Washington University
Department of Economics

Fall 2009
James Morley

Economics 487

Homework #3 Solution Key
Diversification and Computing Efficient Portfolios

A. Excel Exercises: *(10 points)*

1. In this exercise, you will pretend that you are a pension fund manager for Washington University. You have 3 billion dollars in the fund, which you can invest in any combination of US stocks, German stocks, and US Treasury Bills. The idea is to use your knowledge of portfolio theory to make an argument for having an internationally diversified portfolio, rather than just holding domestic assets.

   The data you are given are from *Stocks, Bonds, Bills, and Inflation 1989 Yearbook*, Ibbotson Associates, Inc. The data are monthly returns and the relevant sample statistics are summarized in the following table:

Stock	E[R]	Var(R)	Cov(US,GER)
US Index	0.00959	0.00222	0.00088
GERMAN Index	0.00727	0.00348	
US T-bill	0.003	0	

   a. Using the results of portfolio theory and the estimates above, compute the tangency mutual fund (portfolio) between US and German stocks (i.e., the optimal split between US and German stocks). Recall, the tangency portfolio solves the maximization problem

$$\max_{t_A, t_B} \frac{\mu_p - r_f}{\sigma_p}$$

$$s.t.$$

$$\mu_p = t_A \mu_A + t_B \mu_B,$$

$$\sigma_p^2 = t_A^2 \sigma_A^2 + t_B^2 \sigma_B^2 + 2 t_A t_B \sigma_{AB},$$

$$t_A + t_B = 1$$

   (*Note: these equations are generated using the Equation Editor in Microsoft Word. It is the most convenient way to work in Word with mathematical expressions that have many Greek letters and may be useful for you homework answers and your term project.*) Find the tangency portfolio using the analytic formula from the notes. You should check your result using the Solver in Excel. There is a tutorial on using Solver on the class webpage. Also, I will discuss using Solver in class. Paste the table used with Solver to your Word document.

Solve the tangency portfolio problem for 2-risky assets and a risk free rate of 0.3%,

x_a	x_b	x_rf	constraint	mu_p	sigma_p	sharpe ratio
0.838981926	0.161018	0	1	0.009216	0.043481	0.142968

The solver solution is given above. See Zivot textbook for analytical formulas for tangency portfolio weights in a world with 2-risky assets.

b. Suppose WU would like to achieve an average return of 0.5% per month in excess of the T-bill rate with the smallest possible risk. What is the optimal split between US stocks, German stocks, and T-bills? That is, how much of the $3 billion should you invest in each country and how much should you borrow or lend? What is the standard deviation of this portfolio?

Combine the tangency portfolio with the risk-free asset to obtain mu_p=0.008.

x_a	x_b	x_rf	constraint	mu_p	sigma_p	sharpe ratio
0.67480913	0.12951	0.195681	1	0.008	0.034973	0.142968

A	B	C
$2,024,427,389	$388,530,092	$587,042,520

Lend $587,042,520. Standard deviation is 0.034973.

c. After a bad year on the German stock market, some people at WU try to influence you to divest (i.e., sell all of) the holdings of German stocks. How much should you invest in US stocks and T-bills alone to obtain the same level of risk as you obtained in part b.? (*Hint: you want the standard deviation of the divested portfolio to be the same as the nondivested portfolio.*)

Target standard deviation optimization problem.

x_a	x_b	x_rf	constraint	mu_p	sigma_p	sharpe ratio
0.742257168	0	0.257743	1	0.007891	0.034973	0.139865

Answer can be in terms of solver or formulas.

d. What would be the cost in terms of expected monthly return from divesting in the German stocks? What would be the cost in terms of annual return (note: the returns are continuously compounded)? What would be the cost in dollar terms on the $3 billion portfolio each year?

cost	annual cost	portfolio expected cost
0.000108525	0.001302303	$4,297,767

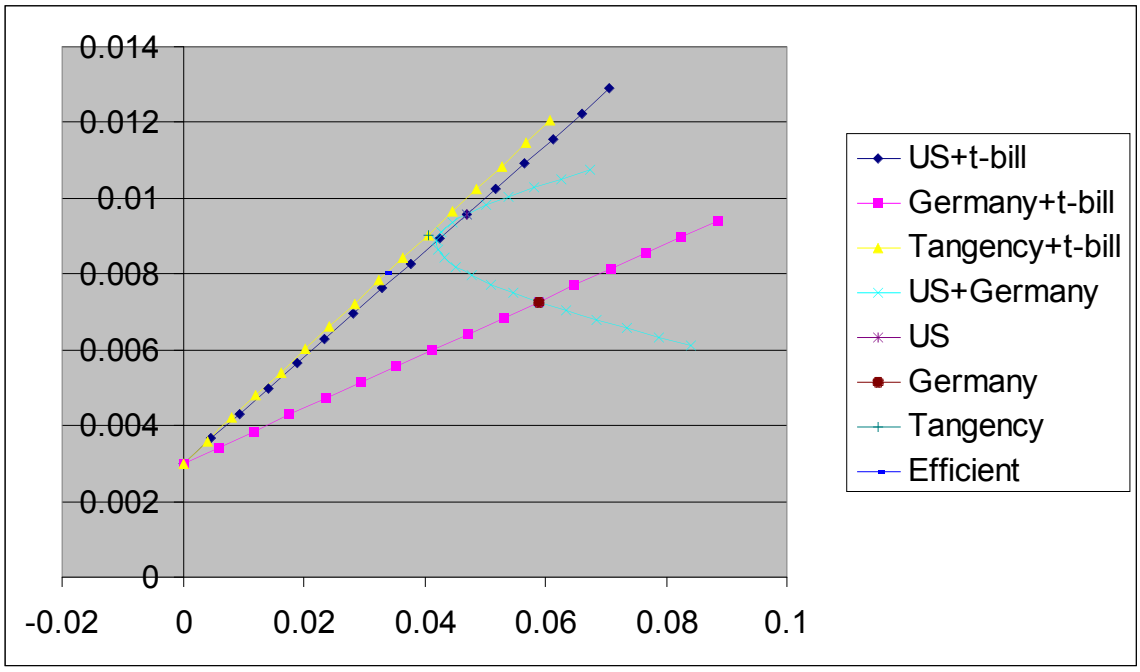
e. Calculate the mean and standard deviations for the following portfolios using the US/Germany data given above:
   i. US stocks and T-bills (no German stocks), with the US stock shares varying from 0% to 200% (10% increments). If plotted, this should be a straight line with a positive slope.

ii. German stocks and T-bills (no US stocks), with the German stock shares varying from 0% to 200%. If plotted, this should be a straight line with a positive slope.

iii. The tangency mutual fund and T-bills, with the mutual fund shares varying from 0% to 200%. If plotted, this should be a straight line with a positive slope.

iv. US stocks and German stocks (no T-bills), with the US stock shares varying from -100% to 200%. If plotted, this should be a tilted parabola.

Plot these four separate curves (actually, three are straight lines and one is a parabola) on the same graph. Mark the following points of interest: US stocks only, German stocks only, the tangency portfolio, and the optimal international portfolio of stocks and T-bills from part b. Comment on the graph. E.g., what can you say about investing in German stocks only? Also, how does the graph relate to the results in parts c. and d.?



German stocks only is inefficient. The Sharpe ratio gets higher as we go from investing in Germany/Bonds to US/Bonds to US/Tangency portfolio, respectively. For a given level of risk, then, the expected return will be reduced from divesting in German assets in the tangency (mutual fund) portfolio.