

Coordination in the Network Minimum Game

Johannes Hoelzemann^{*}

Hongyi Li[†]

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Abstract

Motivated by the problem of organizational design, we study coordination in the *network minimum game*: a version of the minimum-effort game where players are connected by a directed network. We show experimentally that acyclic networks such as hierarchies are most conducive to successful coordination. Introducing a single link to complete a network cycle may drastically inhibit coordination. Further, acyclic networks enable resilient coordination: initial coordination failure is often overcome (exacerbated) after repeated play in acyclic (cyclic) networks.

JEL Classification: C72, C92, D85

Keywords: organizational design, weak-link game, minimum-effort game, coordination failure, quantal response equilibrium

^{*}Department of Economics, University of Toronto: www.johanneshoelzemann.com & j.hoelzemann@utoronto.ca.

[†]School of Economics, UNSW Business School: www.hongyi.li & hongyi@hongyi.li.

1 Introduction

Organizations generate coherent, intricate patterns of coordinated activity. This is no mean feat. An extensive experimental literature documents that coordination failure is almost inevitable in sufficiently large groups. These papers model coordination problems using the minimum-effort game (Van Huyck, Battalio, and Beil 1990), where a group of players stand to benefit if they successfully coordinate on high actions, but each player is incentivized to match the lowest action in the group.

The classic minimum-effort game captures a stylized organization where incentives are coarse and untargeted: each individual is rewarded based on the entire group’s performance, and thus is held responsible for coordinating with everyone else. In practice, the scope and complexity of task and incentive interdependencies within most organizations is limited – by design. Production may be organized so that workers on an assembly line coordinate amongst themselves but operate almost independently of the rest of the organization. Incentives may be tailored so that a team member is responsible only for completing his own assignments while the manager is responsible for the team’s overall performance.

Suppose we represent the set of interdependencies in an organization’s task and incentive design as a network across agents. Put loosely, if agent 2’s payoff depends on agent 1’s actions, we draw a link from $1 \rightarrow 2$. Given a network representation of organizational structure, we seek to understand how organization-wide coordination emerges from the ensemble of network interactions.

To do so, we introduce the *network minimum game*, a generalization of the minimum-effort game. In the network minimum game, players are linked by a directed network, and each player is incentivized to match the lowest action amongst his direct links.

The classic minimum-effort game corresponds to the special case of a complete network. In contrast, our framework allows for incomplete networks, thus capturing the notion of limited within-organization interdependencies. Further, by considering directed networks, we allow for asymmetric interdependencies between players: player i ’s payoff may depend on player j ’s actions, but not vice versa. Such asymmetries in responsibility are common, especially in organizational hierarchies. An entry-level worker may be tasked with mechanically following procedures and instructions, so that his payoff is independent of others’ actions. In contrast, a senior manager may be penalized for coordination failures even when her subordinates are at fault.

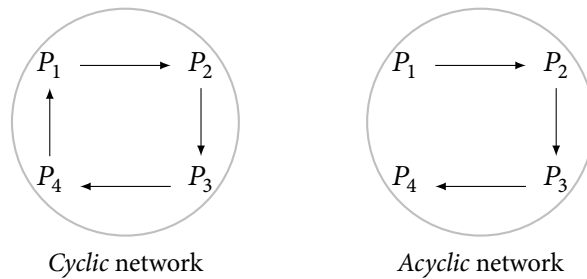


Figure 1: Cyclic vs. Acyclic Network (Examples)

Our experimental setting represents repeated interactions within long-lived organizations with persistent structure: subjects play in fixed groups, with fixed network structure, for ten rounds.

The treatments vary two aspects of network structure: network *cyclicity* (the existence of cycles of dependencies in the network; see Figure 1) and network *density* (the number of links per player).

We find that network cycles inhibit coordination. Table 1 summarizes our key findings. On acyclic networks of all densities, players coordinate on almost-maximal actions (the maximum action being 7). Cyclic networks perform worse than acyclic networks. Dense cyclic networks perform worst: they generate almost-minimal actions (the minimum action being 1). That is, cycles matter, especially for dense networks. Indeed, the difference between the dense acyclic and dense cyclic network in Table 1 is the addition of a *single* link which, by creating network cycles, generates *catastrophic* effects on overall coordination.

Table 1: Mean Final-Round Action, by Network Structure

	acyclic	cyclic
sparse	6.52 [.81]	4.90 [2.02]
dense	6.72 [.63]	2.08 [1.33]

Mean [st. dev.] action for each network structure, averaged over groups of size $n = 6$.

Action set = $\{1, 2, \dots, 7\}$.

Our experiment’s acyclic networks are hierarchical: they take the form of a chain-of-command where higher-indexed subjects depend on lower-indexed subjects. This construction captures the essence of an organizational hierarchy, where supervisors are held responsible for their subordinates’ activities and each worker has a unique chain-of-command leading to the CEO. Some papers study the efficiency of hierarchical structures from the perspective of organizational design (e.g., Sah and Stiglitz 1986, Radner 1993, Harris and Raviv 2014). Our results highlight that hierarchies, being acyclic, are particularly effective at fostering coordination.¹

What mechanisms underlie our results? We have in mind that dependency cycles create feedback loops that allow ‘seeds’ of strategic uncertainty to circulate and amplify, potentially leading to coordination failure: eventually, each player selects a low action simply because he anticipates that the next player in the cycle may do the same, ad infinitum. In contrast, destructive feedback loops do not arise in acyclic networks, thus enabling successful coordination.

This logic may be starkly interpreted in terms of Nash equilibrium. For acyclic networks, the unique Nash equilibrium is for all players to take a maximum action. Whereas, for our cyclic networks, every common action level is a Nash equilibrium – that is, the strategic uncertainty associated with network cycles translates to equilibrium multiplicity. Such multiplicity, however, also means that Nash equilibrium is silent about why sparse cyclic networks coordinate more successfully than dense cyclic networks. (Indeed, the set of pure-strategy Nash equilibria for cyclic networks is independent of network density.) Nor does it speak to some of our other experimental findings. We find, for instance, that in cyclic networks, the level of coordination is independent of cycle length. We also find that in acyclic networks, subjects who are higher in the pecking order – that is, subjects whose payoffs depend more on others’ actions – take lower actions.

¹Less commonly, some organizations are structured in matrix form, which is also acyclic. In a matrix organization, a subordinate may have multiple superiors, and there may be multiple chains-of-command from each employee to the top, but no “cycles of responsibility”. We view our findings as applying to matrices and other acyclic structures as well.

To address these experimental findings, and to enrich our intuitions about how network structure amplifies or dampens strategic uncertainty, we analyze logit (quantal-response) equilibria of the network minimum game (in Appendix A). In logit equilibrium, seeds of strategic uncertainty are introduced by assuming that each player inevitably makes small mistakes when choosing actions (McKelvey and Palfrey 1995; Anderson, Goeree, and Holt 2001; Goeree, Holt, and Palfrey 2016). This modeling device allows us to tractably capture the feedback-loop mechanisms discussed above. Further, logit equilibrium produces sharp comparative static predictions that match our experimental findings.

We also examine how subjects learn to coordinate over time. We find that coordination is more resilient in acyclic networks. In initial rounds, play is noisy and average actions are intermediate for both cyclic and acyclic networks. Subjects in acyclic networks tend to overcome such initial miscoordination: average actions increase towards the maximum level over time. In contrast, in cyclic networks, initial miscoordination is exacerbated: average actions decrease over time. We interpret our results as being largely consistent with a notion of “learning to coordinate” where subjects gradually improve at predicting others’ actions and at making optimal decisions. Viewed in this light, acyclic networks empower players to develop and maintain coordinated outcomes.

Related Literature

This paper relates most closely to existing work that studies coordination problems using the minimum-effort game.

Cooper, DeJong, Forsythe, and Ross (1992), Charness (2000), and Blume and Ortmann (2007) demonstrate that pre-play cheap-talk communication significantly improves coordination. For example, in Blume and Ortmann (2007), groups of size $n = 9$ play the minimum-effort game with pre-play cheap-talk messages, and generate average actions of around 6 out of 7. Relatedly, Van Huyck, Gillette, and Battalio (1992), Brandts and MacLeod (1995), and Weber, Camerer, Rottenstreich, and Knez (2001) model leadership by allowing one subject to send a pre-play cheap-talk message, and find that such recommendations can be effective at fostering coordination. Avoyan and Ramos (2020) consider pre-play communication that is partially binding, and show that such partial commitment improves coordination over the case of cheap talk. We think of these papers as demonstrating that pre-play communication is effective in routine settings or activities where each individual’s optimal actions are clearly defined *ex ante* and thus are easy to describe. On the other hand, in turbulent or unexpected circumstances – for example, during organizational crises – choice sets are often idiosyncratic, complex and thus difficult to describe. In such circumstances, succinctly communicating the appropriate actions for every individual in a large group or organization in a timely fashion becomes almost impossible. We think of our paper’s setting, which excludes pre-play communication while taking network structure as fixed, as capturing such circumstances.

Weber (2005, 2006) shows that gradual organizational growth serves to “build” successful coordination in large groups in the minimum-effort game. In his experiment, efficient coordination was initially induced in groups of $n = 2$, then additional group members who observed the entire history of play were sequentially introduced into the group until size $n = 12$ was achieved. Remarkably, efficient coordination was maintained in two out of nine such groups. Relatedly, Weber and Camerer (2003) run experimental games where groups of subjects have to develop a common

communication language for coordination, and show that coordination may fail following group mergers due to lack of a common language. We view this insight as consistent with our point above that communication in novel circumstances is difficult and thus cannot completely alleviate coordination problems.

Brandts and Cooper (2006) show that coordination failure in the minimum game can be overcome by increasing the payoff from successful coordination, and that – interestingly – the consequent coordination success persists even if coordination payoffs are subsequently lowered. Weber, Camerer, and Knez (2004) study a version of the minimum game where players move sequentially, but without observing the choices of preceding players. Interesting, although this variation (which the authors call ‘virtual observability’) has no impact on the information and payoff structure, it results in a moderate improvement in coordination.

Chen and Chen (2011) consider the effect of social identity on coordination. They induce group identity in the laboratory with some cleverly-designed treatments, and find that a common identity amongst a group substantially increases coordination in the minimum-effort game.

A number of experimental papers study coordination on networks.² Riedl, Rohde, and Strobel (2016) introduce endogenous network formation to the minimum-effort game. In each round, subjects choose who they are willing to connect with. The paper shows that efficient coordination emerges in this setting: subjects are motivated to take high actions by the threat of exclusion (from others’ networks). In comparison, our model with fixed networks captures settings where network structure is formal and difficult to change; this is the case, for example, in large and / or bureaucratic organizations.

Other experimental papers study coordination on fixed networks (like us), but focus on undirected networks (unlike us). Note that any undirected network is equivalent to a directed cyclic network: every pair of connected players depends on each other and thus forms a network cycle. Consequently, these papers are necessarily silent on our comparison between cyclic and acyclic networks. Instead, they study how network characteristics such as clustering and path length affect coordination in the undirected-network setting. Cassar (2007) finds that small-world networks coordinate more successfully than local networks, which have long path lengths relative to group size, or random networks. Keser, Ehrhart, and Berninghaus (1998) find that “circle” networks coordinate more successfully than small complete networks; Berninghaus, Ehrhart, and Keser (2002) extend this analysis to consider “lattice” networks. Charness, Feri, Meléndez-Jiménez, and Sutter (2014) consider a class of network games, including some coordination games; in these coordination games, they find that networks with more clustering tend to coordinate more successfully.³

²Choi, Gallo, and Kariv (2016) provides a comprehensive overview of the broader literature on network experiments.

³Moving slightly away from our coordination-game setting, Gallo and Yan (2015) implement a network game in strategic complements with a unique (but inefficient) Nash equilibrium, and shows that subjects can successfully cooperate to improve on the inefficient equilibrium – but only for simple, symmetric network structures.

2 Framework

2.1 Background: the (Classic) Minimum-Effort Game

The classic *Minimum-Effort Game* models coordination amongst an n -player group under a “weakest-link” production technology where output is determined by the group’s worst performer. Each player P_i in the group $\{P_1, \dots, P_n\}$ simultaneously chooses an action x_i from a compact action set $X \subset \mathbb{R}$, and receives the minimum group action less a private action cost:

$$\pi_i(x_1, \dots, x_n) = \min\{x_1, \dots, x_n\} - cx_i \text{ with } c < 1.$$

In most existing experimental implementations, the action set X is discrete: each subject chooses an integer action between 1 and 7. We follow this convention in the laboratory, but impose a continuous action set in our theoretical analysis (Appendix A).

With at least two (self-interested) players, any symmetric action profile (x, \dots, x) is a Nash equilibrium. These equilibria are Pareto-ranked, with higher-action equilibria being more efficient. The natural interpretation is that low-action equilibria represent coordination failure.

Table 2: Mean Actions in Classic Minimum-Effort Game (Final Round)

	$n = 2$	$n = 3$	$n = 6$
<i>Action</i>	6.39	3.63	1.45
	[1.79]	[2.34]	[1.02]
Mean [st. dev.] action, averaged over groups. ⁴			
Action set = $\{1, 2, \dots, 7\}$.			

Table 2 aggregates existing experimental results to highlight one of the most consistent findings about the classic minimum-effort game: coordination deteriorates dramatically with group size, and failure is almost inevitable in groups with more than three players. Average actions drop from 6.39 with $n = 2$ to 1.45 with $n = 6$.

2.2 The Network Minimum Game

Let’s augment the classic minimum game with a (connected) directed network \mathbf{g} over the group $\{P_1, \dots, P_n\}$. If there is a link $P_j \rightarrow P_i$, then we say that P_i *depends on* P_j . We require that players always depend on themselves: $P_i \rightarrow P_i$ for all P_i . Each player P_i ’s *neighborhood* S_i is the subset of players that P_i depends on.

The network minimum game differs from the classic minimum game in just one respect: each player P_i ’s payoff depends only on actions of those in P_i ’s neighborhood,

$$\pi_i(x_1, \dots, x_n) = \min\{x_j : j \in S_i\} - cx_i \text{ with } c < 1. \quad (1)$$

⁴Observations were aggregated across multiple experiments. Sources: Van Huyck, Battalio, and Beil (1990) for $n = 2$; Knez and Camerer (1994) and Knez and Camerer (2000) for $n = 3$; Knez and Camerer (1994) and Dufwenberg and Gneezy (2005) for $n = 6$.

Notice that the classic minimum-effort game is a special case of the network minimum game where \mathbf{g} is the complete network: $P_i \rightarrow P_j$ for all $i, j \in \{1, \dots, n\}$.

Some terminology: a network *path* is a sequence of players where each player depends on, and is distinct from, his predecessor. A network *cycle* is a network path that starts and ends with the same player. A network without network cycles is *acyclic*.

In any acyclic network, the unique Nash equilibrium is for everyone to play the maximum action, $x_i \equiv \max X$. Network cycles introduce equilibrium multiplicity: for any network cycle, any common action $x_i \equiv x \in X$ is a Nash equilibrium. Taken together, these observations hint at the central point of our paper: network cycles introduce strategic uncertainty, potentially leading to coordination failure. However, this multiplicity also implies that Nash equilibrium is silent about how coordination varies across different cyclic networks. Instead, we appeal to an alternative solution concept.

In Appendix A, we analyze logit equilibria of the (one-shot) network minimum game. Logit equilibrium introduces seeds of strategic uncertainty into the interactions between players. Each player best-responds “noisily” – by playing a distribution over actions where higher-payoff actions are chosen (exponentially) more frequently – to the similarly noisy play of others. Such noise captures the strategic uncertainty inherent in our coordination-game setting. Further, as in Anderson, Goeree, and Holt (2001)’s logit-equilibrium analysis of the classic minimum-effort game, the introduction of noise shrinks the set of equilibria relative to the Nash case, and thus serves as an effective equilibrium-selection device. Consequently, logit equilibria produce intuitive predictions about the circumstances under which coordination failure occurs. When discussing the intuitions underlying our experimental results in Section 4, we will often refer the interested reader to theoretical results in Appendix A.

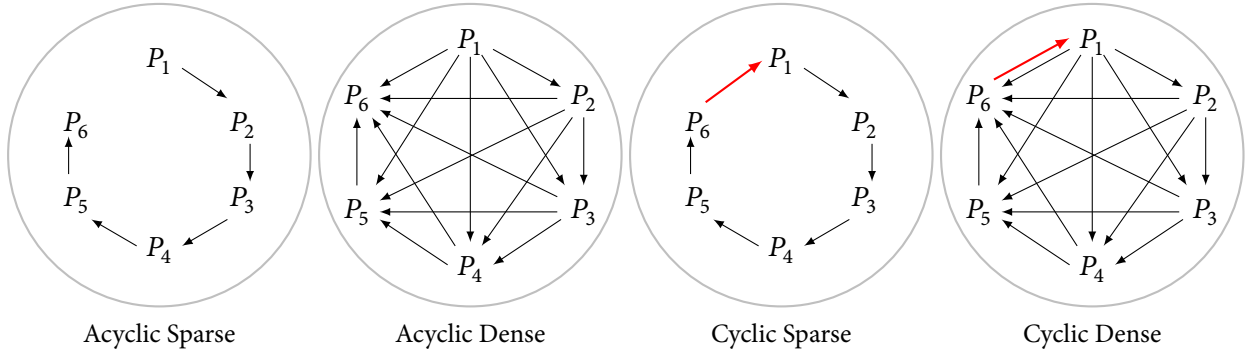
3 Experimental Design

3.1 Network Treatments

In the experiment, groups of subjects played a version of the network minimum game where actions are integers between one and seven, i.e., $X = \{1, 2, \dots, 7\}$.

The experiment adopted a between-subject design: each subject in each session was assigned to a specific network position within a group of fixed size and network structure, and each group played ten rounds of the network minimum game. Groups differed in size: $n \in \{2, 3, 4, 6, 12\}$. Modulo size, network structure took one of four ($= 2 \times 2$) forms, which differed along two dimensions: density and cyclicity. Each treatment thus corresponded to some combination of group size \times network density \times network cyclicity. This taxonomy is illustrated in Figure 2 for groups of size $n = 6$.

Some combinations of group size \times network density \times network cyclicity were not implemented. Table 3 summarizes which, and how many, network treatments were run; Appendix D illustrates each such treatment.



Cyclic networks are constructed by adding a single (red) link to the corresponding acyclic networks.

Figure 2: Six-Player Networks

Table 3: Experimental Design

	<i>acyclic</i>		<i>cyclic</i>	
	<i>sparse</i>	<i>dense</i>	<i>sparse</i>	<i>dense</i>
$n = 2$				✓ _[10]
$n = 3$	✓ _[9]		✓ _[10]	
$n = 4$	✓ _[10]		✓ _[10]	
$n = 6$	✓ _[10]	✓ _[10]	✓ _[10]	✓ _[10]
$n = 12$		✓ _[2]		

✓_[m] = treatment was run; [m] = number of groups.

Consider our acyclic networks. Both the sparse and dense acyclic networks are hierarchical: subject P_i depends on subject P_j only if $i \geq j$, so higher-indexed subjects depend (directly or indirectly) on lower indexed subjects. The sparse acyclic network is minimally connected, in the sense that removing any link will partition the group into two distinct components. The dense acyclic network is maximally connected, in the sense that adding any link will introduce a network cycle. So, our sparse and dense networks represent extremes in density amongst the set of connected acyclic networks.

Our cyclic networks are defined in relation to our acyclic networks. To create the sparse / dense cyclic network of size n , we add a single link – chosen judiciously to complete a network cycle – to the sparse / dense acyclic network of size n . Each cyclic network thus is ostensibly identical to its acyclic counterpart, except for the single additional link.

3.2 Procedural Details

Experimental sessions ran from April – August 2017 at UNSW Sydney’s BizLab. Subjects were recruited from the university’s subject pool and administered via ORSEE (Greiner 2015); the experiment was programmed in zTree (Fischbacher 2007). Overall, 421 subjects participated in 33 sessions plus a pilot study, with 12 to 30 subjects per session. Each treatment was played by ten groups of subjects.⁵ Groups were fixed throughout.

⁵There were two exceptions to this ten-group-per-treatment rule: the cyclic sparse network with $n = 3$ (nine groups) and the acyclic dense network with $n = 12$ (two groups). Each session consisted of multiple groups. In some sessions,

Subjects faced a version of the payoff function from Equation (1). Specifically,

$$\pi_i(x_1, \dots, x_n) = 6 + 3 \min\{x_j : j \in S_i\} - 2x_i,$$

where payoffs were denominated in AUD. Payoff information was presented to subjects in the form of Table 4. Subjects were paid their experimental earnings from one randomly-selected round plus a show-up fee of AUD 5. No subject was allowed to participate in more than one session. On average, each session lasted about 50 minutes and each subject earned AUD 16.27.

Table 4: Network Minimum Game Payoffs

Your Action	Minimum Action in Neighborhood						
	7	6	5	4	3	2	1
7	13.00	10.00	7.00	4.00	1.00	-2.00	-5.00
6	—	12.00	9.00	6.00	3.00	0.00	-3.00
5	—	—	11.00	8.00	5.00	2.00	-1.00
4	—	—	—	10.00	7.00	4.00	1.00
3	—	—	—	—	9.00	6.00	3.00
2	—	—	—	—	—	8.00	5.00
1	—	—	—	—	—	—	7.00

The experimental design corresponds to a complete-information setting. At the start of the experiment and at the start of each round, each subject was reminded about the network structure and his position within the network. At the end of each round, each subject was informed about every subject’s action within his group and every subject’s neighborhood’s minimum action in that round. Each subject only participated in one session and was only exposed to one network structure and one position within that network.

Participants received written instructions and were then shown a 5-minute video which explained each step of the experiment.⁶ Before the start of the experiment, subjects had to pass two on-screen comprehension tests, after which all subjects in the session started the experiment simultaneously.

4 Experimental Findings

This section studies how network density and network cyclicity affect coordination. Here, the analysis is static: we report mean final-round actions for each treatment, averaged over groups. In contrast, Section 5 will study dynamics: that is, how mean actions evolve over the ten rounds. Throughout our analysis, the mean action for one group is treated as a single observation. The approach controls for potential within-group correlations in a conservative fashion.

groups corresponding to different treatments were run in the same session; this was done to optimize the use of subjects given room-size constraints. Table 3 in the Appendix summarizes the experimental design.

⁶These written instructions are reproduced in the Online Appendix.

Acyclic Networks Our first experimental finding is that groups coordinate well in acyclic networks regardless of group size or network density.

Table 5: Mean Actions in Acyclic Networks (Final Round)

	$n = 3$	$n = 4$	$n = 6$	$n = 6$
	<i>sparse</i>	<i>sparse</i>	<i>sparse</i>	<i>dense</i>
<i>Action</i>	6.52	6.70	6.52	6.72
	[.53]	[.50]	[.81]	[.63]

Mean [st. dev.] action, averaged over groups.

There were 10 observations (i.e., 10 groups) per treatment.

Table 5 lists average final-round actions for sparse and dense acyclic networks of various group sizes. Subjects achieved close to the maximum action in the final round of each treatment. Across the various acyclic networks, the final-round average action ranged from 6.52 to 6.72 out of 7.⁷

To highlight the point that high density does not hinder coordination in acyclic networks, we collected data for two “super-dense” acyclic networks with $n = 12$ (Figure 3). These networks coordinated remarkably successfully. In the final round, all subjects played the maximum action 7.

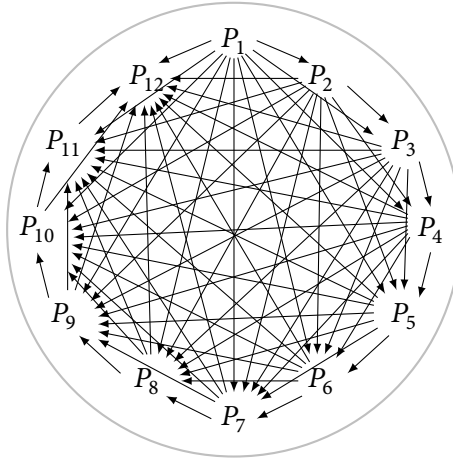


Figure 3: Large Dense Acyclic Network

To sum up, neither group size nor network density made a substantial dent on coordination in acyclic networks. Such insensitivity is in stark contrast with the fact that coordination deteriorates rapidly with group size (and thus with network density) in the classic minimum-effort game (see Table 2). But the same insensitivity is consistent with the Nash equilibrium prediction that subjects in any acyclic network will coordinate well. Pushing this point further, we find in Proposition A.1 of Appendix A that under logit equilibrium, subjects in any acyclic network will choose close-to-maximal actions *as long as player's mistakes are not too large*. We will elaborate on this point shortly, when we compare our experimental results for cyclic versus acyclic networks.

⁷In every acyclic treatment, most subjects played the maximum action: the number of such subjects in the sparse $n = 3, 4, 6$ and the dense $n = 6$ treatments is 18 of 27, 32 of 40, 46 of 60, and 56 of 60, respectively.

Differences in average final-round actions between sparse acyclic treatments of different group sizes are not significant: (two-sided) Wilcoxon rank-sum tests produce p -values of $p_{34} = 0.3412$, $p_{36} = 0.4888$, and $p_{46} = 0.7980$ (where $p_{n_1 n_2}$ is the p -value from comparing group sizes n_1 vs. n_2). The difference in average final actions between the sparse vs. dense $n = 6$ acyclic treatments is not significant either: $p = 0.4016$.

Cyclic (vs. Acyclic) Networks Our second experimental finding is that coordination weakens when cycles are introduced into a network, especially when the existing network is dense. Recall that in our experimental design, the only difference between each acyclic network and its cyclic counterpart is the addition of a single link.

Table 6: Mean Actions in Cyclic Networks (Final Round)

	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>
<i>Action</i>	4.13 [2.22]	4.13 [1.93]	4.90 [2.02]	2.08 [1.33]

Mean [st. dev.] action, averaged over groups

There were 10 observations (i.e., 10 groups) per treatment.

Table 6 shows the mean final-round action in sparse and dense cyclic networks of various group sizes. Sparse cyclic networks produce intermediate levels of coordination, but perform substantially worse than sparse acyclic networks. Average final-round actions are significantly lower in each size- n sparse cyclic network than in the corresponding size- n sparse acyclic network: Wilcoxon rank-sum tests produce p -values of 0.0208, 0.0033, and 0.0571 for $n = 3, 4$, and 6, respectively.

Comparing Tables 5 and 6, the difference in final-round actions between the dense acyclic treatment and the dense cyclic treatment is particularly dramatic. With $n = 6$, the mean final-round action in the dense cyclic network is 2.08 [s.d. 1.33], compared to 6.72 [s.d. .63] in the dense acyclic network ($p < 0.0001$). That is, the addition of a single link – to a dense network – that completes network cycles has devastating effects on coordination.⁸

Relatedly, actions decrease with network density in cyclic networks. With $n = 6$, the mean final-round action in the sparse cyclic network is 4.90 [s.d. 2.02], versus 2.08 [s.d. 1.33] in the dense cyclic network ($p = 0.0035$).

Moving beyond point estimates, Figure 4 shows the empirical distribution of action distributions by network treatment for $n = 6$. Action distributions are significantly higher in stochastic dominance in each acyclic treatment than in its cyclic counterpart: two-sample Kolmogorov-Smirnov (KS) tests and two-sample Epps-Singleton (ES) tests both produce $p < 0.001$ for final-round actions.⁹

What forces underlie coordination failure in cyclic networks, and why are these forces muted in acyclic networks? We have in mind that small seeds of strategic uncertainty are amplified by strategic interactions in cyclic networks, but not in acyclic networks. We formalize this intuition in Propositions A.1 and A.2 of Appendix A. Informally, consider the following tâtonnement process over which ‘reverberant doubt’ unfolds. Suppose that players on a network cycle all (initially) play the maximum action. Now, inject a small ‘seed’ of strategic uncertainty by adding noise to some player P_i ’s action. In the subsequent tâtonnement process, each player who depends on P_i best-responds by lowering his action; in this way, the negative shock propagates along network paths from P_i . In fact, this negative shock will circle back to i along the network cycle, inducing

⁸We obtain similar – indeed, even more pronounced – differences across acyclic and cyclic treatments when we consider each player’s neighbourhood’s minimum action rather than each player’s action.

⁹Empirical distributions for networks with $n = 2, 3, 4$ can be found in Figure B.2 of Appendix C.

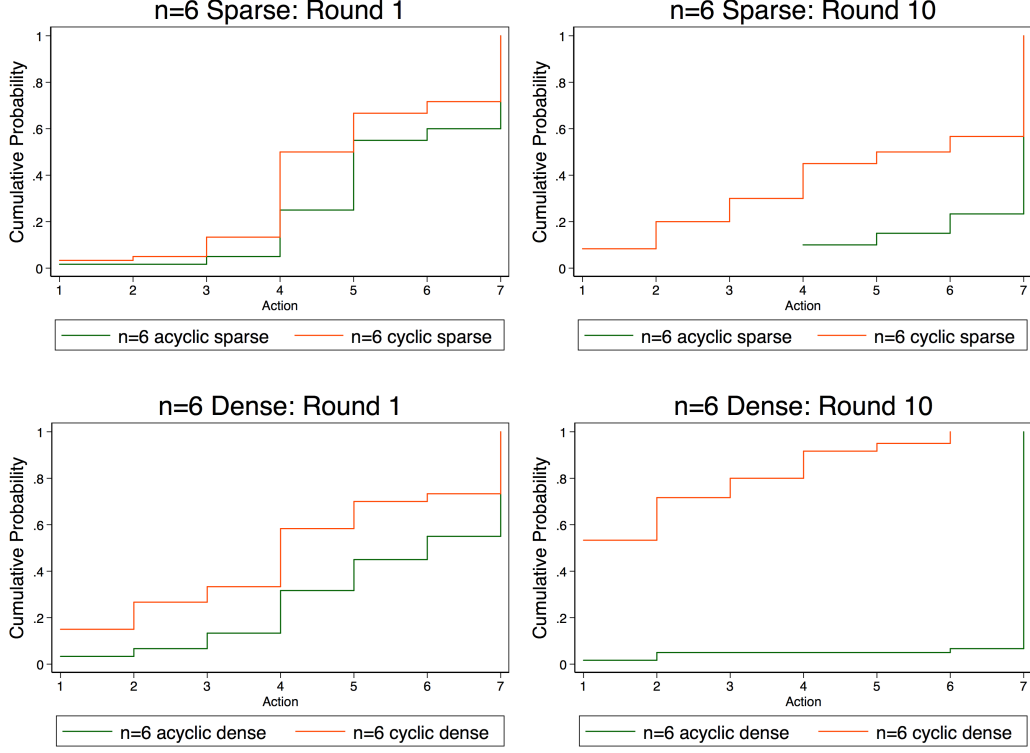


Figure 4: Empirical Distributions of Acyclic vs. Cyclic Networks with $n = 6$

P_i to further lower his action. In other words, the network cycle serves as a feedback loop for the initial seed of strategic uncertainty. Indeed, players may be embedded in multiple network cycles, in which case the feedback effect is multiplied. In dense cyclic networks where many players are embedded in many cycles, the feedback effect is sufficiently strong that any small shock becomes self-reinforcing and eventually leads to coordination failure, where everyone involved plays almost-minimal actions. In contrast, seeds of strategic uncertainty may propagate across acyclic networks but are eventually dampened due to the absence of feedback loops, and thus do not substantially damage coordination.

Cycle Length Our third experimental finding is that cycle length has little, if any, effect on coordination. Consider the sparse cyclic networks. In a sparse cyclic network of size n (Figure 5), there is a single cycle of length n and every neighbourhood has size $|S_i| \equiv 2$. To wit: we can study how cycle length affects coordination, keeping density fixed at $\ell = 2$, by comparing the sparse cyclic networks with different group sizes.

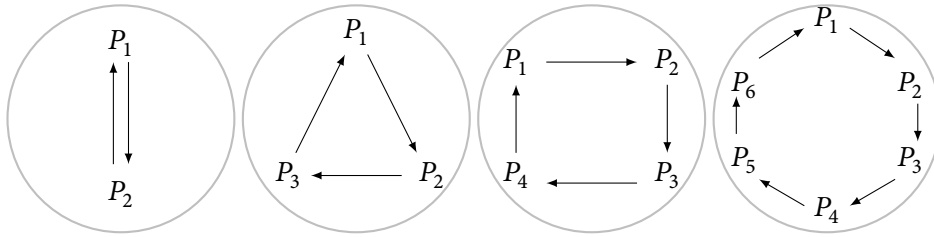


Figure 5: Sparse Cyclic Networks

Table 7 shows average final-round actions in each of the sparse cyclic treatments. Average final-round actions in the various sparse cyclic networks are not significantly different: testing for differences between pairs of treatments, we get p -values of $p_{23} = 0.7596$, $p_{24} = 0.6212$, $p_{26} = 0.7010$, $p_{34} = 0.9095$, $p_{36} = 0.4452$, and $p_{46} = 0.3620$ (where $p_{n_1 n_2}$ is the p -value from comparing group sizes n_1 vs. n_2).¹⁰

Table 7: Mean Actions in Sparse Cyclic Networks (Final Round)

	$n = 2$	$n = 3$	$n = 4$	$n = 6$
<i>Action</i>	4.50	4.13	4.13	4.90
	[2.60]	[2.22]	[1.93]	[2.02]

Mean [st. dev.] action, averaged over groups.

There were 10 observations (i.e., 10 groups) per treatment.

This experimental result is consistent with Proposition A.2 in Appendix A, which implies that logit equilibrium action distributions are independent of cycle length for sparse cyclic networks.¹¹

These findings may shed some light on the mechanisms leading to coordination failure in large groups in the classic minimum-effort game, which (as we recall) corresponds to the complete network. Given a complete network, an increase in group size corresponds to both (i) an increase in network density and (ii) the introduction of longer cycles into the network. What role do each of these factors play in inducing coordination failure in the classic minimum-effort game? Our finding that cycle length has little effect on coordination suggests that (ii) is not a major factor; and thus that coordination failure in large groups in the classic minimum-effort game is due to high network density.

Pecking Order Our fourth experimental finding is that subjects that are “higher-up” in a hierarchical network take lower actions. The sparse and dense acyclic networks of our experimental design are implicitly hierarchical, in the sense that higher-indexed subjects depend on lower-indexed subjects but not vice versa (see Appendix D): to wit, higher-indexed players are assigned greater responsibility for coordination and thus are “higher-ups” in the implied hierarchy.

Table 8 shows average final-round actions for each treatment by subject position. For each acyclic treatment, average actions generally decreased with subject position. The differences across subject positions are relatively small – on average, the highest-indexed subjects in each acyclic treatment played higher actions than the average subject, at any position, in each corresponding cyclic treatment. But the pattern of decreasing actions is statistically significant: a battery of Wilcoxon rank-sum tests as well as directional Jonckheere tests yield p -values of 0.0954, 0.0702, 0.0328, and 0.2691 for sparse $n = 3, 4, 6$ and dense $n = 6$, respectively.¹² In contrast, we did not discern any

¹⁰Two-sample Kolmogorov-Smirnov (two-sample Epps-Singleton) tests produce p -values of $p_{23} = 0.976$ (0.628), $p_{24} = 0.660$ (0.528), $p_{26} = 0.660$ (0.277), $p_{34} = 0.660$ (0.842), $p_{36} = 0.976$ (0.771), and $p_{46} = 0.294$ (0.957) for final-round actions for networks with $n = 2, 3, 4$ and $n = 6$, respectively.

¹¹More generally, action distributions are independent of cycle length for networks where all subjects have the same neighbourhood size ℓ . Sparse cyclic networks correspond to the special case $\ell = 2$.

¹²The p -values of the Jonckheere test are reported. For dense acyclic $n = 6$, the Jonckheere test yields $p \leq 0.0426$ for all periods except the final round.

Table 8: Mean Actions by Network Position (Final Round)

Network Structure		P_1	P_2	P_3	P_4	P_5	P_6
acyclic	$n = 3$ sparse	6.78	6.56	6.22			
		[.44]	[.73]	[.97]			
	$n = 4$ sparse	6.90	6.70	6.90	6.30		
		[.32]	[.67]	[.32]	[1.06]		
	$n = 6$ sparse	6.80	6.80	6.80	6.40	6.30	6.00
		[.63]	[.42]	[.42]	[1.26]	[1.16]	[1.41]
	$n = 6$ dense	7.00	6.90	6.40	7.00	6.50	6.50
		[0]	[.32]	[1.90]	[0]	[1.58]	[1.58]
cyclic	$n = 3$ sparse	3.80	4.20	4.40			
		[2.35]	[2.35]	[2.22]			
	$n = 4$ sparse	4.50	3.90	4.30	3.80		
		[2.01]	[2.18]	[2.11]	[2.10]		
	$n = 6$ sparse	4.90	5.40	4.60	4.30	4.90	5.30
		[2.38]	[2.17]	[2.50]	[2.31]	[1.97]	[2.16]
	$n = 6$ dense	1.90	2.80	2.60	1.80	1.80	1.60
		[1.73]	[1.81]	[1.84]	[1.03]	[1.14]	[1.07]
Mean [st. dev.] action. Refer to Appendix D for network position of each P_i .							

corresponding pattern for cyclic networks (p -values of 0.7118, 0.2758, 0.4648, and 0.1458 for sparse $n = 3, 4, 6$ and dense $n = 6$, respectively.).

Intuitively, in acyclic networks, higher-indexed players find it more costly to take high actions. They do so because the players in their neighbourhood, being themselves relatively high-indexed, play lower actions than the players in the neighbourhood of lower-indexed players. Further, in the case of dense acyclic networks, higher-indexed players have larger neighbourhoods than lower-indexed players, and thus face a greater risk of a low action somewhere in their neighbourhood. In broader terms: within a hierarchy, higher-ups face greater responsibility for coordination, which induces them to take lower actions.¹³ We formalize these points in Proposition A.3 of Appendix A.

Additional Tests We conduct two robustness tests on this section’s results. Our first test excludes subject P_1 from the analysis of acyclic networks. Unlike the other subjects, P_1 in acyclic networks has an empty neighbourhood, so faces no strategic uncertainty – thus their dominant strategy is always to play the maximum action, $x_{i,t} = 7$. Excluding P_1 from the analysis of acyclic networks thus serves, in a sense, to “level the playing field” between cyclic and acyclic networks. Table B.4 reports mean actions for acyclic networks without P_1 , and Table B.5 tests for differences between cyclic and acyclic treatments after excluding P_1 . Our results remain qualitatively unchanged.

Our second test is to consider, instead of the mean action, the mean neighbourhood-minimum action; i.e., the minimum action in each subject’s neighbourhood, averaged over subjects. Each subject’s neighbourhood-minimum action captures the extent to which coordination failure affects the subject. Table C.4 compares cyclic versus acyclic treatments, and shows that the differences

¹³Practically speaking, a number of countervailing factors that are absent from our model may mitigate or even reverse this effect. For example, organizations may assign more highly-motivated personnel to higher-up positions, and offer them stronger incentives, to ensure that they take high actions.

in such mean neighbourhood-minimum actions are, if anything, even more pronounced than the differences in mean actions.

5 Experimental Findings: Dynamics

Section 4’s analysis was static: it focused on final-round actions. Complementarily, Appendix A studies logit equilibria of the one-shot minimum effort game. In this section, we step away from the static setting and discuss how coordination evolved over the ten rounds of play in our experiment.

Coordination Levels Table 9 shows average action choice by treatment, for round 1; rounds 1–5; rounds 6–10; and round 10.¹⁴ A clear pattern emerges: actions increase over time (culminating in relatively high actions) in acyclic networks, but decrease over time (culminating in relatively low actions) in cyclic networks.¹⁵

Table 9: Time Trends in Mean Actions – Acyclic vs. Cyclic Networks

	<i>n</i> = 3 <i>sparse</i>		<i>n</i> = 4 <i>sparse</i>		<i>n</i> = 6 <i>sparse</i>		<i>n</i> = 6 <i>dense</i>	
	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>
<i>Action</i> ₁	5.81	4.47	5.80	4.90	5.52	4.90	5.45	4.23
	[.44]	[1.33]	[.73]	[.77]	[.80]	[.62]	[.53]	[.52]
<i>Action</i> _{1–5}	5.99	4.43	6.18	5.08	5.92	5.11	5.70	3.32
	[.91]	[1.68]	[.68]	[.98]	[.89]	[.98]	[.88]	[1.23]
<i>Action</i> _{6–10}	6.41	3.98	6.45	4.56	6.43	5.14	6.52	2.29
	[.88]	[2.05]	[.63]	[1.66]	[.85]	[1.64]	[.98]	[1.23]
<i>Action</i> ₁₀	6.52	4.13	6.70	4.13	6.52	4.90	6.72	2.08
	[.53]	[2.22]	[.50]	[1.93]	[.81]	[2.02]	[.63]	[1.33]

Mean [st. dev.] action, averaged across groups.

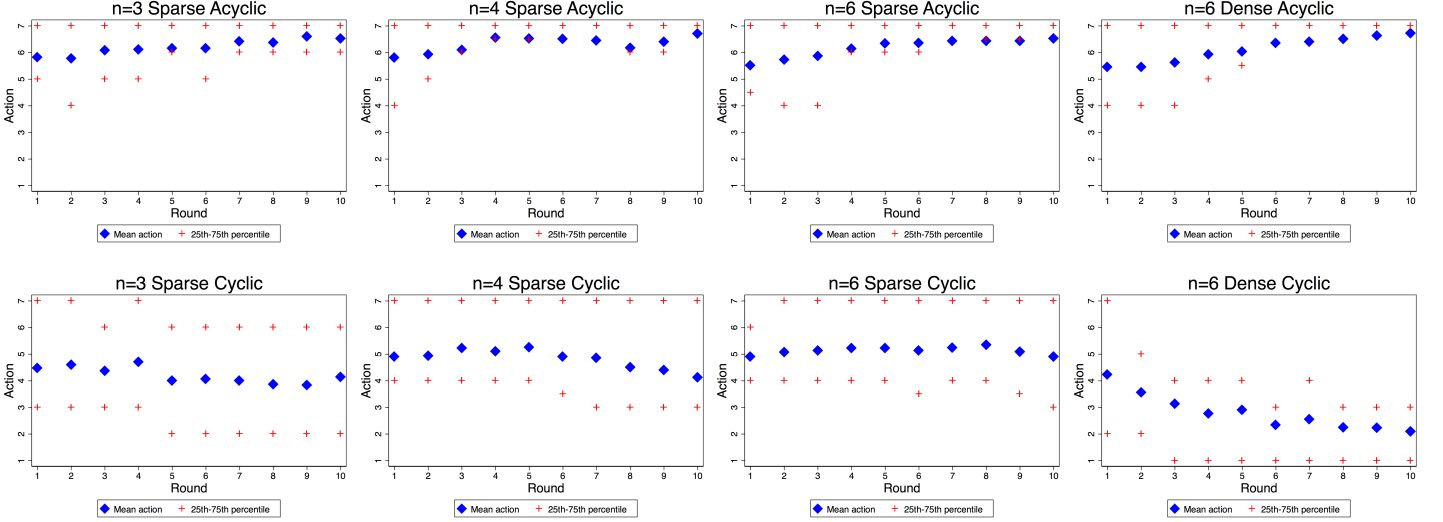
Figure 6 paints a richer picture – showing means and 25th to 75th percentiles of action distributions round-by-round.¹⁶ In the acyclic networks, actions generally increased over time. In contrast, in cyclic networks, action distributions decrease over time. This is especially so in dense networks.¹⁷

¹⁴Tables B.1 and B.2 report statistical tests for differences between cyclic and acyclic treatments. Further, Table 8 omits the cyclic dense *n* = 2 and acyclic dense *n* = 12 treatments; these are relegated to Table B.3.

¹⁵These trends are statistically significant. For acyclic networks, Jonckheere tests for ascending order produce *p*-values of 0.0008, 0.0007, 0.0001, and 0.0001 for sparse *n* = 3, 4, 6 and dense 6, respectively. In contrast, testing for descending order in cyclic networks yields *p*-values of 0.1288, 0.0554, 0.9877, and 0.0001 for sparse *n* = 3, 4, 6 and dense *n* = 6, respectively.

¹⁶A more detailed figure (B.1) can be found in Appendix C, where means and 25th to 75th percentiles of action distributions round-by-round and by subject position are shown.

¹⁷As Figure B.1 shows, in the acyclic networks, actions at each position generally increased over time, especially “higher-up” (higher-indexed) subjects. Further, consistent with the patterns recorded in Table 8, higher-indexed subjects play lower actions than lower-indexed subjects. In contrast, in cyclic networks, action distributions at each position generally decrease over time, and there is no clear trend across positions in action distributions.



Mean actions averaged across treatment are displayed on the vertical axis and rounds are depicted on the horizontal axis. The 25th-75th percentiles of the empirical distributions are highlighted in red. The first (second) column from the left illustrates sparse networks of size $n = 3$ (4). The two columns on the center-right show networks of size $n = 6$, differing in density with sparse and dense. The top (bottom) row always shows acyclic (cyclic) networks.

Figure 6: Evolution of Mean Actions in Acyclic & Cyclic Networks

Overall, these time trends suggest that subjects may be playing adaptively, adjusting their actions over time in response to recent play. Next, we consider whether such adaptive play reduces the uncertainty that subjects face.

Learning We provide suggestive evidence that subjects in both cyclic and acyclic networks learn to coordinate over time, in the sense that they improve their predictions of others' actions. Note that a subject P_i who perfectly anticipates the play of other group members should optimally match the minimum action elsewhere in his neighbourhood by choosing period- t action

$$x_{i,t}^* = \begin{cases} \min\{x_{j,t} : P_j \in S_i \setminus P_i\} & \text{if } |S_i \setminus P_i| \geq 1, \\ 7 & \text{if } |S_i \setminus P_i| = 0. \end{cases}$$

Accordingly, call the difference between the subject P_i 's optimal action and realized action, $|x_{i,t} - x_{i,t}^*|$, the subject's *prediction error*.

Table 10 shows time trends in average prediction errors by treatment.¹⁸ Two patterns emerge.

First, prediction errors are generally substantially larger in cyclic treatments than in the corresponding acyclic treatments.¹⁹ Using two-sided Wilcoxon rank-sum tests to compare prediction errors between acyclic and cyclic treatments – where each observation is the average prediction error in one treatment over a specific time range – we find that, for most network structures and

¹⁸In acyclic networks, subject P_1 has a particularly easy prediction problem: $x_{1,t}^* = 7$. We obtain similar results if we drop subject P_1 from all (acyclic and cyclic) treatments.

¹⁹We can further disaggregate mean prediction errors by *subject position*. Table B.6 lists prediction error by subject position, averaged over all ten rounds. As with Table 10, prediction errors are generally larger in cyclic networks than in the corresponding acyclic networks and at the corresponding network positions. Analogous to our discussion of Table 6, we find in acyclic networks that prediction errors generally increase as we move higher up the hierarchy (i.e., as subject index increases).

most time ranges, prediction error is significantly higher in a given cyclic treatment than the corresponding acyclic treatment.²⁰

Table 10: Time Trends in Prediction Errors

	<i>n</i> = 3 <i>sparse</i>		<i>n</i> = 4 <i>sparse</i>		<i>n</i> = 6 <i>sparse</i>		<i>n</i> = 6 <i>dense</i>	
	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>
<i>Error</i> ₁	1.44	2.07	1.15	1.90	1.28	1.77	1.90	2.70
	[.87]	[1.19]	[.72]	[.57]	[.51]	[.47]	[.76]	[.68]
<i>Error</i> _{1–5}	1.02	1.31	.80	1.70	.96	1.53	1.34	1.78
	[.84]	[1.10]	[.68]	[.85]	[.59]	[.62]	[.91]	[1.03]
<i>Error</i> _{6–10}	.44	.72	.66	1.16	.41	1.09	.41	.77
	[.51]	[.83]	[.87]	[.92]	[.62]	[.88]	[.85]	[.74]
<i>Error</i> ₁₀	.41	.40	.28	.90	.23	.83	.52	.43
	[.33]	[.84]	[.55]	[.91]	[.35]	[.74]	[1.28]	[.48]

Mean [st. dev.] prediction error, averaged across groups. In the *n* = 6 dense acyclic network, eliminating a single outlier reduces final-round mean prediction error to 0.13 [0.39].

Second, prediction error decreases over time in both cyclic and acyclic networks: the average prediction error over the last five rounds is lower than over the first five rounds in every treatment.²¹ For acyclic networks, such improvements in prediction go hand-in-hand with improvements in actions, in the sense that actions increase over time (Table 8) – but not for cyclic networks, where actions decrease over time.

We prefer to interpret prediction error as arising from a combination of strategic uncertainty and noisy decision-making: subjects make smaller prediction errors when they face less strategic uncertainty about actions in their neighbourhood, and when they understand better how to make optimal choices given others' actions. That is, our experimental findings from Table 10 suggest that over repeated play, subjects improve their decision-making and strategic uncertainty diminishes. In this context, it is not surprising that our results from Section 4 – which focus on final-round play – are consistent with the predictions of low- μ logit equilibrium (Appendix A), which posits that agents optimize with a small amount of noise.

Concluding Remarks

This paper argues that introducing cycles of interdependencies into an organization's design may trigger coordination failure. In this sense, the network minimum game provides a perspective that highlights the downsides of interdependencies within organizations; indeed, Lemma A.2 states that performance decreases monotonically whenever interdependencies are introduced. This stark result arises because the network minimum game framework essentially assumes that interdependencies produce no benefits for organizations. While this assumption is clearly unrealistic, we prefer

²⁰See Table B.7 for details. We find that the differences between acyclic and cyclic treatments are statistically significant for the sparse *n* = 4 and *n* = 6. For the dense *n* = 6 treatments, the differences are statistically significant for all time ranges – except that, in the final round, the acyclic treatment has a higher mean prediction error than the cyclic treatment (due to a single outlier in the acyclic treatment). For the sparse *n* = 3 treatments, differences for some time ranges for the sparse *n* = 3 treatment are not significant.

²¹Indeed, Jonckheere tests for descending order produce *p*-values of 0.0001 (0.0001), 0.0022 (0.0003), 0.0001 (0.0006), and 0.0001 (0.0001) for sparse *n* = 3, 4, 6 and dense *n* = 6 acyclic (cyclic) networks, respectively.

not to take a stand on how to model such benefits (see, e.g., Becker and Murphy 1992 and Dessein and Santos 2006 for some options). Instead, we show that the disadvantages of additional interdependencies may be mitigated by avoiding network cycles. In other words, if interdependencies are necessary, an acyclic structure is preferable to a cyclic structure.

A Appendix: Logit Equilibria

To guide Section 4's discussion of our experimental findings, we analyze logit equilibria of the one-shot network minimum game. As Anderson, Goeree, and Holt (2001) do for the classic minimum-effort game, let's consider a version of the network minimum game where the action set is continuous rather than discrete. Specifically, suppose that each player P_i chooses an action x_i from a bounded interval $X = [0, \bar{x}] \subset \mathbb{R}$.

Logit equilibria are a special case of quantal-response equilibria with exponentially-distributed noise (McKelvey and Palfrey 1995; Goeree, Holt, and Palfrey 2016). Consider a profile of action distributions $F(x) = \{F_1(x), \dots, F_n(x)\}$. Given the payoff function (1), we can calculate player P_i 's expected payoff from taking action x to be

$$\mathbb{E}[\pi_i(x; F_{-i})] = \int_0^x \prod_{j \in S_i \setminus P_i} (1 - F_j(y)) dy - cx.$$

Logit equilibrium asserts that players optimize noisily, in the sense that higher-expected-payoff actions are (exponentially) more likely to be played. Specifically, given F , each player P_i plays a *logit response*: an action distribution $L_i(x; F) \in \Delta(X)$ with density

$$l_i(x; F) = \frac{\exp \{ \mathbb{E}[\pi_i(x; F_{-i})] / \mu \}}{\int_0^{\bar{x}} \exp \{ \mathbb{E}[\pi_i(y; F_{-i})] / \mu \} dy}. \quad (\text{A.1})$$

The parameter $\mu > 0$ captures the exogenous noisiness of play; higher μ corresponds to more noise, so (A.1) corresponds to (i) a perfectly-targeted best-response function at the limit $\mu \rightarrow 0$, versus (ii) a uniform distribution over X at the limit $\mu \rightarrow \infty$. Notice from (A.1) that logit responses account for the noisiness of others' play, and thus incorporate strategic uncertainty in a natural way. A logit equilibrium is simply a fixed point of the logit-response correspondence, i.e., a profile F of distributions such that $F \equiv L(\cdot, F)$.

The following lemmas make preliminary observations about logit equilibria in the network minimum game. Players' logit responses are strategic complements in the network minimum game. Such complementarity then implies that extremal logit equilibria exist.²² Some notation: we write $G \succeq H$, and say G *dominates* than H , iff G_i first-order stochastically dominates H_i for all $i \in \{1, \dots, n\}$; equivalently, $G_i(x) \leq H_i(x)$ for all $x \in [0, 1]$. Further, we write $G \succ H$ if $G \succeq H$ and $G \neq H$.

Lemma A.1a. *If $F \succ G$, then $L(\cdot, F) \succ L(\cdot, G)$.*

Proof. Suppose $F_j(x)$ decreases for $P_j \in S_i \setminus P_i$. Notice that for $x > y$,

$$\frac{f_i(x)}{f_i(y)} = \frac{\exp(\pi_i(x)/\mu)}{\exp(\pi_i(y)/\mu)} = \exp \left\{ \mu^{-1} \left(\int_y^x \prod_{j \in S_i \setminus P_i} (1 - F_j(s)) ds - c(x - y) \right) \right\}$$

²² Anderson, Goeree, and Holt (2002) provide a general proof that logit equilibria exist for games where payoffs are continuous in actions. Anderson, Goeree, and Holt (2001) demonstrate equilibrium existence for continuous potential games with bounded actions, such as the classic minimum-effort game. Lemma A.1c provides a slightly stronger result, albeit for our specific setting: it exploits the monotonicity of the logit best-response to construct a smallest, and a largest, equilibrium.

increases. That is, the monotone likelihood ratio property (Milgrom 1981) is satisfied. It follows that $F_i(\cdot)$ decreases, that is, player P_i increases his action in the sense of first-order stochastic dominance. ■

Lemma A.1b. *Given an increasing sequence $\{F^1, F^2, \dots\}$ of distribution profiles, let $F^\infty = \sup_{k \rightarrow \infty} F^k$, i.e., $F_i^\infty(x) \equiv \lim_{k \rightarrow \infty} F_i^k(x)$. Then $L(F^k) \Rightarrow L(F^\infty)$ pointwise.*

Proof. $\{F^1, F^2, \dots\}$ converges pointwise, and thus converges in distribution, to F^∞ . Defining

$$\pi_i^k(x) = \int_0^x \prod_{j \in S_i \setminus P_i} (1 - F_j^k(s)) ds - cx, \quad (\text{A.2})$$

$$\pi_i^\infty(x) = \int_0^x \prod_{j \in S_i \setminus P_i} (1 - F_j^\infty(s)) ds - cx, \quad (\text{A.3})$$

Lebesgue's monotone convergence theorem implies that $\pi_i^k(x) \rightarrow \pi_i^\infty(x)$ for all $x \in [0, \bar{x}]$. That is, $\pi_i^k(\cdot)$ converges pointwise to $\pi_i^\infty(\cdot)$. A second application of Lebesgue's monotone convergence theorem then yields, for all $x \in [0, \bar{x}]$,

$$L_i(F^k)(x) = \frac{\int_0^x \exp(\pi_i^k(s)/\mu) ds}{\int_0^{\bar{x}} \exp(\pi_i^k(s)/\mu) ds} \rightarrow \frac{\int_0^x \exp(\pi_i^\infty(s)/\mu) ds}{\int_0^{\bar{x}} \exp(\pi_i^\infty(s)/\mu) ds} = L_i(F^\infty);$$

thus yielding the desired result. ■

Lemma A.1c. *There exist smallest and largest logit equilibria of the network minimum game.*

Proof. We demonstrate existence of a smallest equilibrium; the case of the largest equilibrium proceeds similarly. Consider the sequence of distribution profiles $\{F^0, F^1, F^2, \dots\}$ where $F^{k+1} = L(F^k)$ and $F^0 = (1, \dots, 1)$, which corresponds to all players playing $x = 0$. By Lemma A.1a, this sequence of distributions is decreasing in dominance. Applying Lemma A.1b, we have $L(F^\infty) = F^\infty$; thus, F^∞ constitutes a logit equilibrium. Furthermore, we claim that F^∞ is the smallest equilibrium. For any equilibrium G , note that

$$G^k \geq F^k \text{ and } \lim_{k \rightarrow \infty} G^k \geq \lim_{k \rightarrow \infty} F^k$$

where $G^{k+1} = L(G^k)$, $G^0 = G$, and $F^0 = (1, \dots, 1)$; in other words, $G \geq F^\infty$. ■

Another useful observation is that logit equilibrium actions decrease as links are added to the network. Let \underline{G} and \overline{G} be the smallest and largest equilibria under network \mathbf{g} . Write $g_{ij} = 1$ if there is a link $P_j \rightarrow P_i$, and $g_{ij} = 0$ otherwise. Further, write $\mathbf{g} \geq \mathbf{h}$ if $g_{ij} \geq h_{ij}$ for all $i, j \in \{1, \dots, n\}$, with strict inequality if $g_{ij} > h_{ij}$ for some $i, j \in \{1, \dots, n\}$.

Lemma A.2. *If $\mathbf{g} > \mathbf{h}$, then $\underline{G} < \underline{H}$ and $\overline{G} < \overline{H}$.*

Proof. Denote $L(x, F, \mathbf{g})$ as the vector of logit responses to an action distribution F , given network \mathbf{g} . Correspondingly, denote $L_i(x, F, S_i)$ as player P_i 's logit response to action distribution F given his neighborhood S_i . Notice that shrinking P_i 's neighborhood from S_i to $\hat{S}_i \subset S_i$ is tantamount to

increasing the actions of all players in $S_i \setminus \hat{S}_i$ to the maximum level \bar{x} : that is, $L_i(x, F, \hat{S}_i) = L_i(x, \hat{F}, S_i)$, where

$$\hat{F}_j = \begin{cases} F_j & : P_j \in \hat{S}_i \\ 0 \text{ for all } x < \bar{x} & : P_j \in S_i \setminus \hat{S}_i \end{cases}.$$

From Lemma A.1a, $L_i(x, \hat{F}, S_i) > L_i(x, F, S_i)$. So, $L_i(x, F, S_i)$ decreases in dominance as S_i increases. It follows that $L(x, F, \mathbf{g})$ decreases in dominance as \mathbf{g} increases. By our construction of the smallest and largest equilibria from Lemma A.1c, the result follows. ■

In words, as directed links are added to the network \mathbf{g} , coordination deteriorates in the sense that logit equilibrium actions decrease – directly for the linked-from player, and indirectly for other players in the network. Intuitively, a new dependency for player P_i lowers the (distribution of the) minimum action in P_i 's neighbourhood, and thus induces P_i to lower his logit response. This negative shock may spread beyond P_i : by lowering his action, P_i induces players who depend on P_i to lower their responses as well.

Our main results focus on the case where the exogenous noise level μ is low. Our first result is about acyclic networks.

Proposition A.1. *For an acyclic network, a unique logit equilibrium exists where*

$$\lim_{\mu \rightarrow 0} F_i(x) = 0 \text{ for each } i \text{ and for all } x \in [0, \bar{x}).$$

Proof. Without loss of generality, (re)label the set of players $\{P_1, \dots, P_n\}$ so that each player depends only on lower-indexed players. (There may exist multiple such labelings.) Consider the sequence of distribution profiles $\{F^0, F^1, F^2, \dots\}$ where $F^{k+1} = L(F^k)$ and F^0 is an arbitrary action distribution. Notice that $F_1^k(\cdot)$ is constant in k and independent of F^0 for $k \geq 1$: player P_1 depends on nobody else, and so his logit response is independent of the initial action distribution. Furthermore, inspection of (A.1) reveals that as $\mu \rightarrow 0$, $F_1^k(\cdot) \rightarrow 0$ for $x < \bar{x}$. Similarly, $F_2^k(\cdot)$ is independent of $F^0(\cdot)$ for $k \geq 2$, because player P_2 depends only on player P_1 and himself (and because $F_1^k(\cdot)$ is constant in k for $k \geq 1$). Indeed, by induction, $F_i^k(\cdot)$ is independent of $F^0(\cdot)$ for $k \geq i$, with $F_i^k(x) \rightarrow 0$ for $x < \bar{x}$ as $\mu \rightarrow 0$. We conclude that the smallest and largest equilibria coincide. The result follows. ■

In words, if the degree μ of exogenous noise is small, then everyone in an acyclic network chooses almost-maximal actions – consistent with our experimental findings from Table 5.

Our second result is about cyclic networks. For tractability, we restrict attention to networks where all players have the same neighbourhood size, $|S_i| \equiv \ell$; we interpret ℓ as the *density* of such a network.

Proposition A.2. *Suppose $|S_i| \equiv \ell$ for some $\ell \geq 2$. Then a unique equilibrium exists, and it is symmetric: $F_i \equiv F_j$ for all $i, j \in \{1, \dots, n\}$. Further,*

$$\lim_{\mu \rightarrow 0} F_i(x) = \begin{cases} 0 & : \ell < 1/c \\ 1 & : \ell > 1/c \end{cases} \text{ for all } x \in [0, \bar{x}). \quad (\text{A.4})$$

Proof. Consider the sequence of distribution profiles $\{F^0, F^1, \dots\}$ where $F^{n+1} = L(F^n)$ and $F^0 = (1, \dots, 1)$. This sequence converges to the smallest logit equilibrium of the network game. But notice that this sequence coincides with the corresponding sequence for the classic ℓ -player minimum game, and thus also converges to an equilibrium of the classic ℓ -player minimum game. Similarly, the largest logit equilibrium of the network game coincides with an equilibrium of the classic ℓ -player minimum game.

Anderson, Goeree, and Holt (2001) show that the classic ℓ -player minimum game has a unique logit equilibrium satisfying Equation (A.4). This must equal both the largest and smallest logit equilibrium of the network game; thus it is also the unique logit equilibrium of the network game. ■

Note that any such network with density $\ell \geq 2$ is cyclic. Proposition A.2 tells us that with such networks, there exists a threshold of network density below which actions are near-maximal, and above which actions are near-minimal.

Proposition A.2 highlights two broad points. First, coordination may fail dramatically in cyclic networks, even for arbitrarily low noise levels – in contrast to acyclic networks. Second, network density has a dramatic impact on coordination in cyclic networks; in contrast, Proposition A.1 tells us that network density has little effect on coordination in acyclic networks when noise levels are low. Put another way, the comparative static effect of Lemma A.2 – that increased network density reduces coordination – is relatively weak in acyclic networks and relatively strong in cyclic networks. These predictions find support in our experimental results (Table 5 and 6).

The mapping from Proposition A.2 to our experimental implementation of dense cyclic networks is imperfect. The dense cyclic networks of our experiment do not satisfy the equal-neighbourhood-size condition, so Proposition A.2 does not apply directly. Nonetheless, both our theory and experiment capture the essential property of dense cyclic networks: that there are many players each embedded in many cycles.

Proposition A.2 also implies that, consistent with our experimental findings in Table 7, the level of coordination is independent of cycle length. To highlight this point, the following corollary focuses on sparse cyclic networks, which correspond to the case $\ell = 2$.

Corollary A.1. *Every sparse cyclic network has a unique symmetric logit equilibrium where F_i is (for given μ) independent of cycle length n .*

Our final result is about how subjects' actions vary with their hierarchical position in acyclic networks. It matches our experimental finding from Table 8 that “higher-up” (higher-indexed) subjects in our acyclic networks play lower actions.

Proposition A.3. *In any of the sparse or dense acyclic networks, in the unique logit equilibrium,*

$$\begin{aligned} & \text{for all } i > j, \\ & F_i(\cdot) \text{ is strictly dominated by } F_j(\cdot). \end{aligned}$$

Proof. Notice that both our sparse and dense acyclic networks satisfy the following *pecking-order* property: If $i > j$, then there is an injection φ from S_j to S_i such that for any player $P \in S_j$, the corresponding player $\varphi(P) \in S_i$ has weakly higher index than P . Further, the pecking order is *strict*

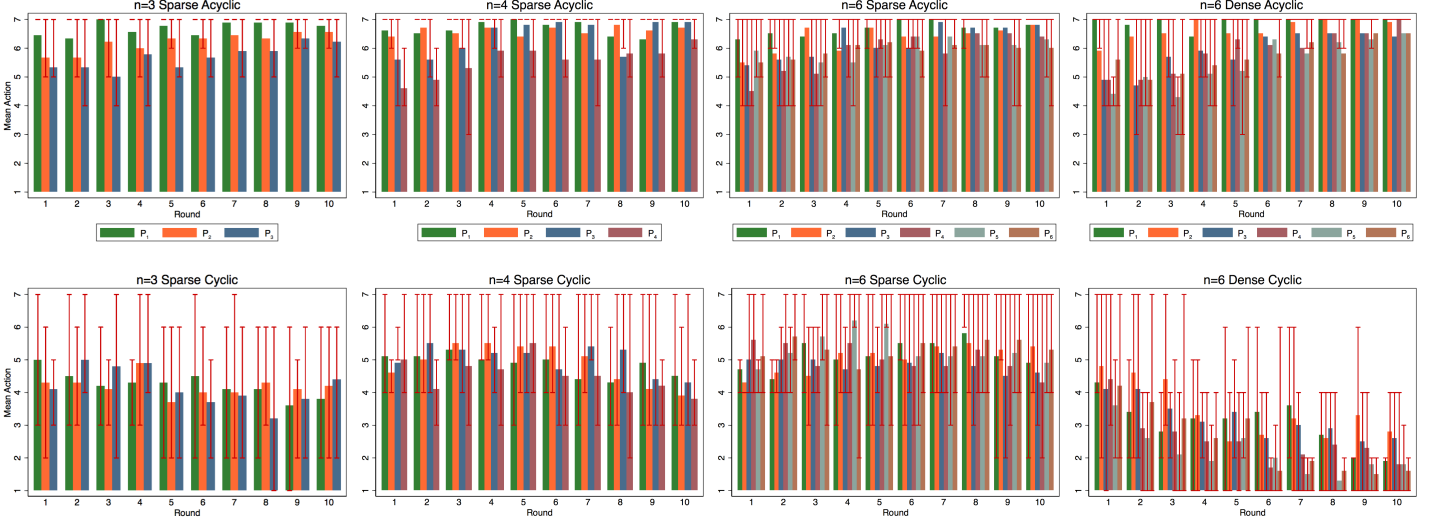
in the sense that $S_j \neq S_i$ for $i \neq j$. In addition, because the network is acyclic, any $P \in S_j$ is also in S_i only if $i \geq j$.

Consider the sequence of distribution profiles $\{F^0, F^1, \dots\}$ where $F^{n+1} = L(F^n)$ and $F^0 = (1, \dots, 1)$, and note that this sequence converges to the unique logit equilibrium. We claim that

$$F_i^k(x) \geq F_j^k(x) \text{ for all } i > j \text{ and all } x \in [0, \bar{x}]. \quad (\text{A.5})$$

The inequality (A.5) holds for $k = 0$. Thus, given the pecking-order property, the inequality holds by induction for all $k \geq 0$. This implies that $F_i^\infty(x) \geq F_j^\infty(x)$ for all $i > j$ and $x \in [0, \bar{x}]$: that is, the inequality in the Proposition holds weakly for all $i > j$. It remains to show that the dominance is strict. But this follows straightforwardly by induction. Consider the claim that F_j strictly dominates F_{j+1} for all $j \leq k$. The claim is clearly true for $k = 1$; and if the claim is true for k , then F_k strictly dominates F_{k+1} by the pecking order property, and thus the claim is true for $k + 1$ also. ■

B Appendix: Additional Figures and Tables



Mean actions by network position, averaged across treatment are displayed on the vertical axis and rounds are depicted on the horizontal axis. Each bar color indicates a player's position within the network: e.g., *green = first player, orange = second player, blue = third player* etc. The 25th-75th percentiles of the empirical distributions are highlighted in red. The first (second) column from the left illustrates sparse networks of size $n = 3$ (4). The two columns on the center-right show networks of size $n = 6$, differing in density with sparse and dense. The top (bottom) row always shows acyclic (cyclic) networks.

Figure B.1: Evolution of Mean Actions in Acyclic & Cyclic Networks

Table B.1: Tests for Differences between Acyclic & Cyclic Networks

	$n = 3$ sparse	$n = 4$ sparse	$n = 6$ sparse	$n = 6$ dense
$Action_1$.023	.023	.080	.001
$Action_{1-5}$.001	.001	.001	.001
$Action_{6-10}$.001	.001	.001	.001
$Action_{10}$.021	.003	.057	.001

Wilcoxon rank-sum test: reported numbers are p -values.

Table B.2: Tests for Differences in Empirical Distributions between Acyclic & Cyclic Networks

	$n = 3$ sparse		$n = 4$ sparse		$n = 6$ sparse		$n = 6$ dense	
	KS	ES	KS	ES	KS	ES	KS	ES
$Action_1$.038	.057	.004	.090	.031	.092	.018	.012
$Action_{1-5}$.001	.001	.001	.001	.001	.001	.001	.001
$Action_{6-10}$.001	.001	.001	.001	.001	.001	.001	.001
$Action_{10}$.001	.001	.001	.001	.001	.001	.001	.001

KS: Two-sample Kolmogorov-Smirnov test; ES: Two-sample Epps-Singleton test.

Reported numbers are p -values.

Table B.3: Time Trends in Mean Actions for Groups of Size $n = \{2, 12\}$

	$n = 2$	$n = 12$
$Action_1$	4.30 [.80]	5.67 [.43]
$Action_{1-5}$	4.39 [1.60]	6.03 [.64]
$Action_{6-10}$	4.55 [2.33]	6.90 [.16]
$Action_{10}$	4.50 [2.53]	7.00 [0]

Mean [st. dev.] action, averaged over groups.

Table B.4: Time Trends in Mean Actions – Acyclic Networks without P_1

	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>
$Action_1$	5.50 [.66]	5.53 [1.01]	5.36 [.87]	5.14 [.63]
$Action_{1-5}$	5.67 [1.12]	6.00 [.86]	5.80 [.99]	5.47 [.99]
$Action_{6-10}$	6.22 [1.10]	6.37 [.72]	6.35 [.99]	6.42 [1.18]
$Action_{10}$	6.39 [.74]	6.63 [.58]	6.46 [.88]	6.66 [.75]

Mean [st. dev.] action, averaged over groups.

Table B.5: Tests for Differences between Acyclic & Cyclic Networks without P_1

	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>
$Action_1$.096	.126	.323	.007
$Action_{1-5}$.001	.001	.002	.001
$Action_{6-10}$.001	.001	.001	.001
$Action_{10}$.034	.003	.057	.001

Wilcoxon rank-sum test: reported numbers are p -values.

Table B.6: Average Prediction Error by Network Position (All Rounds)

Network Structure		P_1	P_2	P_3	P_4	P_5	P_6	ALL
<i>sparse</i>	$n = 3$ <i>acyclic</i>		.64	1.24				.94
			[1.10]	[1.34]				[1.26]
	$n = 3$ <i>cyclic</i>	1.02	.87	1.15				1.01
		[1.38]	[1.27]	[1.28]				[1.31]
<i>sparse</i>	$n = 4$ <i>acyclic</i>		.49	.77	1.34			.87
			[1.31]	[1.29]	[1.44]			[1.34]
	$n = 4$ <i>cyclic</i>	1.48	1.14	1.57	1.53			1.43
		[1.64]	[1.39]	[1.56]	[1.41]			[1.51]
<i>sparse</i>	$n = 6$ <i>acyclic</i>		.75	.82	.89	.68	.63	.75
			[1.41]	[1.31]	[1.36]	[1.29]	[0.91]	[1.27]
	$n = 6$ <i>cyclic</i>	1.46	0.83	1.35	1.41	1.43	1.38	1.31
		[1.64]	[1.09]	[1.25]	[1.56]	[1.44]	[1.51]	[1.44]
<i>dense</i>	$n = 6$ <i>acyclic</i>		.38	1.07	1.05	1.24	1.42	1.03
			[1.18]	[1.77]	[1.68]	[1.70]	[1.74]	[1.66]
	$n = 6$ <i>cyclic</i>	1.52	1.64	1.30	1.39	0.85	1.27	1.33
		[1.79]	[1.73]	[1.64]	[1.51]	[1.17]	[1.82]	[1.64]

Mean [st. dev.] difference. See Appendix D for an illustration of network positions.

Table B.7: Tests for Differences in Prediction Errors $x_{i,t}^*$ between Acyclic & Cyclic Networks

	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>
$Error_1$	0.2441	0.0269	0.0201	0.0269
$Error_{1-5}$	0.3406	0.0001	0.0001	0.0224
$Error_{6-10}$	0.2589	0.0009	0.0001	0.0004
$Error_{10}$	0.2556	0.0334	0.0570	0.1156

Wilcoxon rank-sum test: reported numbers are p -values.

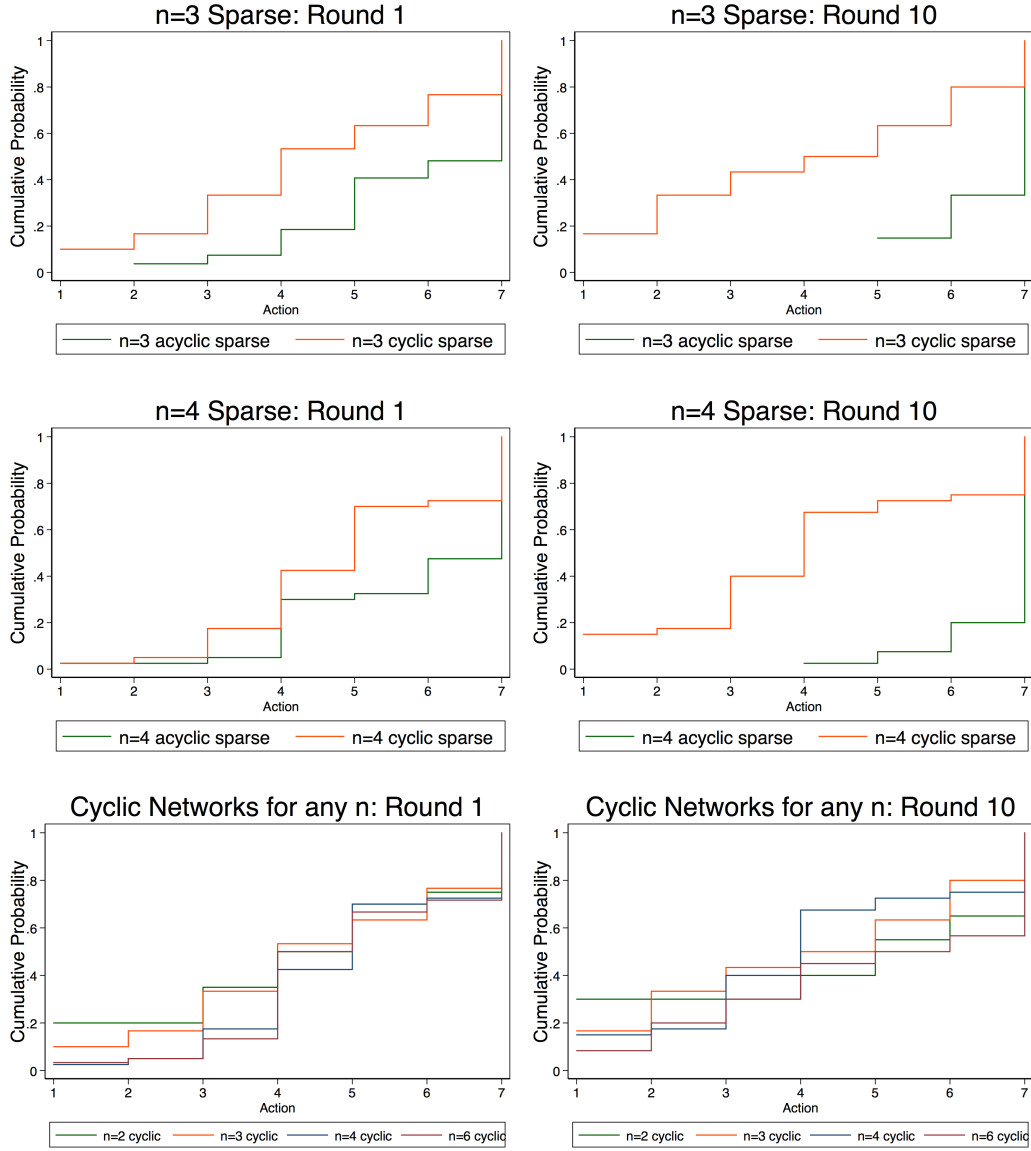


Figure B.2: Empirical Distributions of Acyclic & Cyclic Networks

C Appendix: Neighbourhood-Minimum Action Analysis

Table C.1: Neighbourhood-Minimum Action by Position, Acyclic Networks (Final Round)

	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>	$n = 12$ <i>dense</i>
$Minimum_{p_1}$	6.78 [.44]	6.90 [.32]	6.80 [.63]	7.00 [0]	7.00 [0]
$Minimum_{p_2}$	6.44 [.73]	6.70 [.67]	6.70 [.67]	6.90 [.32]	7.00 [0]
$Minimum_{p_3}$	6.11 [.93]	6.70 [.67]	6.80 [.42]	6.30 [1.89]	7.00 [0]
$Minimum_{p_4}$		6.30 [1.06]	6.40 [1.27]	6.30 [1.89]	7.00 [0]
$Minimum_{p_5}$			6.20 [1.32]	5.90 [2.33]	7.00 [0]
$Minimum_{p_6}$			6.00 [1.41]	5.90 [2.32]	7.00 [0]
\vdots					
$Minimum_{p_{12}}$					7.00 [0]

Mean [st. dev.] neighbourhood-minimum action by position, averaged across groups.

Table C.2: Neighbourhood-Minimum Action by Position, Cyclic Networks (Final Round)

	$n = 2$ <i>dense</i>	$n = 3$ <i>sparse</i>	$n = 4$ <i>sparse</i>	$n = 6$ <i>sparse</i>	$n = 6$ <i>dense</i>
$Minimum_{p_1}$	4.40 [2.55]	3.80 [2.35]	3.80 [2.10]	4.70 [2.54]	1.60 [1.07]
$Minimum_{p_2}$	4.40 [2.55]	3.80 [2.35]	3.90 [2.18]	4.90 [2.38]	1.90 [1.73]
$Minimum_{p_3}$		4.20 [2.35]	3.60 [2.37]	4.40 [2.46]	1.90 [1.73]
$Minimum_{p_4}$			3.40 [2.22]	4.20 [2.44]	1.60 [1.07]
$Minimum_{p_5}$				4.30 [2.31]	1.60 [1.07]
$Minimum_{p_6}$				4.40 [2.32]	1.60 [1.07]

Mean [st. dev.] neighbourhood-minimum action by network position, averaged across groups. Table C.3 reports statistical tests for differences in minimum actions in each player's neighborhood between cyclic and acyclic treatments.

Table C.3: Tests for Differences in Neighbourhood-Minimum Actions, Acyclic vs. Cyclic Networks

	<i>n</i> = 3 <i>sparse</i>	<i>n</i> = 4 <i>sparse</i>	<i>n</i> = 6 <i>sparse</i>	<i>n</i> = 6 <i>dense</i>
<i>Minimum</i> ₁	.012	.001	.019	.001
<i>Minimum</i> _{1–5}	.001	.001	.001	.001
<i>Minimum</i> _{6–10}	.001	.001	.001	.001
<i>Minimum</i> ₁₀	.021	.002	.048	.001

Wilcoxon rank-sum test: reported numbers are *p*-values.

Table C.4: Time Trends in Neighbourhood-Minimum Actions, Acyclic vs. Cyclic Networks

	<i>n</i> = 3 <i>sparse</i>		<i>n</i> = 4 <i>sparse</i>		<i>n</i> = 6 <i>sparse</i>		<i>n</i> = 6 <i>dense</i>	
	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>	<i>acyclic</i>	<i>cyclic</i>
<i>Minimum</i> ₁	5.37	3.43	5.26	3.95	5.00	4.02	4.27	2.28
	[.81]	[1.69]	[.82]	[.88]	[.86]	[.80]	[.96]	[.69]
<i>Minimum</i> _{1–5}	5.75	3.77	5.99	4.23	5.53	4.34	4.95	2.04
	[1.11]	[1.88]	[.84]	[1.30]	[1.10]	[1.14]	[1.37]	[.96]
<i>Minimum</i> _{6–10}	6.36	3.62	6.27	3.98	6.31	4.59	6.34	1.75
	[.94]	[2.04]	[.97]	[1.82]	[1.04]	[1.95]	[1.23]	[1.03]
<i>Minimum</i> ₁₀	6.44	3.93	6.65	3.68	6.48	4.48	6.38	1.70
	[.53]	[2.27]	[.64]	[2.08]	[.87]	[2.32]	[1.37]	[1.29]

Mean [st. dev.] neighbourhood-minimum action, averaged across groups. For acyclic networks, Jonckheere tests for time trends and for ascending order yield *p*-values of 0.0002, 0.0005, 0.0001, and 0.0001 for sparse *n* = 3, 4, 6 and dense 6, respectively. Conversely, testing for descending order in cyclic networks generates *p*-values of 0.3715, 0.1586, 0.8028, and 0.0048 for sparse *n* = 3, 4, 6 and dense *n* = 6, respectively.

D Appendix: Experimental Implementation of Networks

For each network, we illustrate the interface that subjects experienced. Our experimental implementation in z-Tree indicates a subject's position within the network, and also highlights his neighbourhood (which we called his *watch-list*). The subject's position is highlighted in red, and his watch-list is circled in red.

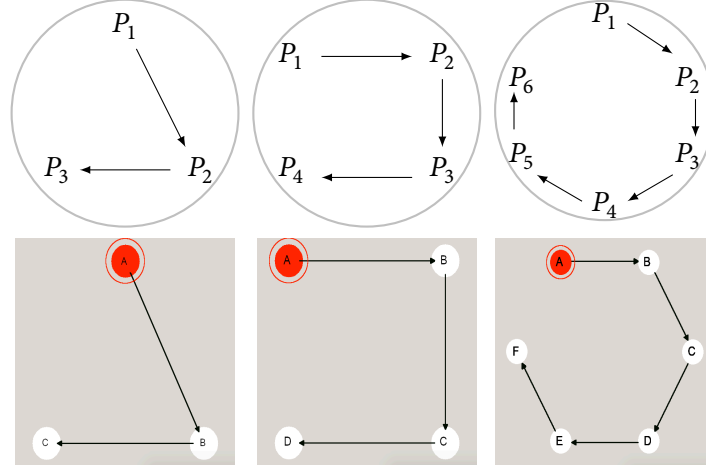


Figure D.1: Acyclic Sparse Networks

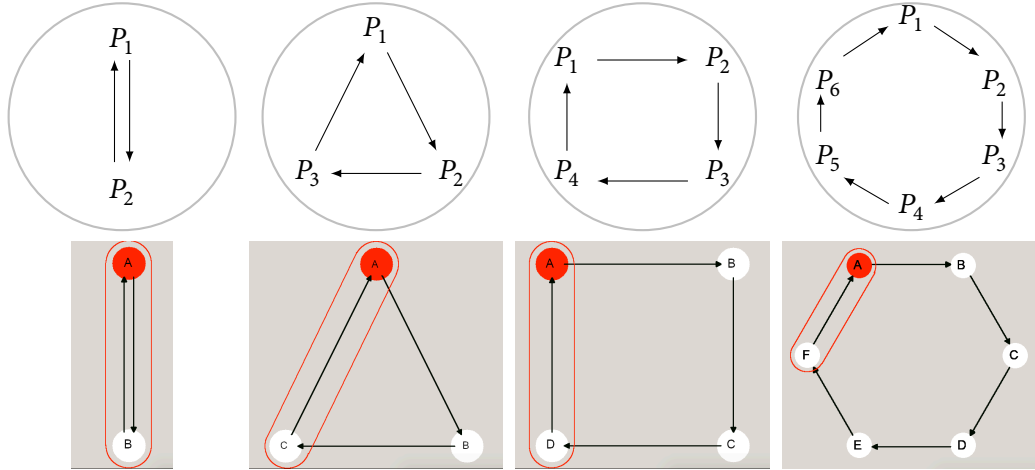


Figure D.2: Cyclic Sparse Networks

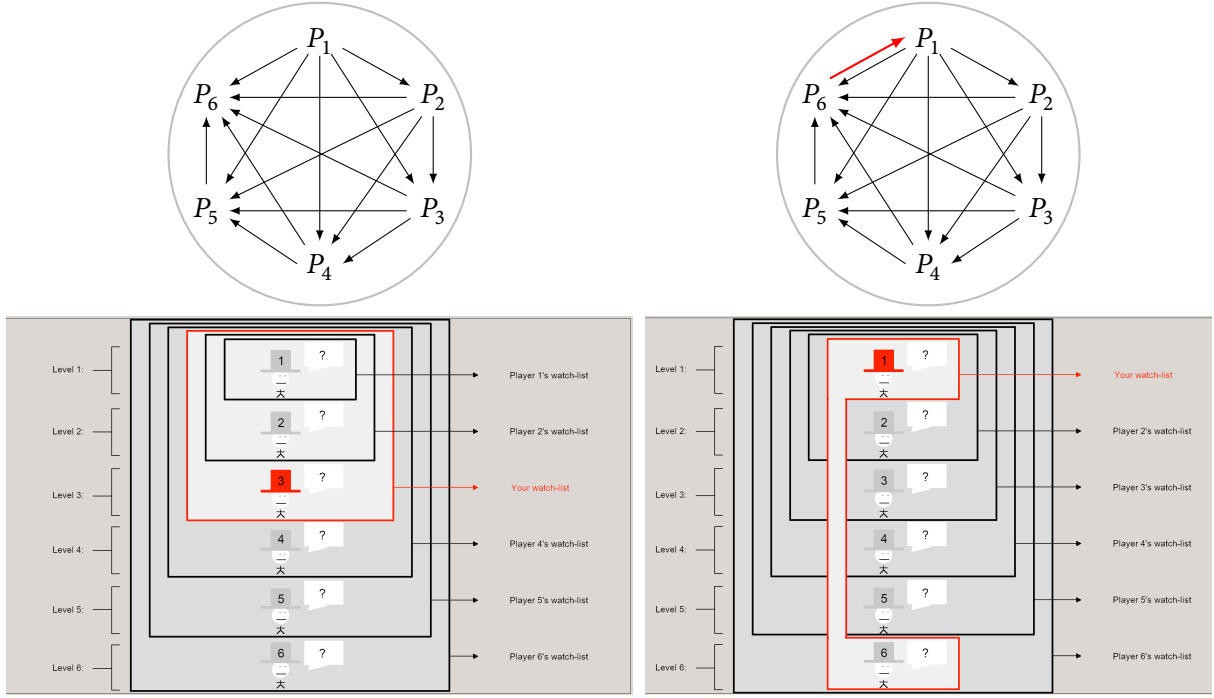


Figure D.3: Dense Acyclic & Cyclic 6-Player Networks

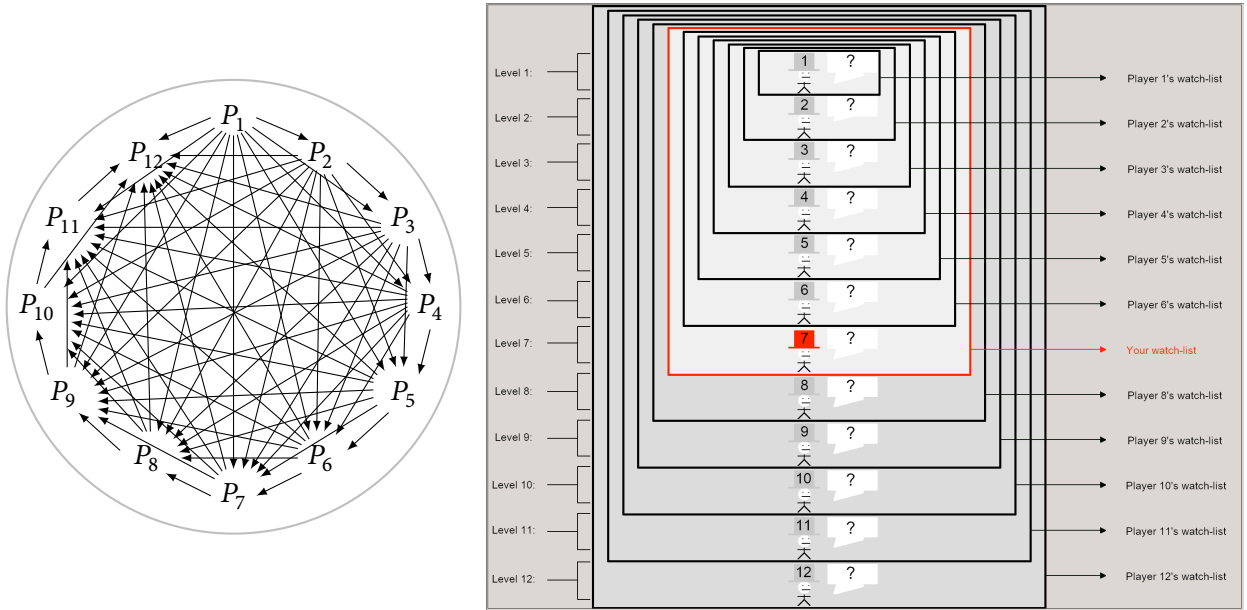


Figure D.4: Acyclic Dense 12-Player Networks

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