Washington University — Fall 2009
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Economics 487

Homework #5

The Market Model and the Capital Asset Pricing Model

Due: Thursday 11/19

For this homework assignment, it is helpful to read through Chapter 10 ("Estimating Betas and the Security Market Line") in Benninga. Again, the Excel exercises are related to those developed by Eric Zivot for his textbook.

The data for the Excel exercises are available in the hw5.xls file. All series are continuously compounded returns:

dec	Monthly returns on Digital Electronics from 1983:1-1987:12
gerber	Monthly returns on Gerber Foods from 1983:1-1987:12
genmil	Monthly returns on General Mills from 1983:1-1987:12
datgen	Monthly returns on Data General from 1983:1-1987:12
ibm	Monthly returns on IBM from 1983:1-1987:12
delta	Monthly returns on Delta Airlines from 1983:1-1987:12
psnh	Monthly returns on Public Service of New Hampshire from 1983:1-1987:12
coned	Monthly returns on Connecticut Edison from 1983:1-1987:12
market	Monthly returns on the S&P 500 market index from 1983:1-1987:12
rkfree	Monthly returns on 3 month T-bills from 1983:1-1987:12

Excel Tips: Regression analysis in Excel is available through the Data Analysis Add-in from the Tools menu. To perform a linear regression in Excel, name the ranges of data associated with the dependent and independent variables. In the exercises, the dependent variable is a return (regular or excess) on a particular stock and the independent variable is the return (regular or excess) on the market portfolio. Then, select Tools/Data Analysis/Regression to bring up the regression dialogue box. For input options, in the input Y Range, type the name of the dependent variable and in the input X range, type the name of the independent variable. For output options, specify a new worksheet and give a name for the new worksheet. For residual options, specify that a line fit plot be created. Click OK.

A. Excel Exercises: *(20 points)*

1. In this exercise, you will look at estimates of β for the Market Model, construct 95% confidence intervals for β, test $H_0 : \beta = 1$, and, using the measured R^2, compute the proportion of risk that is systematic and the proportion that is diversifiable. Four of the assets are "safe": two are public utilities (Connecticut Edison and Public Service of New Hampshire) and two are food companies (Gerber and General Mills). Four of the assets are "risky": three are computer companies (Digital Electronics, Data General, and IBM) and one is an airline (Delta).

a. Using the 60 observations from January 1983 to December 1987, estimate by OLS (ordinary least squares) the parameters α and β in the single index model regression:

$$r_{it} = \alpha_i + \beta_{i,M} r_{Mt} + \varepsilon_{it},$$

for each of the eight stocks. Report the estimated regression line (i.e., $\hat{r}_{it} = \hat{\alpha}_i + \hat{\beta}_{i,M} r_{mt}$) with the estimated standard error underneath the estimated coefficients and the R-squared statistic. E.g.,

$$\hat{r}_{it} = \underset{(0.008)}{0.003} + \underset{(0.059)}{0.673}\ r_{mt}, \quad R^2 = 0.432$$

Do the estimates of β correspond well with your prior intuition or beliefs about these stocks? Why or why not?

b. For each company, make a scatter plot with the company return on the vertical axis, the return on the market portfolio on the horizontal axis, and the estimated single index model regression line drawn through the scatter. Based on the scatter plot, comment on the fit of the regression by interpreting the R^2 of the regression.

c. For each company, construct a 95% confidence interval for β. Then, using a 5% significance level, test the null hypothesis that the company's risk is the same as the risk of the market portfolio against the alternative that it is different (i.e., a two-tailed test with $H_1 : \beta \neq 1$). Also, if the estimated β is less than 1, conduct a one-tailed test with $H_1 : \beta < 1$ or, if the estimated β is greater than 1, conduct a one-tailed test with $H_1 : \beta > 1$. Did you find any surprises in these tests? (Note: when you cannot reject the null hypothesis, do not say "accept the null". Just say that you fail to reject the null.)

d. For each of the companies, report the proportion of total risk that is market risk, also called systematic or nondiversifiable risk. William F. Sharpe, winner of the Nobel Prize in Economics, states that "Uncertainty about the overall market… accounts for only 30% of the uncertainty about the prospects for a typical stock." Does evidence from the eight companies you have examined correspond to Sharpe's typical stock? Why or why not? What is the proportion of total risk that is specific and diversifiable for the eight stocks? Do these proportions surprise you? Why?

e. In your sample, do large estimates of β correspond with higher R^2 values? Would you expect this always to be the case? Why or why not?

2. In this exercise, you will use a modified version of the market model, the *excess return market model*, to test the CAPM. In this model, the dependent variable is the excess return on a particular stock (return minus T-bill return) and the independent variable is the excess return on the market portfolio (market return minus T-bill return). That is, the regression equation is now

$$r_{it} - r_{ft} = \alpha_i^* + \beta_{i,M}^* (r_{Mt} - r_{ft}) + \varepsilon_{it}.$$

a. What are the various statistical and economic assumptions made regarding the MM and the CAPM? Assuming the risk-free rate is constant over time, $r_{ft} = r_t$, how would α_i and α_i^* be related? What restrictions does the CAPM place on α_i and α_i^*?

b. Using the 60 observations from January 1983 to December 1987, construct the excess return variables and then use regression analysis to estimate α_i^* and $\beta_{i,M}^*$ for each company. Report the estimated regression lines with the estimated standard errors underneath each variable and the R-squared for the regression.

c. For each company, test the null hypothesis that $\alpha_i^* = 0$ against the alternative that $\alpha_i^* \neq 0$ using a significance level of 5%. Would rejection of this null hypothesis imply that the CAPM has been invalidated? Why or why not? (Hint: there are both reasons "why" and "why not". Read Benninga, Ch. 10)

3. In this exercise, you will show that β for a portfolio is a linear combination of the β's for the stocks in the portfolio.

a. Consider the MM equation for two stocks A and B:

$$R_A = \alpha_A + \beta_A R_M + \varepsilon_A,$$
$$R_B = \alpha_B + \beta_B R_M + \varepsilon_B.$$

For each regression, it is assumed ε_A and ε_B are independent of R_M. Create a portfolio of stocks A and B with share of wealth x_A invested in stock A and share x_B invested in B such that $x_A + x_B = 1$. Show that the beta of the portfolio, denoted β_p, satisfies

$$\beta_p = x_A \beta_A + x_B \beta_B.$$

(*Hint:* $\beta_p = \dfrac{\text{cov}(R_p, R_M)}{\text{var}(R_M)}$ and $R_p = x_A R_A + x_B R_B$.)

b. Using the eight assets from Exercises 1 and 2, form an equally weighted portfolio: i.e., each weight is $x_i = \dfrac{1}{8}$. Estimate the basic MM regression equation for the portfolio return and numerically verify that the estimated β of the equal-weighted portfolio is equal to an equally-weighted average of the individual β's in Exercise 1.

c. Using the regression output, compute the proportion of total risk that is systematic and the portion that is specific to the eight firms in the portfolio. Is the proportion of systematic risk for the portfolio greater than the estimated proportion of systematic risk for each of the eight stocks that you computed in Exercise 1? Why or why not?

B. Reading Assignment: *(10 points)*

Read Andre Perold, 2004, The Capital Asset Pricing Model, Journal of Economic Perspectives 18, 3-24, which is in the reading package. Provide a 2-3 page summary of the article, with particular reference to what the CAPM says about how investors should behave and how they actually behave.