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Time-variation of CAPM betas across market volatility regimes for Book-to-market and Momentum portfolios

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Capital Asset Pricing Model

$$\mathsf{E}[r_i] = \beta_i \mathsf{E}[r_m]$$

where

- r_i is the excess return of asset i;
- *r_m* is the market excess return;
- β_i is the measure of asset's *i* risk.

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

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- CAPM performs poorly (Fama and French 1992,1993,1996);
- CAPM cannot explain some pricing anomalies:
 - "Size" effect : stocks of small firms outperform those of large firms;
 - "B/M" effect : stocks with high B/M ratios outperform those with low B/M ratios;
 - "Momentum" effect: stocks with high returns in past year outperform those with low past returns .

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Many papers reports that β is time-varying: Jagannathan and Wang(1996), Lettau and Ludvigson(2001).

Conditional CAPM (CCAPM):

$$E_{t-1}[r_{i,t}] = \beta_{i,t-1}E_{t-1}[r_{m,t}]$$

applying iterated expectation:

$$E[r_{i,t}] = \overline{\beta}_i E[r_{m,t}] + Cov(\beta_{i,t-1}, E_{t-1}[r_{m,t}])$$

• CCAPM needs conditioning information

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- Use of rolling windows and/or exogenously defined instrumental variables (IV);
- Common IVs to proxy the conditional market premium are related to BC: default spread, term spread;
- Lewellen and Nagel(2006) argue: CCAPM based on cross-sectional regressions do not impose important theoretical restrictions;
- Choice of IV may be subject to data mining concerns (results are somewhat sensitive to the choice of IV).

- Focus : investigate time-variation in βs across different states of the economy;
- States: low and high market volatility regimes;
- Market volatility regimes are related to BC;
 - Evidence of stock risk variations over BC (Perez-Quiros and Timmermann(2000) and Guidolin and Timmermann(2008)).



- Market volatility switches between two regimes identified by MS model;
 - Many papers show that market volatility can be modeled by MS and it is related to BC;

- Not subject to data mining concerns:
 - we do not use exogenously defined IV;
- Not subject to Lewellen and Nagel(2006) argument:
 - we do not use of cross-sectional estimation.

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Finding	<u>j</u> S							

- Strong time-variation of βs across the market volatility regimes for those portfolios for which the unconditional CAPM is rejected;
- Accounting for variation of β s over states of the economy helps to explain some risk premium not captured by the unconditional CAPM

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New information available to agents at time t: $\varepsilon_t \sim N(0, \sigma_{S_m t}^2)$

$$\sigma_{S_{m,t}}^2 = \sigma_{m,0}^2 (1 - S_{m,t}) + \sigma_{m,1}^2 S_{m,t} \qquad \sigma_{m,0}^2 < \sigma_{m,1}^2$$

 $S_{m,t} = 0$ and $S_{m,t} = 1$ in low and high market volatility regimes

Transition probabilities: $Pr[S_{m,t} = 0|S_{m,t-1} = 0] = q_m$ $Pr[S_{m,t} = 1|S_{m,t-1} = 1] = p_m$

Assuming that agents observe $S_{m,t}$:

$$E[r_{m,t}|S_{m,t}] = \mu_{m,0} + \mu_{m,1}S_{m,t}$$



Assume β switchs between two market volatility regimes:

$$E[r_{i,t}|S_{m,t}] = \beta_{i,S_{m,t}}E[r_{m,t}|S_{m,t}]$$

Empirical joint model of the market volatility and CCAPM:

$$\begin{cases} r_{m,t} = \mu_{m,0} + \mu_{m,1}S_{m,t} + \varepsilon_t & \varepsilon_t \sim N(0,\sigma_{S_{m,t}}^2) \\ r_{i,t} = \alpha_{i,S_{m,t}} + \beta_{i,S_{m,t}}r_{m,t} + u_t & u_t \sim N(0,\sigma_{S_{i,t}}^2) \end{cases}$$

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Data								

- Monthly data on stock returns for value weighted decile portfolios (NYSE, AMEX, NASDAQ);
- Sorted by ratios of book equity to market capitalization (B/M portfolios) and previous year returns ("Momentum" portfolios);
- Returns are cts. compounded in excess of cts. compounded one-month TB (in percent)

• Period 1963:07-2007:12.

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Tabl	e1: Sun	ımary sta	itistics of I	Book-to-mar	ket and Momer	ntum portfolios

	Low	2	3	4	5	6	7	8	9	High
	Panel A	: B/M por	tfolios (m	onthly %)						
Excess return std. dev.	0.24 (5.14)	0.35 (4.72)	0.41 (4.69)	0.44 (4.62)	0.41 (4.37)	0.53 (4.32)	0.60 (4.22)	0.64 (4.22)	0.69 (4.56)	0.77 (5.27)
α std. error	-0.17 (0.10)	-0.04 (0.07)	0.02 (0.07)	0.08 (0.10)	0.07 (0.10)	0.19 (0.08)	0.29 (0.11)	0.32 (0.11)	0.35 (0.11)	0.40 (0.16)
β std. error	1.09 (0.03)	1.03 (0.02)	1.02 (0.02)	0.98 (0.03)	0.91 (0.03)	0.90 (0.03)	0.84 (0.04)	0.84 (0.04)	0.90 (0.04)	0.98 (0.05)
	Panel B	: Moment	um portfo	lios (mont	hly %)					
Excess return std. dev.	-0.59 (7.29)	0.07 (5.81)	0.24 (4.95)	0.31 (4.57)	0.23 (4.29)	0.33 (4.43)	0.37 (4.35)	0.59 (4.40)	0.64 (4.82)	0.99 (6.20)
α std. error	-1.10 (0.18)	-0.35 (0.14)	-0.12 (0.11)	-0.04 (0.11)	-0.11 (0.09)	-0.02 (0.06)	0.03 (0.07)	0.24 (0.08)	0.27 (0.09)	0.53 (0.14)
β std. error	1.36 (0.07)	1.12 (0.06)	0.97 (0.05)	0.93 (0.04)	0.90 (0.03)	0.93 (0.03)	0.91 (0.03)	0.92 (0.03)	1.00 (0.04)	1.21 (0.05)

Sample period 1963:07-2007:12. Data on the value-weighted portfolios sorted by deciles of B/M ratio and previous 11 month return. Newey and West (1987) HAC standard errors are reported in parentheses for α and β . Sample standard deviations are reported in parentheses for excess returns. Statistically significant alphas at the 5 percent level are in bold.

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Figure 1: Excess market stock returns and smoothed probabilities of the high volatility regime

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- LR rejects CAPM with single β and α :
 - for 7-10 decile B/M portfolios;
 - for 2-3, 5-6, 8-10 decile Momentum portfolios;
- ARCH-LM test cannot reject the null : <u>no-ARCH in residuals;</u>
- Jarque-Berra test cannot reject the null : residuals are Normally distributed;
- Residuals from the unconditional CAPM: Both tests reject Normality and no-ARCH effect.

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Estimation results for B/M portfolios

Table2: Estimation results for the joint model of regime-switching market excess returns and CAPM for the B/M portfolios

	Low	2	3	4	5	6	7	8	9	High
Panel A	: α from t	he regime-s	witching m	odel						
α ₀	-0.10	-0.20	-0.08	-0.02	0.00	0.18	0.14	0.23 (0.10)	0.25	0.30
std. error	(0.12)	(0.08)	(0.09)	(0.08)	(0.03)	(0.10)	(0.09)		(0.15)	(0.22)
α ₁	-0.18	0.19	0.13	0.18	0.06	0.05	-0.16	0.13	0.18	0.20
std. error	(0.26)	(0.13)	(0.14)	(0.17)	(0.14)	(0.13)	(0.27)	(0.15)	(0.15)	(0.27)
Panel B	: β from t	he regime-s	witching m	odel						
β ₀	1.08	1.09	1.04	1.01	0.93	0.95	0.99	0.94	1.14	1.20
std. error	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.04)	(0.06)	(0.10)
β ₁	1.08	1.05	1.05	1.05	0.96	0.94	0.70	0.84	0.89	0.89
std. error	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.04)	(0.03)	(0.03)	(0.04)

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Figure 2: B/M portfolios in different regimes

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Figure 3: Excess returns of 1st, 5th, and 10th deciles B/M portfolios and smoothed probabilities of a high market volatility.



- High B/M portfolios demonstrate strong time-variation of β s;
- Regimes are identified as low market volatility / high β and high market volatility / low β;

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Table3: Estimation results for	or the joint model o	f regime-switching	market excess	returns and	CAPM :	for the
Momentum portfolios						

Momentum portionos										
	Low	2	3	4	5	6	7	8	9	High
Panel	A: α from t	he regime-s	witching m	odel						
α ₀ std. error	-0.56 (0.20)	-0.15 (0.11)	-0.12 (0.08)	-0.10 (0.09)	-0.04 (0.07)	-0.06 (0.13)	-0.10 (0.08)	0.20 (0.09)	0.16 (0.09)	-0.01 (0.03)
α ₁ std. error	-2.49 (0.40)	-0.57 (0.24)	0.17 (0.20)	-0.01 (0.06)	-0.10 (0.16)	0.22 (0.25)	1.00 (0.72)	0.09 (0.14)	0.30 (0.13)	0.68 (0.15)
Panel	B: β from t	he regime-s	witching m	odel						
β ₀ std. error	1.30 (0.07)	0.93 (0.03)	0.87 (0.03)	0.96 (0.04)	0.86 (0.03)	0.91 (0.03)	0.96 (0.03)	1.04 (0.03)	1.19 (0.03)	1.68 (0.08)
β ₁ std. error	1.15 (0.07)	1.37 (0.07)	1.20 (0.03)	0.92 (0.04)	0.98 (0.02)	1.04 (0.02)	1.06 (0.04)	0.87 (0.03)	0.82 (0.03)	1.07 (0.04)

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Figure 3: Momentum portfolios in different regimes

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Figure 5: Excess returns of 2nd, 5th, and 10th deciles Momentum portfolios and smoothed probabilities of a high market volatility.

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Results for Momentum portfolios									

- Low ("losers") and high ("winners") Momentum portfolios demonstrate strong time-variation of βs;
- For "losers" regimes are identified as low market /low β volatility and high market volatility /high β ;
- For "winners" regimes are identified as low market volatility /high β and high market volatility /low β ;

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Conclus	sion							

- We find evidence of strong time-variation across the market volatility regimes for:
 - high B/M portfolios;
 - low and high Momentum portfolios;
- These are portfolios for which the unconditional CAPM is rejected;
- Accounting for variation of β s over states of the economy helps to explain some risk premium not captured by the unconditional CAPM