What's Wrong with Annuity Markets?*

Stéphane Verani[†] Pei Cheng Yu[‡]

August 11, 2020

Abstract

The annuity market in the US has been historically small. What drives this fact? The annuity market could be small because of adverse selection or supply-side frictions in insurance markets. Identifying demand- and supply-side frictions is difficult without data separately measuring exogenous shocks and endogenous responses. In this paper, we provide a novel identification using annuity price data and regulatory capital requirements. Using publicly available data, we document a robust relationship between shocks originating in the corporate bond market and annuity price markups. We show that this relationship supports a standard model of adverse selection with an incomplete bond market.

JEL CODES: G22; H55; H24; H63; D86; G32
KEYWORDS: life insurance; annuities; corporate bond market; limited liability; interest rate risk management

---

*We would like to thank, without implication, Celso Brunetti, Dean Corbae, Stefan Gissler, Jim Hines, Ivan Ivanov, Narayana Kocherlakota, Borghan Narajabad, and Coco Ramirez for helpful conversations, seminar participants at the University of Michigan, and Sam Dreith for exceptional research assistance. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

[†]stephane.h.verani@frb.gov, +1-202-912-7972, 20^{th} & C Streets, NW, Washington, D.C. 20551.

[‡]pei-cheng.yu@unsw.edu.au, +61-2-9385-3704, West Lobby Level 4. UNSW Business School building. Kensington Campus UNSW Australia

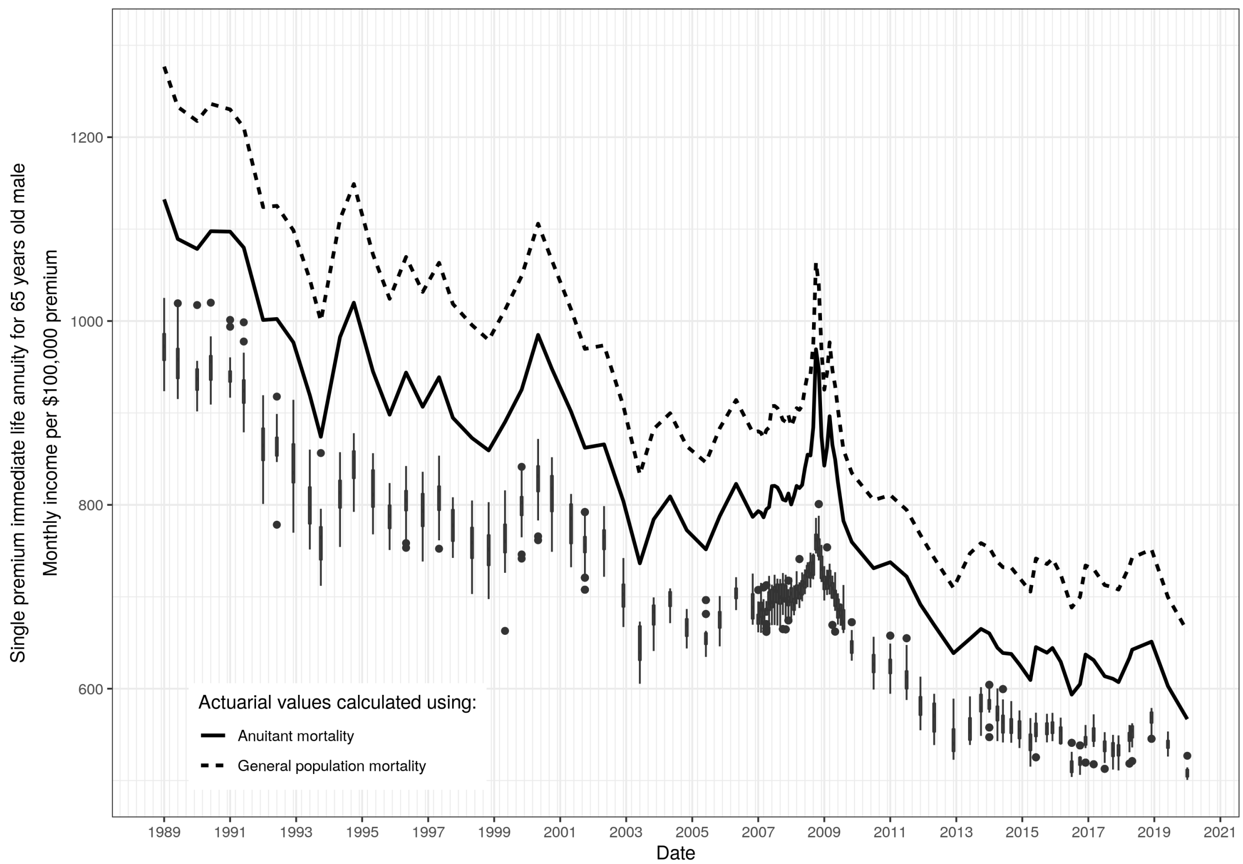
Introduction

The U.S. annuity market has been historically small. Individuals can manage the risk of outliving their financial wealth by purchasing a life annuity from a life insurer. In the U.S., the annuity market is dominated by deferred annuities that are purchased by individuals before their retirement. Deferred annuities allow individuals to accumulate wealth on a tax deferred basis during the contract period and give them the option to receive their accumulated wealth as a lump sum or a life annuity at the end of the contract. Although life annuities pay a rate of return in excess of that received on conventional assets (Mitchell, Poterba, Warshawsky & Brown 1999), relatively few deferred annuity contract holders choose the life annuity payout option and Americans generally annuitize less wealth than what standard economic theory predicts (Yaari 1965, Davidoff, Brown & Diamond 2005). For example, Americans between 50 and 65 years accumulated about $42,500 per person in fixed deferred annuity balances in 2018 and those aged 65 years and above only annuitized about $12,700 of their total per person wealth with a life insurer in the same year. Why is the annuity market so small?

A standard explanation for the low wealth annuitization rate in the U.S. is the presence of market inefficiencies. Inefficiencies may arise if life insurers do not observe the mortality risk of individuals seeking longevity insurance, leading to adverse selection. This type of informational friction may lead to insurers charging a markup over the annuities' actuarial values (Eichenbaum & Peled 1987, Finkelstein & Poterba 2004). Other types of inefficiencies may arise from financial and market frictions that affect life insurers' product design and capital structure decisions (Koijen & Yogo 2015, Foley-Fisher, Narajabad & Verani forthcoming). These supply-side frictions may also be reflected in relatively high annuity markups. Identifying the effect of demand- and supply-side frictions on annuity markups is difficult without data separately measuring exogenous shocks and endogenous responses. Nevertheless, understanding the source of inefficiencies in private longevity insurance markets is crucial to analyze the effects of retirement policies, such as social security reforms.

In this paper, we provide a novel identification that exploits bond market shocks and

Figure 1: **Distribution of actual and actuarial monthly payment for a nominal $100,000 Single Premium Immediate Annuity offered to a 65 year old male.**



the U.S. insurance regulatory framework. We develop an algorithm for annuity valuation to decompose the contribution of demand- and supply-side frictions in annuity markups. Figure 1 plots the distribution of actual monthly payment offered by U.S. life insurers to a 65 year old male for a $100,000 single premium immediate life annuity from 1989 to 2019 (the box plots) together with the monthly payments implied by the actuarial value of the contract calculated using the general population mortality (the dashed line) and annuitant mortality (the solid line). The difference between the dashed and solid lines is a measure of the industry's average *adverse selection pricing* (henceforth, *AS pricing*) and the difference between the actual monthly payment offered and the solid line is an *adverse selection-adjusted annuity price markup* (henceforth, *AS-adjusted markup*). Although the latter consistently accounts for about 50 percent of the overall annuity markup, the literature typically focuses on the former through the examination of "money's worth" calculations

(Mitchell et al. 1999) and attributes the latter to residual unobserved administrative costs, monopoly profits and other transaction costs (Einav & Finkelstein 2011).

We show that life insurer interest rate risk management drives the AS-adjusted markup. We begin by showing theoretically how life insurers set prices in an annuity market with adverse selection and interest rate risk using a simple three-period model. Interest rate risk arises in the model because there is aggregate uncertainty over future interest rate and corporate debt maturity is constrained to be relatively short (Bolton & Scharfstein 1990, 1996, Hart & Moore 1994, 1998, Huang et al. 2019). Life insurers are exposed to interest rate risk because the duration of their assets is less than the duration of their insurance liabilities. This negative duration gap means that a decrease in the interest rate increases the present value of a life insurer's fixed rate liabilities faster than the present value of its fixed income assets, which could lead to insolvency. Since the prospect of ex-post insolvency is incompatible with the sale of long-term longevity insurance ex-ante, limited liability life insurers credibly manage interest rate risk by choosing an observable optimal level of net worth. The cost of financing this net worth is reflected in relatively high AS-adjusted markups.

Our model differs from the textbook model of adverse selection in insurance markets—e.g., Einav & Finkelstein (2011)— because the Modigliani & Miller (1958) theorem does not hold, even when the bond market is complete. With limited liability, insurers must credibly show to annuity shoppers that they are managing interest rate risks with a unique ex-ante capital structure. When the bond market is complete, insurers optimally choose to hold zero net worth and charge the same annuity price as insurers in the textbook adverse selection model. Limited liability leads to a different optimal annuity price from the textbook model when the supply of long-term bonds is constrained, which is an additional source of financial friction in our model. When the bond market is incomplete, insurers finance a positive level of net worth to hedge against interest rate movement, which is reflected in a higher AS-adjusted markup.

We then identify the risk management channel in annuity price data by comparing the change in AS-adjusted markup for annuity contracts with higher average cost to those with lower average cost as a response to the same shock to the average bond demand.

Our identification strategy exploits the property that, under the hypotheses of our model, the AS-adjusted markup is determined by the difference between the insurers' average cost curve and average bond demand curve. We find that conditional on their average cost of funding, insurers tend to raise their adverse-selection adjusted markups when annuity reserve requirements become more binding, but do so significantly less when the spread on long-duration investment grade corporate bonds widens.

We find further evidence of the risk management channel in the cross-sectional variation of AS-adjusted markups. We show that life insurers that are relatively more adversely affected by an unexpected change in the yield curve as a result of their ex-ante interest rate derivative positions disproportionately increase their AS-adjusted markup ex-post. We identify this second effect using the universe position-level interest rate swap derivatives data and the 2009-2015 zero lower bound period, during which all of the movements in the treasury yield curve came from fluctuations in the long end of the curve.

Finally, we identify a relationship between shocks that originate in the corporate bond market and adverse selection in the annuity market. It is well known that higher annuity prices are associated with more severe adverse selection (Rothschild & Stiglitz 1976). We use our model to show that adverse selection in the annuity market depends on the severity of frictions in the corporate bond market, as a higher level of optimal net worth exacerbates adverse selection by increasing the annuity prices. We then present empirical evidence consistent with this effect by exploiting differences in life insurers' pricing of single premium immediate annuities with different types of "period certain" guarantees and by measuring life insurers' AS pricing for these products. We show that an exogenous increase in statutory reserve requirements disproportionately increases the AS pricing for life annuities with 10 and 20 year period certain guarantees relative to life annuities with no period certain guarantees. Since individuals choosing life annuities with 10 or 20 year period certain guarantees think they are at a higher risk of dying within the next 10 or 20 years, respectively, our results suggest that changes in corporate bond market conditions may have a direct effect on adverse selection in annuity markets.

This paper provides new answers to old questions surrounding the seemingly unpopular life annuity contracts in the U.S. by bridging the economic literature on adverse selection in the annuity markets and the finance literature on risk management of financial institutions. Our paper fills the gap in the economic literature that followed the seminal work of Yaari (1965) and Blanchard (1985). This literature models life insurers as profit maximizing firms operating in frictionless financial markets—e.g., Hosseini (2015). Under this assumption, life insurers perfectly match the duration and cash flow of their assets and insurance liabilities, and therefore can costlessly hedge interest rate risk. In practice, the duration of life insurers' assets is typically lower than the duration of their insurance liabilities due to an incomplete bond market. From a theoretical point of view, these frictions require insurers to manage interest rate risk via an ex-ante capital structure policy, which is a departure from the Modigliani & Miller (1958) theorem and is similar to the risk management rationale emphasized by a relatively small but active finance literature—e.g., Froot & Stein (1998), Foley-Fisher, Narajabad & Verani (2016), Rampini, Viswanathan & Vuillemey (2017).

The remainder of the paper proceeds as follows. In section 1, we provide a short overview of the life annuity market. Section 2 presents and analyzes our theoretical model. Sections 3 and 4 compare the predictions of our model to the "textbook" model of adverse selection in insurance market and explain our identification strategy. Section 5 reviews our data sources. Our empirical analysis is contained in Sections 6, 7, and, 8 . We conclude in Section 9.

1 Selling and managing fixed annuities

New retirees can manage the risk of outliving their financial wealth by purchasing a life annuity from a life insurer either directly or through their employer's pension plan. An individual purchasing a life annuity contract transfers its idiosyncratic longevity risk to the life insurer by surrendering his or her wealth in exchange for a stream of payments while he or she is alive. In this section, we briefly discuss the life annuity market.

1.1 The U.S. market

Roughly half of the U.S. life insurance industry's $600 billion aggregate income in 2018 came from annuity considerations.[1] About half of these annuity considerations relate to fixed annuity sales, which are annuity contracts for which the principal is backed by the general account of the life insurer.[2]

The vast majority of fixed annuities sold in the U.S. are actually deferred fixed annuities, which are very similar to banks' certificates of deposit. Deferred fixed annuities are purchased by individuals before their retirement age to accumulate wealth on a tax deferred basis during the contract period. At the end of the deferred fixed annuity contract period and after reaching the age of 59.5 years of age, contract holders have the option of receiving their accumulated wealth as a lump sum, a term annuity, or a life annuity.

New retirees annuitize a relatively small share of their wealth. We calculate that individuals in the U.S. accumulated about $2.5 trillion in the form of deferred fixed annuities using 2018 insurer statutory fillings. This corresponds to roughly $42,500 per American aged between 50 and 65 years. By contrast, a back of the envelop calculation using the aggregate payment from life insurers to life annuity contract holders assuming a 6 percent average yield reveals that Americans annuitize only about $625 billion of their wealth with life insurers, or approximately $12,700 per person aged 65 years and above. This last observation is consistent with the view in the literature that the market for immediate annuities in the U.S. is small–e.g., Mitchell et al. (1999).

1.2 The fixed annuity business model

Life insurers' overall business model consists of earning a spread between the yield they promise on their insurance liabilities and the yield on the assets backing these liabilities. Life annuities and life insurance contracts are fixed rate liabilities that are illiquid, as they are not transferable from one individual to another. Consequently, life insurers tend to invest their annuity considerations and premiums primarily in fixed-income securities

[1] The other half is roughly split between life and health insurance premiums. See the ACLI's 2018 Life Insurers Fact Book https://www.acli.com/posting/rp18-007.

[2] The other type of annuity is variable annuity for which the principal is backed by assets segregated from the life insurer's general account and remains the property of the annuity contract holder.

in an effort to match their asset and liability cash flows. The illiquidity of life insurance liabilities allows insurers to invest considerations and premiums in relatively illiquid fixed income securities, such as corporate bonds, to offer a competitive return to policyholders.

U.S. Life insurers have been the largest institutional investor in corporate bonds issued by U.S. corporations since the 1930s. At the end of 2017, U.S. life insurers held about $2.1 trillion of corporate bonds in their general account, which accounts for about half of their general account assets and roughly one-third of the total corporate bond amount outstanding in the U.S. (ACLI Life Insurers Fact Book 2018).[3] By comparison, the rest of the life insurers' general account assets includes 8 percent in U.S. government securities and 14 percent in mortgage backed securities, including those backed by the U.S. government.[4]

1.3 Interest rate risk management

The duration of life insurers' assets is usually less than the duration of their insurance liabilities because the maturity of corporate debt is typically much shorter than the length of life insurance liabilities. For example, over 90 percent of corporate bonds issued by corporations each year have an initial maturity that is 10 years or less. This contrasts with the duration of a life annuity which is approximately 10 years. This negative duration gap means that a decrease in the interest rates increases the present value of a life insurer's fixed rate liabilities faster than the present value of its fixed income assets, which could lead to insolvency. Because the prospect of insolvency is incompatible with the sale of life annuities, interest risk management is at the heart of the annuity business model.

Life insurers manage interest rate risk by maintaining a suitable level of net worth, which is also referred to as statutory surplus in the industry. Net worth helps cushion the effect of interest rate drops that disproportionately affect the value of the life insurer's insurance liabilities. Because earnings are the primary source of life insurer capital, the cost of preserving net worth is reflected in annuity prices. That said, the effect of interest rate risk management on annuity pricing is typically absent from economic models

[3] General account assets are those assets used by a life insurer to back its insurance liabilities.

[4] https://www.acli.com/Industry-Facts/Life-Insurers-Fact-Book.

with life annuities that assume frictionless financial markets—e.g., Yaari (1965), Davidoff et al. (2005), Hosseini (2015). We formalize the relationship between interest rate risk management and annuity prices in the next section.

2 A model of adverse selection and interest rate risk

In this section, we show how life insurers set prices in an annuity market with adverse selection and interest rate risk. We introduce two additional frictions, which are absent from the adverse selection literature. The first friction is that life insurers are protected by limited liability, which means the insurers' owners' payoff—e.g., the insurers' stock price—cannot be negative. The second friction is that the bond market is incomplete, so debt maturity is relatively short. Competitive life insurers have an incentive to manage interest rate risk with net worth, because annuity shoppers can rationally anticipate the prospect of insolvency associated with certain portfolio and capital structure choices. The cost of net worth is reflected in annuity prices that are above what the adverse selection literature refers to as actuarially fair prices.

2.1 Economic environment

The economy lasts for three periods, $t \in \{0, 1, 2\}$, and it is populated by a continuum of individuals with wealth normalized to 1. Each individual survives from period to period with survival probability α, which is drawn at the beginning of $t = 0$ from a c.d.f of $G(\alpha)$ and p.d.f of $g(\alpha)$ with support $[\underline{\alpha}, \overline{\alpha}] \subset [0, 1]$. The survival probability α is the individual's private information. Every individual is deceased at the end of $t = 2$.

There are two types financial instruments in the economy that can be used to transfer wealth across periods. First, there are one- and two-period corporate bonds issued by non-financial firms, which we do not model explicitly. One unit of the one-period bond returns $R_1 \geq 1$ in $t = 1$ and R_2 in $t = 2$, where $R_2 \in [1, \overline{R}]$ with mean $\mathbb{E}(R_2)$ and is realized in $t = 1$. The long-term bond is priced efficiently in $t = 0 : \frac{1}{R_l} = \frac{1}{R_1}\mathbb{E}\left(\frac{1}{R_2}\right)$. Second, there is cash, which is a riskless asset that can be used as storage across periods but does not accrue any value over time.

Individuals can also transfer wealth across periods by buying an annuity contract from a life insurer in $t = 0$. An annuity contract pays one unit of consumption in each period the contract holder is alive in exchange for a lump sum payment of q in $t = 0$. We do not make explicit assumptions about the individuals' consumption and investment decisions. Instead, we require that the annuity demand $a(\alpha, q)$ of individuals with survival probability α satisfies Assumption 1.

Assumption 1 *The individual annuity demand $a(\alpha, q)$ satisfies: (i) $a(\alpha, q)$ is differentiable in α and q, with $\frac{\partial a}{\partial \alpha} > 0$ and $\frac{\partial a}{\partial q} < 0$; (ii) There exists $\alpha \in (\underline{\alpha}, \overline{\alpha})$ such that $a(\alpha, q) > 0$ when $q = \frac{\overline{\alpha}}{R_1}(1 + \overline{\alpha})$; and (iii) $a(\alpha, q) = 0$ for all α and q if there is a positive probability that the insurer is insolvent in period $t = 1$ or $t = 2$ and $a(\alpha, q) \geq 0$ otherwise.*

The first condition of Assumption 1 follows from the adverse selection literature. It requires that agents with lower mortality risk have a higher demand for annuities and that the demand for annuities is downward sloping. The second condition ensures that there is a market for annuities regardless of the level R_2 and the competitive equilibrium price for annuities exists.[5] The third condition requires that the demand for annuities provided by insurers who might become insolvent after the aggregate shock R_2 is zero. A stark interpretation of this condition is that an annuity contract is worthless ex-ante to an individual if there is a positive probability that the insurer may not always honor its contractual obligations. A less stark interpretation is that an un-modelled insurance regulator and/or ratings agency requires insurers to hold minimum annuity reserves to ensure solvency in all states of the world.

2.2 Interest rate risk management

With the aggregate shock R_2, the insurer could become insolvent in $t = 1$ if the present value of its insurance liabilities exceeds the present value of its assets. Life insurers are concerned about their solvency because they operate under limited liability and, by Assumption 1, will not sell any annuities if they might become insolvent.

[5] Specifically, this condition requires that annuity demand is strictly positive even when insurers break even on agents with the lowest mortality rate and interest rate is at its lowest level—$R_2 = 1$.

2.2.1 The life insurers' balance sheet

An insurer invests its annuity considerations in a portfolio of cash and bonds:

$$m_1 + b_1 + l_2 = q \int_{\underline{\alpha}}^{\overline{\alpha}} a(\alpha, q)\, g(\alpha)\, d\alpha \,, \tag{1}$$

where m_t, b_t and l_2 denote the insurer's investment in cash, one-period bonds and two-period bonds, respectively. The insurer's balance sheet at $t = 0$ is given by

$$m_1 + b_1 + l_2 = \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{\alpha}{R_1} \left[1 + \alpha \mathbb{E}\left(\frac{1}{R_2} \right) \right] a(\alpha, q)\, g(\alpha)\, d\alpha + NW_0 \,, \tag{2}$$

where the first term on the right hand side is the present value of the insurers' annuity liability and $NW_t \geq 0$ is the insurer's net worth or surplus in t. Note that the weakly positive net worth reflects the insurer's limited liability: The owners of the life insurer are not liable for corporate loss that exceed the value of the insurer's assets.

The aggregate shock R_2 is realized at the beginning of $t = 1$, and the insurers' balance sheet becomes:

$$b_2(R_2) + m_2(R_2) = \frac{1}{R_2} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a(\alpha, q)\, g(\alpha)\, d\alpha + NW_1(R_2) \ , \tag{3}$$

where

$$b_2(R_2) + m_2(R_2) = R_1 b_1 + \frac{R_l l_2}{R_2} - \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha a(\alpha, q)\, g(\alpha)\, d\alpha. \tag{4}$$

The limited liability insurer risks becoming insolvent in $t = 1$ if the present value of its insurance liabilities exceeds the present value of its assets, in which case $NW_1(R_2) < 0$. Under Assumption 1, individuals do not purchase an annuities from an insurer that has a non-zero probability of becoming insolvent in $t = 1$. Therefore, life insurers have an incentive to engage in interest rate risk management (IRM) because they are concerned about the solvency risks associated with their long-term liabilities. Life insurers can manage interest rate risk by choosing an asset portfolio, a price for their annuity, and a capital structure such that $NW_1(R_2) \geq 0$ for all realizations of R_2.

2.2.2 Optimal interest rate hedging strategies

Optimal IRM requires no duration mismatch between the insurers' assets and liabilities. The duration D of an asset or liability is the elasticity of its present value PV with respect to the interest rate: $D = \frac{\partial PV}{\partial R}\frac{R}{PV}$. A negative duration gap occurs when the duration of its insurance liabilities exceeds the duration of its assets. With a negative duration gap, the present value of the insurance liabilities increases more rapidly than the present value of the insurer's assets when the interest rate decreases, leading to insolvency.

Consider first, an economy in which insurers can purchase as much two-period bonds as they need to perfectly hedge their interest rate risk. We refer to this special case as the economy with a complete bond market. The insurer chooses bond holdings such that it remains solvent. The insurer prefers bonds over cash holdings because $R_t \geq 1$. It follows that for a given annuity price q and two-period bond holding l_2, an insurer is guaranteed to remain solvent in $t = 2$ if its asset portfolio satisfies the following condition:

$$b_1\left(l_2\right) = \frac{1}{R_1}\left[\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha\left(1+\alpha\right) a\left(\alpha, q\right) g\left(\alpha\right) d\alpha - R_l l_2\right]. \tag{5}$$

By (3) and (4), if (5) holds, then the insurer's net worth is always weakly positive at $t = 1$, and zero when $R_2 = 1$. Since $R_l = \frac{R_1}{\mathbb{E}\left(\frac{1}{R_2}\right)}$, from (2), (3), and (4), we can show that when

$$l_2 = \frac{1}{R_l}\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q\right) g\left(\alpha\right) d\alpha, \tag{6}$$

then

$$b_1 = \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{\alpha}{R_1} a\left(\alpha, q\right) g\left(\alpha\right) d\alpha$$

and $NW_0 = NW_1\left(R_2\right) = 0$ for all R_2.

With a complete bond market, an insurer can perfectly match the duration of its asset and insurance liabilities with a suitable portfolio of one- and two-period bonds. To see why the bond market is complete when the supply of two-period bonds is unlimited, note that the duration of the insurance liabilities at $t = 0$ is

$$D(L) = \frac{\partial PV_L}{\partial R_2} \frac{R_2}{PV_L} = -\frac{\frac{1}{R_2} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q\right) g\left(\alpha\right) d\alpha}{\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \left(1 + \frac{\alpha}{R_2}\right) a\left(\alpha, q\right) g\left(\alpha\right) d\alpha} .$$

On the asset side, the duration of the one-period bond holding is 0 and the duration of the two-period bond holding l_2 is equal to the duration of the insurance liabilities. This demonstrates that when the economy has an unlimited supply of two-period bonds, the insurers are able to perfectly hedge the interest rate risk and they do not need to hold positive net worth: $NW_t = 0$ for all t when the supply of l_2 is unlimited.

We now consider an economy where the supply of two-period bonds is limited. In this economy, the insurers' optimal two-period bond holding is given by

$$l_2 = \zeta \int_{\underline{\alpha}}^{\overline{\alpha}} \frac{\alpha^2}{R_l} a\left(\alpha, q\right) g\left(\alpha\right) d\alpha \, ,$$

where ζ indexes the constraint on the supply of long-term bonds. When $\zeta = 1$, there is an unlimited supply of two-period bonds, and by (6), the insurers perfectly hedge the interest rate risk. When $\zeta \in [0, 1)$, the insurers' IRM is constrained. In particular, when $\zeta = 0$, the economy only supplies one-period bonds. Theorem 1 characterizes the optimal IRM in an incomplete bond market. Furthermore, the optimal IRM strategy is unique.

Theorem 1 *For a given annuity price q and $\zeta \in [0, 1]$, the unique optimal IRM strategy requires an asset allocation and a capital structure that satisfies:*

i. Asset portfolio:

$$\begin{aligned} b_1 &= \frac{1}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \left[1 + (1 - \zeta)\, \alpha\right] a\left(\alpha, q\right) g\left(\alpha\right) d\alpha, \\ l_2 &= \frac{\zeta}{R_l} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q\right) g\left(\alpha\right) d\alpha, \\ b_2\left(R_2\right) &= \left(1 - \zeta + \frac{\zeta}{R_2}\right) \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q\right) g\left(\alpha\right) d\alpha \, . \end{aligned}$$

ii. Capital structure:

$$NW_0 = \frac{1-\zeta}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 \left[1 - \mathbb{E}\left(\frac{1}{R_2}\right)\right] a\left(\alpha, q\right) g\left(\alpha\right) d\alpha,$$
$$NW_1\left(R_2\right) = \left(1-\zeta\right)\left(1 - \frac{1}{R_2}\right) \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q\right) g\left(\alpha\right) d\alpha \, .$$

When there is an insufficient supply of two-period bonds ($\zeta < 1$), the duration of an insurer's assets is strictly lower than the duration of its insurance liabilities. The insurer can eliminate its duration gap by decreasing the duration of its liabilities, which requires holding a strictly positive amount of net worth.

2.3 Annuity pricing

Life insurers engage in Bertrand competition over the annuity price q : Any insurer charging the lowest q and implementing an optimal IRM strategy captures the entire annuity market. Lemma 1 in the appendix characterizes the basic properties of the Bertrand equilibrium.

We focus our analysis on the decision of life insurers that are implementing an optimal IRM strategy. Let us start by considering how adverse selection contributes to the annuity price markup. To do so, first consider the case when the bond market is complete. By (1) and Theorem 1, the equilibrium annuity price is given by:

$$q^{AF} \int_{\underline{\alpha}}^{\overline{\alpha}} a\left(\alpha, q^{AF}\right) g\left(\alpha\right) d\alpha = \frac{1}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \left[1 + \alpha \mathbb{E}\left(\frac{1}{R_2}\right)\right] a\left(\alpha, q^{AF}\right) g\left(\alpha\right) d\alpha \, .$$

We refer to q^{AF} as the *risk-adjusted actuarially fair price*, which accounts for adverse selection in the annuity market in a complete market setting.

Next, consider a complete information economy. Let $q^{CI}\left(\alpha\right)$ denote the equilibrium price in an economy in which insurers observe individual mortality types and the bond market is complete. The complete information actuarially fair price $q^{CI}\left(\alpha\right)$ is given by:

$$q^{CI}\left(\alpha\right) = \frac{\alpha}{R_1} \left[1 + \alpha \mathbb{E}\left(\frac{1}{R_2}\right)\right].$$

Theorem 2 establishes the classic adverse selection result by illustrating the markup caused by adverse selection.

Theorem 2 $q^{AF} > \int_{\underline{\alpha}}^{\overline{\alpha}} q^{CI}(\alpha) g(\alpha) d\alpha$.

Finally, to understand how IRM affects the markup, we characterize how the equilibrium annuity price is affected by a marginal change in the supply of long-term bonds in an incomplete bond market. From (1) and Theorem 1, when the bond market is incomplete ($\zeta \in [0,1)$), the equilibrium annuity price is characterized by

$$q \int_{\underline{\alpha}}^{\overline{\alpha}} a(\alpha, q) g(\alpha) d\alpha = \frac{1}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \left[1 + \alpha \left(1 - \zeta + \zeta \mathbb{E}\left(\frac{1}{R_2}\right)\right)\right] a(\alpha, q) g(\alpha) d\alpha.$$

Let $z = 1 - \zeta + \zeta \mathbb{E}\left(\frac{1}{R_2}\right)$ and define $V(q, z)$ as the insurers' profit, such that

$$V(q, z) = q \int_{\underline{\alpha}}^{\overline{\alpha}} a(\alpha, q) g(\alpha) d\alpha - \frac{1}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha (1 + \alpha z) a(\alpha, q) g(\alpha) d\alpha ,$$

where $z \in \left[\mathbb{E}\left(\frac{1}{R_2}\right), 1\right]$. This means that the supply of long-term bonds is constrained whenever $z \in \left(\mathbb{E}\left(\frac{1}{R_2}\right), 1\right]$. Similar but inversely related to ζ, the variable z captures the severity of the bond market incompleteness, and, therefore, the need for insurers to manage their interest rate risk with net worth. Let q^* be the equilibrium annuity price—the lowest positive annuity price such that $V(q^*, z) = 0$. Theorem 3 shows that the equilibrium annuity price q^* is higher than the risk-adjusted actuarially fair price q^{AF}.

Theorem 3 *The competitive annuity price is increasing in the severity of the bond market incompleteness:* $\frac{\partial q^*}{\partial z} > 0$.

Theorem 3 demonstrates how IRM affects the AS-adjusted markup: $q^* - q^{AF}$. When bond markets are complete, then the AS-adjusted markup is zero: $q^* = q^{AF}$. The AS-adjusted markup increases as the severity of the bond market incompleteness increases. In the next two sections, we explain how our theory differs from the "textbook" model of adverse selection in insurance market and how we can make progress identifying the risk management channel in annuity price data.

3 The missing ingredient in the textbook environment

It is easiest to grasp the fundamental difference between our model and the textbook model of adverse selection in insurance markets—e.g., Einav & Finkelstein (2011)—through graphical illustration. Figures 2 and 3 are graphical representations of the equilibrium in our model with a complete and incomplete bond market, respectively. The horizontal axis indicates the annuity price q. The vertical axis indexes the total demand of annuities by individuals who differ in their mortality risk α, which is not observable by insurers. The downward-sloping demand curve is denoted as $A(q) = \int a(\alpha, q)\, dG(\alpha)$.

When the bond market is complete and the supply of long-term bonds is unlimited ($\zeta = 1$), the equilibrium price q^* in our model corresponds to the equilibrium price in the textbook model. In the textbook model, the competitive price is determined by the intersection of the downward-sloping demand curve and the insurer average cost curve, which is the cost per dollar of annuities sold. This is because insurers can only offer a single annuity price to heterogeneous individuals when their mortality risk is not observable. Perfect competition drives the equilibrium annuity price to be equal to the insurers' average cost.

Unlike the textbook model, the competitive equilibrium price q^* in our model is determined by the intersection of the demand curve $A(q)$ and the insurers' average bond demand $B(q)/q$, which is the amount of bonds insurers demand per dollar of annuities sold. The intersection of $A(q)$ and the average cost curve $C(q)/q$ pins down the *risk-adjusted actuarially fair price* q^{AF} discussed in the previous section. Recall that when the bond market is complete, life insurers can construct an optimal bond portfolio that perfectly hedges the interest rate risk. This case is shown in Figure 2: Insurers optimally hold zero net worth and the average cost curve $C(q)/q$ is equal to the bond demand curve $B(q)/q$. Therefore, the equilibrium price q^* in our model corresponds to the equilibrium price in the textbook model when the bond market is complete, such that $q^* = q^{AF}$.

An important difference with the textbook model is that the Modigliani & Miller (1958) theorem does not hold in our model even if the bond market is complete ($\zeta = 1$). As explained in Theorem 1, insurers' risk management consists of choosing an optimal ex-

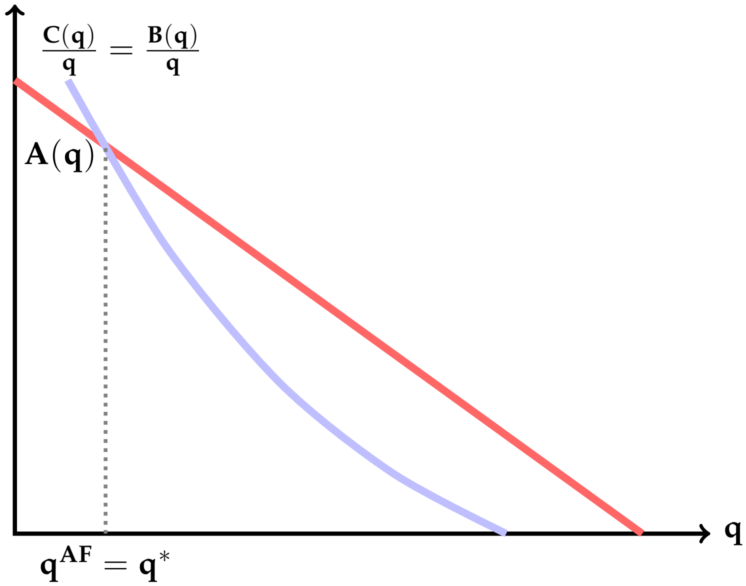


Figure 2: Equilibrium with $\zeta = 1$

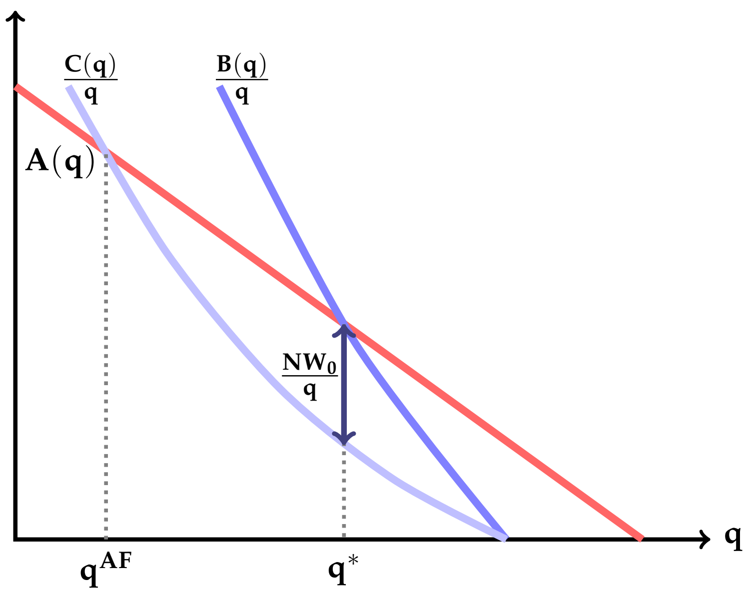


Figure 3: Equilibrium with $\zeta < 1$

ante capital structure and asset portfolio. When the bond market is complete, insurers optimally choose to hold zero net worth—i.e., $NW_0(q)/q = 0$. In contrast, an insurer's capital structure is irrelevant in the textbook model because financial markets are fully efficient. The financial friction that leads to a departure from the Modigliani & Miller

(1958) theorem is the insurers' limited liability. Limited liability implies that insurers must credibly show to annuity shoppers that they are managing their risks, which pins down a unique ex-ante capital structure even when the bond market is complete.

Limited liability leads to a different outcome from the textbook model when the supply of long-term bonds is constrained ($\zeta < 1$), which is an additional source of financial friction in our model. Theorem 1 shows that insurers must finance a positive level of net worth per dollar of annuities sold ($NW_0/q > 0$) to cushion the effect of future interest rate shocks and prevent insolvency. As shown in Figure 3, the average cost curve $C(q)/q$ and average bond demand curve $B(q)/q$ are no longer equal when the supply of long-term bonds is constrained. From Theorem 3, insurers finance their net worth by charging a higher annuity price—the equilibrium annuity price q^* is higher than the *risk-adjusted actuarially fair price* q^{AF}.

The size of the AS-adjusted markup, which we define as $q^* - q^{AF}$, reflects the effects of financial frictions associated with IRM. Using Theorem 3, we can derive the equilibrium relationship between insurers' optimal net worth, which is generally not observable and annuity price markups, which are observable at a high frequency:

$$NW_0 = \int_{\underline{\alpha}}^{\overline{\alpha}} \left(q^* - q^{CI}(\alpha)\right) a(\alpha, q^*)\, d\alpha - \int_{\underline{\alpha}}^{\overline{\alpha}} \left(q^{AF} - q^{CI}(\alpha)\right) a\left(\alpha, q^{AF}\right) d\alpha\,.$$

In the next section, we discuss how we measure annuity markups in the data and how we identify the risk management channel using exogenous shifters to the average cost curve $C(q)/q$ and average bond demand curve $B(q)/q$.

4 Identification

There are two main empirical challenges to identifying the risk management channel in annuity price data. First, it is not possible to directly measure the duration gap between U.S. insurers' assets and insurance liabilities. The reason is that the actual discount rate used by life insurers to value their insurance liabilities is not observable and insurance liabilities are not reported at the contract level in statutory fillings. Second, different types of "supply-side" frictions may lead to observably equivalent annuity markups. For example, annuity markups could be the outcome of monopolistic competition. We overcome these challenges by exploiting shocks that differentially affect the average cost curve $C(q)/q$ and the average bond demand curve $B(q)/q$ of *different annuity contracts* offered by the *same insurer*.

Our identification strategy exploits the property that, under the hypotheses of our model, the $C(q)/q$ and $B(q)/q$ curves determine the AS-adjusted markup when the bond market is incomplete ($\zeta < 1$). On the one hand, an exogenous increase in $C(q)/q$ means that an insurer faces a larger insurance liability for a given pair of annuity price and quantity. Because the interest rate risk is unchanged, competitive insurers finance additional bonds per unit of annuity sold by increasing q^* to maintain the optimal $NW(q)/q$. As a result, $B(q)/q$ increases relatively more than $C(q)/q$ and the AS-adjusted markup $q^* - q^{AF}$ increases. On the other hand, an exogenous decrease in $B(q)/q$ means that the insurer can fund a block of new annuity business with fewer bonds. Competitive insurers respond by offering a higher annuity yield (i.e., a lower price q^*), which increases the average cost along the the $C(q)/q$ curve and $q^* - q^{AF}$ decreases.

The combination of exogenous shocks to the average cost curve $C(q)/q$ and the average bond demand curve $B(q)/q$ is necessary to identify the risk management channel. Loosely speaking, one shock allows us to identify the effect of supply-side frictions while the other shock allows us to tease out the risk management channel from other supply side channels. We can identify the risk management channel by comparing the change in AS-adjusted markup $q^* - q^{AF}$ for annuity contracts with higher average cost to those with lower average cost as a response to the same shock to the average bond demand curve $B(q)/q$. In the remainder of this section, we discuss how we measure $q^* - q^{AF}$, and the two different sources of variation that exogenously shift the average cost curve $C(q)/q$ and the bond demand curve $B(q)/q$. In the next section, we explain how we can implement our test of the risk management channel using a regression framework.

4.1 Annuity price markups measurement

Life insurers reprice their annuities frequently in response to changes in market conditions. It is straightforward to interpret our simple model as the marginal pricing decision of a life insurer. Using this interpretation, a life insurer creates a new block of business at date t, which is added to its existing block of annuities. Therefore, the first step in our identification strategy is to evaluate the insurers' marginal pricing decisions conditional on bond market conditions.

The first input in valuing annuity cash flows is the discount rate. We follow our theory closely and value new annuity cash flows from the perspective of the *owner* of a life insurer operating under limited liability. As discussed in Section 1, annuity contracts are illiquid fixed rate liabilities and life insurers invest their annuity considerations primarily in relatively illiquid fixed-income securities in an effort to match their asset and liability cash flows and offer a competitive return to annuitants. Therefore, our choice of cash flow discount rate needs to be consistent with the yield at which the marginal shareholder of this insurer is willing to commit capital to support the issuance of *illiquid* long-term fixed rate liabilities.[6] Almost all life insurers offering annuities in the U.S. have around an A rating and invest their annuity considerations in a portfolio of A-rated fixed income instruments, on average. Therefore, the discount rate of an average insurer's marginal shareholder should be close to the (duration-matched) yield on A-rated debt securities.

We proxy for the unobserved discount rate of the marginal life insurer shareholder using the zero-coupon High Quality Market (HQM) yield curve produced by the U.S. Treasury.[7] The HQM yield curve is calculated using AAA, AA, and A-rated U.S. corporate bonds and is heavily weighted towards A-rated bonds, consistent with their market share. For example, the sample of bonds used to calculate the HQM yield curve on August 31,

[6] The discount rate of an annuity shopper is likely to be different from the discount rate of the owner of a life insurer. An annuity shopper seeking a safe longevity insurance contract may *perceive* an annuity contract to be relatively "safe" because of the existence, for example, of a state insurance guarantee fund. Consequently, the payoff structure of a life insurer's shareholders and the annuity contract holders are vastly different in the event the life insurer is placed in receivership by its state insurance regulator. For the latter, using a default free discount rate to value annuity contracts may be appropriate. See Novy-Marx & Rauh (2011), Brown & Pennachi (2016) for related discussions about the appropriateness of different discount rates for pension liabilities.

[7] The HQM yield curve data is available at https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Pages/Corp-Yield-Bond-Curve-Papers.aspx.

2011 includes 12 commercial papers, 42 AAA bonds, 299 AA bonds, and 1,345 A bonds.[8]

The second input to valuing annuity cash flows is an assumption about individuals' mortality. Virtually none of the fixed annuities sold by U.S. life insurers are underwritten, which means these contracts require no medical exam and their terms only depend on the date of birth and gender of the individual. We use three different types of mortality assumptions. First, we use a "general" population period mortality table produced by the U.S. Internal Revenue Service that is updated annually with the mortality experience of the entire U.S. general population.[9] Second, we use two different versions of the Individual Annuitant Mortality table produced by the Society of Actuaries (SOA) in collaboration with the National Association of Insurance Commissioners (NAIC). In addition to a "basic" annuitant mortality table, which is estimated from the actual mortality experience of a large pool of annuitants, the SOA produces a "loaded" annuitant mortality table which is used by state insurance regulators to set annuity reserves.[10]

We define the actuarial value of a life annuity contract with an M-year guarantee term per dollar using mortality assumption $i \in \{\text{General}, \text{Basic}, \text{Loaded}\}$ as

$$V_t^i(n, S, M, r) = \underbrace{\sum_{m=1}^{M} \frac{1}{R_t(m, r)^m}}_{\text{M-year term certain annuity}} + \underbrace{\sum_{m=M+1}^{N_S^i - n} \frac{\Pi_{l=0}^{m-1} p_{S,n+l}^i}{R_t(m, r)^m}}_{\text{Life annuity from year } M+1}, \tag{7}$$

where $M \geq 0$ is the number of years the life annuity pays a guaranteed fixed income, $p_{S,n}^i$ is the one-year survival probability for an individual of gender S at age n from the i-th

[8]For more information, see https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Documents/ycp_oct2011.pdf

[9]The general population mortality tables are available at https://www.irs.gov/retirement-plans/actuarial-tables.

[10]The SOA mortality tables are available at https://mort.soa.org/. There are two important differences between the "basic" and the "loaded" annuitant mortality tables. First, the loaded table adds a flat 10 percent loading on estimated survival probabilities which requires insurer to hold more reserve per dollar of annuity sold. Second, statutory regulation did not require insurers to apply the SOA generational mortality improvement factor to the static loaded mortality table for their reserve calculations prior to 2015 when the 2012 table was adopted in most states. As a consequence, regulatory reserves prior to the adoption of the generational table around 2015 became less conservative over time, as the population mortality naturally improved. This phenomenon led the NAIC to update the loaded table in 2000 to essentially "reset" the loading factor. For all our calculations prior to the adoption of the 2012 SOA generational table, we follow industry practice and apply the SOA generational factor to adjust the mortality estimate from the static basic table to the year of observation.

mortality table, and N_S^i is the maximum attainable age for this gender in the i-th mortality table, and $1/R_t(m,r)^m$ is the reference discount factor for period m cash flow evaluated at time t using the HQM yield curve, $r = \text{HQM}$, or the regulatory reference rate , $r = \text{NAIC}$, which we will explain below.

Let $P_t(n, S, M)$ be the price of an M-year guaranteed life annuity offered by a life insurer to an individual of gender S and age n at date t, we decompose the total annuity price markup into an insurer-contract-level AS-adjusted markup and an industry-contract average measure of adverse selection pricing (AS pricing) as:

$$\begin{aligned} P_t(n,S,M) - V_t^{\text{General}}(n,S,M,r=\text{HQM}) \\ = \underbrace{\Big(P_t(n,S,M) - V_t^{\text{Basic}}(n,S,M,r=\text{HQM})\Big)}_{\text{Adverse selection-adjusted markup}} \\ + \underbrace{\Big(V_t^{\text{Basic}}(n,S,M,r=\text{HQM}) - V_t^{\text{General}}(n,S,M,r=\text{HQM})\Big)}_{\text{Average adverse selection pricing}}, \end{aligned}$$

It follows that the insurer-contract-level variable $P_t(n,S,M) - V_t^{\text{Basic}}(n,S,M,r=\text{HQM})$ is the counterpart of the AS-adjusted markup $q^* - q^{AF}$ in our model.

Figure 1, which we discussed in the introduction, plots the distribution of actual monthly payments offered to a 65 year old male for a \$100,000 single premium immediate life annuity from a sample of U.S. life insurers against the monthly payment implied by the the actuarial value of the contract calculated using the general population mortality (dashed) and the annuitant mortality (solid). The difference between the dashed and solid lines measures AS pricing and the difference between the actual monthly payments offered by insurers (the box plot) and the solid line is related to the AS-adjusted markup. We turn to sources of exogenous variation next.

4.2 Regulatory reserve requirements

The first source of exogenous variation comes from annuity contract-level time series variation in the regulatory reserves insurers are required to set aside for each dollar of annuity they sell. As explained by Koijen & Yogo (2015), exogenous time-series variation in reserve requirements across contract maturity arises because regulatory reserves are calculated using a single interest rate that is reset infrequently. We denote this regulatory interest rate by $r = \text{NAIC}$ in the present value discussed above.

Prior to 2018, state insurance regulation required that insurers calculate their annuity reserves—i.e., their insurance liabilities—using a single reference interest rate calculated as "the average over a period of twelve (12) months, ending on June 30 of the calendar year of issue or year of purchase, of the monthly average of the composite yield on seasoned corporate bonds, as published by Moodys Investors Service, Inc."[11] The Moody's composite yield on seasoned corporate bonds is a weighted average yield on all investment grade corporate bonds rated between Baa and Aaa with maturity of at least 20 years.

From 2018, state insurance regulators adopted a new methodology to calculate the single reference interest rate used in regulatory reserve regulations. With the new methodology, the reference interest rate is the sum of a weighted average U.S. Treasuries yield plus a credit spread and an expected default cost. The spread over the reference Treasury rate is calculated by the NAIC using the public bond portion of an average US life insurers asset portfolio. The new reference interest rate varies by the type of annuity contract guarantee period and is reset once a quarter (for retail annuity contracts). For example, the reference rate for a Single Premium Immediate Annuity issued on March 2, 2018 without a guarantee period to a 68 year-old was 3.25 percent, which is about 75 basis points (0.75 percentage points) higher than the reference Treasury rate used in the reference rate calculation.[12] By comparison, Moody's seasoned Aaa and Baa corporate bond yields on the same day are 3.9 and 4.58 percent, respectively.

By construction, the regulatory reference interest rate is close to the average of the longer end of the HQM yield curve. When the actual yield curve is upward sloping, the actuarial value of a life annuity calculated using the average of the long end of a yield curve is mechanically smaller than the corresponding actuarial value calculated using the entire yield curve. Moreover, this difference is greater for life annuities with shorter

[11] https://www.naic.org/store/free/MDL-820.pdf

[12] For more details, see https://www.soa.org/globalassets/assets/library/newsletters/financial-reporter/2018/june/fr-2018-iss113-hance-gordon-conrad.pdf.

expected maturity—i.e., sold to older individuals. Because the regulatory interest rate is reset infrequently (once a year prior to 2018 and once a quarter from 2018 instead of daily), the difference between the ratio of reserves required to the reserve calculated using the daily yield curve exogenously fluctuates over time across annuity contract maturity.

Figure 4: **Reserve ratio for Single Premium Immediate Annuity sold to a 65 year old male (top line) and 70 year old male (bottom line)**

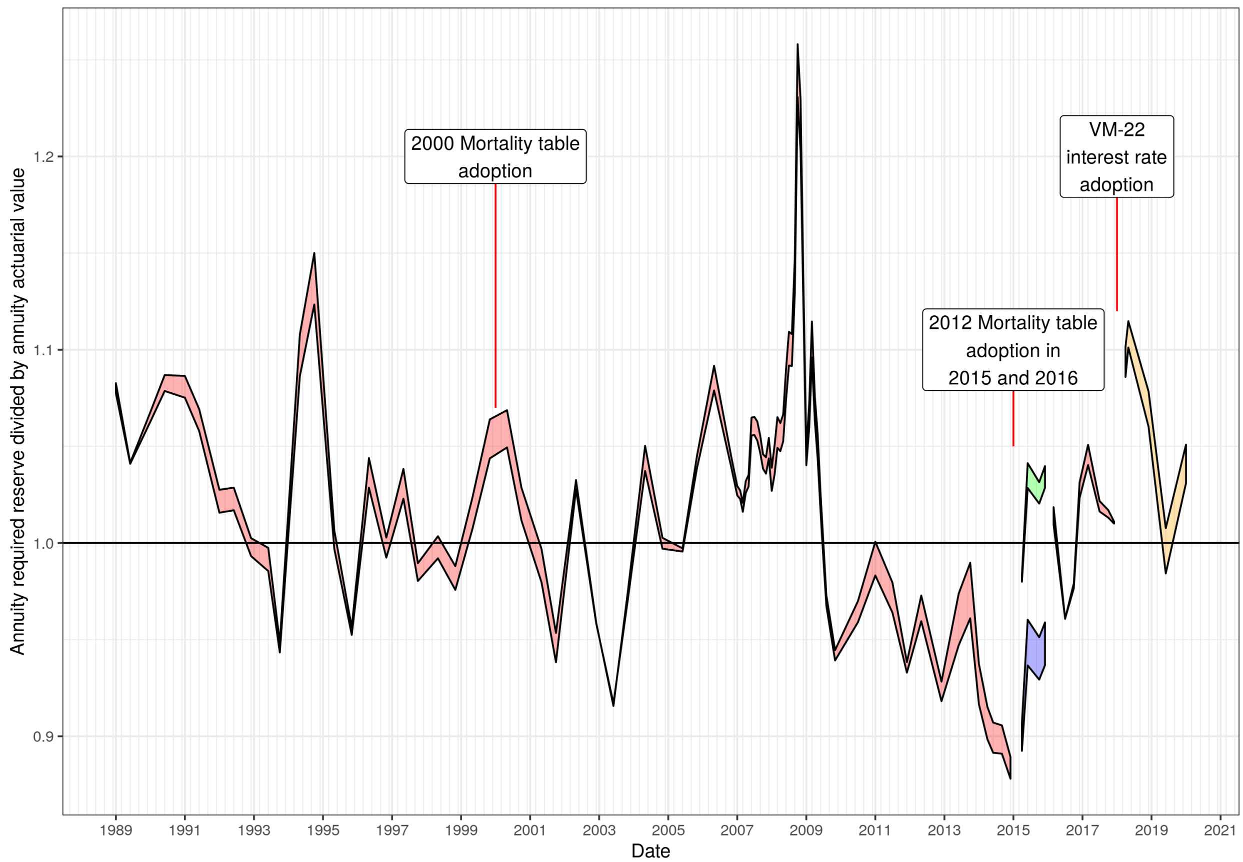


Figure 4 graphically illustrates this source of exogenous variation by plotting the reserve dollars an insurer needs to set aside for each dollar of annuity sold on day t to 65 and 70 year old males. We calculate the regulatory reserve ratio as $V_t^{\text{Loaded}}(n, S, M, r = \text{NAIC}) / V_t^{\text{Basic}}(n, S, M, r = \text{HQM})$. A ratio above 1 indicates that the reserve requirement is *binding*, as the insurer must create a reserve that is greater than the insurance liability warranted by the insurers' yield curve-based actuarial calculation. Conversely, a ratio below 1 indicates that the reserve requirement is not binding because the required reserve is below the insurer's own actuarial calculation. Figure 4 shows the additional source of variation arising from US states staggered adoption of new regulatory mortality assumptions between 2015 and 2016, and the 2018 adoption of the new methodology to calculate the regulatory reference interest rate previously discussed.[13]

Using Figure 3 as a guide, an exogenous increase in reserve requirements increases the average cost curve $C(q)/q$ because the insurer must create a larger insurance liability for a given pair of annuity price and quantity. The insurer matches this larger insurance liability by financing additional bonds per unit of annuity sold, which requires increasing q^* to maintain the optimal $NW(q)/q$ and offset some of the increase in $C(q)/q$. As a result, $B(q)/q$ increases relatively more than $C(q)/q$ and the AS-adjusted markup $q^* - q^{AF}$ rises.

4.3 Yield spreads on long duration investment grade bonds

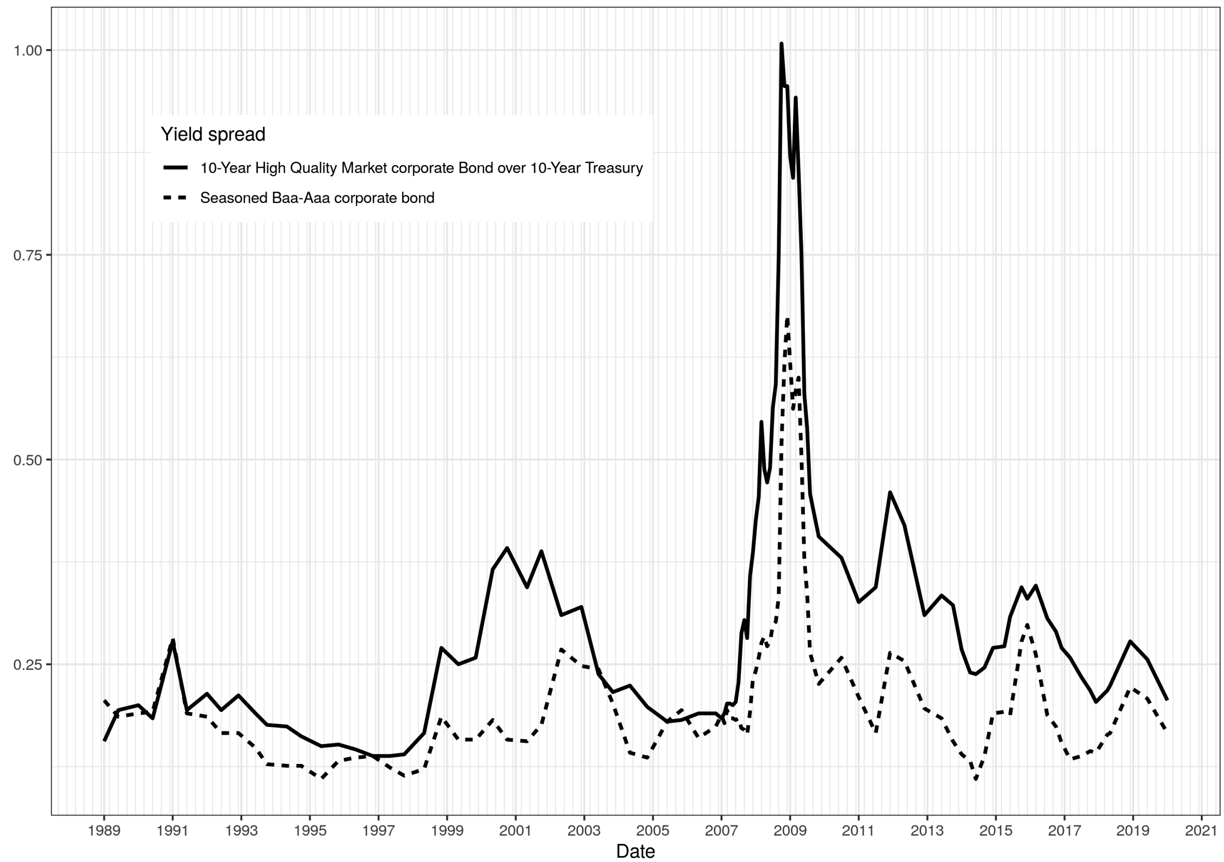
The second source of exogenous variation comes from aggregate time-series variation in the spread between the yield on Baa-rated and Aaa-rated corporate bonds that have at least 20 years of maturity. As we explained above, the regulatory interest rate used prior to 2018 is a weighted average of the corporate bond yields rated above Baa that have at least 20 years of maturity. Under state insurance regulation, corporate bonds rated above Moody's Baa are designated as NAIC 1 and attract the lowest statutory risk-based capital charge. This risk-based capital charge reflects the fact that, historically, the increase in credit risk for a firm moving from A to Aaa is negligible. As explained in Section 1, the life insurer's business model is to earn a spread between the yield on the assets purchased with premiums and considerations and the yield credited on insurance liabilities. Therefore, a widening in Baa-Aaa spread for long duration corporate bonds *conditional* on the insurer's cost of funding corresponds to higher yielding investment opportunities for new annuity money.

Figure 5 graphically illustrates this source of exogenous variation by plotting the Baa-Aaa spread for seasoned corporate bonds and the 10 year HQM yield spread over 10 year US treasury in percentage points. Life insurers can generate more yield per dollar

[13]Roughly half of the U.S. States required insurers to use the 2012 Annuitant mortality table in 2015 and the other half from 2016. We carefully parse each state insurance regulators' website to identify the year at which a new mortality table is implemented for the purpose of regulatory reserve calculation based on the NAIC standard valuation model law 820-1.

of annuity sold when the Baa-Aaa spread increases more than the 10 year HQM yield spread, which is our proxy for the insurers' average cost of funding, since life annuities have an original duration of about 10 years.

Figure 5: **Baa-Aaa spread and insurers' average cost of funding**



Using Figure 3 as a guide once more, an exogenous increase in long duration bond yield decreases $B(q)/q$ because the insurer can fund a block of new annuity business with fewer bonds. Other things being equal, the decrease in $B(q)/q$ results in a level of $NW(q)/q$ that is higher than the optimal level. Competitive insurers respond by lowering the price of their annuity q^*—increasing the yield on their annuities. This corresponds to a movement up along $C(q)/q$ and results in a lower AS-adjusted markup $q^* - q^{AF}$.

That said, our model assumes that the annuity demand curve is fixed. In reality, annuity demand and bond market conditions are likely to be correlated as both financial instruments allow individuals to transfer wealth across periods. In Section 6, we explain how we can implement our test of the IRM channel in a difference-in-differences regression framework that compares the change in AS-adjusted markup for annuity contracts with higher relative reserve requirement to those with lower relative reserve requirement in response to the same shock to the Baa-Aaa spread. We briefly discuss our data next.

5 Data and variable definitions

We focus our analysis on Single Premium Immediate Annuities (SPIA) and SPIA with 10 and 20 year term certain guarantees, which are annuity contracts offered to individuals to fund their retirement. SPIA with 10 and 20 year term certain guarantees promise a payment to a beneficiary during a term period irrespective of the annuitant's survival. Our sample includes quotes from 99 life insurers, with about 20 life insurers per reporting dates. Price quotes are typically reported for male and female individuals aged between 50 and 90 years with five year interval. Annuity prices are collected from the 1989-2019 issues of the *Annuity Shopper Buyer's Guide*.[14] Table 1 reports the summary statistics for the variables used in our analysis.

Our main dependent variable $Annuity\_markup_{ijt}$ is the normalized AS-adjusted markup for product j sold by insurer i, defined as

$$Annuity\_markup_{ijt} = \frac{P_{ijt}(n, S, M)}{V_{jt}^{\text{Basic}}(n, S, M, r = \text{HQM})} - 1 \,.$$

The AS-adjusted markup is just over 15 percent on average and consistently above 10 percent during our sample period. The variable $Reserve\_Ratio_{ijt}$ is the ratio of reserve dollars insurers need to set aside for each dollar of annuity j sold on day t, defined as

$$Reserve\_Ratio_{jt} = \frac{V_{jt}^{\text{Loaded}}(n, S, M, r = \text{NAIC})}{V_{jt}^{\text{Basic}}(n, S, M, r = \text{HQM})} \,.$$

The reserve ratio fluctuates precisely around 1, confirming that the regulatory discount rate and the insurers' discount rate are aligned on average. We also obtain time-varying insurer characteristics data from NAIC statutory filings for 2001-2019. We measure in-

[14] Koijen & Yogo (2015) use the same data source from 1989 to 2011.

Table 1: Summary statistics

	Obs.	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Number of insurers by period		19.8	6.14	16	19	23.2
Number of contracts by period		634	374	294	686	914
Life annuity contract (binary):						
Life only	111,065	0.36				
10 year guarantee	111,065	0.35				
20 year guarantee	111,065	0.29				
50 years old	111,065	0.11				
55 years old	111,065	0.11				
60 years old	111,065	0.14				
65 years old	111,065	0.14				
70 years old	111,065	0.14				
75 years old	111,065	0.13				
80 years old	111,065	0.10				
85 years old	111,065	0.09				
90 years old	111,065	0.04				
Male	111,065	0.50				
Female	111,065	0.50				
$Annuity\_markup_{ijt}$ (%)	111,065	15.68	4.72	12.57	15.30	18.33
$Reserve\_Ratio_{ijt}$	111,065	1.00	0.06	0.96	1.00	1.04
$10Y\text{-}3M\_Treasury\_spread_t$	111,065	1.83	1.02	1.06	1.98	2.56
$Baa\text{-}Aaa\_spread_t$	111,065	0.99	0.36	0.78	0.93	1.13
$10\_HQM\_spread_t$	111,065	1.53	0.58	1.17	1.39	1.72
$Log\_total\_assets_{it}$ (from 2001)	99,737	2.69	1.59	1.76	2.84	3.78
$Leverage\_ratio_{it}$ (from 2001)	99,737	10.55	5.23	6.96	10.05	13.54

surer size as the log of insurers' general account assets and leverage as the ratio between the insurers' general account assets and general account liabilities minus statutory accounting surplus.

We obtain Moody's Seasoned Aaa and Baa corporate bond yields, the 10-Year Treasury Constant Maturity Rate, and 10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Maturity from the Saint Louis Fed's FRED database. We proxy for the insurers' cost of funding by calculating the spread between the 10-year High Quality Market and the 10-Year Treasury Constant Maturity Rate. For all our regressions, we retain the last set of prices observed in a quarter. Our final dataset contains 111,065 insurer-contract-quarter observations with an average of 634 insurer-contract observations per reporting period.

6 Main empirical analysis and results

Adopting a difference-in-differences approach, we test the hypothesis that the AS-adjusted markups on annuity contracts facing higher regulatory reserve requirements are relatively higher in periods when the Baa-Aaa spread is lower. In the main specification, the first difference is between annuity contracts i offered by insurer j with relatively high reserve requirement and different annuity contracts $-i$ offered by the *same* insurer with relatively low reserve requirement. The second difference is between periods in which the Baa-Aaa spread is high and periods in which the Baa-Aaa spread is low. As discussed in Section 4, we condition all our tests on the average cost of funding of the insurer, which we proxy using the 10 year HQM zero coupon yield over the 10 year US Treasury spread.

We implement our test in a linear regression framework. The unit of observation is a life insurer-product-time. The sample of observation extends from 1989 to 2019. The coefficient β_3 on the interaction between $Reserve\_Ratio_{jt}$ and the $Baa\text{-}Aaa\_spread_t$ in the following linear model allows us to trace the difference-in-differences effect of the reduction in reserve requirement on AS-adjusted markup during times of increasing Baa-Aaa spreads *conditional* on the level of the $10\_HQM\_spread_t$:

$$\begin{aligned} Annuity\_markup_{ijt} =& \alpha_1^i + \alpha_2^j + \beta_1 Baa\text{-}Aaa\_spread_t + \beta_2 Reserve\_Ratio_{jt} \\ &+ \beta_3 Baa\text{-}Aaa\_spread_t \times Reserve\_Ratio_{jt} \\ &+ \beta_4 10\_HQM\_spread_t + \mathbf{z}_{it}'\gamma + \epsilon_{ijt}\ . \end{aligned} \tag{8}$$

Specification (8) includes an insurer fixed effect α_1^i to absorb the effects of potentially unobserved fixed insurer characteristics—e.g., difference in state regulations and insurer rating—that may directly affect life insurers' pricing behaviour. We also include a complete set of product fixed effects α_2^j—age, gender, and annuity guarantee type—to absorb the effect of fixed demand characteristics that may influence pricing. The vector z_{it}' includes other insurer-level time varying financial variables, such as insurer size and leverage. We report insurer clustered robust standard errors throughout.

Table 2 summarizes our main result. The coefficient estimate on the interaction term

Table 2: **The effect of investment-grade corporate-bond yield spread on life annuity markups** The unit of observation is a life insurer-product-year. The sample of observation extends from 1989 to 2019. The dependent variable *Annuity markup*$_{ijt}$ is the adverse selection adjusted markup for life annuity j sold by insurer i at date t. Column 1 reports insurer clustered robust standard errors in parentheses and Column 2 and 3 report two-way insurer and date clustered robust standard errors in parentheses. *** $p < 0.01$; ** $p<0.05$; * $p<0.1$.

Dependent variable:	*Annuity_markup*$_{ijt}$		
	(1)	(2)	(3)
Baa-Aaa_spread$_t$ $\times$*Reserve Ratio*$_{ijt}$	-12.68***	-12.68**	-12.87**
	(3.21)	(4.94)	(5.06)
Reserve_Ratio$_{ijt}$	28.93***	28.93***	28.73***
	(3.25)	(5.69)	(6.03)
Baa-Aaa_spread$_t$	11.15***	11.15**	11.29**
	(3.28)	(5.27)	(5.42)
10_HQM_spread$_t$	2.98***	2.98***	3.07***
	(0.40)	(0.77)	(0.82)
Log_total_assets$_{it}$			0.35
			(0.66)
Leverage_ratio$_{it}$			-0.03
			(0.03)
Fixed effects:			
Contract characteristics (j)	Y	Y	Y
Insurer (i)	Y	Y	Y
Observations	111,065	111,065	99,737
Adjusted R^2	0.40	0.40	0.40

suggests that, conditional on the average cost of funding proxied by $10\_HQM\_spread_t$, a one standard deviation increase in $Reserve\_Ratio_{ijt}$ (0.055) raises the AS-adjusted markup by almost one full percentage point when $Baa\text{-}Aaa\_spread_t$ is at its median level (0.93). Moreover, this effect is about 23 percent lower in periods when $Baa\text{-}Aaa\_spread_t$ is in the 3rd quartile of its distribution relative to periods when $Baa\text{-}Aaa\_spread_t$ is in the first quartile of its distribution. This means that, conditional on their average cost of funding proxied by $10\_HQM\_spread_t$, insurers tend to raise their AS-adjusted markups when the reserve requirement becomes binding ($Reserve\_Ratio_{ijt}$ is higher), and do so significantly less when $Baa\text{-}Aaa\_spread_t$ increases. This result suggests that insurers offset some of the higher cost of supplying life annuities due to excess reserve requirement with the extra yield on investment grade bonds.

Column 2 reports the two-way insurer and date clustered robust standard errors that allow for arbitrary types of within insurer correlation as well as contemporaneous correlation of the errors across different insurer clusters. Although onerous in terms of degrees of freedom, allowing for cross-insurer cluster correlation could be important given that insurers reprice their annuity products in response to aggregate bond market shocks. Consistent with this prior, Column 2 shows that the two-way clustered robust standard errors are about twice as large as those reported in Column 1. Nevertheless, our main coefficient estimate of interest remains significant at the less than 5 percent significance level. Finally, Column 3 controls for time varying insurer size measured as the log of the insurer's general account asset and insurer leverage measured as the ratio of the insurer's general account assets to liability minus statutory surplus (statutory surplus is analogous to our definition of net worth in the model in Section 2). Although we only observe these financial variables from 2001, the coefficient estimates in Column 3 are almost identical to those obtained with the full sample in Column 2.

In the next section, we present further evidence of the IRM channel using cross-sectional rather than within-insurer variation in annuity markups.

7 Cross-sectional evidence from interest rate derivatives

The paper so far emphasized building a cushion of net worth to manage future interest rate shocks. Another important aspect of life insurers' risk management is the preservation of net worth, as a long enough sequence of bad shocks could lead to insolvency. This motive is absent from our model because the economy ends at the end of the second period. In this section, we venture outside the predictions of our simple model and look for further evidence of the IRM channel in the cross-section of insurers by studying their interest rate derivative positions.

Life insurers can add positive duration to their balance sheet by entering into a long-term *fixed-for-float* interest rate swap with a counter party. Issuing a fixed-for-float interest rate swap is economically equivalent to financing a fixed maturity bond with short-term floating rate debt. The duration of a fixed-for-float swap contract is then the difference between the (hypothetical) underlying fixed rate instrument (usually a US Treasury bond) and the duration of the floating rate liabilities that finances the fixed rate instrument (usually 3 month LIBOR). An insurer adding more net positive duration with interest rate swaps is more likely to manage the risks associated with a widening negative duration gap between its assets and insurance liabilities.

Although it is not possible to measure a U.S. life insurer's duration gap, we can construct a proxy for the *marginal* net duration added by a life insurer's interest rate swaps. We can then measure how different hedging programs perform facing the *same* sequence of aggregate interest rate shocks and trace out the effect on annuity prices. For example, an insurer adding positive net duration with swaps is relatively more hedged against a flattening yield curve, and vice versa. Although an insurer's swap position is an ex-ante endogenous variable, variations in the the 10 year over 3 month Treasury spread during the period of the *zero lower bound* act as an exogenous shifter of the value of the swap portfolio ex-post. Therefore, we can compare the AS-adjusted markups of insurers that are favorably affected by the aggregate interest rate shock *ex-post* as a result of their *ex-ante* hedging program relative to those that are adversely affected by the shock.

An important issue with this test strategy is that the value of an insurer's swap portfolio may respond differently from a steepening of the yield curve that is driven by lower short rates and higher long rates. By focusing on the period of the *zero lower bound* from 2009 to 2015, we overcome this issue, since all the variations in the yield curve were driven by movements in long rates.

7.1 Interest rate swaps

We use position-level interest rate swap data to calculate a proxy of the net duration added by the swaps as a fraction of an insurer's general account asset portfolio—see Appendix B for details. Our position-level swap data comes from NAIC statutory filings Schedule DB. Schedule DB provides detailed information on each insurer's position-level derivative contracts, including a description of the contract term and its notional amount. We carefully parsed the text of more than 82,000 individual contract-year observations from 44 U.S. life insurers from 2009 to 2015 and extracted the receiving leg, notional amount, and residual maturity of the contracts. Life insurers in our sample have on average 1,416 open interest rate swap contracts at year's end with a standard deviation of 978. The average notional amount of a swap contract is $45 million with a standard deviation of $83 million.

We first calculate the quarter-end individual swap position using each contract's residual maturity. At each quarter-end, we normalize each individual swap contract duration by the duration of a reference 10 year fixed-for-float swap contract and multiply by the contract's notional amount. This number is the dollar amount of duration, which can be positive or negative, added to an individual swap contract. We then sum over an insurer's entire swap portfolio to obtain the aggregate dollar amount of duration added by the swaps. Finally we divide by the insurer's total general account assets to obtain the amount of net duration added by the swaps expressed as a fraction of the insurer's asset portfolio. We obtain 1,700 insurer-quarter observations from 2009 to 2015. A value of zero indicates that the insurer is not adding positive or negative duration using swaps. A value of 0.5 indicates that the insurer is adding net positive duration that is 50 percent of its size. We merge our net duration ratio variable, which we denote as $Net\_swap\_duration_{it}$, to the insurer-level annuity markup data, yielding 31,042 insurer-contract-date observations from 2009 to 2015. The variable $Net\_swap\_duration_{it}$ has a mean of 0.09, a median of 0.01, and a standard deviation of 0.16, which confirms that life insurers add positive duration with interest rate swap on average.

7.2 Regression results

We implement our cross-sectional test by interacting the $Net\_swap\_duration_{it}$ variable with $10Y\text{-}3M\_Treasury\_spread_t$ in the following equation:

$$\begin{aligned} Annuity\_markup_{ijt} =& \alpha_1^i + \alpha_2^j + \alpha_3^t + \beta_1 Net\_swap\_duration_{it} + \beta_2 Reserve\_Ratio_{ijt} \\ & + \beta_3 10Y\text{-}3M\_Treasury\_spread_t \times Net\_swap\_duration_{it} + \mathbf{z}_{it}'\gamma + \epsilon_{ijt}\ . \end{aligned}$$

We focus on the cross-sectional variation in life insurer $Net\_swap\_duration_{it}$ by including date fixed effect α_3^t. We continue to include an insurer fixed effect α_1^i to absorb the effect of fixed unobserved insurer characteristics, a complete set of product fixed effects α_2^j, and time varying insurer-level $\mathbf{z}_{it}'$, which includes insurer size and leverage. We report two-way insurer and date clustered robust standard errors as our benchmark, although the addition of a date fixed effect means that we obtain very similar standard errors using insurer clustered robust standard errors.

Column 1 of Table 3 summarizes our cross-sectional results. The coefficient estimate on the interaction term suggests that, conditional on time t market conditions and any fixed insurer and contract characteristics, a flattening of the yield curve (a one standard deviation decrease in $10Y\text{-}3M\_Treasury\_spread_t$) decreases the AS-adjusted markup of an insurer with a median level of $Net\_swap\_duration_{it}$ (0.013) by about 0.034 percentage point. The small economic magnitude of this average effect suggests that insurers' hedging strategy is effective on average. However, this effect is almost 50 times larger for insurers in the top quartile of the $Net\_swap\_duration_{it}$ distribution relative to those in the bottom quartile of the distribution. Insurers in the top quartile of the $Net\_swap\_duration_{it}$ distribution decrease their AS-adjusted markup by almost a third of a percentage point (1.8 percent of the average AS-adjusted markup) in response to a flatter yield curve.

Table 3: **Cross-sectional evidence of the risk managment channel** The unit of observation is an insurer-product-year. The sample of observation extends from 2009 to 2015, which covers the period of *zero lower bound*. The dependent variable *Annuity markup*$_{ijt}$ is the life annuity markups for product j sold by insurer i in year t. Column 1 is a fixed effect regression with two-way insurer and date clustered robust standard errors reported in parentheses. Column 2 to 4 is a quantile fixed effect regression implemented using the penalized fixed-effects estimation method proposed by Koenker (2004). Percentiles are indicated in the square parenthesis. Clustered bootstrapped standard errors (1,000 replications) are implemented using the generalized bootstrap of Chatterjee & Bose (2005) with unit exponential weights sampled for insurer-contract observations, and reported in parentheses. *** $p < 0.01$; ** $p<0.05$; * $p<0.1$.

Dependent variable: $Annuity\_markup_{ijt}$	(1)	Dependent variable: $Annuity\_markup_{ijt}$	(2) $\tau = 0.25$	(3) $\tau = 0.5$	(4) $\tau = 0.75$
$Net\_swap\_duration_{it} \times$ $10Y\text{-}3M\_Treasury\_spread_t$	5.35**	$Net\_swap\_duration_{it} \times$ $10Y\text{-}3M\_Treasury\_spread_t$	6.98***	4.6***	2.98***
	(2.38)		(0.57)	(0.36)	(0.32)
$Net\_swap\_duration_{it}$	-10.32*	$Net\_swap\_duration_{it}$	-13.89***	-7.44***	-4.85***
	(5.69)		(1.49)	(1.04)	(0.91)
		$10Y\text{-}3M\_Treasury\_spread_t$	-0.76***	-0.73***	-0.31**
			(0.14)	(0.14)	(0.13)
$Reserve\_Ratio_{ijt}$	42.41***	$Reserve\_Ratio_{ijt}$	12.85***	17.1***	24.18***
	(6.79)		(1.31)	(2.1)	(2.29)
		$Baa\text{-}Aaa\_spread_t$	0.86***	0.36	-1.33***
			(0.28)	(0.39)	(0.48)
		$10\_HQM\_spread_t$	1.6***	2.35***	3.72***
			(0.22)	(0.26)	(0.32)
$Leverage\_ratio_{it}$	0.10	$Leverage\_ratio_{it}$	-0.04	-0.05	-0.04
	(0.10)		(0.04)	(0.04)	(0.03)
$Log\_total\_assets_{it}$	-1.61	$Log\_total\_assets_{it}$	-0.14	-0.12	-0.09
	(1.60)		(0.14)	(0.15)	(0.14)
Fixed effects:		Fixed effects:			
Product characteristics (j)	Y	Product characteristics (j)			Y
Insurer (i)	Y	Insurer (i)			Y
Date (t)	Y	Date (t)			N
Observations	31,042	Observations			31,042
Adjusted R^2	0.51	Chi^2 test			54.06***

Columns 2 to 4 dive deeper by estimating a quantile regression with fixed effects. We estimate the conditional quantile functions $Q_{Annuity\_markup_{ijt}}(\tau|x_{ijt})$ of the response of the t-th observation on the j-th annuity contract offered by the i-th insurer's $Annuity\_markup_{ijt}$ given by

$$\begin{aligned} Q_{Annuity\_markup_{ijt}}(\tau|\mathbf{x}'_{ijt}) =& \beta_3(\tau) 10Y\text{-}3M\_Treasury\_spread_t \times Net\_swap\_duration_{it} \\ &+ \beta_1(\tau) Net\_swap\_duration_{it} + \beta_2(\tau) 10Y\text{-}3M\_Treasury\_spread_t \\ &+ \beta_3(\tau) Reserve\_Ratio_{ijt} + \beta_4(\tau) Baa\text{-}Aaa\_spread_t \\ &+ \beta_5(\tau) 10\_HQM\_spread_t + \alpha_1^i + \alpha_2^j + \mathbf{z}'_{it}\gamma(\tau) , \end{aligned} \tag{9}$$

for quantile $\tau \in \{0.25, 0.5, 0.75\}$, where $\mathbf{x}'_{ijt}$ is the vector of covariates, $\mathbf{z}'_{it}\gamma(\boldsymbol{\tau})$ is a vector of insurer level time varying controls, and α_1^i and α_2^j are the insurer and contract fixed effects, respectively. We also control for the effect of corporate bond market shocks $Reserve\_Ratio_{ijt}$, $Baa\text{-}Aaa\_spread_t$, and $10\_HQM\_spread_t$ that we discussed in the previous section.

The bottom row of Column 4 reports a χ^2 test of the null hypothesis that the 25^{th} and 75^{th} percentile coefficients on the interaction term are equal. The difference in the coefficients on the interaction term between the 25^{th} and the 75^{th} percentile is economically meaningful. The counterfactual decrease in AS-adjusted markup in response to a flattening of the yield curve (i.e, a one standard deviation decrease in $10Y\text{-}3M\_Treasury\_spread_t$) would have been about twice as large for an insurer moving from the bottom to the top of the AS-adjusted markup distribution. Moreover, consistent with the results in Column 1, the decrease in AS-adjusted markup by insurers at the top of the $Net\_swap\_duration_{it}$ distribution is almost 50 times larger than the decrease in AS-adjusted markup by insurers at the bottom of the $Net\_swap\_duration_{it}$ distribution in response to a flattening of the yield curve. Combining these two effects, the coefficients on the interaction terms in Columns 2 to 4 suggest that the least competitive insurers (those with relatively high markups) that are beneficially affected by the interest rate shocks as a result of their hedging program disproportionately cut their AS-adjusted markup.

For example, among the insurers in the top quartile of the AS-adjusted markup distribution (the least competitive insurers) who are also in the top of the $Net\_swap\_duration_{it}$ distribution (the better hedged against flattening of the yield curve) cut their markup by about 2 percent in response to a flattening of the yield curve (a one standard deviation decrease in $10Y\text{-}3M\_Treasury\_spread_t$). In contrast, those insurers at the bottom of the $Net\_swap\_duration_{it}$ and the top of the AS-adjusted markup distributions do not significantly cut their markup (0.04 percent).

8 The effect of bond market shocks on adverse selection

For the remainder of the paper, we return to our model in Section 2 to derive another testable hypothesis about the relationship between shocks originating from the corporate bond market and adverse selection in the annuity market. It is well-known that higher annuity prices are associated with more severe adverse selection (Rothschild & Stiglitz 1976). Using our model, we can decompose the effect of a change in the supply of the long-term bond, $z = 1 - \zeta + \zeta \mathbb{E}\left(\frac{1}{R_2}\right)$, on the equilibrium annuity price into a *risk management effect* and an *adverse selection effect*:

$$\frac{\partial q^*}{\partial z} = \underbrace{\frac{\frac{1}{R_1}\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha}{\int_{\underline{\alpha}}^{\overline{\alpha}} a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha}}_{\text{Risk management effect}} + \underbrace{\frac{\partial q^*}{\partial z} \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} -e\left(\alpha, q^*\right)\left[1 - \frac{\frac{\alpha}{R_1}(1+\alpha z)}{q^*}\right] a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha}{\int_{\underline{\alpha}}^{\overline{\alpha}} a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha}}_{\text{Adverse selection effect}}, \quad (10)$$

where $e\left(\alpha, q^*\right)$ is the price elasticity of annuity demand: $e\left(\alpha, q^*\right) = \frac{\partial a(\alpha, q^*)}{\partial q^*} \frac{q^*}{a(\alpha, q^*)}$. Note that each component of $\frac{\partial q^*}{\partial z}$ represents the average marginal effect of IRM on annuity price because each component is normalized by the total amount of annuity supplied.

A marginal increase in IRM raises the equilibrium annuity price because the insurer must finance greater net worth in $t = 0$. By Theorem 1, we can express the optimal amount of net worth at $t = 0$ as a function of $z : NW_0\left(z\right) = \frac{1}{R_1}\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha^2 \left[z - \mathrm{E}\left(\frac{1}{R_2}\right)\right] a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha$. It follows that the risk management effect in equation (10) can be written as

$$\text{Risk management effect} = \frac{\frac{\partial NW_0(z)}{\partial z}}{\int_{\underline{\alpha}}^{\overline{\alpha}} a\left(\alpha, q^*\right) g\left(\alpha\right) d\alpha},$$

showing that higher net worth for IRM requires insurers to charge a higher equilibrium annuity price.

Life insurers' IRM amplifies the effect of adverse selection in the annuity market because the average survival probability of individuals purchasing annuities increases as insurers charge a higher annuity price to finance their net worth. Equation (10) shows that the main determinant of the adverse selection effect is the price elasticity of annuity demand. The adverse selection effect amplifies the price increase if demand is more elastic for agents with lower survival probability $\alpha : -e(\alpha', q^*) > -e(\alpha'', q^*)$ when $\alpha' < \alpha''$. This is because the insurer would lose more agents of high mortality risk than those with lower risk when price increases, which worsens the adverse selection problem and precipitates further increases in price. This phenomenon is often referred to as a death spiral. To see this, first note that among agents purchasing annuities (all α such that $a(\alpha, q^*) > 0$), there exists a $\tilde{\alpha}$ such that $q^* < \frac{\alpha}{R_1}(1 + \alpha z)$ for any $\alpha > \tilde{\alpha}$ and $q^* > \frac{\alpha}{R_1}(1 + \alpha z)$ for any $\alpha < \tilde{\alpha}$.[15] Next, notice that $\frac{\alpha}{R_1}(1 + \alpha z)$ corresponds to the *full information* actuarially fair price when the constraint on the supply of long-term bonds is *binding*. This means insurers make a profit off of mortality types $\alpha < \tilde{\alpha}$, which equates to the loss from types with $\alpha > \tilde{\alpha}$ due to competition. If demand is more elastic for agents with low α, then insurers lose more agents with mortality type $\alpha < \tilde{\alpha}$ than mortality type $\alpha > \tilde{\alpha}$ for an increase in annuity price. This creates a net loss, so the insurer must further raise prices to compensate for more severe adverse selection. This theoretical result establishes a direct link between the supply-side and demand-side frictions, connected by the IRM channel.

8.1 Regression results

The above discussion suggests that corporate bond market shocks may have a direct effect on adverse selection. We look for evidence of this effect by exploiting differences in life insurers' pricing of SPIA with different types of "period certain" guarantees and by measuring the relative change in AS pricing for these products. Note that individuals choosing a life annuity with 10 or 20 year guarantees think they are at a higher risk of dying within the next 10 or 20 years (Finkelstein & Poterba 2004, 2006).

We measure AS pricing as the difference between the total annuity markup and the AS-adjusted markup as follows:

$$AS\_pricing_{ijt} = \frac{P_{ijt}(n, S, M)}{V_{jt}^{\text{General}}(n, S, M, r = \text{HQM})} - \frac{P_{ijt}(n, S, M)}{V_{jt}^{\text{Basic}}(n, S, M, r = \text{HQM})}.$$

[15] Due to competition, $\int_{\underline{\alpha}}^{\overline{\alpha}} \left(1 - \frac{\frac{\alpha}{R_1}(1+\alpha z)}{q^*}\right) a(\alpha, q^*)\, g(\alpha)\, d\alpha = 0$, which implies the existence of $\tilde{\alpha}$.

We then test the hypothesis that $AS\_pricing_{ijt}$ for annuity contracts with longer guarantee periods increase more when regulatory reserve requirements increase in a difference-in-differences framework. In this test, the first difference is between annuity contracts i offered by insurer j with a long guarantee period and annuity contracts $-i$ offered by the same insurer with no guarantee period. The second difference is between periods in which reserve requirements are more binding and periods in which reserve requirements are less binding. We implement our test in a linear regression framework as follows:

$$\begin{aligned} AS\_pricing_{ijt} = & \alpha_1^i + \alpha_2^j + \beta_1 Baa\text{-}Aaa\_spread_t + \beta_2 10\_HQM\_spread_t \\ & + \beta_3 10yr\_guarantee\_period + \beta_4 20yr\_guarantee\_period \\ & + \beta_5 10yr\_guarantee\_period \times Reserve\_Ratio_{ijt} \\ & + \beta_6 20yr\_guarantee\_period \times Reserve\_Ratio_{ijt} \\ & + \beta_7 Reserve\_Ratio_{ijt} + \mathbf{z}'_{it}\gamma + \epsilon_{ijt} \ , \end{aligned} \tag{11}$$

where $10yr\_guarantee\_period$ and $20yr\_guarantee\_period$ are binary variables indicating the length of guarantee period. The coefficients β_5 and β_6 on the interaction terms are measured relative to the effect on SPIA without guarantee period, which is the third type of life annuity contract in our sample and is omitted from this regression. As with our main specification in Section 6, we focus on within-insurer variation using insurer fixed effects and we condition our test on $Baa\text{-}Aaa\_spread_t$ and the average cost of funding of the insurer proxied with $10\_HQM\_spread_t$. The vector $\mathbf{z}'_{it}$ includes other insurer-level time varying financial variables, such as insurer size and leverage.

Table 4 summarizes the results of regression (11). The coefficients in Column 1 show that an exogenous increase in statutory reserve requirement disproportionately increases the AS pricing in life annuities with 10 and 20 year guarantees relative to life annuities without guarantees. For example, a standard deviation increase in the reserve ratio *decreases* the AS pricing of life annuities without a guarantee period by 0.84 percentage points. In contrast the AS pricing of life annuities with 10 and 20 year guarantees *increase* by 0.5 and 0.6 percentage points, respectively, as a response to the same shock. We continue to report two-way insurer and date clustered robust standard errors. The results in Column 2 are broadly similar when the same specification is estimated on a shorter sample period with time-varying insurer-level financial controls. Because individuals choosing life annuities with period certain guarantees think they are at a higher risk of dying within the next few years, this implies that changes in corporate bond market conditions have a direct effect on adverse selection in annuity markets, which is reflected, at least partly, in annuity prices.

Table 4: **The effect of corporate bond market shocks on adverse selection** The unit of observation is a life insurer-product-year. The dependent variable *Adverse selection* $pricing_{ijt}$ is the difference between the markup computed using the general population mortality table and the corresponding markup computed using the annuitant pool mortality table for annuity j sold by insurer i in year t. Two-way insurer and date cluster robust standard errors are reported in parentheses in columns (1) and (2), respectively. *** $p < 0.01$; ** $p<0.05$; * $p<0.1$.

Dep. variable:	$AS\_pricing_{ijt}$	
	(1)	(2)
$10yr\_Guarantee \times Reserve\_Ratio_{ijt}$	24.04***	23.13***
	(3.50)	(3.75)
$20yr\_Guarantee \times Reserve\_Ratio_{ijt}$	26.19***	25.99***
	(3.90)	(4.32)
$Reserve\_Ratio_{ijt}$	-15.18***	-14.38***
	(2.76)	(2.98)
10yr_Guarantee	-28.67***	-27.72***
	(3.56)	(3.78)
20yr_Guarantee	-34.09***	-33.83***
	(3.98)	(4.36)
$Baa\text{-}Aaa\_spread_t$	0.64***	0.71***
	(0.23)	(0.22)
$10\_HQM\_spread_t$	-0.79***	-0.82***
	(0.16)	(0.16)
$Leverage\_ratio_{it}$		-0.01
		(0.01)
$Log\_total\_assets_{it}$		0.56**
		(0.25)
Insurer FE	Y	Y
Observations	111,065	99,737
Adjusted R^2	0.68	0.68

9 Conclusion

In this paper, we show that almost half of the notoriously high life annuity price markups cannot be explained by adverse selection in annuity markets. We develop an algorithm for annuity valuation to decompose the contribution of demand- and supply-side frictions in annuity markups and provide a novel identification that exploits bond market shocks and the U.S. insurance regulatory framework. We show theoretically and empirically that life insurers' interest rate risk management drives the *adverse selection-adjusted* annuity price markup. We also identify a relationship between shocks that originate in the corporate bond market and adverse selection in the annuity market.

This paper bridges the gap between the economic literature on adverse selection in insurance markets and the finance literature on financial institutions' risk management. We hope our findings will motivate researchers to revisit important questions in public finance. For example, a robust result in the public finance literature is that social insurance crowds out private insurance—e.g., Peltzman (1973), Cutler & Gruber (1996), and Hosseini (2015). This result holds even when there are informational asymmetries in insurance and labor markets, as contracts can be designed to overcome this friction (Golosov & Tsyvinski 2007). It is unclear if this seminal result holds when considering the problem of life insurers managing interest rate risk. Moreover, studying life insurers' interest risk management may also shed light on important puzzles surrounding the shrinking US long-term care insurance market in the "low-for-long" interest rate environment—e.g., Ameriks, Briggs, Caplin, Shapiro, Tonetti et al. (2016), Braun, Kopecky & Koreshkova (2019).

References

Ameriks, J., Briggs, J., Caplin, A., Shapiro, M. D., Tonetti, C. et al. (2016), Late-in-life risks and the under-insurance puzzle, Technical report.

Blanchard, O. J. (1985), 'Debt, deficits, and finite horizons', *Journal of Political Economy* **93**(2), 223–247.

Bolton, P. & Scharfstein, D. S. (1990), 'A theory of predation based on agency problems in financial contracting', *American Economic Review* pp. 93–106.

Bolton, P. & Scharfstein, D. S. (1996), 'Optimal debt structure and the number of creditors', *Journal of Political Economy* **104**(1), 1–25.

Braun, R. A., Kopecky, K. A. & Koreshkova, T. (2019), 'Old, Frail, and Uninsured: Accounting for Features of the US Long-Term Care Insurance Market', *Econometrica* **87**(3), 981–1019.

Brown, J. R. & Pennachi, G. G. (2016), 'Discounting Pension Liabilities: Funding Versus Value', *Journal of Pension Economics and Finance* **15**(3), 254284.

Chatterjee, S. & Bose, A. (2005), 'Generalized Bootstrap for Estimating Equations', *The Annals of Statistics* **33**(1), 414–436.

Cutler, D. M. & Gruber, J. (1996), 'Does public insurance crowd out private insurance?', *The Quarterly Journal of Economics* **111**(2), 391–430.

Davidoff, T., Brown, J. & Diamond, P. (2005), 'Annuities and Individual Welfare', *American Economic Review* **95**(5), 1573–1589.

Eichenbaum, M. & Peled, D. (1987), 'Capital accumulation and annuities in an adverse selection economy', *Journal of Political Economy* **95**(2), 334–354.

Einav, L. & Finkelstein, A. (2011), 'Selection in insurance markets: Theory and empirics in pictures', *Journal of Economic Perspectives* **25**(1), 115–38.

Finkelstein, A. & Poterba, J. (2004), 'Adverse selection in insurance markets: Policyholder evidence from the U.K. annuity market', *Journal of Political Economy* **112**(1), 183–208.

Finkelstein, A. & Poterba, J. (2006), 'Testing for Adverse Selection with 'Unused Observables'', *NBER Working Paper 12112* .

Foley-Fisher, N., Narajabad, B. N. & Verani, S. (forthcoming), 'Self-fulfilling Runs: Evidence from the U.S. Life Insurance Industry'.

Foley-Fisher, N., Narajabad, B. & Verani, S. (2016), Securities lending as wholesale funding: Evidence from the us life insurance industry, Technical report, National Bureau of Economic Research.

Froot, K. A. & Stein, J. C. (1998), 'Risk management, capital budgeting, and capital structure policy for financial institutions: an integrated approach', *Journal of Financial Economics* **47**(1), 55–82.

Golosov, M. & Tsyvinski, A. (2007), 'Optimal taxation with endogenous insurance markets', *The Quarterly Journal of Economics* **122**(2), 487–534.

Hart, O. & Moore, J. (1994), 'A theory of debt based on the inalienability of human capital', *The Quarterly Journal of Economics* **109**(4), 841–879.

Hart, O. & Moore, J. (1998), 'Default and Renegotiation: A Dynamic Model of Debt', *The Quarterly Journal of Economics* **113**(1), 1–41.

Hosseini, R. (2015), 'Adverse selection in the annuity market and the role for social security', *Journal of Political Economy* **123**(4), 941–984.

Huang, C., Oehmke, M. & Zhong, H. (2019), 'A Theory of Multi-Period Debt Structure', *Review of Financial Studies* .

Koenker, R. (2004), 'Quantile Regression for Longitudinal Data', *Journal of Multivariate Analysis* **91**(1), 74–89.

Koijen, R. S. & Yogo, M. (2015), 'The cost of financial frictions for life insurers', *American Economic Review* **105**(1), 445–75.

Mitchell, O. S., Poterba, J. M., Warshawsky, M. J. & Brown, J. R. (1999), 'New evidence on the money's worth of individual annuities', *American economic review* **89**(5), 1299–1318.

Modigliani, F. & Miller, M. H. (1958), 'The cost of capital, corporation finance and the theory of investment', *American Economic Review* **48**(3), 261–297.

Novy-Marx, R. & Rauh, J. (2011), 'Public pension promises: how big are they and what are they worth?', *The Journal of Finance* **66**(4), 1211–1249.

Peltzman, S. (1973), 'The effect of government subsidies-in-kind on private expenditures: The case of higher education', *Journal of Political Economy* **81**(1), 1–27.

Rampini, A. A., Viswanathan, S. & Vuillemey, G. (2017), 'Risk management in financial institutions'.

Rothschild, M. & Stiglitz, J. (1976), 'Equilibrium in competitive insurance markets: An essay on the economics of imperfect information', *The Quarterly Journal of Economics* **90**(4), 629–649.

Yaari, M. E. (1965), 'Uncertain lifetime, life insurance, and the theory of the consumer', *Review of Economic Studies* **32**(2), 137–150.

A Theoretical results and proofs

Proof of Theorem 1: By Assumption 1, insurers must remain solvent to be able to sell annuities. Thus, insurers must engage in IRM so that $NW_1(R_2) \geq 0$ for any R_2. The optimal IRM strategy comprises of an optimal asset portfolio and an optimal capital structure, which can be derived following the arguments presented in Section 2.2.2.

Finally, we will argue that the optimal IRM is also unique in a competitive equilibrium. To do this, it is sufficient to show that (5) pins down the unique demand for b_1 for any q and l_2. If $b_1(l_2) < \frac{1}{R_1}\left[\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha(1+\alpha)\, a(\alpha, q)\, g(\alpha)\, d\alpha - R_l l_2\right]$, then there exists $\hat{R}_2$ such that $NW_1(R_2) < 0$ when $R_2 \in \left[1, \hat{R}_2\right]$: The insurer becomes insolvent when R_2 is sufficiently small. On the other hand, if $b_1(l_2) > \frac{1}{R_1}\left[\int_{\underline{\alpha}}^{\overline{\alpha}} \alpha(1+\alpha)\, a(\alpha, q)\, g(\alpha)\, d\alpha - R_l l_2\right]$, then $NW_1(1) > 0$: The insurer has strictly positive net worth even when R_2 is at its lowest level. This implies that the insurer can lower q and buy less b_1 and still perform IRM. This proves uniqueness. ■

Lemma 1 *Under Bertrand competition, no insurer earns a strictly positive profit and at least two insurers implement the optimal IRM strategy.*

Proof The first part of Lemma 1 follows from standard Bertrand competition argument. To see why the equilibrium features at least two insurers managing interest rate risk, suppose that instead none manages interest rate risk according to the strategy in Theorem 1. In this case, Assumption 1 implies that one insurer can earn strictly positive profit by choosing a price q and implement the hedging strategy in Theorem 1, which is a contradiction. ■

Proof of Theorem 2: Rewrite

$$q^{AF} \int_{\underline{\alpha}}^{\overline{\alpha}} a\left(\alpha, q^{AF}\right) g(\alpha)\, d\alpha = \frac{1}{R_1} \int_{\underline{\alpha}}^{\overline{\alpha}} \alpha \left[1 + \alpha \mathbb{E}\left(\frac{1}{R_2}\right)\right] a\left(\alpha, q^{AF}\right) g(\alpha)\, d\alpha\,.$$

as

$$\int_{\underline{\alpha}}^{\overline{\alpha}} \left[q^{AF} - q^{CI}(\alpha)\right] a\left(\alpha, q^{AF}\right) g(\alpha)\, d\alpha = 0.$$

There exists $\alpha^* \in (\underline{\alpha}, \overline{\alpha})$ such that $a\left(\alpha^*, q^{AF}\right) > 0$, and $q^{AF} > q^{CI}(\alpha)$ for any $\alpha < \alpha^*$ and $q^{AF} < q^{CI}(\alpha)$ for any $\alpha > \alpha^*$. This yields

$$\begin{aligned} 0 &= \int_{\underline{\alpha}}^{\alpha^*} \left[q^{AF} - q^{CI}(\alpha)\right] a\left(\alpha, q^{AF}\right) g(\alpha)\, d\alpha + \int_{\alpha^*}^{\overline{\alpha}} \left[q^{AF} - q^{CI}(\alpha)\right] a\left(\alpha, q^{AF}\right) g(\alpha)\, d\alpha \\ &< a\left(\alpha^*, q^{AF}\right) \int_{\underline{\alpha}}^{\overline{\alpha}} \left[q^{AF} - q^{CI}(\alpha)\right] g(\alpha)\, d\alpha\,. \end{aligned}$$

The result follows as $a\left(\alpha^*, q^{AF}\right) > 0$. ■

Proof of Theorem 3: First, we show that there exists a $q^* = \min\{q | V(q,z) = 0\}$ for any z. By Assumption 1, when $q = \frac{\overline{\alpha}}{R_1}(1 + \epsilon\overline{\alpha})$, then $V(q,z) > 0$. Also, when $q = 0$, then $V(q,z) < 0$. Since $V(q,z)$ is continuous in q, the intermediate value theorem implies that there exists q such that $V(q,z) = 0$. Therefore, the set $\{q | V(q,z) = 0\}$ is non-empty. Also, $\{q | V(q,z) = 0\}$ is closed, because $\{0\}$ is closed and V is continuous in q so $V^{-1}(\{0\}, z)$ is closed. Furthermore, $\{q | V(q,z) = 0\}$ is bounded below by zero. Hence, a minimum for $\{q | V(q,z) = 0\}$ exists. Next, through implicit differentiation,

$$\frac{\partial q^*}{\partial z} = \frac{\int_{\underline{\alpha}}^{\overline{\alpha}} \frac{\alpha^2}{R_1} a(\alpha, q^*) g(\alpha)\, d\alpha}{\frac{\partial V(q^*, z)}{\partial q^*}}.$$

Immediately, notice the numerator is strictly positive. Suppose the denominator, $\frac{\partial V(q^*, z)}{\partial q^*}$, is strictly negative. Then, this implies an insurer can deviate by lowering the price to capture the whole market and earn strictly positive profits. However, this contradicts the fact that $q^* = \min\{q | V(q,z) = 0\}$. ■

B Additional details about variable construction

B.1 Mortality assumption

[To be completed]

B.2 Interest rate derivatives

We proxy for the duration of each individual swap contract by assuming that the duration of the hypothetical zero coupon fixed rate bond is $0.75\times$ the residual maturity of the contract and the the interest rate reset on the floating leg of the swap occurs every 3 months. The factor 0.75 is a commonly used ruled of thumb when the actual swap curve is unavailable. Although it is quite crude, this assumption is reasonable to study the variation in average swap duration across insurers in our setting. Assuming that interest rate reset on the floating leg of the swap occurs every 3 months is consistent with the widely used 3-month LIBOR benchmark among life insurers. It follows that that the duration of a fixed-for-float swap is given by $\text{Swap duration}_{it}^{\text{Receive Fixed}} = 0.75 \times \text{Contract residual maturity} - 1/4 \times 1/2$. Similarly, we calculate the swap duration of individual float-for-fixed swap duration as $\text{Swap duration}_{it}^{\text{Receive Float}} = -0.75 \times \text{Contract residual maturity} + 1/4 \times 1/2$.

We then multiply each individual swap contract duration by their respective notional amount and divide this number by the duration of a reference 10 year fixed-for-float swap contract, which is calculated as $0.75 \times 10 - 1/4 \times 1/2$. Taking the average over each individual life insurer's swap portfolio in each year yields how much the insurer buy of the reference 10 year fixed-for-float swap. Finally, we divide by the insurers' total general account asset to obtain the amount of duration added by the swaps as a fraction of the insurer asset portfolio. This ratio is a measure of life insurers' interest rate risk management. A value of zero indicates that the insurer is not adding positive or negative duration to its portfolio using swaps.