Economics 487

Homework #2
Random Variables and the Constant Expected Return Model
Due Tuesday 9/22

The purpose of this homework assignment is to help solidify your understanding of the main statistical concepts needed in this course by using Excel to make calculations related to probability distributions. Also, you will do some "Monte Carlo" simulation and estimate the parameters of the constant expected return model of asset returns. These exercises are based on those developed by Eric Zivot for chapters 2 and 3 of his textbook.

Note: the default Tools menu in Excel does not have Data Analysis in it. To add it to the menu, select Add-Ins. Then, check the box for Analysis ToolPak and press OK. Now, the Tools menu should have Data Analysis near the bottom. If you select it, you will get the choice to do a number of things, including creating a histrogram or generating random numbers.

Also, in your write up, you may find it convenient to sometimes use equation editor in Word.

A. Excel Exercises: *(10 points)*

1. Download the Excel file "hw2a.xls". The file contains the probability distributions for four discrete random variables. Think of the random variables as representing the future annual return on A.G. Edwards, Anaheuser Busch, Boeing, and Monsanto stock. The distributions of these returns are based on (i.e., conditional on) the future state of the economy (e.g., depression, recession, normal, mild boom, or major boom). To help you learn more about Excel, the answers to the below questions have been answered for the A.G. Edwards case (R_A), although you should still comment on the A.G. Edwards results.
   a. Plot the probability distributions for each random variable (make a bar chart). Paste to a Word file. Comment on any differences or similarities between the probability distributions.
   b. For each random variable, compute the expected value, variance, standard deviation, skewness, and kurtosis. Paste your table of these summary statistics to a Word file and comment.
2. Suppose X is a Normally distributed random variables with mean 10 and variance 24. I.e., X~N(10,24).
   a. Using Excel, calculate Pr[X>14]. Write out the Excel function you used to make this calculation.
   b. Calculate Pr[8<X<20]. Again, write out the Excel function.

c. Find the probability that X takes on a value that is at least plus or minus 6 away from its mean. Again, write out how you arrive at your answer.
d. Suppose y is a constant defined such that Pr[X>y]=0.10. What is y? Again, explain how you arrived at this answer.
Hint: Check out the description of functions NORMDIST and NORMINV using the Microsoft Excel Help feature.

3. Consider the possible rates of return that you might obtain over the next year. You can invest in stock A or stock B.

State of economy	Probability	Return on stock A	Return on stock B
Recession	0.3	-10%	0%
Normal	0.4	20%	10%
Boom	0.3	50%	10%

a. Determine the expected return, variance, and standard deviation for stocks A and B.
b. Determine the covariance and correlation between the returns on stocks A and B.
c. Determine the expected return and standard deviation of an equally weighted portfolio of stocks A and B. *Hint: an equally weighted portfolio return is calculated by adding 50% of the return for stock A to 50% of the return to stock B. Think of it as having $50 in asset A and $50 is asset B instead of $100 in one or the other.*

4. Download the Excel file "hw2b.xls". The file contains price data for Microsoft and IBM stock. Compute the continuously compounded monthly returns $r_t \equiv \ln(P_t / P_{t-1})$.

a. Make a time plot of monthly returns (use basic line graph with no markers), putting the returns for Microsoft and IBM on the same plot. Paste to Word and comment on the salient features of the plot.
b. Make a scatter plot of the monthly returns on Microsoft against the returns on IBM. Paste. Briefly comment on the direction and strength of a linear association between the returns on the two stocks.

5. Use **Tools/Data Analysis/Histogram** to compute histograms of the returns on Microsoft and IBM. Paste. Comment on the shape characteristics of the histograms (i.e., center, spread, symmetry, and tail thickness). *Note: you can use the default bins given by Excel.*

6. Compute sample descriptive statistics for the historical returns on Microsoft and IBM. Use Excel functions AVERAGE, VAR, STDEV, SKEW, KURT, COVAR, and CORREL to compute these statistics. Paste a table of results to Word. Briefly compare and contrast the descriptive statistics for the two stocks. *Note: the sample kurtosis reported in Excel is nonstandard. Usually, we think of a normal distribution as having a kurtosis of 3, but Excel would report a kurtosis for a normally distributed random variable of 0. For reporting purposes use the standard definition of kurtosis. That is, add 3 to the sample kurtosis given by Excel.*

7. Using **Tools/Data Analysis/Random Number Generation** simulate 60 observations (number of variables is 1 and number of random numbers is 60) from the constant expected return (CER) model for Microsoft only. Assume that the mean of the normal

distribution is equal to the sample mean and the standard deviation is equal to the sample standard deviation.

a. Make a time plot of the simulated monthly returns. Paste. Briefly comment on the salient features of the plot. Does the simulated data look like the actual data?

b. Compute a histogram of the simulated data. Paste. Does it look like the histogram of the actual data?

c. Compute descriptive statistics for the simulated data. Paste. Are these similar to the descriptive statistics for the actual data?

8. Using the sample statistics computed in exercise 6, give estimates of the parameters of the CER model:

$$R_{it} = \mu_i + \varepsilon_{it}, \quad i = \text{Microsoft, IBM}$$
$$\varepsilon_{it} \sim i.i.d.N(0,\sigma_i^2)$$
$$corr(\varepsilon_{it},\varepsilon_{jt}) = \rho_{ij}$$

a. Compute estimated standard errors and 95% confidence intervals for the μ_i's, σ_i^2's, and ρ_{ij} using the formulas given in class. Comment on the precision of your estimates. *Hint: You can calculate the least squared estimates for mean and variance using the Excel functions AVERAGE, VAR, and COVAR. For example, AVERAGE gives you the sample mean, which is the least squares estimate of the true population mean. You can say how you arrived at the point estimates using the Excel functions, but you do not have to derive the equivalence of the sample mean to the least squares estimate. That is, you can just state that the least squares estimate is the sample mean. In terms of calculating the 95% confidence bands for, say, mu, "mu+/-2*Standard Error(mu)" is the correct formula. Remember the 2 refers to the critical value for a t-distribution with 58 degrees of freedom. That is, only 5% of observations from a t-distributed random variable are greater than 2 in absolute value. This is pretty similar to a standard normal distribution, where only 5% of observations are greater than 1.96 in absolute value. The slightly higher critical value reflects the greater variance in a statistic that makes use of an estimated standard error rather than the true standard error (which has no variance, but is just some number).*

B. Reading Assignment: *(10 points)*

Read through Part Two "How the Pros Play the Biggest Game in Town" of <u>A Random Walk Down Wall Street</u>. As you read, take notes of what each chapter is about. Write up an executive summary of each chapter. An executive summary should cover the main points of raised by the author without assuming that your reader has read the text. You can also comment on how effectively you think points are presented. The total write-up should be between 2-4 pages (typed and double-spaced).