Affirmative action, equal opportunity, or just tax the rich? Development, efficiency, and the pursuit of equity

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Abstract

We examine the efficiency properties of prominent equity policies in a general model of development that incorporates the Lewis model and the Kremer (O-ring) model as limiting cases. We find that reservation (quotas) is the most effective policy in an economy with low technological complexity. Our intuition is that production in such economies is less sensitive to variations in embodied skills. Training to equalize skills becomes more attractive as complexity increases. However, quotas never lose relevance. Tax-transfers supersede the other measures in highly complex and fiscally mature economies. These findings provide a step towards a more informed and robust policy.

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1 Introduction

Tension between efficiency and equity is fundamental to every economy. Even if all persons are created equal, variations in privilege, background and upbringing—not to mention plain luck—render individuals greatly heterogeneous by the time they enter the economy as productive agents. Inequalities in skills and productive capabilities result in unequal earnings, and are thereby transmitted to the next generation. Measures to restore equity, on the other hand, dull production incentives and allocate talents away from their most productive uses, compromising efficiency.[1]

Since the market allocates incomes in the first instance according to productive ability, and abilities are unequally distributed in the natural state, active policy intervention is necessary to reduce undesirable inequalities. Popular policies are classified under three broad categories—tax-and-transfer, affirmative action and equal opportunity. Tax and transfer is self-explanatory and occurs after individuals have earned gross incomes that reflect their productivity. Affirmative action, otherwise known as "quotas" or "reservations", places underprivileged individuals in remunerative jobs (or professional school places) that would not naturally be awarded to them based on native skill or prior qualifications. Equal opportunity attempts to correct the inequality in skills and qualifications before agents reach the employment market by devoting greater resources to the education and training of the disadvantaged groups.[2]

There is no semblance of agreement among experts on the question of optimal policy. Especially intriguing is the fact that the relative desirability of different policies are evaluated differently in different contexts, and even in the same country popular support can shift widely between one policy and another. For example, quotas for underprivileged groups has been a pillar of equity policy in India since independence, but its relevance and effectiveness has been increasingly questioned in recent decades. Quotas were never popular in the US, while northern European countries have had significantly more tolerance for tax-transfer policies than other regions.

This paper takes primitive differences between social groups as given, and examines in some detail the contours of policies that aim to enhance equality of final incomes. We show that the optimal policy is sensitive to the level of development and the state of technology, and hence different policies are appropriate for countries at different stages of development, and indeed for the same country at different times.

A basic intuition driving our approach is that the products demanded and produced in an economy in the earlier stages of development are less complex and require lower embodied

[1]The heterogeneity manifests itself in command of capital as well as productive skills and earning abilities, but for now we will confine our attention to the latter.

[2]Readers may disagree with the interpretations that we have placed on these terms. We are going by the interpretations or descriptions of policies provided above rather than by the nomenclature. In this paper we will call the non-fiscal policies "reservation" for AA and "training" for EO, to keep our meaning clear.

skills to produce. In such economies, preferentially placing less prepared individuals in higher skill jobs does not compromise efficiency to too large an extent. Remedying the disadvantage in skill-acquisition, on the other hand, is often significantly more demanding of resources when education systems are strained and insufficient. Thus affirmative action is likely to be a more efficacious means to reduce unequal outcomes than an attempt to equalize productive capabilities across groups. In economies where production processes incorporate high technology and sophisticated skills, however, placing an unprepared individual in a sensitive production line may severely compromise output, and turn out to be wasteful of valuable human resources. Here it makes more economic sense to adequately train the inductees, even at a relatively high cost.

Of course, the textbook way to attain an efficient outcome is to allow a competitive market to determine levels of training and job allocations, and then use *ex post* taxes and transfers to correct inequality. However, tax-transfer schemes affect incentives, and are costly and constrained by state capacity. Hence these can effectively be used only by a handful of economies with high levels of socialization and extensive state capacity. It is outside the purview of a rudimentary state (Besley and Persson, 2013).

The comparison between affirmative action (reservations) and training (equal opportunity) is illustrated by the following example. Consider the policy of reserving places in medical school for students from disadvantaged backgrounds. This is a standard practice in India, where reservations for "dalits" (historically disadvantaged castes and tribes) has been a mainstay of affirmative action since independence. Reservations are needed because dalit children, on the strength of their educational backgrounds, would not gain admission into medical schools in sufficient numbers, since admission is based on merit as demonstrated by exam scores and past academic performance.[3] Places are reserved in this way not only in professional programs, but also in public administration and the bureaucracy.

How would such a policy affect the performance of the doctors that emerge from medical school? Some doctors who came in with blazing credentials would graduate with great acumen, while a large fraction of those that entered with marginal credentials would possibly only just hold their own.[4]

How much difference would it make if some doctors came into medical school with less-than-ideal prior preparation? This depends on how they spend most of their medical career. In a very underdeveloped country, where life-expectancy is low and most people do not have the luxury of living long enough to develop complex diseases, the bulk of the average doc-

[3] This does not, of course, establish that merit thus measured necessarily determines the subsequent development of ability, but there is good reason to believe that it does. Otherwise, there would surely be instances where places are allocated by lottery, or at least dalits that were admitted would sometimes be selected randomly. In reality, however, the reserved places are also filled on merit, only with a lower entry bar.

[4] Many that entered on quotas would also find it impossible to cope and just drop out, leading to the complaint that reservations programs can cause great waste of scarce resources (see Deshpande and Newman, 2007).

tor's time may be spent organizing inoculation drives and treating ailments like diarrhea and cholera, which claim an appalling number of lives because of lack of basic health information among the public, but can be countered with adequate hydration and general cleanliness. In such a scenario, the large prior variance in preparation may well translate only in a minor variation in outcomes. On the contrary, in advanced economies where complex diseases are the major cause of concern and doctors and surgeons regularly undertake delicate procedures in highly interdependent teams, one underequipped team-member may well impose a severe cost. Accordingly, in more advanced economies, quotas in high-paying occupations are not the favored alternative; in the quest for greater equality, the emphasis is much more on aiding the underprivileged to make the unconditional cut for admission into preferred callings. The focus then is on early childhood and school education (no child left behind), support for specialized institutions (like the traditionally black colleges in the US), and enabling college attendance through scholarships and other means.

In the above example, for the economy in the early stages of development, the constraint on the production of doctors' services is presented not so much on the prior preparation of candidates but by the limited capacity of medical schools. This reflects the conception of development in the Lewis-Ranis-Fei (Lewis, 1954; Ranis and Fei, 1961, henceforth LRF) economy, where development is primarily the process of moving human resources from the less productive agricultural or rural or traditional sector to the more productive industrial or urban or modern one. In LRF, this process is constrained by the availability of capital that determines employment capacity in the modern sector. By default, those who are already in the modern sector are the privileged group, and occupants of the traditional sector are underprivileged.[5] In this sense, the development process in the LRF model is powered by affirmative action subject to capital constraints. Affirmative action, i.e., moving workers from the less-productive to productive occupations, increases output and has no negative consequences. The assumption that industrial profits are reinvested provides the dynamic in that development model.

As development proceeds and production becomes more skill-intensive, this simple representation becomes inadequate. Kremer (1993) has explained that complex production processes can be compromised by low-skill individuals, and catapulting such individuals into high-skill production teams has severe negative externalities. While reservations or "affirmative action" can increase the productivity of disadvantaged individuals in such contexts, they extract a cost by handicapping the more accomplished workers with whom they are teamed. The latter cost can be reduced by upskilling the catapultees before they are elevated, and hence the provision of prior training, or "equal opportunity", begins to look attractive.

We pursue this line of reasoning to compare the effectiveness of different equalization policies at different levels of development. In principle, the best outcome is always attained by di-

[5] Here we ignore the informal sector and Harris-Todaro concerns, which can be accommodated in our analysis in a straightforward way, though it will change some of the quantitative conclusions.

recting investments and productive resources to maximize output, and then reallocating that output through taxes and transfers to attain the desired equity. In practice, however, even when the market effectively implements productive efficiency, states have limited capacity to implement redistribution schemes. Further, taxes affect work-incentives and hence production. We represent these effects in the model as a leakage from the tax-transfer process.

We find that reservations fare better than both training and tax transfer in an economy where skill differences are small, complexity is low and sectoral effects are significant. This resembles the archetypal two-sector underdeveloped economy that Lewis conceived. Here, reservations lead to an increase in per-capita income as well as a decrease in inequality. When production processes become more complex and skill-differences become larger, reservations become more costly and training enters the optimal policy mix. However, training is always used in conjunction with reservations, which never lose relevance.

Reservation has an implicit cost in terms of lost output, and training entails a direct cost which must be covered by taxes. Taxes can alternatively be used in a fiscal redistribution scheme to directly equalize incomes. The choice between the two turns on the costs of reservations and training compared with the cost of raising taxes. Thus as complexity and the skill-gap become larger, and as state capacity increases to make taxation more effective, pure tax-transfer policies become more efficient than the other options.

The effects of affirmative action policies on employment of targeted groups have been studied in the context of the US (Bleemer, 2022) and India (Afridi, Iversen, and Sharan, 2017; Munshi, 2019; Prakash, 2020), and other countries. Equal opportunity and especially early intervention programs have been in the limelight for some time.[6]. The present paper adds to this literature by establishing a framework to evaluate the relative desirability of different inequality-reducing policies according to context, and underlining that effectiveness is conditioned by attributes of the economy that have not previously been explicitly included in this analysis.[7]

A novelty of our approach lies in this twofold linking of optimal policy with population heterogeneity and technological maturity. Researchers have examined the effectiveness and resource cost of equity programs in different contexts. However, circumstances that condition this effectiveness have not been explored in the literature. To this purpose, we integrate the dual-economy model of Lewis (1954) with the insights on production complexity from Kremer (1993). These two classic models comprise the principal analytical frameworks within which economic development is visualized, but until now there has been no attempt to integrate them in one framework. We develop a production function that establishes a continuum between the two, and highlights the regions in which each approach is relevant.

The approach we take in this paper allows us to shift emphasis from the question: "Do the gains from affirmative action justify the accompanying efficiency cost?" to the question "Under

[6]See, for example, Miguel and Kremer (2004); Schultz (2004); Afridi (2010)

[7]An elegant classification of policies along similar lines is proposed in Rodrik and Stantcheva (2021).

what social and technological conditions is affirmative action more cost-efficient than equal opportunity measures to attain specific equity gains?" It will further enable us to determine when it is more advisable to shelve both policies and use progressive taxation and public/welfare spending instead. An unconditional analysis, on the other hand, may conclude that a policy was or was not effective in a certain place or time, but will be unable to explain why it did not obtain similar results in another milieu. Recognizing the effect of critical conditioning variables enables an informed choice of policy.

2 Model

2.1 Production function

Production takes place in teams of n agents each, where n may vary according to the skill levels of the team members and the importance of individual skill and accuracy in the production process. Here we follow Kremer (1993) and adapt the O-ring approach to suit our question.

Further, in order to relate our analysis to some of the dominant paradigms in the development literature, we incorporate the effect of the sector in which a production process operates, where some sectors (i.e., the "modern" sector in the Lewis-Ranis-Fei paradigm) are endowed with a more production-augmenting environment compared to other ("traditional") sectors. We represent this by a sector parameter (s_j in the formalisations below) which embraces the contribution of specific resources such as infrastructure, communications, etc. Some of this is the result of public provision and some are externalities—where we implicitly assume that the externality-generating activities tend to gravitate to the sectors where the publicly provided resources are found. Alternatively, s may be a proxy for industry, with s_j merely representing the capital-intensivity of an industry. The exposition below is consistent with the interpretation that more capital intensive industries attract and recruit more skilled workers.

We write the expected per-capita production function in an n-member team working in sector j as

$$y_j(p, n, \beta) = s_j^{1-\beta} \frac{G(n, \beta)}{n} \Pi_{i=1}^{n} p_i^{\beta} \tag{1}$$

where s_j is the contribution of sector resources, $G(.,.)$ is the value of a unit of final output produced by an n-person team, $p_i \in [0, 1]$ is an index of the skill or precision of the i-th worker, and β is a parameter that captures the importance of that skill or precision in the production process.[8] We specify the value of a unit of final output as

$$G(n, \beta) \equiv n^{1+\mu\beta}$$

[8]We ignore capital, which can be incorporated as in Kremer (1993), see Appendix A.2.

where μ is a positive constant.[9]

As $\beta \longrightarrow 0$, the p_is become inconsequential and the importance of individual precision vanishes. The output of each individual worker production depends only on the sector resources s_j, as visualised by Lewis. Thus β is a factor that tempers the criticality of individual performance, moderating the impact of mistakes in processes according to their skill-sensitivity. This is a direct interpretation of the O-ring production function (Kremer, 1993), and we will not attempt to justify it further. When $\beta = 1$, this production function conforms to the interpretation due to Kremer, that a unit of output is produced if and only if each worker in the process executes her task correctly, and p_i is the probability that the i-th worker does so. Here s_j does not matter. Further $G(.,.)/n$ is increasing in n indicating that more complex processes produce proportionately more valuable output.

Processes where productivity depends only on team size and is not greatly sensitive to individual's accuracy are more likely to incorporate Smithian division of labour in which tasks are split up into small components, and correctly produced components are assembled together. We can categorise such processes as ones where the rate of output is constrained by the speed of the slowest member of the team. Here good management would consist of balancing the numbers of workers at different "stations" such that each station performs its part of the process at the same rate as every other station. Such division of labour produces increasing returns to scale, as Smith famously observed, but these are a result of the organisation of production rather than the interaction of highly-skilled operatives. Increasing returns occur at a qualitatively different scale in "high-tech" complex processes where the marginal productivity of each worker is determined by the precision of all others. The production function above attempts to parametrically represent some aspects of the production process in a developing economy as its technological capabilities go from simple to increasingly complex.

2.2 The economy

The economy consists of two production sectors, which we will often refer to as the *modern* and *traditional* sectors. This is a notional classification. The sectors may be geographically separated, with the urban modern sector equipped with more amenities, public resources and capital, making workers in that sector more productive. They may also be spatially co-located and distinguished by technology, organisation and management methods that make one sector a more productive environment than the other.[10] Where sectoral productivity matters (which is not always), we assume that the modern sector is more productive than the traditional. In

[9] We use a closed form function since it makes the analysis much more transparent, and the general properties of this production function are well understood.

[10] Lewis himself did not espouse the geographical interpretation, or even an identification of the two sectors with "industrial" and "agricultural", see Gollin (2014).

the production function this translates as:

$$s_m \geq s_t \tag{2}$$

where m and t represent the two sectors.

There is a unit mass of workers in the economy. A fraction $1-\theta$ of the workers belong to the privileged group that we call *elits*, while a fraction θ comprise the underprivileged *dalits*. Each elit has an initial skill level p and each dalit has a skill level q, where

$$0 \ < \ q \ < \ p \ < \ 1 \tag{3}$$

2.3 Production

Consider an arbitrary production outfit in an arbitrary sector with n team members, of which a fraction $(1-\gamma)$ are p-types or elits and the remaining fraction γ are q-types or dalits. Average output produced by this team is

$$y(p,q,n,\beta) = s^{1-\beta}\ \frac{G(n,\beta)}{n} p^{\beta n(1-\gamma)} q^{\beta n \gamma}.$$

Letting $G(n,\beta) = n^{1+\mu\beta}$, we have

$$y \ = \ s^{1-\beta}[n^{\mu} p^{n(1-\gamma)} q^{n\gamma}]^{\beta} \tag{4}$$

Higher β reflects increased complexity. Observe that when $\beta = 0$, per-capita output depends only on the sector productivity as in the Lewis-Ranis-Fei economy; while as $\beta \to 1$ we approach a version of Kremer's O-ring production function. As n rises, $\frac{G(n,\beta)}{n}$ rises owing to increasing returns, while the probability of success $p^{\beta n(1-\gamma)} q^{\beta n\gamma}$ declines. Optimal n balances these two effects.

2.3.1 Optimal team size

To find the optimal n, or team size, first write the production function in log form

$$\ln y = (1-\beta)\ln s + \beta[\mu \ln n + n(1-\gamma)\ln p + n\gamma \ln q]$$

and then differentiate $\ln y$ w.r.t n and set it to 0 to obtain optimal n:

$$\frac{\mu}{n} + (1-\gamma)\ln p + \gamma \ln q = 0 \quad \Longrightarrow \quad n^* = -\frac{\mu}{(1-\gamma)\ln p + \gamma \ln q}$$

Rearranging n^* and substituting in y yields:

$$y^* \ = \ s^{1-\beta}(n^{*\mu} p^{n^*(1-\gamma)} q^{n^*\gamma})^{\beta} = s^{1-\beta}(\frac{n^*}{e})^{\beta\mu}$$

Observe that n^* (and hence y^*) is increasing in productivity of p-types, the proportion of p-types $(1-\gamma)$, and the degree of increasing returns (μ).

Now write

$$
\begin{aligned}
q &= \frac{1}{e^k} \quad \text{and} \\
p &= \frac{1}{e^{\frac{k}{r}}}
\end{aligned}
$$

By (3), $k > 0$ and $r > 1$, and r captures the degree of skill-advantage of the p-types. Substituting these values in the expression for n^* yields optimal team size and average per capita output in a mixed team where p and q-types are present in proportions of $(1-\gamma)$ and γ:

$$n = -\frac{\mu}{(1-\gamma)\ln p + \gamma \ln q} = \frac{\mu}{k}\left(\frac{r}{(1-\gamma)+\gamma r}\right) \tag{5}$$

$$y = s^{1-\beta}(\frac{n}{e})^{\beta\mu} = s^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}\left(\frac{1}{(1-\gamma)+\gamma r}\right)^{\beta\mu} \tag{6}$$

2.3.2 Full Separation

Suppose elit p-types and dalit q-types are fully separated, with elits placed in the modern sector and dalits in the traditional sector. Then team size and per-capita output in the traditional and modern sectors, respectively, are given by:

$$n_t^F = \frac{\mu}{k}, \qquad y_t^F = {s_t}^{1-\beta}(\frac{\mu}{ek})^{\beta\mu} \tag{7}$$

$$n_m^F = \frac{\mu r}{k}, \qquad y_m^F = {s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu} \tag{8}$$

where superscript F indicates full separation. Here we assign the elits a double privilege of the more productive sector and the higher skill parameter. Depending on the type of economy we consider, one or the other of these will become more salient.

It is easy to show that, when $\beta > 0$, the full-separation allocation of workers between processes maximises total output. If there are no constraints on assignment of workers across sectors, and $s_m > s_t$, then all workers should be placed in the modern sector, but segregated according to skill. We retain the convention that low-skill processes are operated in the traditional sector and high-skill processes in the modern sector, which may reflect capacity constraints in the modern sector.[11]

Assumption 1. *Teams consisting only of low-skilled workers cannot function in the modern sector, a strictly positive fraction of each modern sector team must be high-skilled.*

Informally, one can imagine that in order to function productively in the modern sector, one must be able to handle codified information and technology to some extent, which becomes feasible only when a team contains at least some trained personnel.

[11] It is possible that creating capacity in the modern sector requires some amount of per-capita investment that repays itself only if the worker is of sufficiently high skill, and skill matters (i.e., β is sufficiently large). In a Lewis economy where $\beta = 0$, accommodating workers in the modern sector is then constrained only by capacity, but when β is sufficiently large the creation of capacity must be justified by the quality of workers being accommodated.

Differences in sectoral resources are important only when β is close to zero, in particular in Proposition 1. The remaining results are driven by skill differences between groups rather than by sectoral differences, and remain valid if we assume that $s_m = s_t$.

3 Reservation, training and income redistribution

In this section, we enumerate the outcomes of different policies, and compare their impacts upon per-capita income and inequality conditional on the level of development of the economy. These policies cause income and inequality to deviate from the full separation outcome described in Section 2.3.2. Recall that full-separation with optimal choice of complexity maximizes total output in the economy conditional on the sectoral allocation of workers with different skills.

Each agent works in a given sector in a production team consisting of some number of workers of each type. In all configurations we consider, production teams will consist only of q-types, only of p-types, or of the two types mixed together. Teams with only q-types always operate in the traditional sector. Teams with only p-types, or with a mix of the two types, operate in the modern sector. This does not require the sector parameters s_t and s_m to be different, nor for there to be a spatial distinction between the sectors.

The total number of workers determines the complexity of the process. Within a team, all members earn the same income, which is the per-capita output produced in the team. Thus the income of an agent that works in an n-person team in sector j, with fractions γ and $1 - \gamma$ of q and p-types, is given by (4), which reduces to (6) when team size is chosen optimally.

We make this modeling choice to keep the analysis streamlined. An alternative would have been to assume that each agent in a multi-type mixed team is paid in proportion to her marginal product. The income of the p-types would then exceed that of the q-types in the team, and the latter in turn would earn more than q-types who are in teams by themselves. That analysis would possibly be more realistic and yield quantitatively different results, but we don't think it would convey a radically different intuition (see Sections 3.4 and 4.3).

We first focus on two policies: *reservation* and *training*, singly and in combination with each other. Then we compare these with *tax-transfer*. *Reservation* refers to placing some of the lower-skill individuals (dalits or q-types) in production processes primarily executed by high-skill workers in the modern sector. *Training* refers to a program in which some number of dalits are provided extra training resources (funded by taxation) so that their skill level increases to p, and are then put in production teams with the elits. Finally, *tax-transfer* refers to a policy of taxing the high-skilled (and hence high-income) modern sector workers, and using the proceeds to increase the income of a subset of the low-skilled traditional sector workers. We consider these against the benchmark of full-separation described in the previous section.

If the fiscal machinery is perfect and costless, then tax-transfer is more efficient than other equalization policies, at least in a static sense, in all but the most underdeveloped economies. However, transfers are seldom costless owing to constraints of state capacity, the shadow cost of public funds, and the effect on work-incentives. We do not explicitly model these in this paper; instead, we collapse these into one indicator that we call *leakage*, which determines the fraction of taxes that are consumed by these costs. We model leakage of tax revenues by a parameter $\lambda \in [0, 1]$. This is the fraction of tax revenues that is dissipated in the process; i.e., of every dollar taxed, only $1 - \lambda$ dollars is actually available to disburse (see, for example, Besley and Persson, 2009).

3.1 Schematics for comparison

In order to make comparisons transparent, we construct the policies in the following way. We start from the full-separation configuration where all dalits are in the traditional sector and all elits are in the modern sector (see Section 2.3.2). We then choose a given mass $\phi \leq \theta$ of dalits (q-types), and apply the relevant policy to raise their income to equal that of the elits (p-types). The cost of the policy is met by uniformly taxing all the workers (elits and newly-elevated dalits) that have the higher income after the application of the policy.

Under a *reservations* policy, ϕ low-skill workers are equally distributed across all modern sector production teams that formerly had only high-skill workers, and the size of the production teams are adjusted to restore optimality. Resulting modern sector production is now represented by the model in Section 2.3.1, with $\gamma = \frac{\phi}{1-\theta+\phi}$. Per-capita income in the modern sector is now lower than before the change.

Under *training*, the ϕ low-skill workers are given training to raise their skill level from q to p, and they are then placed in the modern sector alongside the previously high-skill workers. The cost of training is covered by taxing all the workers that are in the modern sector after the policy is implemented.

Tax-transfer requires no change in either the occupations or the skills of any agents. The reference tax-transfer policy consists of supplementing the incomes of the ϕ dalits by the amount $y_m - y_t$, so that their incomes now equal that of the elits. The subsidy is funded by taxing all high-income workers, including the ϕ dalits that now have higher incomes.

3.1.1 Comparison of mean income and inequality

It follows that, after execution of a policy A, we have $(\theta - \phi)$ of the dalits with unchanged income y_t, while $(1 - \theta + \phi)$ agents have a higher income y_m^A. We expect that for reasonable parameters $y_m^A \in [y_t, y_m]$. In particular, this is always true under reservation.[12]

[12] y_m^A may fall below y_t under tax-transfer and training for extreme values of some parameters. For example, if state capacity is severely limited and λ approaches zero, then neither tax-transfer nor training (funded by taxes)

Now consider two policies A and B, and suppose that $y_m^A > y_m^B$. Recall that low-income earners earn y_t in both cases. It is then clear that per-capita income is higher under A than B, but the comparison of inequality is ambiguous. But now in the distribution corresponding to A, we can redistribute some income δ from the agents receiving y_m^A to those receiving y_t, where δ is calculated to satisfy:

$$\frac{y_m^A - \delta}{y_t + \delta\frac{(1-\theta+\phi)}{(\theta-\phi)}} = \frac{y_m^B}{y_t}.$$

Then the inequality in the post-transfer distribution following A is equal to that following B, and per-capita income is still higher under A, so A leads to a better outcome than B by any criterion that puts positive weight on both higher per-capita income and lower inequality. This is not compromised by the existence of leakage in the tax-transfer process (see next section). Even if the leakage proportion λ is unity, the per-capita income post-transfer is no smaller in A than in B. If such notional transfers are admitted, then it is sufficient to compare per-capita incomes to evaluate the relative merits of different policies, given the implementation scheme we have adopted.[13]

3.2 Reservation

A reservation policy ϕ consists of randomly choosing a mass of agents of size $\phi \in (0, \theta]$ from the q-types in the traditional sector and placing them in the modern sector. They are distributed equally across the modern sector teams, and the team size (complexity) is readjusted to restore optimality (see Section 2.3.1).

Recall that teams consisting only of low-skill workers cannot function in the modern sector. It is easy to see that low-skill workers migrating to the modern sector will not be hired into high-skill teams, because they reduce the productivity of the high-skill co-workers. Thus low-skill workers can be accommodated in the modern sector only if there is some compulsion that they should be included. Reservations achieve precisely this objective.

Let $\gamma(\phi) = \frac{\phi}{1-\theta+\phi}$ denote the proportion of q-types in a modern sector team. $\gamma(\phi)$ ranges from zero to θ as reservation encompasses varying fractions of the dalits. Substituting $\gamma = \frac{\phi}{1-\theta+\phi}$ in (5) and (6) respectively yields employment and average output in the modern sector (with superscript R indicating reservation):

$$n_m^R = n_m\left(\frac{1-\theta+\phi}{(1-\theta)+\phi r}\right); \qquad y_m^R = y_m\left(\frac{1-\theta+\phi}{(1-\theta)+\phi r}\right)^{\beta\mu} \tag{9}$$

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will be effective in delivering higher incomes to the chosen fraction of dalits. Alternatively, if training costs are too high, then y_m^A may even fall below y_t.

[13] In the absence of such transfers, we must resort to using a social welfare function with per-capita income and inequality as arguments. Then it would matter how we measure inequality, and how much weight we put on each argument. However, following the reasoning above, post-policy income distribution will continue to deliver greater social welfare as long as the relative weight on per-capita income is not too small.

There is of course no change in income in the traditional sector, which now has a smaller population than in full separation. From (7), (8), (9) it follows that when $\beta > 0$, $y_m > y_m^R > y_t$.[14] Per capita income in the modern sector is lower under reservation than under full-separation. However, the modern sector now absorbs individuals from the traditional sector, who experience an increase in income. Each of the ϕ dalits that moves from the traditional to the modern sector experiences an increase in income of $y_m^R - y_t$, while each elit loses $y_m - y_m^R$. Income of dalits remaining in the traditional sector remains unchanged. We can write the difference between the full-separation income y_m and the post-reservation mean income y_m^R in terms of these differences:

$$\bar{y}^R - \bar{y} \quad = \quad \phi(y_m - y_t) - (1 - \theta + \phi)(y_m - y_m^R).$$

Observe that $y_m - y_t$ is strictly positive if $s_m > s_t$ or $\beta > 0$, however, $y_m - y_m^R \to 0$ as $r \to 1$ or $\beta \to 0$. As a result, when the skill-advantage of elits is small and the complexity of the production process is low, mean income increases with reservation and there is no tradeoff between equity and efficiency.

Proposition 1 (Economic Development with Unlimited Supplies of Labor)**.** *Mean income in the economy is higher under reservation than under full-separation if complexity is low and/or the skill difference between elits and dalits is small. More formally, suppose $s_m > s_t$. Then,*

for all $r > 1$, there exists $\underline{\beta}(r) > 0$ such that $\bar{y}^R > \bar{y}$ for all $\beta < \underline{\beta}(r)$;

for all $\beta > 0$, there exists $\underline{r}(\beta) > 1$ such that $\bar{y}^R > \bar{y}$ if and only if $r < \underline{r}(\beta)$.

It may sound surprising that mean income can increase with reservation despite a decline in per capita income in the modern sector. The driving force behind the change in income is the difference in sectoral productivities for low values of β and r (where $\bar{y}^R > \bar{y}$). Elevating ϕ dalits to the more productive modern sector and raising their income by $y_m^R - y_t$ outweighs the loss in income $y_m - y_m^R$ suffered by elits.

It is important to note that reservations are instrumental in achieving this improvement. Left to its own devices, the market will not attain this outcome. We know from Kremer (1993) that in a decentralized setting high-skill workers will segregate themselves in different teams from low skill-workers. In our framework this translates to an entry barrier to the modern sector for dalits.

For later reference we express the post-reservation mean income as:

$$\begin{aligned} \bar{y}^R &= (1 - \theta + \phi)y_m^R + (\theta - \phi)y_t \\ &= (1 - \theta + \phi)(1 - \tau^R)y_m + (\theta - \phi)y_t \end{aligned} \tag{10}$$

where

$$\tau^R = 1 - \frac{y_m^R}{y_m} = 1 - \left(\frac{1 - \theta + \phi}{(1 - \theta) + \phi r}\right)^{\beta\mu} \tag{11}$$

[14]When $\beta = 0$ but $s_m > s_t$ we get $y_m = y_m^R$.

To understand the significance of τ^R, suppose that the selected number ϕ of dalits could be elevated to the higher income costlessly. Then the high income would remain y_m (equation 8) after the policy is implemented. Since the process is costly, the new high income instead is a smaller number y_m^R. The fraction by which y_m^R falls short of y_m is τ^R. We use this implicit tax as a measure of the cost of implementing the policy. This cost is larger when the skill-difference is larger, and when this difference matters more in the production process. This is summarized in Lemma 1.

Lemma 1 (Properties of τ^R). *(i) τ^R is increasing in r and β.*

(ii) $\lim_{r\to 1} \tau^R = 0, \quad \lim_{r\to\infty} \tau^R = 1, \quad \lim_{\beta\to 0} \tau^R = 0.$

3.3 Training

Next we consider training a fraction ϕ of the q-types to raise their productivity to p. Training is costly. Training cost per person is $C(p, q)$ which satisfies some standard properties:
(i) $C(q, q) = C(p, p) = 0$, (ii) $C_q(.) < 0,\ C_p(.) > 0$, and $C_{pp}(.) > 0$, and (iii) $\lim_{p\to 1} C(p, q) = \infty$.
A convenient if slightly stronger version is given by

$$C(p, q) \quad = \quad \psi(p) - \psi(q)$$

where $\psi(x)$ is an increasing and convex function that satisfies $\psi'(x) \to 0$ as $x \to 0$ and $\psi'(x) \to \infty$ as $x \to 1$. Total cost of training, $\phi C(p, q)$, is funded by a proportional tax τ^T on the *ex post* high-income individuals (see also footnote 18). Assuming that a fraction λ of the tax receipts are dissipated, the relevant tax rate is:

$$\tau^T = \frac{\phi.C(p, q)}{(1 - \theta + \phi)(1 - \lambda)y_m}. \tag{12}$$

where $q = \frac{1}{e^k}$ and $p = \frac{1}{e^{\frac{k}{r}}}$.
Accounting for training costs, mean income under training can be expressed as

$$\bar{y}^T = (1 - \theta + \phi)(1 - \tau^T)y_m + (\theta - \phi)y_t. \tag{13}$$

Properties of τ^T are listed in the following lemma.

Lemma 2 (Properties of τ^T). *(i)* $\lim_{\beta\to 0} \tau^T > 0$, *and* $\lim_{r\to 1} \tau^T = 0$.

(ii) τ^T is strictly decreasing (increasing) in β if and only if $r > (<) \frac{eks_m^{\frac{1}{\mu}}}{\mu}$.

(iii) τ^T may be increasing or decreasing in r.

Several properties of τ^T are worth noting. First, τ^T can be decreasing in β. An increase in complexity, β, does not affect the cost of training, but it can increase y_m provided r is not too low. This lowers the tax burden in the modern sector which results in lower τ^T. On the other hand, if r is low, y_m is decreasing in β. Since y_m is in the denominator of the expression for τ^T, the latter is increasing in β in this case.

Second, τ^T is not necessarily increasing in r. As r increases, both training costs and modern sector incomes increase. Whether per capita tax increases or not depends on the relative rates of increase of the two. When the value of $\beta\mu$ is below a threshold, training cost increases faster with r than does y_m, so that τ^T increases with r. On the other hand, when complexity and/or degree of increasing returns is high and $\beta\mu$ is large, the tax burden of training can fall as r increases.

Indeed, in the absence of any restriction on μ, it is possible that infinitely costly training can be met with near-zero taxes. Assumption 2 restricts the values of μ to rule out this unrealistic scenario.[15]

Assumption 2. $\lim_{p\to 1} \psi(p)(\ln \frac{1}{p})^{\mu} = \infty$.

An implication of Assumption 2 is $\lim_{r\to\infty} \tau^T = \infty$ (see Claim 1 in Appendix for formal proof). Hereafter we consider parameter values that satisfy Assumption 2.

Finally, recall that τ^R—the implicit tax rate under reservation—approaches zero as $\beta \to 0$. In contrast $\lim_{\beta\to 0} \tau^T$ is positive since training cost per person $C(p,q)$ is strictly positive irrespective of β. Thus, between reservation and training, the former is the preferred policy at early stages of development when the complexity of the production process is low.

3.4 Combining training and reservation

The preceding analysis considers individual policies in isolation, whereas most real economies will normally employ a combination of policies. In this section we examine the conditions under which a combination of training and reservation is the optimal choice. We put tax-transfer aside for this exercise.

Intuitively, reservation is very costly in terms of output when there is a large difference in skills between the two groups, and it may be more efficient to train the dalits before placing them in the modern sector. However, training at the margin becomes increasingly more expensive, and overtakes the marginal benefit as the target skill-level of the dalits increases. Thus it is likely that for any p and q, there is some intermediate level q' such that the optimal strategy is to increase the skill-level of the target subgroup from q to q', and then use reservations to place the q'-skilled dalits in the modern sector.

Consider, therefore, a skill level $q' \leq p$ that has been acquired by the designated mass of ϕ dalits. If this mass was placed in the modern sector as described in Section 3.2, then the team

[15] To appreciate Assumption 2, consider $\psi(x) = \frac{x}{1-x}$. Then $\psi(p) = \frac{p}{1-p}$. We find that $\lim_{p\to 1} \tau^T = \infty$ whenever $\mu < 1$. If $\mu > 1$, $\lim_{p\to 1} \tau^T = 0$ while if $\mu = 1$, $\lim_{p\to 1} \tau^T$ is strictly positive and finite. Assumption (2) restricts $\mu < 1$. For general $\psi(.)$, existence of μ satisfying Assumption 2 follows from noting that $\lim_{(p,\mu)\to(1,0)} (\ln(\frac{1}{p}))^{\mu} = 1$, $\lim_{p\to 1} \psi(p) = \infty$, and consequently $\lim_{(p,\mu)\to(1,0)} \psi(p)(\ln \frac{1}{p})^{\mu} = \infty$

size and per capita income in the modern sector will be

$$n^C = -\frac{\mu}{(1-\gamma)\ln p + \gamma \ln q'} \quad \text{and} \quad y^C = \frac{s_m^{1-\beta}}{e^{\beta\mu}}(n^C)^{\beta\mu} \tag{14}$$

where $\gamma = \frac{\phi}{1-\theta+\phi}$, and the superscript C refers to the combination of policies. An infinitesimally small increase in the skill level q' raises per capita output by

$$\frac{d(y^C)}{dq'} = (\frac{s_m^{1-\beta}}{e^{\beta\mu}})\frac{\gamma\beta\mu(n^C)^{\beta\mu+1}}{q'} \qquad \text{[Marginal Benefit per capita at } q'\text{]} \tag{15}$$

However, increasing skill uses resources, and is funded by taxes. Recall that a consumption sacrifice of $\frac{1}{1-\lambda}$ is required to generate one unit of usable tax revenue. We use the general form cost function $C(p,q) = \psi(p) - \psi(q)$ introduced in Section 3.3, so the marginal cost of increasing q' is $\psi'(.)$ evaluated at q'. Further, the entire modern sector population pays the tax, but the training is given to only a fraction γ of the population. Thus the per capita cost of a marginal increase in skill at q' is

$$\gamma\frac{1}{1-\lambda}\frac{\partial\psi}{\partial q'} \qquad \text{[Marginal Cost per capita at } q'\text{]} \tag{16}$$

As in section 3.3, we assume $\beta\mu$ is less than unity.[16] From the expressions above it is clear that when $q' = 0$, marginal benefit of training is infinitely high, while marginal cost is finite. Thus the optimal q' is positive. On the other hand, marginal benefit is finite for all $q' > 0$, while marginal cost becomes infinitely high as $q' \to 1$. Hence the optimal $q' < 1$. Thus for all $\beta > 0$, there exists at least one $q' \in (0,1)$ where $y^C - C(q',q)$ is maximized.[17] Call this $q^*(p)$. Then $q^*(p)$ is the level of training such that it is more profitable to place dalits with this or higher levels of skill directly in the modern-sector production teams rather than to provide them further training.

Consider an initial configuration of skills (q,p), with $q < p$. If $q^*(p)$ as defined above lies between q and p, then the optimal policy is to train the dalits to improve their skill-level from q to q^*, and then place them in the modern-sector teams. However, this is not always the case. For example, when $q > 0$ and $\beta \to 0$, precision plays little role in the production process and marginal benefit from training becomes vanishingly small. Since marginal cost of training remains strictly positive (and hence exceeds marginal benefit), no amount of training is optimal. Thus a combination of the two policies can yield a strictly better outcome than a single policy for some parameter ranges but not others.

[16] High degree of increasing returns implies that a large amount of training could be funded with near-zero taxes. Assuming $\beta\mu < 1$ rules out that possibility.

[17] At a local maximum the marginal benefit curve described by (15) cuts the marginal cost curve (16) from above. The two curves can intersect at multiple $q' \in (0,1)$ some of which will be local maxima. By assumption the cost $\psi(.)$ is increasing and convex, so the marginal cost in (16) is increasing. Marginal benefit in (15) is decreasing (increasing) in q' if and only if $q' < \hat{q}'$ ($q' > \hat{q}'$), where $q = \hat{q}'$ is the unique solution to $n^C|_{q'=\hat{q}'} = \frac{1}{\gamma\beta(\beta\mu+1)}$.

Finally, if $q^* > p$ then we have the rather paradoxical situation where the most profitable strategy is to train the dalits to q^*, so that they in fact become more skilled than the elits. Intuitively, this indicates that the elits, too, have acquired too little training in spite of their privileged position, and their training should be subsidized further as well. However, we need a more complete model of education acquisition to examine optimal policy in this case, and in fact to even ascertain whether such a contingency (i.e., $q^* > p$) can arise at all when agents in the two groups acquire skills optimally at the individual level. We therefore skirt the additional questions that arise from this possibility, and assume $q^* < p$.

Proposition 2. *For any $p \in (q, 1)$ there exists $q^*(p)$ such that:*

(i) If $q < q^*(p)$, then the optimal policy is to train the target group to $q^*(p)$ and then use reservations to place them in the modern sector.

(ii) If $q \geq q^*(p)$ then the optimal policy is to eschew further training and place the target group directly in the modern sector using reservations.

3.5 Redistribution through tax-transfers

We finally address, for completeness, the purely redistributive tax-transfer policy: tax high-income workers and use the proceeds to raise the incomes of some unskilled workers.

As explained earlier, a mass ϕ of unskilled workers receive transfers $y_m - y_t$ each to lift their incomes, such that the incomes of these workers equal the post-tax incomes of skilled (modern sector) workers. The subsidy is funded by taxing all of $1-\theta+\phi$ high-income workers. Equating taxes (less leakage)

$$(1-\lambda)(1-\theta+\phi)\tau^0 y_m$$

with transfers

$$\phi(y_m - y_t)$$

and simplifying, post tax-transfer average income can be expressed as

$$\bar{y}^0 \quad = \quad (1-\theta+\phi)(1-\tau^0)y_m + (\theta-\phi)y_t \tag{17}$$

where

$$\tau^0 \quad = \quad \frac{\phi(1-\frac{y_t}{y_m})}{(1-\theta+\phi)(1-\lambda)} = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}})}{(1-\theta+\phi)(1-\lambda)}. \tag{18}$$

As before, τ^0 is the fraction by which the per-capita modern-sector income declines as a result of the policy. Lemma 3 records some properties of τ^0.[18]

[18] In the above formulation we assume that all high income workers (including the low-skilled workers that have been elevated) are taxed based on the notional pre-tax income. Our results are not dependent on this exact specification. An alternative formulation is discussed in Appendix A.1.

Lemma 3 (Properties of τ^0). *τ^0 is increasing in r, β, and λ. Furthermore, as long as $s_m > s_t$,*

$$\lim_{\beta \to 0} \tau^0 > 0, \qquad \lim_{r \to 1} \tau^0 > 0.$$

In addition, $\lim_{r\to\infty} \tau^0 < 1$ *for all* $\lambda < \frac{1-\theta}{1-\theta+\phi}$.

Two observations are in order. First, suppose that $s_m > s_t$, i.e. the modern sector is more productive than the traditional sector. Then τ^0 is strictly positive even when complexity plays no role in the production process ($\beta = 0$) or elits have no skill advantage ($r = 1$). In this case, elits receive a higher income purely by virtue of locational advantage, and hence transfers must be made to achieve equalization.

Second, tax-transfer remains a viable candidate for optimal policy as long as leakage λ is lower than a threshold value $\frac{1-\theta}{1-\theta+\phi}$. Otherwise, $(1 - \tau^0)y_m < y_t$ which discourages elits from participating in the modern sector, and reduces the incomes of the selected dalits rather than increasing it. High leakage—reflective of low state capacity—might prompt countries to forego tax-transfer and use more direct equity-enhancing policies (e.g., reservation) in early stages of development. Unless otherwise stated, we will henceforth assume

Assumption 3. $\lambda < \frac{1-\theta}{1-\theta+\phi}$

which ensures that tax-transfer is a viable redistribution policy for some relevant ranges of parameters.[19]

Can tax transfer be better than reservation and training? Yes, but only when r is large (and β is positive). Lemma 3 says that $\lim_{r\to\infty} \tau^0 < 1$. From Lemma 1 we know that $\lim_{r\to\infty} \tau^R \to 1$ So $\lim_{r\to\infty}(\tau^R - \tau^0) > 0$, implying that reduction in income is lower under tax-transfer than reservation when r is larger than some threshold value, say r_1. Similarly, training costs become infinitely large when $r \to \infty$ (Lemma 2). Hence $\lim_{r\to\infty}(\tau^T - \tau^0) > 0$, i.e., the reduction in income is lower under tax-transfer than training when skill-difference is larger than a threshold value, say r_2.

Thus, for $r > \max\{r_1, r_2\}$, tax-transfer fares better than either reservation and training. It remains to show that, even when the two policies are combined, tax-transfer remains the least costly policy for sufficiently high r. We know the optimal combination of training and reservation prescribes training to some level q^* and then placing the q^*-trained agents in the p-teams. Proposition 3 shows that as $r \to \infty$ the cost associated with at least one of these components becomes too large, and overshadows the cost associated with tax-transfer.

Proposition 3 (Optimality of tax-transfer). *Let Assumptions 2 and 3 be satisfied and let $\beta > 0$. Tax-transfer is more efficient than any combination of reservations and training when r is large enough.*

We can summarize our ranking of policies as follows:

[19] Thus tax-transfer can be made viable for any $\lambda < \theta$ by choosing a sufficiently large ϕ. The practicality of such a scale is an empirical matter.

(i) When $\beta = 0$, reservation is a more efficient policy than either training or tax-transfer, regardless of r.

(ii) When β is positive, reservation is the most efficient policy when r is small, but is dominated by a combination of training and reservations as r increases.

(iii) For positive β and sufficiently high values of r, tax-transfer dominates reservation and training as the most efficient policy.

Finally, note that while reservation (for low β and r) and tax-transfer (for high enough β and r) can be efficient when used singly, it is never efficient to use training on its own. When training is efficient, it is always optimal to train dalits to some target level $q^* < q$, after which they are promoted to high-skill teams using quotas or reservations. Thus as long as dalits sit on the other side of a class disadvantage, preferential treatment will always be essential to efficiently attain equity.

4 Discussion

The general conclusions that proceed from the preceding analysis can be summarized as follows. In economies with low complexity, reservation by itself is the most effective policy. Under reasonable parametric specifications, as complexity and skill differences increase, training enters the optimal mix, and policy must turn its attention to providing better facilities for disadvantaged groups to acquire skills, and only then promote the somewhat more qualified individuals by fiat. Quotas still remain relevant. Finally, in highly complex economies, the least costly way to contain inequality is by tax and transfer. The threshold where one regime overtakes another is defined by complexity β and the skill-difference index r, as well as the leakage λ in the fiscal system.

These conclusions are in accordance with some observations from the history of development in the underdeveloped economies, as well as policy choices in the developed ones.[20] In the early stages of development of the developing economies of the mid-twentieth century, the default path to growth was modern-sector expansion fuelled primarily by a migration of "unlimited supplies of labor" from the traditional subsistence sector to the modern sector. The latter sector only required rudimentary skills that were easy to acquire, but generated greater income by virtue of positional and resource advantage (Lewis 1954, see also Gollin 2014). There was discussion around whether growth led to modern sector *enlargement* that would reduce inequality over time, as opposed to modern sector *enrichment* which would enhance it (Fields, 1979, 1987), but the process was not presumed to be hobbled by lack of appropriate skills. For thinkers on the active policy front it was clear that if the gap between earnings of different

[20]Policy choices are influenced by many factors other than efficiency, such as perceptions of social and economic mobility (Alesina, Stantcheva, and Teso, 2018), which we have not considered in this model.

groups was to be bridged, it was first by using reservations and quotas to breach the hold of the elites on the more remunerative occupations. For example, employment and educational quotas for dalits were mandated in the Constitution of India, but predate it by several decades (Deshpande, 2005).

As modern sector growth continued, and some countries moved towards producing more and more complex products using increasingly sophisticated production techniques, reservations become increasingly inefficient, quotas were decried, and the focus turned to the development of skills. Correspondingly, the theory of endogenous growth emerged in the 1980s and emphasis shifted to human capital (see Barro, 1990; Ray, 1998, chapter 4). However, quotas never became completely irrelevant, even in the more developed countries like the US, though they attracted increasing opposition from the more conservative segments of society. In the meantime, countries with extremely well-functioning state mechanisms such as the Scandinavian countries moved on a different track, actively using tax-transfer mechanisms and extensive public goods provision to implement *ex post* egalitarianism, while interfering less with the allocation of labor.

This paper uses a stylized model to underline some fundamental observations about the effectiveness of equity policies at different stages of development. This forces us to ignore a number of considerations that are nevertheless important. It also leaves open the possibility of several extensions that could make the analysis more realistic.[21] Below we discuss the rationale for some of our simplifications.

4.1 Sectoral allocation of labor, and crowding out

We have left the determinants of the sectoral allocation of labor intentionally vague. In our model, low-skill workers always work in the traditional sector when they work on their own, even though a shift to the modern sector would increase their productivity. The development literature provides many indications why such an assumption, even if it is quick and dirty, might be realistic. The modern sector is a placeholder for an assortment of complementary factors that may be subject to crowding, in which case the gain in output from transferring a low-skill worker to the modern sector may be outweighed by the negative externalities on other workers.[22] Alternatively, the agglomeration of resources that constitute the modern sector may go hand-in-hand with a geographical location that is provided with public facilities necessary to produce skilled workers. The traditional sector in this case may be the rural sector that lacks the resources that complement modern production, and these resources may also double as the ones that are needed to mold youths into skilled workers. In this paper we have not addressed the complex of reasons that have led several generations of development

[21] A potential dynamic extension would analyze how the configuration of parameters in one generation can influence the trajectory of outcomes in the future (Mookherjee and Ray, 2010; Galor and Zeira, 1993).

[22] Admittedly, our model does not accommodate this interaction, but uses an assumption to substitute for it.

economists to find it reasonable to equate the rural, traditional and agricultural sector, and distinguish it from the urban, modern and industrial one.

An alternative construction is to think of modern sector places as scarce, with elits having preferential access. In the extreme case, only elits (but presumably not all of them) have access to that sector, and more enter as new places are created. Affirmative action would then reserve some of the newly created places for dalits. This is the formulation that perhaps best matches the affirmative action regime in India.

Much of the discontent with reservations in less-developed countries like India stems from the fact that some out of a fixed number of scarce places—for example in medical schools—are set aside for scheduled caste/tribe candidates. These places become unavailable for elite candidates (who may have scored higher in the medical admission exams), who thereby lose the opportunity that the dalits gain. The problem is further compounded if the dalit candidates are sufficiently lacking in prior preparation that they are unable to complete the program (Deshpande and Newman, 2007).

If some elits are left in the traditional sector along with the dalits, or if there is competition between the two groups for the desirable places, then in any outcome we must have some dalits in each sector, and some elits in each sector. Further, these agents may function in mixed teams (ps and qs) or in segregated teams, resulting in more than two income groups. As soon as the number of groups exceeds two we must face choices about how to measure inequality, which makes some of the conclusions less transparent. We will return to this in Section 4.3.

4.2 Training costs and the initial allocation of skills

In our model, dalits and elits come to the market with different endowments of skills. The paper is motivated by the idea that these differences arise from differences in privilege, family income and community resources, as well as prior discrimination. However, we do not explicitly model this, in part because (as this section discusses) this question needs an extensive analysis of its own.

An economic model of skill acquisition must be motivated by an appropriate definition of "skill", and the costs and benefits of its acquisition. Here we will content ourselves with a notional scale of skill that goes from "unskilled" to some level of "highly skilled". We conceive of the skill-acquisition process as a sequence of stages, which for expositional purposes may be set to two. In the first stage, children within the family and community acquire a base level of ability to learn and develop skills, and in the second stage they enter formal education or training to acquire workplace-relevant productive skills. Those who have higher ability will find it less costly at the margin to acquire a little more training, and as a result will acquire greater skill.

Individuals in the two groups: privileged and underprivileged, have different resources in the early stages that they may receive from family, community or the state. The elits and their communities are much better endowed with these resources than are dalits, and as a result the marginal cost of acquiring training at the second stage is smaller for the elit children. It then follows that, at the individual optimum, elits will acquire a higher level of training, and hence skill, than dalits.[23]

Once an equity policy is introduced, it increases the expected payoff of dalits at each skill-level, and reduces that of the elits. Thus in the presence of a policy, dalits will acquire a higher skill than otherwise, while elits will acquire a lower skill. Thus even dalits that are not chosen for promotion will train more and earn a somewhat higher income, while elits will train less. The net effect of these externalities on GDP per capita is ambiguous, but a full equilibrium model would take them into effect.

4.3 Wage determination

The most obvious simplification in our model is the assumption that, in each team, all workers receive a wage equal to the average product of the team, even when the team is heterogeneously comprised of high- and low-skilled workers. A reasonable market-founded or coalitional game-theory-founded wage functions would allocate higher wages to workers with higher skills. If we incorporate this in the current model, however, reservations would result in at least three income groups: dalits that remain in the low-output teams, dalits that move to high output teams, and elits.

The first group continues to earn a low income y_t, the second group earns a higher income y_m^{low} and the third group earns the highest income y_m^{high}, which is still lower than the full-separation high-income y_m. It is easy to see that the income distribution in the mixed teams now is a mean-preserving spread of the (constant) distribution that we obtained in Section 3.1, and hence there is more inequality than when incomes were equal within teams, though still less inequality than under full-separation.

However, to properly state the results related to inequality in this version, we would need to adopt a specific inequality function to compare inequality between policies, and in general different acceptable functions will yield different results (see also the end of Section 4.1). In the tightly schematic specification of policies that we have adopted in the paper, there are only two income groups at any time, and comparing efficiency across policies is straightforward. We feel it is worthwhile to emphasize the clear results regarding comparison using this simpler formulation, rather than state results that are contingent on specific properties of measurement functions, and obscure the direct tradeoff between inequality and efficiency.

[23]We need to tread carefully here because, with optimal allocation of workers to processes in the O-ring model, the marginal benefit of additional skill is not a downward sloping curve. However, if there is a determinate optimum to the individual problem then the conclusion remains correct.

4.4 Discrimination

A final and obvious omission in our model is that there is no role ascribed to inter-group animosity or discrimination. Dalits in our model are not barred from good jobs because the elits refuse to work alongside them, nor are they denied because of their color or other personal characteristics. A dalit who becomes suitably skilled faces no barriers in our model. This glaring omission is intentional. We feel this is a separate (though closely related) problem which, while possibly more pressing than lack of resources, is best treated in a different conceptual framework. There is a large literature on this subject (see Lang and Lehmann, 2012, for a recent survey), and we are not sure the present paper adds much to that question.

5 Conclusion

This paper analyzes stylized versions of equity-enhancing income policies in a schematic model of a two-class economy. We start from the premise that incomes are unequal in the first place because different classes have systematically different access to opportunities for skill acquisition, and hence to remunerative employment. We focus first on policies that equalize access to such employment, either directly through quotas, or through skill-improvement programs for disadvantaged groups.

In a context where production is conducted in interdependent teams, the main concern with quotas is that lower-skilled personnel not only produce lower output themselves but also produce an externality by compromising the productivity of higher-skilled co-workers. Training, on the other hand, incurs a cost that is likely to be large, for otherwise the relevant workers would presumably have themselves acquired the training. The optimal policy balances these two concerns.

The primary insight of this paper is twofold. First, we observe that the balance between reservation and training changes as the economy becomes more technologically complex. At low levels of complexity reservation is not only more efficient than training, it is efficient in an absolute sense in that it provides access to more productive employment for the hitherto underprivileged classes along with a rise in output. However, as complexity increases, teams grow more interdependent and the externality looms larger, so the optimal policy mix begins to include some positive amounts of training.

The second observation is that it is typically never optimal to eschew reservations (or preferential treatment for the underclass) altogether. To do so would require training the underprivileged workers until their skill-level is at par with the more privileged group. But by a familiar marginal argument, even at the optimum, some skill-difference must continue to be tolerated by an egalitarian employment regime.

Some countries widely use tax-and-transfer policies to keep inequalities under control, rather than quotas or access-enhancement.[24] We find that fiscal equalization policies can be optimal when skill-differences are very large, because quotas are costly in terms of output, and the cost of training increases steeply with the skill-gap. Correspondingly, tax-and-transfer is likely to be optimal only in very technologically advanced and fiscally competent countries.

This paper makes several simplifications, and the conclusions are compromised to that extent. We have assumed that initial inequalities (in ability or skill) result from prior inequalities in income and wealth. Individuals in each class have access to specific developmental resources during their formative period, and are conditioned by a specific cultural, educational and social environment. These coupled with the array of opportunities available in the economy determine the equilibrium level of skills with which they enter the economy. We take these entry levels as given, and do not concern ourselves with the effects of policies on the baseline skills, because our interest lies elsewhere.

Finally, note that we confine ourselves to static analysis. The implicit assumption here is that reducing the initial inequalities in one generation results in smaller initial discrepancies in the next one. We have not examined the mechanisms by which intergenerational effects work. However, the existing literature alerts us to the fact that dynamic trajectories can be far from straightforward (see, e.g., Mookherjee and Ray, 2003). These questions remain to be explored.

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[24]It is possible that this corrects for differences in fortune or preference, and not class disadvantage at all.

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Appendix

A Variations

A.1 Alternative tax-transfer formulations

In Section 3.5 we assumed that all high income workers (including the low-skilled workers that have been elevated) are taxed based on the notional pre-tax income. An alternative formulation of tax transfer is to assume that only p-types are taxed. Then

$$\text{Taxes} = (1-\theta)\tau y_m, \quad \text{Leakage} = (1-\theta)\tau\lambda y_m, \quad \text{Transfers} = \phi((1-\tau)y_m - y_t)$$

where the last equality follows from noting that post tax-transfer income, i.e.,$(1-\tau)y_m$ has to be the same for both p-types and the fraction ϕ of q-types. As taxes (less leakage) must equal transfers, we have

$$(1-\theta)\tau^0(1-\lambda)y_m = \phi((1-\tau^0)y_m - y_t).$$

Rearranging this gives:

$$\tau^0 = \frac{\phi(1-\frac{y_t}{y_m})}{(1-\theta)(1-\lambda)+\phi} = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta}\frac{1}{r^{\beta\mu}})}{(1-\theta)(1-\lambda)+\phi}.$$

The two tax rates - one above and one in the main text - differ only because the incidence of leakage is different in the two cases, they are the same when $\lambda = 0$. There is no qualitative difference between the two specifications.

A.2 Capital

In the formulation in the paper we have ignored capital, since it is peripheral to our concerns. However, capital can be accommodated without complications, as shown below.

Suppose output is produced using both labor and capital (z). Using a Cobb-Douglas formulation, let

$$\begin{aligned} Y(p,q,n,\beta,z) &= \left(s^{1-\beta}(G(n,\beta)p^{\beta n(1-\gamma)}q^{\beta n\gamma}\right)^{\alpha}\left(z\right)^{1-\alpha} \\ &= \left(s^{1-\beta}(n^{\frac{1}{\beta}+\mu}p^{n(1-\gamma)}q^{n\gamma})^{\beta}\right)^{\alpha}\left(z\right)^{1-\alpha}. \end{aligned} \tag{19}$$

Suppose the rental rate of capital is ρ. A n-member team chooses z to maximize average income net of rental payments

$$\frac{X^{\alpha}z^{1-\alpha}-\rho z}{n} = \left(\frac{z}{n}\right)\left((\frac{X}{z})^{\alpha}-\rho\right)$$

where

$$X = s^{1-\beta}(G(n,\beta)p^{\beta n(1-\gamma)}q^{\beta n\gamma}$$

First-order condition corresponding to the maximization problem is given by:

$$(1-\alpha)(\frac{X}{z})^{\alpha} = \rho \Longleftrightarrow z = (\frac{1-\alpha}{\rho})^{\frac{1}{\alpha}} X.$$

Using the equation above we can express the maximand as

$$(\frac{\rho\alpha}{(1-\alpha)})(\frac{z}{n}) = \alpha(\frac{\rho}{(1-\alpha)})^{1-\frac{1}{\alpha}} \frac{X}{n}.$$

Substituting $G(n, \beta) = n^{1+\beta\mu}$ and simplifying, the maximization problem boils down to choosing n to maximize

$$X = s^{1-\beta}(n^{\mu} p^{n(1-\gamma)} q^{n\gamma})^{\beta}.$$

which is the same problem as in the main model.

B Proofs

Proof of Proposition 1

Proof. Dividing $\bar{y}^R - \bar{y}$ by y^m we get

$$\begin{aligned}
\frac{\bar{y}^R - \bar{y}}{y_m} &= \phi\left(1 - \frac{y_t}{y_m}\right) - \left((1-\theta+\phi))(1 - \frac{y_m^R}{y_m})\right) \\
&= \phi\left(1 - (\frac{s_t}{s_m})^{1-\beta} \frac{1}{r^{\beta\mu}}\right) - \left((1-\theta+\phi))(1 - \left(\frac{1-\theta+\phi}{1-\theta+\phi r}\right)^{\beta\mu})\right) \equiv \Delta^R(r, \beta), \text{ say}
\end{aligned}$$

Observe that

$$\lim_{\beta \to 0} \Delta^R(r, \beta) = \phi(1 - (\frac{s_t}{s_m})) > 0$$

which implies that for any r, there exists a cutoff value $\underline{\beta}(r)$ such that $\Delta^R(r, \beta) > 0$ for all $\beta < \underline{\beta}(r)$. This proves the first part of Proposition 1. To prove the second part of the Proposition note that:

$$\begin{aligned}
\lim_{r \to 1} \Delta^R(r,\ \beta) &= \phi\left(1 - (\frac{s_t}{s_m})^{1-\beta}\right) > 0, \\
\lim_{r \to \infty} \Delta^R(r,\ \beta) &= -(1-\theta) < 0.
\end{aligned}$$

The claim - in particular the existence of a unique cutoff $\underline{r}(\beta)$ then follows from noting that

$$\frac{d\Delta^R(r,\ \beta)}{dr} = \frac{\phi\beta\mu}{r^{\beta\mu+1}}\left((\frac{s_t}{s_m})^{1-\beta} - (\frac{1-\theta+\phi}{\frac{1-\theta}{r}+\phi})^{\beta\mu+1}\right) < 0$$

The inequality is due to (a) $\frac{s_t}{s_m} < 1$ and (b) $\frac{1-\theta+\phi}{\frac{1-\theta}{r}+\phi} > 1$. □

Proof of Lemma 1

Proof. (i) That τ^R is increasing in r and β is immediate from observing (11).

(ii) Limiting values of τ^R follows from noting that

$$\lim_{\beta\to 0}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=1, \quad \lim_{r\to 1}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=1, \quad \text{and} \quad \lim_{r\to\infty}(\frac{1-\theta+\phi}{1-\theta+\phi r})^{\beta\mu}=0.$$

□

Proof of Lemma 2

Proof. (i) The limiting values, i.e. $\lim_{\beta\to 0}\tau^T>0$, and $\lim_{r\to 0}\tau^T=0$, follow from the expression of τ^T in (12), once we note that the denominator in the right-hand side of (12) is always positive, while the numerator is positive for all $\beta\in[0,1]$, and approaches zero as $r\to 1$.

(ii) We have

$$y_m={s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}=s_m(\frac{\mu r}{eks_m^{\frac{1}{\mu}}})^{\beta\mu},$$

Differentiating y_m with respect to β gives:

$$\frac{dy_m}{d\beta}=\mu(\frac{\mu r}{eks_m})^{\beta\mu}ln(\frac{\mu r}{eks_m^{\frac{1}{\mu}}})\gtreqless 0\iff r\gtreqless\frac{eks_m^{\frac{1}{\mu}}}{\mu}. \tag{20}$$

From (12) we have $\frac{d\tau^T}{d\beta}\gtreqless 0\iff\frac{dy_m}{d\beta}\lesseqgtr 0$, which, together with (20) implies that τ^T is decreasing in β if and only if

$$r>\frac{ek{s_m}^{\frac{1}{\mu}}}{\mu},$$

and increasing in β if the inequality is reversed.

(iii) That τ^T could be increasing or decreasing in r follows from noting that both $\psi(p)-\psi(q)=\psi(e^{-\frac{k}{r}})-\psi(e^{-k})$ in the numerator and y_m in the denominator are increasing in r. Unless we specify $\psi(.)$ exactly, both possibilities remain: τ^T is increasing in r if the numerator increases with r at a faster rate than the denominator, and decreasing in r in the converse case.

Claim 1: Suppose $\lim_{p\to 1}\psi(p)(\ln\frac{1}{p})^{\mu}=\infty$. Then $\lim_{r\to\infty}\tau^T=\infty$.

Proof: Using $y_m={s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}$ from (8) and noting that $p=e^{-\frac{k}{r}}\iff r=\frac{k}{\ln\frac{1}{p}}$, we can express τ^T in (12) as

$$\tau^T=\frac{(\psi(p)-\psi(q))(\ln\frac{1}{p})^{\beta\mu}}{(1-\theta+\phi)(1-\lambda)s_m^{1-\beta}(\frac{\mu}{e})^{\beta}}$$

Taking limits we get

$$\lim_{p\to 1}\tau^T=\frac{\lim_{p\to 1}\psi(p)(\ln\frac{1}{p})^{\beta\mu}-\psi(q)\lim_{p\to 1}(\ln\frac{1}{p})^{\beta\mu}}{(1-\theta+\phi)(1-\lambda)s_m^{1-\beta}(\frac{\mu}{e})^{\beta}}.$$

Since $\lim_{p\to 1}(\ln\frac{1}{p})^{\beta\mu}=0$ for all $\beta\mu>0$ and $\psi(q)<\infty$, we have

$$\psi(q)\lim_{p\to 1}(\ln\frac{1}{p})^{\beta\mu}=0. \tag{21}$$

We also have

$$\lim_{p\to 1}\psi(p)(\ln\frac{1}{p})^{\beta\mu} \geq \lim_{p\to 1}\psi(p)(\ln\frac{1}{p})^{\mu} = \infty \tag{22}$$

where the first inequality follows from noting that $(\ln\frac{1}{p})^{\beta\mu} > (\ln\frac{1}{p})^{\mu}$ when p is close to 1, and the second inequality follows from applying Assumption 2. Together, (21) and (22) imply that

$$\lim_{p\to 1}\tau^T = \infty$$

Since $p \to 1 \Leftrightarrow r \to \infty$ we have $\lim_{r\to\infty}\tau^T = \infty$.[25]

□

Proof of Lemma 3

Proof. That τ_0 is increasing in r, β and λ is immediate from observing (18). Limiting values of τ_0 follows from noting that

$$\lim_{\beta\to 0}\tau^0 = \frac{\phi(1-\frac{s_t}{s_m})}{(1-\theta+\phi)(1-\lambda)} > 0, \qquad \lim_{r\to 1}\tau^0 = \frac{\phi(1-(\frac{s_t}{s_m})^{1-\beta})}{(1-\theta+\phi)(1-\lambda)} > 0,$$

and

$$\lim_{r\to\infty}\tau^0 = \frac{\phi}{(1-\theta+\phi)(1-\lambda)} < 1.$$

□

Proof of Proposition 3

Proof. For a given p and q satisfying $0 < q < p < 1$, suppose the optimal policy is to raise the skill level of ϕ dalits to $q' \in (q, p)$ and then apply reservation so that they work alongside elit p-types in modern sector teams. As in the main text, let $q = e^{-k}$ and $p = e^{-\frac{k}{r}}$ where $r > 1$. There exists a unique $r' \in (1, r)$ such that $q' = e^{-\frac{k}{r'}}$.

Cost of raising the skills of ϕ dalits from q to q' via training is $\phi C(q', q)$ which requires raising $\frac{\phi C(q',q)}{(1-\lambda)(1-\theta+\phi)}$ in taxes from each individual in the modern sector. Reservation for q'-types reduces per-capita income in a modern sector team from

$${s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu} \equiv y_m$$

to

$${s_m}^{1-\beta}(\frac{\mu r}{ek})^{\beta\mu}\left(\frac{1-\theta+\phi}{1-\theta+\phi(\frac{r}{r'})}\right)^{\beta\mu} = y_m\left(\frac{1-\theta+\phi}{1-\theta+\phi(\frac{r}{r'})}\right)^{\beta\mu}.$$

[25]When $\beta = 0$, $(\ln\frac{1}{p})^{\beta\mu} = 1$. Then the result immediately follows from the expression of τ^T, noting that $\psi(q)$ is finite and $\psi(p) \to \infty$ as $p \to 1$.

Per-capita income in a modern sector team corresponding to optimal combined policy is

$$\begin{aligned} y_m^C &= \left(\left(\frac{1-\theta+\phi}{1-\theta+\phi(\frac{r}{r'})}\right)^{\beta\mu} - \frac{\phi C(q',q)}{(1-\lambda)(1-\theta+\phi)y_m}\right)y_m \\ &= (1-\tau^C)y_m \end{aligned}$$

where

$$\tau^C = 1 - \left(\frac{1-\theta+\phi}{1-\theta+\phi(\frac{r}{r'})}\right)^{\beta\mu} + \left(\frac{\phi C(q',q)}{(1-\lambda)(1-\theta+\phi)y'_m}\right)\left(\frac{y'_m}{y_m}\right) \tag{23}$$

and

$$y'_m = {s_m}^{1-\beta}(\frac{\mu r'}{ek})^{\beta\mu} = y_m\left(\frac{r'}{r}\right)^{\beta\mu}$$

is the per-capita income of an optimally sized team where all team members have skill level q'.

By definition, $\tau^C \leq \tau^R$. The properties of τ^R in Lemma 1 imply that $\tau^R \leq 1$ which in turn implies $\tau^C \leq 1$. We establish that for all $\beta > 0$,

$$\lim_{r\to\infty} \tau^C = 1. \tag{24}$$

Suppose not. That is, suppose $\lim_{r\to\infty} \tau^C < 1$. It must be that $\lim_{r\to\infty} \frac{r'}{r} > 0$, because, otherwise, if $\lim_{r\to\infty} \frac{r'}{r} = 0$, $\left(\frac{1-\theta+\phi}{1-\theta+\phi(\frac{r}{r'})}\right)^{\beta\mu} \to 0$ as $r \to \infty$. Then (23) implies $\lim_{r\to\infty} \tau^C \geq 1$ which contradicts $\lim_{r\to\infty} \tau^C < 1$.

Proposition 3 then follows from noting that for all $\beta > 0$

$$\lim_{r\to\infty} \tau^0 < 1 = \lim_{r\to\infty} \tau^C.$$

where the inequality and the equality are due to Lemma 3 and (24) respectively.[26] $\square$

[26] If $\beta = 0$, choosing $r' = 1$ gives $\tau^C = \tau^R = 0$ irrespective of r. Then, $\lim_{r\to\infty} \tau^C = 0$. Tax-transfer is no longer efficient since $\lim_{r\to\infty} \tau^0 > 0 = \lim_{r\to\infty} \tau^C$.