Partners in Crime: Collusive Corruption and Search

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Abstract

This paper analyzes corruption as a collusive act which requires the participation of two willing partners. An agent intending to engage in a corrupt act must search for a like-minded partner. When many people in the economy are corrupt, such a search is more likely to be fruitful. Thus when an agent engages in a search, he raises the net benefit of searching for other similar agents in the economy, creating an externality. This introduces a non-convexity in the model, which consequently has multiple equilibria. The economy can be in stable equilibrium with a high or low level of corruption.

Starting from the high-corruption equilibrium, a sufficient increase in vigilance triggers a negative cascade, leading the economy to a new equilibrium in which no agent finds it profitable to search for corrupt partners. The no-corruption equilibrium continues to be stable if vigilance is then relaxed. This suggests that the correct way to deal with corruption is to launch a “big push” with large amounts of resources. Once the level of corruption declines, these resources can be withdrawn.

KEYWORDS: corruption, search, coordination, vigilance, multiple equilibria
1. Introduction

Corruption is a common problem around the world, in developed as well as in less-developed countries. In the latter it is often ubiquitous, with bribes changing hands daily over small transactions. In developed countries there are fewer instances of corruption in everyday transactions, though incidents involving large considerations frequently come to light.

Corruption is defined as the use of public office for private gain (Jain 2001, Bardhan 1997). The most common acts of corruption—such as bribery—require collaboration between at least two agents: the individual who needs a dispensation he does not deserve, and the public official who is willing to make that dispensation in return for a payment or other favor.

In poorer countries, corrupt public officials are often in large supply and it is easy for an individual to identify such an official. In turn corrupt officials foresee a steady stream of individuals willing to pay bribes, and hence find it worthwhile to facilitate such identification even if it entails some risk of being apprehended and punished.

Consider, for example, day-to-day activities such as obtaining a driver’s license or buying a train ticket. An individual who attempts to obtain the service legally would go to the relevant government office and stand in line, following which he may still be denied the service with positive probability. Alternatively, having assessed the cost of a legal attempt, he may decide to approach a relevant official and offer a bribe. Many applicants choose the latter course, and a large proportion of officials are willing to dispense the service readily in return for a bribe. On both sides, the willingness to engage in corruption (and to flag the intention) is encouraged by the ready availability of partners on the other side of the table.

In most developed countries, on the other hand, members of the public do not expect to find a corrupt official readily. Rather than engage in costly search for a corrupt official, they therefore go about their business in a lawful way. Correspondingly, officials do not expect that many clients will arrive bearing offers of bribes, and hence find it prudent not to advertise themselves as being open to such advances. Only in cases where large gains are to be made do the respective agents undertake the costly process of searching for potential partners in corrupt activities.

1.1. This paper

This paper analyzes corruption as a collusive act. In order to reap the private benefit from corruption, agents must act in pairs. Thus an agent who wishes to engage in corruption must find a like-minded partner. This calls for search, which is costly. When there are many such agents searching, finding a partner is relatively easy, and
the expected benefit of search is positive. In other words, when an agent engages in search, he raises the net expected benefit of search for other similar agents in the economy, creating an externality. This introduces a non-convexity in the model, which consequently has multiple equilibria. The economy can be in stable equilibrium with a high or low level of corruption.

The government in our economy engages in vigilance against corruption. Vigilance results in some corrupt agents being apprehended, and these agents incur a penalty which outweighs the benefit of the corrupt act. An increase in vigilance increases the probability of being apprehended, and hence reduces the expected payoff of engaging in corruption.

If the government does increase the level of vigilance, then, some erstwhile corrupt individuals find that the net benefit of such behavior becomes negative, and refrain from looking for partners. In turn, this reduces the payoff for the remaining corrupt agents. A sufficient increase in vigilance can trigger a downward cascade, leading the economy to a new equilibrium in which no agent finds it profitable to search for corrupt partners. Once this equilibrium is reached, however, vigilance can be relaxed again—the no-corruption equilibrium continues to be stable. Thus control of corruption needs a intensive but temporary burst of anti-corruption activity—a characteristic common to coordination models.

Since anti-corruption activity yields results with a lag, a democratic government facing a short re-election horizon may be unwilling to commit large amounts of resources to such activity. This may explain why high-corruption equilibria sometimes persist in democratic countries with otherwise well-meaning governments that are fully cognizant of the evils of corruption.

The argument presented here may apply equally well to criminal activities other than corruption. The hallmark of a relevant activity is that it requires cooperation between more than one agent, and these cooperative groups must be formed anew each time the criminal activity needs to be undertaken. Many types of petty corruption readily fit the bill; a private citizen may overstep regulations in a number of directions, but each such transgression requires the cooperation of a government official in a different department. Organized crime, in contrast, also requires cooperation, but engages in actions committed repeatedly by members of the same syndicate, who do not need to find each other every time they transgress.

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1 Diamond (1982) showed that search and coordination failure can give rise to multiple equilibria with different levels of employment. Here we use it to explain the emergence of high- and low-corruption equilibria in economies with the same fundamental characteristics.

2 This is not an endorsement of totalitarian governments or dictatorships, which are founded on a corrupt basis to begin with, and unlikely to embrace anti-corruption activity for quite different reasons. Discussion of a potential flaw in the democratic process does not amount to a ratification of dictatorship.
1.2. The case of Hong Kong

An empirical case to which the present model may usefully be applied is the history of corruption in Hong Kong, which has received some attention in the literature. In the 1960s and 70s the Hong Kong police force was riddled with corruption. Bribery was rampant, and the Anti-Corruption Branch was itself thoroughly corrupt. Despite efforts by the government, corruption was increasing steadily in Hong Kong.

In 1973, the governor of Hong Kong established a new body known as the Independent Commission Against Corruption (ICAC) under the leadership of Jack Cater. Cater instituted a major assault on corruption, which among other things incorporated two features. First, he drastically increased the degree of vigilance, including the establishment of new channels for reporting corrupt acts. Secondly, he recruited experienced police officers from Britain on short contracts to replace local officers.

Klitgaard (1988) observes that the short-term hiring of officers from abroad prevented the development of “buddy-buddy” relationships, and increased the cost of searching for corrupt partners. It thus reduces the probability of success in undertaking illegal transactions. The high vigilance level increases the probability of being caught. Although these steps made the ICAC very expensive in the short run, it was successful in curbing corruption in the long run.

Our model differs formally from the Hong Kong case since we do not distinguish between government officials and members of the public, nor do we require that the agents in a corrupt pairing belong to different types. However, the aspects of the model that are investigated here are not too sensitive to this specification. The conclusions would continue to hold in a model with two populations where search is more effective for agents of one type if there are more corrupt agents among the other type.

1.3. Approaches to the analysis of corruption

Much of the existing literature on corruption analyzes the problem using the principal-agent model (e.g. Bardhan 1997, page 1321). The government cannot perfectly monitor its officials, so the latter have some discretion over their actions. This discretion may be used to promote personal gain, e.g. by accepting a bribe to authorize an application that does not meet relevant guidelines.

Since corruption is the outcome of asymmetric information, the remedy is to reduce information asymmetry (Rose-Ackerman 1978, pp. 17-29) or impose sufficiently high penalties (Becker 1968). If the enforcement authority is itself suscepti-
ble to corruption, then the penalty is not as efficient a deterrent (Basu et. al. 1992), but the problem can be alleviated by using an appropriate reward structure (Maj-jit and Shi 1998). Other work on corruption using the principal-agent approach includes Barro (1973), Becker (1983), Klitgaard (1988), Grossman and Helpman (1994), Rose-Ackerman (1999), Rasmusen and Ramsayer (1994) and Banerjee (1997).

A second approach to corruption analyzes it as a rent-seeking problem (Krueger 1974, Shleifer and Vishny 1993). In its purest form, successful rent-seeking realizes potential surplus by appropriately reallocating resources to high-surplus uses. Such corruption may increase efficiency (Lui 1985, Beck and Maher 1986). However, Shleifer and Vishny (1993) distinguish between corruption “without theft” and corruption “with theft”, and show that the efficiency argument does not hold uniformly. Murphy, Shleifer and Vishny (1993) argue that rent seeking activities exhibit increasing returns. This is reflected in our model described below, where an additional individual engaging in corruption increases the payoff to other corrupt agents.

One of the limitations of the above approaches is that they fail to explain why the incidence of corruption is so different between countries. A possible answer is that different countries have different norms, and therefore what is regarded in one culture as a corrupt act may in another be considered a routine transaction (Bardhan, 1997, page 1330). Multiple equilibria arise from an externality which originates in agents’ perceptions of the prevalence and acceptability of corruption in the economy (Cadot 1987, Sah 1988, Andvig and Moene 1990, Tirole 1996).

Another approach to the analysis of corruption is to investigate governance structures to identify the characteristics which make certain structures prone to corruption (e.g., Mookherjee (1998), Bardhan and Mookherjee (2005)). In this paper we treat the governance structure as exogenous, and hence do not discuss this literature in more detail.

The present paper focuses on the way agents come together to perform corrupt acts, rather than specifically modelling an individual corrupt act. A successful corrupt act in our model yields a positive benefit to the agents who cooperate in performing it, and simultaneously generates a larger cost for the society at large. We do not engage the question whether corruption reduces overall economic efficiency; we assume that it does. In this paper, as in the norm-based approach, individuals who engage in corruption generate an externality for others contemplating it. However, the present model is not a norm-based model of coordination—multiple equilibria in this model arise from complementarities in search rather than changes in popular attitudes towards corrupt activities. The externality is a consequence of the structure of the economy, not of a change in the subjective perceptions of agents.

The next section sets out the model. Section 3 identifies the equilibria and
establishes their characteristics. Section 4 then investigates the consequence of different anti-corruption policies.

2. Model

The economy consists of a government, and a large (countable) number of agents with mass normalized to unity. Agents are identical except in one dimension—the individual cost of being corrupt—discussed in section 2.4 below.

Time is divided into periods. In each period, each agent can potentially undertake one productive transaction. The transaction may be made in an honest manner or in a corrupt manner. The proportion of agents who are corrupt is denoted $e$, subscripted by the period if necessary.

2.1. Honest and corrupt transactions

An agent can undertake an honest transaction on his own in the normal course of activities. An honest transaction generates a private income of $y$ to the agent.

In order to undertake a corrupt transaction, an agent needs a partner who is also corrupt. An agent must search to find a partner, and content himself with an honest transaction if he fails to find a partner.\footnote{For example, suppose a dishonest building contractor wants to substitute low-grade materials for the contracted quality. He must buy these materials from a dishonest supplier, who will certify that material of the contracted quality has been sold and paid for. The two agents need to cooperate to carry out the fraud.}

For an individual agent the probability of finding a partner, $k(e)$, is an increasing function of $e$, the number of corrupt agents in the economy. For simplicity of exposition we assume that $k(e)$ is linear and increasing in $e$, and the probability of finding a partner is 0 when no one is corrupt. The results are robust to much weaker restrictions (see Remark 1 in Section 2.5).

\begin{equation}
    k(e) = ke, \quad k \leq 1
\end{equation}

If $e$ agents are corrupt (i.e., attempt corrupt transactions) then each agent finds a partner with probability $ke$. By the law of large numbers $ke^2$ agents will actually succeed in executing a corrupt transaction.\footnote{In this formulation no distinction is made between the size of the corrupt population and its proportion in the total. This is inconsequential as long as the size of the total population remains constant. If we were to analyze the impact of population growth on corruption, we would need to sharpen the model by specifying whether it is the proportion or the absolute size of the corrupt population that determines the matching probability.}
A corrupt transaction generates a private income of $y + \phi$ to the agent (where $\phi > 0$), but also imposes an external cost of $B$ on the economy. The cost $B$ is shared equally by all agents. Since the number of agents is large, the individual ignores the externality generated by his own corrupt act. However, the net social benefit of a corrupt act is negative:

$$B > \phi.$$  

Thus when an agent undertakes a corrupt act instead of an honest one, the total welfare generated in the economy decreases.

2.2. Vigilance

The government undertakes vigilance to prevent corruption. The quantity of vigilance is determined by expenditure allocated to it, denoted $v$. Vigilance is financed by a tax levied equally on all agents in the economy.

Vigilance results in some corrupt agents being apprehended. The probability that a corrupt agent will be apprehended depends positively on $v$. We denote by $p(v)$ the probability that a corrupt agent will be caught. We assume $p(.)$ is continuous, increasing and concave in $v$. Further, if the level of vigilance is high enough, then a corrupt agent will almost surely be apprehended, i.e.

$$p'(v) > 0, \quad p''(v) < 0, \quad p(v) \to 1 \quad \text{as} \quad v \to \infty$$

If an agent is apprehended, he is fined an amount $\beta$. The consequences of varying the penalty on crime is well known, as are the limitations placed by limited liability and finite wealth, and we do not investigate these issues here. For our purposes, the quantity $\beta$ is exogenous and constant throughout the analysis. We assume $\beta > \phi$; so the punishment is potentially a deterrent against corruption.

2.3. Income from transactions

We can now calculate the expected income of an agent conditional on the type of transaction he chooses, the proportion of agents in the economy that are corrupt, and the vigilance expenditure allocated by the government. Let superscripts $h$ and $n$ denote honest and corrupt agents respectively. The expected incomes of an agent

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6It can be argued that the proportion of corrupt agents would also affect this probability. We have adopted a formulation that simplifies the algebra and does not make a qualitative difference to the analysis.
on choosing the two kinds of transactions are, respectively:

\begin{align*}
Y^h &= y - Bke^2 - v \\
Y^n &= y + ke[\phi - p(v)\beta] - Bke^2 - v
\end{align*}

An honest agent obtains income $y$, pays a tax of $v$ and suffers negative externality of $B$ per corrupt transaction undertaken in the economy. Since $ke^2$ of the agents succeed in undertaking such acts, the total externality generated is $Bke^2$. A corrupt agent in addition succeeds in undertaking a corrupt transaction with probability $ke$, which generates a gain of $\phi$ but attracts a penalty $\beta$ with probability $p(v)$.

The incremental expected benefit of attempting a corrupt transaction rather than an honest one is therefore the difference between $Y^n$ and $Y^h$, which is denoted

\begin{equation}
Z(e, v) = ke[\phi - p(v)\beta]
\end{equation}

For given $v$, $Z(e, v)$ is a straight line from the origin with slope $k[\phi - p(v)\beta]$ when plotted against $e$. Recall $\beta > \phi$, and $p(v)$ tends to unity as $v$ becomes large. Thus, plotted against $e$, $Z$ is positively sloped when $v$ is small. It becomes flatter as $v$ increases, and ultimately becomes negatively sloped for sufficiently large values of $v$.

When $e$ agents are corrupt, the total income or surplus produced in the economy is

\begin{equation}
S = y - (B - \phi)ke^2
\end{equation}

### 2.4. Cost of a corrupt act

There is a cost to undertake a corrupt transaction, and this cost varies across agents. We think of this as a psychological cost of being dishonest, arising from a prejudice in favor of honesty, which is a consequence of the agent’s socialization and upbringing. Thus it reflects the agent’s personal degree of “aversion to corruption”. However, there are other specifications of cost that are consistent with the model, e.g. it could relate to the particular opportunity that is available to an individual agent. For each agent, this cost is given and constant throughout the analysis, and does not vary in response to the prevalence of corruption in the economy. The extent of corruption in the economy does affect the matching probability, captured by the function $k(.)$ described earlier.

Let the $i$-th agent’s cost of being corrupt be denoted $c_i$. $c$ is distributed in an interval $[c_0, c_1]$ with $c_0 > 0$. Without loss of generality, we arrange the agents in the economy in order of increasing cost, i.e.,
\[ i > j \iff c_i \geq c_j \]

The costs are distributed with density function \( f(c) \) and distribution function \( F(c) \). Since the population has been normalized to unity, we have \( F(c_0) = 0 \) and \( F(c_1) = 1 \).

### 2.5. Some assumptions

An agent \( i \) will choose to undertake a corrupt transaction if the expected gain from doing so exceeds the cost, in other words if \( Z(e, v) \geq c_i \). In order to ensure that corruption occurs under some circumstances, we make the following assumption:

**Assumption 1.** If there is no vigilance and if the entire population is corrupt, then even the agent with the highest cost will find it profitable to be corrupt. Formally:

\[ Z(e, v) > c_1 \text{ at } e = 1, v = 0. \]

For ease of exposition, we also assume that the distribution of costs is single-peaked. This would be true if the population is relatively homogeneous.\(^7\)

**Assumption 2.** The distribution of costs is unimodal, i.e., there is \( m \in (0, 1) \) such that \( f(.) \) is increasing in \([0, m)\), reaches a maximum at \( m \), and is decreasing in \((m, 1]\).

\[ G(e) \equiv F^{-1}(e), \quad e \in [0, 1] \]

\( G(e) \) is the cost such that the proportion of the population with costs \( c \leq G(e) \) is exactly \( e \). It follows from assumption 2 that \( F \) is monotonic and s-shaped, hence \( G \) has the following properties:

**Observation 1.** \( G \) is well-defined and strictly increasing. \( \exists \mu \in (0, 1) \) such that \( G(e) \) is concave in the region \([0, \mu]\) and convex in the region \((\mu, 1]\).

**Proof:** Let \( m \) be the (unique) mode of \( f(.) \). It follows from 2 that \( F \) is convex over \([c_0, m]\) and concave over \([m, c_1]\), i.e. \( F(.) \) is S-shaped. Further since \( f \) is non-zero everywhere, \( F \) is strictly increasing over its domain. The observation follows since \( G \) is the inverse of \( F \). \( \blacksquare \)

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\(^7\)For example, if we interpret the cost as a pang of conscience on committing a corrupt act, then unimodality implies that most people feel a similar distaste towards such an act, and relatively few people feel much less or much more distaste.
Remark 1. In the interest of an intuitive and tractable analysis, we have made some assumptions that are more restrictive than necessary. However, the results we obtain below hold true under far more general conditions. The following are sufficient:

(i) $k(.)$ and $p(.)$ are increasing in their arguments, which implies that $Z(e, v)$ is increasing in $e$ and decreasing in $v$,

(ii) $Z(e, 0) < c_0$ at $e = 0$, and

(iii) $Z(e, 0) > G(e)$ for some $0 < e < 1$.

Assumption 1 is a strong version of the last requirement, which ensures that there are some circumstances under which corruption will occur. The remaining properties are part of the model. The specific shapes assumed for $F(.)$ in assumption 2 and for $Z$ (via the linearity of $k$) limit the multiplicity of equilibria and facilitate the diagrammatic exposition which accompanies the analysis, but they are not essential to the qualitative results.

The next section establishes the equilibria for this economy. The following section investigates the dynamics of changing vigilance.

3. Equilibria: existence and stability

Given the number of corrupt agents $e$ and the level of vigilance $v$, the $i$-th agent will be corrupt (attempt to undertake a corrupt transaction) if

$$Z(e, v) \geq c_i$$

By the ordering of agents described in (5), if the $i$-th agent chooses to be corrupt, then so will all agents with indexes smaller than $i$. When $e$ agents are corrupt, an agent will choose to be corrupt if his expected gain from corruption, $Z(e, v)$ exceeds his cost. If the economy is in equilibrium with $e$ agents being corrupt, then the corrupt agents will be precisely the ones with costs between 0 and $Z(e, v)$, while those agents with a higher cost will choose not to be corrupt. Thus the fraction of the population with costs not exceeding $Z(e, v)$ must be exactly $e$.

Definition 1 (equilibrium). Given the vigilance level $v$, an equilibrium is a fraction of the population $e$ such that, when $e$ agents are engaged in corruption, the marginal agent is indifferent between honesty and corruption, i.e.:

$$F(Z(e, v)) = e.$$
For $Z(e,v) \in (c_0,c_1)$, an interior equilibrium corresponds to an intersection of the curves $Z$ and $G$ plotted against $e$ as in figure 1. At the corners, $e = 0$ is an equilibrium if $Z(0,v) \leq c_0$, and $e = 1$ is one if $Z(1,v) \geq c_1$.

An equilibrium $e^*$ is stable if a small deviation of $e$ from $e^*$ is self-correcting. We need that when $e$ exceeds $e^*$ by an infinitesimal amount, the marginal corrupt agent prefers not to be corrupt, while when $e$ falls below $e^*$ the marginal corrupt agent strictly prefers to be corrupt. At $e = 0$ only the first condition is relevant, and at $e = 1$ only the second condition is relevant.

**Definition 2** (Stability). An equilibrium $e^*$ is stable if there is $h > 0$ such that $G(e) < Z(e,v)$ for $e \in (e^* - h, e^*) \cap [0,1]$, and $G(e) > Z(e,v)$ for $e \in (e^*, e^* + h) \cap [0,1]$.

In terms of the diagram, an interior equilibrium $e^*$ is unstable if $Z$ intersects $G$ from below, and stable if the reverse is true. Corner equilibria, if they exist, are stable.

### 3.1. Corner equilibria

Propositions 1 and 2 follow straightforwardly from the assumptions.

**Proposition 1.** For any level of vigilance, there is a stable equilibrium in which no agent is corrupt.

*Proof:* $Z(0,v) = 0$ for all values of $v$, since $k(0) = 0$. Since all agents have strictly positive costs ($c_0 > 0$), no agent will choose to be corrupt when $Z = 0$. In other words, $F(Z(e,v))|_{e=0} = 0$. By continuity of $Z$ and $G$ in $e$, the gain remains less than the cost for small $e$, so the equilibrium is stable. ■

**Proposition 2.** When there is no vigilance, there is a stable equilibrium in which all agents are corrupt.

*Proof:* By assumption 1, the gain from corruption exceeds the cost for all agents, so $e = 1$ is an equilibrium. By continuity of $Z$ and $G$, this remains true when $e$ falls below unity by a small amount, so the equilibrium is stable. ■

### 3.2. Interior equilibria

Next we trace the equilibria that obtain for different values of the vigilance parameter $v$.

The two central relations of the model, $Z$ and $G$ are sketched in figure 1. The horizontal axis measures $e$, and the vertical axis measures incomes, costs, etc.
Figure 1: Cost of corrupt action compared with expected gain given different levels of vigilance

$G(e)$ has the shape discussed in observation 1 and takes values $c_0$ at $e = 0$ and $c_1$ at $e = 1$. $Z(e, v)$ has a value of 0 at $e = 0$ independent of $v$, and is linear by equations (1) and (3) for any given $v$. In the figure, $v'' > \hat{v} > v' > 0$.

**Lemma 1.** There is $\hat{v} > 0$ such that for $v < \hat{v}$, $Z(e, v) > G(e)$ for some $e \in (0, 1)$, and for $v > \hat{v}$, $Z(e, v) < G(e)$ for all $e > 0$.

In other words, when vigilance is higher than $\hat{v}$, corruption is never profitable for anyone in the economy, whereas with lower levels of vigilance there are some circumstances where corruption is profitable for some agents.

**Proof:** When there is no vigilance, the gain function $Z$ lies below the cost function $G$ at $e = 0$, and above it at $e = 1$. By the continuity of both functions, $Z$ must intersect $G$ at least once between 0 and 1. As $v$ increases, $Z$ decreases for all $e > 0$, and falls below the horizontal axis. Thus, for large enough $v$, $Z$ must lie entirely below $G$. By the continuity of both functions, the assertion must be true.
Corollary 1. : It follows that there is some $\hat{e} \in (0, 1]$ such that when $v = \hat{v}$,

$$Z(e, \hat{v}) \begin{cases} = G(e) & \text{for } e = \hat{e} \\ < G(e) & \text{for } e \neq \hat{e} \end{cases}$$

i.e. $Z$ lies below $G$ everywhere except at $\hat{e}$.\(^8\)

Figure 2: Two possible configurations of costs and expected gains ($v' < \hat{v}$)

The two possible configurations of the functions $Z$ and $G$ are shown in figure 2. The pattern of equilibria for different values of $v$ now follows naturally. Define $v_1$ to be the level of vigilance such that $Z(1, v_1) = G(1)$ (i.e., if $v = v_1$ and all agents are corrupt, then the individual with the highest cost of corruption is just indifferent between corruption and honesty). Let $\hat{v}$ be as defined in lemma 1.

Proposition 3 (Equilibria for different levels of vigilance).

(i) When $v \leq v_1$, there are three equilibria, at $e = 0$, $e_1 \in (0, \hat{e})$ and $e = 1$. Of these, 0 and 1 are stable and $e_1$ is unstable.

\(^8\)If $G(e)$ does not become sufficiently steep as $e$ approaches unity, then $\hat{e} = 1$. Note that, by assumption 2, $G$ is strictly convex and then strictly concave, which ensures that $\hat{e}$ is a point and not a segment.
(ii) When \( v_1 < v < \hat{v} \), there are three equilibria, at \( e = 0 \), \( e_1 \in (0, \hat{e}) \) and \( e_2 \in (\hat{e}, 1) \). Of these, 0 and \( e_2 \) are stable and \( e_1 \) is unstable.

(iii) When \( v = \hat{v} \) there are two equilibria, one at \( e = 0 \) which is stable and the other at \( e = \hat{e} \) which is unstable.

(iv) When \( v > \hat{v} \) there is only one stable equilibrium at \( e = 0 \).

Proof: (iii) follows from lemma 1 and observation 1. (i) and (iv) are obvious. The proof of (ii) follows from the shape of \( G \). Since \( v < \hat{v} \), \( Z \) is greater than \( G \) at \( \hat{e} \). Thus it must intersect \( G \) from below at \( e_1 < \hat{e} \) causing an unstable equilibrium. To the right of \( \hat{e} \), \( Z \) intersects \( G \) at \( e_2 \in (\hat{e}, 1) \) causing a stable equilibrium at \( e_2 < 1 \). \( G \) becomes progressively steeper as \( e \) increases beyond \( e_2 \), so \( Z \) remains below \( G \). ■

When \( v = 0 \), there is the stable equilibrium at \( e = 0 \). \( Z \) intersects \( G \) from below at \( e_1 \) which is an unstable equilibrium. A slight displacement from \( e_1 \) to the left will send the proportion of corrupt agents cascading down to zero, a slight displacement to the right will send that proportion to unity. To the right of \( e_1 \), \( Z \) remains above \( G \), and forms another stable equilibrium at \( e = 1 \).

As \( v \) increases from zero, the line \( Z \) pivots downwards, remaining rooted at the origin. Thus \( e_1 \) moves to the right. The point at which \( Z \) intersects the vertical at \( e = 1 \) moves down until it reaches \( c_1 \) when \( v = v_1 \). For higher values of \( v \), \( Z \) cuts \( G \) from above at \( e_2 < 1 \), which forms a stable equilibrium (see the line \( Z(., v') \) in figure 2). For such values of \( v \), the economy will gravitate to an equilibrium at \( 0 \) if the starting point is at some \( e \in [0, e_1) \), and to \( e_2 \) starting from any \( e \in (e_1, 1] \).

As \( v \) increases further, \( e_1 \) and \( e_2 \) move closer to each other, and coincide at \( \hat{e} \) when \( v = \hat{v} \). The equilibrium at \( \hat{e} \) is stable for deviations to the right, but unstable for deviations to the left.

Once the level of vigilance exceeds \( \hat{v} \), there is only one equilibrium at \( e = 0 \), and this equilibrium is stable.

4. Policy considerations

When the extent of vigilance is low, the economy may find itself in a stable equilibrium with a high degree of corruption, or at equilibrium with no corruption. The latter is not a problem and calls for no solution, but in the former case the government may want to take action. In this section we concern ourselves with the costs and benefits of changes in vigilance, and the process by which such changes affect the level of corruption.
4.1. Returns to increasing vigilance

We know from proposition 3 that high and stable levels of corruption can only occur when the level of vigilance is less than \( \hat{v} \). For \( v \in [0, v_1] \) the equilibrium is at \( e = 1 \), and the level of corruption is not sensitive to small changes in \( v \). For \( v > \hat{v} \), the only equilibrium is at \( e = 0 \). For levels of vigilance between \( v_1 \) and \( \hat{v} \), there is a stable equilibrium at \( e_2 < 1 \). A small increase in \( v \) causes \( Z \) to rotate down and \( e_2 \) to move to the left.

For interior equilibria, the equilibrium value of \( e \) is implicitly defined by equality between the costs \( G(e) \) and benefits \( Z(e, v) \) of corruption. Using (3), this can be written as

\[
G(e) = ke[\phi - p(v)\beta]
\]

Note that this is satisfied at two values of \( e \), \( e_1 \) and \( e_2 \). The rate at which \( e \) changes with \( v \) is found by implicitly differentiating the equilibrium condition (7). Rearranging and substituting from (7) we obtain

\[
\frac{de}{dv} = -\frac{\beta ke^2 p'(v)}{eG'(e) - G(e)}
\]

\( \beta, k \) and \( p' \) are positive, so the sign of the derivative depends on the sign of \( eG'(e) - G(e) \). A simple geometrical interpretation assures us that this is positive for \( e \) to the right of \( \hat{e} \), and negative for \( e \) to the left of \( \hat{e} \). Thus \( e_2 \), the stable equilibrium to the right of \( \hat{e} \) moves to the left—i.e. corruption falls—as \( v \) increases.

When \( v = \hat{v} \) the denominator of (8) vanishes. We know that the equilibrium \( e \) is discontinuous in \( v \) at this point, falling abruptly to zero.

When \( e \) agents are corrupt, the number of corrupt acts in the economy is \( ke^2 \), each of which generates a (negative) social surplus \( (\phi - B) \). The rate at which social surplus increases when \( e \) falls is therefore given by

\[
\frac{dS}{de} = 2ke(\phi - B)
\]

As long as \( v \) remains less than \( \hat{v} \), the rate at which social surplus increases with a marginal increase in vigilance can be obtained from (8) and (9). However, an increase in vigilance from less than \( \hat{v} \) to a value greater than \( \hat{v} \) causes corruption to fall discontinuously to zero, restoring the entire amount of social surplus that was previously lost owing to corruption.

**Proposition 4.** Suppose the economy is at equilibrium with \( v^* < \hat{v} \) and \( e^* > \hat{e} \). Then a marginal increase in \( v \) will raise social surplus at the rate

\[
\frac{dS}{dv} = \begin{cases} 
0 & \text{if } v < v_1 \\
\frac{2\beta k^2 (e^*)^3 p'(v)(B - \phi)}{eG'(e) - G(e)} & \text{if } v \in [v_1, \hat{v})
\end{cases}
\]
However, if the increase in vigilance is large enough to raise \( v \) above \( \hat{v} \), then corruption falls to zero and social surplus increases by \((ke^*)^2[B - \phi]\).

**Proof:** The value of \( \frac{dS}{dv} \) for \( v < v_1 \) follows from Proposition 3 part (i). The non-zero expression for the case where \( v \in [v_1, \hat{v}] \) is obtained from (8) and (9). The last part where \( v \) rises above \( \hat{v} \) follows from Proposition 3 part (iv).

The behavior of \( \frac{dS}{dv} \) as \( v \) changes is not immediately obvious. As \( v \) increases, \( p'(v) \) in the numerator falls (see 2). But at the same time \( e \) decreases, so given the shape of \( G(e) \), the denominator also falls. Since both numerator and denominator decline with an increase in \( v \), it is not possible to predict the direction of change in the overall quantity without further assumptions. We do know, however, that as \( v \to \hat{v}^+ \), \( \frac{dS}{dv} \) increases without bound.

### 4.2. Adjustment to equilibrium with change in vigilance

The process of adjustment to a new equilibrium after a change in vigilance in fact takes more than a single period. This section provides a brief exposition of the adjustment process.

To link periods in the dynamic analysis we will assume that, in period \( t \), agents expect the degree of corruption that obtained in period \( t - 1 \) to prevail, that is

Assumption 3. \( E e_t = e_{t-1} \).

Where \( E \) is the expectations operator. The simple adaptive expectations assumption provides a degree of inertia that keeps the analysis intuitive.\(^9\) The adaptive expectations assumption also captures some of the sluggishness that would arise if agents in the population did not fully believe the government’s pronouncements on anti-corruption initiatives. For example, suppose that the government declares that vigilance will be increased from \( v \) to \( v' \), but agents only ascribed a probability less than unity to the event that this will indeed be carried out. Then some agents (those who have high individual cost) would find it optimal in expected terms to desist from corruption, while others would continue to search for corrupt partners. As the government persists with higher vigilance, the latter agents would gradually become convinced of the government’s intent, so corruption would decline to its new equilibrium level over time, but not instantly.

In assumption 3 we have posited the adaptive expectation rule that agents in a given period expect the proportion of corrupt agents to be the same as the proportion

\(^9\)In the present model, fully rational expectations would unnecessarily multiply the multiplicity of equilibria. Adaptive expectations eliminates the possibility that the existing equilibrium might become disrupted purely as a result of the coordinated expectation that all agents may behave differently tomorrow.
they have witnessed in the previous period. They do, however, perceive the level of vigilance correctly, and hence calculate their expected gain based on these two variables:

\[
E_t Z_t = ke_t e_t \left[ \phi - p(v_t) \beta \right] = ke_{t-1} \left[ \phi - p(v_t) \beta \right] = Z(e_{t-1}, v_t)
\]

where subscripts indicate time-periods. Note that expectations are formed at the beginning of the period, before the values of the variables are realized. Actions for that period are taken based on these expectations, which then generate the realized values. The sequence of events within each period is as follows: the government announces a level of vigilance, individuals decide whether to pursue honest or corrupt transactions based on this level and on the previous period’s corrupt proportion, and these decisions then determine the corrupt proportion in the present period.

Suppose that, in some initial period \( t \), the economy is in a position of equilibrium with a high level of corruption \( e_0 > \hat{e} \). Let the vigilance level at this equilibrium be \( v_0 \). In period \( t + 1 \), the government increases the level of vigilance to a higher value \( v_1 \). Corruption will fall to a new lower equilibrium level \( e_1 \). However, this adjustment will occur in steps over a sequence of periods.

In period \( t + 1 \), agents calculate the gain from corruption as \( Z(e_0, v_1) \). Accordingly, the number of agents who decide to seek a corrupt partner is \( e_{t+1} \), given by the solution to

\[
G(e_{t+1}) = Z(e_0, v_1)
\]

Since \( e_0 \) is the equilibrium corresponding to \( v_0 \), which is smaller than \( v_1 \), it follows that \( e_{t+1} < e_0 \). In successive periods, the levels of corruption \( e_{t+n} \) are given by the iterative formula:

\[
G(e_{t+n}) = Z(e_{t+n-1}, v_1) \quad n \geq 1
\]

Note that \( e_{t+n} \) is a decreasing sequence which converges to \( e_1 \), the equilibrium level corresponding to \( v_1 \). Convergence takes an infinite number of steps if \( e_1 \) is to the right of \( \hat{e} \) (i.e. \( v_1 < \hat{v} \)), as is usual in equilibrium dynamics.

However, it is noteworthy that, if \( e_1 = 0 \) (i.e. \( v_1 > \hat{v} \)) full convergence occurs in a finite number of steps. Figure 3 illustrates the two processes. The reader can readily convince herself that, the larger is \( v_1 \), the smaller is the number of steps or periods it takes for corruption to fall to zero.
4.3. Local vs. global anti-corruption measures

If there is a high level of corruption in the economy, is it socially desirable for the government to reduce corruption by increasing vigilance? In light of proposition 4, the answer to this question may depend on the size of the proposed anti-corruption campaign. It is possible that a small increase in vigilance does not pay for itself in terms of increased social surplus, but a large onslaught on the problem yields more than proportionate benefits. This is a common property of coordination models.

For the purpose of this discussion we abstract from considering the lag in adjustment following a change in vigilance. As a simplification, suppose that full adjustment to equilibrium occurs immediately, in the same period as the change in vigilance is implemented. The level of corruption in period \( t, e_t \), solves the equi-
librium condition (7) with \( v = v_t \). It will be clear that any modifications that arise from full consideration of the adjustment process are quantitative, not qualitative.

Suppose that initially \( v < \hat{v} \), and the economy is in stable equilibrium at a high level of corruption \( e^* > \hat{e} \). Now consider two different policy initiatives.

In the first initiative—which we will call the “small” initiative, the government increases vigilance marginally to \( v' \), where \( v < v' < \hat{v} \). In the second “large” initiative, vigilance is increased drastically to \( v'' > \hat{v} \).

If the small initiative is implemented, then in each period the government incurs a cost of \( \Delta v = v' - v \), and obtains an increase in social surplus of \( \Delta S_{\text{small}} = 2 \beta k^2(e) p'(v)(B - \phi) e G'(e) - G(e) \Delta v \) (see Proposition 4), for a net gain in each period of

\[
\Delta \pi_{\text{small}} = \left[ \frac{2 \beta k^2(e^*)^3 p'(v)(B - \phi)}{e^* G'(e^*) - G(e^*)} - 1 \right] \Delta v
\]

Note that the benefit is obtained only in those periods that the expenditure is undertaken.

If the large initiative is implemented, however, vigilance rises above \( \hat{v} \) leading to a decrease in corruption to zero. Once this is attained, the level of corruption will remain at zero in subsequent periods even if vigilance is eliminated altogether, since \( e = 0 \) is a stable equilibrium for all values of \( v \) (proposition 1). The per-period benefit from implementing the large initiative is \( \Delta S_{\text{large}} = k(e^*)^2[B - \phi] \), and the one-time cost is \( \Delta v'' - \hat{v} \).

Thus with a large initiative the government spends a significant amount on vigilance for a short period, and then enjoys substantially lower corruption forever. With a small initiative, the reduced level of corruption endures only as long as vigilance remains high. Of course, a particular government may not care about “forever”, but only about a number of periods that is relevant to its own political purposes. The large initiative will look attractive when this relevant horizon—which we may call the government’s “planning horizon”—is long and the future is not heavily discounted.

**Proposition 5.** (i) If the government has a long enough planning horizon and a sufficiently high discount factor, then the large initiative is economically viable (i.e., yields positive net benefit).

(ii) If the government has a long enough planning horizon and a sufficiently high discount factor, then it prefers the large initiative to the small initiative.

**Proof:** (i) Let the government’s planning horizon \( T \) satisfy

\[
T > \frac{\frac{\Delta v'' - \hat{v}}{k(e^*)^2[B - \phi]}}.
\]

The
discounted net benefit from the large initiative is

\[ \Delta \pi^{\text{large}} = -[v'' - v] + \sum_{t=0}^{T} \delta^t (k(e^*)^2 [B - \phi]), \]

which must become positive as \( \delta \to 1 \) by definition of \( T \).

(ii) The per-period welfare gain with the large initiative \( \Delta S^{\text{large}} \) is strictly bigger than the per-period welfare gain under the small initiative \( \Delta S^{\text{small}} \), since under the former corruption is eliminated completely, but under the latter it is only eliminated partially. Let the government’s horizon \( T \) be large enough that \( T \) times \( \Delta S^{\text{large}} - \Delta S^{\text{small}} \) exceeds the one-time incremental cost \( v'' - v \) of the large initiative. Then in the limit as \( \delta \to 1 \), the discounted sum \( \sum_{t=0}^{T} \delta^t [\Delta S^{\text{large}} - \Delta S^{\text{small}}] \) must also exceed \( v'' - v \). Hence for such \( T \) and \( \delta \), the government must prefer the large initiative even if the cost of the small initiative is 0. ■

Finally, note that in (14), \( \Delta \pi^{\text{small}} \geq 0 \) only if

\[ 2 \beta k^2 (e^*)^3 p'(v)(B - \phi) + G(e^*) \geq e^* G'(e^*). \]

We know \( k \) and \( e \) are bounded above by unity and \( G \) by \( c_1 \), so the left-hand-side is bounded above by \( 2 \beta p'(v)(B - \phi) + c_1 \). At high values of \( e \), \( G'(e) \) on the right-hand-side will be steep if at high values of \( c \) the density \( f \) is small. Thus for appropriate specifications of the functions \( p \) and \( f \), the inequality above may not obtain, i.e., the benefit from a small initiative may well be negative.

The fact that a large initiative leads to a zero corruption equilibrium which then remains stable even in the absence of vigilance is reminiscent of norms-based models of aggregate behavior. We hasten to point out, however, that in norm-based models it is the attitude of agents towards certain types of behavior that changes when they observe that such behavior is no longer the norm. In the present model there is no change in attitudes—the ‘distaste for corruption’ captured by \( c_i \) remains constant for each individual \( i \). However, the decline in aggregate corruption creates an externality which renders corruption unprofitable. This is perhaps important to note, since the difference between a corrupt economy and a non-corrupt one is thus not made by “changing people”, as many idealists may believe necessary.

5. Conclusion

In this paper we analyzed corruption which requires collaboration between agents. In order to undertake a corrupt act, an agent must find a willing partner. This description includes some of the most common categories of corruption, such as obtaining an undeserved building permit by bribing an officer in the municipality, or
obtaining information on tender bids made by rival firms by bribing the appropriate government official.

Undertaking search is costly for agents, and search is more likely to yield results when there are more agents in the economy who are willing to cooperate in corruption. Thus when many agents are corrupt, the expected benefit of corruption is high, and even agents with high costs of search find it profitable to search. Similarly, when few agents are corrupt the potential benefits low and even low cost agents find that search is not worthwhile. Thus the economy has multiple equilibria, some with high levels of corruption and some with low levels of corruption.

We have shown that, if a high-corruption equilibrium initially obtains in the economy, then an effective policy consists of a very high intensity (and possibly high-cost) vigilance program. If such a program is pursued for a sufficient length of time, the economy will converge to the low-corruption equilibrium, where it will then stay even if the vigilance program is subsequently eliminated.

For some kinds of corruption, it is reasonable to assume that in each corrupt partnership the two agents must be drawn from two distinct populations: government officials and members of the public. The present model can be extended to incorporate this detail. The search efficiency of each type of agent would then depend on the corrupt fraction among the other type. However, this complication does not alter the qualitative results obtained in this paper. Such a formulation, however, may be useful for investigating other related questions: for example whether it is more effective to target anti-corruption efforts at government officials or at members of the public. It may also be a natural framework within which to explore the consequences of corrupt relationships which are long-term, and the effectiveness of rotating officials among jobs in such contexts.

References