

Homework #4 Solution Key
Portfolio Calculations and the Markowitz Algorithm

A. Excel Exercises: (10 points)

1. Download the Excel file “hw4.xls” from the class website. This file contains monthly closing price data on four stocks (Boeing, Microsoft, Nordstrom, and Starbucks) over the period February 1995 – January 2006. For this exercise, you will use matrix algebra to simplify the running of solver in Excel.
 - a. Use the Excel functions AVERAGE, VAR, STDEV, COVAR, and CORREL to estimate the expected returns, variances, standard deviations, covariances, and correlations for the four stocks using the historical returns. (*Hint: you can also use the covariance feature under **tools/data analysis.***) Report your results in Table format in your Word file. Comment on the estimates, paying particular attention to the correlations. Compute estimated standard errors for the means, standard deviations, and the correlations. Comment on the precision of your estimates. Arrange the covariance estimates into a covariance matrix.

	mu_a	mu_b	mu_c	mu_d	sigma_a	sigma_b	sigma_c	sigma_d
Estimate	0.00997	0.01134	0.02230	0.01481	0.08631	0.10978	0.11909	0.10687
SE	0.00729	0.00928	0.01006	0.00903	0.00518	0.00658	0.00714	0.00641
	rho_AB	rho_AC	rho_AD	rho_BC	rho_BD	rho_CD		
Estimate	0.22126	0.14877	0.07254	0.24794	0.25191	0.27059		
SE	0.08067	0.08294	0.08437	0.07960	0.07944	0.07861		

The standard deviations are fairly precise. But the expected returns and correlations are much less precise. E.g., the expected return for Nordstrom is quite low, but quite imprecisely estimated.

Covariance matrix:

Sigma			
0.007449361	0.002096514	0.001529196	0.000669081
0.002096514	0.012052169	0.003241543	0.002955494
0.001529196	0.003241543	0.014182421	0.00344392
0.000669081	0.002955494	0.00344392	0.011421368

- b. Compute the global minimum variance portfolio allowing for short-sales. The minimization problem is

$$\min_m m' \Sigma m$$

$$s.t.$$

$$m' 1_4 = 1$$

where m is the vector of portfolio weights, Σ is the covariance matrix, and 1_4 is a 4x1 vector of ones. Are there any negative weights in this portfolio? If so, interpret them. Compute and report the expected return, variance, and standard deviation of this portfolio.

2b.

m	ones	constraint	variance	E[Rp]	Std. Dev
0.450538691	1	1	0.004076481	0.013135951	0.063847322
0.161299765	1				
0.142278843	1				
0.245882701	1				

No negative weights.

- c. Of the four stocks, determine the stock with the largest average historical return. Use this maximum average return as the target return for the computation of an efficient portfolio allowing for short-sales. That is, find the minimum variance portfolio that has an expected return equal to this target return. The minimization problem is

$$\begin{aligned} \min_x & x' \Sigma x \\ \text{s.t.} & \\ & x' \mu = \mu_{p,0} \\ & x' \mathbf{1}_4 = 1 \end{aligned}$$

where x is the vector of portfolio weights, μ is the vector of expected returns, and $\mu_{p,0}$ is the target expected return. Are there any negative weights in this portfolio? Compute and report the expected return, variance, and standard deviation of this portfolio. Finally, compute and report the covariance between the global minimum variance portfolio and the above efficient portfolio using the formula $\text{cov}(R_{p,m}, R_{p,x}) = m' \Sigma x$.

Target portfolio optimization.

2c.

x	ones	constraint	constraint	variance	E[Rp]	Std. Dev
-0.061334583	1	1	0.022302607	0.013343763	0.022302607	0.115515209
-0.111033399	1					
0.908873046	1					
0.263494935	1					

Cov(x,m)

0.004076481

The weights on Boeing and Nordstom are negative. This means an investor should short-sell them.

- d. Using the fact that all efficient portfolios can be written as a convex combination of two efficient portfolios, compute efficient portfolios as convex combinations of the global minimum variance portfolio and the efficient portfolio computed in question 3. That is, compute

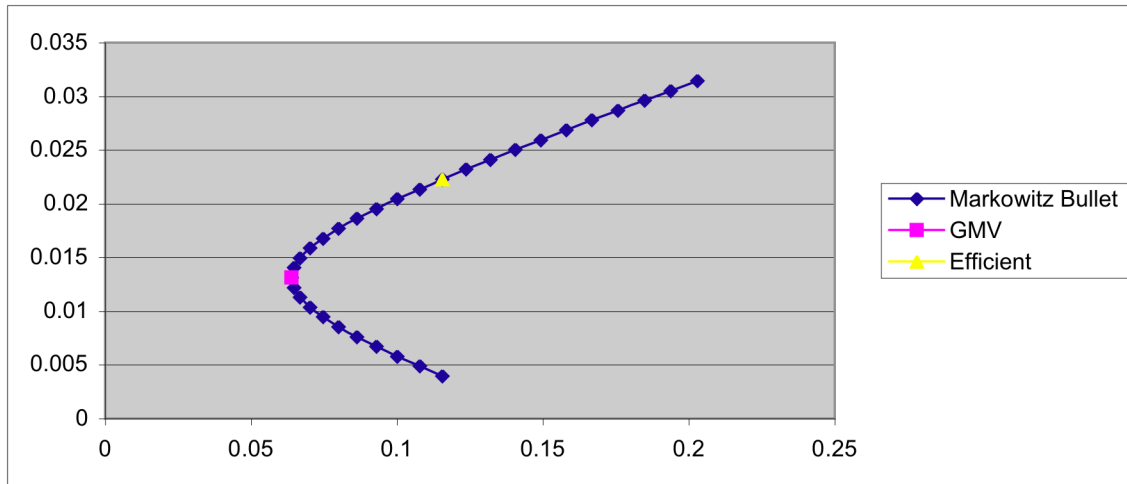
$$z = \alpha \cdot x + (1 - \alpha) \cdot m$$

for values of α between -1 and 2 (make a grid for $\alpha = -1, -0.9, \dots, 0, 0.1, \dots, 1.9, 2$). Compute and report the expected return, variance, and standard deviation of these portfolios in a Table.

2d.

alpha	sigma_p	mu_p
0	0.063847322	0.013135951
-1	0.115515204	0.003969295
-0.9	0.107624248	0.004885961
-0.8	0.100037697	0.005802626
-0.7	0.092830213	0.006719292
-0.6	0.086097049	0.007635958
-0.5	0.079958119	0.008552623
-0.4	0.074560348	0.009469289
-0.3	0.070075215	0.010385954
-0.2	0.066687118	0.01130262
-0.1	0.064568981	0.012219285
-1.38778E-16	0.063847322	0.013135951
0.1	0.064568982	0.014052617
0.2	0.06668712	0.014969282
0.3	0.070075218	0.015885948
0.4	0.074560351	0.016802613
0.5	0.079958123	0.017719279
0.6	0.086097053	0.018635944
0.7	0.092830218	0.01955261
0.8	0.100037701	0.020469275
0.9	0.107624253	0.021385941
1	0.115515209	0.022302607
1.1	0.123652306	0.023219272
1.2	0.131990029	0.024135938
1.3	0.140492663	0.025052603
1.4	0.149132005	0.025969269
1.5	0.157885613	0.026885934
1.6	0.166735492	0.0278026
1.7	0.175667093	0.028719266
1.8	0.184668558	0.029635931
1.9	0.19373015	0.030552597
2	0.20284381	0.031469262

- e. Plot the Markowitz bullet based on these efficient portfolios that you computed above. On the plot, indicate the location of the minimum variance portfolio and the location of the efficient portfolio that you found in part c. Paste this graph to your Word file.



- f. Compute the tangency portfolio assuming the risk-free rate is 0.005 (i.e., $r_f = 0.5\%$) per month. That is, solve

$$\max_t \frac{\mu_{p,t} - r_f}{\sigma_{p,t}}$$

$$s.t.$$

$$\mu_{p,t} = t' \mu$$

$$\sigma_{p,t}^2 = t' \Sigma t$$

$$t' \mathbf{1}_4 = 1$$

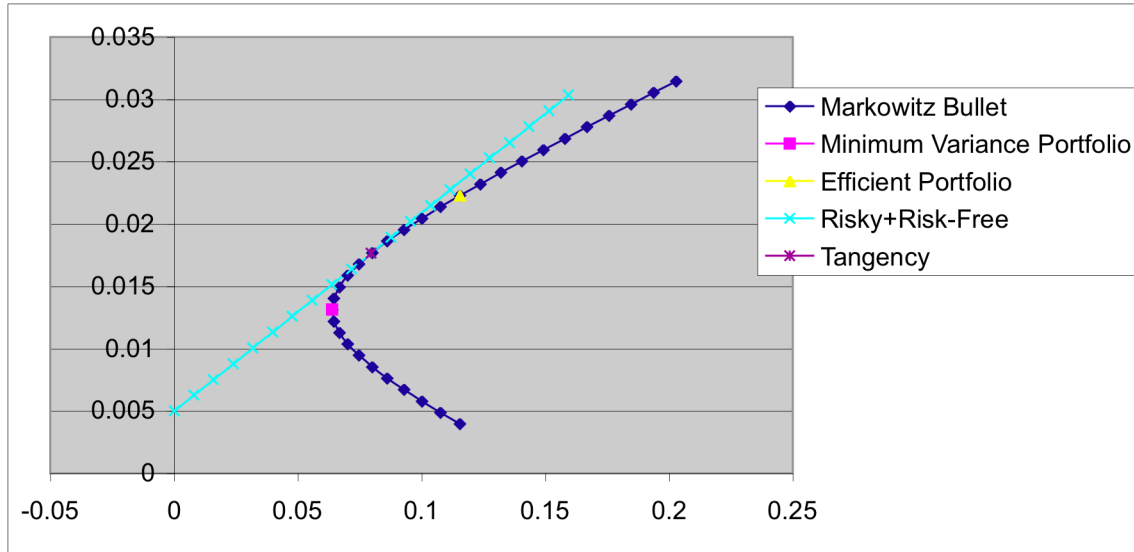
where t denotes the portfolio weights in the tangency portfolio. Are there any negative weights in the tangency portfolio? If so, interpret them. Compute and report the expected return, variance, and standard deviation of the tangency portfolio.

2f.

t	ones	constraint	std. Dev	E[Rp]	Sharpe Ratio
0.196850107	1	1	0.079704212	0.017679002	0.159075684
0.026331171	1				
0.522207642	1				
0.254611079	1				

No negative weights.

- g. On the graph with the Markowitz bullet, plot the efficient portfolios that are combinations of T-bills and the tangency portfolio. Indicate the location of the tangency portfolio on the graph. Paste this graph to your Word file.



- h. For 4 bonus points, use the Benninga handout as a guide to compute and plot the Markowitz bullet for the data in hw4.xls when there are no short sales allowed. On the same set of axes, plot the efficient frontiers with and without short sale restrictions.

Calculate the tangency portfolio, imposing nonnegativity constraints on weights, for a variety of risk free rates. The result is that the efficient frontier is the same as with short sales from the Global Min portfolio (which had only positive weights) up to a certain expected return. Then, the efficient frontier is lower than the short sales case beyond that point. See graph.

