CHAPTER 7

Economic Growth I: Capital Accumulation and Population Growth

MACROECONOMICS SIXTH EDITION

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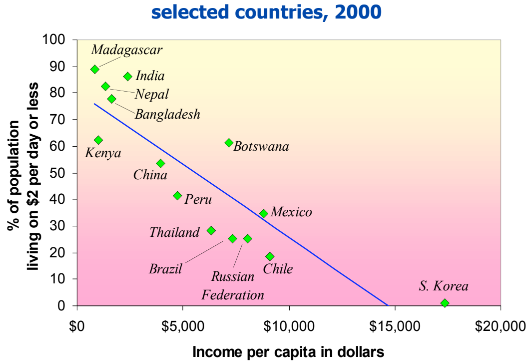
PowerPoint® Slides by Ron Cronovich

In this chapter, you will learn…

- the closed economy Solow model
- how a country's standard of living depends on its saving and population growth rates
- how to use the "Golden Rule" to find the optimal saving rate and capital stock

Income and poverty in the world

selected countries, 2000



Why growth matters

- Anything that effects the long-run rate of economic growth – even by a tiny amount – will have huge effects on living standards in the long run.

annual growth rate of income per capita	percentage increase in standard of living after…		
	…25 years	…50 years	…100 years
2.0%	64.0%	169.2%	624.5%
2.5%	85.4%	243.7%	1,081.4%

Why growth matters

- If the annual growth rate of U.S. real GDP per capita had been just one-tenth of one percent higher during the 1990s, the U.S. would have generated an additional $496 billion of income during that decade.

The Solow model

- due to Robert Solow, won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policy making
  - benchmark against which most growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run

How Solow model is different from Chapter 3's model

1. ***K*** is no longer fixed:
   investment causes it to grow,
   depreciation causes it to shrink
2. ***L*** is no longer fixed:
   population growth causes it to grow
3. the consumption function is simpler

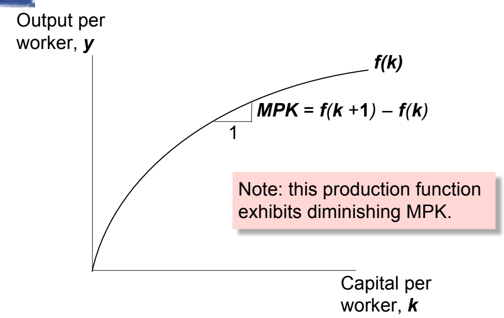
How Solow model is different from Chapter 3's model

4. no ***G*** or ***T***
   (only to simplify presentation;
   we can still do fiscal policy experiments)
5. cosmetic differences

The production function

- In aggregate terms: $Y = F(K, L)$
- Define: $y = Y/L$ = output per worker
  $k = K/L$ = capital per worker
- Assume constant returns to scale:
  $zY = F(zK, zL)$ for any $z > 0$
- Pick $z = 1/L$. Then
  $Y/L = F(K/L, 1)$
  $y = F(k, 1)$
  $y = f(k)$ where $f(k) = F(k, 1)$

The production function



The national income identity

- $Y = C + I$ (remember, no ***G***)
- In "per worker" terms:
  $y = c + i$
  where $c = C/L$ and $i = I/L$

The consumption function

- ***s*** = the saving rate,
  the fraction of income that is saved
  (***s*** is an exogenous parameter)

  Note: ***s*** is the only lowercase variable that is not equal to its uppercase version divided by ***L***
- Consumption function: $c = (1–s)y$
  (per worker)

Saving and investment

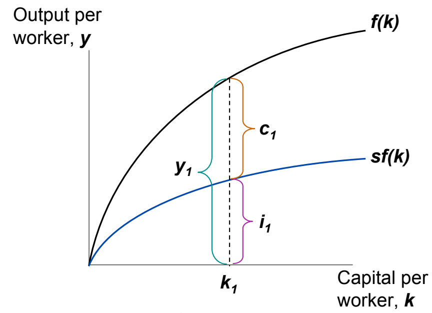
- saving (per worker) $= y - c$
  $= y - (1\!-\!s)y$
  $= sy$
- National income identity is $y = c + i$

  Rearrange to get: $i = y - c = sy$

  (investment = saving, like in chap. 3!)
- Using the results above,

  $i = sy = sf(k)$

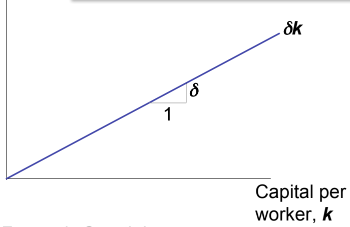
Output, consumption, and investment



Depreciation

Depreciation per worker, δk

δ = the rate of depreciation
= the fraction of the capital stock that wears out each period



Capital accumulation

The basic idea: Investment increases the capital stock, depreciation reduces it.

Change in capital stock = investment – depreciation

$\Delta k = i - \delta k$

Since $i = sf(k)$, this becomes:

$$\Delta k = sf(k) - \delta k$$

The equation of motion for *k*

$$\Delta k = sf(k) - \delta k$$

- The Solow model's central equation
- Determines behavior of capital over time…
- …which, in turn, determines behavior of all of the other endogenous variables because they all depend on ***k***. *E.g.*,

  income per person: $y = f(k)$

  consumption per person: $c = (1\!-\!s)\, f(k)$

The steady state

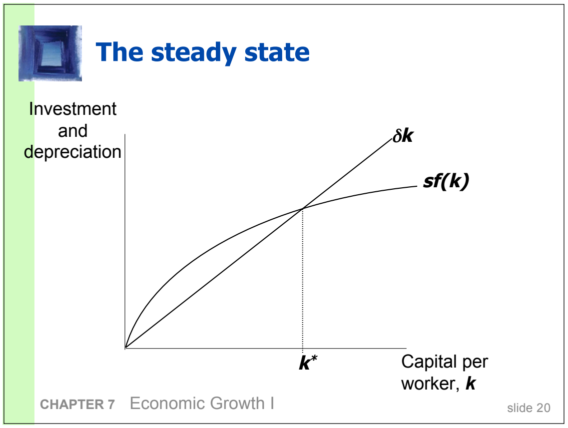
$$\Delta k = sf(k) - \delta k$$

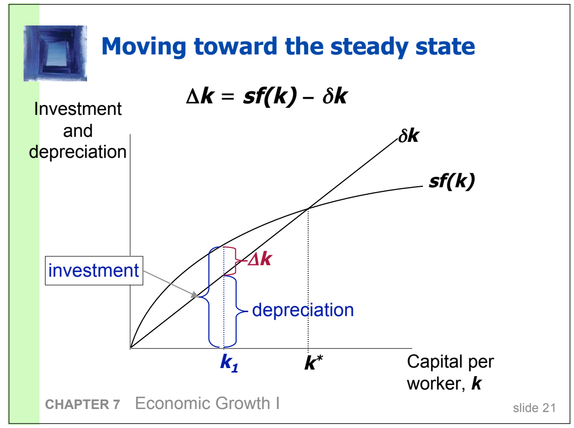
If investment is just enough to cover depreciation $[sf(k) = \delta k]$,

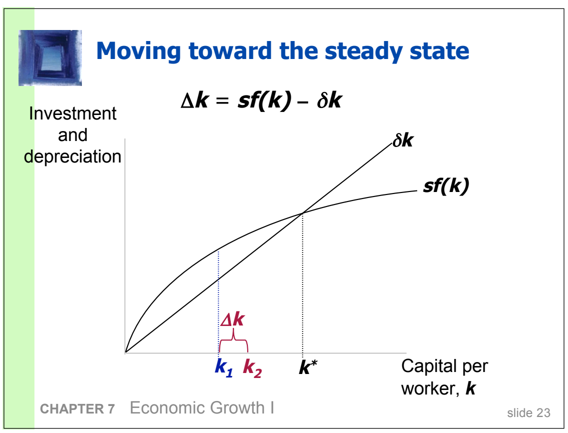
then capital per worker will remain constant:

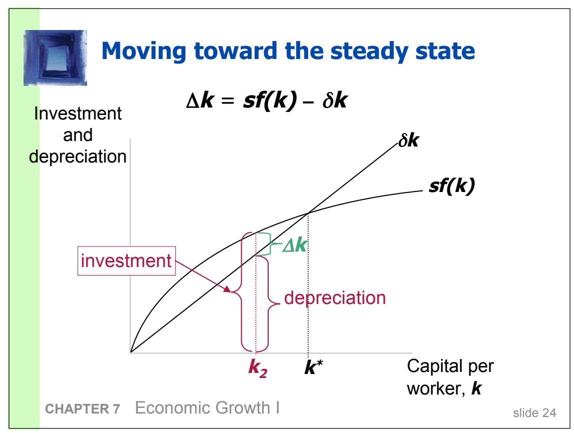
$\Delta k = 0.$

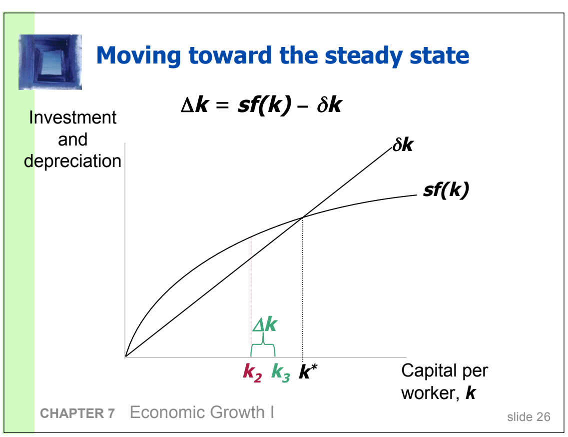
This occurs at one value of ***k***, denoted k^*, called the ***steady state capital stock***.


The steady state
Investment and depreciation
δk
sf(k)
k*
Capital per worker, k
CHAPTER 7 Economic Growth I
slide 20


Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
investment
Δk
depreciation
k1
k*
Capital per worker, k
CHAPTER 7 Economic Growth I
slide 21


Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Δk
k1 k2
k*
Capital per worker, k
CHAPTER 7 Economic Growth I
slide 23


Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
investment
Δk
depreciation
k2
k*
Capital per worker, k
CHAPTER 7 Economic Growth I
slide 24


Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Δk
k2 k3 k*
Capital per worker, k
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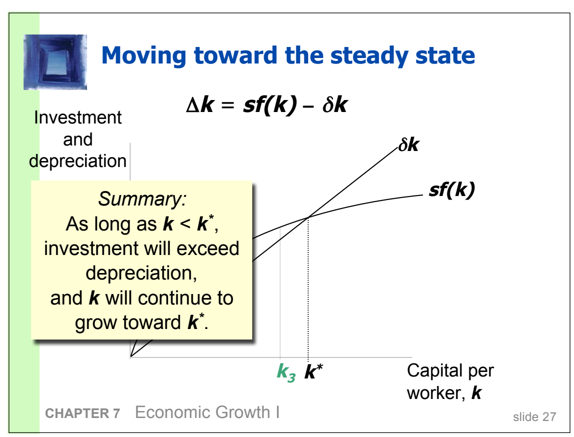
slide 26


Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Summary:
As long as k < k*,
investment will exceed
depreciation,
and k will continue to
grow toward k*.
k3 k*
Capital per worker, k
CHAPTER 7 Economic Growth I
slide 27

Now you try:

Draw the Solow model diagram, labeling the steady state k^*.

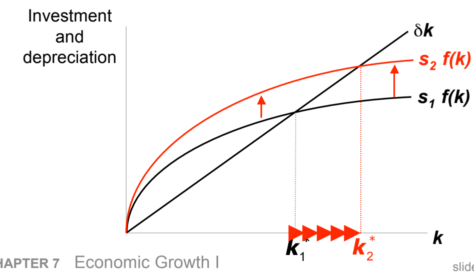
On the horizontal axis, pick a value greater than k^* for the economy's initial capital stock. Label it k_1.

Show what happens to k over time. Does k move toward the steady state or away from it?

CHAPTER 7 Economic Growth I slide 28

An increase in the saving rate

An increase in the saving rate raises investment…
…causing k to grow toward a new steady state:



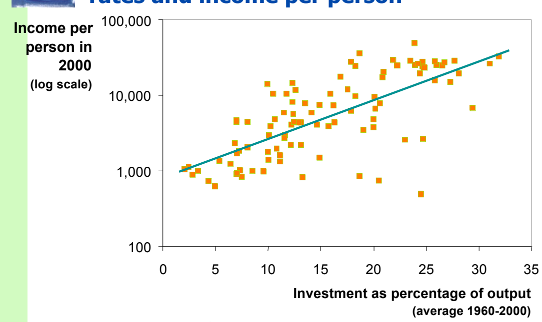
CHAPTER 7 Economic Growth I slide 34

Prediction:

- Higher s ⇒ higher k^*.
- And since $y = f(k)$, higher k^* ⇒ higher y^* .
- Thus, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.

CHAPTER 7 Economic Growth I slide 35

International evidence on investment rates and income per person



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The Golden Rule: Introduction

- Different values of s lead to different steady states. How do we know which is the "best" steady state?
- The "best" steady state has the highest possible consumption per person: $c^* = (1–s) f(k^*)$.
- An increase in s
  - leads to higher k^* and y^*, which raises c^*
  - reduces consumption's share of income (1–s), which lowers c^*.
- So, how do we find the s and k^* that maximize c^*?

CHAPTER 7 Economic Growth I slide 37

The Golden Rule capital stock

k^*_{gold} = the **Golden Rule level of capital**, the steady state value of k that maximizes consumption.

To find it, first express c^* in terms of k^*:

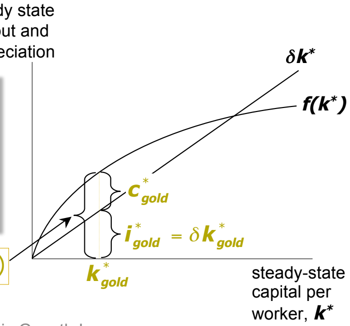
$$c^* = y^* - i^*$$
$$= f(k^*) - i^*$$
$$= f(k^*) - \delta k^*$$

In the steady state: $i^* = \delta k^*$ because $\Delta k = 0$.

CHAPTER 7 Economic Growth I slide 38

The Golden Rule capital stock

Then, graph $f(k^*)$ and δk^*, look for the point where the gap between them is biggest.

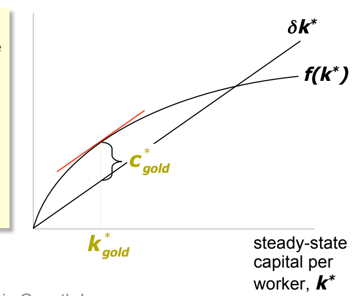


$y^*_{gold} = f(k^*_{gold})$

The Golden Rule capital stock

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production function equals the slope of the depreciation line:

MPK = δ



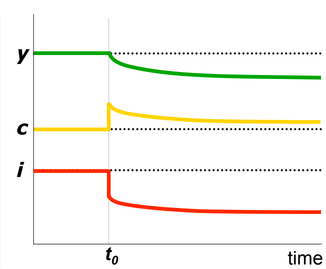
The transition to the Golden Rule steady state

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k^* > k^*_{gold}$ then increasing c^* requires a fall in ***s***.

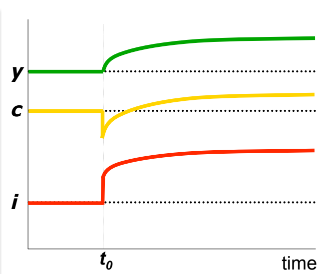
In the transition to the Golden Rule, consumption is higher at all points in time.



Starting with too little capital

If $k^* < k^*_{gold}$ then increasing c^* requires an increase in ***s***.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Population growth

- Assume that the population (and labor force) grow at rate ***n***. (***n*** is exogenous.)

$$\frac{\Delta L}{L} = n$$

- EX: Suppose ***L*** = 1,000 in year 1 and the population is growing at 2% per year (***n*** = 0.02).
- Then $\Delta L = nL = 0.02 \times 1{,}000 = 20$, so ***L*** = 1,020 in year 2.

Break-even investment

- $(\delta + n)k$ = **break-even investment**, the amount of investment necessary to keep k constant.
- Break-even investment includes:
  - δk to replace capital as it wears out
  - $n k$ to equip new workers with capital

    (Otherwise, k would fall as the existing capital stock would be spread more thinly over a larger population of workers.)

The equation of motion for k

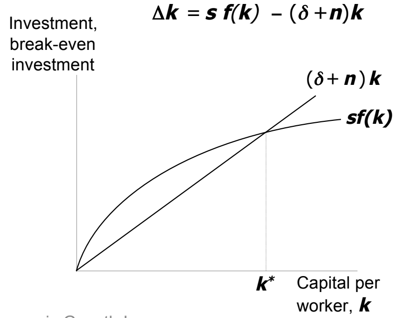
- With population growth, the equation of motion for k is

$$\Delta k = s f(k) - (\delta + n) k$$

$s f(k)$ = actual investment; $(\delta + n) k$ = break-even investment

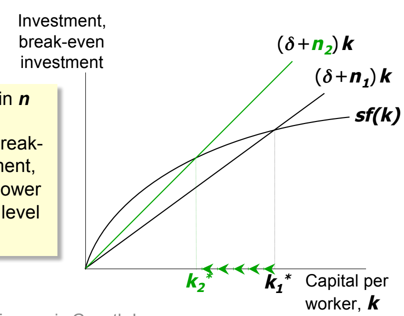
The Solow model diagram

$$\Delta k = s f(k) - (\delta + n) k$$



The impact of population growth

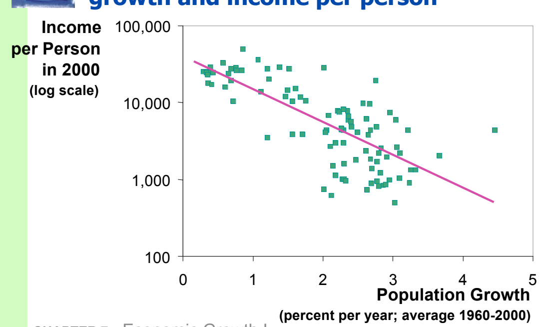
An increase in n causes an increase in break-even investment, leading to a lower steady-state level of k.



Prediction:

- Higher n ⇒ lower k^*.
- And since $y = f(k)$, lower k^* ⇒ lower y^*.
- Thus, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

International evidence on population growth and income per person



The Golden Rule with population growth

To find the Golden Rule capital stock, express c^* in terms of k^*:

$$c^* = y^* - i^*$$
$$= f(k^*) - (\delta + n)\, k^*$$

c^* is maximized when

$$\text{MPK} = \delta + n$$

or equivalently,

$$\text{MPK} - \delta = n$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Alternative perspectives on population growth

The Malthusian Model (1798)

- Predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since Malthus, world population has increased sixfold, yet living standards are higher than ever.
- Malthus omitted the effects of technological progress.

Alternative perspectives on population growth

The Kremerian Model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists & engineers, so faster technological progress.
- Evidence, from very long historical periods:
  - As world pop. growth rate increased, so did rate of growth in living standards
  - Historically, regions with larger populations have enjoyed faster growth.

Chapter Summary

1. The Solow growth model shows that, in the long run, a country's standard of living depends
   - positively on its saving rate
   - negatively on its population growth rate
2. An increase in the saving rate leads to
   - higher output in the long run
   - faster growth temporarily
   - but not faster steady state growth.

Chapter Summary

3. If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.

   If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.