SHORT-TERM GENERATION ASSET VALUATION: A REAL OPTIONS APPROACH

CHUNG-LI TSENG
Department of Civil & Environmental Engineering, University of Maryland, College Park, Maryland 20742, chungli@eng.umd.edu

GRAYDON BARZ
Electricity & Natural Gas Practice, McKinsey & Co., Two Houston Center, 909 Fannin, Suite 3500, Houston, Texas 77010, gbarz@yahoo.com

(Received July 1999; accepted September 2000)

This paper discusses using real options to value power plants with unit commitment constraints over a short-term period. We formulate the problem as a multistage stochastic problem and propose a solution procedure that integrates forward-moving Monte Carlo simulation with backward-moving dynamic programming. We assume that the power plant operator maximizes expected profit by deciding in each hour whether or not to run the unit, that a certain lead time for commitment and decommitment decisions is necessary to start up and shut down a unit, and that these commitment decisions, once made, are subject to physical constraints such as minimum uptime and downtime. We also account for the costs associated with starting up and shutting down a unit. Last, we assume that there are hourly markets for both electricity and the fuel used by the generator and that their prices follow Ito processes. Using numerical simulation, we show that failure to consider physical constraints may significantly overvalue a power plant.

With deregulation of the electricity industry a global trend, utilities and power generators must adjust to the new risks of volatile spot prices in the competitive marketplace. Because they are no longer able to rely on embedded cost recovery regulation, these organizations must fundamentally change the way they view power plant operation. For example, the deterministic and cost-based unit commitment problem (e.g., Sheble and Fahd 1994, Tseng 1996) that schedules power plants to satisfy demand should be replaced by an optimization problem which is, at least, price-based and takes into account price stochastics. Solving such an optimization problem not only yields the optimal commitment decision, but also reveals the power plant's value over the operating period.

Unfortunately, recent work that has tackled this power plant valuation problem using financial options theory has overlooked the plant's unit-commitment constraints. Specifically, these financial options approaches implicitly assume (i) a zero startup time, i.e., a unit can be started up immediately when favorable prices are observed; and (ii) no minimum up/downtime constraints, e.g., an online unit can be turned down whenever the prices become unfavorable. Consequently, these approaches always result in a nonnegative payoff for operating a power plant and suggest the operator faces no risk of loss. However, as this paper shows, this is not the case; the error made by failing to consider such physical constraints is often significant.

To incorporate these unit constraints, we formulate the power plant valuation problem as a multistage stochastic problem with the prices for electricity and the fuel consumed characterized as uncertainties. Specifically, as in the financial options approaches, we assume that there are hourly markets for both electricity and fuel and that their prices follow Ito processes. Over a short-term period, the operator must decide when to run the generator so as to maximize expected profit. In our approach, however, the commitment decision must be made before the uncertain prices are revealed, and, once the power plant is in operation, it is subject to physical limitations such as minimum uptime and downtime constraints. For example, an online unit cannot be turned back down even if market prices become unfavorable before the minimum uptime constraint is fulfilled. Similarly, an offline unit cannot be turned back on before the minimum downtime constraint is fulfilled. Additionally, in our approach, we include the startup/shutdown cost associated with turning on/off a unit.

To solve this multistage stochastic problem, we use Monte Carlo simulation in conjunction with backward dynamic programming. Specifically, we use simulation to determine the optimal decision strategy for plant operation in the last period and then repeat the process moving backward, having obtained all optimal decision strategies for the subsequent time periods. Thus, the complete procedure involves repeated backward-moving dynamic-programming recursion and forward-moving simulation.

Because the valuation method presented in this paper takes price processes as inputs, it can be applied to general Ito price processes. In a later section, however, we also suggest specific processes for both electricity and fuel prices based on observed characteristics from existing markets such as mean reversion, seasonality, and lognormal

Subject classifications: Natural resources: energy. Decision analysis: applications. Finance: investment.
Area of review: ENVIRONMENT, ENERGY, AND NATURAL RESOURCES.

0030-364X/02/5002-0297 $05.00
1526-5463 electronic ISSN

Operations Research © 2002 INFORMS
Vol. 50, No. 2, March–April 2002, pp. 297–310

distributions. In addition, using such processes enables us to numerically evaluate our proposed methodology.

This paper is organized as follows. In §2, we provide a background review of the financial options approaches and illustrate the need for incorporating the physical constraints of power plant operation into the valuation methodology. In §3, we describe these physical constraints, along with a simplified deterministic model that contains no price uncertainties, and we provide the corresponding solution procedure. In §4, we generalize the model and its solution to the multistage stochastic case. We present numerical results in §5 and conclude the paper in §6.

1. BACKGROUND

The recent application of financial concepts in electric power markets—for example in demand-side management (Gedra and Vayaiya 1993, Gedra 1994), transmission pricing (e.g., Oren et al. 1995), and the growing importance of real options and risk management in power plant operations (Kaminski 1997)—suggests such concepts may also be of use in project valuation. Indeed, capitalizing on the physical optionality of power plant operation, Hsu (1998a, 1998b) and Deng et al. (1998) use financial options techniques to tackle the problem of power plant valuation.

The fundamental idea of these approaches is as follows: A power plant, with its associated *heat rate*, converts a particular fuel into electricity. Because this conversion involves two commodities with different market prices, owning a power plant can be regarded as holding call options of *spark spreads*, defined as the electricity price less the product of the heat rate associated with the generator and the fuel price. When the electricity price is high but the fuel price is low (so that their ratio is greater than the unit's heat rate), the power plant should run to capitalize on the profitable price spread. When the spark spread is negative, the power plant should not run. Because the power plant profits increase as the spread increases, and because its losses are bounded, the power plant resembles a call option on the price spread. Similar price spread concepts have also been applied to other industries such as oil refining (e.g., Shimko 1994).

More specifically, the heat rate of a power plant, H, is defined as the conversion ratio between the electricity and fuel (or heat content). The units of H are MMBtu/MWh so that a higher heat rate implies a lower operating efficiency. For example, a generator with a heat rate H, generating 1 MWh electricity, requires H MMBtu of fuel heat content. Because this conversion involves two marketed commodities, the payoff of a generator can be modeled as a linear system of their market prices. Assuming H is known, for every 1 MWh electricity generation:

$$\text{Payoff } = p^E - H \cdot p^F, \tag{1}$$

where p^E (\$/MWh) and p^F (\$/MMBtu) stand for electricity and fuel prices, respectively. Given this situation, a rational plant operator will decide to run a unit only if $p^E \geqslant H \cdot p^F$, because in this case the operator will profit by purchasing fuel, using the generator to convert the fuel to electricity, and then selling the electricity in the market.

In the works of Hsu (1998a, 1998b) and Deng et al. (1998), the authors estimate the Linear Payoff Function (1) of a power plant to determine the plant's value. Specifically, over a period $[0, T]$, they propose:

$$\text{Power plant value} = \sum_{t=1}^{T} E_0[\max(p_t^E - Hp_t^F, 0)], \tag{2}$$

so that given the price processes of electricity and the fuel, a power plant's value may be estimated by a series of (European) spark-spread call options (expiring at t) as in (2). Moreover, for price processes such as geometric Brownian motion (GBM) and "geometric mean-reversion" (GMR), Deng et al. (1998) provide analytical solutions to (2). Because of the nature of these methodologies, we shall refer to them as the financial options approaches in this paper. Note that because these approaches value a power plant on a per MWh output basis, they are also known as "unit capacity valuation."

While using option theory to value a power plant is novel, such approaches necessarily overlook a power plant's operational constraints. In particular, using (2) to value a power plant assumes that:

(1) Unit-commitment decisions are made after the prices p_t^E and p_t^F are observed. Or, equivalently, a unit can be immediately started up if the market prices are favorable, and vice versa. This implies that there is no commitment-decision lead time required.

(2) There are no intertemporal constraints for the commitment: A unit can be committed/decommitted at any time.

(3) The unit heat rate H is a constant at all levels of power generation.

In reality, a power plant does have physical constraints such as nonzero startup/shutdown times (lead time), nonzero minimum uptime and minimum downtime, and a heat rate that varies over different levels of generation output. Ignoring these physical constraints thus overestimates a power plant's value. And, in the increasingly volatile electricity market, such assumptions may involve considerable risk.

To address these concerns, we specifically incorporate these physical constraints into our model. In the next section, we begin by describing these physical constraints in detail and subsequently propose our valuation methodology.

2. POWER-GENERATION UNIT COMMITMENT

In this paper we mainly focus on valuing fossil-fueled steam (thermal) units whose unit-operation dynamics are relatively complex. The power-generation process of a steam unit begins with heating water in its boiler and, therefore, requires time to start up or shut down the generator. Additionally, a thermal-generation unit cannot switch between the online mode and the offline mode at an arbitrary frequency, due to both the nonzero response time of the unit and the damaging effects of fatigue. Consequently, once a thermal unit is shut down (or started up), it is required to say offline (or online) for a minimum period, known as the minimum down- (or up-) time, before it can be started up (or shut down) again. These constraints are modeled in the following section.

2.1. Physical Constraints for a Power Plant

In the model development, we first introduce the following standard notation. Additional symbols will be introduced when necessary.

- t: index for time ($t = 0, \ldots, T$).
- τ: unit startup time, i.e., commitment decision lead time ($\tau \geqslant 1$ and $\tau \in Z$, a set of integers).
- ν: unit shutdown time, i.e., commitment decision lead time ($\nu \geqslant 1$ and $\nu \in Z$).
- x_t: state variable whose sign indicates whether the unit is started up (+) or shut down (−), and whose magnitude indicates the length of time at time period t the unit has been in this mode.
- u_t: zero-one generation unit-commitment decision variable made in time period t.
- t^{on}: the minimum number of periods the unit must remain on after it has been turned on ($t^{\text{on}} \in Z$).
- t^{off}: the minimum number of periods the unit must remain off after it has been turned off ($t^{\text{off}} \in Z$).
- t^{cold}: the number of periods after a unit has been turned off until its boiler has completely cooled down ($t^{\text{cold}} \in Z$).
- q_t: decision variable indicating the amount of power the unit is generating in time period t.
- $q^{\min}$: minimum rated capacity of the unit.
- $q^{\max}$: maximum rated capacity of the unit.
- R: unit ramp rate.
- p_t^E: electricity price (\$/MWh) in time period t.
- p_t^F: fuel price (\$/MMBtu) in time period t.
- $C(q_t, p_t^F)$: fuel cost for operating the unit at output level q_t in time period t when the fuel price is p_t^F.
- $S_u(x_t)$: startup cost associated with turning on the unit in time period t.
- $S_d(x_t)$: shutdown cost associated with turning off the unit.

In this paper, unless otherwise stated, the unit of time period is hours. The physical constraints of a power plant are modeled below.

Minimum Up/Downtime Constraints

$$u_t = \begin{cases} 1, & \text{if } 1 \leqslant x_t < t^{\text{on}}, \\ 0, & \text{if } -t^{\text{off}} < x_t \leqslant -1, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \tag{3}$$

State-Transition Constraints

$$x_t = \begin{cases} \min(t^{\text{on}}, x_{t-1}+1), & \text{if } 0 < x_{t-1} \\ & \quad \text{and } u_{t-1} = 1, \\ -1, & \text{if } x_{t-\nu} = t^{\text{on}} \\ & \quad \text{and } u_{t-\nu} = 0, \\ \max(-t^{\text{cold}}, x_{t-1}-1), & \text{if } x_{t-1} < 0 \\ & \quad \text{and } u_{t-1} = 0, \\ 1, & \text{if } x_{t-\tau} \leqslant -t^{\text{off}} \\ & \quad \text{and } u_{t-\tau} = 1. \end{cases} \tag{4}$$

The state-transition diagram is given in Figure 1. In (4) the state space of x_t is summarized as a finite set $\{x \in Z \mid -t^{\text{cold}} \leqslant x \leqslant t^{\text{on}}, x \neq 0\}$, where $t^{\text{cold}} \geqslant t^{\text{off}}$ represents the time period beyond which the temperature of the boiler is so low that the unit state can be approximated by $-t^{\text{cold}}$. Consequently, referring to (3) we define

$$\Phi \equiv \{x \in Z \mid x = t^{\text{on}} \text{ or } -t^{\text{cold}} \leqslant x \leqslant -t^{\text{off}}\}$$

as the set of states in which the unit commitment may be adjusted (i.e., on-off). If $x_t > 0$ we say the unit is *online* at time t, and is *offline* if $x_t < 0$.

Unit-Capacity Constraints

$$q^{\min} \cdot \operatorname{sign}(x_t) \leqslant q_t \leqslant q^{\max} \cdot \operatorname{sign}(x_t), \quad t = 1, \ldots, T, \tag{5}$$

where

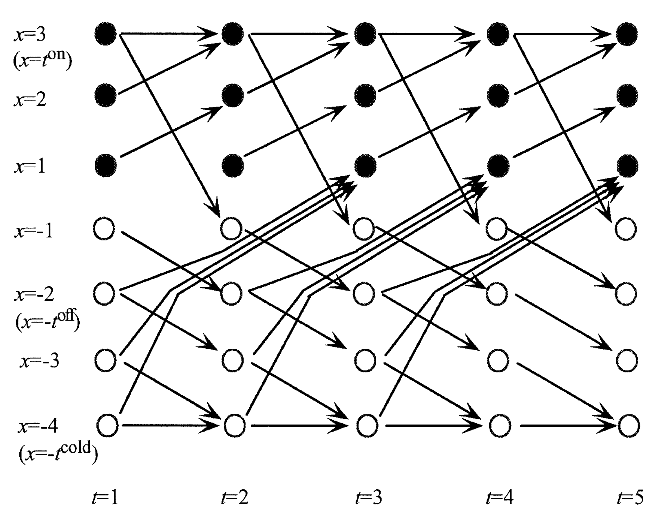
$$\operatorname{sign}(x) \equiv \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Note that (5) implies (i) $q_t = 0$ if $x_t < 0$; (ii) $q_t \in [q^{\min}, q^{\max}]$ if $x_t > 0$.

Ramp Constraints

$$|q_t - q_{t-1}| \leqslant R, \quad \text{if } u_t \cdot u_{t-1} = 1, t = 1, \ldots, T, \tag{6}$$

Figure 1. An example of the state transition diagram: $t^{\text{on}} = 3$, $t^{\text{off}} = 2$, $t^{\text{cold}} = 4$, $\tau = 2$, and $\nu = 1$.



where $R > 0$ is called the *ramp rate* of the unit. The ramp constraints are only applied to the generation levels of any two successive online periods.

Note that when an offline unit is started up, it takes time to gradually increase generation to reach the minimum rated capacity $q^{\min}$. Similarly, when an online unit is turned off, its generation level has to reduce to $q^{\min}$ subject to Ramp Constraints (6), then from $q^{\min}$ gradually reducing to 0. In the state-transition diagram in Figure 1, the startup period (shutdown period) is implicitly accounted for by the transaction arc from $x_t = 0$ to $x_{t+\tau} = 1$ (from $x_t = t^{\text{on}}$ to $x_{t+\nu} = -1$).

Initial conditions on x_t and q_0 at $t = 0$. That is, x_0 and q_0 equal to some initial state, say $\tilde{x}_0$ and $\tilde{q}_0$.

2.2. Modeling the Cost Functions

As aforementioned, the input-output characteristic of a generating unit is captured by $H(q)$ (MMBtu), which is a function describing the heat required to generate q (MW) of power; see Wood and Wollenberg (1984). As is standard, we model $H(q)$ as a quadratic function of the output q, so that the fuel cost may be represented as follows:

$$C(q_t, p_t^F) \equiv H(q_t)p_t^F \equiv (a_0 + a_1 q_t + a_2 q_t^2)p_t^F.$$

Moreover, we take each coefficient a_j $(j = 1, 2, 3)$ of $H(\cdot)$ to be nonnegative. In general, $a_2 > 0$, so that the cost function is convex; $a_1 > 0$ since $a_1 q_t$ (MMBtu) is the major component of the heat-power conversion; and $a_0 > 0$ because $a_0 p_t^F$ captures the so-called "no-load cost" associated with keeping the plant running with no power output in order to maintain immediate availability. Note that this approach significantly generalizes the previously mentioned financial options approach, in which q_t is either 0 or a fixed quantity, and H is linear in q (i.e., $a_0 = a_2 = 0$).

In addition to fuel costs, the generating costs of a thermal unit also include startup and shutdown costs. The startup costs $S_u(x_t)$ vary with the temperature of the boiler. The longer the generator is down, the more heat is lost from its boiler and the greater the expense to reheat the water. In practice, a boiler is assumed to cool at an exponential rate inversely proportional to a cooling constant γ, and we denote this function by

$$S_u(x_t) = \begin{cases} b_1(1-\exp(x_t/\gamma)) + b_2, & \text{if } -t^{\text{cold}} \leqslant x_t \leqslant -t^{\text{off}}, \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

where b_1 represents the cold-start fuel cost for the unit and b_2 combines the labor costs plus the fixed operating and maintenance expenses of the plant amortized over the unit. To limit the size of the state space, we assume that $S_u(x_t)$ can be approximated by $S_u(-t^{\text{cold}})$ when $x_t < -t^{\text{cold}}$.

Finally, in this paper we model the shutdown costs S_d as a function of x_t:

$$S_d(x_t) = \begin{cases} S_d \text{ (a constant)}, & \text{if } x_t = t^{\text{on}}, \\ 0, & \text{otherwise,} \end{cases}$$

where S_d characterizes the labor and maintenance costs involved in plant shutdowns.

2.3. Ramp Constraints

Ramp Constraints (6), which limit the capacity changes of a generator from one online period to the next, may significantly impact the value of the generation asset. Detailed models for incorporating ramp constraints into power plant operations can be found in Svoboda et al. (1997) and Tseng (1996). Because we describe the generation level q_t by a continuous variable, to incorporate ramp constraints into the finite state valuation problem requires discretizing q_t, which significantly increases the dimension of the dynamic-programming state space. Thus, to simplify the exposition of our approach, we initially leave out ramp constraints from the model development. However, in a later section, we revisit these constraints, discussing both an exact and a heuristic method for incorporating them into the valuation problem.

2.4. The Deterministic Case

The purpose of this paper is to determine the value of a power plant, where this value is determined by the maximum possible profit that the unit can achieve from operation over the entire planning period. Thus, the power plant valuation problem is, by default, also an optimal-operation problem. Consequently, we present a solution procedure for the valuation problem by solving the optimal-operation problem. In this section, we assume that prices for electricity and the fuel are fully and perfectly known.

The objective is to maximize the total profit subject to Constraints (3), (4), and (5) and the initial conditions, omitting (6), as mentioned above.

$$J^* = \max_{u_t, q_t} \sum_{t=0}^{T} f_t(x_t, q_t; p_t^E, p_t^F) - S_u(x_t)u_t - S_d(x_t)(1-u_t), \tag{P}$$

where

$$f_t(x_t, q_t; p_t^E, p_t^F) \equiv (p_t^E q_t - C(q_t, p_t^F))\text{sign}(x_t) \tag{8}$$

is the profit associated with the state x_t at time t, and where the last two terms in the objective depend on the commitment and decommitment decisions.

In the problem formulation, the operator needs to make a commitment decision in every (feasible) time period, i.e., determine u_t, and dispatch the generator if online by determining q_t. In reality, a generator is dispatched by the Automatic Generation Control (AGC) system in real time.

Therefore, we assume the dispatch problem can be solved *instantaneously* and *optimally* after the prices are revealed. Assuming a unit is online at time t, its optimal dispatch problem is as follows:

$$\max\ p_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2) p_t^F$$
$$\text{s.t.}\quad q^{\min} \leqslant q_t \leqslant q^{\max}. \tag{9}$$

Thus, the optimal generation denoted by g_t can be obtained by

$$g_t \equiv \min\left(q^{\max}, \max\left(q^{\min}, \frac{1}{2a_2}\left(\frac{p_t^E}{p_t^F} - a_1\right)\right)\right). \tag{10}$$

In the sequel, the decision variables q_t will be dropped out of the formulation and replaced by g_t. Note that g_t is a function of the prices p_t^E and p_t^F. Consequently the optimization problem, originally formulated as a mixed-integer program, is reduced to a 0-1 integer program.

To solve the deterministic case, let $F_t(x_t)$ be the power plant value if the period starts at t in state x_t. We employ backward dynamic programming to solve the deterministic problem. The recurrence equations are as follows:

- If $x_t = t^{\text{on}}$, $t \leqslant T - \nu$,

$$F_t(x_t) = f_t(x_t, g_t; p_t^E, p_t^F) + \max_{u_t}[F_{t+1}(t^{\text{on}})u_t + (F_{t+\nu}(-1) - S_d(x_t)) \times (1 - u_t)]. \tag{11a}$$

- If $-t^{\text{cold}} \leqslant x_t \leqslant -t^{\text{off}}$, $t \leqslant T - \tau$,

$$F_t(x_t) = f_t(x_t, g_t; p_t^E, p_t^F) + \max_{u_t}[(F_{t+\tau}(1) - S_u(x_t))u_t + F_{t+1}(x_{t+1})(1 - u_t)]. \tag{11b}$$

- If (t, x_t) is not in either one of the two cases above,

$$F_t(x_t) = f_t(x_t, g_t; p_t^E, p_t^F) + F_{t+1}(x_{t+1}), \tag{11c}$$

where x_{t+1} in the above equations is subject to state-transition constraints (4). The boundary conditions are

$$F_T(x_T) = f_T(x_T, g_T; p_T^E, p_T^F). \tag{12}$$

In both (11a) and (11b), the right-hand side of the equation contains $F_{t+\nu}$ and $F_{t+\tau}$, respectively, where we generally assume that ν, $\tau \geqslant 1$. While somewhat atypical, this representation of Bellman's equation allows us to show the effect of nonzero startup or shutdown time. Alternatively, one may introduce additional states to capture subperiods within the startup or shutdown period so as to formulate (11a) and (11b) in a more standard form with only F_{t+1} on the right-hand side.

The optimal value J^* of this deterministic problem can be obtained from the last step of the dynamic-programming algorithm as

$$J^* = F_0(\tilde{x}_0),$$

where we use a *tilde* to denote that the value of x_0 is known with certainty at Time 0.

Note that because we focus on short-term periods (hours to days), discount factors are not applied in the recurrence relations. Should the period be long enough that the time value of money becomes important, future values of $F_{t'}(\cdot)$ $(t' > t)$ should be discounted, for example, be multiplied by $\exp(-r(t' - t))$, where r is the interest rate.

3. A MULTISTAGE STOCHASTIC MODEL

Obviously, price uncertainties plead for a stochastic model. In this section, we accommodate this need by formulating the generation asset valuation problem as a multistage decision-making problem.

3.1. Model Statement

In our stochastic model, a commitment decision must be made before the commitment becomes effective. For example, the decision variable u_t is made based on observed p_t^E and p_t^F. If $u_t = 1$, the unit will be committed "on" at time $t + \tau$, although the prices at time $t + \tau$ remain uncertain. However, we continue to assume that the generation level q_t can be adjusted optimally in real time (i.e., to g_t) to maximize profit within the capacity range $[q^{\min}, q^{\max}]$. Last, we assume that the operator is risk neutral so that his objective is to maximize expected profit with respect to the random price vectors $(\mathbf{p}^{\text{E}}, \mathbf{p}^{\text{F}})$, representing some subjective price probability distributions believed by the operator.

At time t, the operator observes the prices p_t^E and p_t^F and he either has no commitment decision to make, because the minimum uptime or downtime constraints are not yet satisfied, or he can decide to turn on or turn off the unit. However, once committed, a unit needs to fulfill the minimum uptime or downtime requirement regardless of whether prices turn out to be favorable or unfavorable to the power plant's profitability.

Let $J_t(x_t; p_t^E, p_t^F)$ denote the power plant value for the remaining period starting at state x_t at time t. The dispatcher's problem is then one of solving the recursive relationship:

- If $x_t = t^{\text{on}}$, $t \leqslant T - \nu$ (cf. (11a)),

$$J_t(x_t; p_t^E, p_t^F) = f_t(x_t, g_t; p_t^E, p_t^F) + \max_{u_t} E_t[J_{t+1}(t^{\text{on}}; p_{t+1}^E, p_{t+1}^F)u_t + (J_{t+\nu}(-1; p_{t+\nu}^E, p_{t+\nu}^F) - S_d(x_t))(1 - u_t)]; \tag{13a}$$

- If $-t^{\text{cold}} \leqslant x_t \leqslant -t^{\text{off}}$, $t \leqslant T - \tau$ (cf. (11b)),

$$J_t(x_t; p_t^E, p_t^F) = f_t(x_t, g_t; p_t^E, p_t^F) + \max_{u_t} E_t[(J_{t+\tau}(1; p_{t+\tau}^E, p_{t+\tau}^F) - S_u(x_t))u_t + J_{t+1}(x_{t+1}; p_{t+1}^E, p_{t+1}^F) \times (1 - u_t)]; \tag{13b}$$

• If (t, x_t) is not in either of the two cases above (cf. (11c)),

$$J_t(x_t; p_t^E, p_t^F) = f_t(x_t, g_t; p_t^E, p_t^F) + E_t[J_{t+1}(x_{t+1}; p_{t+1}^E, p_{t+1}^F)], \tag{13c}$$

where E_t denotes the expectation operator given the price information available at time t. Again, the above equations are subject to (3), (4), and (5), and similar to (12), the boundary conditions are

$$J_T(x_T; p_T^E, p_T^F) = f_T(x_T, g_T; p_T^E, p_T^F). \tag{14}$$

Note that because the maximization in the equations compares the values of no more than two cases, $u_t = 1$ or 0, in the rest of the paper we will also use the notation $J_t(x_t; \tilde{u}_t, p_t^E, p_t^F)$ with some known $\tilde{u}_t$ to evaluate $J_t(x_t; p_t^E, p_t^F)$ at $\tilde{u}_t$ chosen in the maximization term of the right-hand sides of (13) or (13b). That is,

$$J_t(x_t; p_t^E, p_t^F) = \max(J_t(x_t; u_t = 1, p_t^E, p_t^F), J_t(x_t; u_t = 0, p_t^E, p_t^F)).$$

3.2. Valuation Using Simulation

In any time period t such that $x_t \in \Phi$, there is a commitment decision to make. Optimal commitment decisions at x_t are determined based on current information (p_t^E, p_t^F) and its impact on the expectations of future gas and electricity prices. We assume that both prices follow Ito processes and are Markov. At each state $x_t \in \Phi$, the *optimal* decision can be described by a function of p_t^E and p_t^F, say $d_t(p_t^E, p_t^F; x_t)$, such that the optimal commitment decision to make in state x_t at time t is

$$u_t = \begin{cases} 1, & \text{if } d_t(p_t^E, p_t^F; x_t) > 0, \\ 0, & \text{if } d_t(p_t^E, p_t^F; x_t) < 0. \end{cases} \tag{15}$$

When the prices (p_t^E, p_t^F) are on the "locus" corresponding to $d_t(p_t^E, p_t^F; x_t) = 0$, the commitment decisions "on" and "off" are equivalent in terms of expected profit. We call this locus the *indifference locus* (IL) in state $x_t \in \Phi$ at time t.

One approach to obtain the IL at each state x_t is to discretize the state space spanned by all possible prices (p_t^E, p_t^F), then solve (13) and (14) by recursive dynamic-programming iterations. This is the approach taken in Tseng (2000). The challenges of this approach include the potentially large dimension of the price state space, and the difficulty to model correlation between prices p_t^E and p_t^F into the state space. In this paper, we use Monte Carlo simulation to obtain the IL, which has the flexibility to avoid these difficulties.

Thus, supposing that at time t_0 in state x_{t_0}, one knows all the IL for all $x_t \in \Phi$, $t = t_0 + 1, \ldots, T$, one can determine the power plant value over the period $[t_0, T]$ by making commitment decision u_{t_0} at t_0, i.e., $J_{t_0}(x_{t_0}; u_{t_0}, p_{t_0}^E, p_{t_0}^F)$, using simulation.

Simulation Algorithm for Obtaining $J_{t_0}(x_{t_0}; u_{t_0}, p_{t_0}^E, p_{t_0}^F)$ with All Future IL Known

Data: Initial conditions t_0, $(p_{t_0}^E, p_{t_0}^F)$, u_{t_0} and x_{t_0} are given. $N \gg 0$ is given.

Step 0: Set $t \leftarrow t_0$, $i \leftarrow 1$ and $J^{(i)} \leftarrow 0$. Go to Step 3.

Step 1: If $i > N$, stop. Otherwise obtain a pair of sample prices $(p_t^{E(i)}, p_t^{F(i)})$.

Step 2: Determine $u_t^{(i)}$ using (15).

Step 3: $J^{(i)} \leftarrow J^{(i)} + f(x_t^{(i)}, g_t^{(i)}; p_t^{E(i)}, p_t^{F(i)}) - S_u(x_t^{(i)})u_t^{(i)} - S_d(x_t^{(i)})(1 - u_t^{(i)})$.

Step 4: Determine $x_{t+1}^{(i)}$, $x_{t+\tau}^{(i)}$, or $x_{t+\nu}^{(i)}$ based on $u_t^{(i)}$ and (4).

Step 5: Update $t \leftarrow t+1$, $t \leftarrow t+\tau$, or $t \leftarrow t+\nu$ based on $u_t^{(i)}$. If $t > T$, set $t \leftarrow t_0$, $i \leftarrow i+1$ and $J^{(i)} \leftarrow 0$, then go to Step 3. Otherwise, go to Step 1. □

In the algorithm, the superscript (i) denotes the simulation iteration. Note that in the algorithm, u_{t_0} is an input parameter. When the algorithm terminates, the expected asset value by choosing u_0 initially is given by

$$J_{t_0}(x_{t_0}; u_{t_0}, p_{t_0}^E, p_{t_0}^F)] \approx \frac{\sum_{i=1}^N J^{(i)}}{N}.$$

The expected asset value is optimized over both possible initial decisions, and is thus

$$J_{t_0}(x_{t_0}; p_{t_0}^E, p_{t_0}^F) = \max_{u_{t_0} \in \{0,1\}} J(x_{t_0}; u_{t_0}, p_{t_0}^E, p_{t_0}^F), \tag{16}$$

where u_{t_0} is subject to (3). By setting $t_0 = 0$ and x_{t_0} to be $\tilde{x}_0$, (16) gives the power plant value sought by this paper. What remains to be shown is how to determine the IL at all $x_t \in \Phi$, $t = 1, \ldots, T$. In the following section, we show that one can use the simulation algorithm recursively to determine the IL of $x_{t_0} \in \Phi$ at time t_0, provided all IL for all $x_t \in \Phi$, $t = t_0 + 1, \ldots, T$ are known. Thus, by repeating this process starting from $t_0 = T - 1$, one may obtain all IL at all $x_t \in \Phi$ of all t.

3.3. Generating IL Using Simulation

At time t, if a price pair $(\hat{p}_t^E, \hat{p}_t^F)$ is said to be on the IL corresponding to state $\hat{x}_t \in \Phi$, it implies $d_t(\hat{p}_t^E, \hat{p}_t^F; \hat{x}_t) = 0$ and

$$J_t(\hat{x}_t; u_t = 1, \hat{p}_t^E, \hat{p}_t^F) = J_t(\hat{x}_t; u_t = 0, \hat{p}_t^E, \hat{p}_t^F). \tag{17}$$

Equivalently,

• If $\hat{x}_t = t^{\text{on}}$, $t \leqslant T - \nu$ (cf. (13a)),

$$E_t[J_{t+1}(t^{\text{on}}; \hat{p}_{t+1}^E, \hat{p}_{t+1}^F)] = E_t[J_{t+\nu}(-1; \hat{p}_{t+\nu}^E, \hat{p}_{t+\nu}^F) - S_d]. \tag{18a}$$

• If $-t^{\text{cold}} \leqslant \hat{x}_t \leqslant -t^{\text{off}}$, $t \leqslant T - \tau$ (cf. (13b)),

$$E_t[J_{t+\tau}(1; \hat{p}_{t+\tau}^E, \hat{p}_{t+\tau}^F) - S_u(\hat{x}_t)] = E_t[J_{t+1}(x_{t+1}; \hat{p}_{t+1}^E, \hat{p}_{t+1}^F)], \tag{18b}$$

again; x_{t+1} in (18b) is subject to u_t and (4).

To determine a locus theoretically calls for an infinite number of price pairs satisfying (17). In practice, we must identify only as many price pairs as necessary to obtain an acceptable approximation of the locus. To obtain one price pair $(\hat{p}_t^E, \hat{p}_t^F)$ satisfying (17), we initially fix either of these prices, say the electricity price $\hat{p}_t^E$, and then search for the root of the following one-variable equation $h(y) = 0$:

$$h(y) = J_t(\hat{x}_t; u_t = 1, \hat{p}_t^E, y) - J_t(\hat{x}_t; u_t = 0, \hat{p}_t^E, y) = 0.$$

Because $h(y)$ is continuous and smooth, we can apply root-finding techniques such as curve fitting to improve convergence.

In solving the equation $h(y) = 0$, each function evaluation involves the difference of two terms, where each of these terms is obtained by simulations using the algorithm presented in the previous section. For example, to evaluate $J_{t'}(\hat{x}_{t'}; u_{t'} = 1, \hat{p}_{t'}^E, \hat{y})$, we employ the simulation algorithm by setting $t_0 \leftarrow t'$, $(p_{t_0}^E, p_{t_0}^F) \leftarrow (\hat{p}_{t'}^E, \hat{y})$, $u_{t_0} \leftarrow u_{t'} = 1$ and $x_{t_0} \leftarrow \hat{x}_{t'}$, provided all the IL for all $x_t \in \Phi$, $t > t'$ are available. Therefore, to obtain the IL, we move backward starting from $t = T - 1$.

Note that for each $x_t \in \Phi$ at some t close to T, there is no IL because the remaining time to T is already shorter than the startup or shutdown time required. This corresponds to the third case covered by (13c). Thus, in these cases, we start either from $t = T - \tau$ or $t = T - \nu$. For example, assuming $x_{T-\tau} = t^{\text{off}}$, a Monte Carlo simulation can be initiated for any given $\hat{p}_{T-\tau}^E$ to determine a corresponding gas price $\hat{p}_{T-\tau}^F$ such that (17) is satisfied with $t = T - \tau$, where

$$\begin{aligned} &J_{T-\tau}(t^{\text{off}}; u_{T-\tau} = 1, \hat{p}_{T-\tau}^E, \hat{p}_{T-\tau}^F) \\ &\quad = 0 - S_u(t^{\text{off}}) + E_{T-\tau} J_T(x_T; \hat{p}_T^E, \hat{p}_T^F) \quad \text{(from (13b))} \\ &\quad = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} f(1, g_T^{(i)}; \hat{p}_T^{E(i)}, \hat{p}_T^{F(i)}) - S_u(t^{\text{off}}), \end{aligned} \tag{19}$$

and

$$\begin{aligned} &J_{T-\tau}(t^{\text{off}}; u_{T-\tau} = 0, \hat{p}_{T-\tau}^E, \hat{p}_{T-\tau}^F) \\ &\quad = 0 + E_{T-\tau}[J_{T-\tau+1}(t^{\text{off}}; \hat{p}_{T-\tau+1}^E, \hat{p}_{T-\tau+1}^F)] \quad \text{(from (13b))} \\ &\quad = E_{T-\tau}[0 + E_{T-\tau+1}[J_{T-\tau+2}(t^{\text{off}}; \hat{p}_{T-\tau+2}^E, \hat{p}_{T-\tau+2}^F)]] \\ &\quad = \qquad \vdots \\ &\quad = E_{T-\tau}[E_{T-1}[J_T(t^{\text{off}}; \hat{p}_T^E, \hat{p}_T^F)]] \\ &\quad = 0. \end{aligned}$$

In (19), $p_T^{E(i)}, p_T^{F(i)}$, and $g_T^{(i)}$ are the values of p_T^E, p_T^F, and g_T in scenario i based on some assumed price model using Monte Carlo simulation with initial prices $(\hat{p}_{T-\tau}^E, \hat{p}_{T-\tau}^F)$. Once a sufficient number of $(\hat{p}_{T-\tau}^E, \hat{p}_{T-\tau}^F)$ pairs satisfying (17) are obtained, we can construct an IL. Similarly, we can obtain the corresponding IL for other $x_{T-\tau} \in \Phi$.

With all the IL for all $x_{T-\tau} \in \Phi$ at time $T - \tau$ obtained, we repeat the same process beginning at time $T - \tau - 1$. Likewise, for each given $\hat{p}_{T-\tau-1}^E$, we determine corresponding $\hat{p}_{T-\tau-1}^F$ such that (17) is satisfied with $t = T - \tau - 1$. Because the IL of $t = T - \tau$ are available, we can employ the simulation algorithm presented in the previous section to evaluate the expected power plant value in the period $[T - \tau - 1, T]$.

By repeating this process, working backward in time to Time 1, we identify the IL for each t and $x_t \in \Phi$. The last simulation, which begins with the initial conditions at Time 0, provides an estimate of the value of the power plant during the operating period. In summary, this approach integrates a forward-moving Monte Carlo method into a backward-moving dynamic program. Our approach corresponds closely to that presented in Grant et al. (1997) for valuing path-dependent options, in which the authors obtain a critical locus in each period to determine an optimal early exercise policy for American-style Asian options. In this paper, we extend the approach to a more complicated situation involving multistage decision making and intertemporal constraints.

3.4. An Analytical Upper Bound

If we relax some of the physical constraints described in §2, we may (more easily) obtain an upper bound of the power plant value. To that end, in this section, we focus on a special case in which an analytical solution is possible.

If we let $t^{\text{on}} = t^{\text{off}} = t^{\text{cold}} = 1$, $\tau = \nu = 1$, and $S_u(x_t) = S_d(x_t) = 0 \; \forall\, t$, we have an upper bound, because the actual constraints t^{on}, t^{off}, and t^{cold} are tighter, and the actual costs are greater. These assumptions imply $\Phi = \{+1, -1\}$, so that Equations (13) and (13b) become identical and can be reduced to

$$\begin{aligned} J_t(x_t; p_t^E, p_t^F) &= f_t(x_t, g_t; p_t^E, p_t^F) \\ &\quad + \max_{x_{t+1}} E_t[J_{t+1}(x_{t+1}; p_{t+1}^E, p_{t+1}^F)], \end{aligned}$$

where the capacity constraints (5) remain imposed. Thus, the IL for either $x_t = +1$ or $x_t = -1$ at each t are identical. For shorthand, we use $f_t(x_t)$ and $J_t(x_t)$ instead of $f_t(x_t, g_t; p_t^E, p_t^F)$ and $J_t(x_t; p_t^E, p_t^F)$.

In period T, we have

$$J_T(x_T) = f_T(x_T).$$

In period $T - 1$, we have

$$\begin{aligned} J_{T-1}(x_{T-1}) &= f_{T-1}(x_{T-1}) + \max_{x_T} E_{T-1}[F_T(x_T)] \\ &= f_{T-1}(x_{T-1}) + \max(E_{T-1}[f_T(1)], 0). \end{aligned}$$

The IL at this time period is $E_{T-1}[f_T(1)] = 0$. In period $T - 2$,

$$\begin{aligned} J_{T-2}(x_{T-2}) &= f_{T-2}(x_{T-2}) + E_{T-2}[\max(E_{T-1}[f_T(1)], 0)] \\ &\quad + \max_{x_{T-1}} E_{T-2}[f_{T-1}(x_{T-1})] \\ &= f_{T-2}(x_{T-2}) + E_{T-2}[\max(E_{T-1}[f_T(1)], 0)] \\ &\quad + \max(E_{T-2}[f_{T-1}(1)], 0). \end{aligned}$$

The IL in this period is $E_{T-2}[f_{T-1}(1)]=0$. In period $T-3$,

$$\begin{aligned}J_{T-3}&(x_{T-3})\\&=f_{T-3}(x_{T-3})+E_{T-3}[\max(E_{T-2}[f_{T-1}(1)],0)]\\&\quad+E_{T-3}[E_{T-2}[\max(E_{T-1}[f_T(1)],0)]]\\&\quad+\max_{x_{T-2}}E_{T-3}[f_{T-2}(x_{T-2})]\\&=f_{T-3}(x_{T-3})+E_{T-3}[\max(E_{T-2}[f_{T-1}(1)],0)]\\&\quad+E_{T-3}[\max(E_{T-1}[f_T(1)],0)]\\&\quad+\max(E_{T-3}[f_{T-2}(1)],0),\end{aligned}$$

where the fact that the price processes for p_t^E and p_t^F are Markov is used to reduce the number of operators for conditional expectation. Furthermore, we can summarize the solution of the unconstrained case as

$$\begin{aligned}J_0(\tilde{x}_0)&=f_0(\tilde{x}_0)+\max(E_0[f_1(1)],0)\\&\quad+\sum_{t=2}^{T}E_0[\max(E_{t-1}[f_t(1)],0)]\\&=(p_0^E g_0-H(g_0)p_0^F)\operatorname{sign}(\tilde{x}_0)\\&\quad+\max(E_0[p_1^E g_1-H(g_1)p_1^F],0)\\&\quad+\sum_{t=2}^{T}E_0[\max(E_{t-1}[p_t^E g_t-H(g_t)p_t^F],0)].\end{aligned}\tag{20}$$

Finally, the IL are the same for all $x_t\in\Phi$, $\forall t$ (because the price processes are Markov) and is

$$\begin{aligned}&E_t[f_{t+1}(1)]=E_t[p_{t+1}^E g_{t+1}-H(g_{t+1})p_{t+1}^F]=0\\&\quad\Leftrightarrow E_0[f_1(1)]=E_0[p_1^E g_1-H(g_1)p_1^F]=0,\end{aligned}$$

which can be shown to be a straight line passing through the origin.

Note that given the more realistic characterization of plant constraints, the value obtained by (20) should better approximate a power plant's value than that obtained by the financial options approach. We will quantitatively evaluate this improvement in a later section of numerical results.

3.5. Price Processes

In this paper, we assume that price processes for both electricity and fuel are given. Although the valuation method to be presented in this paper can be applied to general Ito price processes, we focus on the following two processes for electricity and fuel respectively:

$$d\ln(p_t^E)=-\mu^E(\ln(p_t^E)-m_t^E)\,dt+\sigma^E\,dB_t^E,$$

and

$$d\ln(p_t^F)=-\mu^F(\ln(p_t^F)-m_t^F)\,dt+\sigma^F\,dB_t^F,$$

where B_t^E and B_t^F are two Wiener processes with instantaneous correlation ρ. The above commodity price models are characterized by mean reversion and lognormally distributed seasonal prices. Because, to varying degrees, both electricity and fuel have associated storage costs, their prices are determined to a large degree by the forces of producer supply and consumer demand, and less so by investor speculation. This interplay is manifested in the mean-reverting nature of their price processes. In some sense, the mean-reversion parameter μ represents the storability of the commodity. For electricity, which is quite difficult to store, this parameter is large, implying little autocorrelation between today's price and tomorrow's price. Furthermore, this parameter in conjunction with σ captures the short- and long-term price fluctuations and characterizes the variance of the lognormal price distribution. This distribution resembles that of other traditional price-process models in that price returns are normally distributed and prices are nonnegative. Finally, m_t is a periodic function capturing the cyclical nature of the long-term expected prices. m_t is thus a function of the interplay between the cost of production and consumer demand for the commodity. In the Appendix of this paper, we will describe the procedures for estimating the parameters from historical data using the method of maximum likelihood.

Note that because risk-neutral price processes reflect the present market values of the prices in the future, implied by existing derivative securities, their resolution is limited to that of the derivatives. Thus, in our analysis we utilize actual price processes instead of risk-neutral price processes because we seek to determine the impact of operational constraints of shorter term than the monthly averages upon which derivative contracts are typically based. However, the incorporation of risk-neutral processes is reasonably straightforward. The necessary modification is simply a constant adjustment to the commodity-price-process mean, where the magnitude of the adjustment is determined from the appropriate commodity derivatives (e.g., futures).

3.6. Ramp Constraints Revisited

As previously mentioned, an exact method for incorporating the ramp constraints into the dynamic programming is to discretize the generation levels q_t to the state space. For example, let

$$q_t\in Q\equiv\{q^{\min},q^{\min}+\Delta q,q^{\min}+2\Delta q,\ldots,q^{\max}\},$$

where Δq is a divisor of $q^{\max}-q^{\min}$ and $\Delta q\leqslant R$. After incorporating q_t into the state space, the state space can be thought of as a three-dimensional diagram extended from the one shown in Figure 1, where the transition among the states of different q_t is subject to the ramp constraints (6). Consequently, by changing $J_t(x_t;p_t^E,p_t^F)$ to $J_t(x_t,q_t;p_t^E,p_t^F)$, we may obtain an IL for each state (x_t,q_t), $\forall x_t\in\Phi$, $\forall q_t\in Q$. Thus, the algorithm for obtaining an IL still applies in the ramp-constrained case. Obviously, the major change is in the computational complexity, which is greatly increased due to the increase of the size of the state space.

Without extending the state space to include variable generation levels, we can use heuristics in the Monte Carlo simulation process to account for the ramp constraints. Because in the proposed method the price simulations are forward moving, intuitively one can solve a ramp-constrained dispatch problem sequentially starting from $t = 0$. The ramp constraints are imposed with respect to the generation level obtained for the previous hour. The heuristic ramp-constrained dispatch problem (cf. (9)) is as follows.

$$\begin{aligned} \max\ & p_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2) p_t^F \\ \text{s.t.}\ & q^{\min} \leqslant q_t \leqslant q^{\max}, \\ & |q_t - q_{t-1}| \leqslant R, \end{aligned} \tag{21}$$

where q_{t-1} is known when determining optimal q_t. By definition, the IL of a state describes the optimal commitment strategy at that state, which depends on the dispatch rule. For example, in the nonramp-constrained case, (10) uniquely determines the dispatch g_t for a given price pair (p_t^E, p_t^F). With the presence of ramp constraints, and using the modified dispatch rule (21), determining q_t depends not only on (p_t^E, p_t^F), but also the price history prior to t. An IL of a state obtained by simulation using such a heuristic method for dispatch can be regarded as an "average approximation" of the loci of all $\text{IL}(x_t, q_t)$, $q_t \in Q$ corresponding to the exact method.

In our numerical test, we apply the above heuristics (21) only in the final simulation runs starting from $t = 0$. That is, the ramp constraints are not considered in obtaining the IL of all states. Therefore, the computational complexity is virtually unchanged due to ramp constraints. Again, we emphasize that an IL provides optimal commitment strategy when the generation is at the optimal level g_t, while in the final simulation runs, subject to the ramp constraints, the generation level may not be exactly g_t. In the final simulation runs starting from $t = 0$, the obtained value provides a lower bound of the expected profit of the generator during the operating period. This follows because one dispatches the unit suboptimally using the heuristic method with respect to suboptimal commitment strategy at each stage.

3.7. Incorporating Other Markets: The Case of Spinning Reserves

Thus far, we have only valued a power plant based on its energy output. Other "products" associated with a power plant, such as ancillary services and emission allowances, which are traded in separate markets, can also add value. For example, when a unit is online (spinning) and its generation level, q_t (MW), is not at its maximum rated capacity ($q_t < q^{\max}$), the residual capacity $q^{\max} - q_t$ can be sold to a spinning-reserve market, a capacity-only market. If the unit is called to generate on contingency, it gains additional value from energy. On the other hand, the power plant could sell its emission allowance in exchange for less generation (thus less emission) if it believes doing so is profitable. With the presence of markets other than energy, the valuation problem becomes a complicated optimization problem where one must determine the optimal strategy to allocate resources to different markets simultaneously. As an example, in this section we discuss how the proposed model can be extended to account for the spinning-reserve market.

Assume a unit is online and generates q_t (MW) of power such that $q^{\min} \leqslant q_t \leqslant q^{\max}$, let $\hat{q}_t$ denote the spinning-reserve capability available for the unit:

$$\hat{q}_t \equiv \min(q^{\max} - q_t, \hat{q}^{\max}),$$

where $\hat{q}^{\max}$ is the maximum spinning reserve of the unit. For simplicity, assume there is a spot market for spinning reserve, with prices denoted by p_t^S. We can modify the Profit Function (8) as follows.

$$\begin{aligned} &\hat{f}_t(x_t, q_t; p_t^E, p_t^F, p_t^S, w_t, \tilde{q}_t) \\ &\quad = \big(p_t^E q_t + p_t^S \hat{q}_t + p_t^E \tilde{q}_t w_t \\ &\qquad - C(q_t + \tilde{q}_t w_t, p_t^E)\big)\operatorname{sign}(x_t), \end{aligned} \tag{22}$$

where w_t is a 0-1 random variable indicating whether the unit is called to generate on contingency and where $\tilde{q}_t$ is a nonnegative random number bounded above by $\hat{q}_t$ indicating the additional generation level if, in fact, the unit is called. In (22), the first term is the revenue of selling power from the energy market, the second term is the capacity payment from the spinning-reserve market, and the third term is the payment for energy if called to generate on contingency. Accordingly, (13c) should be replaced by

$$\begin{aligned} &J_t(x_t; p_t^E, p_t^F, p_t^S, w_t, \tilde{q}_t) \\ &\quad = \hat{f}_t(x_t, q_t; p_t^E, p_t^F, p_t^S, w_t, \tilde{q}_t) \\ &\qquad + E_t\big[J_{t+1}(x_{t+1}; p_{t+1}^E, p_{t+1}^F, p_{t+1}^S, w_{t+1}, \tilde{q}_{t+1})\big]. \end{aligned} \tag{23}$$

Given the process of p_t^S and probabilistic distribution of w_t, one can use the proposed procedure to value (23).

For a responsive unit such as a combustion turbine that is usually dispatched on the margin and gets its major revenue from the ancillary service market, the physical constraints (minimum uptime/downtime constraints) modeled in §3 can be ignored. Consequently, a method similar to the financial options approach may be more appropriate for valuing this type of unit. Because a combustion turbine can be started up and shut down very quickly, we assume that at time t the operator observes the prices p_t^E, p_t^F, and p_t^S, and then determines the quantities to sell to the energy market q_t and the spinning-reserve market $\hat{q}_t$. With some chance w_t, the combustion turbine would be dispatched on contingency, so the value of a combustion turbine can be formulated as (cf. (2)),

$$V_0 = \sum_{t=1}^{T} E_0\bigg[\min_{q^{\min} \leqslant q_t \leqslant q^{\max}} \big(p_t^E q_t + p_t^S \hat{q}_t + p_t^E \tilde{q}_t w_t \\ - C(q_t + \tilde{q}_t w_t, p_t^F)\big)\bigg].$$

As a final remark for this section, the markets for ancillary service or emission may require bidding processes to buy or sell to the market. Because bidding rules vary in different areas, the formulations presented in this section may need further modification in practice.

4. NUMERICAL RESULTS

We have implemented the proposed method for valuing a power plant in FORTRAN. This section presents the numerical test results.

4.1. Test System Parameters

The proposed method has been applied to a natural gas-fueled generating unit with the following input-output characteristics:

$$H(q_t) = 600 + 9.121 q_t + 0.00131 q_t^2, \qquad (24)$$

with $q^{\min} = 250$ MW and $q^{\max} = 750$ MW. For the startup costs, we assume $b_1 = \$2{,}300$, $b_2 = \$950$, and $\gamma = 4$ hours in (7).

To obtain the parameters of the price processes of both electricity and fuel, we examine the historical price-data series of NYMEX natural gas prices and electricity prices from the California Power exchange, taking the logarithm of these prices as our basic data series. Because there is no hourly market for natural gas, we assume that m_t^F is constant within a given day and fit the price process to daily data. Because the model time step is hourly, however, we adjust these parameters accordingly. Specifically, because the model is a continuous-time model, we deduce the implied hourly fluctuations from the daily parameters.

For electricity, we use historical daily data from the California Power Exchange and similarly normalize the data to the hourly time step. However, to incorporate a daily price pattern, we adjust m_t^E by overlaying the daily electricity price pattern (in terms of percentage changes). In the Appendix of this paper, we summarize how to use the maximum likelihood method to estimate price parameters.

Using the maximum likelihood method, we obtain $\mu^F = 6.95 \times 10^{-4}$ and $\sigma^F = 0.019$ for fuel, and $\mu^E = 0.072$ and $\sigma^E = 0.27$ for electricity, where $1/\mu$ is in units of hours and characterizes the rate of mean reversion, and σ is in \$/MWh. Obtained m_t^E values, which capture the cyclical nature of the expected electricity prices, are summarized in Table 1, where m_t is in the same units as σ, i.e., \$/MWh. (Note that while m_t reflects the daily pattern of prices, it is not a vector of the average daily values, because each such average is a function of both corresponding m_t and the average value from the preceding time period.) Also, we assume that the instantaneous correlation coefficient between electricity and natural gas prices is $\rho = 0.4$.

4.2. Indifference Loci (IL)

We begin our analysis by valuing the unit of (24) over a one-week (168 hours) period, and investigate how the IL behaves as time changes. We assume that $t^{\text{on}} = t^{\text{off}} = t^{\text{cold}} = 10$ and $\tau = \nu = 2$. Therefore $\Phi = \{+10, -10\}$.

Table 1. Values of hourly m_t^E.

t	m_t^E	t	m_t^E	t	m_t^E
1	1.8874	9	4.8613	17	3.7233
2	2.6557	10	4.7100	18	1.4573
3	1.9348	11	5.8114	19	1.3220
4	2.3402	12	4.7363	20	2.5106
5	3.5027	13	5.0440	21	3.6167
6	3.8568	14	5.7383	22	0.6446
7	3.7583	15	5.9166	23	1.8328
8	4.6602	16	4.7126	24	1.8328

In Figure 2, we show all IL corresponding to $x_t \in \Phi$, $t = 1, \ldots, 168$. Each IL divides the price plane (p_t^E, p_t^F) into two regions: online region (the lower right half) and offline region (the upper left half). At some state x_t, if (p_t^E, p_t^F) falls into the online region of the corresponding IL, the optimal decision is to turn on the unit, and vice versa. The unit, once turned on, will be in full operation two (τ) hours later. The IL are clustered into two groups: The upper group corresponds to $x_t = +10$, and the lower one to $x_t = -10$. This implies that for each t, the online region of some $x_t < 0$ is a subset of the online region of some $x_t > 0$, i.e.,

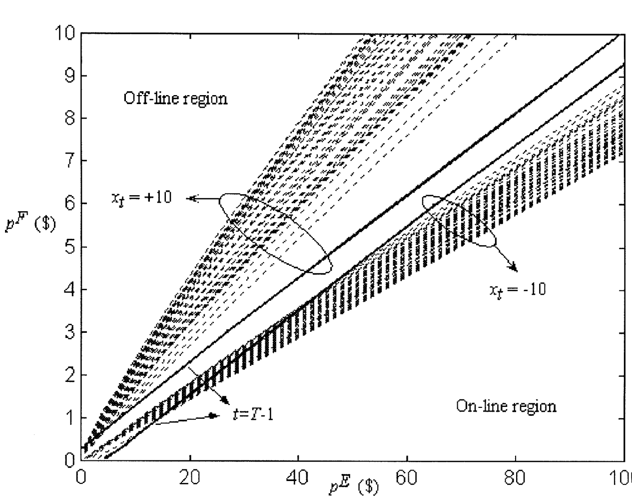
$$\begin{aligned}&\{(p_t^E, p_t^F) \mid d_t(p_t^E, p_t^F; x_t < 0) > 0, x_t \in \Phi\}\\ &\quad \subseteq \{(p_t^E, p_t^F) \mid d_t(p_t^E, p_t^F; x_t > 0) > 0, x_t \in \Phi\}. \qquad (25a)\end{aligned}$$

Similarly, because the online region and the offline region are mutually exclusive,

$$\begin{aligned}&\{(p_t^E, p_t^F) \mid d_t(p_t^E, p_t^F; x_t > 0) < 0, x_t \in \Phi\}\\ &\quad \subseteq \{(p_t^E, p_t^F) \mid d_t(p_t^E, p_t^F; x_t < 0) < 0, x_t \in \Phi\}. \qquad (25b)\end{aligned}$$

This result is intuitive: If an observed price set (p_t^E, p_t^F) is worth starting up the unit from an offline state ($x_t < 0$ and

Figure 2. Example of indifference loci.



$x_t \in \Phi$), it is certainly optimal for the unit to remain online if it were already online, without incurring the startup cost. In Figure 2, we highlight the two IL corresponding to $t = T-1$. The upper one corresponding to $x_t = +10$ has a positive y-intercept, which reflects the nonzero shutdown cost S_d. That is, it allows a small loss if the loss is smaller than the shutdown cost. The lower highlighted IL corresponding to $x_t = -10$ has a positive x-intercept, which represents nonzero startup cost S_u. The nonzero x-intercept reflects the minimum profit required to turn on the unit. As the time index t moves toward the starting point, the impact due to the startup or shutdown cost is gradually alleviated, i.e., the x-intercept and y-intercept are both approaching zero. As t is decreased, while the y-intercept is reducing to zero, the IL corresponding to $x_t = +10$ move upward. This shows that the IL are not linear, but more like asymptotically linear. The movement for the IL corresponding to $x_t = -10$ is opposite to that of $x_t = +10$. Equations (25a) and (25b) are with respect to the same t, so Figure 2 should not be taken to imply that the two clusters of IL do not overlap. Indeed, in various tests with different price parameters, they do.

4.3. Power Plant Value vs. Physical Constraints

To demonstrate the relation between a power plant value and physical constraints, we applied the proposed method to the generator of (24) under various physical constraints. Five cases are tested over a one-week period (168 hrs). The first case considers no physical constraints and no decision lead time (i.e., $\tau = \nu = 0$, $t^{\text{on}} = t^{\text{off}} = t^{\text{cold}} = 0$, $S_u = S_d = 0$), which corresponds to the approaches using financial options (e.g., in Deng et al. 1998; Hsu 1998a, 1998b). Case 2 assumes all physical constraints are at their minimum values (i.e., $\tau = \nu = 1$, $t^{\text{on}} = t^{\text{off}} = t^{\text{cold}} = 1$, $S_u = S_d = 0$) corresponding to the approximate method presented in §4.3. In Case 5, we set t^{on} to infinity, implying the unit is online unconditionally at all hours, much like a nuclear power plant. The other two cases correspond to different minimum uptime and downtime constraints. The purpose is to observe how plant value varies with different physical constraints. The results are summarized in Table 2.

In these test cases, we see that with physical constraints the expected power plant value decreases, but the variance increases. Ignoring the physical constraints may result in a 13.7% difference in power plant value (between Cases 1 and 4). Case 3 has a shorter minimum uptime and downtime, and its performance is between those of Cases 1 and 4. Case 4 has a lower mean, but larger variance due to physical constraints than Case 1. Case 5, with units on at all hours, can be considered to be a measurement of the limiting case for increasing t^{on}. This case, which corresponds to the lowest mean profit and the highest variance among all test cases, is the worst among the five cases and provides a poor lower bound of the power plant value.

Table 2. Power plant value vs. physical constraints.

Case no.	1	2	3	4	5
t^{on} (hr)	0	1	4	10	∞
t^{off} (hr)	0	1	4	10	—
τ (hr)	0	1	1	2	—
ν (hr)	0	1	1	2	—
S_u ($)	0	0	0	3,061	—
S_d ($)	0	0	1,000	1,000	—
x_0	1	1	4	10	—
R (WM)	∞	∞	100	75	75
Mean power plant value ($\$ \times 10^6$)	2.82	2.75	2.64	2.48	2.40
Std ($\$ \times 10^6$)	1.49	1.43	1.50	1.52	1.59
Skewness	1.19	1.26	1.03	1.30	1.13
Kurtosis	5.30	5.72	4.34	6.10	5.29
Mean per MWh profit ($/MWh)	35.24	35.03	34.72	31.34	23.71

4.4. Power Plant Value vs. Time

In this section we focus on the relation of the power plant value and time. By repeatedly running the program with different lengths of planning horizon T, we obtain the expected power plant values summarized in Table 3. The unit parameter set corresponding to Cases 1 to 4 in Table 2 are tested. Cases 1 and 2 are tested using the financial options approach (recorded as Value 1 in Table 3) and the approximate method presented in §4.3 (Value 2), respectively. To incorporate ramp constraints into the testing, we adopt the heuristic method presented in §4.5. The power plant values before and after applying the heuristic method are recorded in Table 3 corresponding to Value 3 and Value 4, respectively. It is clear that Values 1 to 4 are in a decreasing order because of the increasing physical constraints. That is, both Value 1 and Value 2 provide upper bounds for the true power plant values, and Value 2 is a better bound than Value 1 as stated in §4.3. Comparing Value 1 and Value 4, the percentage overestimate of the power plant value of the financial options approach decreases as T increases, and seems to converge approximately to 7% and 14% for Case 3 and Case 4, respectively. Comparing Value 3 and Value 4, the effect of the ramp constraints upon the power plant value can be observed, and this proves to be critical. In the two test cases, about 50% of the overestimate of the financial options approach is due to the ramp constraints (i.e., the ratio of (Value 3 − Value 4) to (Value 1 − Value 4)). Additionally, each of these methods reveals approximately linear relations between the power plant value and the length of the planning horizon; see Figure 3, which ranges in our tests from one day to one week.

We also record the per MWh value ($/MWh) of the power plant in Table 4, as well as depict it in Figure 4. The percentage overestimate of the per MWh value of the power plant obtained by the financial options approach seems to converge to approximately 1.5% in Case 3 and 12% in Case 4, as T increases. The numerical results also show

Table 3. Power plant value ($) vs. time.

	T (hr)	24	48	72	96	120	144	168
Case 1	Value 1 ($\$ \times 10^6$)	0.30	0.71	1.13	1.55	1.97	2.39	2.82
Case 2	Value 2 ($\$ \times 10^6$)	0.29	0.68	1.10	1.51	1.92	2.34	2.75
	Value 3 ($\$ \times 10^6$)†	0.28	0.68	1.09	1.50	1.91	2.32	2.73
	Value 4 ($\$ \times 10^6$)‡	0.27	0.65	1.05	1.44	1.84	2.24	2.64
Case 3	Overestimate (%)*	11.1	9.2	7.6	7.6	7.1	6.7	6.8
	Overestimate due to ramp (%)**	33.3	50.0	50.0	54.5	53.8	53.3	50.0
	Value 3 ($\$ \times 10^6$)†	0.26	0.65	1.05	1.45	1.85	2.26	2.66
	Value 4 ($\$ \times 10^6$)‡	0.22	0.59	0.97	1.34	1.72	2.10	2.48
Case 4	Overestimate (%)*	36.4	20.3	16.5	15.7	14.5	13.8	13.7
	Overestimate due to ramp (%)**	50.0	50.0	50.0	52.4	52.0	55.2	52.9

†: by the proposed method without ramp constraints.
‡: by the proposed method with ramp constraints handled by the heuristic method in Section 4.5.
*: (Value 1/Value 4) − 1.
**: (Value 3 − Value 4)/(Value 1 − Value 4).

that the per MWh value of the power plant seems less sensitive to the physical constraints compared with the measure of dollar value. Clearly, results for different markets would vary as well with the price parameters μ and σ.

5. CONCLUSION AND FUTURE DIRECTIONS

In this paper we present a method for valuing a power plant using Monte Carlo simulation. As opposed to the popular approach using financial options, we incorporate physical constraints into the problem modeling, without which the potential risk due to operational limits is overlooked, and the payoff is nonnegative.

The real-world physical constraints which we model into the power plant valuation problem include the minimum uptime/downtime constraints and the unit ramp constraints. We present both the exact and the heuristic methods to handle ramp constraints. We observe that the power plant value in some test case is overestimated by approximately 14%.

Although this paper focuses on the short-term power plant valuation, the proposed method can also be used to aid long-term valuation. When dealing with long-term generation-asset valuation, it is debatable whether one wants to run such a spot-price model, say hour to hour, for a one-year period. Monthly forward prices seem more appropriate. How to integrate the proposed short-term model to a better long-term price structure such as a forward price curve is a new research direction. To sum up, the proposed method can be used to estimate how a power plant value can be affected by the physical constraints during a short-term period. This component, if overlooked, may result in sizable risks.

Our numerical results also suggest an approximate form of formulation when length of time period to be valued is much larger than the ranges of physical constraints, and suggest a correction term to the financial options approaches to account for the physical constraints. In these cases, a power plant can be valued accurately and efficiently.

Valuing a power plant using dynamic programming and Monte Carlo simulation is quite flexible, and so is readily conducive to incorporating new price processes or even new uncertainties. This method, however, requires massive computations in repeated forward and backward processes. To improve the efficiency, our research direction has turned to developing a price lattice that can represent both fuel and electricity price processes, so that we may solve the problem using backward stochastic dynamic programming (Tseng 2000). We shall present these additional research results in a future paper.

Figure 3. Power plant value vs. time.

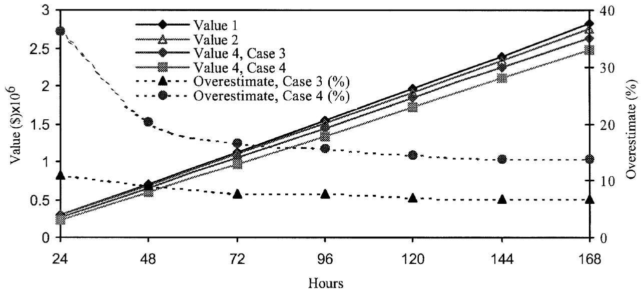


Figure 4. Power plant per MWh value vs. time.

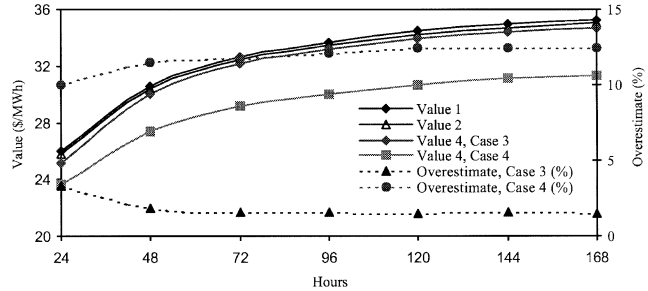


Table 4. Power plant value (\$/MWh) vs. time.

	T (hr)	24	48	72	96	120	144	168
Case 1	Value 1	26.03	30.59	32.63	33.70	34.47	34.96	35.24
Case 2	Value 2	25.77	30.40	32.40	33.43	34.21	34.72	35.03
	Value 3	25.37	30.17	32.30	33.34	34.12	34.60	34.92
	Value 4	25.19	30.03	32.13	33.19	33.95	34.43	34.72
Case 3	Overestimate (%)	3.3	1.9	1.6	1.5	1.5	1.5	1.5
	Overestimate due to ramp (%)	27.3	33.3	51.5	41.7	48.6	47.2	62.5
	Value 3	23.89	28.25	30.17	31.59	31.83	32.30	32.57
	Value 4	23.67	27.43	29.20	30.05	30.65	31.09	31.34
Case 4	Overestimate (%)	10.0	11.5	11.7	12.1	12.5	12.4	12.4
	Overestimate due to ramp (%)	9.3	25.9	28.3	42.2	30.9	31.3	31.5

APPENDIX A. FITTING MODEL PARAMETERS

Studies of the dynamic behavior of electricity and natural gas prices suggest that their dynamic price-process behavior may be reasonably represented by "geometric" mean-reverting processes. (The interested reader is referred to Barz 1999 for a formal comparison of various electricity price-process models.) Denoting the price at time t by p_t, we represent the dynamic price behavior as

$$d\ln(p_t) = -\mu(\ln(p_t) - m_t)\,dt + \sigma\,dB_t,$$

where B_t represents a Weiner process and m_t is time-varying to capture the effects of seasonality. The procedure for fitting the data is as follows:

• We begin by constructing a new time-series consisting of the natural logarithm of prices, z_i.

• Next, we deseasonalize this modified data by removing the predictable price patterns to allow the use of a nontime-varying stochastic process:

$$dz_t = -\mu(z_t - \bar{m})\,dt + \sigma\,dB_t$$

Specifically, we subtract the average difference of the value at each hour of the day from the overall mean. Note that this can be shown to be equivalent to fitting m_t directly, using the method of maximum likelihood.

• Last, we fit the deseasonalized, log-price data via the method of maximum likelihood, where the maximum likelihood estimators:

$$\bar{m} = \frac{1}{n}\sum_{i=0}^{n} z_i - \frac{1}{n}\left(\frac{z_0 - e^{-\mu} z_n}{1 - e^{-\mu}}\right)$$

$$\approx \frac{1}{n+1}\sum_{i=0}^{n} z_i \quad \text{for } z_0, z_n \approx \bar{m},$$

$$\mu = -\ln\left[\frac{\sum_{i=1}^{n}(z_i - \bar{m})(z_{i-1} - \bar{m})}{\sum_{i=0}^{n-1}(z_i - \bar{m})^2}\right],$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left((z_i - \bar{m}) - e^{-\mu}(z_{i-1} - \bar{m})\right)^2,$$

$$\sigma^2 = \frac{2\mu\hat{\sigma}^2}{(1 - e^{-2\kappa})},$$

are derived directly from conditional distributions implied by the previous period's prices and the stochastic model (Oksendal 1995, Davidson and MacKinnon 1993).

To simulate prices, these steps are reversed.

• First, nontime varying log-prices are simulated:

$$z_t = e^{-\mu}(z_{t-1} - \bar{m}) + \bar{m} + \sigma N[0, 1].$$

• Next, seasonal (daily) fluctuations are reincorporated into z_t by adding to z_t its respective hour-of-day adjustment. This reproduces the time-varying effect of m_t.

• Last, prices are derived: $p_t = e^{z_t}$.

The correlation of gas and electricity prices is incorporated via the $N[0, 1]$ random variables.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to B. F. Hobbs and two anonymous reviewers for several useful comments. The first author was supported by a GRB summer research award from the University of Maryland.

REFERENCES

Barz, G. 1999. Stochastic financial models for electricity derivatives. Ph.D. dissertation, Department of Engineering-Economic Systems and Operations Research, Stanford University, Stanford, CA.

Davidson, R., J. G. MacKinnon. 1993. *Estimation and Inference in Econometrics*. Oxford University Press, New York, NY.

Deng, S., B. Johnson, A. Sogomonian. 1998. Exotic electricity options and the valuation of electricity generation and transmission. Presented at the Chicago Risk Management Conference, Chicago, IL.

Gedra, T. W. 1994. Optional forward contracts for electric power markets. *IEEE Trans. Power Systems* **9** 1766–1773.

——, P. Varaiya. 1993. Markets and pricing for interruptible electric power. *IEEE Trans. Power Systems* **8** 122–128.

Grant, D., G. Vora, D. Weeks. 1997. Path-dependent options: Extending the Monte Carlo simulation approach. *Management Sci.* **43** 1589–1602.

Hsu, M. 1998a. Spark spread options are hot! *Electricity J.* **11** 28–39.

——. 1998b. Using financial options theory to value and hedge natural gas power assets. Presented at the EPRI's Pricing Energy in a Competitive Market Conference, Washington, D.C.

Kaminski, V. 1997. The challenge of pricing and risk managing electricity derivatives. Chapter 10 in *The U.S. Power Market*. Risk Publications, Financial Engineering Ltd, London, U.K.

Oksendal, B. K. 1995. *Stochastic Differential Equations: An Introduction with Applications*. Springer, Berlin, Germany.

Oren, S. S., P. T. Spiller, P. Varaiya, F. Wu. 1995. Nodal prices and transmission rights: A critical appraisal. *Electricity J.* **8** 24–35.

Sheble, G. B., G. N. Fahd. 1994. Unit commitment synopsis. *IEEE Trans. Power Systems* **9** 128–135.

Shimko, D. 1994. Options on futures spreads: Hedging, speculation and valuation. *J. Futures Markets* **14** 183–213.

Svoboda, A. J., C. L. Tseng, C. A. Li, R. B. Johnson. 1997. Short term resource scheduling with ramp constraints. *IEEE Trans. Power Systems* **12** 77–83.

Tseng, C. L. 1996. On power system generation unit commitment problems. Ph.D. dissertation, Department of Industrial Engineering and Operations Research, University of California at Berkeley, Berkeley, CA.

——, K. Lin. 2001. A framework using two-factor price lattices for generation asset valuation. Working paper, University of Maryland, College Park, MD. Submitted to *Oper. Res.*

Wood, A. J., B. F. Wollenberg. 1984. *Power Generation, Operation and Control*. John Wiley, New York.