Capital Asset Pricing Model

Econ 487

Outline

- CAPM Assumptions and Implications
- CAPM and the Market Model
- Testing the CAPM
- Conditional CAPM

CAPM Readings

- Zivot, Ch. 8 (pp. 185-191) (page #'s at top of page)
- Benninga, Ch. 10 (pp. 221-228)
- Perold (2004) (pp. 288-289)

What is the CAPM?

- Theory of asset price determination for firms
- Based on portfolio theory and Market Model
- The only thing that matters is Beta (co-movement with the market)
- Alternative to valuation theory for individual firms

CAPM Assumption #1

- Many investors who are all price takers
  - I.e., financial markets are competitive
  - Returns provide full summary of investment opportunities

CAPM Assumption #2

- All investors plan to invest over the same time horizon
  - Abstracts from heterogeneity in investors (i.e., risk averse have different time preferences than the risk tolerant)
  - Helps address any deviations from CER model

CAPM Assumption #3

- No distortionary taxes or transaction costs
  - Clearly a false assumption (debt vs. equity)

CAPM Assumption #4

- All investors can borrow/lend at same risk-free rate
  - Again, clearly false
  - But we can consider Zero-Beta version of CAPM with short-sales

CAPM Assumption #5

- Preferences: Investors only care about expected return (like) and variance (dislike)
  - Consistent with portfolio theory and CER model under Normality
  - False if assets have different co-movements with state of the economy (i.e., recession vs. boom)

CAPM Assumption #6

- All investors have same information and beliefs about distribution of returns
  - E.g., CER model, with same beliefs about parameters

CAPM Assumption #7

- Market portfolio that determines Beta consists of all publicly traded assets

Implication #1

- Investors will use Markowitz algorithm to determine same set of efficient portfolios

Implication #2

- Risk-averse investors will put most of their wealth in risk-free asset
- Risk-tolerant investors will put most of their wealth in risky assets
- In equilibrium, no net borrowing

  => risk-free rate and tangency portfolio

Implication #3

- Tangency portfolio = market portfolio
- Note: implies positive weights on all assets in tangency portfolio, even allowing for short sales

Implication #4

- Market portfolio is mean-variance efficient
- I.e., highest Sharpe Ratio

Implication #5

- "Security Market Line"(SML) pricing holds for all assets and portfolios
- I.e., expected return on asset *i* is fully determined by three things:

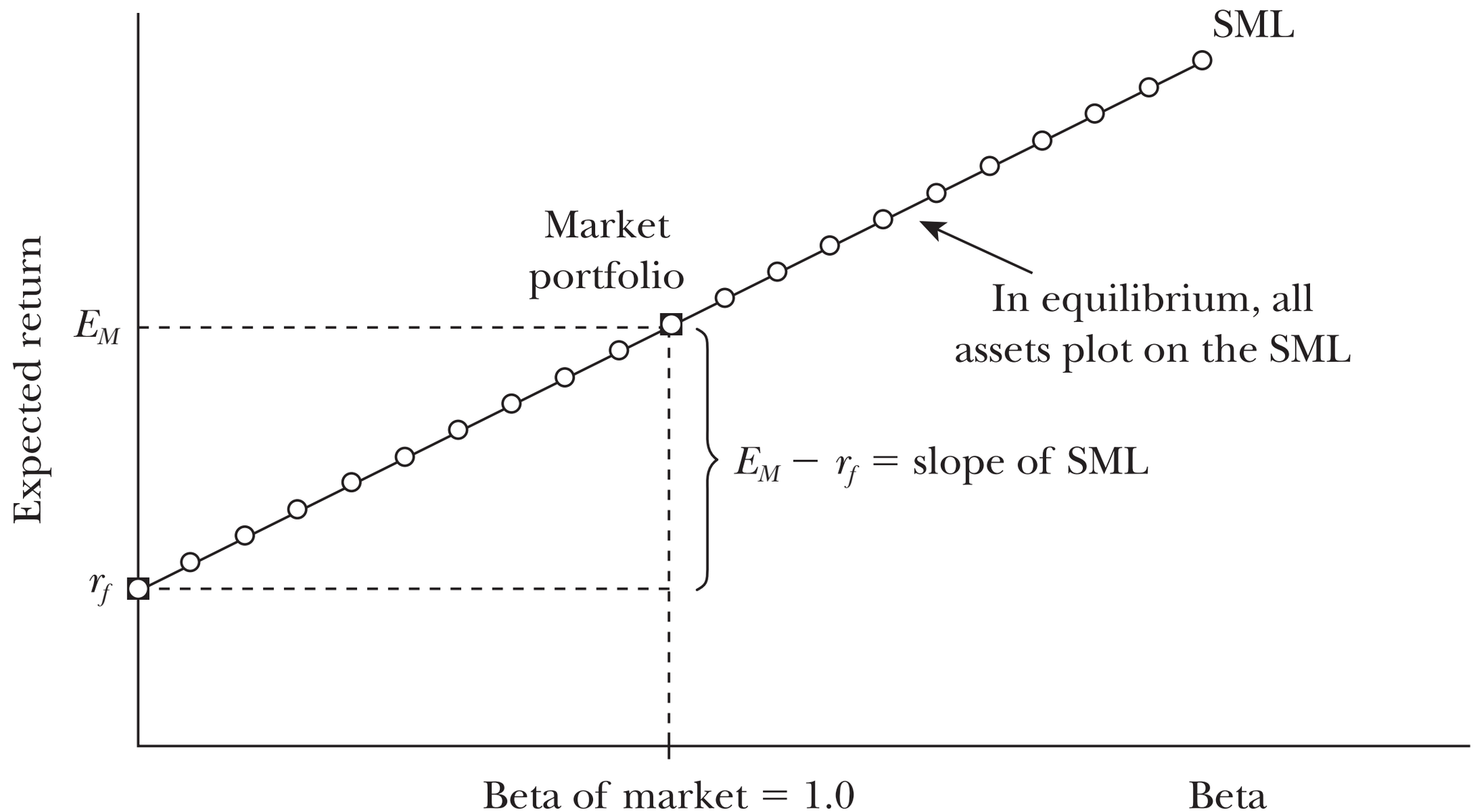
1. risk-free rate
2. market risk premium
3. Beta for asset *i*

Security Market Line (SML)

$$E[R_{it}] = r_f + \beta_{i,M}(E[R_{Mt}] - r_f)$$

where *i* refers to an individual asset or portfolio

The Securities Market Line (SML)



Intuition for CAPM

- Investors should not be compensated for diversifiable risk

E.g., Beta=0

- If expected return > risk-free, borrow at risk-free to buy zero-beta asset
- If expected return < risk-free, sell zero-beta asset short and buy more risk-free
- Both cases imply higher portfolio return without higher risk (at the margin)

Equilibrium for Beta=0

- Risk-free rate and price of zero-beta asset adjust to equate expected return and risk-free rate
- E.g., if expected return < risk-free rate, price falls today to make future expected returns higher
  - recall log-linear present-value relationship between price and expected returns

Log-Linear Present Value Relationship

$$p_t = \frac{c}{1-\gamma} + E_t\left[\sum_{j=1}^{\infty}\gamma^{j-1}(1-\gamma)d_{t+j}\right] - E_t\left[\sum_{j=1}^{\infty}\gamma^{j-1}r_{t+j}\right]$$

E.g., Beta=1

- If expected return > market, sell other assets to buy high-return asset
- If expected return < market, sell asset to buy more of market portfolio
- Both cases imply higher portfolio return without higher risk (at the margin)
- Prices adjust to bring about equilibrium

In general

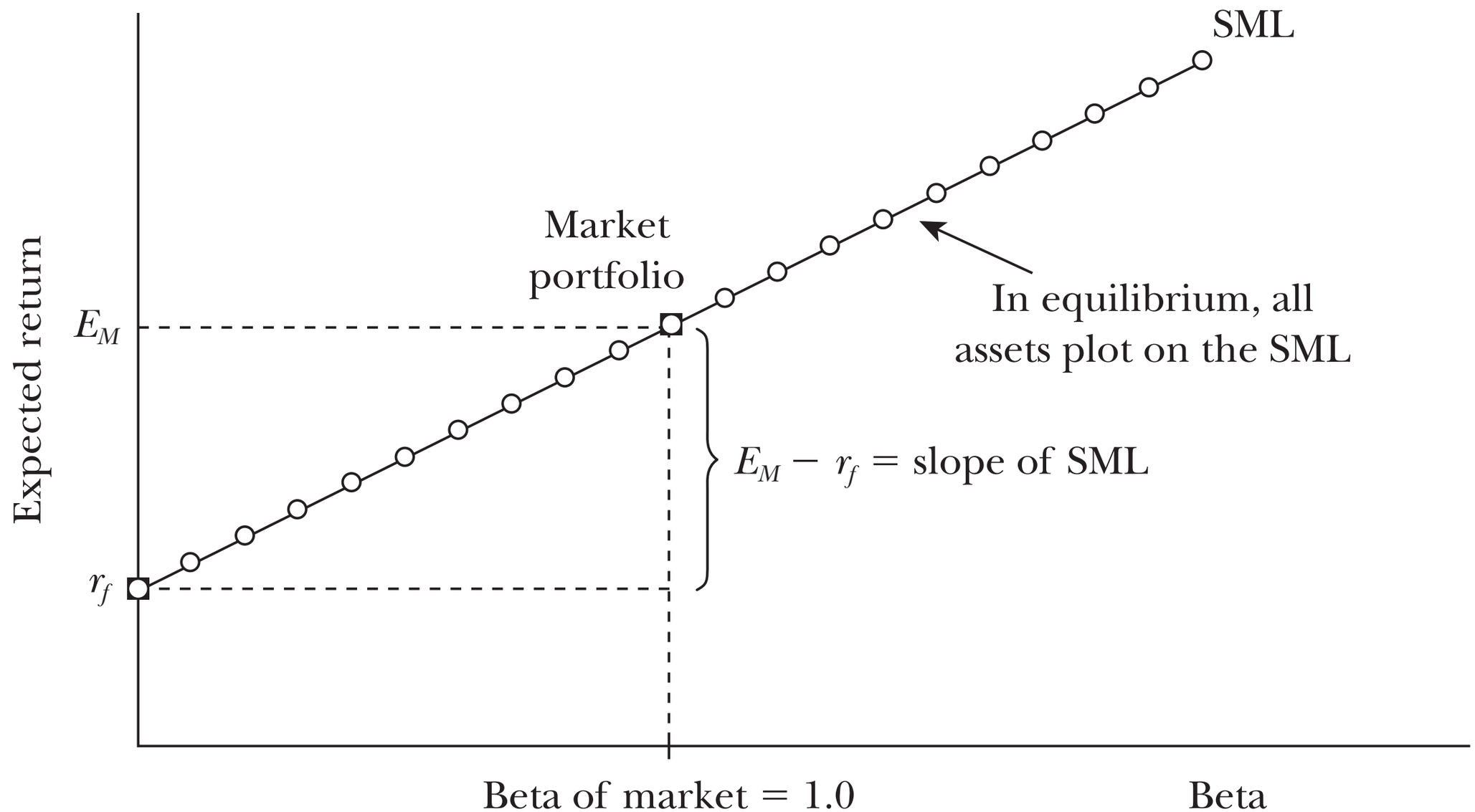
- Investors can choose mix of risk-free asset and market portfolio to achieve any desired expected return => “Beta Portfolio”
  - Weight on market portfolio is Beta in SML
- If expected return on asset *i* is different than SML, prices will adjust as investors buy/sell Beta portfolio and asset *i*.

Beta-Portfolio

$$E[R_{pt}] = (1 - x_m)r_f + x_m E[R_{Mt}]$$
$$= r_f + x_m(E[R_{Mt}] - r_f)$$

Set $x_m = \beta_{i,M}$ for a given asset *i* to compare $E[R_{pt}]$ on Beta-Portfolio to $E[R_{it}]$.

The Securities Market Line (SML)



Key Point

- Even if there is an implicit present-value model of stock price determination, there is no need to forecast future dividends for firm *i*
- Given SML, all that matters for pricing firm *i* is Beta (i.e., responsiveness to market return)
- Easier to estimate Beta than to forecast future dividends

CAPM and the Market Model

- Market Model is a statistical model
- CAPM is a theory that places parameter restrictions on Market Model
- Consider "excess return" version of Market Model

Market Model

$$R_{it} = \alpha_i + \beta_{i,M} R_{Mt} + \varepsilon_{it}$$

$$\varepsilon_{it} \sim iidN(0, \sigma_{\varepsilon i}^2)$$

$$\text{cov}(R_{Mt}, \varepsilon_{it}) = 0$$

Subtract r_f from both sides

$$R_{it} - r_f = \alpha_i - r_f + \beta_{i,M} R_{Mt} + \varepsilon_{it}$$

Add and subtract $\beta_{i,M} r_f$ from right-hand-side:

$$R_{it} - r_f = \alpha_i - (1 - \beta_{i,M}) r_f + \beta_{i,M} (R_{Mt} - r_f) + \varepsilon_{it}$$

Excess-Return Market Model

$$R_{it} - r_f = \alpha_i^* + \beta_{i,M}(R_{Mt} - r_f) + \varepsilon_{it},$$

where $\alpha_i^* = \alpha_i - (1 - \beta_{i,M})r_f$.

Take expectations of both sides and take r_f to right-hand-side:

$$E[R_{it}] = r_f + \alpha_i^* + \beta_{i,M}(E[R_{Mt}] - r_f)$$

The SML for CAPM implies $\alpha_i^* = 0$ or, equivalently, $\alpha_i = (1 - \beta_{i,M})r_f$.

Testing the CAPM

1. Test H0: $\alpha^*=0$ for excess-return Market Model (t-test for one asset or F-test for a joint test for a set of assets)
2. Check if market portfolio is efficient and equal to tangency portfolio for assets in market portfolio
3. Check predictions for expected returns based on Beta and SML

Why might tests reject CAPM in practice? (I)

- CAPM does not hold
  - restrictions on short sales
  - heterogeneous probability assessments

Why might tests reject CAPM in practice? (II)

- Maybe CAPM only holds for portfolios, not individual assets
  - liquidity issues for individual assets

Why might tests reject CAPM in practice? (III)

- Set of assets in market portfolio proxy incomplete
  - foreign assets, bonds, real estate
  - explains if market portfolio appears less efficient than tangency portfolio
  - Roll critique (only test is if true market portfolio is mean-variance efficient)

Why might tests reject CAPM in practice? (IV)

- Non-tradable assets matter
  - human capital
  - co-movement of traded assets with business cycle (multiple factor model)
    - given same Beta, prefer less correlation with labour income

Early Studies

- Black, Jensen, Scholes (72), Fama and MacBeth (73), Blume and Friend (73) support mean-variance efficiency of market portfolio
- But estimated mean return on zero-beta portfolio > risk-free rate

Anomalies Literature

- Portfolios based on firm characteristics appear to have higher ex-post Sharpe ratios than market proxy
- Portfolios emphasizing smaller size, lower P/E ratios, and higher book/market ratios appear to be better

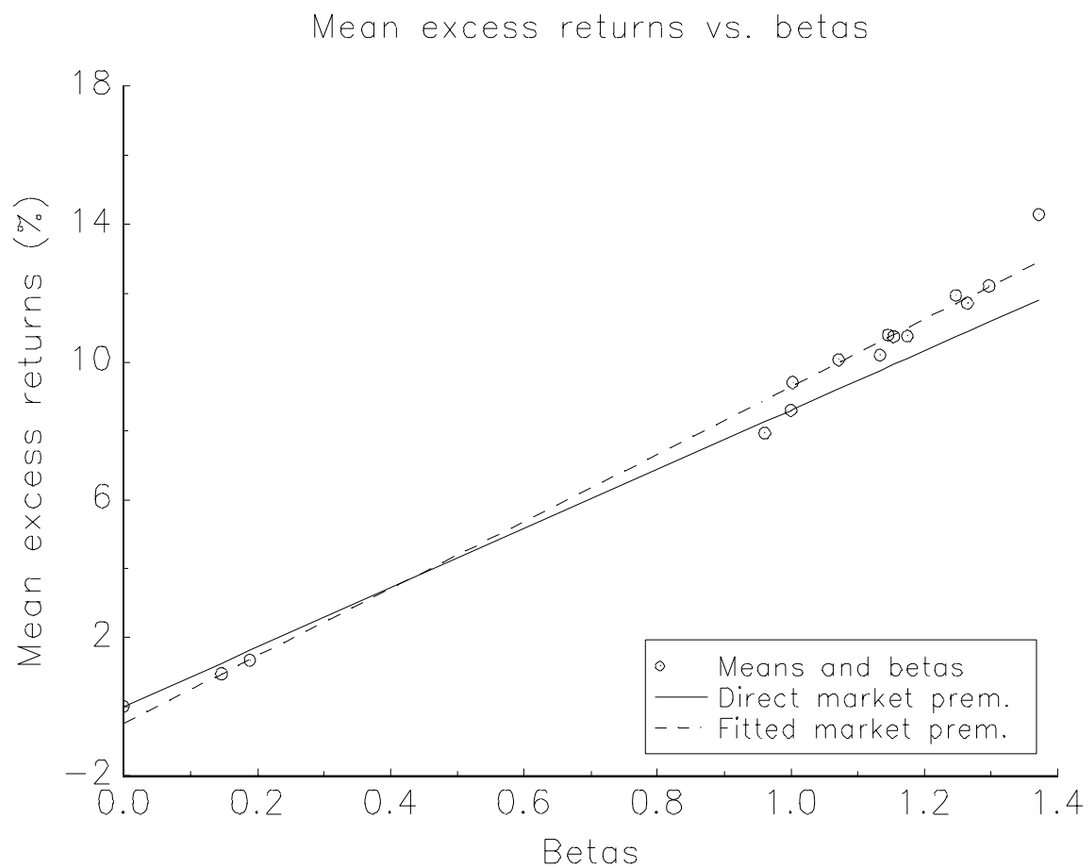


Figure 1: The CAPM. Average returns vs. betas on the NYSE value-weighted portfolio for 10 size-sorted stock portfolios, government bonds, and corporate bonds. Sample 1947-1996. The solid line draws the CAPM prediction by fitting the market proxy and treasury bill rates exactly (a time-series test) and the dashed line draws the CAPM prediction by fitting an OLS cross-sectional regression to the displayed data points (the second-pass or cross-sectional test). The small firm portfolios are at the top right. As you move down and to the left, you see increasingly large-firm portfolios, and the market index. The points far down and to the left are the government bond and treasury bill returns.

Counter-Attack

- Little theory behind additional factors
  - data snooping? (alphabet experiment)
  - firms with long data on BM ratios have survivorship bias
  - large econometric effects of even small measurement error for true market portfolio

Statistical Issues

- We will consider validity of CER model assumptions
- For example, Beta not stable over time
- Conditional CAPM?

Conditional CAPM

$$E_t[R_{it+1}] = r_{ft} + \beta_{it}(E_t[R_{Mt+1}] - r_{ft})$$

where $\beta_{it} = \dfrac{\mathrm{cov}_t(R_{it+1}, R_{Mt+1})}{\mathrm{var}_t(R_{Mt+1})}$

In practice $\mathrm{var}_t(R_{Mt+1})$ changes a lot over time.

Unconditional Expectations

$$E[R_{it+1}] = r_{ft} + \beta_i(E[R_{Mt+1}] - r_{ft}) + \mathrm{cov}(\beta_{it}, E_t[R_{Mt+1}])$$

I.e., $\alpha_i^* = \mathrm{cov}(\beta_{it}, E_t[R_{Mt+1}])$ if conditional CAPM holds

Conditional CAPM

- Lewellaen and Nagel (06) find that short-horizon estimates of time-varying betas and the market risk premium do not explain α^*'s for B/M or momentum portfolios

Conclusions (I)

- CAPM does not hold in simple form
- Yet it can explain a large portion of variation in expected returns
- Normative model (investors should not needlessly expose themselves to diversifiable risk)

Conclusions (II)

- Modern finance still mostly based on factor approach instead of individual firm valuation
- Need to address statistical problems with Market Model
- For conditional CAPM, need good estimates of time-varying betas and market risk premium