

Economics 487

Homework #4

**Portfolio Calculations and the Markowitz Algorithm**

Due Tuesday 10/20

**A. Excel Exercises: (10 points)**

1. Download the Excel file “hw4.xls” from the class website. This file contains monthly closing **price** data on four stocks (Boeing, Microsoft, Nordstrom, and Starbucks) over the period February 1995 – September 2006. You must convert the data to continuously compounded returns. For this exercise, you will use matrix algebra to simplify the running of solver in Excel.

- a. Use the Excel functions AVERAGE, VAR, STDEV, COVAR, and CORREL to estimate the expected returns, variances, standard deviations, covariances, and correlations for the four stocks using the historical returns. (*Hint: you can also use the covariance feature under **tools/data analysis.***) Report your results in Table format in your Word file. Comment on the estimates, paying particular attention to the correlations. Compute estimated standard errors for the means, standard deviations, and the correlations. Comment on the precision of your estimates. Arrange the covariance estimates into a covariance matrix.
- b. Compute the global minimum variance portfolio allowing for short-sales. The minimization problem is

$$\begin{aligned} \min_m m' \Sigma m \\ \text{s.t.} \\ m' 1_4 = 1 \end{aligned}$$

where  $m$  is the vector of portfolio weights,  $\Sigma$  is the covariance matrix, and  $1_4$  is a 4x1 vector of ones. Are there any negative weights in this portfolio? If so, interpret them. Compute and report the expected return, and standard deviation of this portfolio.

- c. Of the four stocks, determine the stock with the largest average historical return. Use this maximum average return as the target return for the computation of an efficient portfolio allowing for short-sales. That is, find the minimum variance portfolio that has an expected return equal to this target return. The minimization problem is

$$\begin{aligned} \min_x x' \Sigma x \\ \text{s.t.} \\ x' \mu = \mu_{p,0} \\ x' 1_4 = 1 \end{aligned}$$

where  $x$  is the vector of portfolio weights,  $\mu$  is the vector of expected returns, and  $\mu_{p,0}$  is the target expected return. Are there any negative weights in this

portfolio? Compute and report the expected return, and standard deviation of this portfolio. Finally, compute and report the covariance between the global minimum variance portfolio and the above efficient portfolio using the formula  $\text{cov}(R_{p,m}, R_{p,x}) = m' \Sigma x$ .

- d. Using the fact that all efficient portfolios can be written as a convex combination of two efficient portfolios, compute efficient portfolios as convex combinations of the global minimum variance portfolio and the efficient portfolio computed in question 3. That is, compute

$$z = \alpha \cdot x + (1 - \alpha) \cdot m$$

for values of  $\alpha$  between -1 and 2 (make a grid for  $\alpha = -1, -0.9, \dots, 0, 0.1, \dots, 1.9, 2$ ). Compute and report the expected return, variance, and standard deviation of these portfolios in a Table.

- e. Plot the Markowitz bullet based on these efficient portfolios that you computed above. On the plot, indicate the location of the minimum variance portfolio and the location of the efficient portfolio that you found in part c. Paste this graph to your Word file.
- f. Compute the tangency portfolio assuming the risk-free rate is 0.005 (i.e.,  $r_f = 0.5\%$ ) per month. That is, solve

$$\begin{aligned} \max_t & \frac{\mu_{p,t} - r_f}{\sigma_{p,t}} \\ \text{s.t.} & \\ & \mu_{p,t} = t' \mu \\ & \sigma_{p,t}^2 = t' \Sigma t \\ & t' \mathbf{1}_4 = 1 \end{aligned}$$

where  $t$  denotes the portfolio weights in the tangency portfolio. Are there any negative weights in the tangency portfolio? If so, interpret them. Compute and report the expected return, variance, and standard deviation of the tangency portfolio.

- g. On the graph with the Markowitz bullet, plot the efficient portfolios that are combinations of T-bills and the tangency portfolio. Indicate the location of the tangency portfolio on the graph. Paste this graph to your Word file.
- h. For 2 bonus points, use the Benninga text in the class reader (see Chapter 11 of the Benninga text, section 11.3) as a guide to compute and plot the Markowitz bullet for the data in hw4.xls when there are no short sales allowed. On the same set of axes, plot the efficient frontiers with and without short sale restrictions.

**B. Reading Assignment:** (10 points) (\*\*\*\*\*Due Thursday, 10/29\*\*\*\*\*)

Read through Part Three of [A Random Walk Down Wall Street](#). As you read, take notes of what each chapter is about. Write up an executive summary of each chapter. An executive summary should cover the main points of raised by the author without assuming that your reader has read the text. You can also comment on how effectively you think points are presented. The total write-up should be between 2-4 pages (typed and double-spaced).