Highway Development Decision-Making under Uncertainty: A Real Options Approach

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Abstract: A highway system development involves huge irreversible investments, and requires rigorous modeling and analysis before the implementation decision is made. This decision-making process is embedded with multiple uncertainties due to changes in political, social, and environmental contexts. In this paper, we present a multistage stochastic model for decision making in highway development, operation, expansion, and rehabilitation. This model accounts for the evolution of three uncertainties, namely, traffic demand, land price, and highway deterioration, as well as their interdependence. Real options in both development and operation phases of a highway are also incorporated in the model. A solution algorithm based on the Monte Carlo simulation and least-squares regression is developed. Numerical results show that the proposed model and solution algorithm are promising. This model makes a radical and conceptual step towards optimal decision making in highway engineering, which achieves decision-making *optimality* that is generally not well defined in traditional policy-based approaches for highway planning.

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Introduction

The highway construction boom from the 1950s to 1970s, as well as the highway rebuilding in the 1980s, established the foundation of today's national highway network, which has also advanced the practices of highway planning and development. However, many facilities in highway systems in the U.S., especially around older cities, are still in disrepair. For example, as of December 2001, about 14.2% of highway bridges in the U.S. were considered structurally deficient, and another 13.8% were deemed functionally obsolete (Federal Highway Administration 2001). Ongoing operation decisions about capacity expansion, maintenance/rehabilitation, and regular maintenance have been based merely on experience or perceived urgency of failure. As a result, highway service may not be provided at an appropriate level, and highway may be aging faster than predicted. Very often, key decisions during planning and design, such as the selection of the right-of-way width and number of lanes, are made without considering the uncertainties in demand, revenue, user benefits, etc. Highway system development involves huge irreversible investments, and requires rigorous modeling and analysis before the implementation decision is made. In addition, increasing private sector participation in highway developments demands a prudent approach in sharing the commercial, financial, and development risks among various agencies. The demand and revenue projections for the life cycle of a highway are embedded with multiple uncertainties due to changes in political, societal, and environmental contexts. Thus, an effective model that incorporates alternatives analysis and accounts for evolution of uncertainties is required.

Life-cycle analysis has been widely used in infrastructure management. Abaza (2002) developed a flexible pavement life-cycle model to yield an optimum maintenance and rehabilitation plan. Recently, stochastic methods have also been employed in life-cycle analysis. For example, Zayed et al. (2002) used a Markov decision process in selecting a rehabilitation plan among predetermined decision policies. However, the optimality of decision making was limited to predetermined polices or plans. The uncertainties such as demand, costs and revenues, and service quality are often interrelated and cannot be dealt with in isolation. To the best of the authors' knowledge, an integrative approach that models optimal decision making under uncertainty in highway development, operation, rehabilitation, and expansion has not yet been attempted.

In this paper, a real-options approach is developed for optimal decision making in highway design, operation, rehabilitation, and expansion, which incorporates life-cycle analysis. Decisions on land acquisition and land use are also included. Recently, the concept of real options, stemming from financial options theory, has attracted research attention. Real options refer to flexibility embedded in *real* operational processes, activities, or investment opportunities that are not financial instruments (e.g., Trigeorgis 1996). Real-options-based decision making recognizes the value of flexibility (or flexible alternatives) due to timely exercise of the flexibility. That is, a flexible alternative may turn uncertainty to opportunity. Therefore, such a valuation approach promotes flexible designs. In the life cycle of a highway system, right-of-way acquisition and land development can be viewed as real options.

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To fully evaluate their flexibility, the evolutions of three key uncertainties, namely, traffic demand, land price, and highway service quality, are included, and each of these is modeled as a stochastic process.

In this paper, the decision-making process is modeled as a multistage stochastic problem. The decision maker (DM) is assumed to maximize the overall net benefit of the highway system. The main challenge lies in valuing the profitable opportunities due to proper exercise of the real options. To fully capture the evolution of the profitable opportunities, the uncertainties are simulated based on the Monte Carlo (MC) method, and the optimal decision criteria are approximated by regression, integrated with backward dynamic programing steps. In a case study, we have applied the proposed model and the solution algorithm to a 50-mile-long highway section. We demonstrate that the proposed method not only can select the optimal design alternative in the design phase, but also can provide a timely decision on additional right-of-way acquisition, expansion, and rehabilitation during the operation phase. The proposed model, as well as the solution algorithm, can be directly extended to handle larger cases such as a network of roads.

This paper addresses the decision making for a highway system under uncertainty in a nontraditional method using the real-options approach. Although the approach attempts for a radical shift in the decision process, its practical significance is substantiated by its benefits. This approach may be very useful in developing and developed countries, especially with the private sector participating in infrastructure development, to ascertain the viability of projects.

Modeling a Highway System

The process to develop a highway usually consists of five stages: planning, preliminary design, final (detailed) design, right-of-way acquisition, and construction. After the highway is completed, ongoing operation, maintenance, and rehabilitation activities continue throughout the life of the highway facility. When the demand increases and approaches the existing capacity, expansion of the highway becomes a main alternative. The actions of rehabilitation and expansion should also be properly timed to benefit users and preserve the service life and quality of the highway. To achieve a sustainable development requires proper quantitative modeling taking the uncertainties of varying demand, costs, and land availability into account. Before we present the multistage stochastic model, we first explore the major real options and underlying uncertainties involved in the life cycle of a highway system.

Review of Real Options

In finance, an *option* is defined as the right, but not the obligation, to buy (or sell) an asset under specified terms (e.g., Luenberger 1998; Hull 1999). For example, an option that gives the right to purchase something is called a *call option*; an option that gives the right to sell something is called a *put*. Usually, there is a specified price (called an *exercise price*) at which the underlying asset can be purchased upon exercise of the option and a specified period of time over which the option is valid. There are two primary conventions regarding acceptable exercise dates before expiration. An *American*-style option allows exercise at any time before and including the expiration date. A *European*-style option allows exercise only on the expiration date. Consider an American call option that allows one to buy a specified stock at $50 before some future time T. At some time before T, if the stock price is higher than $50, say $70, the call option will be valuable because the call option can be exercised by buying the stock at $50 and then reselling the stock back to the market at a profit of $20. A closely related problem would be to determine the optimal timing for exercising the option.

Recently, the concept of options has been applied extensively in a variety of areas other than financial instruments such as calls and puts. This subject is known as *real options* valuation. Real options refer to the options embedded in *real* operational processes, activities, or investment opportunities that are not financial instruments (e.g., Trigeorgis 1996). In fact, a real option provides the option owner the right but not the obligation, or the *flexibility*, to take an action. An example of a real option in infrastructure expansion is given next.

Zhao and Tseng (2003) present a case study for constructing a public parking facility. Because the public parking facility may face insufficient capacity as demand increases, instead of seeking land to build another parking facility, the local government considers the possibility of expanding the parking structure vertically in the future. However, unless the original foundation has been designed to support expansion, it is normally infeasible (both technically and economically) to enhance the foundation after the construction is completed. An enhanced foundation provides a real option for future expansion but incurs additional construction time and costs. The authors determine the optimal size of the foundation that provides the maximal option value. They concluded that the flexibility value of an enhanced foundation in the case study is so significant that failure to account for flexibility is unjustifiable. Other literatures touching upon the application of option theory to infrastructure investment include Neely and de Neufville (2001), Gifford (2003, pages 100–110), and Zhao (2003).

Note that the decision to exercise a real option, such as expanding a parking garage, is made *after* uncertainties are revealed, which can eliminate some of the risk. Conventional valuation methods do not consider the value of being able to adjust after observing uncertainty, or simply the strategic value of flexibility. Therefore, neglecting the value of flexibility in the analysis may result in suboptimal decisions. Interested readers may find additional discussion of the concept of real options and valuation by Zhao (2003), and Zhao and Tseng (2003).

In this paper, the two terms, real option(s) and flexibility, will be used interchangeably. In addition, an option will be distinguished from an *alternative*.

Embedded Real Options

Many complicated decisions must be made during the life cycle of a highway system. For example, in the development phase, the DM must account for many factors such as the character of the area, the needs of the highway users, the benefits to the users, and the challenges and opportunities. The decisions that must be made include design of highway parameters, such as alignment, design speed, number of lanes, width of right of way, geometric shape, drainage, and intersections. Each design parameter may provide a different level of flexibility. After the highway is completed, decisions involving highway operation, maintenance, expansion, and rehabilitation activities may also be exercised to cope with the changing environment. By real options embedded in the life cycle of the highway system, we refer to the decision alternatives that may provide flexibility for future decision making or the decision alternatives that may be exercised flexibly in time to cope with uncertainty. In this paper, we focus on the following three real options:

Right of Way
A right-of-way contract is apparently a real option of expansion. Acquiring the required right of way is needed for every highway expansion (widening) process. Acquiring additional right-of-way width beyond immediate need may be viewed as reserving land. This may reduce the risks associated with land availability and price in future highway expansion.

Highway Expansion
With an acquired right of way, the DM may exercise the expansion real option. The decision making regarding exercising this real option involves the determination of the optimal timing and the number of expansion lanes at different stages in the life cycle.

Rehabilitation Decisions
These decisions may be viewed as real options, because they can be made flexibly to cope with highway deterioration. Because these options are readily available to the DM without any cost for acquisition, the focus will be on the exercise timing and the opportunity profit due to proper exercise of the option.

Note that the above real options are American-style options, because they are usually exercisable at any time during the highway service life.

Underlying Uncertainties

There is no doubt that a highway system is subjected to many uncertainties in its life cycle, such as changing requirements of users in terms of traffic demand, changing social and economic environment, changes in technology, and deterioration of the highway. They can be categorized as internal and external uncertainties. The internal uncertainties refer to those embedded in the evolution of the highway per se, such as aging and deterioration. The external uncertainties correspond to the variability of factors in the external environment that may affect decision making, such as land price, labor cost, demand, political and socioeconomic environment, land availability, and natural hazards, including earthquakes, hurricanes, and floods.

In this paper, we focus on the exercise of the three real options specified in the previous section subject to the following three uncertainties: traffic demand, land price, and highway condition/service quality. Each uncertainty is denoted by an upper-case letter and is discussed below.

Traffic Demand
The fundamental measure of traffic volume is the annual average daily traffic (ADT), which is defined as the number of vehicles that pass a particular point on a roadway during a period of 24 consecutive hours, averaged over a period of 365 days. Using empirical relation, ADT values can be converted to other measures of traffic, such as peak hourly volumes (e.g., Wright and Paquette 1979). The demand for traffic volume, denoted by Q, is represented by the ADT values. In particular, for toll roads, forecasting the demand accurately for the life cycle is an important task for economic purposes. Though traffic demand has been forecast for every highway system, potential pitfalls are prevalent in such forecasting, including data quality and model accuracy, system stability over time, land use, travel behavior, value of time, etc. Other pitfalls could include development of competing facilities, and changes in political and economic environments. These pitfalls exacerbate the demand forecasting inaccuracy and eventually become an underlined uncertainty for the highway system over its life cycle. Because of the wide variability of traffic flow over time, the demand Q is modeled as the following stochastic process:

$$\frac{dQ}{Q} = \mu_Q(Q,t)dt + \sigma_Q dz_Q \tag{1}$$

where z_Q=Wiener process. In particular, $\mu_Q(Q,t)$ is called the drift function, and σ_Q is the volatility. Without the noise z_Q, the demand pattern can be obtained by solving the following differential equation:

$$\frac{dQ}{Q} = \mu_Q(Q,t)dt \tag{2}$$

A positive drift term means the uncertainty tends to drift up over time; whereas the greater the volatility is, the uncertainty evolution is more volatile. The advantage of using Eq. (1) is to show the volatility of the traffic demand. Although Eq. (1) is a *continuous* stochastic process, we do not intend to argue that the traffic demand per se is continuous or can be best captured by a continuous model. Instead, in our implementation using Eq. (1), the traffic demand is simulated at discrete time points.

Land Price
Land prices vary over time. They depend on land use, which is used as an input to forecast traffic demand. The market value of a land parcel should be estimated at its highest and best use. According to the Uniform Appraisal Standards for Federal Land Acquisitions (Appraisal Institute 2000), land appraisal is usually implemented by one of the following three approaches: cost, sales comparison, and income capitalization. Land price, denoted by P, is assumed to follow the following stochastic process (Devuyst et al. 1995; Roebeling and Hendrix 2002):

$$\frac{dP}{P} = \mu_P(P,t)dt + \sigma_P dz_P \tag{3}$$

where z_P=Wiener process, and $\mu_P(P,t)$ and σ_P=drift function and volatility of land price, respectively.

Highway Service Quality
The highway service quality (or performance) can be defined as the degree to which the highway serves users and fulfills the purpose for which it was built (Hudson et al. 1997). It can be represented by a time series of quality measurements (or condition indices), on a scale of 5 to 1, corresponding to the conditions of excellent, good, fair, poor, and very poor, respectively. In this paper, the condition index at time t is denoted as I_t, and $\{I_t, t = 0,1,2,...\}$ is a (discrete time) Markov chain, which takes value in $\{1,2,3,4,5\}$ and $\{I_t\}$ decreases over time. The stochastic process $\{I_t\}$ may be viewed as the deterioration process of the highway with its value decreasing over time, if no maintenance or rehabilitation is applied. The factors that cause physical deterioration of a highway include traffic demand, load, environment, construction quality, material degradation, etc. Markov processes have been used to model infrastructure deterioration, to name a few, by Cesare et al. (1992), Madanat et al. (1995), Micevski et al. (2002) and Li et al. (1996).

Interdependency of the Uncertainties

There are well-pronounced interdependencies existing among various uncertainties, such as demand, land price, and service quality. An improved service quality highway system improves the "induced traffic." The improved economic condition increases the "developed traffic." Both induced and developed traffic improves social and economic condition of the region, which in turn increase the land use and its price. For example, the evolutions of traffic demand and land price may bear some positive correlation due to regional development. To model such an interdependency, a correlation can be imposed to the two Wiener processes that govern the uncertainty evolutions, e.g.,

$$\mathrm{cov}(z_Q, z_P) = \rho_{QP} \tag{4}$$

where ρ_{QP}=constant. Similarly, the increase of demand may also accelerate highway deterioration and reduce service quality. One can model the state transition probabilities of the Markov chain to be dependent on μ_Q and σ_Q.

Note that in this paper, for simplicity we have implicitly assumed that the uncertainties considered are independent of the DM's decisions. This may not be true in reality. For example, traffic flows may increase in response to a highway expansion decision, and land prices may react, too. To fully account for the interactions between the decision variables and the underlying uncertainties, the problem becomes an *equilibrium* problem and is much more complicated to solve in general.

Note on Uncertainty Discretization

While we modeled the traffic demand and the land price as continuous-time stochastic processes, we do not argue that each of them per se is continuous, or is best captured by a continuous model. Conversely, having an approximated continuous-time random process, its drift function and volatility function provide some insight into the model that uncertainty evolves over time. In real implementation, decision making and option exercise are considered only at discrete time points. Therefore, these two continuous random processes must be discretized in implementation (as detailed in a later section). It may be evident that a detour has been taken, namely, data observed at discrete time points are transformed to a continuous-time random process, and then transformed back to a discrete-time-based implementation for decision-making, but such an approach has been commonly adopted in practice.

The parameters for modeling the evolution of uncertainties can be estimated based on historical data. For example, Zhao and Tseng (2003) demonstrate how to estimate the drift and volatility of the parking demand for a parking facility. More technical issues about parameter estimation, such as data requirement, computation efficiency, and estimation consistency, can be found in Matasov (1998).

Multistage Stochastic Model

Under the three uncertainties presented in the previous section, the DM must constantly assess the system value (or profit) and cope with them with all available options. A mathematical model is presented below.

Mathematical Formulation

Previously, we have defined three uncertainties Q_t, P_t, and I_t for the traffic demand, land price, and condition index at time t, respectively. Additional standard notations for other parameters and variables are introduced next.

t=index for time ($t=0,\ldots,T$) in years, where T=length of the planning horizon over the life cycle of the highway system. n_t=state variable indicating the number of lanes of the highway at time t, where $n_t \in \{2,4,6,8\}$. Δn_t=decision variable indicating the number of lanes of the highway to be expanded at time t, where $\Delta n_t \in \{2,4,6\}$. w_t=state variable indicating the right-of-way width at time t. Assume that the width of the right of way along the highway is uniform and $w_t \in \{150,175,200\}$ (ft). Δw_t=decision variable indicating the width of the right of way of the highway to be acquired at time t, $\Delta w_t \geqslant 0$. $h_t = 0-1$ decision variable for rehabilitation. $\mathbf{v}_t$=a vector (collection) of state variables at time t, $\mathbf{v}_t = (n_t, w_t)$. $\mathbf{u}_t$=a vector (collection) of decision variables at time t, $\mathbf{u}_t = (\Delta n_t, \Delta w_t, h_t)$. $\mathbf{X}_t$=a vector (collection) of the underlying uncertainties at time t, $\mathbf{X}_t = (Q_t, P_t, I_t)$. $\mathbf{f}_t(\mathbf{v}_t;\mathbf{X}_t)$=revenue function of the highway system in time period t under state $\mathbf{v}_t$, conditioned on the uncertainty realization of $\mathbf{X}_t$ at time t. (Note the semicolon (;) distinguishes variables from parameters. In this case, $\mathbf{X}_t$ is a parameter.) $c_t(\mathbf{u}_t,\mathbf{v}_t)$=cost incurred for making decision $\mathbf{u}_t$ under state $\mathbf{v}_t$ at time t.

The objective is to develop an integrative framework that supports optimal decision-making in right-of-way acquisition, highway expansion, and rehabilitation under the uncertainties. The problem is modeled as a multistage stochastic program. The timing of the event occurrence is as follows. Assume that at state $\mathbf{v}_t$ at time t, the uncertainty vector $\mathbf{X}_t$ is revealed. Upon observing $\mathbf{X}_t$, the DM (i) must realize the current system revenue $f_t(\mathbf{v}_t;\mathbf{X}_t)$; and (ii) can strategically utilize available flexibility by making decisions $\mathbf{u}_t$ with a cost of $c_t(\mathbf{u}_t,\mathbf{v}_t)$ incurred.

Let $F_t(\mathbf{v}_t;\mathbf{X}_t)$ be the value-to-go function indicating the total value (expected profit) of the system for the remaining period at state $(\mathbf{v}_t)$ at time t. This problem can be formulated as the following recursive relation:

$$F_t(\mathbf{v}_t;\mathbf{X}_t) = f_t(\mathbf{v}_t;\mathbf{X}_t) + \max_{\mathbf{u}_t}\{e^{-r}E_t[F_{t+1}(\mathbf{v}_{t+1};\mathbf{X}_{t+1})] - c_t(\mathbf{u}_t,\mathbf{v}_t)\} \tag{5}$$

where E_t=expectation operator and subscript t=expectation based on the available information for the uncertainty $\mathbf{X}_t$ at time t, and $t \in [0, T-1]$. The maximization in Eq. (5) is subject to the following constraints.

State Transition Constraints

$$n_{t+1} = n_t + \Delta n_t \leqslant 8 \quad \forall t \tag{6}$$

$$w_{t+1} = w_t + \Delta w_t \leqslant 200 \ \ (\mathrm{ft}) \quad \forall t \tag{7}$$

Expansion Constraint

$$n_t \omega \leqslant w_t \quad \forall t \tag{8}$$

where ω=lane width. In general, the service quality of the existing lanes could be refreshed simultaneously when the new lanes are added. The service quality of a highway after expansion may be a function of the number of the existing lanes, number of the new lanes added, and the service quality before expansion. To relieve the computational complexity, we assume that expansion does not improve the service quality of the existing highway in this paper.

Rehabilitation Constraints

$$h_t = 1 \quad \text{if } I_t = 1 \tag{9}$$

$$I_{t+1} = 5 \quad \text{if } h_t = 1 \tag{10}$$

$$h_t \in \{0,1\} \quad \forall t \tag{11}$$

The rehabilitation constraints state that when the highway service level is in the "poor" condition, rehabilitation is mandatory. After the rehabilitation, the highway service level is upgraded to the "excellent" condition.

Initial Conditions

At time $t=0$, $\mathbf{v}_0=\tilde{\mathbf{v}}_0$, $\mathbf{X}_0=\tilde{\mathbf{X}}_0$

The first term in Eq. (5), following the expectation operator, defines another stochastic program to be considered in the subsequent year, which is discounted by e^{-r}, where r=risk-adjusted discount rate over one year, determined based on the riskiness of the project (Copeland et al. 1990). From the state transition constraints, we implicitly assume that all decisions require a one-year lead time. That is, highway expansion or rehabilitation requires one year to complete.

The expectation operator in Eq. (5) is not measured in the *risk-neutral* framework commonly adopted in financial option valuation. To determine the risk-neutral probability measure for the underlying uncertainties, such as the traffic demand, land price, and service level, is not trivial. The difficulty arises because there are no traded derivative securities dependent on the values of these uncertainties. Without these derivative securities, the "dynamic hedging" approach used in the financial options valuations cannot be applied (see Hull 1999). If there were derivative securities dependent on the traffic demand, land price, and highway service level, one would be able to obtain the *market price of risk* of these uncertainties using the dynamic hedging, and then obtain their corresponding risk-neutral processes [currently, Eqs. (1) and (3) describe the *true* probabilities of event occurrence, not the risk-neutral probabilities]. If the risk-neutral processes of the uncertainties were available, the discount rate would be the risk-free rate. To obtain a risk-adjusted discount rate, one may apply the capital asset pricing model (CAPM). One example using the CAPM can be found by Leviakangas and Lahesmaa (2002).

The optimal value F^*, representing the maximal expected system value, can be obtained from the last step of the recursive relation, represented by (5), $F_0(\tilde{\mathbf{v}}_0;\tilde{\mathbf{X}}_0)$ and the alternative that yields the maximal F^* will be recommended.

Detailed Modeling

In this section, we present a model for revenue and cost functions. Typically, in evaluating the feasibility of a highway project, decision variables are expressed in terms of revenues and cost functions. Due to the inherent complexity of modeling, each function is handled separately to address the corresponding variables.

Modeling the Revenue Function

The highway capacity is assumed to be a linear function of the number of lanes. For a highway to be constructed or under construction, the revenue function $f_t(\cdot)$ is assumed to be zero. For an existing highway, for the sake of simplicity, we consider below only two sources of revenue: traffic flow and land use.

$$f_t(\mathbf{v}_t;\mathbf{X}_t)=\text{revenue from traffic flow}+\text{revenue from land} \tag{12}$$

Assume the highway is a toll road. The revenue is modeled as a linear function

$$\text{Revenue from traffic flow}=\gamma \min[\alpha \cdot n_t \cdot x(I_t), Q_t], \tag{13}$$

where γ=average yearly revenue per vehicle; α=lane capacity of ADT; and $x(I_t)$=weighting factor of the revenue in terms of the highway service level. Since the capacity of a highway is measured by its ability to accommodate traffic, a lower service level may also lower the highway capacity. We model the weighting factor $x(I_t)$ using the following form:

$$x(I_t)=\beta^{5-I_t} \quad \beta \in (0,1) \tag{14}$$

It can be seen that the highway capacity will be discounted when the service level is not at the "excellent" condition.

$$\text{Revenue from land development}=\ell(w_t-\omega n_t)d, \tag{15}$$

where ω=lane width; and d=total distance of the highway. Because the excess right of way along the highway may also be used for purposes such as planting crops, parking lots, or further commercial development, a constant ℓ is used to denote the per mile revenue that the DM may obtain from the land use.

Modeling the Cost Function

The cost function $c_t(\cdot)$ is assumed to be a linear function, which is the summation of expansion cost, land acquisition cost, and rehabilitation cost.

$$\begin{aligned} c_t(\mathbf{u}_t,\mathbf{v}_t) &= \text{expansion costs}+\text{acquisition cost for right of way} \\ &\quad +\text{cost for rehabilitation} \\ &= d(c_n\Delta n_t + P_t\Delta w_t + c_m n_t h_t) \end{aligned} \tag{16}$$

where c_n=construction cost and c_m=rehabilitation cost. Both costs are measured per mile and per lane. Note that at time t, $c_t(\cdot)$ is known with certainty, but the future costs are uncertain.

Challenges of the Solution Algorithm

The proposed integrative highway planning model is a difficult stochastic optimization problem. It involves a constrained integer program at each stage subject to multiple correlated stochastic processes. As mentioned previously, the options involved in our model are American options. It is well known that the difficulty in valuing American options lies mainly in the determination of the optimal exercise strategy, which is likely to be different from stage to stage. Although there has been a rich body of finance literature devoted to American options valuation, solving the optimization problem addressed in this paper remains a challenging task. Two distinctions between the proposed optimization problem and the American financial option valuation are summarized as follows:

- Financial options normally involve only two alternatives, exercise or not exercise. The proposed integrative highway planning problem involves many more decision alternatives. The alternatives include: Whether to acquire right of way, and if so, how much? Whether to expand the highway, and if so, how much? And, whether to rehabilitate the highway or not. Also, constraints exist for exercising these options in the proposed problem, whereas there is no exercise constraint in the financial options valuation.
- The cash flows and payoff of a financial option are immediately known once it is exercised. Accordingly, once a financial option is exercised, the cash flows terminate. However, the decision making in the proposed model, as well as the associated cash flows, need not stop and may continue to the end of the planning horizon. The effectiveness of a decision in the highway system may not be known until many years later.

In the following section, we shall introduce an algorithm for solving the problem. The proposed algorithm may be viewed as an extension of the least-squares Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001). However, we extend the LSMC method to solve a much more complex problem.

Algorithm Development

To tackle the recursive relation in Eq. (5), we refer to the Bellman's "principle of optimality," which states that an optimal policy must contain optimal subpolicies (Bellman and Dreyfus 1962). Therefore, an inductive procedure to determine optimal (sub)strategies *backward* in time is necessary. However, the MC methods simulate state variables forward moving over time. Therefore, the challenge of the MC methods resides in determining optimal (sub)strategies at different stages to guide forward-moving simulations. The proposed approach will integrate forward-moving simulation iterations with backward moving dynamic programing steps to solve the recursive relation in Eq. (5).

First, we define

$$\pi_t(\mathbf{X}_t;\mathbf{u}_t,\mathbf{v}_t)=E_t[F_{t+1}(\mathbf{v}_{t+1};\mathbf{X}_{t+1})] \tag{17}$$

which appears in Eq. (5). Note that in Eq. (17) $\mathbf{u}_t$ and $\mathbf{v}_t$=parameters. This implies that there is a separate $\pi_t(\cdot)$ for each possible realization of $(\mathbf{u}_t,\mathbf{v}_t)$. If $\pi_t(\mathbf{X}_t;\mathbf{u}_t,\mathbf{v}_t)$ is available, at time t under $(\mathbf{u}_t,\mathbf{v}_t)$, one could know the expected system profit for the next time period when the uncertainty $\mathbf{X}_t$ is revealed at time t. One would also know how to make the optimal decision at t as well. An analytic form of $\pi_t(\cdot)$ is either nonexistent or very difficult to obtain. We will employ numerical methods based on MC simulation and the least-squares regression to approximate $\pi_t(\cdot)$.

To illustrate the idea, consider a set of N sample points of a random variable Z, $\{Z^{(1)},Z^{(2)},\ldots,Z^{(N)}\}$. It can be easily shown that the arithmetic mean of these N samples is the best representation of the samples in the sense of the least-squares error. To see that, consider the following minimization problem $\min_z\sum_{i=1}^N(z-Z^{(i)})^2$. It can be verified that $\sum_{i=1}^N Z^{(i)}/N$, the arithmetic mean, is the optimal solution. When N is sufficiently large, the arithmetic mean approaches the mean of Z.

In Eq. (17), to approximate $E_t[F_{t+1}(\mathbf{v}_{t+1};\mathbf{X}_{t+1})]$ we generate N data samples $(\mathbf{X}_t^{(i)},\mathbf{X}_{t+1}^{(i)})$, $i=1,...,N$ based on the uncertainty model of $\mathbf{X}_t$. Given fixed $\mathbf{u}_t$ and $\mathbf{v}_t$, we obtain $F_{t+1}(\mathbf{v}_{t+1};\mathbf{X}_{t+1}^{(i)})$, denoted by $F^{(i)}$. The expected value of $F_{t+1}(\mathbf{v}_{t+1};\mathbf{X}_{t+1}^{(i)})$ can be approximated by the function that best regresses $F^{(i)}$ on $\mathbf{X}_t^{(i)}$. Here, the least-squares regression is used to achieve two goals simultaneously: to approximate the functional relation between $F_{t+1}(\cdot)$ and $\mathbf{X}_t$ and to approximate the expected value of $F_{t+1}(\cdot)$ in the sense discussed previously.

According to the Bellman's principle of optimality, $\pi_t(\cdot)$ is obtained backward in time from $t=T$. At time $t=T$, first let

$$\pi_T(\mathbf{X}_t;\mathbf{u}_T,\mathbf{v}_T)=0, \quad \forall \mathbf{u}_T,\mathbf{v}_T,\mathbf{X}_T \tag{18}$$

Based on known $\pi_t(\cdot)$, the following algorithm is used to determine $\pi_{t-1}(\cdot)$, in backward iterations:

Algorithm: Obtaining $\pi_{t-1}(\mathbf{X}_{t-1};\mathbf{u}_{t-1},\mathbf{v}_{t-1})$, with $\pi_t(\mathbf{X}_t;\mathbf{u}_t,\mathbf{v}_t)$ known for all $\mathbf{u}_t$, $\mathbf{v}_t$

Data: $\mathbf{v}_{t-1}$ and $\mathbf{u}_{t-1}$ are given.

- Step 0: Set $i\leftarrow 0$, $F^{(i)}\leftarrow 0$.
- Step 1: If $i>N$, go to step 4. Otherwise, generate a random vector $\mathbf{X}_{t-1}^{(i)}$.
- Step 2: Evaluate

$$F^{(i)}\leftarrow f_t(\mathbf{v}_t^{(i)};\mathbf{X}_t^{(i)})+\max_{\mathbf{u}_t}\{e^{-r}\pi_t(\mathbf{X}_t^{(i)};\mathbf{u}_t,\mathbf{v}_t)-c_t(\mathbf{u}_t,\mathbf{v}_t)\} \tag{19}$$

- Step 3: Update $i\leftarrow i+1$ and $F^{(i)}\leftarrow 0$, then go to step 1.
- Step 4: Regress $F^{(i)}$ on $\mathbf{X}_{t-1}^{(i)}$ to obtain $\pi_{t-1}(\mathbf{X}_{t-1};\mathbf{u}_{t-1},\mathbf{v}_{t-1})$.

In the algorithm, the superscript (i) denotes the simulation iteration.

In the last iteration when $t=0$, setting $\mathbf{u}_{t_0}$, $\mathbf{v}_{t_0}$, and $\mathbf{X}_{t_0}$ to be $\overline{\mathbf{u}}_0$, $\tilde{\mathbf{v}}_0$, and $\tilde{\mathbf{X}}_0$, respectively, the maximization in Step 2 gives the optimal decision that yields the maximal expected system profit at $t=0$.

Remarks

1. Depending on how the initial condition $\tilde{\mathbf{v}}_0$ at $t=0$ is set, the proposed model can be applied in different phases of the life cycle of the highway development. A numerical example is demonstrated in the next section.
2. The interdependency is considered through the generation of the sample points. Therefore, this approach is very flexible in handling multiple and intercorrelated uncertainties.
3. A more efficient implementation can be achieved by modifying Step 1 of the algorithm for generating $\mathbf{X}_t^{(i)}$ at each iteration i. A fixed number of uncertainty sample *paths* ($\mathbf{X}_t^{(i)}$ from $t=0$ to T) can be predetermined prior to the algorithm. The uncertainty sample data can be stored in some array or database. The least-squares regression will be performed to fit these sample data in the database $\mathbf{X}_t^{(i)}$ at time t to their corresponding responses $F^{(i)}$. Because the sample data are fixed and $\pi_t(\cdot)$ are determined backward in time, evaluations of $f_t(\mathbf{v}_t^{(i)};\mathbf{X}_t^{(i)})$ and elements of the regression function for fitting $\pi_t(\mathbf{X}_t^{(i)};\mathbf{u}_t,\mathbf{v}_t)$ can be saved for determining $\pi_{t-1}(\cdot)$.

Additional implementation issues, such as the selection of the functional form for the regression are discussed below.

Numerical Examples

A multistage stochastic model has been presented for highway development decision making. This section presents numerical examples.

Consider a highway (without frontage) with available widths of right of way summarized in Table 1. The maximum width of right of way of the test system is 200 ft, and the maximum number of lanes is 8. System parameters are summarized in Table 2. Note that in Table 2 the cost data are based on a highway cost survey by the Washington State Department of Transportation (2002).

Table 1. Available Right of Way and Corresponding Minimum Width

Number of Lanes	Width (ft)
2	150
4	150
6	175
8	200

For the three uncertainties considered in this test system, traffic demand, land price, and highway service quality, assume that their current values are: 4,200 vehicles of ADT, $70,000 per acre, and "excellent." As mentioned previously, for implementation we discretize the stochastic processes corresponding to the traffic demand (1) and the land price (3) as follows:

$$\frac{\Delta Q}{Q}=0.05\Delta t+0.2\epsilon_1\sqrt{\Delta t} \tag{20}$$

Table 2. Test System Parameter Values

Parameter	Value
γ	\$14,000
α	1,000 vehicles
ℓ	\$10,000 per acre per year
ω	12 ft
d	50 miles
c_n	\$750,000
c_m	\$200,000
T	25 years
β	0.7

$$\frac{\Delta P}{P}=0.1\Delta t+0.2\epsilon_2\sqrt{\Delta t} \quad (21)$$

where ϵ_1 and ϵ_2=standard normal random variables with a correlation 0.2:

$$\text{cov}(\epsilon_1,\epsilon_2)=0.2 \quad (22)$$

Since the time units are in years, $\Delta t=1$ year. The discrete Markov chain of the highway deterioration is assumed as follows: When $I_t>1$, I_{t+1} has probability 0.5 to stay unchanged as I_t, and probability 0.5 to be I_t-1; when $I_t=1$, I_{t+1} remains unchanged $(=1)$. We assume the discount rate to be 8%. As discussed previously, estimating the (risk-adjusted) discount rate, or equivalently the probability measure, is not trivial. We do not elaborate the determination of the discount rate here because it is beyond the scope of this paper.

The function $\pi_t(\mathbf{X}_t;\mathbf{u}_t,\mathbf{v}_t)$, which defines a *recourse* stochastic program in Eq. (17), must be determined using regression for each possible realization of $(\mathbf{u}_t,\mathbf{v}_t)$ at each time t. That is, at each time t and each state $\mathbf{v}_t$ (there are nine possible cases, as shown in

Table 3. Highway System Value for Each Design Alternative

(n,w)	System value (\$$10^6$)
(2 lanes, 150 ft)	257.02
(2 lanes, 175 ft)	258.40
(2 lanes, 200 ft)	258.42
(4 lanes, 150 ft)	261.45
(4 lanes, 175 ft)	262.85
(4 lanes, 200 ft)	262.90
(6 lanes, 175 ft)	249.80
(6 lanes, 200 ft)	249.90
(8 lanes, 200 ft)	215.67

Table 3), and for each possible and feasible decision $\mathbf{u}_t$ (no more than eight other cases) to make, one must determine a function $\pi_t(\cdot)$ (no more than 72 cases). Since the condition index I_t is a discrete integer ranging from 1 to 5, instead of simulating it we determine a $\pi_t(\cdot)$ for each index value. This brings the (maximum) number of functions $\pi_t(\cdot)$ to be determined at each time t to be 360 $(=72\times5)$. The computation requirement of the proposed approach is intense.

Another challenge of using regression is to determine the functional form to be regressed. Some commonly used functional forms include polynomial, Hermite, Legendre, and Chebyshev. Based on our experience, the following polynomial functional form gives satisfactory results: for each given decision $(\Delta n_t,\Delta w_t,h_t)$ under each $\mathbf{v}_t$ and I_t,

$$\pi_t(\cdot)=a_1+a_2P_t+a_3Q_t+a_4Q_t^2+a_5Q_t^3+a_6Q_t^4 \quad (23)$$

where a_i, $i=1,\ldots,6$=constant. Although the same functional form is applied to each possible realization of parameters and each stage, the coefficients are likely to be different from case to case and from stage to stage.

Fig. 1 illustrates the basic idea of the LSMC. Sample points

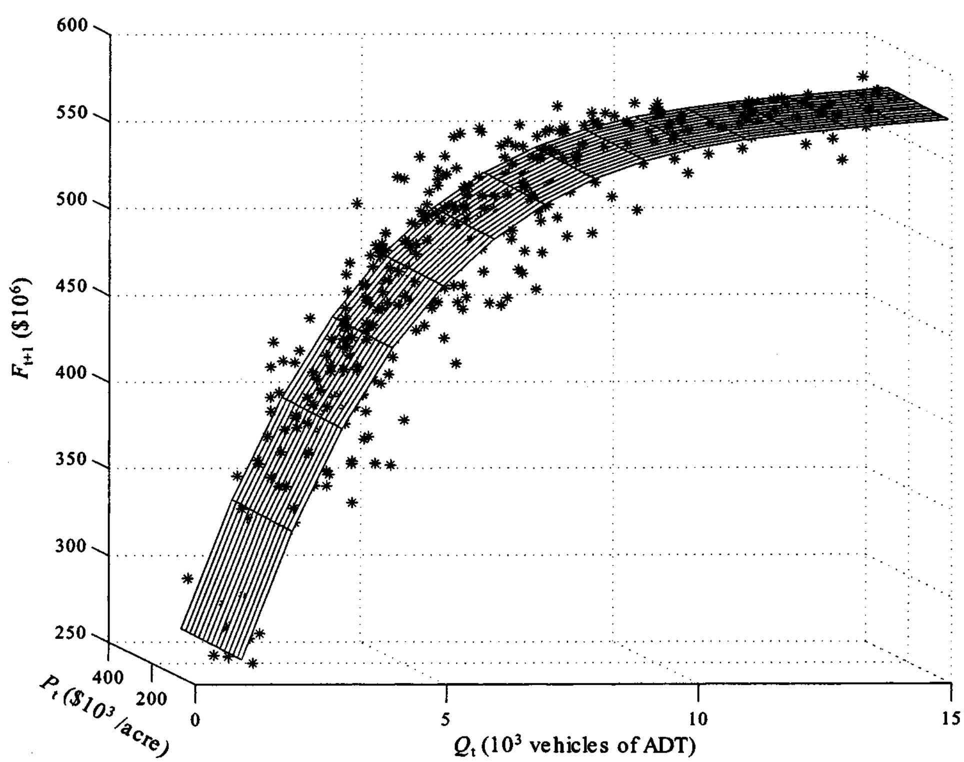

Fig. 1. Regression of $\pi_t(\cdot)$ when $t=10$, $n_t=2$ lanes, $w_t=200$ ft, $I_t=2$, and $h_t=1$

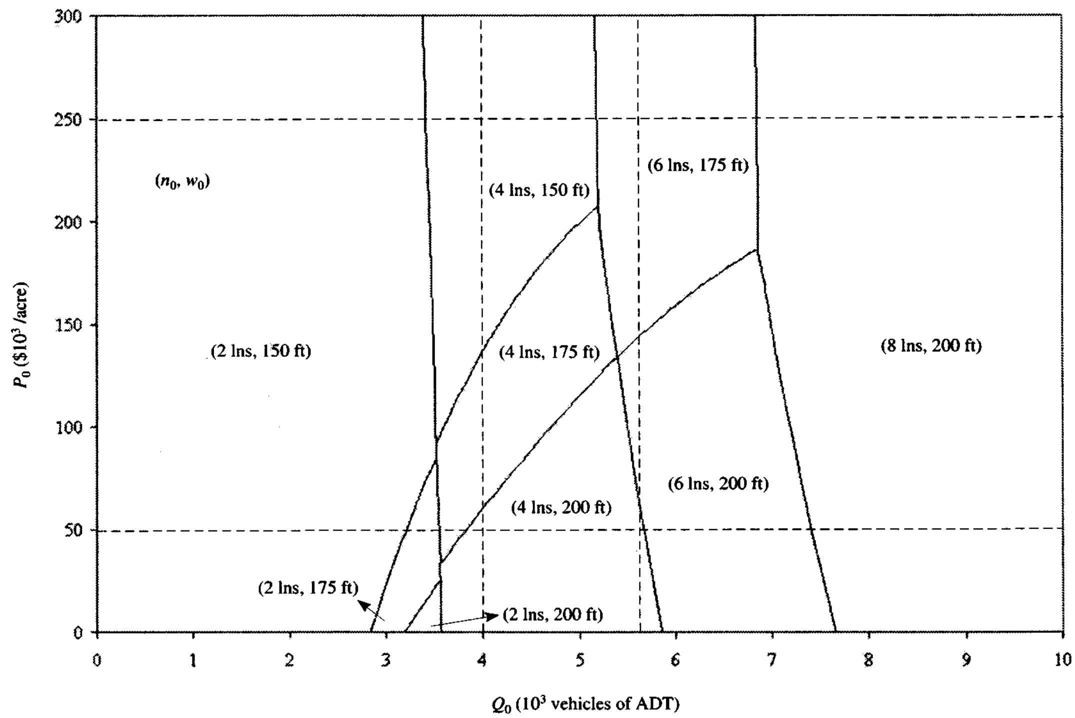

Fig. 2. Relation between prevailing design alternatives and the initial conditions

for the uncertainties P_t and Q_t are generated (or taken from a database) and evaluated. Their values are then regressed with respect to P_t and Q_t. Note that the regression function $\pi_t(\cdot)$ not only does regression fitting but averaging to approximate some expected value function. It can be seen from Fig. 1 that the LSMC obtains a reasonable approximation.

Numerical Test Results

As mentioned previously, the proposed multistage stochastic optimization model can be used for decision making in the design and operation phases. Next, we demonstrate its usefulness in different situations.

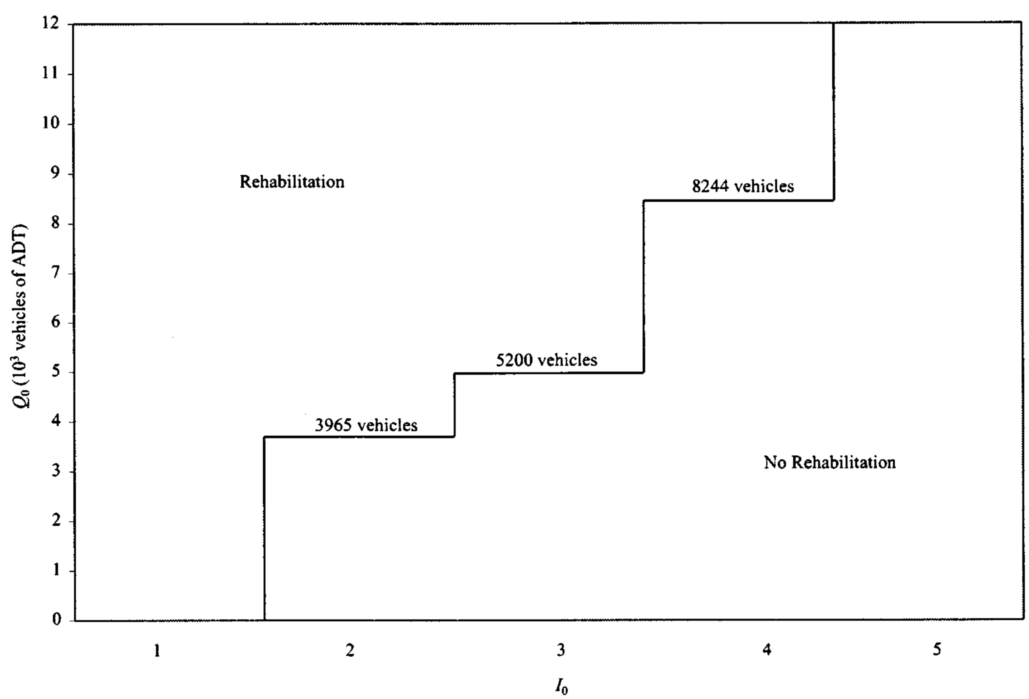

Fig. 3. Relationship between initial demand and rehabilitation decision

Table 4. Highway System Values for Expansion and Acquisition Alternatives

$(\Delta n_0, \Delta w_0)$	Expected profit ($\$10^6$)
(0 lanes, 0 ft)	407.64
(0 lanes, 25 ft)	407.67
(2 lanes, 0 ft)	412.09
(2 lanes, 25 ft)	412.14
(4 lanes, 0 ft)	399.04
(4 lanes, 25 ft)	399.14
(6 lanes, 25 ft)	364.91

Case 1: Selecting Design Alternatives

In the design phase, one decision that must be made is to determine the optimal number of lanes and the width of right of way. Consider a 25-year planning horizon and assume the uncertainties at time 0 are $P_0 = 70{,}000$/acre and $Q_0 = 4{,}200$ vehicles of ADT. Using the LSMC algorithm, the expected profit for each feasible design alternative is given in Table 3. The design alternative that yields the maximum expected profit should be recommended, which is to design a four-lane highway with 200 ft right of way purchased.

Next, we explore the relation between the optimal design alternative and the initial conditions of the traffic demand and the land price with all other parameters fixed, by repeatedly running the program. The result is depicted in Fig. 2. Clearly, the optimal alternative varies when the initial condition changes. In general, a higher traffic demand requires more lanes and encourages a greater land reserve if the land price is low. Although this general trend is very intuitive, the detailed quantitative relation between the initial conditions and the optimal design is nonlinear and not obvious. Consider the horizontal dotted line at $P_0 = \$50{,}000$. When the initial demand Q_0 increases, the corresponding change of the optimal design initially increases the land reserve, and then expands the highway. For example, 25 additional feet of width is acquired from 175 ft before the two-lane highway is expanded to a four-lane highway. When the maximum right-of-way width (200 ft) is reached, demand increases can only be accommodated by increasing the number of lanes. Consider another horizontal dotted line at $P_0 = \$250{,}000$. Because the land price is quite high, no land reserve is worthwhile. That is, the right-of-way width is always kept at its minimum level.

On the other hand, when the traffic demand is fixed, increasing the land price does not change the number of lanes in the highway design. For example, consider the vertical dotted line at $Q_0 = 4{,}000$, where increasing land price drives down the land reserve. However, at $Q_0 = 5{,}700$, the dotted line shows that increasing land price triggers highway expansion from four to six lanes. While this may seem somewhat unreasonable, it is because at this demand level (quite high), widening the highway can increase the service quality, which turns out to be profitable, according to our model (13). Another way to reason this is to recognize that the profit from the highway system is much greater than the value of the land. Therefore, additional revenue due to a higher service level can offset the increased cost of the land acquisition.

Case 2: Making Rehabilitation Decisions

In this case, we assume that the highway has been built and used for some periods. Currently, the highway has eight lanes with the right-of-way width of 200 ft. Again, considering a planning period of 25 years, the DM can use the proposed model to make rehabilitation decisions. In this situation, the land price will not affect the rehabilitation decisions since no right-of-way purchase will be needed. Therefore, the land price is kept fixed ($P_0 = \$70{,}000$) in this analysis. By repeated simulations, we determine the relation between the optimal rehabilitation decisions and the initial conditions (including the index and initial traffic demand), which is illustrated in Fig. 3. The relation depicted in Fig.

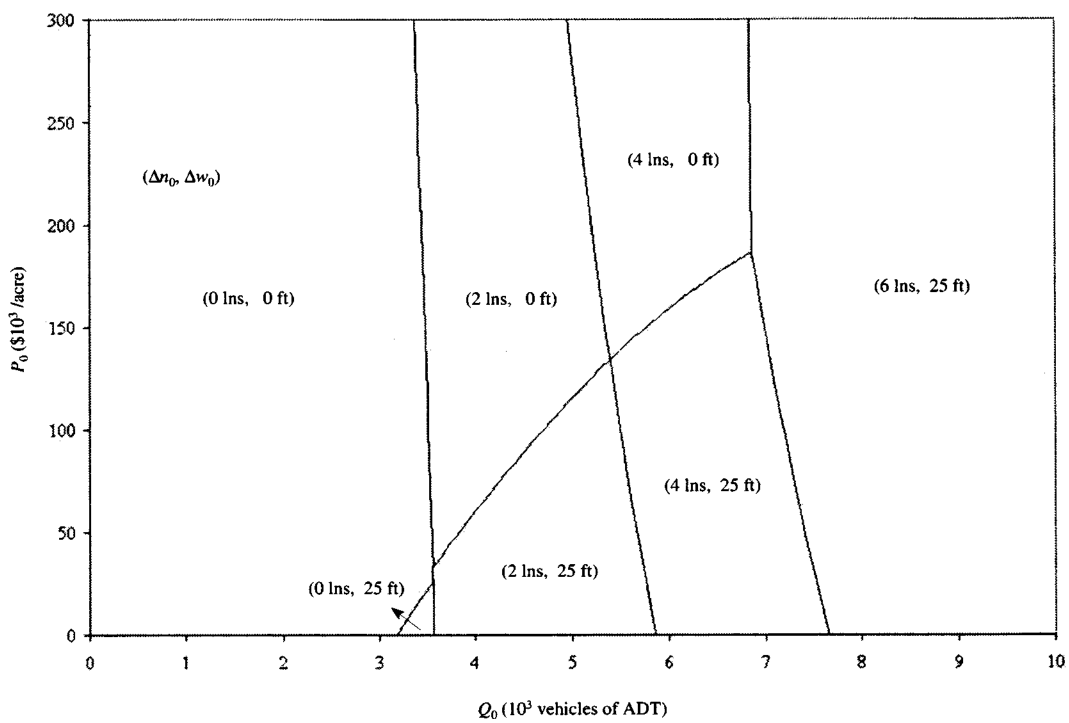

Fig. 4. Expansion and acquisition versus initial traffic demand and land price

3 is straightforward. It indicates that rehabilitation may still be needed even when the highway is in very good condition, if the traffic demand is sufficiently high.

Case 3: Making Decisions in Highway Expansion and Land Acquisition

In this case, the highway is assumed to have been built and used for a number of years. Currently, the highway has two lanes with a 175 ft wide right of way, and a condition index of 5. The DM can use the proposed model to determine whether to expand the highway and/or to acquire additional land and, if so, the width of land to be acquired and the number of lanes to be expanded. Assume a 25-year planning period and the initial conditions $P_0 = \$80{,}000$/acre and $Q_0 = 4{,}200$. The expected profits for all feasible alternatives are summarized in Table 4, in which the optimal alternative is to expand two lanes and to acquire 25 additional feet of right of way.

Similarly to cases 1 and 2, using repeated simulation, the relation between optimal expansion and acquisition decision and the initial conditions (including the initial traffic demand and land price) is depicted in Fig. 4. The relation is similar to that of case 1 and can be interpreted similarly.

Conclusions

In this paper we have presented a multistage stochastic model for decision making in highway development, operation, and rehabilitation. A solution algorithm based on the Monte Carlo simulation is developed. Numerical results indicate that the proposed model and solution algorithm are promising.

The model proposed in this paper offers a radical conceptual step towards optimal decision making in highway engineering, especially for highway expansion and rehabilitation decisions that are essential in highway systems subject to uncertainties. The proposed approach achieves decision-making *optimality*, which is generally not well defined in the traditional policy-based approaches. Furthermore, this model can very flexibly accommodate many more uncertainties than those considered in this paper, such as future highway status, future cost information, changing users' requirements, and rapid development of technology, and can be directly extended to handle larger cases such as a network of roads.

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