Affirmative Action, Equal Opportunity, or just tax the rich? Development, efficiency, and the pursuit of equity

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Abstract

Tension between efficiency and equity is fundamental to every economy. Historical differences between groups translate into inequality in skills and hence earnings. Measures to correct inequalities affect incentives and misallocate talent, therefore compromising efficiency. This paper examines the efficiency properties of the three most common classes of equity policies: affirmative action, equal opportunity and tax-transfer. Our focus is to examine how the effectiveness of policies vary with the level of development and technology and the political maturity of the state. We argue that the optimal policy is likely to be different for different countries, and indeed for the same country at different stages of development.

The intuition driving our approach is that the products produced in a less-developed economy are less complex and require lower embodied skills. Here, preferentially placing less prepared individuals in higher skill jobs does not compromise efficiency to too large an extent. In high-technology production processes, however, skills are more critical and productivities are interdependent, so it makes more economic sense to adequately train the inductees even at a relatively high cost. The most efficient outcomes are yielded by competitive markets accompanied by appropriate tax-transfer schemes. However, such schemes can effectively be used only by economies with the highest levels of socialisation and state capacity.

We find that, in a low-complexity economy, reservation fares better than both training and tax transfer. As complexity of the production process increases training becomes more attractive and is in turn superseded by tax-transfers in the most complex and politically mature economies. These findings provide a step towards more informed and robust policy. We discuss several omissions and directions for further development.

JEL Classifications: D2, D3, D63, I3, O2

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1 Introduction

Tension between efficiency and equity is fundamental to every economy. Even if all persons are created equal, variations in privilege, background and upbringing—not to mention plain luck—renders individuals greatly heterogeneous by the time they enter the economy as productive agents. Deep differences between groups are endemic in most countries as a result of historical, geographical and socio-cultural reasons. These inequalities translate into inequality in skills and productive capabilities, resulting in unequal earnings, and in turn generate inequalities between individuals in the next generation. Measures to correct such inequalities inevitably result in the dulling of production incentives or the allocation of talents away from their most productive uses, and therefore compromise efficiency.[1]

Since the market allocates incomes in the first instance according to productive ability, and these abilities are unequally distributed in the natural state, active policy intervention is necessary to reduce undesirable inequalities. Popular policies can be naturally classified under three broad categories—tax-and-transfer, affirmative action and equal opportunity. Tax and transfer is self-explanatory and occurs after individuals have earned gross incomes that reflect their productivity. Affirmative action, otherwise known as "quotas" or "reservations" works by placing underprivileged individuals in jobs (or professional school places) that would not naturally be awarded to them based on native skill or prior qualifications, by requiring less stringent qualifications from such candidates in the competition for more remunerative jobs. Equal opportunity, on the other hand, attempts to correct the inequality in skills and qualifications before agents reach the employment market, by devoting greater resources to the education and training of the disadvantaged groups.[2]

The adequacy, effectiveness and opportunity costs of these various policies are topics for perpetual debate. The investigations conducted to date make it clear that the question of what is the optimal policy does not have an unequivocal answer. Especially intriguing is the fact that the relative desirability of different policies are evaluated differently in different contexts, and even in the same country popular support can shift widely between one policy and another. For example, reservations for scheduled caste/scheduled tribe candidates has been a pillar of equity policy in India since independence, but its relevance and effectiveness has been increasingly questioned in recent decades. In the US, on the other hand, quotas have never been popular, while northern European countries have had significantly more tolerance for tax-transfer policies than other regions.

This paper takes primitive differences between social groups as given, and examines in some

[1] The heterogeneity manifests itself in command of capital as well as productive skills and earning abilities, but for now we will confine our attention to the latter.

[2] Readers may disagree with the interpretations that we have placed on these terms. We are going by the interpretations or descriptions of policies provided above rather than by the nomenclature. In this paper we will call the non-fiscal policies "reservation" and "training", to keep our meaning clear.

detail the contours of policies that aim to enhance equality of final incomes. The question has both static and dynamic aspects. In this paper we confine ourselves to statics. We aim to show that the optimal policy is sensitive to a host of parameters, some of which we can associate with the level of development and the state of technology, and hence the optimal policy is likely to be different for countries at different stages of development, and indeed for the same country at different times. Some of the results may be useful as a basis for a dynamic extension.[3]

A basic intuition driving our approach is that the products demanded and produced in an economy in the earlier stages of development are less complex and require lower embodied skills to produce. In such economies, preferentially placing less prepared individuals in higher skill jobs does not compromise efficiency to too large an extent. Remedying the disadvantage in preparation, on the other hand, may be significantly more demanding of resources when education systems are strained and insufficient. Thus affirmative action may be a more efficacious means to reduce outcome inequalities than a "no child left behind" policy. In advanced and high-technology production processes that abound in richer economies, however, placing an unprepared individual in a sensitive production line may severely compromise output, and turn out to be wasteful of valuable human resources. Here it makes more economic sense to adequately train the inductees, even at a relatively high cost. Of course, the most notionally efficient outcome is to allow a market (presumed competitive) to determine levels of training and job allocations, and then use *ex post* taxes and transfers to correct inequality. However, tax-transfer schemes affect incentives, and are costly and constrained by state capacity. Hence these can effectively be used only by a handful of economies with high levels of socialisation and extensive state capacity, largely concentrated in Northern Europe. It is outside the purview of a rudimentary state (Besley and Persson, 2013).

The comparison between affirmative action (reservations) and training (equal opportunity) is illustrated by the following example. Consider the policy of reserving places in medical school for students from disadvantaged backgrounds. This is a standard practice in India, where reservations for "dalits" (historically disadvantaged castes and tribes) has been a mainstay of affirmative action since independence. The reason why reservations are needed is that dalit children, on the strength of their educational backgrounds, would not gain admission into medical schools in sufficient numbers, since admission is based on merit as demonstrated by exam scores and past academic performance.[4] Places are reserved not only in medical and engineering schools, but also in positions in administration and the bureaucracy, where appointees are entrusted with many responsible roles in government and public service.

[3]A potential dynamic extension of the model would analyse how the configuration of parameters in one generation can influence the trajectory of outcomes in the future (Mookherjee and Ray, 2010; Galor and Zeira, 1993).

[4]This does not, of course, establish that merit thus measured necessarily determines the subsequent development of ability, but there is good reason to believe that it does. Otherwise there would surely be instances where places are allocated by lottery, or at least dalits that were admitted would sometimes be selected randomly. In reality, however, the reserved places are also filled on merit, only with a lower entry bar.

Now think of how such a policy would affect the performance of doctors that complete medical school. Some doctors who came in with blazing credentials would on average emerge with great acumen, while a large fraction of those that entered with marginal credentials would possibly only marginally hold their own.

How much difference would it make that some doctors came into medical school with less-than-ideal prior preparation? This depends on how they spend most of their medical career. In a very underdeveloped country, where life-expectancy is low and most people do not have the luxury of living long enough to develop complex diseases, the bulk of the average doctor's time may be spent organising inoculation drives and treating diseases like diarrhoea and cholera, for which the main treatment is often adequate hydration and general cleanliness, but which claim an appalling number of lives because of lack of basic health information among the public. In such countries, the large prior variance in preparation may well translate only in a minor variation in outcomes. In advanced economies where complex diseases are the major cause of concern, and doctors spend their time performing complicated diagnoses and surgeons undertake intricate procedures, difference in preparation may well have a severe cost. Accordingly, in more advanced economies, quotas in high-paying occupations are not the favoured alternative; in the quest for greater equality the emphasis is much more on aiding the underprivileged to make the unconditional cut for admission into preferred callings. The focus then is on early childhood and school education (no child left behind), support for traditionally black colleges, and enabling college attendance through scholarships and other means.

In the above example, for the economy in the early stages of development, the constraint on the production of doctors' services is presented by the capacity of medical schools, not so much on the prior preparation of candidates. Conceptually, this reflects the conception of development in the Lewis-Ranis-Fei (Lewis, 1954; Ranis and Fei, 1961, henceforth LRF) economy, where development is primarily the process of moving human resources from the less productive agricultural or rural or traditional sector to the more productive industrial or urban or modern one. In LRF, this process is constrained by the availability of capital that determines employment capacity in the modern sector. By default, those who are already in the modern sector are the privileged group, and occupants of the traditional sector are underprivileged.[5] In this sense, the development process in the LRF model is powered by affirmative action subject to capital constraints. Affirmative action increases output and has no negative consequences. The assumption that industrial profits are reinvested provides the dynamic in that development model.

As development proceeds and production becomes more skill-intensive, this simple representation may not continue to suffice. Kremer (1993) has explained that complex production

[5]Here we ignore the informal sector and Harris-Todaro concerns, which can be accommodated in our analysis in a straightforward way, though it will change some of the quantitative conclusions.

processes can be compromised by low-skill individuals, and catapulting such individuals into production teams above their skill-class can have severe negative externalities. While reservations or "affirmative action" can increase the productivity of disadvantaged individuals in such contexts, they extract a cost by handicapping the more accomplished workers with whom they are teamed. The latter cost can be reduced by upskilling the catapultees before they are elevated, and hence "equal opportunity", or the provision of prior training, begins to look attractive.

We pursue this line of reasoning to compare the effectiveness of different equalisation policies at different levels of development. We also compare the outcomes of different classes of policies with those of a pure tax-transfer scheme. In principle, the best outcome is always attained by directing investments and productive resources to maximise output, and then reallocating that output through taxes and transfers to attain desired equity. In practice, however, states have limited capacity to implement such schemes, both owing to the reaction of taxpayers and the reticence of politicians focused on re-election. Taxes also affect work-incentives and hence production. We represent these effects as a leakage from the tax-transfer process.

We find that reservation fares better than both training and tax transfer in an economy where skill differences are small, complexity is low and sectoral effects are significant. This resembles the archetypal two-sector underdeveloped economy that Lewis conceived. Here, reservations lead to an increase in per-capita income as well as a decrease in inequality. However, as complexity of the production process increases, reservation becomes more costly. In fact, for a high enough skill difference and complexity, reservation is the least appealing of the three policy options. Notwithstanding its lack of appeal, reservations may persist longer in developing economies as the fiscal capacity to fund training costs or engage in redistributive tax-transfers remains limited.

Training and tax-transfers (naturally) require taxes. As we mentioned earlier, both tax transfer and training beat reservation in a complex economy with large skill differences. However, the ranking between tax-transfers and training depends, among other things, on the properties of training cost, and the effectiveness of the fiscal system. Under reasonable conditions, training becomes the most efficient policy in economies with intermediate levels of skill differences and complexity, while high state-capacity and large skill-differences skew the advantage in favour of tax-transfers.

The effects of affirmative action policies on employment of targeted groups have been studied in context of the US (Bleemer, 2022) and India (Afridi, Iversen, and Sharan, 2017; Munshi, 2019; Prakash, 2020), and other countries. Equal opportunity and especially early intervention programs have been in the limelight for some time (Miguel and Kremer, 2004; Schultz, 2004; Afridi, 2010). Our project will add to this literature by establishing a framework to evaluate the relative desirability of different inequality-reducing policies according to context, and underlining that effectiveness is conditioned by attributes of the economy that have not previously been explicitly included in this analysis.[6]

A novelty of our approach lies in this twofold linking of optimal policy with population heterogeneity and technological maturity. Researchers have examined the effectiveness and resource cost of equity programs in different contexts, e.g., Deshpande and Weisskopf (2014) for Indian Railways, Bleemer (2022) for the (removal of) affirmative action in the University of California system, and Holzer and Neumark (2000) for race-based affirmative action in the US more generally. However, circumstances that condition this effectiveness have not been explored in the literature. To this purpose, we integrate the dual-economy model of Lewis (1954) with the insights on production complexity from Kremer (1993). We develop a production function that establishes a continuum between these two classic models which comprise the principal analytical frameworks within which economic development is visualised, but until now there has been no attempt to integrate them in one framework that highlights the regions in which each approach is relevant.

The approach we take in this paper allows us to shift emphasis from the question: "Do the gains from affirmative action justify the accompanying efficiency cost?" to the question "Under what social and technological conditions is affirmative action more cost-efficient than equal opportunity measures to attain specific equity gains?" It will further enable us to determine when it is more advisable to shelve both policies and use progressive taxation and public/welfare spending instead. An unconditional analysis, on the other hand, may conclude that a policy was or was not effective in a certain place or time, but will be unable to explain why it did not obtain similar results in another milieu. Recognising the effect of critical conditioning variables enables an informed choice of policy.

2 Model

2.1 Production function

Production takes place in teams of n agents each, where n may vary according to the skill levels of the team members and the importance of individual skill and accuracy in the production process. Here we follow Kremer (1993) and adapt the O-ring approach to suit our question.

Further, in order to relate our analysis to some of the dominant paradigms in the development literature, we incorporate the effect of the sector in which a production process operates, where some sectors (i.e., the "modern" sector in the Lewis-Ranis-Fei paradigm) are endowed with a more production-augmenting environment compared to other ("traditional") sectors. We represent this by a sector parameter (s_j in the formalisations below) which embraces the contribution of specific resources such as infrastructure, communications, etc. Some of this is

[6] An elegant classification of policies along similar lines is proposed in Rodrik and Stantcheva (2021).

the result of public provision and some are externalities—where we implicitly assume that the externality-generating activities tend to gravitate to the sectors where the publicly provided resources are found. Alternatively, s may be a proxy for industry, with s_j merely representing the capital-intensivity of an industry. The exposition below is consistent with the interpretation that more capital intensive industries attract and recruit more skilled workers. There is scope to further disentangle the sector parameter.

We write the expected per-capita production function in an n-member team working in sector j as

$$y_j(p, n, \beta) = s_j^{1-\beta} \frac{G(n, \beta)}{n} \Pi_{i=1}^{n} p_i^{\beta} \tag{1}$$

where s_j is the contribution of sector resources and $G(., .)$ is the contribution of an n-person team of labour to a unit of final output, $p_i \in [0, 1]$ is an index of the skill or precision of the i-th worker, and β is a parameter that captures the importance of skill and precision in the production process.[7] We specify the value of a unit of final output as

$$G(n, \beta) \equiv n^{1+\mu\beta}$$

where μ is a positive constant.[8]

As $\beta \longrightarrow 0$, the p_is become inconsequential and the importance of individual precision vanishes. The output of each individual worker production depends only on the sector resources s_j, as visualised by Lewis. Thus β is a factor that tempers the criticality of individual performance, moderating the impact of mistakes in processes according to their skill-sensitivity. This is a direct interpretation of the O-ring production function (Kremer, 1993), and we will not attempt to justify it further. When $\beta = 1$, this production function conforms to the interpretation due to Kremer, that a unit of output is produced if and only if each worker in the process executes her task correctly, and p_i is the probability that the i-th worker does so. Here s_j does not matter. Further $G(., .)/n$ is increasing in n indicating that more complex processes produce proportionately more valuable output.

Processes where productivity depends only on team size and is not greatly sensitive to individual's accuracy are more likely to incorporate Smithian division of labour in which tasks are split up into small components, and correctly produced components are assembled together. We can categorise such processes as ones where the rate of output is constrained by the speed of the slowest member of the team. Here good management would consist of balancing the numbers of workers at different "stations" such that each station performs its part of the process at the same rate as every other station. Such division of labour produces increasing returns

[7]We ignore capital, which can be incorporated as in Kremer (1993), see Appendix A.3.

[8]We use a closed form function since it makes the analysis much more transparent, and the general properties of this production function are well understood.

to scale, as Smith famously observed, but these are a result of the organisation of production rather than the interaction of highly-skilled operatives. Increasing returns occur at a qualitatively different scale in "high-tech" complex processes where the marginal productivity of each worker is determined by the precision of all others. The production function above attempts to parametrically represent some aspects of the production process in a developing economy as its technological capabilities go from simple to increasingly complex.

2.2 The economy

The economy consists of two production sectors, which we will often refer to as the *modern* and *traditional* sectors. This is a notional classification. The sectors may be geographically separated, with the urban modern sector equipped with more amenities, public resources and capital, making workers in that sector more productive. They may also be spatially co-located and distinguished by technology, organisation and management methods that make one sector a more productive environment than the other.[9] Where sectoral productivity matters (which is not always), we assume that the modern sector is more productive than the traditional. In the production function this translates as:

$$s_m \geq s_t \tag{2}$$

where m and t represent the two sectors.

There is a unit mass of workers in the economy. A fraction $1-\theta$ of the workers belong to the privileged group that we call *elits*, while a fraction θ comprise the underprivileged *dalits*. Each elit has an initial skill level p and each dalit has a skill level q, where

$$0 \; < \; q \; < \; p \; < \; 1 \tag{3}$$

2.3 Production

Consider an arbitrary production outfit in an arbitrary sector with n team members, of which a fraction $(1-\gamma)$ are p-types or elits and the remaining fraction γ are q-types or dalits. Average output produced by this team is

$$y(p,q,n,\beta) = s^{1-\beta}\,\frac{G(n,\beta)}{n} p^{\beta n(1-\gamma)} q^{\beta n \gamma}.$$

Letting $G(n,\beta) = n^{1+\mu\beta}$, we have

$$y \;\; = \;\; s^{1-\beta}[n^{\mu} p^{n(1-\gamma)} q^{n\gamma}]^{\beta} \tag{4}$$

[9] Lewis himself did not espouse the geographical interpretation, or even an identification of the two sectors with "industrial" and "agricultural", see Gollin (2014).

Higher β reflects increased complexity. Observe that when $\beta = 0$, per-capita output depends only on the sector productivity as in the Lewis-Ranis-Fei economy; while as $\beta \to 1$ we approach a version of Kremer's O-ring production function. As n rises, $\frac{G(n,\beta)}{n}$ rises owing to increasing returns, while the probability of success $p^{\beta n(1-\gamma)}q^{\beta n\gamma}$ declines. Optimal n balances these two effects.

2.3.1 Optimal team size

To find the optimal n, or team size, first write the production function in log form

$$\ln y = (1-\beta)\ln s + \beta[\mu \ln n + n(1-\gamma)\ln p + n\gamma \ln q]$$

and then differentiate $\ln y$ w.r.t n and set it to 0 to obtain optimal n:

$$\frac{\mu}{n} + (1-\gamma)\ln p + \gamma \ln q = 0 \quad \Longrightarrow \quad n^* = -\frac{\mu}{(1-\gamma)\ln p + \gamma \ln q}$$

Rearranging n^* and substituting in y yields:

$$y^* \;=\; s^{1-\beta}(n^{*\mu}p^{n^*(1-\gamma)}q^{n^*\gamma})^{\beta} = s^{1-\beta}(\frac{n^*}{e})^{\beta\mu}$$

Observe that n^* (and hence y^*) is increasing in productivity of p-types, the proportion of p-types $(1-\gamma)$, and the degree of increasing returns (μ).

Now write

$$q = \frac{1}{e^k} \quad \text{and}$$
$$p = \frac{1}{e^{\frac{k}{r}}}$$

By (3), $k > 0$ and $r > 1$, and r captures the degree of skill-advantage of the p-types. Substituting these values in the expression for n^* yields optimal team size and average per capita output in a mixed team where p and q-types are present in proportions of $(1-\gamma)$ and γ:

$$n = -\frac{\mu}{(1-\gamma)\ln p + \gamma \ln q} = \frac{\mu}{k}\left(\frac{r}{(1-\gamma)+\gamma r}\right) \tag{5}$$

$$y = s^{1-\beta}(\frac{n}{e})^{\beta\mu} = s^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}\left(\frac{1}{(1-\gamma)+\gamma r}\right)^{\beta\mu} \tag{6}$$

2.3.2 Full Separation

Suppose elit p-types and dalit q-types are fully separated, with elits placed in the modern sector and dalits in the traditional sector. Then team size and per-capita output in the traditional and modern sectors, respectively, are given by:

$$n(q,t) = \frac{\mu}{k}, \qquad y_t^F = {s_t}^{1-\beta}(\frac{\mu}{ek})^{\beta\mu}; \tag{7}$$

$$n(p,m) = \frac{\mu r}{k}, \qquad y_m^F = {s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}. \tag{8}$$

where superscript F indicates full separation. Here we assign the elits a double privilege of the more productive sector and the higher skill parameter. Depending on the type of economy we consider, one or the other of these will become more salient.

It is easy to show that, when $\beta > 0$, the full-separation allocation of workers between processes maximises total output. If there are no constraints on assignment of workers across sectors, and $s_m > s_t$, then all workers should be placed in the modern sector, but segregated according to skill. We retain the convention that low-skill processes are operated in the traditional sector and high-skill processes in the modern sector, which may reflect capacity constraints in the modern sector.[10] However, most of the results below are driven by skill differences between groups, and remain valid if we assume that $s_m = s_t$. Differences in sectoral resources become important only when β is close to zero.

The objective of this paper is to examine the per-capita output and inequality of income under different policy regimes and organisations of production. As a benchmark, we write down the mean income and the variance of the income distribution under full separation.

$$\begin{aligned} \text{Mean: } \bar{y}^F &= (1-\theta)y_m^F + \theta y_t^F \\ \text{Variance: } \sigma^{2F} &= (1-\theta)y_m^2 + \theta y_t^2 - ((1-\theta)y_m + \theta y_t)^2 \\ &= \theta(1-\theta)(y_m^F - y_t^F)^2 \end{aligned}$$

where y_m^F and y_t^F are as in (8) and (7). Henceforth we will drop the superscript and let $\bar{y}$ and σ^2 represent mean and variance in the (reference) full-separation economy, while y_t and y_m represent full-separation income in the two sectors.

3 Reservation, training and income redistribution

Next, we enumerate the outcomes of different policies, and compare their impacts upon per-capita income and inequality conditional on the level of development of the economy. These policies cause income and inequality to deviate from the full separation outcome described in Section 2.3.2. Recall that full-separation with optimal choice of complexity maximises total

[10] It is possible that creating capacity in the modern sector requires some amount of per-capita investment that repays itself only if the worker is of sufficiently high skill, and skill matters (i.e., β is sufficiently large). In a Lewis economy where $\beta = 0$, accommodating workers in the modern sector is then constrained only by capacity, but when β is sufficiently large the creation of capacity must be justified by the quality of workers being accommodated. These considerations would be better addressed in a dynamic model.

output in the economy conditional on the sectoral allocation of workers with different skills.

Each agent works in a given sector in a production team consisting of some number of workers of each type. In all configurations we consider, production teams will consist only of q-types, only of p-types, or of the two types mixed together. Teams with only q-types always operate in the traditional sector. Teams with only p-types, or with a mix of the two types, operate in the modern sector. This does not require the sector parameters s_t and s_m to be different, nor for there to be a spatial distinction between the sectors.

The total number of workers determines the complexity of the process. Within a team, all members earn the same income, which is the per-capita output produced in the team. Thus the income of an agent that works in an n-person team in sector j, with fractions γ and $1-\gamma$ of q and p-types, is given by (4), which reduces to (6) when team size is chosen optimally.

We make this modelling choice to keep the analysis streamlined. An alternative would have been to assume that each agent in a multi-type mixed team is paid in proportion to her marginal product. The income of the p-types would then exceed that of the q-types in the team, and the latter in turn would earn more than q-types who are in teams by themselves. That analysis would possibly be more realistic and yield quantitatively different results, but we don't think it would convey a radically different intuition (see Sections 4.3 and 4.4).

Our focus is on three policies: *reservation*, *training* and income redistribution through fiscal means, or *tax-transfer*. We consider these against the benchmark of full-separation described in the previous section.

Reservation refers to placing some of the lower-skill individuals (dalits or q-types) in production processes primarily executed by high-skill workers. We saw in Section 2.1 that in mixed teams the per-capita output is higher than that in teams consisting entirely of the low types, and lower than those consisting entirely of the high types, when team sizes are always chosen optimally. Thus inequality in the economy will typically be lower with reservations compared to full separation, and per-capita income will also be lower. However, there is an important exception to this in a range of parameter values that correspond to the LRF economy.

Training refers to a program in which some number of dalits are provided extra training resources so that their skill level increases to p. They are then put in production teams with the rest of the p-types. We assume that training uses resources, which is funded by a tax on the incomes of the high types (elits and the newly upskilled dalits) so that their average incomes are reduced from that given in (8). The net effect of this is negative on inequality and ambiguous on per-capita income (net of training costs). The analysis of training is complex because it calls into play the composition of resources that shape the initial abilities of agents in our economy. This is discussed later in Section 4.2.

Finally, *tax-transfer* refers to a policy of taxing the high-skilled (and hence high-income) modern sector workers, and using the proceeds to increase the income of a subset of the low-skilled traditional sector workers. If the fiscal machinery is perfect and costless, then this is potentially more efficient than other equalisation policies at least in a static sense. However, transfers are seldom costless, owing to constraints of state capacity, the shadow cost of public funds, and the effect on work-incentives (which are not explicitly modeled in this paper). We collapse these into one indicator that we call *leakage*, which determines the fraction of taxes that are consumed by these costs. The zero-leakage tax-transfer case is the benchmark against which the effectiveness of all policies is evaluated.

This section enumerates and compares these effects in order to answer the key question: which policies should be used, and when?

3.1 Schematics for comparison

In order to make the policies comparable, we construct them in the following way. Starting from the full-separation configuration (see Section 2.3.2), we choose a given mass $\phi \leq \theta$ of low-skill workers (dalits or q-types). We then apply the relevant policy to raise the income of this segment of workers to equal that of the high-skilled workers. The cost of the policy is then met by uniformly withdrawing income from all the workers (elits and newly-elevated dalits) that now have higher incomes.

Tax-transfer consists of supplementing the incomes of the ϕ dalits by the amount $y_m - y_t$, so that their incomes now equal that of the elits. The subsidy is funded by taxing all high-income workers, including the now elevated dalits.

With reservations, ϕ low-skill workers are equally distributed across all modern sector production teams that formerly had only high-skill workers, and the size of the production teams are adjusted to restore optimality. Resulting modern sector production is now represented by the model in Section 2.3.1, with $\gamma = \frac{\phi}{1-\theta+\phi}$. All modern sector workers now receive a lower income than when there were only high-skilled workers in that sector.

With training, the ϕ low-skill workers are given training to raise their skill level from q to p, and they are then placed in the modern sector alongside the previously high-skill workers. The cost of training is covered by taxing all the workers that are in the modern sector after the policy is implemented.

3.1.1 Comparison of mean income and inequality

It follows that, after execution of a policy A, we have $(\theta - \phi)$ of the dalits with unchanged income y_t, while $(1 - \theta + \phi)$ agents have a higher income y_m^A. We expect that for reasonable parameters $y_m^A \in [y_t, y_m]$. In particular this is always true under reservation.[11]

[11] y_m^A may fall below y_t under tax-transfer and training for extreme values of some parameters. For example, if state capacity is severely limited and λ approaches zero, then neither tax-transfer nor training (funded by taxes) will be effective in delivering higher incomes to the chosen fraction of dalits. Alternatively, if training costs are

Now consider two policies A and B, and suppose that $y_m^A > y_m^B$. Recall that low-income earners earn y_t in both cases. It is then clear that per-capita income is higher under A than B, but the comparison of inequality is ambiguous.

However, in the distribution corresponding to A, now redistribute some income δ from the agents receiving y_m^A to those receiving y_t, where δ is calculated to satisfy:

$$\frac{y_m^A - \delta}{y_t + \delta\frac{(1-\theta+\phi)}{(\theta-\phi)}} = \frac{y_m^B}{y_t}.$$

Then the inequality in the post-transfer distribution following A is equal to that following B, and per-capita income is still higher under A, so A leads to a better outcome than B by any criterion that puts positive weight on both higher per-capita income and lower inequality.[12]

If such notional transfers are admitted, then it is sufficient to compare per-capita incomes to evaluate the relative merits of different policies, given the implementation scheme we have adopted.

In the absence of such transfers, we must resort to using a social welfare function, in this case with per-capita income and inequality as arguments, with, a positive and a negative coefficient respectively. It matters then how we measure inequality, and how much weight we put on each argument. However, following the reasoning above, a post-policy income distribution will continue to deliver greater social welfare as long as the relative weight on per-capita income is not too small.

3.2 Redistribution through tax-transfers

We start with a purely redistributive tax-transfer policy: tax high-income workers and use the proceeds to raise the incomes of some unskilled workers. As explained above, a mass ϕ of unskilled workers receive transfers $y_m - y_t$ each to lift their incomes, such that the incomes of these workers equal the post-tax incomes of skilled (modern sector) workers.

The subsidy is funded by taxing all of $1 - \theta + \phi$ high-income workers. A fraction λ of tax revenues is dissipated in the process, reflecting imperfect state-capacity (i.e., of every dollar taxed, only $1 - \lambda$ dollars is actually available to disburse).[13] Equating taxes (less leakage)

$$(1-\lambda)(1-\theta+\phi)\tau^0 y_m$$

too high, then y_m^A may even fall below y_t. However, reservation will turn out to be a more efficient policy in these cases.

[12] This is not compromised by the existence of leakage in the tax-transfer process (see next section). Even if the leakage proportion λ is unity, the per-capita income post-transfer is no smaller in A.

[13] λ is a proxy for state-capacity (see, e.g., Besley and Persson, 2009).

with transfers

$$\phi(y_m - y_t)$$

and simplifying, post tax-transfer average income can be expressed as

$$\bar{y}^0 \quad = \quad (1-\theta+\phi)(1-\tau^0)y_m + (\theta-\phi)y_t \tag{9}$$

where

$$\tau^0 \quad = \quad \frac{\phi(1-\frac{y_t}{y_m})}{(1-\theta+\phi)(1-\lambda)} = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}})}{(1-\theta+\phi)(1-\lambda)}. \tag{10}$$

To understand the significance of τ^0, suppose that the selected number ϕ of dalits could be elevated to the higher income costlessly. Then the high income after the tax would be y_m (equation 8). Since the process is costly, the new high income instead is a smaller number y_m^0. τ^0 is the fraction by which y_m^0 falls short of y_m. We use this implicit tax as a measure of the cost of implementing the policy. Lemma 1 records some properties of τ_0.[14]

Lemma 1 (Properties of τ^0)**.** *τ_0 is increasing in r, β, and λ. Furthermore, as long as $s_m > s_t$,*

$$\lim_{\beta\to 0}\tau^0 > 0, \qquad \lim_{r\to 1}\tau^0 > 0.$$

In addition, $\lim_{r\to\infty}\tau^0 < 1$ *for all* $\lambda < \frac{\phi}{1-\theta+\phi}$.

Two observations are in order. First, suppose that $s_m > s_t$, i.e. the modern sector is more productive than the traditional sector. Then τ^0 is strictly positive even when complexity plays no role in the production process ($\beta = 0$) or elites have no skill advantage ($r = 1$). In this case elits receive a higher income purely by virtue of locational advantage, and hence transfers must be made to achieve equalisation. Second, tax-transfer remains a viable candidate for optimal policy as long as leakage λ is lower than a threshold value $\frac{\phi}{1-\theta+\phi}$. Otherwise, $(1-\tau^0)y_m < y_t$ which discourages elits from participating in the modern sector, and reduces the incomes of the selected dalits rather than increasing it. High leakage—reflective of low state capacity—might prompt countries to forego tax-transfer and use more direct equity-enhancing policies (e.g., reservation) in early stages of development.

Unless otherwise stated, we will henceforth assume

$$\lambda < \frac{\phi}{1-\theta+\phi} \tag{11}$$

which ensures that tax-transfer is a viable redistribution policy for some relevant ranges of parameters.

[14] In the above formulation we assume that all high income workers (including the low-skilled workers that have been elevated) are taxed based on the notional pre-tax income. Our results are not dependent on this exact specification. An alternative formulation is discussed in Appendix A.2.

3.3 Reservation

A reservation policy ϕ consists of randomly choosing a mass of agents of size $\phi \in (0, \theta]$ from the q-types (traditional sector), and placing them in the modern sector. They are then distributed equally across the modern sector teams, and team sizes (complexity) is readjusted to restore optimality.

Let $\gamma(\phi) = \frac{\phi}{1-\theta+\phi}$ denote the proportion of q-types in a modern sector team. When there is no reservation, $\phi = 0$ and $\gamma(0) = 0$, while $\phi = \gamma(\theta) = \theta$ when reservation encompasses all the $q-$types in the population.

Substituting $\gamma = \frac{\phi}{1-\theta+\phi}$ in (5) and (6) respectively yields employment and average output in the modern sector (with superscript R indicating reservation):

$$
\begin{aligned}
n_m^R &= \frac{\mu}{k}\left(\frac{(1-\theta+\phi)r}{(1-\theta)+\phi r}\right) &&= n_m\left(\frac{(1-\theta+\phi)}{(1-\theta)+\phi r}\right), && (12)\\
y_m^R &= s_m^{1-\beta}(\frac{\mu}{ek})^{\beta\mu}\left(\frac{(1-\theta+\phi)r}{(1-\theta)+\phi r}\right)^{\beta\mu} &&= y_m\left(\frac{(1-\theta+\phi)}{(1-\theta)+\phi r}\right)^{\beta\mu}. && (13)
\end{aligned}
$$

There is of course no change in income in the traditional sector (which now has a smaller population than in full separation)

$$n_t^R = \frac{\mu}{k}, \qquad y_t^R = {s_t}^{1-\beta}(\frac{\mu}{ek})^{\beta\mu}. \tag{14}$$

From (7), (8), (13) it follows that when $\beta > 0$,[15]

$$y_m > y_m^R > y_t.$$

Compared to full separation, per capita income in the modern sector is lower under reservation. Nevertheless, the modern sector continues to employ more individuals and yield higher per capita income than the traditional sector. Each newly elevated dalit enjoys an increase in average income by $y_m^R - y_t$, while each elit loses $y_m - y_m^R$. Income of dalits remaining in the traditional sector remains unchanged. Mean income in the economy changes from

$$\bar{y} \; = \; (1-\theta)y_m + \phi y_t + (\theta - \phi)y_t$$

to

$$\bar{y}^R \; = \; (1-\theta)y_m^R + \phi y_m^R + (\theta - \phi)y_t$$

under reservation.

[15]When $\beta = 0$ but $s_m > s_t$ we get $y_m = y_m^R$.

Express the difference in mean income as

$$\bar{y}^R - \bar{y} = \phi(y_m - y_t) - (1 - \theta + \phi)(y_m - y_m^R).$$

Observe that $y_m - y_t$ is strictly positive if $s_m > s_t$ or $\beta > 0$, however, $y_m - y_m^R \to 0$ as $r \to 1$ or $\beta \to 0$. As a result, when the skill-advantage of elits is small and the complexity of the production process is low, mean income increases with reservation and there is no tradeoff between equity and efficiency.

Proposition 1 (Development with unlimited supplies of labor)**.** *Compared to the full separation benchmark, mean income in the economy is higher under reservation when complexity is low and/or skill differences between elits and dalits are small. More formally, suppose $s_m > s_t$. Then,*

for all $r > 1$, there exists $\underline{\beta}(r) > 0$ such that $\bar{y}^R > \bar{y}$ for all $\beta < \underline{\beta}(r)$;

for all $\beta > 0$, there exists $\underline{r}(\beta) > 1$ such that $\bar{y}^R > \bar{y}$ if and only if $r < \underline{r}(\beta)$.

It may sound surprising that mean income can increase with reservation despite a decline in per capita income in the modern sector. However, note that the driving force behind the difference in income is the difference in sectoral productivities for low values of β and r (where $\bar{y}^R > \bar{y}$). Elevating ϕ dalits to productive modern sector and raising their income by $y_m^R - y_t$ outweighs the loss in income $y_m - y_m^R$ suffered by elits.

How does reservation compare with tax-transfer? To answer that we express the post-reservation mean income as:

$$\begin{aligned}\bar{y}^R &= (1 - \theta + \phi) y_m^R + (\theta - \phi) y_t \\ &= (1 - \theta + \phi)(1 - \tau^R) y_m + (\theta - \phi) y_t \end{aligned} \tag{15}$$

where

$$\tau^R = 1 - \frac{y_m^R}{y_m} = 1 - \left(\frac{1 - \theta + \phi}{(1 - \theta) + \phi r} \right)^{\beta\mu} \tag{16}$$

denotes the implicit tax on modern-sector income associated with reservation. Lemma 2 records some properties of τ^R.

Lemma 2 (Properties of τ^R)**.** *τ^R is increasing in r and β. Furthermore,*

$$\lim_{r \to 1} \tau^R = 0, \quad \lim_{r \to \infty} \tau^R = 1, \quad \lim_{\beta \to 0} \tau^R = 0.$$

Since $\lim_{\beta \to 0} \tau^0 > 0$ (by Lemma 1) and $\lim_{r \to 1} \tau^0 > 0$, Lemma 2 implies

$$\lim_{r \to 1} (\tau^R - \tau^0) < 0, \quad \text{and} \quad \lim_{\beta \to 0} (\tau^R - \tau^0) < 0.$$

From (9) and (15) it follows that the lower the tax rate the higher the mean income:

$$\bar{y}^R \gtreqless \bar{y}^0 \iff \tau^R \lesseqgtr \tau^0.$$

Thus reservation is the less costly pursuit when r and β are low. Proposition 2 gives a more complete comparison.

Proposition 2 (Reservation versus Tax-Transfer)**.** *When complexity is low and/or skill differences between elits and dalits are small, mean income in the economy is higher under reservation than under tax-transfer. More formally, suppose $s_m > s_t$ and (11) holds. Then,*

(i) for all $r > 1$, there exists $\beta^R(r, \lambda) > 0$ such that $\bar{y}^R > \bar{y}^0$ for all $\beta < \beta^R(r, \lambda)$;

(ii) for all $\beta > 0$, there exists $r^R(\beta, \lambda) > 1$ such that $\bar{y}^R > \bar{y}^0$ if and only if $r < r^R(\beta, \lambda)$.

Both thresholds $\beta^R(r, \lambda)$ and $r^R(\beta, \lambda)$ are increasing in λ.

To see Proposition 2 consider once again small values of r and β. From Proposition 1 we know that mean income strictly increases under reservation for all such values as ϕ dalits move from traditional to relatively more productive modern sector. In contrast, mean income never increases under tax-transfer. By design, tax-transfer redistributes income by taxing elits and transferring it to a fraction ϕ of dalits. If $\lambda = 0$, mean income does not change. If $\lambda > 0$, part of the tax-receipts is dissipated and mean income strictly declines. Thus, for low levels of skill advantage and complexity, reservation is strictly preferred to tax-transfer.

This preference is reversed for high values of r. Lemma 1 and Lemma 2 imply that

$$\lim_{r \to \infty} (\tau^R - \tau^0) > 0$$

which in turn implies that $\bar{y}^R < \bar{y}^0$ when skill-difference between the elits and dalits is larger than a threshold value $r^R(\beta, \lambda)$. In the appendix we show that this threshold is unique. Naturally, the threshold is increasing in λ as an increase in leakage does not affect reservation but makes tax-transfer less appealing by requiring higher taxes to fund same amount of transfer.

3.4 Training

Next we consider training a fraction ϕ of the q-types to raise their productivity to p. Training is costly. Training cost per person is $C(p, q)$ which satisfies some standard properties (i) $C(q, q) = C(p, p) = 0$, (ii) $C_q(.) < 0, C_p(.) > 0$, and $C_{pp}(.) > 0$, and (iii) $\lim_{p \to 1} C(p, q) = \infty$. Total cost of training, $\phi C(p, q)$, is funded by a proportional tax τ^T on the *ex post* high-income individuals (see footnote 14). Assuming that a fraction λ of the tax receipts are dissipated, the relevant tax rate is:

$$\tau^T = \frac{\phi C(p, q)}{(1 - \theta + \phi)(1 - \lambda) y_m}.$$

We will work with a specific cost function which satisfies properties (i) - (iii) mentioned above:

$$C(p,q) = \alpha\bigg(\frac{p}{1-p} - \frac{q}{1-q}\bigg). \tag{17}$$

Accounting for training costs, mean income under training can be expressed as

$$\bar{y}^T = (1-\theta+\phi)(1-\tau^T)y_m + (\theta-\phi)y_t \tag{18}$$

where τ^T, given by (19) below, follows from substituting $q = \frac{1}{e^k}$ and $p = \frac{1}{e^{\frac{k}{r}}}$ in (17):

$$\tau^T = \frac{\phi\alpha\bigg(\frac{1}{e^{\frac{k}{r}}-1} - \frac{1}{e^k-1}\bigg)}{(1-\theta+\phi)(1-\lambda)y_m} = \frac{\phi\alpha\bigg(\frac{1}{e^{\frac{k}{r}}-1} - \frac{1}{e^k-1}\bigg)}{(1-\theta+\phi)(1-\lambda){s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}}. \tag{19}$$

Properties of τ^T are noted below.

Lemma 3 (Properties of τ^T)**.** *τ^T is strictly increasing (decreasing) in β for all $r > (<)\frac{eks_m^{\frac{1}{\mu}}}{\mu}$. Furthermore, (i) $\lim_{\beta\to 0}\tau^T > 0$, (ii) $\lim_{r\to 1}\tau^T = 0$, and (iii) $\lim_{r\to\infty}\tau^T = \infty(0)$ if and only if $\beta\mu < (>)1$. When $\beta\mu = 1$, $\lim_{r\to\infty}\tau^T$ is positive and finite.*

Several properties of τ^T are worth noting. First, unlike τ^0 and τ^R, τ^T can be decreasing in β. An increase in complexity does not affect the cost of training but it increases y_m, the pre-tax average income in the modern sector. This lowers the tax burden for high-income individuals which results in lower τ^T provided r is not too low. Second, τ^T is not necessarily increasing in r either. As r increases, both training costs and modern sector income increase. Whether per capita tax increases or not depends on the rate of increase of the two. When complexity of the production process or degree of increasing returns is high, y_m increases at a faster rate than does training costs, leading to a lower tax burden per high-income individual. The opposite is true when $\beta\mu$ is strictly lower than unity.

How does training compare with tax-transfer? Applying Lemmas 1 and 3 we get

$$\lim_{r\to 1}(\tau^T - \tau^0) < 0$$

which implies training yields higher mean income than tax-transfer when skill-difference is small. For large skill-differences, the comparison between the two policies depends on complexity and the degree of increasing returns, or more specifically on the value of $\beta\mu$, since

$$\beta\mu < (>)1 \iff \lim_{r\to\infty}(\tau^T - \tau^0) > (<)0.$$

Observe that when $\beta\mu > 1$, τ^T is strictly lower than τ^0 both for sufficiently low and sufficiently high values of r (but may be higher for intermediate values). Consequently, mean income is higher under training both when skill-differences are small and when skill-differences are large. Indeed, depending on parameters in the training cost function, training may dominate tax-transfer for all values of r. If tax-transfer indeed yields higher income for some intermediate degree of skill difference the preference between the two policies will be non-monotone, with training preferred for large and small values of r and tax-transfer for intermediate values. Proposition 3 collects these observations and provides a comparison between the two policies on the basis of skill-differences and complexity of the production process.

Proposition 3 (Training versus Tax-Transfer)**.** *For all $\beta < \frac{1}{\mu}$, there exists $\underline{r}^T(\beta)$ such that training yields higher mean income than tax-transfer if and only if $r < \underline{r}^T(\beta)$. For $\beta > \frac{1}{\mu}$, either $\underline{r}^T(\beta)$ is infinitely large so that training is always the preferred policy between the two, or there exists $\bar{r}^T(\beta) > \underline{r}^T(\beta)$ such that mean income under training is higher for $r < \underline{r}^T(\beta)$ and $r > \bar{r}^T(\beta)$.*[16]

3.5 Comparison

Equipped with the findings from binary comparisons of policies, now we turn to address the key question of the paper: which one among the three—reservations, training, or tax transfer—is the least costly pursuit of equity? The answer, as reflected in the propositions, depends on the skill-difference (r) and the complexity of the production process (β).

We make some simplifying assumptions to avoid the discussion becoming taxonomic. We continue to assume that (11) holds, so that tax-transfers and training (which requires taxes) are viable alternatives to reservation.[17] Hereafter, we also assume that $\mu < 1$. Recall $\mu(> 0)$ captures the degree of increasing returns. For large values of μ, training tax rate $\tau^T \to 0$ even when $r \to \infty$. An implication is that infinitely costly training can be met with near-zero taxes. Restricting $\mu < 1$ rules out this unrealistic scenario.[18]

We start with optimality of tax-transfer partly because both binary comparisons in Propositions 2 and 3 involve tax-transfers. Proposition 2 says that tax-transfer fares better than reservation if and only if the skill-difference exceeds a threshold level $r^R(\beta, \lambda)$. In a similar vein, Proposition 3 implies that, for $\mu < 1$, tax-transfer fares better than training if and only if the skill-difference exceeds a threshold level $\underline{r}^T(\beta)$. Proposition 4 establishes that there is a non-empty region where tax-transfer is optimal.

Proposition 4 (Optimality of Tax-Transfer)**.** *Let (11) be satisfied and let $\mu < 1$. Then*
(i) Tax-transfer is the most (least) preferred policy when skill differences are large (small). For all $\beta > 0$,

[16]We have not established sufficient regularity of the functions τ^T and τ^0, hence it is possible that the two tax-rates change position in the interval $(\underline{r}^T(\beta), \bar{r}^T(\beta))$. Monotonicity can be established by placing suitable restrictions on the underlying functions, but this does not contribute either to the model or to our understanding of the relevant phenomena.

[17]if λ is too large in Proposition 5 below, then reservation wins but without any competition from other policies.

[18]This simplification comes at the cost that it weakens the case for training. However, as we see later in Proposition 6, training continues to be the optimal policy for a range of parameter values.

there exists $r_1(\beta, \lambda)$ and $r_2(\beta, \lambda)$ satisfying

$$1 < r_1(\beta, \lambda) \leq r_2(\beta, \lambda) < \infty$$

such that average income is

- *highest under tax-transfer if and only if $r > r_2(\beta, \lambda)$,*
- *lowest under tax-transfer if and only if $r < r_1(\beta, \lambda)$.*

(ii) For all $r > 1$, there exists $\beta(r) > 0$ such that tax-transfer is never optimal for $\beta < \beta(r)$.

Part (i) of Proposition 4 follows immediately once we define

$$r_1(\beta, \lambda) = \min\{r^R(\beta, \lambda), \underline{r}^T(\beta)\}, \quad \text{and} \quad r_2(\beta, \lambda) = \max\{r^R(\beta, \lambda), \underline{r}^T(\beta)\} \tag{20}$$

To understand the second part, recall that $\lim_{\beta \to 0} \tau^0 > 0$ (Lemma 2), i.e., no matter how small β is, equalizing income of ϕ dalits in the traditional sector with the elits in more productive modern sector requires strictly positive transfers and consequently strictly positive taxes. In contrast, by Lemma 1, $\lim_{\beta \to 0} \tau^R = 0$. When β is small, reservation is almost costless as there is little loss in per capita income in the modern sector.

On the other hand, at sufficiently low β, reservation is preferred not only to tax-transfer, but also to training since $\lim_{\beta \to 0} \tau^T > 0$ (Lemma 3). Proposition 5 records conditions under which reservation is optimal.

Proposition 5 (Optimality of Reservation). *(i) Reservation is the most preferred policy if the degree of complexity is sufficiently small. For any $r > 1$, there exists $\beta_0(r, \lambda)$ such that mean income is highest under reservation if $\beta < \beta_0(r, \lambda)$.*

(ii) For low complexity, mean income is highest under reservation when skill difference is small. In other words, there is a $\hat{\beta} > 0$ such that for all $\beta \leq \hat{\beta}$, there exists $r_3(\beta) > 1$ such that mean income is highest under reservation for all $r < r_3(\beta)$.

The first part captures the essence of the discussion preceding the proposition: reservation is optimal when complexity is sufficiently low, regardless of the skill-difference. The second part says that if the production process is not too complex, reservation continues to be optimal when skill-differences are sufficiently low. To see why, first note that, by Proposition 4, tax-transfer is the least preferred option for small values of r. However, the preference between reservation and training is not clear-cut, as Lemmas 2 and 3 respectively imply that both $\lim_{r \to 1} \tau^R = 0$ and $\lim_{r \to 1} \tau^T = 0$. Whether reservation or training is better for low values of r

then depends on the rate of increase of τ^R and τ^T at $r = 1$. We have

$$\frac{d\tau^R}{dr}|_{r=1} = \frac{\phi\beta\mu}{1-\theta+\mu} \quad (21)$$

$$\frac{d\tau^T}{dr}|_{r=1} = \frac{\phi\alpha k^{\beta\mu+1}}{(1-\theta+\mu)(1-\lambda)(\frac{\mu}{e})^{\beta\mu}(e^k-1)^2} \quad (22)$$

Observe that, as $\beta \to 0$, $\frac{d\tau^R}{dr}|_{r=1}$ is smaller than $\frac{d\tau^T}{dr}|_{r=1}$. This establishes the second part of Proposition 5.

Given Propositions 4 and 5, is there a case for training? To see this, define $\hat{\beta}$ to be the smallest value of β that equates the right-hand-sides of (21) and (22). It can be checked that $\hat{\beta}$ exists and is less than unity if

$$\alpha < (\frac{\mu}{k})^{\mu+1}(e^k-1)^2 e^{\mu}(1-\lambda). \quad (23)$$

Further, for small values of r, $\beta > \hat{\beta} \iff \tau^T < \tau^R$.

From the preceding arguments it is immediate that training is preferred to reservation for low values of r when $\beta > \hat{\beta}$. Thus, a small skill-advantage of elits does not necessarily make a case for reservation. When complexity is high, these differences amplify and result in a larger loss of output under reservation. Despite similar profiles of skill differences, two economies might opt for different equity enhancing policies if they are at different stages of development. Further, from Proposition 3, training is preferred to tax-transfer when $\beta < \frac{1}{\mu}$ and $r < \bar{r}^T$, and it is always preferred to tax-transfer if $\beta > \frac{1}{\mu}$. More generally,

Proposition 6 (Optimality of training). *For all β and $\lambda < \frac{\phi}{1-\theta+\phi}$ there is α small enough such that training is optimal for a non-empty range of values of r.*

In other words, if training is not too costly, then there is a region in the viable parameter space such that training is optimal.

4 Discussion

The general conclusions that proceed from the preceding analysis can be summarised as follows. In economies with low complexity, reservation is the most effective policy. Under reasonable parametric specifications, as complexity and skill differences increase, reservation is superseded by training, and policy must turn its attention to providing better facilities for disadvantaged groups to acquire skills, rather than place them into more productive jobs by fiat. Finally, in highly complex economies, the least costly way to contain inequality is by tax and transfer. The threshold where one policy overtakes another is defined by complexity β and the skill-difference index r, as well as the leakage λ in the fiscal system.

These conclusions are in accordance with some observations from the history of development in the underdeveloped economies, as well as policy choices in the developed ones.[19] In the early stages of development in the developing economies of the mid-twentieth century, the default path to growth was modern-sector expansion fuelled primarily by a migration of "unlimited supplies" of workers from the traditional subsistence sector to the modern sector. The latter sector only required rudimentary skills that were easy to acquire, but generated greater income by virtue of positional and resource advantage (Lewis 1954, see also Gollin 2014). There was discussion around whether growth led to modern sector *enlargement* that would reduce inequality over time, as opposed to modern sector *enrichment* which would enhance it (Fields, 1979, 1987), but the process was not presumed to be hobbled by lack of appropriate skills. For thinkers on the active policy front it was clear that if the gap between earnings of different groups was to be bridged, it was first by using reservations and quotas to breach the hold of the elites on the more remunerative occupations. For example, employment and educational quotas for dalits were mandated in the Constitution of India, but predate it by several decades (Deshpande, 2005).

As modern sector growth continued, and some countries moved towards producing more and more complex products using increasingly sophisticated production techniques, reservations become increasingly inefficient, quotas were decried, and the focus turned to the development of skills. Correspondingly, the theory of endogenous growth emerged in the 1980s and emphasis shifted to human capital (see Barro, 1990; Ray, 1998, chapter 4). Many of these threads were brought together conceptually in Kremer (1993). In the meantime, countries with extremely well-functioning state mechanisms such as the Scandinavian countries moved on a different track, actively using tax-transfer mechanisms and extensive public goods provision to implement *ex post* egalitarianism, without tampering with the allocation of labour.

This paper uses a stylised model to underline some fundamental observations about the effectiveness of equity policies at different stages of development. This forces us to ignore a number of considerations that are nevertheless important. It also leaves open the possibility of several extensions that could make the analysis more realistic. Below we discuss the rationale for some of our simplifications and explore the contours of a more complete model. We have intentionally not pursued that model here because it obscures some important relationships to which we want to draw attention.

4.1 Sectoral allocation of labour, and crowding out

We have left the determinants of the sectoral allocation of labour intentionally vague. In our model, low-skill workers always work in the traditional sector when they work on their own, even though a shift to the modern sector would increase their productivity. The development literature provides many indications why such an assumption, even if it is quick and dirty,

[19]Policy choices are influenced by many factors other than efficiency, such as perceptions of social and economic mobility (Alesina, Stantcheva, and Teso, 2018), which we have not considered in this model.

might be realistic. The modern sector is a placeholder for an assortment of complementary factors that may be subject to crowding, in which case the gain in output from transferring a low-skill worker to the modern sector may be outweighed by the negative externalities on other workers.[20] Alternatively, the agglomeration of resources that constitute the modern sector may go hand-in-hand with a geographical location that is provided with public facilities necessary to produce skilled workers. The traditional sector in this case may be the rural sector that lacks the resources that complement modern production, and these resources may also double as the ones that are needed to mould youths into skilled workers. In this paper we have not addressed the complex of reasons that have led several generations of development economists to find it reasonable to equate the rural, traditional and agricultural sector, and distinguish it from the urban, modern and industrial one.

An alternative construction is to think of modern sector places as scarce, with elits having preferential access. In the extreme case, only elits (but presumably not all of them) have access to that sector, and more enter as new places are created. Affirmative action would then reserve some of the newly created places for dalits. This is the formulation that perhaps best matches reservations in professional education programs like medical schools.

Much of the discontent with reservations in less-developed countries like India stem from the fact that places in the more remunerative occupations are scarce, and some out of a fixed number of places—for example in medical schools—are set aside for scheduled caste/tribe candidates. These places become unavailable for elite candidates (who may have scored higher in the medical admission exams), who thereby lose the opportunity that the dalits gain. The problem is further compounded if the dalit candidates are so lacking in prior preparation that they are unable to complete the program (Deshpande and Newman, 2007).

If some elits are left in the traditional sector along with the dalits, or if there is competition between the two groups for the desirable places, then in any outcome we must have some dalits in each sector, and some elits in each sector. Further, these agents may function in mixed teams (ps and qs) or in segregated teams, resulting in more than two income groups. As soon as the number of groups exceeds two we must face choices about how to measure inequality, which makes some of the conclusions less transparent. We will return to this in Section 4.4.

4.2 Training costs and the initial allocation of skills

In our model, dalits and elites come to the market with different endowments of skills. The paper is motivated by the idea that these differences arise from differences in privilege, family income and community resources, as well as prior discrimination. However, we do not explicitly model this, in part because (as this section discusses) this question needs an extensive analysis of its own.

[20] Admittedly, our model does not accommodate this interaction, but uses an assumption to substitute for it.

An economic model of skill acquisition must be motivated by an appropriate definition of "skill", and the costs and benefits of its acquisition. Here we will content ourselves with a notional scale of skill that goes from "unskilled" to some level of "highly skilled". We conceive of the skill-acquisition process as a sequence of stages, which for expositional purposes may be set to two. In the first stage, children within the family and community acquire a base level of ability to learn and develop skills, and in the second stage they enter formal education or training to acquire productive skills. Those who have higher ability will find it less costly at the margin to acquire a little more training, and as a result will acquire greater skill.

Individuals in the two groups: privileged and underprivileged, have different resources in the early stages that they may receive from family, community or the state. Individual and family resources include finance and levels of parental education. The level of general education in the neighbourhood, local patterns or norms of school attendance, and community expectations are some of the factors that comprise community resources. Finally public goods such as school facilities and the provision of teachers, transport, health and sanitation play an important role.

The elites and their communities are much better endowed with these resources than are dalits, and as a result the marginal cost of acquiring training at the second stage is smaller for the elite children. It then follows that, at the individual optimum, elites will acquire a higher level of training, and hence skill, than dalits.[21]

In an optimization/equilibrium model, the level of skill an individual acquires is determined by her marginal cost of skill-acquisition and the composition of the team in which she expects to work. Thus in the full-separation model, each individual takes the skill-level of her own group h as given, and acquires skill than maximises her own payoff given that her colleagues will acquire skill h. The equilibrium h^* is then the level of skill that maps into itself, given the cost-function. The dalits choose a lower skill-level than the elits because they expect to work with other dalits with a lower skill level, and because their marginal cost of skill-acquisition is higher.

Once an equity policy is introduced, it increases the expected payoff of dalits at each skill-level, and reduces that of the elits. Thus in the presence of a policy, dalits will acquire a higher skill than otherwise, while elits will acquire a lower skill. Thus even dalits that are not chosen for promotion will train more and earn a somewhat higher income, while elits will train less. The net effect of these externalities on GDP per capita is ambiguous, but an equilibrium model must take them into effect.

[21] We need to tread carefully here because, with optimal allocation of workers to processes in the O-ring model, the marginal benefit of additional skill is not a downward sloping curve. However, if there is a determinate optimum to the individual problem then the conclusion remains correct.

4.3 Combining policies or using partial policies

The analysis above considers individual policies in isolation, whereas most real economies will normally use a combination of policies. For example, when the difference in skill levels between groups is very high, using only training or only reservations may each turn out to be very expensive, while pure-tax transfer may require large taxes that evoke resentment. A reasonable government policy may well incorporate some amount of training until the marginal cost increase becomes very steep, and then institute reservations having used the training to reduce the degree of output loss this induces.

Alternatively, training may lift some of the dalits to an intermediate level of skill between q and p, and put them to work in a production process with intermediate complexity. This would reflect the development experiences of many countries that have recently approached the status of developed countries or are well on the way to development, and have created large and thriving middle classes in the process. It is also the path to development suggested in Kremer's original paper, only given a helping hand by policy.

4.4 Wage determination

The most obvious simplification in our model is the assumption that, in each team, all workers receive a wage equal to the average product of the team, even when the team is heterogeneously comprised of high- and low-skilled workers. Without speculating what a specific wage model should look like, it is clear that most reasonable market-founded or coalitional game-theory-founded wage functions would allocate higher wages to workers with higher skills.

It is not difficult to accommodate a wage function that has this property in the model. However, in this case, reservations would result in at least three income groups: dalits that remain in the low-output teams, dalits that move to high output teams and elits.

The first group continues to earn a low income y_t, the second group earns a higher income y_m^{low} and the third group earns the highest income y_m^{high}, which is still lower than the full-separation high-income y_m. It is easy to see that the income distribution in the mixed teams now is a mean-preserving spread of the (constant) distribution that we obtained in Section 3.1, and hence there is more inequality than when incomes were equal within teams, though still less inequality than under full-separation.

However, to properly state the results related to inequality in this version, we would need to adopt a specific inequality function to compare inequality between policies, and in general different acceptable functions will yield different results. In the tightly schematic specification of policies that we have adopted in the paper (a fixed proportion ϕ of low-income agents raised to the high-income group), there are only two income groups at any time and comparing efficiency across policies is straightforward. We feel it is worthwhile to emphasise the clear results regarding comparison using this simpler formulation, rather than state results that are contingent on specific properties of measurement functions, and obscure the direct tradeoff between inequality and efficiency.

4.5 Discrimination

A final and obvious omission in our model is that there is no role ascribed to inter-group animosity or discrimination. Dalits are not barred from good jobs because the elits refuse to work alongside them, nor are they denied because of their colour or other personal characteristics. A dalit who becomes suitably skilled faces no barriers in our model. This omission is intentional. We feel this is a separate (though closely related) problem which, while possibly more pressing than lack of resources, is best treated in a different conceptual framework There is a large literature on this subject (see Lang and Lehmann, 2012, for a recent survey), and we are not sure the present paper adds much to that question.

5 Conclusion

This paper analyses very stylised versions of equity-enhancing policies in a schematic model. We find that reservation fares better than both training and tax transfer in a low complexity Lewis-Ranis-Fei (LRF) type economy, and leads to an increase in per-capita income in low-skill economies. As complexity of the production process increases, reservation becomes more costly. In fact, for a high enough skill difference and complexity, reservation is the least appealing policy option. At higher levels of complexity training becomes more attractive, and is superseded by tax-transfers in the most complex economies.

We have assumed that initial inequalities (in ability or skills) result from prior inequalities in income and wealth. Individuals in each class have access to specific developmental resources during their formative period, and are conditioned by a specific cultural and educational social environment. These coupled with the array of opportunities available in the economy determine the equilibrium level of skills with which they enter the economy. We take these entry levels as given, and do not concern ourselves with the effects of policies on the baseline skills, because our interest lies elsewhere.

In order to keep the analysis tractable and the observations sharp, we assume a two-class economy and consider very specifically defined policies. However, it should be easy to see how more nuanced policies can be incorporated into this framework. In the discussion section and in other appropriate places, we have tried our best to point out the simplifications that we have made and possible ways in which the analysis can be generalised.

Finally note that this paper confines itself entirely to static analysis. The implicit assumption here is that reducing the initial inequalities to some extent in one generation results in smaller initial discrepancies in the next one. However, the existing literature alerts us to the fact that dynamic trajectories can be far from straightforward (see, e.g., Mookherjee and Ray, 2003). These questions remain to be explored.

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Appendix

A Variations

A.1 Alternative formulations of training

Suppose a fraction α of training costs are paid by p types. Then

$$(1-\theta)\tau(1-\lambda)y_m = \phi\alpha C(p,q) \tag{24}$$

Post-training income should be the same for p-types and the trained q-types, i.e.

$$\begin{aligned}(1-\tau)y_m &= y_m - (1-\alpha)C(p,q)\\ &= y_m - C(p,q) + \frac{(1-\theta)\tau(1-\lambda)y_m}{\phi}\end{aligned}$$

where the second equality follows from applying (24). Dividing both sides by y_m and simplifying we get:

$$-\tau = -\frac{C(p,q)}{y_m} + \frac{(1-\theta)\tau(1-\lambda)}{\phi}.$$

Rearranging further we get:

$$\tau = \frac{\phi C(p,q)}{[(1-\theta)(1-\lambda)+\phi]y_m}.$$

The two tax rates related to training - one above and one in the main text - differ only because the incidence of leakage is different in the two cases; they are the same when $\lambda = 0$. There is no qualitative difference between the two specifications.

A.2 Alternative tax-transfer formulations

In Section 3.2 we assumed that all high income workers (including the low-skilled workers that have been elevated) are taxed based on the notional pre-tax income. An alternative formulation of tax transfer is to assume that only p-types are taxed. Then

- Taxes = $(1-\theta)\tau y_m$
- Leakage = $(1-\theta)\tau\lambda y_m$
- Transfers = $\phi((1-\tau)y_m - y_t)$

where the last equality follows from noting that post tax-transfer income, i.e.,$(1-\tau)y_m$ has to be the same for both p-types and the fraction ϕ of q-types. As taxes (less leakage) must equal transfers, we have

$$(1-\theta)\tau^0(1-\lambda)y_m = \phi((1-\tau^0)y_m - y_t).$$

Rearranging this gives:

$$\tau^0 = \frac{\phi(1-\frac{y_t}{y_m})}{(1-\theta)(1-\lambda)+\phi} = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}})}{(1-\theta)(1-\lambda)+\phi}.$$

The two tax rates - one above and one in the main text - differ only because the incidence of leakage is different in the two cases, they are the same when $\lambda = 0$. There is no qualitative difference between the two specifications.

A.3 Capital

In the formulation in the paper we have ignored capital, since it is peripheral to our concerns. However, capital can be accommodated without complications, as shown below.

Suppose output is produced using both labor and capital (z). Using a Cobb-Douglas formulation, let

$$\begin{aligned} Y(p,q,n,\beta,z) &= \left(s^{1-\beta}(G(n,\beta)p^{\beta n(1-\gamma)}q^{\beta n\gamma}\right)^{\alpha}\left(z\right)^{1-\alpha} \\ &= \left(s^{1-\beta}(n^{\frac{1}{\beta}+\mu}p^{n(1-\gamma)}q^{n\gamma})^{\beta}\right)^{\alpha}\left(z\right)^{1-\alpha}. \end{aligned} \tag{25}$$

Suppose the rental rate of capital is ρ. A n-member team chooses z to maximize average income net of rental payments

$$\frac{X^{\alpha}z^{1-\alpha}-\rho z}{n} = \left(\frac{z}{n}\right)\left(\frac{X}{z})^{\alpha}-\rho\right)$$

where

$$X = s^{1-\beta}(G(n,\beta)p^{\beta n(1-\gamma)}q^{\beta n\gamma}$$

First-order condition corresponding to the maximization problem is given by:

$$(1-\alpha)(\frac{X}{z})^{\alpha} = \rho \Longleftrightarrow z = (\frac{1-\alpha}{\rho})^{\frac{1}{\alpha}}X.$$

Using the equation above we can express the maximand as

$$(\frac{\rho\alpha}{(1-\alpha)})(\frac{z}{n}) = \alpha(\frac{\rho}{(1-\alpha)})^{1-\frac{1}{\alpha}}\frac{X}{n}.$$

Substituting $G(n,\beta) = n^{1+\beta\mu}$ and simplifying, the maximization problem boils down to choosing n to maximize

$$X = s^{1-\beta}(n^{\mu}p^{n(1-\gamma)}q^{n\gamma})^{\beta}.$$

which is the same problem as in the main model.

B Proofs

Proof of Lemma 1

Proof. That τ_0 is increasing in r, β and λ is immediate from observing (10). Limiting values of τ_0 follows from noting that

$$\lim_{\beta\to 0}\tau^0 = \frac{\phi(1-\frac{s_t}{s_m})}{(1-\theta+\phi)(1-\lambda)} > 0, \qquad \lim_{r\to 1}\tau^0 = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta})}{(1-\theta+\phi)(1-\lambda)} > 0,$$

and

$$\lim_{r\to\infty}\tau^0 = \frac{\phi}{(1-\theta+\phi)(1-\lambda)} < 1.$$

□

Proof of Proposition 1

Proof. Dividing $\bar{y}^R - \bar{y}$ by y^m we get

$$\begin{aligned}\frac{\bar{y}^R-\bar{y}}{y_m} &= \phi\left(1-\frac{y_t}{y_m}\right) - \left((1-\theta+\phi))(1-\frac{y_m^R}{y_m})\right) \\ &= \phi\left(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}}\right) - \left((1-\theta+\phi))(1-\left(\frac{1-\theta+\phi}{1-\theta+\phi r}\right)^{\beta\mu})\right) \equiv \Delta^R(r,\beta), \text{ say}\end{aligned}$$

Observe that

$$\lim_{\beta\to 0}\Delta^R(r,\beta) \ = \ \phi(1-(\frac{s_t}{s_m})) > 0$$

which implies that for any r, there exists a cutoff value $\underline{\beta}(r)$ such that $\Delta^R(r,\beta) > 0$ for all $\beta < \underline{\beta}(r)$. This proves the first part of Proposition 1. To prove the second part of the Proposition note that:

$$\begin{aligned}\lim_{r\to 1}\Delta^R(r,\ \beta) &= \phi\left(1-(\frac{s_t}{s_m})^{1-\beta}\right) > 0, \\ \lim_{r\to\infty}\Delta^R(r,\ \beta) &= -(1-\theta) < 0.\end{aligned}$$

The claim - in particular the existence of a unique cutoff $\underline{r}(\beta)$ then follows from noting that

$$\frac{d\Delta^R(r,\ \beta)}{dr} = \frac{\phi\beta\mu}{r^{\beta\mu+1}}\left((\frac{s_t}{s_m})^{1-\beta} - (\frac{1-\theta+\phi}{\frac{1-\theta}{r}+\phi})^{\beta\mu+1}\right) < 0$$

The inequality is due to (a) $\frac{s_t}{s_m} < 1$ and (b) $\frac{1-\theta+\phi}{\frac{1-\theta}{r}+\phi} > 1$. □

Proof of Lemma 2

Proof. That τ^R is increasing in r and β is immediate from observing (16). Limiting values of τ^R follows from noting that

$$\lim_{\beta\to 0}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=1, \quad \lim_{r\to 1}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=1, \quad \text{and} \quad \lim_{r\to\infty}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=0.$$

□

Proof of Proposition 2

Proof. Average income is higher under reservation if and only if the implied tax rate is lower under tax-transfer, i.e.

$$\bar{y}^R > \bar{y}^0 \iff \tau^0 > \tau^R.$$

Substituting the expressions for τ^R and τ^0 from (16) and (10) and rearranging we get:

$$\begin{aligned}
\tau^0 > \tau^R \quad &\iff \quad \frac{\phi}{(1-\theta+\phi)(1-\lambda)}\left(1-\frac{y_t}{y_m}\right) > 1-\frac{y_m^R}{y_m} \\
&\iff \quad \frac{\phi}{1-\lambda}\left(1-\frac{y_t}{y_m}\right)-(1-\theta+\phi)(1-\frac{y_m^R}{y_m}) > 0 \\
&\iff \quad \phi\left(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}}\right)-\left((1-\theta+\phi))(1-\left(\frac{1-\theta+\phi}{1-\theta+\phi r}\right)^{\beta\mu})\right) > 0.
\end{aligned}$$

Define

$$\Delta^{R0}(r,\beta,\lambda)=\frac{\phi}{1-\lambda}\left(1-\frac{y_t}{y_m}\right)-(1-\theta+\phi)(1-\frac{y_m^R}{y_m})$$

which is exactly the same as $\Delta^R(r,\beta)$ in the proof of Proposition 1 except for the term $1-\lambda$. Hereafter, the proof is analogous to the proof of Proposition 1 and hence omitted. The last statement of the claim follows from noting that τ^0 is increasing in λ while τ^R is independent of λ. □

Proof of Lemma 3

Proof. That τ^T could increase or decrease with r follows from noting that

$$\frac{d(\frac{\mu r}{eks_m})^{\beta\mu}}{d\beta}=\mu(\frac{\mu r}{eks_m})^{\beta\mu}ln(\frac{\mu r}{eks_m}) \gtreqless 0 \iff r \gtreqless \frac{eks_m}{\mu}.$$

where $x=\frac{k}{r}$. Parts (i) and (ii) follow from the expression of τ^T in (19) once we note that both numerator and denominator in the right-hand side of (19) remain positive as $\beta\to 0$, while the numerator approaches zero as $r\to 1$. Finally, to understand how τ^T behaves as $r\to\infty$ note that

$$\lim_{r\to\infty} \tau^T = \frac{\phi}{(1-\theta+\phi)(1-\lambda)s_m^{1-\beta}(\frac{\mu}{e})^{\beta}} \left(\lim_{x\to 0} \frac{x^{\beta\mu}}{e^x - 1} \right)$$

where $x = \frac{k}{r}$. Observe that both $x^{\beta\mu} \to 0$ and $e^x - 1 \to 0$ as $x \to 0$. Applying L'Hôpital's rule we get

$$\lim_{x\to 0} \frac{x^{\beta\mu}}{e^x - 1} = \lim_{x\to 0} \frac{\beta\mu x^{\beta\mu - 1}}{1}.$$

The claim in the Lemma then follows from noting that

$$\lim_{x\to 0} \frac{\beta\mu x^{\beta\mu-1}}{1} = \infty(\beta\mu, 0) \Longleftrightarrow \beta\mu > (=, <)1.$$

□

Proof of Proposition 3

Proof. Suppose $\beta < \frac{1}{\mu}$. Using the limit values of τ^0 and τ^T from Lemmas 1 and 3 we get:

$$\begin{aligned} \lim_{r\to 1}(\tau^0 - \tau^T) &= \frac{\phi(1 - (\frac{s_t}{s_m})^{1-\beta})}{(1-\theta+\phi)(1-\lambda)} > 0, \\ \lim_{r\to\infty}(\tau^0 - \tau^T) &= -\infty < 0. \end{aligned}$$

Continuity of $\tau^0 - \tau^T$ in r implies that there exists $r = \underline{r}^T(\beta)$ for which $\bar{y}^T - \bar{y}^0 = 0$. Below we show that $\underline{r}^T(\beta)$ is unique. Cancelling the common term $\frac{\phi}{(1-\theta+\phi)(1-\lambda)}$ from τ^0 and τ^T we get

$$\tau^0 - \tau^T \gtreqless 0 \Longleftrightarrow \Delta^T(r,\beta) \gtreqless 0.$$

where

$$\Delta^T(r,\beta) = s_m^{1-\beta}(\frac{\mu}{ek})^{\beta\mu}\left(r^{\beta\mu} - (\frac{s_t}{s_m})^{1-\beta}\right) - \left(\frac{\alpha}{e^{\frac{k}{r}} - 1} - \frac{\alpha}{e^k - 1}\right)$$

From the two limit values—$\lim_{r\to 1}(\tau^T - \tau^0)$ and $\lim_{r\to\infty}(\tau^T - \tau^0)$—it follows that:

$$\lim_{r\to 1} \Delta^T(r,\beta) > 0 > \lim_{r\to\infty} \Delta^T(r,\beta).$$

Fix $\beta > 0$. Suppose there exists r_1 and r_2, satisfying $1 < r_1 < r_2 < \infty$ such that $\Delta^T(r_1,\beta) = \Delta^T(r_1,\beta) = 0$. Then, there must exist at least two critical points, i.e. two values of r where

$$\frac{\partial\Delta^T(r,\beta)}{\partial r} = s_m^{1-\beta}(\frac{\mu}{ek})^{\beta\mu}\beta\mu r^{\beta\mu-1} - \frac{\alpha k e^{\frac{k}{r}}}{r^2(e^{\frac{k}{r}} - 1)^2} = 0,$$

i.e.,

$$\beta\mu s_m^{1-\beta}(\frac{\mu}{e})^{\beta\mu} - \frac{\alpha e^x x^{\beta\mu+1}}{(e^x - 1)^2} = 0.$$

where $x = \frac{k}{r}$. It can be shown that $\frac{e^x x^{\beta\mu+1}}{(e^x-1)^2}$ is strictly decreasing in x for all $\beta\mu < 1$ which implies there is at most one value of r such that $\frac{\partial \Delta^T(r,\beta)}{\partial r} = 0$. This in turn implies that $\underline{r}^T(\beta)$ is unique.

Now suppose $\beta\mu > 1$. As in $\beta\mu < 1$ case, $\lim_{r\to 1}(\tau^0 - \tau^T) = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta})}{(1-\theta+\phi)(1-\lambda)} > 0$ which implies there exists $r = \underline{r}^T(\beta)$ such that $r < \underline{r}^T(\beta) \iff \bar{y}^T - \bar{y}^0 > 0$. In addition, $\lim_{r\to\infty}(\tau^0 - \tau^T) = \frac{1}{(1-\theta+\phi)(1-\lambda)} > 0$ which implies that there exists $r = \bar{r}^T(\beta)$ such that $\bar{y}^T - \bar{y}^0 > 0$ for all $r > \bar{r}^T(\beta)$. When α is small, $\underline{r}^T(\beta) = \bar{r}^T(\beta)$.

□

Proof of Proposition 4

Proof. Part (i) follows from the definition of $r_1(\beta,\lambda)$ and $r_2(\beta,\lambda)$ in (20) and the discussion following the Proposition. Part (ii) follows from noting that, for any $r > 1, \lim_{\beta\to 0}\tau^0 > 0$ (Lemma 1) while $\lim_{\beta\to 0}\tau^R = 0$ (Lemma 2). □

Proof of Proposition 5

Proof. (i) That reservation is the most preferred policy for low degree of complexity follows from noting that for any $r > 1, \lim_{\beta\to 0}\tau^R = 0$ (Lemma 2) while both $\lim_{\beta\to 0}\tau^0 > 0$ (Lemma 1) and $\lim_{\beta\to 0}\tau^T > 0$ (Lemma 3).

(ii) From Proposition 2 we already know that for all $\beta > 0$, there exists $r^R(\beta,\lambda) > 1$ such that $\bar{y}^R > \bar{y}^0$ if and only if $r < r^R(\beta,\lambda)$. To complete the proof, below, we show that $\bar{y}^R - \bar{y}^T > 0$ when β and r are suitably small. As established in the main text, $\bar{y}^R - \bar{y}^T > 0 \iff \tau^R - \tau^T > 0$. We have

$$\tau^R - \tau^T = 1 - \left(\frac{1-\theta+\phi}{(1-\theta)+\phi r}\right)^{\beta\mu} - \frac{\phi\alpha\left(\frac{1}{e^{\frac{k}{r}}-1} - \frac{1}{e^k-1}\right)}{{s_m}^{1-\beta}(1-\theta+\phi)(1-\lambda)(\frac{\mu r}{ek})^{\beta\mu}}.$$

Observe that $\lim_{r\to 1}\tau^R - \tau^T = 0$. Then the claim follows from noting that

$$\lim_{\beta\to 0}\frac{d\tau^R}{dr}|_{r=1} = 0, \qquad \lim_{\beta\to 0}\frac{d\tau^T}{dr}|_{r=1} > 0,$$

and consequently

$$\lim_{\beta\to 0}\frac{d(\tau^R - \tau^T)}{dr}|_{r=1} < 0,$$

where the expressions for $\frac{d\tau^R}{dr}|_{r=1}$ and $\frac{d\tau^T}{dr}|_{r=1}$ are given by (21) and (22) respectively. □

Proof of Proposition 6

Proof. The Proposition follows from two observations. First, note the parameter α scales the training cost, and hence for any (β, r) there is α small enough to ensure that τ^T is non-negative and as small as desired. So for any β consider $r^R(\beta, \lambda)$ as defined in part (ii) of Proposition 2. Then $\tau^R \lesseqqgtr \tau^0$ as $r \lesseqqgtr r^R(\beta, \lambda)$. Next, lower α to $\hat{\alpha}$ such that $\tau^T = \tau^R = \tau^0$ at parameter values β and $r^R(\beta, \lambda)$. Then for all $\alpha < \hat{\alpha}$, there is an interval around $r^R(\beta, \lambda)$ such that training is optimal in this interval given β. $\square$