Search and Intermediation: Toward a Model of the Merchant Trader

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Abstract

This paper develops a dynamic model of an economy in which homogeneous agents choose between specializing as producers or as merchants, and can change occupation at any time. Merchants operate alongside a decentralized search market and provide immediacy in exchange in return for a price. We characterize equilibria in symmetric Markov strategies, and derive conditions under which merchants and their clients form a repeated relationship. We analyze welfare and discuss the prospect of an endogenous rise of an institution of intermediation.

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1. Introduction

Merchants and traders—agents who mediate the transfer of goods and services between producers and consumers—are central to the economic process. Specialization, the source of the wealth of nations, cannot proceed unless some agents mediate exchange.[1] In an economy with any degree of specialization, agents must trade to acquire goods that they wish to consume in exchange for goods that they produce. Searching for trading partners can be costly and time-consuming. Some agents recognize that there is profit in facilitating exchange and specialize as intermediaries or merchants. They reduce search costs and provide immediacy in exchange. Further, merchants derive profits from repeat business with returning clients rather than from random encounters with one-time clients.

The present paper focuses on these characteristics of the merchant. Our objective is a parsimonious but self-contained model of the merchant trader that reflects some essential features of an institution of intermediation in its "early and rude state". We are especially interested in the prospect of endogenous emergence of specialist merchant traders.

We explore a variant of Diamond's (1982) "coconut economy" in which homogeneous agents choose whether to specialize as producers or as merchants, and can change occupation at any time. A producer must exchange his output each period before he can consume. Exchanges are made with other producers or with merchants—the producer finds these partners through search. A merchant sets up a trading post; producers arriving at her post can trade by paying a commission that the merchant she sets each period. Locating a merchant through search may not be *a priori* easier than locating another producer. However, a merchant can predict her own location from one period to the next, whereas a producer cannot. A producer who succeeds in finding a merchant may thus return to her in the next period, avoiding search. The viability of specialized intermediation is predicated on the ability of intermediaries to form such ongoing relationships with their clients.

Producers sometimes forget the locations of their merchants, so a search market remains active in parallel. A producer may therefore credibly decline to return to his merchant and choose instead to search anew for a trading partner. The continued existence of a search market also affords incipient merchants a pool from which they can draw clients. The process by which a new merchant acquires clients is endogenous to the dynamics of the model.

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[1]Thus Hicks, who considered the merchant trader the "principal character" in economic history, wrote that "it is specialization upon trade that is the beginning of the new world" (Hicks, 1969, p.25).

An equilibrium determines the occupational choice of each agent, and the commissions charged by merchants. We find that there are two classes of equilibria in symmetric Markov strategies. In *bandit equilibria*, merchants act as bandits and claim the entire output of their clients as "commission". In this case, producers understandably never return to these merchants, but search for trading partners in each period. This of course bears no resemblance to an institution of intermediation.[2]

Of greater interest to us are *intermediation equilibria*, in which merchants charge a commission that induces existing clients to return in succeeding periods. Intermediation equilibria exist only if producers remember the location of their merchants with sufficiently high probability. In such equilibria, the average merchant makes a supernormal profit: her payoff remains larger than that of a producer, even though producers can choose to start up as merchants at any time.

An agent who specializes as a merchant facilitates exchange, but does not produce output. The optimal proportion of merchants in the economy must balance these two effects. We find that, in general, an equilibrium is not optimal. We give conditions under which an intermediation equilibrium improves welfare compared to an economy with no merchants.

Finally, we discuss conditions under which an institution of intermediation can be expected to arise endogenously when the *status quo ante* is an economy with no merchants. We provide a heuristic interpretation of the parameters in our model in terms of technological and socio-political conditions that determine when intermediation rises and flourishes and when it declines under threat of brigandry and disorder.

The investigation of the role of intermediaries in speeding up search was initiated by Rubinstein and Wolinsky (1987).[3] In their model, buyers, sellers, and intermediaries are randomly matched: trading with an intermediary is not a choice. In equilibrium, intermediaries are active if buyers and sellers encounter an intermediary at least as often as each other.

Gehrig (1993) presents a static model in which buyers and sellers, who differ in valuations and costs that are private information, can choose to search for trading partners and negotiate price, or access intermediaries whose locations and prices are publicly observable. Yavas (1994) allows heterogeneous agents to choose the intensity of private search, or to opt for the service of

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[2]It is perhaps no accident that, in many historical contexts, merchants and brigands possessed similar enforcement capabilities. Even today unscrupulous merchants may, if they choose, defraud an unsuspecting client once with ease.

[3]Various other functions of intermediaries have been investigated in the literature: Spulber (1999) has an extensive survey.

an intermediary. Spulber (1996) presents a dynamic model in which buyers, sellers, and intermediaries are heterogeneous in several respects, and intermediaries and their prices have to be found through search; there is no parallel unmediated search market in which buyers and sellers can trade directly. The focus is on deriving the equilibrium bid-ask spread and comparing the outcome with Walrasian prices. Rust and Hall (2003) extend the model of Spulber (1996) by adding a second type of intermediaries who post publicly observable prices. **?** examines the role of fiat money in a search market where merchants organize exchange.

The papers cited above can accommodate rich heterogeneity among agents, but intermediaries are exogenously present in the economy and do not choose their calling. In contrast, a key concern of the present paper is to generate an endogenous distribution of occupational assignments in equilibrium starting from a homogeneous population.

We are aware of only a few papers that explicitly model endogenous occupational choice between production and intermediation. Li (1998) does so in the context of a friction quite different from ours: the function of intermediaries is to assess the quality of goods that are traded. In Bhattacharya and Hagerty (1987), producers may trade only with intermediaries; thus, the viability of intermediation is never in question. In Hellwig (2002) and Shevchenko (2004), the role of intermediaries is to resolve the problem of double coincidence of wants. Intermediaries achieve this by complementing money in Hellwig's model, and by stocking a variety of goods in Shevchenko's model. In both these models, there is also an unmediated search market, as in ours. Prices set by intermediaries in Hellwig's model are publicly observable; terms of trade with an intermediary are determined by Nash bargaining in Shevchenko's model.

Our paper is most closely related to Masters (2007). Masters also investigates the endogenous emergence of intermediaries in the context of Diamond's "coconut" model. In his model, intermediaries enter the market with a unit of a good they have acquired after exchange. This gives them an advantage in Nash bargaining with producers because the intermediary has the option of consuming the good he holds whereas the producer does not. He finds that, when all producers have identical production costs, intermediaries uniformly reduce welfare in the economy; however, when production costs are *ex ante* unequal intermediation can increase welfare. In contrast, the advantage of intermediaries in our model comes from the potential of repeated trade which Masters does not allow. As a consequence, in our model, intermediation can improve welfare even though all producers face identical costs (which we normalize to zero).

In all but one (Bhattacharya and Hagerty, 1987) of the papers mentioned above, agents search for trading partners from scratch in each period: intermediaries do not establish durable links with their clients. In our paper, the benefit of establishing a trading link with an intermediary is that search costs can be avoided in *future* periods. At an intermediation equilibrium, clients and their merchants form ongoing relationships.

Durable client relations is accommodated in some papers that investigate price-setting by sellers in a market where consumers search for prices. The pricing component of the model here is similar to Benabou (1997), but simpler as our agents are homogeneous while Benabou allows heterogeneous agents. Burdett and Coles (1997) presents a model of noisy search in which a searching producer can observe more than one price with positive probability.

The next section sets out the model. Equilibrium is derived in Sections 3 and 4 for general matching functions. We then adopt a specific form for the matching functions for the remainder of the paper. We derive closed-form expressions for equilibrium values of key variables in Section 5 and analyze welfare in Section 6. Section 7 discusses the possibility of an endogenous rise of merchants *ab initio*. Section 8 concludes with comments.

2. Model

2.1. Context

The setting is a highly stylized model of production, search, and exchange, adapted from Diamond (1982). The economy operates over an infinite succession of discrete periods. Agents are homogeneous and risk-neutral; they live for ever; the set of agents is a continuum of unit measure. Each agent has access to a technology for production that generates one unit of a homogeneous, divisible good at no cost in each period. A taboo against consuming the output of one's own production ensures that exchange must precede consumption. This artifice allows, within a one-commodity framework, a representation of the reality that agents in an economy consume very little of their own output and the need for specialization and trade is paramount.[4]

We imagine that the pursuit of production takes producers to random locations so that the search for trading partners has to be undertaken anew after each episode of production. Thus, every period each agent sets out to trade units with any other agent he may encounter. The probability that a

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[4]Although the model is in effect one of pure exchange, it will be convenient for terminological clarity to interpret it as a special model with production.

given agent will meet a trading partner during the period is $\lambda \in (0, 1)$. We interpret λ as a measure of the efficacy of unmediated search.[5] Once two agents meet, the units are traded one for one,[6] consumption takes place, and the agents are free to return to production, which will generate another unit of the good next period. Throughout the paper we assume that untraded good cannot be carried as inventory.[7] Thus, an agent unsuccessful in effecting exchange foregoes consumption and returns in the next period with a newly produced unit.

We normalize payoffs so that the utility of consuming x units in a period is x. Letting $\delta \in (0, 1)$ represent the common discount factor of the agents, the present value of expected payoff of any agent is given by $v = \lambda + \delta v$, so that

$$v = \frac{\lambda}{1 - \delta}. \tag{1}$$

Is there scope in such a setting for some agents to set up as specialist intermediaries and offer the service of immediacy in exchange in return for a price?

2.2. Producers and Merchants

We now allow each agent, in every period t, the choice of specializing either as a *producer*, or as a *merchant*. A specialist producer can access the production technology described above to produce output, but cannot commit to be available for trade at a specified location. In contrast, a specialist merchant cannot produce output, but can commit to be available for trade at a specified location. A specialist merchant operates a *trading post* where producers can exchange their output. For this service, a merchant charges a price that she sets each period.[8] Agents can switch occupation at any time.

A merchant's location or price is not public information and must be initially found by a producer during search. Merchants may be peripatetic, but they are not subject to random displacements in location. A merchant

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[5]More generally, $1-\lambda$ may be interpreted as a measure of any transaction cost associated with coordinating trades without an organized market. In the language of search models the transaction cost is a lost trading opportunity.

[6]As agents are symmetric, any reasonable bargaining solution would prescribe an equal share of the gains from trade.

[7]Alternatively, we could assume that production cannot be carried out with a unit of the good already in hand. What we need is that a producer is not in possession of more than one unit at any given time.

[8]Since units are traded one for one in the unmediated search market, the merchant's price is also her commission or bid-ask spread.

decides each period where she will locate in the following period and communicates this information to her clients. Thus, once a producer makes contact with a merchant, it opens for him the prospect of a long-term relationship for future trade without further search.

If a producer trades with a merchant in period t, then he learns where the merchant will locate in period $t+1$. With probability $\gamma \in (0,1)$ he remembers this information at $t+1$; with the complementary probability $1-\gamma$ he forgets this information before $t+1$. If a producer has not traded with a merchant in period t, then he does not know the location of any merchant at $t+1$.

The non-persistence of memory embodied in $\gamma \in (0,1)$ is meant to reflect the unavoidable frictions in continuing business relations, perhaps more pervasive in a nascent market than in a mature one. For example, the pursuit of production may take a producer too far from his merchant. In the model the assumption ensures that an unmediated search market remains viable.

An agent is *informed* in period t if he knows the location of a merchant's trading post at the beginning of t, and *uninformed* if he does not.

An uninformed producer must search for a trading partner. The search may yield one of three outcomes in a given period: either he does not meet a trading partner, or he meets another producer who is also searching, or he finds a trading post. If he fails to find a partner, he cannot trade. If he finds another producer, units are exchanged one for one. If he comes upon a trading post, he concludes trade at the merchant's set price, and learns where the merchant will locate at $t+1$.

An informed producer has two options: he may proceed directly to his merchant's trading post, pay the merchant's commission, and immediately conclude trade; alternatively, he may search anew for a trading partner. In the latter case, he forsakes the knowledge of his erstwhile merchant and is exactly in the same position as an uninformed producer. We assume that informed producers on their way to a merchant are unavailable to other producers searching for a partner.

Note that a producer can meet at most one trading partner in a period. Thus, if he meets a merchant, he may as well trade—regardless of how adverse the price set by the merchant is—since his current unit of the good will become obsolete after the present period. However, a producer will not return to a merchant who offers an unacceptable price even if he remembers her location in the following period.

Let m_t denote the measure of agents who specialize as merchants in period t and s_t denote the measure of producers in the search market. For a producer who searches for a trading partner, the probability of success is governed by two *matching functions*, λ^m and λ^s: $\lambda^m(m_t, s_t)$ is the probability that he finds

a merchant and $\lambda^s(m_t, s_t)$ the probability that he meets another producer who is also searching in period t. Thus, the probability that he is able to trade within the period is given by $\lambda^m(m_t, s_t) + \lambda^s(m_t, s_t)$. Throughout the paper we assume that λ^m is increasing in the first argument, λ^s is increasing in the second argument,

$$\lambda^m(m, s) + \lambda^s(m, s) \in (0, 1),$$
$$\lambda^m(0, 1) = 0, \quad \lambda^s(0, 1) = \lambda,$$
$$\lambda^m(m, s) > 0 \quad \text{for } m > 0, \quad \lambda^s(m, s) > 0 \quad \text{for } s > 0.$$

Merchants may meet and serve multiple clients within a period. The clients of a given merchant in a given period t come from two sources: informed clients from period $t - 1$ who choose to return, and new clients—producers who discover her trading post during period t in the course of search. Merchants do not meet other merchants; such meetings are inconsequential in this model.

The mechanics of intermediation could be modelled in at least two different ways. In one formulation, a merchant starts a period with sufficient inventory to conduct the first trade and funds successive trades out of the proceeds of the previous one. A merchant who plans to set a price p_t in period t must carry from period $t - 1$ an inventory of $1 - p_t$ units to offer her first client at t in return for the client's single unit. She can then consume p_t and use the remaining $1 - p_t$ to conduct the next trade. Thus, in this formulation, pricing decisions must be made one period in advance. Indeed, occupational choice decisions must also be made one period ahead. An agent who was a producer in $t - 1$ and wishes to switch occupation and become a merchant at t must carry over the necessary inventory by foregoing $1 - p_t$ units of consumption. Correspondingly, a merchant who decides to switch to production in period t can enjoy an extra $1 - p_{t-1}$ units of consumption in $t - 1$.

In the formulation we have adopted, all decisions—in particular, occupational choice and pricing—pertaining to period t are made within the period. The merchant acts as a market-maker who organizes exchange between producers. Producers who show up at her trading post pay a commission to the merchant to trade with each other. The merchant therefore does not need to carry inventory between periods; nor does a new merchant need capital to start up business.

The two formulations lead to qualitatively identical results and only small differences in the explicit expressions for equilibrium values of some variables. We have opted for the second formulation since it yields a more streamlined model and a simpler exposition.

2.3. Solution Concept

The interaction among the agents in this economy over time is modelled as a stochastic game.[9] The sequence of events unfolds as follows. At the beginning of period t, each agent observes his *information state*: whether he is informed, or uninformed. Having observed his information state, each agent chooses an occupation: whether to be a merchant (choice M) or a producer (choice Π). Each merchant μ sets a price $p_t^\mu \in [0,1]$ and each informed producer (having produced output) decides whether to return to the merchant that he traded with in the previous period (choice R) or to undertake search for a trading partner (choice S). An uninformed producer has no option but to search. The outcomes of the search processes are realized; trade and consumption take place. Finally, each producer who traded with a merchant at t forgets where the merchant will locate at $t+1$ with probability $1-\gamma$. The ones who do not will be the only informed agents in period $t+1$.

Thus, the set of *actions* available to an agent in a period depends on the agent's information state at the beginning of the period: an informed agent may choose an action in $A^i = \big\{\{M\} \times [0,1], \{\Pi\} \times \{R,S\}\big\}$; an uninformed agent must choose an action in $A^u = \big\{\{M\} \times [0,1], \{\Pi\} \times \{S\}\big\}$.

In any period t, agents have imperfect information of the history of play up to t. The *personal history observed* by an agent in period t consists of (a) the prices she set and the size of the clientele she served in every period up to $t-1$ that she operated as a merchant, (b) the prices he paid in every period up to $t-1$ that he was a producer and traded with a merchant, and (c) the outcome in every period up to $t-1$ that he searched for a trading partner.[10]

An agent's *strategy* is a sequence of functions (indexed by t) that prescribe, for every period t, an action in A^i or A^u as a function of the personal history observed by the agent at t and his information state at the beginning of t. We call an agent's strategy *Markov* if the sequence of functions is time-invariant, and if the action it prescribes in period t is determined entirely by the outcomes observed by the agent at $t-1$, and by the agent's information state at the beginning of t. In particular, for a merchant μ following a Markov strategy, the choice of price at t, p_t^μ, can depend only on the size of her clientele k_{t-1}^μ and her price p_{t-1}^μ at $t-1$; for an informed producer following a Markov strategy, the decision of whether to return to his merchant

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[9]The description here is informal: we do not furnish the measure-theoretic structure to make it entirely precise; but the details are standard.

[10]In particular, a merchant does not observe the identity of an individual customer and thus cannot give discounts to returning customers. Merchants do not observe the history of prices set by other merchants. Producers do not observe the client size of any merchant.

at t can depend only on the price the merchant had charged at $t-1$.

We call a profile of Markov strategies *symmetric* if the function prescribing the pricing rule is the same for every merchant and the function prescribing the return-decision rule is the same for every informed producer. The occupational choice decisions may vary across agents.

For a producer, the period-t (Bernoulli) payoff is 1 if at t he trades with another producer, $1-p_t$ if he trades with a merchant who charges a price p_t, and 0 if he fails to execute a trade at t. The period-t payoff of a merchant μ who sets a price p_t^μ and serves k_t^μ clients is $p_t^\mu k_t^\mu$.

Definition 2.1. An *equilibrium* is a profile of symmetric Markov strategies such that, given the strategy choices of other agents,
- the occupational choice of each agent in every period t is optimal;
- the price set by each merchant in every period t maximizes that merchant's expected continuation payoff at t;
- for each informed producer, the return decision in every period t maximizes his expected continuation payoff at t;
- $m_t = m$, $s_t = s$ for every period t.

The focus of the paper is the class of equilibria in which each merchant and her (informed) clients form a repeated relationship: we call these intermediation equilibria.

Definition 2.2. An *intermediation equilibrium* with $m^* \in (0,1)$ merchants is an equilibrium such that an informed producer has no incentive to search for a trading partner: his (weakly) optimal choice in every period t is to return to the merchant he had dealt with at $t-1$.

We do not *a priori* restrict an agent's set of strategies to be symmetric or Markov. However, we are only able to completely characterize the set of equilibria in which agents' strategy choices are symmetric and Markov.

2.4. Preliminary Observations

Observation 2.1. It is important to note that the size of the clientele that a given merchant μ serves in period t, k_t^μ, is stochastic. It is perfectly possible that most—or even all—of the clients served by a particular merchant at $t-1$ forget her location at t. The realization of k_t^μ can vary across merchants at t, and over time for the same merchant μ.

Observation 2.2. At an intermediation equilibrium, by definition, only uninformed producers search. Thus, at an intermediation equilibrium with

$m \in (0, 1)$ merchants, the expected size of clientele for a given merchant μ, who had set a price p_{t-1}^{μ} and served k_{t-1}^{μ} clients at $t - 1$, evolves according to the equation

$$E\left(k_t^{\mu}|k_{t-1}^{\mu}, p_{t-1}^{\mu}, m\right) = \gamma k_{t-1}^{\mu} + \frac{\lambda^m(m, s)s}{m}, \tag{2}$$

where E is the expectation operator. A fraction γ of a merchant's clients from period $t - 1$ retain the knowledge of her location. At an intermediation equilibrium, these informed clients return to deal with her. Moreover, each of the s producers in the search market discovers a merchant with probability λ^m. Since there are m merchants, the expected size of searching producers that arrive at the trading post of a given merchant in any period t is $\lambda^m(m, s)s/m$. This yields equation (2).

The above equation also shows how an incipient merchant—an erstwhile producer who sets up as a merchant in period t—can start from a base of $k_{t-1}^{\mu} = 0$ and acquire clients over time.

The next two sections develop the analysis of the class of equilibria in which the occupational choices of agents result in a proportion $m \in (0, 1)$ of merchants.

3. Pricing

In this section, we take a fixed stationary occupational assignment with $m \in (0, 1)$ merchants as given, and characterize the sequence of prices that the merchants set and the decision rules that the informed producers follow in any equilibrium. Section 4 analyzes the occupational choices in equilibrium.

In any given period t, a merchant must determine the price to charge. We want to look for equilibria in which the price can only depend on the size of the client base she served and the price she charged at $t - 1$. An informed producer must correspondingly decide in period t whether to return to the merchant he dealt with at $t - 1$ or to search for a trading partner. We want to look for equilibria in which this decision can depend only on the price he observed at $t - 1$. Moreover, these decision rules of the agents are time-invariant (by Markov property) and symmetric (by assumption).

Thus, in equilibrium, the strategy of an informed producer can be represented as a time-invariant partition $\{P^R, P^S\}$ of $[0, 1]$: by definition, a producer returns to his merchant in period t if and only if the price charged by the merchant in period $t - 1$ was in P^R; otherwise, the producer searches for a trading partner.

Lemma 3.1. *In any equilibrium with $m < 1$, P^S is nonempty, and $\sup P^S = 1$. In an intermediation equilibrium, if one exists, P^R is nonempty, $\sup P^R < 1$ and $\sup P^R \in P^R$.*

Proof. In any period, there are at least $(1-\gamma)(1-m)$ agents searching. Hence a producer's per period expected payoff from search is at least $\lambda^s\big(m, (1-\gamma)(1-m)\big) > 0$. Thus, a price greater than $1 - \lambda^s\big(m, (1-\gamma)(1-m)\big)$ cannot be in P^R. It follows that $\big(1 - \lambda^s\big(m, (1-\gamma)(1-m)\big), \ 1\big] \subset P^S$, and $\sup P^S = 1$.

By definition of an intermediation equilibrium, P^R is non-empty. From the argument in the previous paragraph, $\sup P^R \leq 1 - \lambda^s\big(m, (1-\gamma)(1-m)\big)$.

If $\sup P^R \notin P^R$, then a merchant charging $p \in P^R$ can increase her current period profit, without affecting the return-decision of any client, by raising her price slightly. $\qquad\square$

In the rest of the paper, we will be primarily interested in intermediation equilibria with an ongoing relationship between a merchant and its informed clients. First, we dispense with a case in which all merchants always charge a price of unity and producers never return. This is really a description of a search equilibrium with *bandits*, not of an institution of intermediation. These bandits live off appropriating the entire endowments of any searching producers who unluckily encounter them, and the producers obviously do not return to trade with them.

Lemma 3.2 (Bandit Pricing). *Each merchant always setting a price of 1 and each producer always choosing to search, even when informed, constitute mutual best responses for any given proportion of merchants.*

Proof. Since clients do not return, it is never optimal for a merchant to set a price below 1. Even if a merchant deviates and sets a price below 1 in some period, the strategy profile calls for her to revert to a price of 1 in the following period. Hence, it is never optimal for an informed producer to return to his merchant.[11] $\qquad\square$

The next lemma identifies the only two price paths that are compatible with equilibrium. A merchant will set her price at 1 or $\sup P^R$ depending on whether it is more profitable to act as a bandit and take her clients' entire endowment, or to induce her clients to return when they remember her location.

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[11] A producer is indifferent between trading at a price of 1 and declining trade and foregoing consumption. However, there is no equilibrium in which a producer declines trade since her merchant would be better off lowering the price slightly.

Lemma 3.3 (Optimal Pricing). *A merchant's optimal pricing rule is to set price in each period t as follows:*

$$p_t = \begin{cases} \sup P^R & \text{if} \quad \sup P^R > 1 - \gamma\delta \\ \sup P^R \ or \ 1 & \text{if} \quad \sup P^R = 1 - \gamma\delta \\ 1 & \text{if} \quad \sup P^R < 1 - \gamma\delta \ or \ P^R = \emptyset. \end{cases} \tag{3}$$

The intuition behind the lemma is transparent. Suppose a merchant deviates from the constant price sequence of $\sup P^R$ for one period and charges a price of 1. She would gain $1 - \sup P^R$ from each client in that period, but lose her client base. The expected discounted loss on each client is $1/(1-\gamma\delta)$. The lemma follows from a comparison of these magnitudes.

Proof of Lemma 3.3. First, note that it cannot be optimal for a merchant to set a price other than $\sup P^R$ or 1 since she can increase her current period profit, without affecting the return-decision of any client, by raising her price slightly.

We start with the price sequence $p_t = \sup P^R$ for every period t, and show that the merchant cannot gain by deviating from this sequence if $\sup P^R > 1 - \gamma\delta$.

STEP 1: Consider a one-period deviation in some period τ in which the merchant sets $p_\tau = 1$ (the best one-period deviation). The merchant's net gain from this deviation discounted to period τ is:

$$\begin{aligned} \Delta_1 &\equiv k_\tau\big(1 - \sup P^R\big) \ - \ \gamma\delta k_\tau \sup P^R\big(1 + \gamma\delta + \ldots\big) \\ &= k_\tau\left(1 - \frac{\sup P^R}{1 - \gamma\delta}\right). \end{aligned}$$

$\Delta_1 < 0$ since $\sup P^R > 1 - \gamma\delta$. Thus if there is a profitable deviation from the constant sequence $\sup P^R$, then the price must deviate from $\sup P^R$ in at least two periods.

STEP 2: So let τ be a period in which $p_\tau = 1$ and $\tau' = \tau + n$ the next period such that $p_{\tau'} = 1$, with $p_t = \sup P^R$ for the intermediate periods $\tau < t < \tau'$ (there may be no such intermediate periods).

Now replace p_τ with $\sup P^R$, and calculate the change in the merchant's profit. The merchant loses $k_\tau(1 - \sup P^R)$ in period τ owing to the lower price she charges. However, of these k_τ producers, the ones that remain informed return in the periods $\tau + 1, \ldots, \tau + n$, which they would not have done if the merchant had charged p_τ. The merchant's net gain discounted to period τ

12

is:

$$\Delta_2 = k_\tau(1 - \sup P^R) \; + \; \sum_{t=\tau+1}^{\tau+n-1}(\gamma\delta)^{t-\tau}\, \sup P^R k_\tau + \; (\gamma\delta)^n k_\tau$$

$$= -k_\tau[1 - (\gamma\delta)^n] \; + \; \sup P^R k_\tau\left(\sum_{t=0}^{n-1}(\gamma\delta)^t\right)$$

$$= -k_\tau[1 - (\gamma\delta)^n] \; + \; \sup P^R k_\tau\left(\frac{1 - (\gamma\delta)^n}{1 - \gamma\delta}\right),$$

which is positive since $\sup P^R > 1 - \gamma\delta$. Thus the merchant's profit increases if she sets $p_\tau = \sup P^R$.

By Step 1 and Step 2, if $\sup P^R > 1 - \gamma\delta$, then the merchant's profit is maximized by setting $p_t = \sup P^R$ in each period t. The proofs for the cases when $\sup P^R \leq 1 - \gamma\delta$ are analogous. $\qquad\square$

Thus, in equilibrium, each merchant sets a price that remains constant over time. At an intermediation equilibrium, merchants set a constant price $p \in P^R$ and informed producers return to their merchants. Further, p is in fact the highest price for which the clients would be willing to return. We compute this next by determining the optimal return-decision rule for informed producers.

Consider a configuration (p, m) with $p \in P^R$. The expected number of producers who search at such a configuration is given by

$$s(m) = (1 - \gamma)\big(1 - m - s(m)\big) + \big[1 - \gamma\lambda^m(m, s)\big]s(m). \tag{4}$$

In each period, a fraction $(1 - \gamma)$ of the $(1 - m - s)$ informed producers forget the location of their merchants and return as searchers. Of the s searching producers, a fraction λ^m discover a merchant's trading post; of those, a fraction γ return as informed; all others return as searchers. This gives (4), which simplifies to[12]:

$$s(m) = \frac{(1 - \gamma)(1 - m)}{1 - \gamma(1 - \lambda^m)}. \tag{5}$$

The proportions m and $s(m)$ are together sufficient to implicitly determine the matching probabilities λ^s and λ^m.

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[12]We suppress the arguments of the functions λ^m and λ^s in what follows: no confusion should arise.

Let $V^r(p, m)$ denote the expected continuation payoff of an informed producer who returns to his merchant, and let $V^s(p, m)$ denote the corresponding expected continuation value of a producer who searches. Then, $V^r(p, m)$ and $V^s(p, m)$ are given by

$$V^r(p, m) = (1 - p) + \delta\big[(1 - \gamma)V^s(p, m) + \gamma V^r(p, m)\big] \qquad (6)$$
$$V^s(p, m) = \lambda^s(1 + \delta V^s(p, m)) + \lambda^m V^r(p, m) + (1 - \lambda^m - \lambda^s)\delta V^s(p, m). \qquad (7)$$

A returning producer concludes trade immediately with his merchant at the price p, consumes $1-p$, and returns again with another unit next period with probability γ. With probability $1 - \gamma$ he forgets his information and must search. This yields equation (6). A searcher encounters another searcher with probability λ^s, trades units one for one, and will again search in the following period. With probability λ^m he finds a trading post with a merchant charging price p, and then is precisely in the same position as a returning producer. With probability $1 - \lambda^m - \lambda^s$ he does not find a trading partner during the period, receives no utility, and returns as a searcher next period. This yields equation (7).

The solutions to equations (6) and (7) yield the continuation values for the informed and uninformed producers, and are given by

$$V^r(p, m) = \frac{(1 - \gamma)\delta\lambda^s + \big[1 - \delta(1 - \lambda^m)\big](1 - p)}{(1 - \delta)\big[1 - \gamma\delta(1 - \lambda^m)\big]} \qquad (8)$$
$$V^s(p, m) = \frac{(1 - \gamma\delta)\lambda^s + \lambda^m(1 - p)}{(1 - \delta)\big[1 - \gamma\delta(1 - \lambda^m)\big]}. \qquad (9)$$

Suppose an individual merchant sets a price p' different from p every period while all other merchants charge the returning price p. Then the continuation value of an informed client who decides to return to this merchant in each period that he is informed is:

$$\bar{V}^r(p'|p, m) = (1 - p') + \delta\big[(1 - \gamma)V^s(p, m) + \gamma\bar{V}^r((p'|p, m)\big] \qquad (10)$$

from which we can solve for $\bar{V}^r(p'|p, m)$ in terms of $V^s(p, m)$ and p'. $\bar{V}^r$ is decreasing in p' as expected.

Define $f(p, m)$ to be the value of p' which equates $\bar{V}^r((p'|p, m)$ and $V^s(p, m)$. Some algebra yields

$$f(p, m) = 1 - \frac{(1 - \gamma\delta)[1 - \lambda^s - \lambda^m] + p\lambda^m}{1 - \gamma\delta(1 - \lambda^m)}. \qquad (11)$$

For any p' lower than $f(p, m)$ the client is better off returning to the merchant, while at a higher price he would do better to search.

Let $p^*(m)$ solve $p = f(p, m)$. From (11), we obtain p^* in terms of m:

$$p^*(m) = \frac{1 - \lambda^s - \lambda^m}{1 - \lambda^m}. \tag{12}$$

Lemma 3.4 (Intermediation Pricing). *Suppose at an intermediation equilibrium $m = \hat{m}$ and $p = \hat{p}$. Then it must be true that*
(a) $\hat{p} \geq 1 - \gamma\delta$.
(b) $\sup P^R = f(\hat{p}, \hat{m})$
(c) $\hat{p} = p^*(\hat{m})$.

Proof. Part (a) follows from Lemma 3.3. Also by Lemma 3.3, $\hat{p} = \sup P^R$.

If $\sup P^R \geq 1 - \gamma\delta$, then it is optimal for each merchant to set $p = \sup P^R$ in every period (by Lemma 3.3). Hence, regardless of the price observed in the current period, each client expects his merchant to charge $\sup P^R$ in the following period.

(i) Thus if $\sup P^R < f(\hat{p}, \hat{m})$ then, regardless of the price set by the merchant in the current period, a client is strictly better off returning to his merchant in the next period than searching (by definition of f). But then it cannot be optimal for the merchant to charge any $p < 1$ in the current period, and the equilibrium price $\hat{p}$ must be unity. By observation 3.1, $1 \notin P^R$ and this cannot be part of an intermediation equilibrium.

(ii) If all merchants charge $\hat{p}$, and $\hat{p} > f(\hat{p}, \hat{m})$, then $\hat{p}$ cannot be in P^R, since each client is better off searching than returning at this price.

Thus we must have $\hat{p} = f(\hat{p}, \hat{m}) \Leftrightarrow \hat{p} = p^*(\hat{m})$. Note that the argument in (i) does not apply in this case, since the client is indifferent between returning to the merchant and searching.

This establishes (b) and (c). □

The pricing model here extends the classic model of search by Diamond (1971) to a repeated environment and incorporates a parallel search market. As in Diamond's model, each merchant enjoys local monopoly; but the monopoly power is tempered by the coexistence of the search market. The highest price at which a merchant has repeat clients is less than the pure monopoly price of 1. However, the merchant appropriates the entire

15

information rent from her clients. Consequently, the continuation payoffs of informed and uninformed producers are equal.[13]

We have modelled the merchants as price-setters. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer as in Rubinstein and Wolinsky (1987) or Masters (2007). Then, the informed producers would retain some of the gains from reduced search costs.

4. Occupational Choice and Equilibria

4.1. Occupational Choice

Let a stationary configuration (p, m), in which a measure m of merchants set a constant price $p \in P^R$, be given. Let $V_t^\mu(k_{t-1}, p, m)$ denote the continuation value in period t of a merchant who had served k_{t-1} clients at $t-1$. An agent who was a producer at $t-1$, and decides to start up as a merchant at t, will begin with no established client base and must acquire clients over time. (A proportion of the searching population in period t will chance upon her trading post.) A continuing merchant may also find herself with no clients, since with positive probability all her clients may forget her location. The continuation value of such an *incipient merchant* is then $V_t^\mu(0, p, m)$.

The next lemma identifies the restrictions imposed by the requirement that the occupational choices of agents are optimal in equilibrium.

Lemma 4.1 (Occupational Choice). *At a configuration (p, m), the occupational choice of each agent is optimal at every period t if and only if*

$$V_t^\mu(0, p, m) = V^s(p, m). \tag{13}$$

Proof. An agent who was a producer at $t-1$ can start up as a merchant at t and obtain the continuation value $V_t^\mu(0, p, m)$. Correspondingly, an agent who was a merchant at $t-1$ can switch to production at t—in which case he must start as an uninformed producer and search for a trading partner—and obtain the continuation value $V^s(p, m)$.

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[13]The model of competition among merchants here is qualitatively similar to models of sequential search without recall. In models of sequential search, an agent who encounters an unacceptable price simply defers consumption and continues to search—the cost of additional search may be a delay in consumption or represented as a fixed amount. In our model, the cost of a bad search outcome is a reduction in current period consumption. The agent (producer) trades at the unfavourable price, consumes, but would resume search next period with a new unit of the good.

Since there must be some searching producers at any t, some producer will want to start up as a merchant if $V_t^\mu(0, p, m) > V^s(p, m)$, but no merchant will want to switch to production. Thus, the proportion of merchants cannot remain constant. On the other hand, if $V_t^\mu(0, p, m) < V^s(p, m)$, a merchant μ who experiences $k_{t-1}^\mu = 0$ will have an incentive to switch to production, but no producer will have an incentive to become a merchant. Again, the proportion of merchants cannot remain constant. When $V_t^\mu(0, p, m) = V^s(p, m)$, each agent's optimal choice in period t is to continue with the occupation chosen in period $t - 1$. □

Observation 4.1. It follows from Lemma 4.1 that, at an equilibrium, almost all incumbent merchants earn "supernormal profits" in almost all periods in the sense that

$$V_t^\mu\big(k_{t-1}, p, m\big) > V_t^\mu(0, p, m) = V^s(p, m) \quad \text{whenever } k_{t-1} > 0. \qquad (14)$$

Although agents are free to switch occupation at any time, an incumbent merchant with an established base of clients enjoys an advantage over an incipient merchant who must acquire clients over time.

4.2. Equilibria

Proposition 4.1 below combines Lemma 3.4 and Lemma 4.1 to provide a complete characterization of all equilibria in which some but not all agents are merchants.

Definition 4.1. Define a *bandit strategy profile* as one in which
- a proportion $m \in (0, 1)$ of agents always specializes as merchants and each merchant sets the price $p_t = 1$ in every period t,
- the remaining agents always specialize as producers and each producer always searches for a trading partner (i.e., P^R is empty).

Define an *intermediation strategy profile* as one in which
- a proportion $m \in (0, 1)$ of agents always specializes as merchants, the remainder of the agents always specializes as producers,
- the informed producers' return strategy specifies P^R such that $\sup P^R = p^*(m)$,
- merchants set $p_t = p^*(m)$ in each period t.

Proposition 4.1 (Equilibria). *There are at most two classes of equilibria with $m \in (0, 1)$ merchants in every period t.*

(a) *A bandit strategy profile constitutes an equilibrium if and only if*

$$\frac{\lambda^m(m, 1-m)}{\lambda^s(m, 1-m)} = \frac{m}{1-m}. \tag{15}$$

(b) *An intermediation strategy profile constitutes an equilibrium if and only if*

$$1 - \gamma\delta = \frac{\left[1 - \lambda^m(m, s) - \lambda^s(m, s)\right]\lambda^m(m, s)\, s}{\lambda^s(m, s)\, m}, \quad and \tag{16}$$

$$\gamma\delta \geq \frac{\lambda^s(m, s)}{1 - \lambda^m(m, s)}, \tag{17}$$

where s is given by (5).

Condition (15) follows from (13) by noting that the proportion of searchers $s = 1 - m$ if all producers search regardless of information state. Condition (16) is a restatement of occupational choice equilibrium condition (13) given the equilibrium price p^* defined in (12): it implicitly determines m. Condition (17) ensures that merchants find it profitable to charge the returning price p^* according to Lemma 3.3.

Proof of Proposition 4.1. Lemma 3.4 shows that the pricing decisions of merchants and the return decisions of producers given in (a) and (b) above are the only ones that are consistent with equilibrium. It remains to verify the optimality of the occupational choices.

(a)   If agents follow the bandit strategy profile, we have $s = 1 - m$ at every t, and a merchant extracts 1 from each of the $\lambda^m(m, 1-m)[1-m]/m$ of searching producers who come upon her trading post. Further, since informed clients don't return, every merchant is in the same position as one who served no clients in the previous period. Thus, for any merchant μ at every t

$$V_t^\mu(0, 1, m) = \frac{\lambda^m(m, 1-m)(1-m)}{(1-\delta)m}. \tag{18}$$

Further, at the given strategy profile, the continuation value at any t for any producer, whether informed or uninformed, is given by

$$V^s(1, m) = \frac{\lambda^s(m, 1-m)}{1 - \delta}. \tag{19}$$

By Lemma 4.1, the above two continuation values must be equal. This yields (15).

(b) If agents follow the intermediation strategy profile, we have

$$V_t^\mu\big(0, p^*(m), m\big) = \sum_{t=1}^{\infty} \delta^{t-1} p^*(m) k_t, \tag{20}$$

where k_t evolves according to (2) with $k_0 = 0$. This yields

$$V_t^\mu\big(0, p^*(m), m\big) = \frac{p^*(m)\lambda^m s}{(1-\delta)(1-\gamma\delta)m} \tag{21}$$

$$= \frac{(1-\lambda^m-\lambda^s)\lambda^m s}{(1-\delta)(1-\gamma\delta)(1-\lambda^m)m} \quad \text{using (12).} \tag{22}$$

It follows from equations (7) and (12) that

$$V^s\big(p^*(m), m\big) = \frac{\lambda^s}{(1-\delta)(1-\lambda^m)}. \tag{23}$$

By Lemma 4.1, the two continuation values in (21) and (23) must be equal. This gives (16). Condition (17) follows from the requirement for merchants to charge a returning price in Lemma 3.3, and the specification of the equilibrium price in equation (12). $\qquad\square$

5. Closed-Form Solutions

In our analysis so far, the matching functions λ^m and λ^s were quite arbitrary. In the remainder of the paper, we focus on a specific pair of matching functions in the interest of obtaining closed-form expressions for the equilibrium values of the key variables in terms of the parameters of the model: γ, δ, and λ. The matching functions we focus on are given by

$$\lambda^m(m, s) = \frac{\lambda\, m^{1/2}}{m^{1/2}+s^{1/2}} \tag{24}$$

$$\lambda^s(m, s) = \frac{\lambda\, s^{1/2}}{m^{1/2}+s^{1/2}}, \qquad \lambda \in (0,1). \tag{25}$$

Observe that merchants and producers are treated symmetrically by the matching technology: merchants enjoy no *a priori* advantage in this respect. The matching functions are also 0-homogeneous so that there are no thick-market externalities.[14]

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[14]The search literature often employs matching functions in which the probability of

Proposition 5.1 (Closed-Form Solutions). *Let the matching functions λ^m and λ^s be given by (24) and (25). Then,*

(a)   *There is a unique bandit equilibrium. At this equilibrium, the proportion of agents specializing as merchants is given by $m = 1/2$.*

(b)   *An intermediation equilibrium exists if and only if*

$$\gamma\delta(1-\lambda)^2 \geq (1-\gamma\delta)(\lambda-\gamma\delta). \tag{26}$$

If it exists, it is unique. The proportion of agents specializing as merchants is given by

$$m^* = \frac{(1-\gamma)(1+\alpha^{1/2})}{(1-\gamma)(1+\alpha^{1/2})(1+\alpha)+\gamma\lambda\alpha}, \tag{27}$$

$$where \quad \alpha = \frac{(1-\gamma\delta)^2}{(1-\lambda)^2}; \tag{28}$$

and the price set by each merchant in every period is given by

$$p^* = \frac{(1-\lambda)^2 + (1-\lambda)(1-\gamma\delta)}{(1-\lambda)^2 + (1-\gamma\delta)}. \tag{29}$$

Proof. (a)   Using (24) and (25) in (15), and recognizing that $s = 1 - m$ at a bandit equilibrium, we get

$$\frac{\lambda\, m^{1/2}(1-m)}{m^{1/2}+(1-m)^{1/2}} = \frac{\lambda\,(1-m)^{1/2}m}{m^{1/2}+(1-m)^{1/2}},$$

which reduces to $m = 1/2$. This proves part (a).

(b)   Substitute (24) and (25) in (16) to get

$$\frac{m^{1/2}}{s^{1/2}} = \frac{1-\lambda}{1-\gamma\delta}. \tag{30}$$

Using (24) and (25) in (17), we get

$$\gamma\delta \geq \frac{\lambda\, s^{1/2}}{(1-\lambda)\, m^{1/2}+s^{1/2}}, \qquad or,$$

$$\frac{m^{1/2}}{s^{1/2}} \geq \frac{\lambda-\gamma\delta}{\gamma\delta(1-\lambda)}. \tag{31}$$

---

meeting a particular type of agents is linear in the relative proportion of that type in the searching population. In our model, this would translate to $\lambda^m = \lambda\, m/(m+s)$ and $\lambda^s = \lambda\, s/(m+s)$. This specification generates a knife-edge intermediation equilibrium in the present model. We can eliminate the knife-edge character by introducing an arbitrarily small cost to set up as a merchant. The matching functions used here give more compact results without requiring additional assumptions.

Combining (30) and (31) gives (26).

By (30),

$$\frac{s}{m} = \frac{(1 - \gamma\delta)^2}{(1 - \lambda)^2}. \tag{32}$$

Defining $\alpha = \frac{s}{m}$ gives the value of α in (28).

Using $\alpha = s/m$, (24) reduces to

$$\lambda^m = \frac{\lambda}{1 + \alpha^{1/2}}, \tag{33}$$

and (5) becomes

$$\alpha m = \frac{(1 - \gamma)(1 - m)}{1 - \gamma\left(1 - \dfrac{\lambda}{1 + \alpha^{1/2}}\right)},$$

which yields the value m^* in terms of the parameters given in (27). It is easily seen that $m^* \in (0, 1)$.

Finally, (29) follows from substituting (24), (25) and (30) in (12). $\qquad\square$

Part (b) of Proposition 5.1 identifies the range of parameter values for which there is an intermediation equilibrium in the economy described by (24) and (25).

Observation 5.1. There is a threshold value $\lambda^* \in (\gamma\delta, 1)$ such that the economy has an intermediation equilibrium if and only if $\lambda \leq \lambda^*$.

Proof. Rewrite (26) as

$$\phi(\lambda) \equiv \gamma\delta(1 - \lambda)^2 - (1 - \gamma\delta)(\lambda - \gamma\delta) \geq 0 \tag{34}$$

Inequality (34) is strict for $\gamma\delta \geq \lambda$. Also, we have $\phi' < 0$ for all $\lambda \in [0, 1]$, and $\phi(1) = -(1 - \gamma\delta)^2 < 0$. Since ϕ is continuous, the result follows. $\qquad\square$

The intuition is transparent: the rate at which searchers can be found cannot be too high compared to the rate at which former clients return to their merchants. Otherwise, it would be more profitable for merchants to act as bandits than to induce clients to return.

6.   Welfare

Merchants in this model provide a beneficial trading externality: an encounter with a merchant opens up the prospect of a long-term relationship for

future trade, and potentially reduces search costs. However, specialization by merchants in the service of exchange comes at the expense of the production of the physical good. Merchants also create negative externalities—as the proportion of merchants rises, the search market gets thinner affecting the trade prospects of the searching population. Also, the clientele of a new merchant in steady-state is not drawn entirely from the hitherto searching population; some of her clients would otherwise have been clients of the merchants already in the market.

The natural measure of social welfare here is the expected aggregate consumption per period. How does welfare at an equilibrium with merchants compare with an economy with no merchants? What is the optimal proportion of merchants in the economy? Is the equilibrium proportion of merchants optimal? This section addresses these questions in the context of the closed-form economy described in Section 5.

It is obvious that the bandit equilibrium outcome is worse for welfare than an economy with no merchants: bandits do not produce; nor do they reduce search cost for other agents.

Suppose that the economy is in steady-state with m merchants who set a price $p \in P^R$. Then, the size of the set of uninformed agents is given by equation (5). With m merchants, $1 - m$ units are produced in a period; of these, a fraction $(1 - \lambda^m - \lambda^s)s$ fails to get traded and consumed. Letting W denote the welfare, we have

$$W(m) = 1 - m - (1 - \lambda^m - \lambda^s) \, s(m). \tag{35}$$

Proposition 6.1 (Welfare). *Let the matching functions be given by (24) and (25).*

(a) *Welfare is maximized at an interior proportion $\tilde{m}$ of merchants. At $\tilde{m}$,*

$$s'(\tilde{m}) = -\frac{1}{1 - \lambda}. \tag{36}$$

(b) *The welfare associated with the intermediation equilibrium characterized in Proposition 5.1(b) is greater than the welfare in the pure-search economy if and only if*

$$\gamma(1 - \gamma\delta)^2 > (1 - \gamma)[(1 - \gamma\delta) + (1 - \lambda)]. \tag{37}$$

Proof. (a)   Using equations (24) and (25) in (35),

$$W(m) = 1 - m - (1 - \lambda) \, s(m). \tag{38}$$

22

$W(0) = \lambda$ and $W(m)$ must fall below λ for values of m in excess of $1 - \lambda$: at least λ units of output must be produced in the economy for welfare to exceed λ. Since W is continuous in m, it attains a maximum over $[0, 1 - \lambda]$. We now verify that the derivative of $W(m)$ is positive at $m = 0$.

From (38), we have

$$W'(m) = -1 - (1 - \lambda)\, s'(m)$$

so that $W'(m) > 0$ if $s'(m)$ is negative and larger in absolute value than $1/(1 - \lambda)$. Some tedious algebra yields

$$s'(m) = -\frac{(1 - \gamma)\,(m^{1/2} + s^{1/2})^2 + (1/2)\,\gamma\lambda s^{2/3} m^{-1/2}}{(1 - \gamma)\,(m^{1/2} + s^{1/2})^2 + \gamma\lambda\, m^{1/2}\,(m^{1/2} + s^{1/2}) + (1/2)\,\gamma\lambda m^{1/2} s^{1/2}},$$

which is negative for all values of s and m between 0 and 1. Using the fact that $s \longrightarrow 1$ as $m \longrightarrow 0$, we find that $s'(m)$ increases without bound in absolute value as $m \longrightarrow 0$. Thus, W attains an interior maximum and (36) follows from the first-order condition.

(b)   For welfare in intermediation equilibrium with m^* merchants, identified in Proposition 5.1(b), to be greater than that in the pure-search economy, we need

$$1 - m^* - (1 - \lambda)s > \lambda,$$

or,

$$\frac{1}{m^*} - \frac{s}{m^*} > \frac{1}{1 - \lambda}. \tag{39}$$

Recalling that we defined $\alpha = \frac{s}{m}$ and substituting the value of m^* from (27), (39) reduces to

$$\gamma\alpha(1 - \lambda) > (1 - \gamma)(1 + \alpha^{1/2}). \tag{40}$$

Substituting $\alpha = \frac{(1 - \gamma\delta)^2}{(1 - \lambda)^2}$ from equation (30) in (40) and simplifying yields condition (37). $\qquad\square$

Observation 6.1. Condition (37) holds for a wide range of parameter values that are consistent with Proposition 5.1 (b). For example try $\gamma = 9/10$, $\delta = 5/8$, $\lambda = 9/16$. Note that $\lambda = \gamma\delta$, so the existence condition is satisfied. Similarly, there are also ranges of values for which an intermediation equilibrium exists, but condition (37) does not hold. Thus in general there is no correspondence between equilibria and optima, or even a presumption that welfare at an equilibrium is necessarily greater than in the pure search economy.

7.   The Rise of Merchants

Suppose that we start with an economy in which all agents specialize as producers. Under what conditions can we expect an institution of intermediation to endogenously arise in this economy? We show below that, if γ is sufficiently large, then it will be strictly profitable for an arbitrarily small measure of producers to deviate and set up as merchants. In this sense, intermediation will arise endogenously in such an economy.

Consider therefore an economy in which each agent specializes as a producer every period, and searches for trading partners. Each agent's per-period payoff is $\lambda^s(0,1) = \lambda$. Let each producer's return-decision-rule—when informed—be given by $P^R = [0, 1 - \lambda]$.

Now, suppose a small measure of agents deviates, starts up as merchants, and sets a price less than $1 - \lambda$. Producers who come upon their trading posts will want to return. Thus the merchants, beginning with a client base of zero, will acquire clients over time. Proposition 7.1 below shows that this deviation is profitable provided that a merchant's retention rate of clients, γ, is sufficiently high relative to λ.

Proposition 7.1 (Emergence of Merchants). *Let the matching functions be specified by (24) and (25), and let $\gamma\delta > \lambda$. Suppose that each agent's strategy is to specialize as producer in every period and, if informed, use the return-rule $P^R = [0, 1 - \lambda]$. Then*

(a) *there exists $\bar{m} \in (0,1)$ such that, for all $m' \in (0, \bar{m})$ any subset of agents of measure m' would find it profitable to start up as merchants and set a price $\tilde{p} \in (1 - \gamma\delta, 1 - \lambda]$.*

(b) *The payoffs of deviating merchants increases without bound as $m' \to 0$.*

Proof. Let a subset of agents of measure m simultaneously start up as merchants and set a price $\tilde{p} \in (1 - \gamma\delta, 1 - \lambda]$ every period. Since $\gamma\delta > \lambda$, $\tilde{p} \in P^R$. Then, the continuation value of this deviation for an individual agent is given by

$$
\begin{aligned}
V_t^\mu(0, \tilde{p}, m) &= \frac{\tilde{p}\,\lambda^m(m, 1-m)\,(1-m)}{(1-\delta)(1-\gamma\delta)\,m}, \quad \text{[see (21)]} \\
&= \frac{\tilde{p}\,\lambda\,(1-m)}{(1-\delta)(1-\gamma\delta)\left[m + m^{1/2}(1-m)^{1/2}\right]}, \quad \text{by (24) and (25)} \\
&> \frac{\lambda\,(1-m)}{(1-\delta)\left[m + m^{1/2}(1-m)^{1/2}\right]}, \quad \text{since } \tilde{p} > 1 - \gamma\delta. \qquad (41) \\
&> \frac{\lambda}{1-\delta}, \quad \text{for } m < 1/4, \text{ since } m^{1/2}(1-m)^{1/2} \leq 1/2.
\end{aligned}
$$

24

which establishes part (a). Moreover, from (41), $V_t^\mu(0, \tilde{p}, m)$ increases without bound as $m \to 0$ which is part (b). □

Observation 7.1. The condition $\gamma\delta > \lambda$ ensures that $\sup P^R > 1 - \gamma\delta$; thus, the payoff for the deviating agents is higher than their payoff if they were to become bandits (see Lemma 3.3). Moreover, if $\tilde{p} < 1 - \lambda$, the deviation makes all agents—not only those in the deviating subset—strictly better off.

The prospect of an endogenous rise of an institution of intermediation *ab initio* thus depends on the relative values of the parameters γ and λ (for a fixed δ). These parameters, in turn, are arguably determined by social and technological conditions.

Exchange for personal consumption between producers has occurred since prehistory within local circles, and formed the basis for division of labor and specialization in village economies. The ambit of such exchange, for which λ is a proxy, is likely to remain limited and evolve slowly in the absence of professional traders.

The parameter γ, which captures the ability of merchants to communicate with their clients and of the clients to return to their merchants, is likely to be more sensitive to social, political, and technological conditions. Communication and commerce may be rendered impossible between one period and the next by natural calamities or bandits or unreliable transportation; rulers may prevent access or impose tolls; local wars may intervene. Viewed in this way, γ is likely to rise with improvements in law and order and in the technology of communication and transport. Thus, when order deteriorates disrupting communication and transportation networks, even erstwhile reliable merchants may turn to banditry; but professionally mediated trade will arise again as order is restored and communication improves.[15]

8. Conclusion

Intermediaries perform many roles in facilitating trade. They may variously exploit advantages in the technology of transaction and trade, costs of storing inventory, aggregating information, assessing quality of goods or some attributes of agents or a market, matchmaking, and so forth. We focused on only two aspects that are interrelated in our model—reducing the

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[15]This interpretation is not inconsistent with European history. In the second half of the first millennium AD there was a general decline of law and order, accompanied by a contraction of trade. As stability was re-established early in the second millennium, professional merchants flourished and trade expanded as well, both within Europe and across the Mediterranean.

cost of search, and fostering long-term trading relationships with clients. Our primary objective was to develop a self-contained, if rudimentary, account of an emergent institution of intermediation. Thus, the important modelling concern was to start with a homogeneous population, endogenize the choice between the two occupations of production and intermediation, and investigate the configuration of parameters that predicate the rise of intermediation as a sustainable occupation.

In focusing on these, we have marginalized several other concerns that may legitimately claim attention in the context of this paper. We briefly comment on some of these below.

We supposed that the price at which a producer trades with a merchant is set by the merchant. In our model, this allowed merchants to extract all the rent. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer, with the consequence that the latter would retain some of the gains from reduced search costs.

Our treatment of competition among merchants was minimalist. In particular, a producer knows at most one merchant; he cannot maintain his link with a merchant and simultaneously search for a better price. It may be of interest to investigate the consequences of allowing producers to randomly observe a second price, as in Burdett and Coles (1997). It is worth reiterating, however, that even the simple model elaborated here incorporates the full extent of competition that is afforded by standard models of sequential search without recall (see 13).

Our formulation of intermediation abstracted from the important question of inventory. As we had suggested (toward the end of Section 2.2), an elementary treatment of inventory can be accommodated without any qualitative changes in our analysis. However, a more detailed modelling would likely produce greater insights into the mediation process.

As presented, the present model is one of pure exchange: the production process is entirely mechanistic in that it involves no choice variable. As Diamond (1982) has shown in a model of search, reducing anticipated delays in exchange can influence production decisions. In future work, we plan to extend the present model by incorporating production to yield richer general equilibrium interactions. This would also provide the bridge between the analysis of the microstructure of exchange and the formulation of macroeconomic policy, which was the intention of Diamond's original article.

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