Are there complementarities in educational peer effects?

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Abstract

Many recent studies have shown that students benefit in the form of improved educational outcomes from having high-quality peers around them. Yet little evidence exists regarding whether peer attributes complement or substitute for own inputs into education. If they complement own inputs, then assortative matching into learning groups is the optimal outcome both from an individual point of view and from a societal point of view, and would arise from voluntary sorting. If peer inputs substitute for own inputs, then reverse assortative matching into peer groups is optimal even though this would not come about via voluntary sorting. We introduce this issue in a simple model and explore its theoretical implications when utility is a function of both the ability and the effort level of self and peers. We then use students' responses to questionnaires and actual choice-making in student groups to shed light on this unknown. The empirical evidence strongly suggests that students believe in social complementarities in educational production. Based on our model and our additional empirical evidence, we find support for the proposition that streaming students according to ability is the socially optimal design of educational institutions.

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People are people through other people. Xhosa saying.

1 Introduction

The recent literature on peer effects in education has found evidence that high-quality peers make a positive contribution to the educational outcomes of others.[1] While the main effect of peers may be positive, the question remains whether peer attributes complement or substitute for own inputs into education. Closely related to this are the questions of whether effort is endogenous and whether better peers can substitute for own effort.

If the peer externality complements own inputs, then the highest aggregate education outcomes are attained when individuals of greatest talent are streamed together and those of lowest talent are also streamed together. Moreover, individuals themselves would sort this way if given the choice. However, if the peer externality is a substitute for own inputs (for instance by leading to a reduction in own effort), then the allocation of students that leads to the highest average education outcome is one where the highest ability individuals are matched with individuals of the lowest ability. Yet in this state of the world, if the main effect of peers is increasing in peer ability, high-ability individuals would still prefer to be streamed together rather than grouped with low-ability peers. Hence, an enforced allocation mechanism would be needed to enable the socially optimal negative assortative matching to take place.

While the economic literature has not yet addressed this issue to any great extent, education debates about whether society should aim for positive or negative assortative matching in education have raged for decades. The English comprehensive system deliberately aims to mix abilities at the

[1]Recent studies include Hanushek, Kain, Markman & Rivkin (2003), Lefgren (2004),Kang (2007), Ding & Lehrer (2007), Zimmerman (2003), Hoxby & Weingarth (2005) ,Henry & Rickman (2007), Ammermueller & Pischke (2006), Vandenberghe (2002), Betts & Zau (2004), and Entorf & Lauk (2006).

primary and secondary school levels under the belief that this negative assortative matching is optimal in aggregate. On the other hand, the system of specialized academic and vocational schools in much of mainland Europe, most notably Germany, is predicated on the belief that the aggregate outcome is higher under positive assortative matching. The elementary and secondary schooling system in the United States, whose *de facto* allocation reflects both parental choice and the choices of individual states and localities, is a mixture of these two extremes.

Only a minority of the population enters tertiary education, which in most countries operates under positive assortative matching by default at the population level, due primarily to ability-based sorting of students across universities with admissions requirements of varying strictness. Even within universities, positive assortative matching is more likely than negative assortative matching at the university-wide level. This is because in countries that do not follow the liberal education system, students enter specific programs of study, each with potentially different entry requirements. Hence, students are likely to find themselves broadly matched by default to program- or cohort-level peers of similar ability. Yet even in this case, where cohort-level student heterogeneity is lower than population-level heterogeneity, the issue of matching surfaces in the form of how students within a given program or cohort are allocated into tutorial groups to learn.

In this paper, we offer a new, simple, and economically sensible theoretical model of educational outcomes to highlight the social choice aspects of the peer externality. Whilst the qualitative results are already well-known in the literature on team production (Becker 1973, Durlauf & Seshadri 2003), we offer explicit solutions for our case, as well as explicit pricing mechanisms.[2] In our model, individual students make a choice about effort based on their own talent and on the talent and effort levels of their peers. We label the product of effort and talent the 'net input' into education, which flows from two sources: both self and peers. We then derive the optimal choice of effort for each student given

[2]We provide explicit pricing solutions for the negative assortative matching case, which we believe is new in this literature.

peer ability, and we show which peer matches and resulting levels and distributions of outcomes arise from various social allocation mechanisms of students into peer groups. The mechanisms we consider include voluntary matching in the absence of side payments; voluntary matching with side payments; and enforced random or negative assortative matching via a social planner. We map our framework to existing achievement models typically used in the peer effects literature.

We then introduce three questionnaires we posed over the course of the 2008 university year to which we received responses from over a thousand Australian undergraduate students, wherein we directly queried them about their beliefs regarding whether peer externalities complement or substitute for their own inputs. Students responded with a wide distribution of answers, but the dominant belief is that peer externalities are complements rather than substitutes. We investigate whether heterogeneity in these beliefs is related to actual effort levels in the way we hypothesize, and find that it is, again suggesting a weak complementarity in educational production. Finally, we are able to measure actual behavioral choices, ability levels, and expected performance in the classroom, and we apply these data to specifications implied by our model to explore the strength of peer interaction in the classroom. Our evidence indicates that this strength is quite limited if we define the relevant peers as the universe of students in a classroom.

The main novelty of this paper is that we accommodate the possibility that individuals anticipate and react to peer influences by varying their own inputs to education. This responds in a theoretical sense to a widespread concern in the literature for the endogeneity of peer outcomes (e.g., Brock & Durlauf (2001), Brock & Durlauf (2007)) as well as to more recent empirical evidence that agents' effort levels vary in response to the environment and are important to academic outcomes (Stinebrickner & Stinebrickner 2007, Houtenville & Conway 2008). Our evidence is furthermore consistent with recent findings that behavior in the classroom cannot be well forecasted on the basis of only fixed student and school characteristics (Segal 2008). Finally, we show how student preferences regarding selection into learning environments can result from an educational production function with a social component, and how selection is therefore a public welfare concern.

The main limitation of our study is that we only provide direct empirical evidence about peer effects that Australian students believe operate during tutorials. This implies that our results do not necessarily carry over to other forms of education, and that they are conditional upon the assumption that students are generally aware of how peer effects operate upon them.

The next section introduces the models we use to think about peer effects, and discusses some relevant recent literature using our simple model as a conceptual framework. We then introduce our survey data and present our empirical results on students' beliefs about peer influence in Section 3. In Section 4 we estimate the equations of effort and educational production that our theoretical model implies, to try to verify students' beliefs regarding peer influence. Section 5 concludes.

2 Models of peer effects

We begin with a model in which each student receives utility from educational attainment and disutility from effort, an input into educational production. Each individual is endowed with ability and chooses effort. Each student's ability and effort level enter her educational production function directly, and also enter other students' educational production functions.

Utility takes the simple form

$$u_i(e_i, E_i) = e_i - \eta E_i^2 \tag{1}$$

where e_i is the educational attainment of student i, E_i is her effort, and η is assumed positive. We presume that individuals are matched into pairs to learn, with each individual subscripted by i matched to a peer subscripted by j.[3] Once matched to a peer, every student chooses her effort in a

[3] The assumption that all peer groups contain two students is restrictive, but it enables an exposition of the model's implications using the familiar concept of a marketplace for (peer) services that is populated by individual autonomous agents.

one-shot game.

In this simplest model, educational production takes the form

$$e_i = \gamma_0 + \gamma_1(\alpha_i E_i) + \gamma_2(\alpha_j E_j) \tag{2}$$

Here, $\boldsymbol{\alpha}$ is ability, each element of which is drawn independently from a continuous cumulative distribution function $Q(.)$ that is bounded above and below and defined on the positive axis.[4] In line with standard economic thinking about production models in general, this equation allows both peers' effort and ability to influence a given student's educational outcome, rather than only peers' ability.

The term $\gamma_1(\alpha_i E_i)$, where we assume throughout that $\gamma_1 > 0$, captures the individual's net input into her own education and implies that we conceptualize own ability as the marginal productivity of own effort. The term $\alpha_j E_j$ refers to net peer input into own education, such that $\gamma_2(\alpha_j E_j)$ denotes the total contribution of net peer input into creating own educational outcomes. γ_2 represents a pure externality that does not interact with own choices or endowments. We will look later at what happens if we allow for complementarity or substitution between own and peer inputs.

Individuals are matched into peer groups via a first stage procedure. In the next subsection, we consider in detail the consequences of several voluntary and compulsory matching procedures. For every type of matching algorithm, all individuals are presumed to have full information about the distribution of ability in the group of potential peers, the form of the educational production function for all players, and the matching process.

We solve the model by backward induction, beginning with the second stage of the individual's problem, after matches to peers are made. Conditional on a match, optimal effort can be determined by solving the individual's utility maximization problem. The first-order condition with respect to

[4] It may help to think of $Q(.)$ as the uniform distribution. γ_0 should be interpreted as not subject to choice, but could contain linear functions of own ability α_i or even peer ability α_j without qualitatively affecting any further results.

effort is

$$\frac{d}{dE_i}(u_i(e_i, E_i)) = 0 = \gamma_1\alpha_i - 2\eta E_i, \tag{3}$$

so optimal effort is defined by

$$E_i^* = \frac{\alpha_i\gamma_1}{2\eta} \tag{4}$$

which is increasing in own ability. Solving and substituting in for the optimal effort level of student i's peer, maximized utility for student i now equals

$$u_i^*(\alpha_i, \alpha_j) = \gamma_0 + \frac{\gamma_1^2\alpha_i^2}{4\eta} + \frac{\gamma_1\gamma_2\alpha_j^2}{2\eta}$$

In this expression, the term $\frac{\gamma_1\gamma_2\alpha_j^2}{2\eta}$ is the own utility value of peers in education, and we will label it the "peer transfer." If $\gamma_2 > 0$, then learning is enhanced by high-ability peers. If $\gamma_2 < 0$, then individuals benefit relatively more from low ability peers than from high ability peers. Unlike Durlauf & Seshadri (2003), we consider group sized fixed by institutional factors (rather like marriage is legally limited to two persons) and do not discuss general production functions with variable group membership. As a result, the basic results on the optimality of assortative matching is entirely standard and hinges on the sign of $\frac{\delta^2 u_i^*(\alpha_i,\alpha_j)}{\delta\alpha_i\delta\alpha_j}$.

Given the reduced-form utility function, the preference ordering for peers is completely determined by γ_2. If pairwise matching is required, and if $\gamma_2 > 0$, then all individuals want to match with the individuals of highest ability. The reverse holds if $\gamma_2 < 0$: if they must match to a peer, individuals all want the lowest-ability peer available. It is natural to see $\gamma_2 > 0$ as the generic case—and in Sections 3 and 4 we provide some empirical support for this proposition—so we focus on the implications of the model when the peer transfer is positive.

2.1 Matching and outcomes in the simple model

We initially consider voluntary matching without side payments as the allocation mechanism used to produce matched pairs of peers. Matching takes place in sequential bidding rounds and is comprehensive, open, and voluntary, in the sense that each individual evaluates all others and is able to announce publicly her preference ordering for peers and to reject proposed partners. Only those pairings where both individuals agree to the match form. Each round, all matched pairs are taken out from the pool over which remaining unmatched individuals announce their preference orderings, until all individuals are matched.

When $\gamma_2 > 0$, the outcome of self-selection under this bid-based sequential matching process is simple: there will be complete positive assortative matching of students based on ability. The two highest-ability individuals remaining in the student pool, who hold the most power in the marketplace for peer services, will match with each other in every round. With sufficient support in the ability distribution, this sequential bid-based matching process will result in each individual being matched with someone of whose ability level is epsilon close to her own ability. Approximating α_j with α_i, realized utility for each individual is then $\gamma_0 + \frac{\gamma_1 \alpha_i^2 (\gamma_1 + 2\gamma_2)}{4\eta}$.

If, alternatively, a social planner were to match individuals into pairs randomly, then each individual would in expectation receive a peer transfer of $\int \frac{\gamma_1 \gamma_2 \alpha_j^2}{2\eta} dQ$. In expectation, this would benefit all those with low ability who would otherwise have ended up with lower peer transfers, and adversely affect all those with high ability who would otherwise have ended up with higher peer transfers. A similar redistributive effect would be seen with respect to education achieved, as the education produced for lower-ability (higher-ability) students would be higher (lower) under forced random matching than under self-selection. The cost of this redistribution could be approximated in either educational or utility terms by subtracting total outcomes under self-selection from total outcomes under forced random matching, but it might be costless or even produce overall utility and education gains, depending on the exact distribution of student ability.

A social planner could alternatively enforce negative assortative matching by pairing the highest ability individual with the lowest ability individual in each matching round, creating another and possibly larger net transfer of utility and education from the high ability to the low ability. Again, this redistributive transfer might be costless, depending on the underlying student ability distribution and the consequent outcome of the counterfactual voluntary matching process.

Finally, a market-based allocation mechanism could be established by allowing for compensating utility transfers between individuals under otherwise perfect market circumstances. In the model above, individual i's willingness to pay for a peer equals the peer transfer term $\frac{\gamma_1\gamma_2\alpha_j^2}{2\eta}$. Under voluntary matching without transfers, student i would expect a peer transfer of $\frac{\gamma_1\gamma_2\alpha_i^2}{2\eta}$, since she expects to match with a peer of ability epsilon close to her own. Thus, student i's willingness to pay for a peer under an enhanced voluntary matching scheme equals the utility she would derive from the proposed peer j less her default expected peer transfer under simple voluntary matching, or $\frac{\gamma_1\gamma_2[\alpha_j^2-\alpha_i^2]}{2\eta}$. For all j such that $\alpha_i < \alpha_j$, such that student i is considering "trading up," this willingness-to-pay is positive. Conversely, student i will only consider "trading down" if she is given a positive utility transfer to compensate her for doing so, as her willingness to pay for this sort of trade is negative. Since the same can be said for all students, then if transfer costs are zero and individuals' resources are sufficient to offer transfers, individuals will be observed to match randomly in expectation. All observed compensating transfers, in the amount $\frac{\gamma_1\gamma_2[\alpha_1^2-\alpha_2^2]}{2\eta}$, will flow from the lower-ability to the higher-ability student in the pair. If resources are constrained, then the actual matches observed will depend upon the distribution of initial resources available with which individuals might compensate partners with higher ability than themselves.[5] In terms of aggregate utility outcomes, any observed match under a compensating transfers scheme leads to identical outcomes as the voluntary matching case, although the distribution of education achieved may differ. Lower-ability individuals will be observed with equal or better educational outcomes depending on their ability and choice to buy high ability peers, and by contrast, higher-ability individuals will be observed with equal or worse

[5]If lower-ability individuals face higher costs of borrowing than higher-ability individuals, for example, then we would again observe some degree of assortative matching because lower-ability individuals are outbid for high-ability peers by higher-ability individuals.

educational outcomes depending on their ability and choice to buy high ability peers.

To summarize, under our simplest model without complementarities or substitution, we observe that a matching algorithm where no utility transfers are possible results in positive assortative matching. Peer-sourced educational benefits flow disproportionately to higher-ability individuals. With transfers, this disproportionality in the distribution of benefits from peers can be potentially eliminated with no net change in utility, but only if lower-ability students can make costless transfers to compensate higher-ability partners. The uncompensated random or negative assortative matching of individuals implemented by a social planner will progressively redistribute the educational benefits of peers and also make higher-ability individuals worse off, and lower-ability individuals better off, in utility terms.[6]

As a final note, recent literature in economics (Bandiera, Barankay & Rasul 2007, Foster 2005) as well as older work from other social sciences (Lazarsfeld & Merton 1954) suggests that people choose to befriend others on the basis of social elements that are not well-proxied by strict ability. Indeed, in most real-life situations, we do not observe strict ability-based matching. Following this line of thought, students may benefit in utility terms from a social element of their peers that is orthogonal to ability levels. This type of orthogonal social dimension to utility would simply dilute, but not erase, the assortative matching on the basis of ability that our model predicts. Students may in fact use social appeal as a tradeable asset with which to compensate peers for their own ability deficits in a compensated matching process similar to what we describe above.

2.2 Previous literature

With our basic modeling framework in place, we now review some recent work in two areas. First, we discuss the modeling approaches used by authors who have empirically estimated peer effects.

[6]Note that forced random or negative assortative matching would produce a *regressive* redistribution of education and utility were $\gamma 2 < 0$.

Second, we review papers whose authors have used social effects models or empirical results to discuss, explicitly or implicitly, the social choice element implied by different group allocation mechanisms.

2.2.1 Peer effects estimation

We can re-write equation (2) as a traditional endogenous effects equation:

$$
\begin{aligned}
e_i &= \gamma_0(1 - \frac{\gamma_2}{\gamma_1}) + (\gamma_1 - \frac{\gamma_2^2}{\gamma_1})(\alpha_i E_i) + \frac{\gamma_2}{\gamma_1} e_j && (5) \\
&= \gamma_0^* + \gamma_1^*(\alpha_i E_i) + \gamma_2^* e_j && (6)
\end{aligned}
$$

where the education outcome of individual i is written as a function of own inputs, $\alpha_i E_i$, and the educational outcome (rather than inputs) of the peer, e_j. If this is the correct model and if all items are well-measured, then γ_1^* and γ_2^* in this "pure endogenous effects" equation are identified.

However, were a researcher to try to estimate a pure endogenous effects model such as that shown in equation 6, variation in student effort is conventionally not observed distinct from ability, so $\alpha_i E_i$ would be afflicted by measurement error; and own effort level, which we expect to be correlated with peer outcomes, would be in the error term. In the presence of measurement error in α_i, the issue is further compounded by the probability that students endogenously select themselves into groups, such that the peer outcome is not just correlated with unobserved effort, E_i, but also with α_i, via the relation between peer ability and own ability—a common concern in the peer effects literature (see, amongst many others, Evans, Oates & Schwab (1992), Krauth (2006), and Arcidiacono, Foster, Goodpaster & Kinsler (2009)). Even in the absence of unobserved ability-based self-selection, however, if student effort enters educational production as it does in our model, then even a pure endogenous-effects specification cannot consistently estimate the peer effect without data on effort.

In the economics of education literature the pure endogenous effects specification has been applied only rarely, primarily due to the belief that contextual and/or correlated effects, in the terminology of Manski (1995), are also present in education. As also discussed in Manski (1995), it is impossible to identify all three types of social effects when the identifying variation in peer outcomes is only a function of changes in the mix of fixed student characteristics across groups, and so many researchers have chosen to use their variation to try to identify contextual and/or correlated effects rather than endogenous effects. A common specification is one where only peer background characteristics—not observed outcomes—enter own educational production. This specification has sometimes been thought to yield a lower bound for the "full" effect of peers in own education, under the assumption that self-selection will generally result in correlated and endogenous effects pushing in the same direction as the contextual effect. To generate a contextual effects specification using our model, where students choose effort conditional on peer and own ability, we can substitute optimal effort for self and peers into equation (2) to yield

$$\begin{aligned} e_i &= \gamma_0 + \frac{\gamma_1^2 \alpha_i^2}{2\eta} + \frac{\gamma_1 \gamma_2 \alpha_j^2}{2\eta} \\ &= \gamma_0 + b_1 \alpha_i^2 + b_2 \alpha_j^2 \end{aligned} \tag{7}$$

Perhaps the most surprising corollary of equation (7) is that endogenous effort in our model can be accommodated by means of including quadratics rather than levels of ability in the estimation of educational outcomes. The squared terms prohibit a representation of this equation as a linear combination of a traditional contextual-effects model—where only levels of peer and own ability appear—and additional terms. It is nonetheless clear that estimating own education as a function of levels rather than squares of own and peer ability will not necessarily yield a lower bound on the effects of peers in own education. If we take the α_i used in previous research to be a noisy measure of α_i^2, then both the measurement error of peer ability and selection concerns afflict the peer effect estimate, even if the actual measurement of own ability α_i is perfect.[7] If the correlation of α_j with α_i^2, such as would be observed under assortative matching, dominates the measurement error problem

[7]See Arcidiacono et al. (2009) for a detailed empirical investigation of these competing influences in peer effects estimation.

pertaining to α_j, then this specification would yield an overestimate of b_2 and an underestimate of b_1. In the case of random peer matching, α_i and α_j should be uncorrelated, and so here as well as in the case of a weaker influence of selection than of measurement error we should get the classical result of an underestimate of both b_1 and b_2.[8] Although not included in our model, the educational production function may also include correlated effects. While such effects would have no impact on our theoretical results, they form an additional source of bias in the estimation of spillovers if excluded.[9]

Papers trying to estimate Equation (7) for grades and other educational outcomes include large-sample empirical studies on elementary school students (Hanushek et al. 2003, Hoxby 2000, Hoxby & Weingarth 2005, Ammermueller & Pischke 2006, Nechyba & Vigdor 2004), high school students (Ding & Lehrer 2007), undergraduates (Sacerdote 2001, Betts & Morell 1999, Zimmerman 2003, Foster 2006, Carrell, Malmstrom & West 2008, Han & Li 2009), and postgraduate students (Arcidiacono & Nicholson 2005). Using a variety of techniques and measures, these generally find a modest but significantly positive b_2.[10] Due to the absence of reliable data on ability levels, such papers usually

[8]It is interesting to note that the estimate of the educational returns to *own* ability as well as the estimated peer effects yielded by this specification are biased. A full exploration of this problem and its consequences for the educational returns literature is beyond the scope of this paper.

[9]A common 'correlated effect' that influences both partners would imply $e_i = \gamma_0 + \gamma_1(\alpha_i E_i) + \gamma_2(\alpha_j E_j) + \gamma_{ij}$ where γ_{ij} is a fixed unpredictable common shock to both e_i and e_j. Because it is a shock, it does not affect choices or transfers and hence does not affect any of the findings in this paper. However, it will matter for empirical estimation because γ_{ij} is by design correlated with e_j though not with α_j. Hence in estimating equations of peers' outcomes on each other, it is important to control for factors involved in producing γ_{ij}.

[10]Many recent papers have also estimated specifications similar to Equation (7) that allow returns to peers to be nonlinear. Some results (Hoxby & Weingarth 2005, Zimmerman 2003, Carrell et al. 2008) support the presence of nonlinearities. In our model, the peer externality has no effect on the behavior of the individual, and hence its functional form can be changed at will. What matters in estimation

exploit very special empirical circumstances in order to identify peer effects, implying that the external validity of their results is somewhat questionable. Moreover, as shown above, a key weakness of all of these studies is that they both fail to measure effort E_i and fail to circumvent the lack of data on effort, which leads to estimation of the wrong functional form from which either underestimates or overestimates may result.

2.2.2 Allocation as social choice

The theoretical literature on team production has mainly focussed on whether or not assortative matching will arise out of voluntary matching without side payments, and whether the allocation mechanism is efficient. When group sizes are fixed, as in the early work of Becker (Becker 1973), then assortative matching hinges on the cross-derivative of the reduced-form utility function. When there are externalities and effort is endogenous, individual choices can differ from socially optimal choices, as demonstrated by Durlauf & Seshadri (2003). When group size is also variable, the efficiency of allocation starts to depend on whether there are increasing returns to having larger groups (in which case the optimal team size is the whole economy) and, if not, what the precise relation between group size and productivity is. Then, the distribution of abilities also matters (see, e.g., Bhattacharya (2008) and Epple & Romano (1998)).

Most empirical studies by economists working in the field of social effects seem somewhat disconnected from this theoretical literature and only address concerns about optimal grouping mechanisms briefly, if at all, and rarely are these treatments grounded in a theoretical model of behavior. There is little academic consensus regarding what type of grouping mechanism is best either for individuals or for society as a whole. As an example of early work in this area, Henderson, Mieszkowski & Sauvageau (1978) estimate an array of outcome specifications in lieu of offering a behavioral model, and conclude from their results that

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is that the correct functional form, including both peer ability and peer effort, is used.

> ...if the objective of society is to maximize the *overall* achievement level of its students, or mean achievement, *a uniform mixing of students by achievement will be optimal.*

(emphasis in original). In a study of peer effects in a work environment, Mas & Moretti (2006) parenthetically mention the implications for total productivity of different worker grouping mechanisms in the presence of social effects on individual productivity. They interpret their empirical results to imply that in order to maximize total productivity, high- and low-productivity workers should be mixed, such that diversity in worker groups is maximized. In stark contrast, Duflo, Dupas & Kremer (2008) exploit experimental evidence from Kenyan schools and find that ability-streaming is preferable to ability-mixing for students at all points on the ability distribution. Falk & Ichino (2006) find that grouping workers is preferable to having workers work alone, in terms of total productivity on an unskilled task, but they do not compare different worker-grouping paradigms. Drawing on empirical results derived from a Chinese secondary school context, Ding & Lehrer (2007) briefly consider different possible student grouping mechanisms and find that students at the top of the ability distribution would benefit most from streamed (ability-homogeneous) classes, while those at the bottom would benefit most from mixed classes. This result, also found in Kang (2007), implies that optimal student grouping is a concern that demands treatment from a public welfare perspective rather than a production-maximization perspective.

To address the twin possibilities that students of different ability levels may be subject to different peer transfers, and that there may exist a conflict between the individually optimal and socially optimal peer allocation mechanism, we now augment our basic model by allowing for peer and own inputs to education to interact in each student's educational production function. Our expanded model again aims for explicit solutions, and takes group sizes as given.

2.3 Adding interactions between self and peers

The simple model above reflects the dominant assumption in the applied literature that peers affect own educational outcomes in a linear, additive manner. Here, we expand on that framework by looking at what happens if we allow for interaction effects between own and peers' inputs to educational production. We use an expanded educational production function that includes cross-effects:

$$e_i = \gamma_0 + \gamma_1(\alpha_i E_i) + \gamma_2(\alpha_j E_j) + \gamma_3(\alpha_j E_j)(\alpha_i E_i) \tag{8}$$

Here, the term $\gamma_3(\alpha_j E_j)(\alpha_i E_i)$ denotes the interaction between own net input and the net input of the peer. This function allows for peer inputs to be either substitutes (in which case $\gamma_3 < 0$) or complements (in which case $\gamma_3 > 0$) to own inputs in own educational production.[11]

Optimal effort level expended by i and j then becomes:

$$\begin{aligned} E_i^* &= \frac{\alpha_i(\gamma_1 + \gamma_3(\alpha_j E_j))}{2\eta} \\ E_j^* &= \frac{\alpha_j(\gamma_1 + \gamma_3(\alpha_i E_i))}{2\eta} \end{aligned}$$

which solve to

$$\begin{aligned} E_i^* &= \frac{\alpha_i \gamma_1(2\eta + \alpha_j^2 \gamma_3)}{4\eta^2 - \alpha_i^2 \alpha_j^2 \gamma_3^2} \\ E_j^* &= \frac{\alpha_j \gamma_1(2\eta + \alpha_i^2 \gamma_3)}{4\eta^2 - \alpha_i^2 \alpha_j^2 \gamma_3^2} \end{aligned}$$

[11] We think of the term $\gamma_3(\alpha_j E_j)(\alpha_i E_i)$ as a reduced-form effect from a possibly much more complex functional form. While this term in principle allows for education to become decreasing in the net input of the peer (when γ_3 is highly negative), we will in the remainder of the paper think of the term $\gamma_3(\alpha_j E_j)(\alpha_i E_i)$ as 'small' relative to $\gamma_2(\alpha_j E_j)$, such that $\frac{\delta e_i}{\delta(\alpha_j E_j)} > 0$ and $\frac{\delta e_i}{\delta(\alpha_i E_i)} > 0$ for the range of values taken on by $\alpha_i E_i$. This restriction is not really necessary to explain the model, but allows us to focus on the most plausible states of the world.

These expressions imply that maximized utility equals

$$u_i^*(\alpha_i, \alpha_j, E_j) = \{\gamma_0 + \frac{\gamma_1^2 \alpha_i^2}{4\eta}\} + \alpha_j E_j \{\frac{\gamma_1 \gamma_3 \alpha_i^2}{2\eta} + \gamma_2\} + \alpha_j^2 E_j^2 \{\frac{\gamma_3^2 \alpha_i^2}{4\eta}\} \tag{9}$$

for any given values of own and peer ability. Linearizing this equation around $\gamma_3 = 0$ and inserting for E_j yields

$$u_i^*(\alpha_i, \alpha_j) \approx \{\gamma_0 + \frac{\gamma_1^2 \alpha_i^2}{4\eta}\} + \gamma_2 \{\frac{\gamma_1 \alpha_j^2}{2\eta}\} + \gamma_3 \{(\gamma_1 + \gamma_2) \frac{\gamma_1 \alpha_i^2 \alpha_j^2}{4\eta^2}\} \tag{10}$$

Again recalling our assumption that $\gamma_2 > 0$, the key thing to note about this equation is that $\frac{\delta^2 u}{\delta \alpha_j \delta \alpha_i} > 0$ iff $\gamma_3 > 0$. That is, when the input of peers is complementary to own input in educational production, higher ability individuals benefit more in utility terms from higher ability peers. Now, if $\gamma_3 < 0$, such that peer inputs have decreasing returns with own input, then $\frac{\delta^2 u}{\delta \alpha_j \delta \alpha_i} < 0$, implying that high-ability individuals' utility returns to high-ability peers are less than those of lower ability individuals.

In the case that $\gamma_3 > 0$, the positive dependence of own optimal effort on peer effort implies that the process we consider is a supermodular game, as analyzed at length in the game theory literature (Milgrom & Roberts 1990, Shimer & Smith 2000, Vives 2005). The core defining element of such a game is that the returns to behavior for each agent increase in the like behavior of the other (either partner or competing) agent. In many settings, this leads to the theoretical possibility of multiple equilibria. Which equilibrium the system arrives at is strongly dependent upon initial conditions and specific aspects of game context. In our application, however, due to the cross-agent heterogeneity in willingness to pay and matching-market power generated by agents' underlying differences in ability endowments, we can derive one full-information equilibrium per assignment mechanism. We derive these in the next subsection.

2.4 Matching and outcomes with net input interactions

We now consider the amount and distribution of outcomes that are implied if we apply to this expanded model the same set of possible peer allocation mechanisms as we did above using the simple model.

In the case of the voluntary bid-based sequential matching process without transfers, the resulting allocation is complete assortative matching iff $\frac{\delta u}{\delta \alpha_j} > 0$, since in this case all students wish to be matched to the highest-ability peer possible. This first derivative is positive when $\gamma_2\{\frac{\gamma_1 \alpha_j}{\eta}\} + \gamma_3\{(\gamma_1 + \gamma_2)\frac{\gamma_1 \alpha_i^2 \alpha_j}{2\eta^2}\} > 0$ which trivially holds when $\gamma_2 > 0$ and γ_3 is positive or small and negative,[12] or in other words when net peer effects are positive. Given the evidence from prior literature that net peer effects are positive, this is the case we assume and that we will discuss further.

Given positive assortative matching brought about by voluntary selection without transfers, the net utility of an individual with ability α_i can once again be found by approximating α_j with α_i yielding $u_i^*(\alpha_i, \alpha_i) \approx \gamma_0 + \alpha_i^2 \frac{\gamma_1^2 + 2\gamma_2\gamma_1}{4\eta} + \alpha_i^4 \frac{\gamma_3\gamma_1(\gamma_1+\gamma_2)}{4\eta^2}$. This individual would see net education produced in the amount of $e_i \approx \gamma_0 + \alpha_i^2 \frac{\gamma_1^2 + \gamma_2\gamma_1}{2\eta} + \alpha_i^4 \frac{\gamma_3(2\gamma_1^2+\gamma_2\gamma_1)}{4\eta^2}$.

The second matching scheme we consider is forced random matching by a social planner. The actual utility of an individual then depends entirely on the ability of the allocated peer. What we can say in general is that low-ability individuals can expect to gain from random matching since they can expect to be allocated a higher ability peer than otherwise, while high-ability individuals can expect to lose from random matching since they can expect to be allocated a lower ability peer. This general conclusion also held under our simple model, in the absence of interactions between own and peers' inputs. The expected change in utility for any individual, and hence for the population, will again depend on the actual distribution of ability. An individual's expected net benefit from being forced into a random match can be computed as the difference between utility expected under forced

[12] This will always be true if both γ_2 and γ_3 are positive. If γ_3 is negative, then this will hold if $|\gamma_3| < \frac{2\eta\gamma_2}{(\gamma_1+\gamma_2)\alpha_i^2}$.

random assignment versus that obtainable via voluntary assortative matching:

$$\begin{aligned}\Delta U(\alpha_i) &= \int_{a_{min}}^{a_{max}} u_i^*(\alpha_i,\alpha)dQ(a) - u_i^*(\alpha_i,\alpha_i)\\ &\approx (\sigma_j^2-\alpha_i^2)\{\frac{\gamma_2\gamma_1}{2\eta}+\frac{\gamma_3(\gamma_1+\gamma_2)\gamma_1\alpha_i^2}{4\eta^2}\}\end{aligned}$$

where $\sigma_j^2 = \int a^2 dQ(a)$. This expected change will be positive for low α_i individuals and negative for high α_i individuals.

A final issue worth noting with regard to the forced allocation mechanism is that its appeal from a social perspective depends trivially on γ_3. If $\gamma_3 > 0$, then total utility and education outcomes are clearly maximized under positive assortative matching, which will come about voluntarily. Yet, when $\gamma_3 < 0$, then the exact opposite holds since the effect of having a high-ability peer is then higher for a low-ability person than for a high-ability person. From a social perspective, high-ability peers should be allocated to low-ability individuals in order to produce the most utility and education in aggregate. As a result, despite the fact that all individuals wish to be matched to high-ability peers, and thus that voluntary sorting will yield positive assortative matching, aggregate utility and education are maximized by negative assortative matching. Forced mixing is then one way to achieve this social objective, although it will also produce a regressive redistribution of both utility and education.

The final matching protocol we consider is the voluntary market allocation mechanism with side payments. If $\gamma_3 > 0$, the compensated matching process yields a straightforward outcome: the higher-ability individuals will outbid others to match with other high-ability individuals. The net transfer will be zero, since same-ability types match up in sequential rounds of matching and pay each other the same transfer.

The more complicated case under compensating transfers occurs when $\gamma_3 < 0$, where low-ability types would have an incentive to offer the most appealing peer transfers, because they gain more from having a high-ability peer than other types. The resulting equilibrium is negative assortative matching. In this case, the exact level of the transfers involved is non-trivial to calculate, and has to be solved by backward induction from the resulting equilibrium allocation.

In the last round of matching, individuals with median ability pay each other a zero transfer. In the penultimate round of matching, the remaining population still includes individuals of median ability and individuals with abilities just above and below the median. The individual who is just below the median by ability level ε would be willing to pay to be matched with peer just ε above the median rather than at the median. The amount that an individual at the median would be willing to pay to be matched with a peer ε above the median rather than the median equals $u_i^*(Q^{-1}(0.5), Q^{-1}(0.5) + \varepsilon) - u_i^*(Q^{-1}(0.5), Q^{-1}(0.5))$. Since the lower-ability individual only needs to bid this amount to overbid the median person, this is the transfer that will occur. Simplifying this expression yields

$$u_i^*(Q^{-1}(0.5), Q^{-1}(0.5) + \varepsilon) - u_i^*(Q^{-1}(0.5), Q^{-1}(0.5)) \approx \varepsilon \frac{\delta u_i^{*\prime}(Q^{-1}(0.5), \alpha_j)}{\delta \alpha_j}|^{\alpha_j = Q^{-1}(0.5)}$$

Having solved for the transfer seen in the penultimate round of matching, we can now consider the round immediately before this. If we extend the line of thought above to the individuals with ability 2ε above and below the median, we find the transfer to the former from the latter has to be $\varepsilon u_i^{*\prime}(Q^{-1}(0.5), Q^{-1}(0.5)) + \varepsilon u_i^{*\prime}(Q^{-1}(0.5) - \varepsilon, Q^{-1}(0.5) + \varepsilon)$. Generalizing this, we obtain a formula for the transfer $t(\alpha_i)$ paid by individual with ability $\alpha_i < Q^{-1}(0.5)$ to her peer of ability $\alpha_j = Q^{-1}(1 - Q(\alpha_i))$:

$$t(\alpha_i) = \int_{\alpha_i}^{Q^{-1}(0.5)} \frac{\delta u(\alpha, \alpha_j = Q^{-1}(1 - Q(\alpha)))}{\delta \alpha_j} d\alpha$$

In the case where $Q(.)$ is the uniform distribution, this formula simplifies to

$$t(\alpha_i) = \int_{\alpha_i}^{Q^{-1}(0.5)} \frac{\delta u(\alpha, 2Q^{-1}(0.5) - \alpha)))}{\delta \alpha} d\alpha.$$

In summary, adding an interaction between own and peers' inputs into educational production yields a much more complicated problem, and one where the sign of the interaction is crucial in determining the default matching pattern that the system will produce when individuals match voluntarily.

We see that the market mechanism obviates the need for a social planner to force students to match in order to achieve the social optimum, essentially because the peer externality is fully priced by means of side payments. Social allocation mechanisms only appear to be useful when side payments cannot be made and when $\gamma_3 < 0$.

3 Data

Empirical peer effects papers almost without exception use recorded data on peer group assignment and behavior to deduce the strength and direction of peer effects. However, as noted above, this pursuit is plagued with a host of empirical obstacles—including problems with both selection bias and measurement error in ability—such that authors using behavioral data frequently must resort to artificial and/or complex identification strategies that call into question the generalizability of their results.

In this paper, we pursue a different strategy. Having set out above a theoretical framework of peer effects that would explain facts like systematic sorting by ability and an association of effort level with ability, we proceed to validate our theory not using behavioral data but rather using survey data about beliefs. From an economic theory perspective, real students' beliefs about the parameters γ_2 and γ_3, even more than their actual values, are what should matter for choice behavior. Importantly, beliefs are unaffected by selection into peer groups or measurement error in ability.[13] Moreover, since undergraduate students have usually experienced full-time education for 12 consecutive years already, we can be confident that these students will have had ample time to learn about γ_2 and γ_3 from

[13] One assumption we make is that the self-reported data we use measure true beliefs, which students actually use as utility-maximizers in deciding upon behavior such as their effort level and sorting preferences. While some students may lie, we see no reason to expect systematic lying in this case, since the questions we pose are not personal or embarrassing in any way.

previous peer interactions. This means we can view their opinions about these coefficients as the reasonable, rational estimates of primary producers of education outcomes. Furthermore, students are exactly the group we would want to have as survey responders in order to address our research question; the usual reasons for critiquing the use of students in surveys do not apply in our case.

With this goal in mind, we now turn to three collections of survey data on a sub-sample of two large populations of undergraduate students in which selected questions from the surveys were specifically designed to shed light on the signs and relative magnitudes of γ_2 and γ_3. Two surveys were online and one was administered in class. In total, we obtained 1733 responses. We draw evidence from each of the three surveys on specific dimensions of our problem. Further details regarding the surveys are provided in Appendix A.

3.1 Answers on questions pertaining to γ_2 and γ_3

We now introduce the survey questions that pertain directly to estimating the parameters of our model, including both direct effects and interactions of own and peer inputs.

To ascertain whether students believe that they benefit from having high ability peers, we asked the following question in the two online surveys, answered by 1275 students in total:

> Q1: *(response codes 1, Strongly Agree, through 11, Strongly Disagree)* "If a tutorial is full of more capable students, I will learn more than I would if it were full of less capable students."

This question directly addresses the issue of whether individuals benefit from high ability peers or not. In all the variants of the model above, we assume that they do, for all levels of own ability. In Figure 1, we show the distribution of answers to this question, depending on whether the individual self-reported being in the lowest relative ability category (self-rated as "not as smart/capable" as other students), medium ability category (self-rated as "about as smart/capable" as other students),

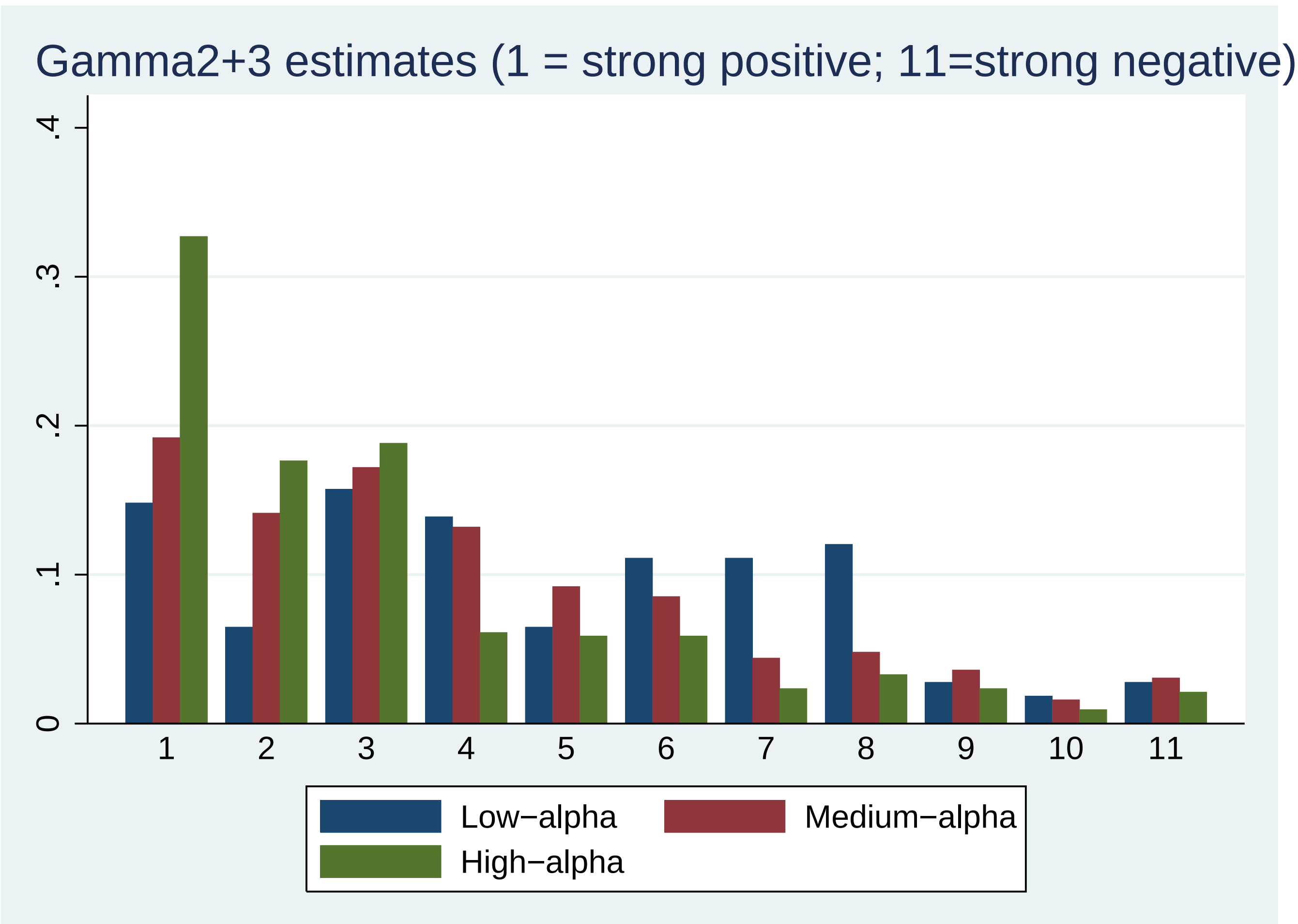

Figure 1: Estimates of $\gamma_2 + \gamma_3$

or highest ability category (self-rated as "more smart/capable" or "much more smart/capable" than other students).

We see, for all ability levels, that the density of individuals who answer this question positively (a 5 or lower) outweighs the density answering negatively, which we take as direct evidence that the combined effect of γ_2 and γ_3 is positive. We also see that the question is answered more positively for the higher ability individuals, giving the preliminary suggestion that γ_3 may also be positive.

Two additional questions were posed in the two online surveys to see whether students believe that peer inputs matter in terms of choice behavior. One question was concerned with peer ability and the other with peer effort:

> Q2: *(response codes 1, Strongly Agree, through 11, Strongly Disagree)* "If other students in a tutorial work hard, it makes me work hard too. "
>
> Q3: *(response codes 1, Strongly Agree, through 11, Strongly Disagree)* "The smarter the other students are in a tutorial, the harder I work in that tutorial. "

Question 2 pertains to the behavioral response to peer effort, whereas question 3 asks for the behavioral response to peer ability. Both questions directly identify whether or not γ_3 is positive. From the theoretical model we know that if γ_3 is positive, then the answer to both questions should be positive and identical for both questions. If γ_3 is negative, then the answer to both questions should be negative.

We show the distribution of the answers to these two questions, combining responses from the two online surveys together, in Figure 2. Once again we see much more density in the positive range or responses (0 to 5), confirming a positive sign overall.[14] We take this as a direct indication that the belief amongst students is that γ_3 is positive.

[14] At the respondent level, the Spearman correlation between responses to these two questions is .62 and Kendall's τ-a is .4472.

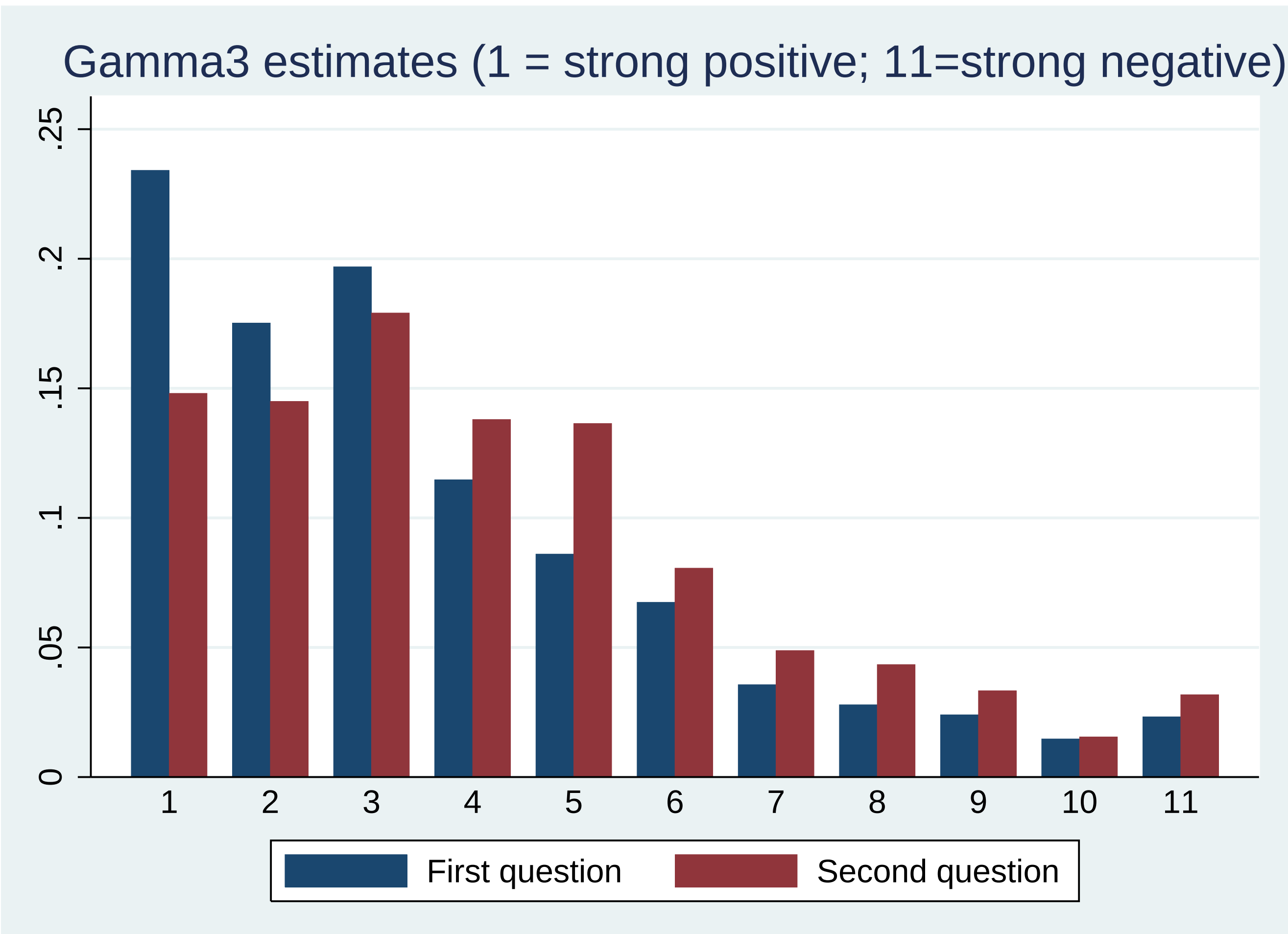

Figure 2: Estimates of γ_3

3.2 Do γ_2 and γ_3 vary over individuals?

We now briefly investigate the determinants of the variance we document above in students' perceptions of peer effects. Peer effects researchers would be interested to know whether peer effect parameters are heterogeneous across people, which in our context corresponds to the question of whether γ_2 and γ_3 vary systematically. To this point in the paper we have presumed that they do not, simply because our model of peer effects becomes too complex to solve analytically in terms of peer sorting if one presumes parameter heterogeneity. Yet, the empirical truth might well be that there is parameter heterogeneity.

In order to see whether we should take the variance in responses to the questions above as evidence of actual heterogeneity instead of simply reflecting a single coefficient with classical measurement error in the responses, we relate responses to these questions to students' self-reported relative ability, effort levels and outcomes. In the case that there is in actuality a population-wide single γ_2 and γ_3 then there should be no valid variation in the responses above and hence they should not predict any outcome, nor should they be predictable on the basis of other student-specific measures.

To address this question, we use responses to the questions that we asked students on both online surveys regarding their relative ability, relative effort, and expected grades, as reproduced in Appendix A.

Table 1 shows the results of our investigation into whether the variance in γ_2 and γ_3 reflects actual parameter heterogeneity. The first column of Panel A shows the results of a naive regression of average expected grades (across all courses for which the respondent reported expected grades) onto the three questions above that we use to estimate γ_2 and γ_3. In Column 2, we add own effort and ability measures to the equation, and in Column 3 we add a further array of control variables taken from matched records on survey respondents extracted from university data banks. In Panel B of Table 1, we use a range of independent variables about students, including expected grades and self-reported ability and effort, to predict students' answers to the questions targeting perceptions of γ_2 and γ_3.

We see in Panel A of this table that the stronger the perception that γ_2 and/or γ_3 are positive—i.e., the lower is a student's coded answer to the three questions eliciting information about these parameters—the higher the student's average expected grades. This relationship is still statistically significant for the peer-ability-based measure of γ_3 and for our measure of $\gamma_2 + \gamma_3$ even when we control for own effort and own ability in the regression, implying that the heterogeneity in perceived peer influence is not fully captured by either of these presumably first-order influences in education. As expected, both self-reported effort level and self-reported ability are powerfully, positively and significantly associated with expected grades.[15]

Panel B of Table 1 shows that the variance in perceptions of peer effects is systematically predicted by expected grades, own effort, and own ability. In each reported equation, some combination of own effort, ability, and expected outcomes is statistically significantly associated with perceived peer effects, in the expected direction: students who have higher grade expectations and/or who self-rate as more able or more hardworking than their fellow students report perceiving more positive peer effects. This implies a potential need to accommodate heterogeneity by ability and/or effort in γ_2 and/or γ_3 in our models of educational production.

4 Is effort endogenous?

We now use our combination of survey and administrative data to ascertain whether the key aspect of our model, the endogeneity of effort, is indeed borne out by the data.

[15]The excluded ability category is students who self-rate as "Not as smart/capable" as other students, and the excluded effort category is students who self-rate as "Not as hard-working" as other students.

Table 2 reports the results from simple regressions on self-reported effort levels. The first panel in the table shows the effect on effort levels of individuals' own beliefs and ability levels. The middle panel of the table, which is based on the in-tutorial data, adds in the effort and ability level of peers. The bottom panel of the table is also based on the in-tutorial data, but adds course fixed effects.

The top panel reveals that own ability is a strong predictor of own effort, exactly in line with the both the simple and the extended theoretical model. The results imply a strong degree of endogeneity of effort. Working from the results in the second column, a student with the highest level of ability expends about 0.59 more effort than a student with the lowest level of ability. Since effort is measured on a 1 to 3 scale, this means that a student's level of effort increases by about 30% by going from the bottom ability rung to the top ability rung. The third column in the panel shows that this result is robust to the inclusion of additional controls such as age, sex, class time, and institution. Interestingly, students' beliefs about the influence of peers are all of the right sign and are sometimes significant. These questions are coded as shown in earlier figures, such that a negative sign on the peer variable implies that the more strongly a student believes that high-ability peers improve outcomes, the higher that student's effort. According to the first column, those who strongly believe that $\gamma_2 + \gamma_3$ is positive put out an effort that is about 0.19 (=11*0.0172) scaled points higher than those who strongly disbelieve that $\gamma_2 + \gamma_3$ is positive. Together, the results in the top panel give some evidence for the existence of complementarities in educational production via endogenous effort.

Panel B of Table 2 shows that in actual tutorials, individual effort and ability are again strongly positive related—indeed even more so than for the general student population in Panel A. The coefficient on the highest ability level indicator increases from about 0.59 in Panel A to 0.93 in Panel B, meaning that effort increases by up to 50% as one moves from the bottom to the top of the ability distribution. This finding is robust to adding more controls, as witnessed by the steadfastness of the coefficients on ability as we move from Panel B to Panel C.

By contrast, the effects on own effort of peer effort and peer ability, as measured by the ability and effort of the other students in the same tutorial, are very small and relatively unimportant. The point estimate of peer effort is positive but insignificant, while peer ability enters negatively. Total peer input (effort times ability) is equally small and insignificant. Hence, whilst effort is strongly endogenous to own ability and to students' beliefs concerning the effect of peers, other students in the tutorials do not seem to significantly affect effort even in regressions that do not control for student sorting, despite the expected upward pressure on the estimator for the peer externality. This is suggestive evidence that students' interactions with their entire tutorial group may be modest, such that any tutorial-level peer effects on behavior are outweighed by other influences.

4.1 Outcomes: Expected grades

We now look at whether own inputs and inputs from other students in tutorials help to explain a measure of outcomes, which in our case is the level of expected grades. Our data on expected grades are taken from our in-class survey for which we received approximately 440 responses. Table 3 shows that there is a good deal of variation in expected grades in this sample for students individually, as well as in the distribution of average expected grades of tutorial-level peers.

Table 4 shows the estimated effects of own inputs and peer inputs on these expected grades using different specifications of an endogenous-effects model, where we gradually add in more controls. The first column shows results from the naive peer effects regression, implicitly presuming that there is no effort endogeneity problem. Using this specification, we get a very large and significant effect of expected peer grades on own grades. If we then add self-reported own ability (Column 2) we see that there is still a highly significant, positive peer effect, increasing from 0.176 to 0.184. If we then add own effort, we still see that peer inputs have a strong positive effect. As expected, own effort and ability are strong predictors of own grades. Of note, when we simply add own effort times own ability in place of separate effort and ability indicators (column 4), the R^2 actually slightly increases from

0.0722 to 0.0799 and the F-statistic more than doubles, indicating that the theoretical assumption of a multiplicative effect of own effort and ability appears to hold out well in this sample. Again, the peer effect remains strongly positive.

The story changes, however, when we add further controls in Columns 5 and 6 of Table 4 for age, sex, course, international student status, time of day of classes, and day of week of classes. While the predicted effects of own inputs remain highly positive and virtually unchanged from the previous columns, the estimated peer effect becomes negative, though at weak levels of significance. This indicates a strong influence of correlated effects on grades in this setting which appears to dominate the heterogeneity in grades resulting from the social effort choice problem, as found in (Lyle 2007). We hypothesize from these results that students only interact with a selected subset of the entire tutorial, and that we should therefore put more faith in the importance of beliefs about peer effects rather than behavioral evidence based on the performance of all students in the same tutorials.

5 Discussion

In this paper we have suggested an explicit model of peer effects that explicitly incorporates the notion that students choose their effort level depending upon the peer attributes they encounter in their learning environment. We use this model to show three things. First, we derive closed-form solutions of the model for utility and education under a variety of different peer-group sorting mechanisms. When peer and own inputs interact in educational production, we get the standard finding that voluntary matching is preferable to forced random matching unless the peer-own input interaction is negative—such that peer inputs substitute for, rather than complement, own educational inputs—*and* cost-free compensatory side payments to potential peers are not possible. Since voluntary matching will result in positive assortative matching when peer inputs complement own inputs to education, this result implies that with peer complementarities, ability streaming in education is preferable both individually and socially to ability mixing, at least when group sizes are institutionally fixed.

Second, we show new survey-based evidence that undergraduate students in fact believe that the peer-own input interaction term is positive. This provides some empirical support for believing the recommendation above based on the presumption of complements in educational production.

Finally, we show how reported effort is indeed endogenous to own ability and own beliefs about the effects of peers, again confirming a weak level of complementarity in education. We also briefly consider whether we can identify actual peer effects through matching individual students to others in the same tutorials, in which students have some limited interaction. However, in our sample, we did not observe a robust peer effect, which we take as evidence that the fairly limited interaction in these tutorials does not produce results strong enough to be picked up statistically.

At face value, our evidence of a weak level of complementarities in the education production function validates the widespread, but not universal, practice of assortative matching in universities and would mean that negative assortative matching could only come about involuntarily and has an efficiency cost. The question remains whether the same is true for secondary and primary school education.

A Data appendix: Surveys

In this paper we use data drawn from three surveys of Australian undergraduates. The population from which our survey data are drawn is the universe of students enrolled internally in undergraduate programs in the business faculties of two Australian universities: the University of South Australia (UniSA) in Adelaide, and the University of Technology Sydney (UTS), during the first and second semesters of 2008. Three surveys were administered: the first occurred in April to May of 2008 and was online; the second in late August 2008 was in paper-and-pencil format; and the third in September to October 2008 was online.

For the first survey, we designed a battery of questions to tap into beliefs about peers as well as on effort expended by respondents on academic work. Regarding personal effort levels, we asked students how many hours per week they studied, and also asked them to rank themselves relative to other students in terms of how hard they work. We also ask respondents about their self-assessed ability level relative to other students, and their expected grades in all courses in which they were enrolled. The first survey was administered online to all undergraduate business students at both institutions in April 2008. Due to ethics protocols, we could not offer a particularly strong incentive for students to complete the survey,[16] and hence the response rate to the survey was fairly low. 675 students responded to the first survey out of a combined universe of approximately 10,000 students.

The second survey was implemented in the week 6 lectures of seven business-division courses at the University of South Australia, and hence captured data on students sharing the same courses and tutorials. In this short survey we simply asked students about their self-assessed ability and effort levels relative to other students, their absolute effort levels both within the course in question and overall at university, and their expected grade in the course in question. Of students who attended class, very few refused the survey. We hence obtained this information from 436 respondents. Although we did not capture the universe of enrolled students in these courses due to some students not coming to class, our model implies that the sample of enrolled students who actually attend class and are hence exposed to their peers is in fact our population of interest.

The final survey was administered to students in both universities online in September 2008, and included questions identical to those on the first survey regarding self-assessed effort and ability, expected grades, and perceived peer effects. We obtained 622 responses to this survey.[17]

The questions common to each survey that we used to measure self-assessed effort levels, ability,

[16] We included respondents' names in a random draw for $200.

[17] Once again, the participation incentive was that we included respondents' names in a random draw, this time for $500.

and expected grades were as follows:

Q5: "Overall, with respect to academic work, would you rate yourself as . . . (choose one)

- Not as capable as other [UTS/UniSA] students
- About as capable as other [UTS/UniSA] students
- More capable than other [UTS/UniSA] students
- Much more capable than other [UTS/UniSA] students

Q6: "Overall, with respect to academic work, would you rate yourself as . . . (choose one)

- Not as hardworking as other [UTS/UniSA] students
- About as hardworking as other [UTS/UniSA] students
- More hardworking than other [UTS/UniSA] students
- Much more hardworking than other [UTS/UniSA] students

Q7: "Please list the courses you are enrolled in at [UTS/UniSA] this semester and the final course marks (percentages out of 100) that you expect in each.

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Table 1: Is the variance in perceived peer effects systematic?

Panel A	(1)	(2)	(3)
Dep. Var.: Average expected grade			
Gamma2+gamma3	-0.4480***	-.2722***	-.2643***
	(0.0869)	(.0830)	(.0853)
Gamma3 (item 1: peer effort)	-0.0931	.0050	-.0988
	(0.1128)	(.1066)	(.1098)
Gamma3 (item 2: peer ability)	-.2430**	-.3115**	-.2600***
	(0.1128)	(.1062)	(.1095)
Own effort step 2	-	2.8042***	2.7529***
		(.5338)	(.5514)
Own effort step 3	-	4.0352***	3.9388***
		(.6024)	(.6266)
Own ability step 2	-	3.8158***	3.7061***
		(.7737)	(.8080)
Own ability step 3	-	7.0319***	6.9305***
		(.8257)	(.8745)
N	1194	1194	1137
F	20.80	33.24	9.47
Adj R^2	.0474	.1591	.1571
Panel B	(1)	(2)	(3)
Dep. Var.:	$\gamma_2 + \gamma_3$	γ_3 **(item 1)**	γ_3 **(item 2)**
Average expected grade	-.0458***	-.0346***	-.0472***
	(.0107)	(.0100)	(.0103)
Own effort step 2	-.1095	-.5929***	-.3863**
	(.2016)	(.1877)	(.1925)
Own effort step 3	-.1930	-.5597***	-.4044
	(.2309)	(.2150)	(.2204)
Own ability step 2	-.4578	.2436	-.2372
	(.2955)	(.2752)	(.2822)
Own ability step 3	-1.1418***	.3364	-.3039
	(.3241)	(.3018)	(.3094)
N	1137	1137	1137
F	3.59	2.50	2.92
Adj R^2	.0498	.0295	.0374

Note: In Panel A, the dependent variable is the average of expected grades in all classes as reported in the survey; in Panel B, the dependent variables used are responses to the questions regarding γ_2 and γ_3, and additional controls in all regressions in Panel B and in Column (3) of Panel A are year and quarter of birth, sex, international student status, time of day and day of week of classes, and institution. 'Own effort step 2' is a dummy set to 1 if the respondent self-reported as 'about as hard-working' as other students, and 0 otherwise; 'Own effort step 3' is a dummy set to 1 if the respondent self-reported as being 'more' or 'much more' hardworking than others, and 0 otherwise. 'Own ability step 2' is a dummy set to 1 if the respondent self-reported as being 'about as smart/capable' as other students, and 0 otherwise; 'Own ability step 3' is a dummy set to 1 if the respondent self-reported as being 'more' or 'much more' smart/capable than others, and 0 otherwise. See text for more detail. In this and ensuing tables, one asterisk denotes significance at the 10% level; two asterisks, significance at the 5% level; and three asterisks, significance at the 1% level.

Table 2: Predicting own effort

Panel A: Self-reported relative effort, 1 to 3 scale - Sample 1	(1)	(2)	(3)
Gamma2+gamma3	-0.0172**	-.0067	-.0007
	(0.0080)	(.0079)	(.0081)
Gamma3 (item 1: peer effort)	-0.0135	-.0199*	-.0175*
	(0.0104)	(.0101)	(.0104)
Gamma3 (item 2: peer ability)	-.0087	-.0037	-.0031
	(0.0104)	(.0101)	(.0103)
Own ability step 2	-	.3325***	.3345***
		(.0696)	(.0719)
Own ability step 3	-	.5912***	.6239***
		(.0740)	(.0772)
N	1297	1297	1225
F	4.93	17.69	5.94
Adj R^2	.0090	.0605	.0850
Panel B: Self-reported relative effort, 1 to 3 scale - Sample 2	(1)	(2)	(3)
Peer average effort	0.1472	.1140	-
	(0.1045)	(.0957)	
Peer average ability	-0.0237	-.0230	-
	(0.1069)	(.0981)	
Peer average effort*peer average ability	-		.0265
			(.0250)
Own ability step 2	-	.6046***	.6013***
		(.0858)	(.0858)
Own ability step 3	-	.9337***	.9334***
		(.1006)	(.1006)
N	442	442	442
F	1.12	22.38	29.73
Adj R^2	.0005	.1624	.1635
Panel C: Self-reported relative effort, 1 to 3 scale - Sample 2	(1)	(2)	(3)
Peer average effort	0.0289	.0377	-
	(0.1086)	(.1000)	
Peer average ability	-0.1117	-.0793	-
	(0.1139)	(.1052)	
Peer average effort*peer average ability	-		-.0061
			(.0273)
Own ability step 2	-	.6259***	.6250***
		(.0904)	(.0903)
Own ability step 3	-	.9330***	.9356***
		(.1064)	(.1062)
N	440	440	440
F	1.54	6.45	6.85
Adj R^2	.0262	.1751	.1761

Note: Panel A displays results from regressions run on Sample 1 (the two online surveys); Panels B and C display results from regressions estimated on data from Sample 2 (the in-class survey). ‘Own ability step 2’ is a dummy set to 1 if the respondent self-reported as being ‘about as smart/capable’ as other students, and 0 otherwise; ‘Own ability step 3’ is a dummy set to 1 if the respondent self-reported as being ‘more’ or ‘much more’ smart/capable than others, and 0 otherwise. Actual peer effort and ability in Panels B and C are constructed from the 1-to-3 scaled measures of self-reported effort and ability. The final column of Panel A includes controls for year and quarter of birth, sex, international student status, time of day and day of week of classes, and institution. Panel C includes analogous controls plus course fixed effects; Panel B does not. See text for more detail.

Table 3: Summary statistics: In-class survey

	Mean	Standard deviation	Minimum	Maximum
Average expected grade	72.26	(8.13)	50	100
Peer average expected grade	72.20	(4.47)	57.5	85

Notes: Statistics are calculated across all useable respondents to the second, in-class survey: N=419 for own expected grade, and N=417 for peers' expected grades.

Table 4: Predicting expected grades

Dep. Var.: Average expected grade						
	(1)	(2)	(3)	(4)	(5)	(6)
Peer avg expected grade	.1763**	.1840**	.1806**	.1886**	-.1906*	-.1885*
	(.0884)	(.0867)	(.0863)	(.0852)	(.0984)	(.0978)
Own ability * Own effort	-	-	-	1.0472***	-	.9468***
				(.1825)		(.1797)
Own ability step 2	-	2.7034**	1.7748	-	2.4025**	-
		(1.1576)	(1.1825)		(1.1582)	
Own ability step 3	-	6.1623***	4.8158***	-	5.5171***	-
		(1.3488)	(1.4148)		(1.3985)	
Own effort step 2	-	-	2.8563***	-	1.6855	-
			(1.0686)		(1.0302)	
Own effort step 3	-	-	3.9939***	-	2.6426**	-
			(1.2937)		(1.2482)	
N	418	418	418	418	417	417
F	3.98	8.90	7.49	18.61	5.18	6.19
Adj R^2	.0071	.0538	.0722	.0779	.1940	.1966

Notes: Effort and ability are measured as self-reported relative quantities, either in step form (for the main effects) or in 1-to-3 scaled form (for the interaction of own ability and own effort). Columns (5) and (6) include additional controls for course, year and quarter of birth, sex, international student status, time of day and day of week of classes.