Applied Financial Economics, 2011, **21**, 1463–1478

Time variation of CAPM betas across market volatility regimes

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We investigate time variation in Captial Asset Pricing Model (CAPM) betas for Book-to-Market (B/M) and momentum portfolios across stock market volatility regimes. For our analysis, we jointly model market and portfolio returns using a two-state Markov-switching process, with beta and the market risk premium allowed to vary between 'low' and 'high' volatility regimes. Our empirical findings suggest strong evidence of time variation in betas across volatility regimes in almost all the cases for which the unconditional CAPM can be rejected. Although the regime-switching conditional CAPM can still be rejected in many cases, the time-varying betas help explain portfolio returns much better than the unconditional CAPM, especially when market volatility is high.

Keywords: conditional CAPM; Markov-switching model; book-to-market; momentum

JEL Classification: C32; G12

I. Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) remains a benchmark asset pricing model in the academic literature. According to the CAPM, the risk of an asset is measured by its 'beta', which is the covariance between the asset's return and the return on the market portfolio per unit of variance for the market return. A number of studies (e.g. Fama and French, 1992, 1993, 1996) have examined the CAPM with constant betas (i.e. the unconditional CAPM) and reported that the model performs poorly and is unable to explain certain asset pricing anomalies. In particular, they find that the unconditional CAPM cannot explain why (i) portfolios of small firms outperform those of large firms (the 'size' effect), (ii) portfolios of firms with high Book-to-Market (B/M) ratios outperform those for firms with low B/M ratios (the 'B/M' effect) and (iii) portfolios of firms with relatively high returns in the past year outperform those for firms with relatively low past returns (the 'Momentum' effect).

One of the explanations for the failure of the CAPM is its assumption that beta for a given portfolio and the market risk premium are both constant over time. Many papers report that betas are time varying (e.g. Jagannathan and Wang, 1996; Fama and French, 1997, 2006; Lettau and Ludvigson, 2001; Lewellen and Nagel, 2006; Ang and Chen, 2007). Jagannathan and Wang (1996) show that 'alpha' from test regressions for the unconditional CAPM, where 'alpha' corresponds to the expected excess return for the portfolio over what

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Applied Financial Economics ISSN 0960–3107 print/ISSN 1466–4305 online © 2011 Taylor & Francis
http://www.informaworld.com
DOI: 10.1080/09603107.2011.577010

would be predicted by the unconditional CAPM, is theoretically related to the covariance between a time-varying beta and a time-varying market risk premium. Many studies find that capturing this covariance can help explain the size and B/M anomalies, although it is harder to explain the momentum effect.

Many of the previous studies on the conditional CAPM (e.g. Shanken, 1990; Clare *et al*., 1996; Jagannathan and Wang, 1996; Ferson and Harvey, 1999; Lettau and Ludvigson, 2001; Petkova and Zhang, 2005; Avramov and Chordia, 2006) use instrumental variables to proxy time-variation in CAPM betas and market risk premium and to specify the covariance between them. However, Harvey (2001) shows that the results of the time variation in betas can be highly sensitive to the choice of instrumental variables. Also, Lewellen and Nagel (2006) argue that tests of the conditional CAPM based on cross-sectional regressions do not impose important theoretical restrictions in the estimation of the covariance between beta and the market risk premium. Thus, it can be useful to check the robustness of past findings on the conditional CAPM to consideration of alternative approaches to modelling time variation in beta and to testing the CAPM.

Our study contributes to the conditional CAPM literature by (i) investigating time variation in betas for the B/M and momentum portfolios across states of the economy corresponding to discrete changes in the level of stock market volatility and the market risk premium, and (ii) studying the performance of the conditional CAPM within these different states of the economy. In contrast to the traditional approach used in the pervious literature on the conditional CAPM, we do not use instrumental variables to capture time variation in betas. Instead, for our analysis, we use a Markov-switching specification to estimate discrete changes in betas.

Our consideration of discrete changes in CAPM betas is motivated by numerous previous studies that find large discrete changes in the level of stock market volatility. For example, Hamilton and Susmel (1994) find that the persistent low frequency changes in volatility can be captured by a discrete Markov-switching process that appears somewhat related to discrete changes in business cycle phases between periods of expansion and recession.[1] Thus, if low frequency changes in volatility are abrupt and priced by market participants, one might expect the market risk premium to change in a discrete way too. Then, according to the idea of the conditional CAPM, if any changes in beta are correlated with the discrete changes in market volatility, they could explain the empirical failure of the unconditional CAPM.

For our analysis, we follow Turner *et al*. (1989) and Kim *et al*. (2004) by assuming that (i) stock market volatility follows a two-state Markov-switching process, with the market risk premium varying across these 'low' and 'high' volatility regimes and (ii) the processing of information about the prevailing volatility regime generates a volatility feedback effect that needs to be accounted for in order to reveal a positive underlying relationship between market volatility and the market risk premium. According to the idea of volatility feedback, an exogenous and persistent increase in the volatility of market news leads to additional return volatility as stock prices adjust in response to higher future expected returns.[2] We then jointly model the market return and the conditional CAPM, with time variation in beta driven by the market volatility regimes.

Our approach to testing the conditional CAPM has three benefits over the traditional approaches taken in the literature. First, we do not have to find exogenous observable variables to identify time variation in the market risk premium, thus helping us to avoid any data mining concerns with an instrumental variables approach. Second, the timing of changes in beta, which correspond to changes in the market risk premium, is determined directly by the returns data through the Markov-switching specification, rather than being imposed exogenously. For example, this has a benefit over a rolling window approach, which will naturally smooth out discrete changes in beta and the results for which will depend highly on the choice of window length. Third, our model is based on time-series regressions, in contrast to cross-sectional models traditionally used in previous literature, therefore it is not subject to the limitations of the cross-sectional approach discussed in Lettau and Ludvigson (2001).

[1] Schwert (1989), Chu and Tang Liu (1996), Schaller and van Norden (1997), Assoe (1998), Kim *et al*. (1998, 2001, 2004), Hess (2003) and Mayfield (2004), among many others, have modeled monthly stock return volatility using a Markov-switching specification, with high volatility regimes typically corresponding to periods of recession and low volatility regimes typically corresponding to periods of expansion. Perez-Quiros and Timmermann (2000) and Guidolin and Timmermann (2008) also find evidence of discrete changes in stock return risk across business cycle phases. In an instrumental variables setting, Avramov and Chordia (2006) also show that betas are significantly correlated with the business cycle.

[2] French *et al*. (1987), Turner *et al*. (1989), Campbell and Hentschell (1992) and Kim *et al*. (2004), among many others, account for volatility feedback to study the relationship between stock returns and volatility.

Our modelling approach is conceptually close to that of Ang and Chen (2007) and Adrian and Franzoni (2009), who study the conditional CAPM by modelling time variation in betas as stationary latent variables in order to address the limitations of the approach based on instrumental variables. Our approach differs from these papers by modelling large and discrete changes in betas rather than assuming smooth and continuous changes. Also, our study is similar to Huang (2000), who considers a Markov-switching beta for a single stock, but he does not relate it to market volatility regimes or business cycle phases and does not study the asset pricing anomalies.

Consistent with the basic idea of the conditional CAPM, our empirical findings suggest strong evidence of time variation in betas across market volatility regimes in almost all the cases for which the unconditional CAPM can be rejected. For 'value' portfolios of stocks for firms, which have relatively high B/M ratios, and 'winner' portfolios of stocks, which have relatively strong returns over the previous year, the regimes alternate between periods of low market volatility/high beta and periods of high market volatility/low beta. For 'loser' portfolios of stocks, which have relatively weak returns over the previous year, the regimes alternate between periods of low market volatility/low beta and periods of high market volatility/high beta. Although the regime-switching conditional CAPM can still be rejected in many cases, the time-varying betas help explain portfolio returns much better than the unconditional CAPM, especially when market volatility is high. Our findings are consistent with many previous studies, including Ang and Kristensen (2010), who report strong time variation in betas for the 'value', B/M strategy, and momentum strategy portfolios using a nonparametric method. However, it is useful to confirm the robustness of past findings by considering a very different approach to modelling time-varying betas than considered in previous studies using instrumental variables, smooth and continuous latent variables or nonparametric methods. At the same time, the reasonably good performance of the CAPM in periods of high volatility is an important new result that the regime-switching version of the conditional CAPM allows us to uncover from the data.

The rest of this article is organized as follows. Section II presents the model. Section III describes the data and reports the empirical results. Section IV concludes this article.

II. Model

According to the Sharpe–Lintner CAPM, the expected excess return on a portfolio of assets over a risk-free rate depends on a simple measure of the portfolio's risk relative to the market portfolio:

$$E[r_{i,t}] = \beta_i E[r_{m,t}] \tag{1}$$

where $r_{i,t}$ is the return for portfolio i in excess of the risk-free return, $r_{m,t}$ the market return in excess of the risk-free return and β_i the measure of the portfolio's risk, defined as

$$\beta_i = \frac{\mathrm{cov}(r_{i,t}, r_{m,t})}{\mathrm{var}(r_m)} \tag{2}$$

Fama and French (1992) examine the performance of the unconditional CAPM and find that estimated betas do not explain variation in average returns across different portfolios. A possible explanation for this failure of the CAPM is its assumption that the market risk premium and beta are both constant over time.[3] Relaxing this assumption we get the conditional CAPM, which holds period by period

$$E[r_{i,t}|I_{t-1} = \beta_{i,t-1} E[r_{m,t}|I_{t-1}] \tag{3}$$

where I_{t-1} denotes information available to market participants in the previous period and $\beta_{i,t-1}$ denotes beta conditional on information I_{t-1}. In this model, market participants price assets for the period t conditional on information available in the previous period. Following Jagannathan and Wang (1996) and applying iterated expectations on both sides of Equation 3, we get

$$E[r_{i,t}] = \bar{\beta}_i E[r_{m,t}] + \mathrm{cov}(\beta_{i,t-1}, E[r_{m,t}|I_{t-1}]) \tag{4}$$

where $\bar{\beta}_i$ is the unconditional expectation of beta. If beta is constant, which means that the covariance term is zero, then this equation is equivalent to Equation 1 describing the unconditional CAPM. However, it is straightforward to see that the unconditional CAPM would fail if beta were correlated with the market risk premium.

[3] Another explanation for the failure of the CAPM has to do with the basic structure of the model. For example, Zou (2006) introduces an alternative definition of beta and shows that doing so theoretically produces a smaller alpha than the standard version of the CAPM. Raei and Mohammadi (2008) consider fractional returns and show that their model has higher R^2 than the standard model. However, the interpretation of parameters for these alternative approaches is somewhat different than it is in the traditional CAPM, so it can be difficult to compare them to the standard approach.

The conditional CAPM requires specifying the information available to market participants when they form conditional expectations of the market risk premium and beta. In our analysis, we assume that market participants know that risk changes discretely, distinguishing only between 'good' and 'bad' states of the economy related to market volatility. It should be noted that, in principle, we could consider more than two states of the economy. However, as we show in the empirical analysis, a model with two regimes is sufficient to capture persistent changes in the volatility of market excess returns for the sample period under consideration.

Following many studies, including Turner *et al.* (1989) and Kim *et al.* (2004), we model states of the economy with a two-state Markov-switching variance for the market excess return

$$\varepsilon_{m,t} \sim N(0, \sigma_{m^2, S_{m,t}}), \tag{5}$$

$$\sigma^2_{m, s_{m,t}} = \sigma^2_{m,0}(1 \sim S_{m,t}) + \sigma^2_{m,1} S_{m,t} \tag{6}$$

$$\sigma^2_{m,0} < \sigma^2_{m,1} \tag{7}$$

$$\Pr\left[S_{m,t} = 0 | S_{m,t-1} = 0\right] = q_m \quad \text{and} \quad \Pr\left[S_{m,t} = 1 | S_{m,t-1} = 1\right] = p_m \tag{8}$$

where $\varepsilon_{m,t}$ denotes the market news at time t, $\sigma^2_{S_{m,t}}$ the variance of $\varepsilon_{m,t}$, $S_{m,t}$ a Markov-switching state variable that takes value 0 in the low volatility regime and 1 in the high volatility regime and q_m and p_m the continuation probabilities for the regimes. The regimes are assumed to be given exogenously and are observable to market participants. However, it should be noted that the regimes are not directly observable by econometricians, who instead need to make inferences about the regimes based on the dynamics of observable returns data. The normalization constraint in Equation 7 ensures that estimated regimes are correctly labelled as 'low' and 'high' volatility regimes.

Given the two-state specification for market volatility, one possible informational assumption for the conditional CAPM is that market participants immediately observe the market volatility regime. Under this assumption, the period-by-period market risk premium can be expressed as

$$E[r_{m,t} | S_{m,t}] = \mu_{m,0} + \mu_{m,1} S_{m,t} \tag{9}$$

where $\mu_{m,0}$ denotes the market risk premium in the low volatility regime and $\mu_{m,1}$ determines the marginal effect of the high volatility regime on the market risk premium. However, consistent with past findings, we find a negative estimate for $\mu_{m,1}$ in our empirical analysis. This result runs contrary to the theoretical positive relationship between risk and return, suggesting that, although market participants may react to information inherent in the true volatility regimes, it would be more realistic to assume that they take time to process information about the prevailing volatility regime.

In order to capture the processing of information, we follow Campbell and Hentschell (1992) and Kim *et al.* (2004), among many others, by allowing for volatility feedback in our model. According to the idea of volatility feedback, an exogenous and persistent increase in the volatility of market news generates additional return volatility as stock prices adjust in response to higher future expected returns. We follow Kim *et al.* (2004) and consider a Markov-switching model of the market excess return with volatility feedback, which is specified as

$$r_{m,t} = E[r_{m,t} | S_{m,t-1}] + f_{m,t} + \varepsilon_{m,t} \tag{10}$$

where

$$E[r_{m,t} | S_{m,t-1}] = \mu_{m,0} + \mu_{m,1} \Pr[S_{m,t} = 1 | S_{m,t-1}] \tag{11}$$

$$f_{m,t} = \delta\{S_{m,t} - \Pr[S_{m,t} = 1 | S_{m,t-1}]\} \tag{12}$$

The $f_{m,t}$ term captures an unpredictable volatility feedback effect on the market return due to period-by-period revisions in future expected returns, where $E\,[f_{m,t} | S_{m,t-1}] = 0$. The δ coefficient in the volatility feedback term is related to the other model parameters based on a discounted sum of log-linear future expected returns, as shown in Kim *et al.* (2004). Specifically, the coefficient is equal to $\delta = \frac{-\mu_{m,1}}{1-\rho\lambda}$, where $\lambda = p_m + q_m - 1$ and ρ denotes the parameter of linearization for the log-linear present value model, which is the average ratio of the stock price to the sum of the stock price and the dividend and, in practice, has the value of 0.997, as reported in Kim *et al.* (2004). In this specification, it is assumed that market participants observe the previous volatility regime $S_{m,t-1}$ at the beginning of the current period, but learn about the current volatility regime $S_{m,t}$ during the current period. This specification helps to reveal a positive relationship between risk and return. See Kim *et al.* (2004) for further details.

Similar to the market excess return, we assume that the portfolio excess return is specified as

$$r_{i,t} = E[r_{i,t} | S_{m,t-1}] + f_{i,t} + \varepsilon_{i,t} \tag{13}$$

where $E[r_{i,t} | S_{m,t-1}]$ is defined by the conditional CAPM, $f_{i,t}$ is the volatility feedback term for the portfolio return and $\varepsilon_{i,t}$ the news about portfolio i. Because the conditional CAPM time-varying beta may covary with the time-varying market risk premium, which in our setting takes on two discrete values conditional on the market volatility regimes, we allow for different values of beta in these two regimes.[4] Thus, the regime-switching conditional CAPM is given by

$$E[r_{i,t}|S_{m,t-1}] = \beta_{i,S_{m,t-1}}, E[r_{m,t}|S_{m,t-1}] \tag{14}$$

where $\beta_{i,s_{m,t-1}}$ takes on two values depending on the market volatility regime at period $t-1$.[5] Also, substituting for $r_{i,t}$ and $r_{m,t}$ into Equation 14 based on Equations 13 and 10, we can show that

$$E[f_{i,t}|S_{m,t-1}] = \beta_{i,S_{m,t-1}} E[f_{m,t}|S_{m,t-1}] = 0$$

which is consistent with the CAPM notion that the expected excess return for a portfolio depends only on its beta and the market risk premium.

Based on Equations 10 and 14, our joint model of market and portfolio excess returns is given as follows:

$$\begin{aligned} r_{m,t} = \mu_{m,0} + \mu_{m,1} \Pr[S_{m,t} = 1|S_{m,t-1}] \\ + \delta\{S_{m,t} - \Pr[S_{m,t} = 1|S_{m,t-1}]\} + \varepsilon_{m,t} \end{aligned} \tag{15}$$

$$\begin{aligned} r_{i,t} = \alpha_{i,S_{m,t-1}} + \beta_{i,S_{m,t-1}} r_{m,t} + u_t \\ \varepsilon_{m,t} \sim N(0, \sigma^2_{m,S_{m,t}}) \quad \text{and} \quad u_t \sim N(0, \sigma^2_{i,S_{i,t}}) \end{aligned} \tag{16}$$

where u_t denotes idiosyncratic news for portfolio i, which according to the CAPM should be uncorrelated with market news. In this model, the regime-switching process for market volatility and the alpha and beta for portfolio i is driven by a common unobservable state variable $S_{m,t}$ that takes on discrete values of 0 in the low market volatility regime and 1 in the high market volatility regime. If the conditional CAPM holds, $\alpha_{i,S_{m,t-1}} = 0$ in both regimes. In addition to regime-switching market volatility, we also control for heteroscedasticity in the residual for the portfolio return by assuming that the variance $\sigma^2_{i,S_{i,t}}$ of idiosyncratic news u_t follows a two-state Markov-switching process that is assumed to be independent of the process for market volatility. Estimates are based on maximum likelihood estimation following the procedure developed by Hamilton (1989).

III. Empirical Results

Data

We consider monthly data for stock returns on value-weighted decile portfolios of all stocks listed on the NewYork Stock Exchange (NYSE), AMEX and NASDAQ sorted separately by B/M ratios (B/M portfolios) and by the previous year's returns (momentum portfolios).[6] The B/M portfolios are constructed at the end of June each year based on the ratio of the book equity of stocks for the previous fiscal year to their market capitalization in December of the previous year. The portfolios are formed annually by sorting stocks using decile breakpoints of B/M ratios for the NYSE stocks only. Momentum portfolios are constructed each month using the previous 11-month-return decile breakpoints for NYSE stocks. The portfolio returns are value-weighted monthly average returns on the stocks in deciles. We define the 'market' return by considering the return on a value-weighted portfolio of all stocks listed on the NYSE, AMEX and NASDAQ. All returns are continuously compounded in excess of the continuously compounded one-month Treasury bill rate and expressed in percentage terms. Most previous empirical studies only consider data for July 1963 and afterwards in order to focus on a period for which the unconditional CAPM fails to explain B/M and momentum effects (e.g. Ang and Chen (2007) find that the unconditional CAPM cannot be rejected for B/M portfolios over the longer sample period of 1926 to 2001). Therefore, we consider the sample period of July 1963 to December 2010 in our analysis. For this sample period, we do not observe a strong size effect for portfolio returns double-sorted by size and B/M ratios, which is a common way of sorting portfolios in the literature, so we consider the overall B/M sorting.[7]

Table 1 reports summary statistics for the returns on B/M and momentum portfolios and estimates for

[4] An alternative approach would be to assume that beta has its own Markov-switching process. However, from the theory of the conditional CAPM, the relevant issue for the failure of the unconditional CAPM is whether beta covaries with the market risk premium, which in this case is driven by market volatility. Therefore, for simplicity, we consider a specification with common regimes for beta and market volatility.

[5] The beta used to price a portfolio depends on expectations of $S_{m,t}$. Thus, the beta will depend on the sensitivity of the portfolio to market news in both regimes, with the weights on the two regimes depending on the continuation probabilities for the Markov-switching state variable. Analytically, given fixed continuation probabilities, this assumption is equivalent to specifying beta to be a function of $S_{m,t-1}$, as this will capture the weighted-average sensitivity for the portfolio conditional on $S_{m,t-1}$.

[6] We are grateful to Kenneth French for making these data available at his data library at dhttp://mba.tuck.dartmouth.edu/pages/faculty/ken.french/DataLibrary/. Detailed description of portfolio formation is provided in Fama and French (2006).

[7] By considering overall B/M-sorted portfolios, we are following Ang and Chen (2007). Although the average returns for the largest size portfolios are always smaller than average returns for other size portfolios, the average returns for size portfolios other than largest size portfolio are not always decreasing with size. Also, preliminary analysis, not reported to conserve space, suggests that the unconditional CAPM cannot be rejected for the size-sorted portfolios for the sample period under consideration.

Table 1. Summary statistics for B/M and momentum portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
Panel A: B/M portfolios											
$\bar{r}_i$	0.21	0.35	0.38	0.38	0.38	0.46	0.52	0.56	0.63	0.72	0.51
SD	(5.24)	(4.82)	(4.74)	(4.86)	(4.55)	(4.59)	(4.45)	(4.65)	(4.85)	(5.91)	(4.64)
α_i	−0.15	0.00	0.04	0.04	0.07	0.14	**0.23**	**0.26**	**0.31**	**0.36**	**0.50**
SE	(0.10)	(0.06)	(0.07)	(0.09)	(0.09)	(0.08)	(0.11)	(0.12)	(0.11)	(0.16)	(0.20)
β_i	1.06	1.01	0.98	0.99	0.91	0.93	0.86	0.89	0.92	1.07	0.00
SE	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.05)	(0.04)	(0.06)	(0.13)
Panel B: Momentum portfolios											
$\bar{r}_i$	−0.58	0.05	0.25	0.30	0.24	0.31	0.36	0.55	0.57	0.90	1.48
SD	(8.06)	(6.29)	(5.37)	(4.86)	(4.53)	(4.62)	(4.48)	(4.55)	(4.98)	(6.35)	(6.86)
α_i	**−1.07**	**−0.35**	−0.10	−0.02	−0.07	−0.01	0.06	**0.24**	**0.23**	**0.50**	**1.57**
SE	(0.20)	(0.14)	(0.12)	(0.11)	(0.08)	(0.06)	(0.08)	(0.07)	(0.09)	(0.14)	(0.28)
β_i	1.44	1.18	1.00	0.95	0.91	0.94	0.90	0.92	0.99	1.18	−0.26
SE	(0.09)	(0.07)	(0.05)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)

Notes: Data are for the value-weighted portfolios sorted into deciles of B/M ratios and the previous 12-month returns for the sample period of July 1963 to December 2010. HML denotes a 'High minus Low' portfolio; $\bar{r}_i$ the average excess return for portfolio i. Sample SD for excess returns are reported in parentheses. Estimates of α_i and β_i for the unconditional CAPM regression model are based on Ordinary Least Squares (OLS). Newey and West (1987) heteroscedasticity and autocorrelation consistent SEs are reported in parentheses for α and β. Statistically signicant alphas at the 5% level are in bold.

the unconditional CAPM regression model. The results suggest a pattern of increasing average returns with increasing B/M ratios and momentum. Based on the estimated alphas, the unconditional CAPM can be rejected for the last four deciles of the B/M portfolios and for the first two and last three deciles of the momentum portfolios.

Regime-switching volatility and the estimated market risk premium

In order to determine the under of regimes that is sufficient to capture changes in market volatility, we estimate and compare the two-state and three-state Markov-switching models of market excess returns. Incorporation of volatility feedback in the three-state model is fairly complicated. Thus, for simplicity, we consider models without volatility feedback, although, as we discuss below, this has little impact on the estimated regimes for the two-state model. At first glance, it appears that the fit of the three-state model is better, with a log likelihood of −1623.85 versus −1635.72 for the two-state model.[8] However, we find that the estimated regimes for the three-state model suggest that the highest volatility regime is merely capturing a few extreme negative outliers in the data, rather than persistent changes in volatility. This result is consistent with the finding of Hamilton and Susmel (1994) that 'extremely large shocks, such as the October 1987 crash, arise from quite different causes and have different consequences for subsequent volatility than do small shocks'.

As an alternative way to address negative outliers, we modify the model given in Equation 15 and consider the two-state model with a dummy variable to capture the most volatile negative spikes in market returns. The modified model is described as follows:

$$\begin{aligned} r_{m,t} = \mu_{m,0} + \mu_{m,1}\Pr[S_{m,t}=1|S_{m,t-1}] \\ + \delta\{S_{m,t} - \Pr[S_{m,t}=1|S_{m,t-1}]\} + \gamma D_t + \varepsilon_{m,t} \end{aligned} \tag{17}$$

where D_t denotes the dummy variable that is equal to zero for most observations and equal to one for selected negative outliers and γ captures the additional volatility for these outlier observations. We rank the negative outlier observations by their magnitude and find that a model with only the two largest negative outliers in the sample period – October 1987 and October 2008 – produces a much higher likelihood value than the three-state model, even though the three-state model has many more parameters. Thus, we argue that the two-state model with the dummy variable provides a better description of the data than a three-state model. Meanwhile, it is important to emphasize that our estimates of parameters and regimes are extremely robust to including the dummy variable or not. Indeed, our

[8] Note, however, that it is difficult to evaluate the significance of the third volatility regime based on these likelihood values due to violations of standard testing conditions for Markov-switching models.

residual diagnostic checks suggest that a two-state process without the dummy variable is generally sufficient to capture ARCH effects and non-Normalities in the data. However, in the remainder of this article, we focus on results for the model with the dummy variable because it appears to provide the best description of the data. Meanwhile, it should be noted that the dummy variable is introduced only into the market return equation, while the CAPM equation remains the same as in Equation 16, ensuring that alpha and beta estimates will still depend on all of the sample data, including October 1987 and October 2008.

In terms of the volatility regimes for the two-state model, the estimates of the continuation probabilities suggest that both regimes are very persistent, with 95% and 96% month-to-month probabilities of remaining in the low and high volatility regimes, respectively. From Fig. 1, which displays the smoothed probabilities of the high volatility regime over the sample period, we observe that the periods of high stock market volatility include all of the National Bureau of Economic Research (NBER) recessions, suggesting a link between market volatility and the state of the economy.

Table 2 reports estimates for regime-switching volatility and market risk premium based on the specifications of the market return given in Equations 15 and 17. In this case, the model is estimated separately from consideration of portfolio returns and we consider both a restricted version of the model without volatility feedback (i.e. $\delta = 0$) and a version that allows for volatility feedback. The model without volatility feedback has a negative estimated market risk premium in the high volatility regime, as discussed in the previous section. Notably, whether or not the market risk premium is actually negative, its estimate is significantly lower in the high volatility regime than in the low volatility regime. This result does not accord with a basic theoretical positive relationship between risk and return.[9] However, after accounting for volatility feedback, the estimates are consistent with a positive relationship. Meanwhile, a Likelihood Ratio (LR) test rejects the restricted model without volatility feedback with a p-value of < 0.001 based on an asymptotic $\chi^2(1)$ distribution, suggesting that volatility feedback is an important feature of stock returns. This is true whether or not the dummy variable is included in the model, suggesting that the feedback term is not merely

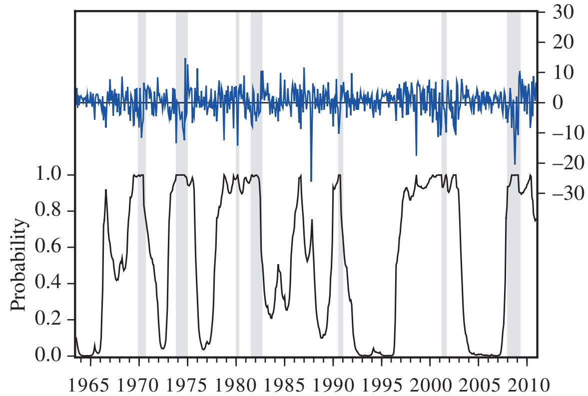

Fig. 1. Monthly stock market excess returns and smoothed probabilities of the high volatility regime

Notes: Returns are continuously compounded monthly value-weighted returns for all stocks listed on the NYSE, AMEX and NASDAQ in excess of continuously compounded one-month Treasury bill yields for the sample period of July 1963 to December 2010. Shaded areas correspond to NBER recessions.

proxying for large negative outliers. In addition to the significant feedback term, the estimates for market volatility are quite different across the two regimes. Thus, taken together, these results provide evidence of significant time variation in the market risk premium related to changes in market volatility.

Despite the differences in estimates of the market risk premium for the two models with and without volatility feedback, the estimates related to volatility are quite similar. Also, smoothed probabilities of the volatility regimes for both models are similar, with a correlation of 0.95, suggesting that the regimes are mainly identified by changes in variance rather than by changes in the mean of excess returns. However, given the significance of the feedback parameter, we focus on models with volatility feedback in the remainder of this article.

Regime-switching betas for B/M portfolios

Do the betas for the B/M portfolios vary across market volatility regimes? To test for a regime-switching beta for a given portfolio, we consider another LR test. In this case, the LR test statistic is constructed based on the likelihood for a restricted version of the joint model of portfolio and market returns described by Equations 16 and 17 in which alpha is allowed to be regime switching and beta is assumed to be constant across volatility regimes

[9] Breen *et al*. (1989), Campbell (1987), Nelson (1991) and Glosten *et al*. (1993), among many others, find a negative relationship between market volatility and the market risk premium. Glosten *et al*. (1993) argue that market participants may not require a larger risk premium in more risky periods because they may need to save relatively more for a future that may be even riskier.

Table 2. Parameter estimates for regime-switching volatility and market risk premium

Model	$\mu_{m,0}$	$\mu_{m,1}$	δ	γ	$\sigma_{m,0}$	$\sigma_{m,1}$	q_m	p_m	$\log L$
Model without	1.00	−1.90			3.15	6.24	0.95	0.92	−1635.72
volatility feedback	(0.22)	(−1.05)			(0.24)	(0.55)	(0.02)	(0.07)	
Model with	0.30	0.80	−7.00		2.96	5.80	0.96	0.93	−1630.69
volatility feedback	(0.20)	(0.32)	(1.46)		(0.23)	(0.46)	(0.02)	(0.05)	
Model with volatility	0.27	0.60	−6.81	−24.89	2.59	5.07	0.95	0.96	−1614.08
feedback and dummy	(0.20)	(0.24)	(1.13)	(3.15)	(0.28)	(0.28)	(0.02)	(0.05)	

Notes: The model for the market return is described by Equations 15 and 17, where $\delta = 0$ for the version of the model without volatility feedback. The model with volatility feedback and dummy includes the dummy parameter, denoted γ. The SE for the volatility feedback parameter estimate was obtained using the Delta method. $\log L$ denotes the log likelihood.

Table 3. Likelihood ratio tests for regime-switching betas and residual diagnostics for B/M portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
LR stat.	1.19	16.72	18.24	0.05	9.91	0.12	0.84	2.69	6.75	7.52	5.52
p-value	(0.28)	(0.00)	(0.00)	(0.83)	(0.00)	(0.73)	(0.36)	(0.10)	(0.01)	(0.01)	(0.02)
Residual diagnostic tests: portfolio return with constant beta and variance											
ARCH-LM	3.37	22.71	44.89	95.30	50.49	9.74	28.40	65.40	12.64	7.98	5.17
p-value	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)
JB stat.	5.42	13.68	403.27	494.45	266.36	186.01	289.70	549.59	168.51	310.39	62.74
p-value	(0.07)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Residual diagnostic tests: portfolio return with regime-switching beta and variance											
ARCH-LM	0.03	0.77	0.41	0.83	0.73	2.08	3.63	4.25	0.94	0.33	0.58
p-value	(0.85)	(0.38)	(0.52)	(0.36)	(0.39)	(0.15)	(0.06)	(0.04)	(0.33)	(0.57)	(0.45)
JB stat.	0.72	3.73	3.66	2.57	3.29	0.41	0.62	2.26	1.35	8.50	1.44
p-value	(0.70)	(0.16)	(0.16)	(0.28)	(0.19)	(0.81)	(0.74)	(0.32)	(0.51)	(0.01)	(0.49)

Notes: To test for a regime-switching β, we use likelihood ratio (LR) test statistics constructed based on the likelihood for the joint model of market and portfolio returns with regime-switching α and constant β (null) and the likelihood for the model with regime-switching α and β (alternative). The residual diagnostic tests are conducted for the residuals in the portfolio return equation of the joint model. The ARCH-LM statistics are constructed using R^2 from an auxiliary regression of squared standardized residuals on their lag and have a $\chi^2(1)$ asymptotic distribution under the null of no ARCH effects. The JB – Jarque and Bera (1980) test statistics of Normality of residuals have a $\chi^2(2)$ asymptotic distribution under the null of Normality. HML denotes a 'High minus Low' portfolio.

relative to the likelihood for a less restrictive version of the model in which both alpha and beta are allowed to be regime switching. Because both models have Markov-switching market volatility under the null hypothesis, they are nested without nuisance parameters and the LR statistic should have an asymptotic $\chi^2(1)$ distribution. The test results, reported in Table 3, strongly support regime-switching betas for five out of ten B/M portfolios. Amongst the B/M portfolios for which the unconditional CAPM is rejected, the LR tests support regime-switching betas at the 1% level for the ninth and tenth decile portfolios, the 5% level for the 'High minus Low' portfolio, and just shy of the 10% level for the eighth decile portfolio. The LR tests also support regime-switching betas for three of the B/M portfolios for which the unconditional CAPM cannot be rejected. It should be noted that the fact that the LR tests cannot reject a constant beta for the first, fourth, sixth, and seventh decile portfolios only suggests that the betas for these portfolios do not have large changes over the market volatility regimes, but they may still be time varying. However, importantly for the conditional CAPM, they appear not to be time varying in a way that corresponds to changes in the market risk premium.

The residual diagnostics, also reported in Table 3, suggest that, after accounting for time variation in beta and a regime-switching variance, there are no remaining significant ARCH effects in the portfolio residuals and, for the most of the B/M portfolios, conditional Normality of returns cannot be rejected based on the Jarque and Bera (1980) test. These results are consistent with Hamilton and

Table 4. Estimates for the regime-switching model of market and portfolio returns for BM portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
Panel A: Regime-switching alphas											
$\alpha_{i,0}$	−0.20	**−0.17**	−0.11	0.01	0.03	0.20	**0.30**	**0.30**	**0.27**	0.25	0.32
SE	(0.11)	(0.07)	(0.06)	(0.01)	(0.19)	(0.11)	(0.10)	(0.11)	(0.13)	(0.18)	(0.33)
$\alpha_{i,1}$	0.07	0.23	0.23	−0.01	0.06	0.04	−0.15	0.13	0.22	0.21	0.33
SE	(0.16)	(0.12)	(0.13)	(0.02)	(0.11)	(0.13)	(0.19)	(0.12)	(0.13)	(0.23)	(0.25)
Panel B: Regime-switching betas											
$\beta_{i,0}$	1.02	1.12	1.10	1.02	1.03	0.97	0.90	0.96	1.12	1.29	0.41
SE	(0.04)	(0.02)	(0.02)	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)	(0.05)	(0.08)	(0.11)
$\beta_{i,1}$	1.07	0.95	0.93	1.03	0.90	0.95	0.96	0.86	0.89	0.86	−0.17
SE	(0.03)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.03)	(0.05)	(0.06)
Panel C: Other parameters											
$\mu_{m,0}$	0.22	0.33	0.44	0.27	0.25	0.28	0.33	0.29	0.34	0.33	0.27
SE	(0.20)	(0.20)	(0.18)	(0.21)	(0.20)	(0.20)	(0.21)	(0.19)	(0.18)	(0.19)	(0.20)
$\mu_{m,1}$	0.69	0.80	0.55	0.59	0.63	0.57	0.67	0.57	0.39	0.53	0.54
SE	(0.26)	(0.30)	(0.23)	(0.25)	(0.21)	(0.23)	(0.28)	(0.22)	(0.16)	(0.22)	(0.19)
δ	−6.75	−5.71	−7.04	−6.81	−6.98	−6.96	−6.80	−6.78	−6.95	−6.13	−6.39
SE	(1.09)	(1.33)	(1.56)	(1.15)	(1.24)	(1.12)	(1.46)	(1.14)	(1.35)	(1.17)	(1.07)
γ	−25.06	−25.44	−25.27	−24.81	−25.55	−24.84	−25.30	−23.98	−23.90	−23.72	−24.00
SE	(2.98)	(2.70)	(2.89)	(3.23)	(2.37)	(3.21)	(2.81)	(3.70)	(3.61)	(3.84)	(3.58)
$\sigma_{m,0}$	2.58	3.02	3.32	2.58	2.60	2.58	2.91	2.71	2.86	2.87	2.51
SE	(0.21)	(0.19)	(0.17)	(0.31)	(0.23)	(0.27)	(0.23)	(0.25)	(0.17)	(0.20)	(0.23)
$\sigma_{m,1}$	5.06	5.53	5.63	5.06	5.03	5.06	5.32	5.12	5.06	5.17	4.92
SE	(0.25)	(0.36)	(0.36)	(0.30)	(0.26)	(0.26)	(0.32)	(0.27)	(0.23)	(0.29)	(0.22)
$\sigma_{i,0}$	1.15	1.19	1.16	1.23	1.30	1.33	1.39	1.53	1.71	2.38	3.48
SE	(0.13)	(0.06)	(0.04)	(0.06)	(0.09)	(0.06)	(0.08)	(0.07)	(0.12)	(0.10)	(0.15)
$\sigma_{i,1}$	2.21	2.33	3.12	3.55	3.09	3.14	3.66	4.51	3.94	6.12	7.01
SE	(0.11)	(0.39)	(0.39)	(0.38)	(0.39)	(0.30)	(0.34)	(0.46)	(0.53)	(0.71)	(0.56)

Notes: Data are for value-weighted B/M decile portfolios for the sample period of July 1963 to December 2010. HML denotes a 'High minus Low' portfolio. Panels A and B report alphas and betas from the regime-switching model of portfolio and market returns described by Equations 16 and 17. Statistically significant alphas at the 5% level are in bold face.

Susmel (1994). They find that most of the ARCH effects in weekly stock returns die out at the monthly horizon and the remaining volatility changes that persist over longer period of time can be captured by a two-state Markov-switching process. For comparison, the residuals for the unconditional CAPM regression model exhibit strong ARCH effects and Normality is strongly rejected.

For each of the B/M portfolios, Table 4 reports estimates for the regime-switching model of portfolio and market returns described by Equations 16 and 17. The estimates of the betas for the two portfolios with the highest B/M ratios (i.e. 'value' portfolios) vary considerably across the two market volatility regimes; in particular, the betas for these portfolios in the low volatility regime are higher than in the high volatility regime. This result appears contrary to some theoretical models (e.g. Zhang, 2005) that suggest betas for value portfolios should be higher during bad times when marginal utility is high than in good times. However, our findings are similar to Ang and Kristensen (2010), who find using nonparametric estimates that betas for value portfolios are higher during bad times than in good times. Our findings are also consistent with Lakonishok *et al.* (1994), who report that betas for value portfolios are higher (lower) than betas for growth portfolios (i.e. portfolios with low B/M ratios) in good times (bad times). They explain the B/M anomaly by 'contrarian' investment behaviour, whereby certain market participants overinvest in stocks that are 'underpriced' and underinvest in stock that are 'overpriced'. By contrast, Petkova and Zhang (2005) find a positive relationship between betas for value portfolios and the market risk premium. However, as discussed by Ang and Kristensen (2010), this result is presumably driven by the specification of both beta and the market risk premium as linear functions of the same instrumental variables. Meanwhile, our finding that the dispersion of betas for B/M portfolios is considerably higher in the high volatility regime than in the low volatility regime is consistent with the theoretical findings in Gomes *et al.* (2003), who show that the dispersion of conditional betas should be countercyclical to the business cycle.

Table 5. Long-run expected alphas for B/M portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
Panel A: Alphas for the unconditional CAPM regression model											
α_i	−0.15	0.00	0.04	0.04	0.07	0.14	**0.23**	**0.26**	**0.31**	**0.36**	**0.50**
SE	(0.10)	(0.06)	(0.07)	(0.09)	(0.09)	(0.08)	(0.11)	(0.12)	(0.11)	(0.16)	(0.20)
Panel B: Long-run expected alphas for the regime-switching model											
α_i	−0.06	−0.03	−0.01	0.00	0.04	0.11	0.12	0.21	0.24	0.23	0.33
SE	(0.09)	(0.06)	(0.06)	(0.02)	(0.11)	(0.08)	(0.09)	(0.07)	(0.09)	(0.13)	(0.17)

Notes: Panel A repeats estimates of α from the unconditional CAPM regression model, also reported in Table 1, for comparison purposes. HML denotes a 'High minus Low' portfolio. Panel B reports estimates of long-run expected alphas for the regime-switching model of portfolio and market returns described by Equations 16 and 17. The long-run expected alpha for each portfolio is constructed as the weighted average of alphas in the two market volatility regimes, with weights equal to the steady-state probabilities of each regime. The standard errors for these expected alphas are computed using the Delta method.

From Table 1, the unconditional CAPM can be rejected for the value portfolios, while we find that the regime-switching alphas for these portfolios are closer to zero in both regimes. Although the alphas for the second, seventh, eighth, and ninth decile portfolios remain statistically significant at the 5% level in the low volatility regime, the alphas for all decile portfolios are statistically insignificant in the high volatility regime. We note that, for the second decile portfolio, the unconditional CAPM regression model has economically and statistically insignificant alpha, while the regime-switching model has statistically significant alpha in one of the regimes. This result illustrates that, while CAPM may appear to hold unconditionally, it could still fail in some states of the economy.

Table 5 reports estimates of the long-run expected alphas for the B/M portfolios. These long-run alphas are computed as weighted-averages of alpha in the two market volatility regimes, with weights equal to the steady-state probabilities of the regimes. The estimates suggest that the values of most of the long-run alphas are closer to zero than the alphas for the unconditional CAPM regression model, although most of them are still statistically significant. To be clear, then, we do not claim that the conditional CAPM explains the entire behaviour of excess returns for the B/M portfolios; point estimates of alphas for the last three portfolios are still large. Yet, we find evidence that portfolios with high B/M return premia demonstrate strong time variation of betas in the two market volatility regimes. We also find that accounting for time variation in the betas for the B/M portfolios over different states of the economy helps to explain some of the excess returns not captured by the unconditional CAPM. For example, the long-run expected alpha for the 'High minus Low' portfolio strategy declined from 0.50 for the unconditional CAPM regression model to 0.33 for the regime-switching model.

Figure 2 illustrates the relative performance of the unconditional CAPM and the regime-switching conditional CAPM for the B/M portfolios. If the CAPM provided a useful qualitative prediction for the behaviour of returns, then one should observe points scattered along the 45° line, which corresponds to excess returns fitted by the CAPM being equal to average realized excess returns. As reported in many studies (e.g. Fama and French, 1992; Jagannathan and Wang, 1996), the unconditional CAPM performs very poorly. The unconditional CAPM predicts flat excess returns for the B/M portfolios, while the average realized excess returns vary significantly across the portfolios. The correlation coefficient between the excess returns predicted by the unconditional CAPM and average realized excess returns for the different portfolios has a value of −0.24, confirming the poor performance of the unconditional CAPM.

The performance of the regime-switching conditional CAPM for the B/M portfolios is different across the two regimes. Although there is not much visual improvement in the regime-switching conditional CAPM performance in the low volatility regime, there is an apparent improvement in the high volatility regime, where we can observe a fairly linear relationship between the CAPM-predicted excess returns and the average realized excess returns.[10]

[10] The fitted excess returns for B/M portfolios in high market volatility regime are negative because they are computed based on realized market excess returns (see details in the note to Fig. 2), which are negative. We use realized market excess returns to compute fitted excess returns because we compare them with actual realized excess returns of portfolios.

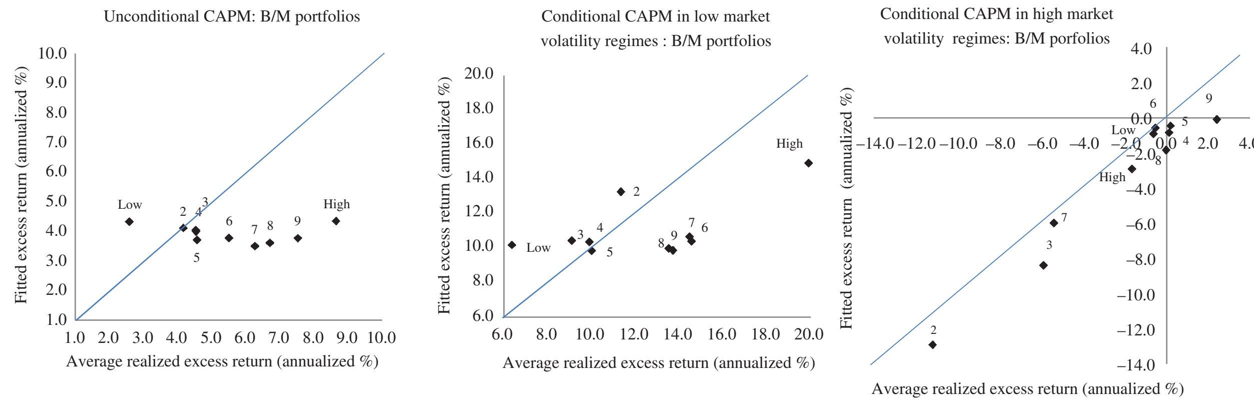

Fig. 2. CAPM fitted excess returns versus average realized excess returns for book-to-market portfolios

Notes: The returns are expressed as annualized percentages. The left scatter plt displays points with the average realized excess returns on the horizontal axis and the fitted excess returns from the unconditional CAPM on the vertical axis. The scatter plot in the middle (on right) displays points with the average realized excess returns conditional on smoothed probabilities of the high market volatility regime being lower (higher) than 0.5 on the horizontal axis and the fitted excess returns in the low (high) volatility regime from the regime-switching conditional CAPM on the vertical axis. The fitted excess return for each portfolio at low (high) market volatility regime is computed as an average of fitted excess returns calculated as a product of estimated betas in a previous period regime and realized market excess returns for observations with smoothed probabilities of high market volatility lower (higher) than 0.5. The straight lines on the graphs are 45° lines from the origins.

The correlation coefficients between the excess returns fitted by the conditional CAPM and the average realized excess returns for different B/M portfolios have values of 0.58 and 0.98 in the ‘low’ and ‘high’ market volatility regimes, respectively. This result suggests that, in the high volatility regime at least, the regime-switching conditional CAPM provides a much better qualitative prediction for excess returns on the B/M portfolios than provided by the unconditional CAPM.

Although there is some variation in the evidence for regime-switching betas across different B/M portfolios, the estimates of parameters related to the market return when considered jointly with the different B/M portfolios are in the same range as those for a model of the market return with regimes identified using only market volatility and not jointly estimated with portfolio betas. The correlation coefficients between smoothed probabilities of the high volatility regime for the market-only model and the joint market/CAPM model for the different deciles of the B/M portfolios range from 0.75 to 0.99. This finding suggests that the regimes are mainly identified by changes in market volatility rather than by changes in the betas. Figure 3 displays portfolio excess returns and smoothed probabilities of the high market volatility regime for the first, fifth and tenth B/M decile portfolios. Consistent with the regimes being identified by changes in market volatility, the smoothed probabilities appear quite similar to those in Fig. 1 and to each other across the different portfolios.

Regime-switching betas for momentum portfolios

For the analysis of the momentum portfolios, we proceed as before with the B/M portfolios. The LR tests for the null hypothesis of a constant beta, the results for which are reported in Table 6, support regime-switching betas at the 5% level for all but the fifth and sixth decile portfolios. Indeed, the tests are significant at the 1% level in the majority of cases. Thus, there is strong evidence for regime-switching betas for all of the momentum portfolios for which the unconditional CAPM can be rejected, as well as for some of the portfolios for which it cannot be rejected.

The residual diagnostic tests, also reported in Table 6, suggest that, for most of the momentum portfolios, there are no remaining ARCH effects in the portfolio residuals. The Normality of the residuals cannot be rejected for the majority of the portfolios based on the Jarque and Bera (1980) test and the test statistics for other portfolios declined considerably relative to those for the unconditional CAPM regression model. Again, the residuals for the unconditional CAPM regression model exhibit strong ARCH effects and their Normality is strongly rejected.

Table 7 reports estimates for the regime-switching model of the market and portfolio returns for each of the momentum portfolios. The estimates of the betas for most of the momentum portfolios vary considerably across the two volatility regimes. Betas for the four portfolios of stocks, which have relatively strong

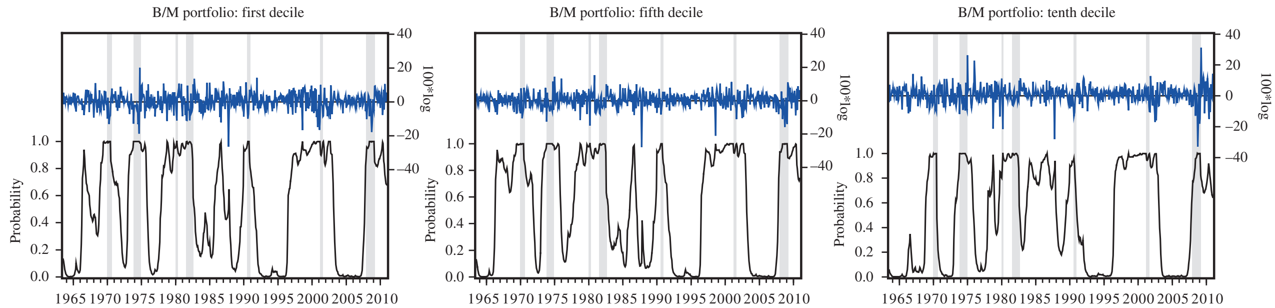

Fig. 3. Monthly returns for selected B/M portfolios and smoothed probabilities of the high market volatility regime
Notes: Returns are continuously compounded monthly value-weighted returns for B/M portfolios in excess of continuously compounded one-month Treasury bill yields for the sample period of July 1963 to December 2010. Shaded areas correspond to NBER recessions.

Table 6. Likelihood ratio tests for regime-switching betas and residual diagnostics for momentum portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
LR stat.	10.81	69.62	53.60	21.08	1.33	3.36	6.73	15.56	73.28	41.50	28.13
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.07)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)
Residual diagnostic tests: portfolio return with constant beta and variance											
ARCH-LM	84.97	41.05	39.35	64.33	7.49	44.86	28.22	28.77	34.81	19.45	84.78
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
JB stat.	722.12	541.32	914.26	484.24	2266.40	612.24	1038.02	100.31	340.77	165.60	575.03
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Residual diagnostic tests: portfolio return with regime-switching beta and variance											
ARCH-LM	0.70	6.56	0.31	0.03	0.06	0.22	0.91	1.01	0.91	0.39	0.37
p-value	(0.40)	(0.01)	(0.58)	(0.86)	(0.80)	(0.64)	(0.34)	(0.31)	(0.34)	(0.53)	(0.54)
JB stat.	18.60	13.15	3.43	1.26	3.70	1.22	1.99	0.39	7.82	8.95	17.38
p-value	(0.00)	(0.00)	(0.18)	(0.53)	(0.16)	(0.54)	(0.37)	(0.82)	(0.02)	(0.01)	(0.00)

Notes: To test for a regime-switching β, we use LR test statistics constructed based on the likelihood for the joint model of market and portfolio returns with regime-switching α and constant β (null) and the likelihood for the model with regime-switching α and β (alternative). The residual diagnostic tests are conducted for the residuals in the portfolio return equation of the joint model. The ARCH-LM statistics are constructed using R^2 from an auxiliary regression of squared standardized residuals on their lag and have a $\chi^2(1)$ asymptotic distribution under the null of no ARCH effects. The JB – Jarque and Bera (1980) test statistics of Normality of residuals have a $\chi^2(2)$ asymptotic distribution under the null of Normality. HML denotes a 'High minus Low' portfolio.

returns in the previous year (the 'winner' portfolios), are higher in the low volatility regime than in the high volatility regime. By contrast, betas for the four portfolios of stocks, which have relatively weak returns in the previous year (the 'loser' portfolios), are lower in the low volatility regime than in the high volatility regime.

Table 8 reports estimates of the long-run expected alphas for the momentum portfolios. Compared to the B/M portfolios, we observe smaller differences from the unconditional CAPM alphas. For example, the long-run expected alpha for the 'High minus Low' portfolio strategy declined only from 1.57 for the unconditional CAPM regression model to 1.49 for the regime-switching model. Therefore, we do not argue that the regime-switching conditional CAPM explains the failure of the unconditional CAPM for the momentum portfolios.[11] However, as shown in Fig. 4, allowing for changes in beta still helps the CAPM in terms of its qualitative predictions. In particular, similar to Fig. 2 for the B/M portfolios, Fig. 4 illustrates the relative performance of the unconditional CAPM and the regime-switching conditional CAPM for the momentum portfolios. As with the B/M portfolios, the unconditional

[11] Jegadeesh and Titman (1993) and Fama and French (1996) suggest that the momentum anomalies may reflect a short-run underreaction to news. To the extent this is the case, we should not expect the conditional CAPM to explain alpha for the momentum portfolios.

Table 7. Estimates for the regime-switching model of market and portfolio returns for momentum portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
Panel A: Regime-switching alphas											
$\alpha_{i,0}$	−0.34	−0.14	−0.12	−0.11	−0.05	0.00	−0.11	**0.24**	**0.18**	**0.56**	**1.44**
SE	(0.18)	(0.10)	(0.08)	(0.07)	(0.08)	(0.06)	(0.11)	(0.07)	(0.09)	(0.14)	(0.27)
$\alpha_{i,1}$	**−1.94**	**−0.61**	0.05	0.16	−0.13	−0.03	0.14	0.10	**0.29**	0.30	**1.64**
SE	(0.23)	(0.22)	(0.13)	(0.18)	(0.12)	(0.07)	(0.12)	(0.18)	(0.13)	(0.26)	(0.76)
Panel B: Regime-switching betas											
$\beta_{i,0}$	1.11	0.92	0.85	0.86	0.87	0.87	1.04	1.02	1.20	1.35	0.24
SE	(0.05)	(0.03)	(0.03)	(0.02)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)	(0.04)	(0.09)
$\beta_{i,1}$	1.37	1.54	1.23	1.15	0.96	1.00	0.91	0.76	0.82	0.80	−0.96
SE	(0.05)	(0.04)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)	(0.04)	(0.03)	(0.06)	(0.39)
Panel C: Other parameters											
$\mu_{m,0}$	0.24	0.45	0.32	0.26	0.36	0.36	0.28	0.47	0.37	0.40	0.43
SE	(0.21)	(0.18)	(0.19)	(0.19)	(0.20)	(0.19)	(0.21)	(0.18)	(0.18)	(0.18)	(0.18)
$\mu_{m,1}$	0.65	0.70	1.00	1.03	0.58	0.35	0.77	0.58	0.54	0.86	0.72
SE	(0.23)	(0.23)	(0.33)	(0.32)	(0.24)	(0.16)	(0.26)	(0.24)	(0.19)	(0.30)	(0.25)
δ	−6.82	−5.68	−5.66	−4.79	−5.82	−7.25	−5.74	−5.36	−4.38	−5.60	−5.34
SE	(1.19)	(1.13)	(1.04)	(1.07)	(1.77)	(1.30)	(1.27)	(1.47)	(1.09)	(1.14)	(1.31)
γ	−25.53	−25.26	−25.16	−25.07	−24.05	−23.91	−25.50	−25.27	−25.34	−24.86	−25.28
SE	(2.38)	(2.96)	(3.01)	(3.07)	(3.82)	(3.50)	(2.57)	(3.00)	(2.77)	(3.35)	(3.00)
$\sigma_{m,0}$	2.66	3.42	3.35	3.41	2.92	2.39	2.81	3.47	3.19	3.47	3.47
SE	(0.18)	(0.14)	(0.17)	(0.18)	(0.26)	(0.18)	(0.33)	(0.21)	(0.18)	(0.15)	(0.18)
$\sigma_{m,1}$	5.03	5.70	5.72	5.82	5.37	4.91	5.29	5.80	5.55	5.86	5.86
SE	(0.23)	(0.37)	(0.41)	(0.44)	(0.38)	(0.20)	(0.40)	(0.39)	(0.30)	(0.42)	(0.45)
$\sigma_{i,0}$	2.29	1.44	1.37	1.28	1.20	1.14	1.28	1.10	1.44	2.00	3.47
SE	(0.11)	(0.10)	(0.06)	(0.05)	(0.05)	(0.04)	(0.05)	(0.08)	(0.06)	(0.15)	(0.46)
$\sigma_{i,1}$	7.43	3.99	4.94	4.18	3.49	3.41	4.42	2.31	4.14	4.58	9.20
SE	(0.49)	(0.24)	(0.39)	(0.36)	(0.30)	(0.29)	(0.55)	(0.19)	(0.61)	(0.39)	(0.95)

Notes: Data are for value-weighted momentum decile portfolios for the sample period of July 1963 to December 2010. HML denotes a 'High minus Low' portfolio. Panels A and B report alphas and betas from the regime-switching model of portfolio and market returns described by Equations 16 and 17. Statistically significant alphas at the 5% level are in bold face.

Table 8. Long-run expected alphas for momentum portfolios

	Low	2	3	4	5	6	7	8	9	High	HML
Panel A: Alphas for the unconditional CAPM regression model											
α_i	**−1.07**	**−0.35**	−0.10	−0.02	−0.07	−0.01	0.06	**0.24**	**0.23**	**0.50**	**1.57**
SE	(0.20)	(0.14)	(0.12)	(0.11)	(0.08)	(0.06)	(0.08)	(0.07)	(0.09)	(0.14)	(0.28)
Panel B: Long-run expected alphas for the regime-switching model											
α_i	**−1.19**	**−0.26**	−0.08	−0.05	−0.08	−0.02	0.00	**0.20**	**0.22**	**0.50**	**1.49**
SE	(0.20)	(0.10)	(0.07)	(0.07)	(0.07)	(0.04)	(0.06)	(0.07)	(0.07)	(0.12)	(0.23)

Notes: Panel A repeats estimates of α from the unconditional CAPM regression model, also reported in Table 1, for comparison purposes. HML denotes a 'High minus Low' portfolio. Panel B reports estimates of long-run expected alphas for the regime-switching model of portfolio and market returns described by Equations 16 and 17. The long-run expected alpha of each portfolio is constructed as the weighted average of alphas in the two market volatility regimes, with weights equal to the steady-state probabilities of each regime. The standard errors for these expected alphas are computed using the Delta method.

CAPM predicts nearly the same excess returns for the various momentum portfolios, while there is significant variation in average realized excess returns across the portfolios. When the two volatility regimes are considered separately, there appears to be a positive linear relation between the excess returns fitted by the conditional CAPM and the averaged realized excess returns.[12]

[12] The 1st decile portfolio appears to be an outlier from the linear relationship in the high volatility regime. However, the average returns for this portfolio are negative, while it has the highest volatility amongst all momentum portfolios. Because this portfolio comprises assets under financial stress and limited borrowing, we should probably not expect the CAPM to explain the returns for this decile.

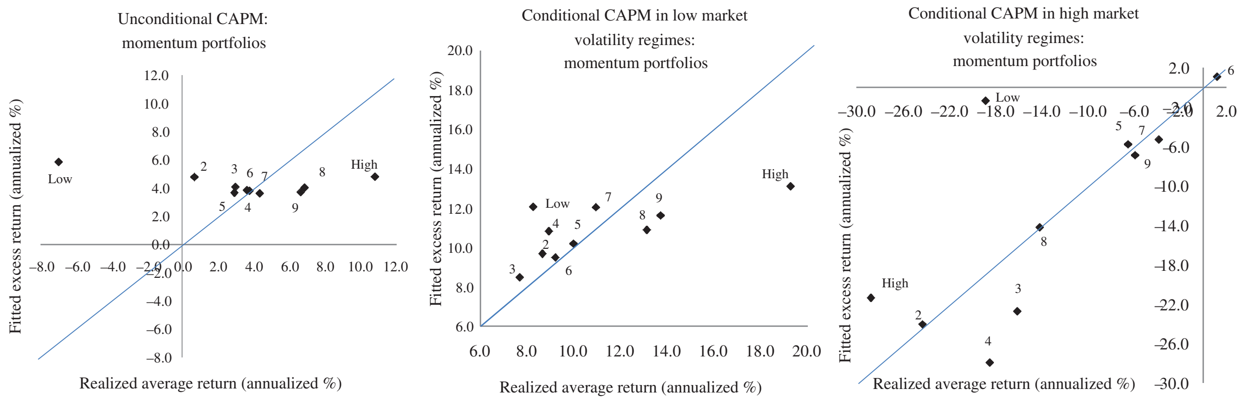

Fig. 4. CAPM fitted excess returns versus average realized excess returns for momentum portfolios

Notes: The returns are expressed as annualized percentages. The left scatter plot displays points with the average realized excess returns on the horizontal axis and the fitted excess returns from the unconditional CAPM on the vertical axis. The scatter plot in the middle (on right) displays points with the average realized excess returns conditional on smoothed probabilities of the high market volatility regime being lower (higher) than 0.5 on the horizontal axis and the fitted excess returns in the low (high) volatility regime from the regime-switching conditional CAPM on the vertical axis. The fitted excess return for each portfolio at low (high) market volatility regime is computed as an average of fitted excess returns calculated as a product of estimated betas in a previous period regime and realized market excess returns for observations with smoothed probabilities of high market volatility lower (higher) than 0.5. The straight lines on the graphs are 45° lines from the origins.

The correlation coefficient between the excess returns fitted by the unconditional CAPM and average realized excess returns for the different portfolios has a value of −0.57, confirming a similarly poor performance of the unconditional CAPM as was found for the B/M portfolios. Meanwhile, the correlation coefficients between the excess returns fitted by the conditional CAPM and the average realized excess returns for different momentum portfolios have values of 0.70 and 0.73 in the 'low' and 'high' market volatility regimes, respectively. Thus, the regime-switching conditional CAPM provides much better qualitative predictions for excess returns on the momentum portfolios than provided by the unconditional CAPM.

Given the lack of tangible improvement in the long-run expected alphas, it might seem surprising that there is such an improvement in the qualitative predictions of the conditional CAPM. This result can be explained by the fact that the alphas, while apparently not equal to zero, are responsible for only relatively small portions of the overall portfolio returns. By contrast, variation in the market return explains sizable portions of the portfolio returns, especially in the high volatility regime. In this sense, the conditional CAPM, while not strictly holding for all portfolios, appears to provide a reasonable approximation of asset pricing behaviour.

The correlation coefficients between smoothed probabilities of the high volatility regime from the market-only model and the joint market/CAPM model for different deciles of the momentum portfolios range from 0.59 to 0.94. Evidently, in some cases, changes in betas are not so strongly related to changes in market volatility. In principle, to resolve this issue, we could consider a joint model that imposes the same market volatility regimes for all momentum portfolios. However, in practice, this is not feasible since it is important to allow for heteroscedasticity in idiosyncratic news for each portfolio, which would require incorporating 2^{11} (i.e. 2048) regime processes in the joint model for all momentum portfolios. In some cases, then, the joint market/CAPM model for each momentum portfolio identifies regimes as joint market volatility/beta regimes rather than as market volatility regimes. For the 'loser' portfolios (first, second, third and fourth deciles), the joint market volatility/beta regimes are identified as low volatility/low beta and high volatility/high beta regimes. For the 'winner' portfolios (seventh, eighth, ninth and tenth deciles), the regimes are identified as low volatility/high beta and high volatility/low beta. Figure 5 displays the excess portfolio returns and smoothed probabilities of the high market volatility regime for the first, fifth and tenth momentum decile portfolios. Although changes in beta for the first momentum decile portfolio do not appear to significantly alter the identification of volatility regimes, as the smoothed probabilities are similar to those in Fig. 1, changes in beta appear to strongly affect the identification of regimes for the tenth momentum decile portfolios. This lack of correspondence may also explain why the regime-switching conditional CAPM can still be rejected for a majority of the momentum portfolios.

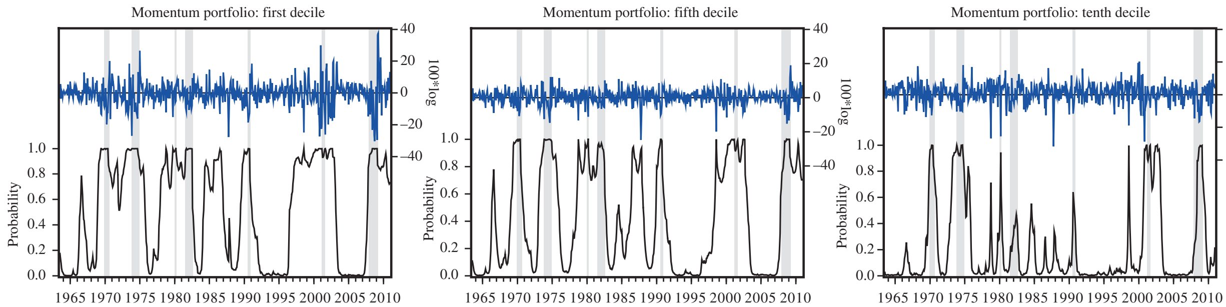

Fig. 5. Monthly returns for selected momentum portfolios and smoothed probabilities of the high market volatility regime
Notes: Returns are continuously compounded monthly value-weighted returns for momentum portfolios in excess of continuously compounded one-month Treasury bill yields for the sample period of July 1963 to December 2010. Shaded areas correspond to NBER recessions.

IV. Conclusion

In this article, we allowed for time variation in CAPM betas for B/M and momentum portfolios according to a two-state Markov-switching process driven by stock market volatility. Our empirical findings suggest strong evidence of time variation in betas across volatility regimes in almost all of the cases for which the unconditional CAPM can be rejected. Somewhat supportive of the regime-switching conditional CAPM, we found that accounting for this time variation in betas helps explain some of the portfolio excess returns that are not captured by the unconditional CAPM. Thus, although the regime-switching conditional CAPM can still be rejected in many cases, it provides much better qualitative predictions about the relationship between risk and return compared to the unconditional CAPM, especially when market volatility is high.

Acknowledgements

We thank the editor and two anonymous referees for helpful comments. The views expressed in this article are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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