may be readily constructed. Moreover, it is straightforward to construct examples that do not yield strict Pareto-improving welfare gains. In these examples the vector $\widehat{\beta}$ has all elements of the same sign, so that all countries respond similarly to additional income.

6 Concluding remarks

This paper explored the role of multilateral transfers in achieving *strict* Pareto improvements in welfare, focusing in particular on identifying conditions under which their use is warranted, or not, when there is a global environmental externality and where there may be nationally set carbon prices and tariff impediments to international trade. The analysis has shown that there is no such case for multilateral transfers when all instruments can be freely deployed and they are set at their Pareto efficient levels. However, Pareto-efficiency does require international income transfers when either carbon or trade taxes in some countries are constrained: its purpose then is to account for the impact on emissions of inappropriate carbon pricing and the trade distortions that exist. Moreover, it is shown that a strict Pareto-improving multilateral lump sum transfer exists if and only if a *generalized normality condition* is violated. In the special case where trade taxes are set at their Pareto-efficient levels (zero), there may exist multilateral income transfers that are strict Pareto-improving in welfare when carbon taxes are sub-optimally set. To illustrate this possibility, we provided a numerical example of such a case when carbon taxes are set to zero in every country and there is free trade.

The analysis here is of course severely limited in several respects. Factors have been assumed internationally immobile, for example, precluding the possibility of carbon leakage through location choices that is a major concern in policy debates (elements of this appear in Kotsogiannis and Woodland (2013)). The analysis, however, suggest a similar set of conditions emerge.

Appendix A: Proofs of Propositions and Lemmas

Proof of Lemma 1 Omitting the market equilibrium for commodity 1 (the numeraire), the system can be written as

$$A^* du + B^* dq + C^* db + E^* dk = 0, \tag{A.1}$$

where the matrices are

$$A^* \equiv \begin{bmatrix} p^\intercal S_{pu}^1 & 0 & \cdots & 0 \\ 0 & p^\intercal S_{pu}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & p^\intercal S_{pu}^J \\ S_{qu}^1 & S_{qu}^2 & \cdots & S_{qu}^J \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; B^* \equiv \begin{bmatrix} p^\intercal S_{pq}^1 + S_q^{1\intercal} \\ \vdots \\ p^\intercal S_{pq}^J + S_q^{J\intercal} \\ S_{qq} \\ 0^\intercal \\ \overline{S}_{tq} \end{bmatrix}, \tag{A.2}$$

$$C^* \equiv \begin{bmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \\ 0_{N-1} & 0_{N-1} & \cdots & 0_{N-1} \\ 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \; D^* \equiv \begin{bmatrix} p^\intercal S_{pk}^{1\intercal} \\ p^\intercal S_{pk}^{2\top} \\ \vdots \\ p^\intercal S_{pk}^{J\top} \\ S_{qk} \\ 0 \\ 1 \end{bmatrix},$$

the vectors of change are

$$du \equiv \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^J \end{bmatrix}, \; dq \equiv \begin{bmatrix} dp_2 \\ \vdots \\ dp_N \end{bmatrix}, \; db \equiv \begin{bmatrix} db^1 \\ db^2 \\ \vdots \\ db^J \end{bmatrix}, \tag{A.3}$$

and

$$S_{qq} \equiv \sum\nolimits_{j \in J} S_{qq}^j, \tag{A.4}$$

$$\overline{S}_{tq} \equiv 1_N^\intercal \sum\nolimits_{j \in J} S_{tq}^j, \tag{A.5}$$

$$S_{qk} \equiv \sum\nolimits_{j \in J}^{J} S_{qk}^j. \tag{A.6}$$

A strict differential Pareto improvement exists if and only if there are du, dq, db, dk satisfying (A.1) and $du \gg 0$. Applying Motzkin's Theorem of the Alternative (Mangasarian, 1969, page 34), there exists du, dq, db, dk satisfying (A.1) if and only if there is no vector $y \in R^{J+N+1}$ such that

$$
\begin{aligned}
y^\intercal A^* &< 0_J^\intercal, && \text{(A.7)}\\
y^\intercal [B^*, C^*, D^*] &= 0_{J+N+1}^\intercal . && \text{(A.8)}
\end{aligned}
$$

It is convenient to partition the vector y as $y = (y_1^\intercal, y_2^\intercal, y_3, y_4)^\intercal$ where $y_1 \in R^J, y_2 \in R^{N-1}, y_3 \in R, y_4 \in R$. It is now straightforward to show that $y^\intercal C^* = 0^\intercal$ implies that $y_1 = y_3 1_J$, where 1_J is the $J-$dimensional vector of ones. (A.7) and (A.8) can then be written as

$$
\begin{aligned}
y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j &\leq 0 \ , \ j = 1, \dots J, && \text{(A.9)}\\
y_2^\intercal S_{qq} + y_3 p^\intercal S_{pq} + y_4 \overline{S}_{tq} &= 0_{N-1}^\intercal \ , && \text{(A.10)}\\
y_2^\intercal S_{qk} + y_3 p^\intercal S_{pk} + y_4 &= 0 \ , && \text{(A.11)}
\end{aligned}
$$

with (A.9) holding with strict inequality for at least one j (in (A.10), (10) has also been used). Solving equation (A.11) for y_4 and substituting into (A.9) and (A.10) one obtains

$$
\begin{aligned}
y_2^\intercal S_{qu}^j + y_3 p^\intercal S_{pu}^j &\leq 0, \ j = 1, \dots J, && \text{(A.12)}\\
y_2^\intercal \left(S_{qq} - S_{qk}\overline{S}_{tq}\right) + y_3 p^\intercal \left(S_{pq} - S_{pk}\overline{S}_{tq}\right) &= 0_{N-1}^\intercal. && \text{(A.13)}
\end{aligned}
$$

If (A.12) and (A.13) have a solution $y_2 \in R^{N-1}, y_3 \in R$, then (A.7) and (A.8) have a solution with

$$
\begin{aligned}
y_1 &= y_3 1_J, && \text{(A.14)}\\
y_4 &= -y_2^\intercal S_{qk} - y_3 p^\intercal S_{pk}, && \text{(A.15)}
\end{aligned}
$$

and conversely, (A.7) and (A.8) have a solution if and only if (A.12) and (A.13) have a solution with (A.14) and (A.15). □

Proof of Proposition 1. The proof of Proposition **1** utilizes Lemma 1. Using the assumption that the matrix $\tilde{S}_{qq} \equiv S_{qq} + S_{qk}S_{sq}$ has full rank (and so its inverse exists), it follows that (28) implies

$$
y_2^\intercal = -y_3 p^\intercal \left(S_{pq} - S_{pk}\overline{S}_{tq}\right)\left(S_{qq} - S_{qk}\overline{S}_{tq}\right)^{-1}. \tag{A.16}
$$

Substituting this into (A.9) gives

$$y_3 \left[p^{\intercal} S^j_{pu} - p^{\intercal}\left(S_{pq} - S_{pk}\overline{S}_{tq}\right)\left(S_{qq} - S_{qk}\overline{S}_{tq}\right)^{-1} S^j_{qu}\right] \leq 0,\ j = 1, \ldots J, \tag{A.17}$$

and with strict inequality for at least one country j. Recalling the definition of the country-specific scalars as

$$\beta^j \equiv p^{\intercal} S^j_{pu} - p^{\intercal}\left(S_{pq} - S_{pk}\overline{S}_{tq}\right)\left(S_{qq} - S_{qk}\overline{S}_{tq}\right)^{-1} S^j_{qu},\ j = 1, \ldots J, \tag{A.18}$$

(A.17) reduces to

$$y_3\beta^j \leq 0,\ j = 1, \ldots J, \tag{A.19}$$

with strict inequality for at least one country j.

Close inspection of (A.19) reveals that the only feasible solution is $y_3 \neq 0$. The reason for this is as follows. If all β^j have the same sign, say positive, then $y_3 < 0$. If they are all negative then $y_3 > 0$. If some are negative and some positive then y_3 is not admissible as it violates the inequalities. This implies that if (A.19) has a solution it must be $y_3 \neq 0$. This implies that y_2 has a non-trivial solution given by (A.16), that $y_1 = y_3 1_J$ and that $y_4 = -y_2^{\intercal} S_{qk} - y_3 p^{\intercal} S_{pk}$, $j = 1, ..., J$. $\square$

Proof of Corollary 2 The proof of Corollary **2** utilizes the proof of Proposition 1 in showing that (34) reduces to $\hat{p}^{\intercal} \equiv p^{\intercal} \gg 0$ if $\tau^j = 0$ and $t^i = \sum_{j=1}^{J} E^j_k 1_N \equiv t1_N$ and so, if $S^j_{pu} < 0$, $j = 1, ..., J$, then $p^{\intercal} S^j_{pu} < 0$. This implies that there is $y_3 \neq 0$ that satisfies (A.19). To see that $\hat{p}^{\intercal} \equiv p^{\intercal} \gg 0$, notice that (34) implies

$$p^{\intercal}\left(S_{pp} - S_{pk}\overline{S}_{tp}\right) = p^{\intercal}\left[\sum\nolimits_{j=1}^{J} S^j_{pp} - \sum\nolimits_{j=1}^{J} S^j_{pk}\left(1_N^{\intercal} \sum\nolimits_{l=1}^{J} S^l_{tp}\right)\right]. \tag{A.20}$$

The homogeneity property of the S^j function implies, since $p^{j\intercal} = p^{\intercal}$, that $p^{\intercal} S^j_{pp} + t^{j\intercal} S_{tp} = 0^{\intercal}$ and that $p^{\intercal} S^j_{pk} = -E^j_k$ and so $-p^{\intercal} \sum_{j=1}^{J} S^j_{pk} = \sum_{j=1}^{J} E^j_k$. Accordingly, (A.20) can be written as

$$-t1_N^{\intercal} \sum\nolimits_{j=1}^{J} S^j_{tp} + \sum\nolimits_{j=1}^{J} E^j_k \left(1_N^{\intercal} \sum\nolimits_{l=1}^{J} S^l_{tp}\right) = -t1_N^{\intercal} \sum\nolimits_{j=1}^{J} S^j_{tp} + t1_N^{\intercal} \sum\nolimits_{j=1}^{J} S^j_{tp} = 0^{\intercal}. \tag{A.21}$$

Using (34), this implies that $\hat{p}^{\intercal} \equiv p^{\intercal}$ as required. $\square$**.**

Proof of Proposition 2 The proof follows the proof of Proposition 1. Since $S_{pk} = 0$ by assumption, the shadow price vector $\hat{p}$ takes the special form given by (38) and

$$p^\intercal S_{pq} S_{qq}^{-1} = (p_1, q^\intercal) \begin{pmatrix} S_{1q} \\ S_{qq} \end{pmatrix} S_{qq}^{-1} = p_1 S_{1q} S_{qq}^{-1} + q^\intercal. \tag{A.22}$$

These results imply that the shadow price vector is now

$$\hat{p}^\intercal \equiv (p_1, q^\intercal) - \left(0, p^\intercal S_{pq} S_{qq}^{-1}\right) = \left(p_1, -p_1 S_{1q} S_{qq}^{-1}\right). \tag{A.23}$$

If S_{pp} exhibits net substitutability (all off-diagonal elements are negative) then $S_{1q} \ll 0$ and $S_{qq}^{-1} \geq 0$. The implication of this is that $\hat{p}^\intercal > 0$. Since all goods are assumed normal and so $S_{pu}^j = -E_{pu}^j \ll 0$, $j = 1, ..., J$, it follows that $\beta^j < 0$ for all countries $j = 1, ..., J$. Thus, by Corollary 1 to Proposition 1 there does not exist a strict Pareto-improving transfer of income. □

Proof of Proposition 3 The proof follows the proof of Proposition 1. Since $S_{pp} = 0$ by assumption, the shadow price vector defined by (34) may be expressed as

$$\hat{p}^\intercal \equiv p^\intercal - \left(0, p^\intercal \left(S_{pk}\overline{S}_{tq}\right)\left(S_{qk}\overline{S}_{tq}\right)^{-1}\right), \tag{A.24}$$

which, in turn, implies that

$$p^\intercal \left(S_{pk}\overline{S}_{tq}\right)\left(S_{qk}\overline{S}_{tq}\right)^{-1} = (p_1, q^\intercal) \begin{pmatrix} S_{1k}\overline{S}_{tq} \\ S_{qk}\overline{S}_{tq} \end{pmatrix} \left(S_{qk}\overline{S}_{tq}\right)^{-1} = p_1 S_{1k}\overline{S}_{tq} + q^\intercal, \tag{A.25}$$

and thus that

$$\hat{p}^\intercal \equiv (p_1, q^\intercal) - \left(0, p^\intercal \left(S_{pk}\overline{S}_{tq}\right)\left(S_{qk}\overline{S}_{tq}\right)^{-1}\right) = \left(p_1, -p_1 \left(S_{1k}\overline{S}_{tq}\right)\left(S_{qk}\overline{S}_{tq}\right)^{-1}\right). \tag{A.26}$$

If $S_{pk}\overline{S}_{tp}$ exhibits net substitutability (all off-diagonal elements are negative) then $S_{1k}\overline{S}_{tq} \ll 0$ and $\left(S_{qk}\overline{S}_{tq}\right)^{-1} \geq 0$. The implication of this is that $\hat{p}^\intercal > 0$. Since all goods are assumed normal and so $S_{pu}^j \ll 0$ $j = 1, ..., J$, it follows that $\beta^j < 0$ for all countries $j = 1, ..., J$. Thus, by Corollary 1 to Proposition 1 there does not exist a strict Pareto-improving transfer of income. □

Appendix B: Structure of the shadow price vector

B.1 Alternative expressions for the shadow price vector

As noted in the paper, the shadow price vector $\hat{p}$ defined by

$$\hat{p}^\intercal \equiv p^\intercal - \left(0, p^\intercal \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}\right), \tag{B.1}$$

may be written as

$$\hat{p}^\intercal = \begin{bmatrix} p_1 & -p_1 \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{bmatrix}. \tag{B.2}$$

To prove this, write the shadow price as $\hat{p}^\intercal = p^\intercal A$, where

$$\begin{aligned} A &\equiv I - \left(0, \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1}\right) \\ &\equiv I - (0, C). \end{aligned} \tag{B.3}$$

Matrix C simplifies in structure to the expressions

$$\begin{aligned} C &\equiv \left(S_{pq} + S_{pk}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ &= \begin{bmatrix} S_{1q} + S_{1k}\kappa_p^\intercal \\ S_{qq} + S_{qk}\kappa_p^\intercal \end{bmatrix} \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ &= \begin{bmatrix} \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ \left(S_{qq} + S_{qk}\kappa_p^\intercal\right) \left(SS_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} \left(S_{1q} + S_{1k}\kappa_p^\intercal\right) \left(S_{qq} + S_{qk}\kappa_p^\intercal\right)^{-1} \\ I_{N-1} \end{bmatrix} \\ &\equiv \begin{bmatrix} d \\ I_{N-1} \end{bmatrix}. \end{aligned} \tag{B.4}$$

Accordingly, the shadow price vector becomes

$$\begin{aligned}
\hat{p}^{\intercal} &= p^{\intercal} A \qquad\qquad (B.5)\\
&= p^{\intercal}\left[I-(0,C)\right]\\
&= p^{\intercal}\left[\begin{bmatrix} 1 & 0 \\ 0 & I_{N-1}\end{bmatrix} - \begin{bmatrix} 0 & d \\ 0 & I_{N-1}\end{bmatrix}\right]\\
&= p^{\intercal}\begin{bmatrix} 1 & -d \\ 0 & I_{N-1}-I_{N-1}\end{bmatrix}\\
&= p^{\intercal}\begin{bmatrix} 1 & -d \\ 0 & 0_{N-1}\end{bmatrix}\\
&= p_1\begin{bmatrix} 1 & -d \end{bmatrix}\\
&= \begin{bmatrix} p_1 & -p_1\left(S_{1q}+S_{1k}\kappa_p^{\intercal}\right)\left(S_{qq}+S_{qk}\kappa_p^{\intercal}\right)^{-1} \end{bmatrix}.
\end{aligned}$$

This completes the proof. □

This shows that the sign structure of $\hat{p}$ depends crucially upon the sign structure of the $(N-1)$-vector $d \equiv \left(S_{1q}+S_{1k}\kappa_p^{\intercal}\right)\left(S_{qq}+S_{qk}\kappa_p^{\intercal}\right)^{-1}$. This vector depends upon the full substitution matrix through the second inverse matrix. The first term is a vector expressing the total effects of a change in world prices upon the net exports of the numeraire good, taking into account the impact of world prices on world emissions and hence on net exports as well as the direct price effects.

B.2 The case of zero tariffs and carbon taxes

Here it is assumed that the countries all have free trade and have zero carbon taxes. If tariffs and carbon taxes are set to zero in every country ($\tau^j = 0,\ t^j = 0$), then domestic prices are the same in every country, $p^j = p$, and the homogeneity conditions

$$\begin{bmatrix} p^{j\intercal} & t^{j\intercal} \end{bmatrix}\begin{bmatrix} S^j_{pp} & S^j_{pt} \\ S^j_{tp} & S^j_{tt}\end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix},\ j = 1, ..., , J \qquad (B.6)$$

imply that

$$p^{\intercal}\begin{bmatrix} S^j_{pp} & S^j_{pt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix},\ j = 1, ..., , J. \qquad (B.7)$$

This result clearly translates to the world substitution matrix so that

$$p^{\intercal}\begin{bmatrix} S_{pp} & S_{pt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}. \qquad (B.8)$$

Using this result, the shadow price vector $\hat{p}$ may be expressed as

$$\hat{p}^{\intercal} = \left[\begin{array}{cc} p_1 & p^{\intercal} S_{pk} \kappa_p^{\intercal} \left(S_{qq} + S_{qk} \kappa_p^{\intercal} \right)^{-1} \end{array} \right]. \tag{B.9}$$

This shows that the sign structure of $\hat{p}$ depends upon the sign structure of the $(N-1)$-vector $S_{pk} \kappa_p^{\intercal} \left(S_{qq} + S_{qk} \kappa_p^{\intercal} \right)^{-1}$.

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