Economics 487

Homework #5 Solution Key

The Market Model and the Capital Asset Pricing Model

A. Excel Exercises: *(20 points)*

1a.

CONED:

$$\hat{r}_{it} = \underset{(0.0063)}{0.0189} + \underset{(0.1059)}{0.0249}\ r_{mt}, \quad R^2 = 0.0010$$

Utilities are usually safe since they have very predictable costs and revenues. Thus, the very low beta corresponds well with my prior.

PSNH:

$$\hat{r}_{it} = \underset{(0.0193)}{-0.0189} + \underset{(0.3217)}{0.1812}\ r_{mt}, \quad R^2 = 0.0054$$

Another utility with a low beta.

GENMIL:

$$\hat{r}_{it} = \underset{(0.0081)}{0.0148} + \underset{(0.1345)}{0.5666}\ r_{mt}, \quad R^2 = 0.2343$$

Food companies are usually safe since consumption of food basics is usually smoothed out over the business cycle. Beta is less than 1, which is the definition of a safe asset.

GERBER:

$$\hat{r}_{it} = \underset{(0.0102)}{0.0122} + \underset{(0.1708)}{0.9155}\ r_{mt}, \quad R^2 = 0.3312$$

Another food company. Beta is less than 1, but it is high and does not correspond to my prior. There is some relationship between the business cycle and birth rates, so perhaps GERBER demand is less immune to the business cycle than GENMIL.

DATGEN:

$$\hat{r}_{it} = \underset{(0.0137)}{0.0010} + \underset{(0.2278)}{1.0991}\ r_{mt}, \quad R^2 = 0.2864$$

A tech company with a beta slightly larger than one. This corresponds well with my prior (although I might imagine an even higher beta).

DEC:

$$\hat{r}_{it} = \underset{(0.0120)}{0.0137} + \underset{(0.2008)}{1.0945}\ r_{mt}, \quad R^2 = 0.3388$$

Another tech company with a similar beta.

DELTA:

$$\hat{r}_{it} = -\,0.0023 + \; 0.6403 \; r_{mt}, \quad R^2 = 0.1653$$
$$(0.0113) \quad (0.1889)$$

I would have anticipated a beta of greater than one for an airline since demand for business travel should be quite sensitive to the business cycle.

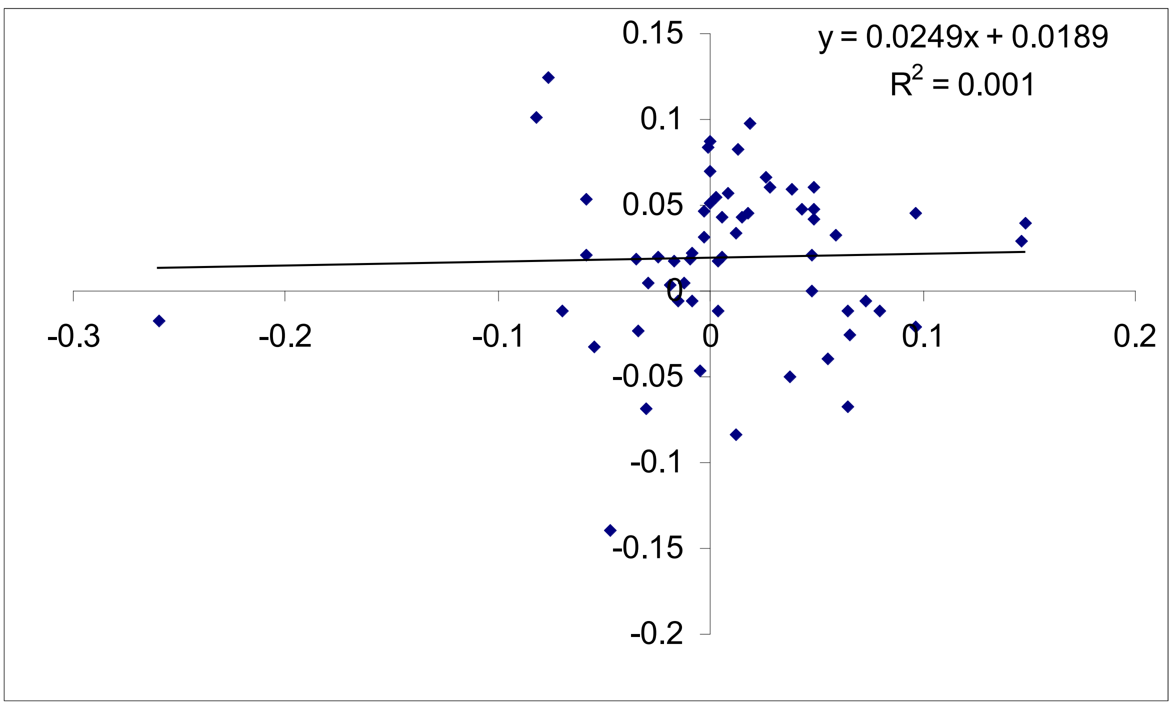
IBM:

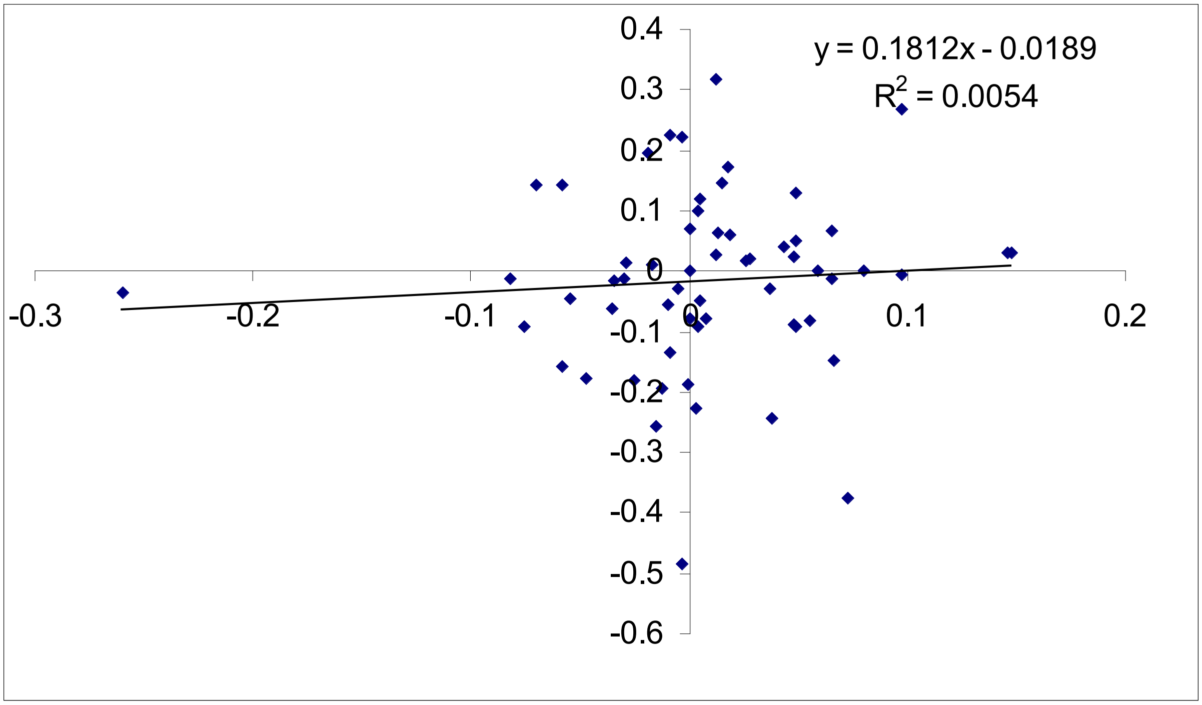
$$\hat{r}_{it} = \; 0.0019 \; + \; 0.6569 \; r_{mt}, \quad R^2 = 0.4130$$
$$(0.0062) \quad (0.1028)$$

Again, I would have anticipated a higher beta. However, IBM was such a large and well-diversified company during this sample period that the low beta is plausible. Also, the low beta may reflect a fitting of the model to the 1987 stock market crash.

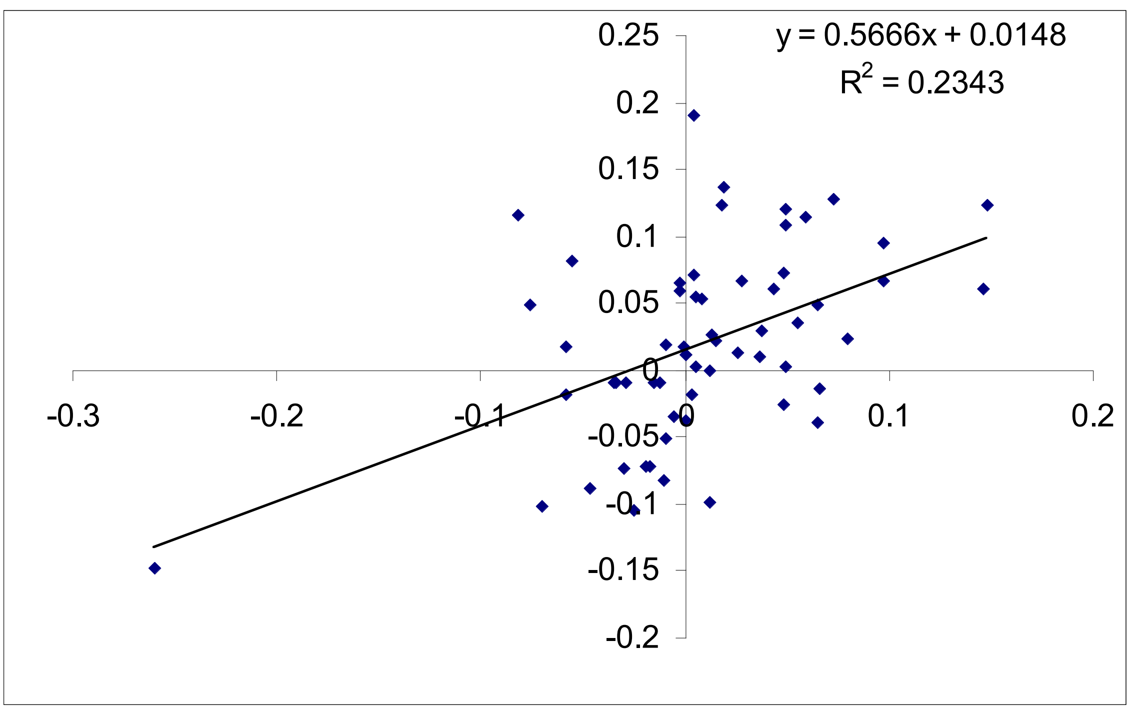
1b.

CONED: The line does not fit the data particularly well, thus the low R-squared.

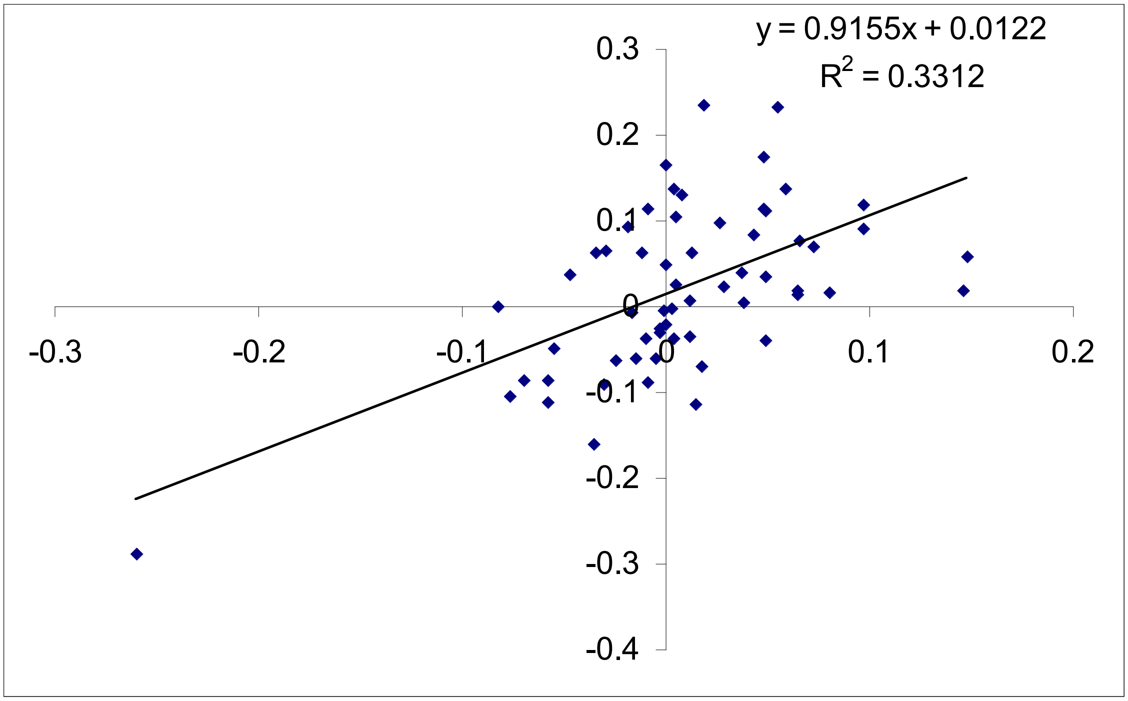




PSNH: The line does not fit much better than before. There is a large distribution of returns about the line. Again R-squared is low.

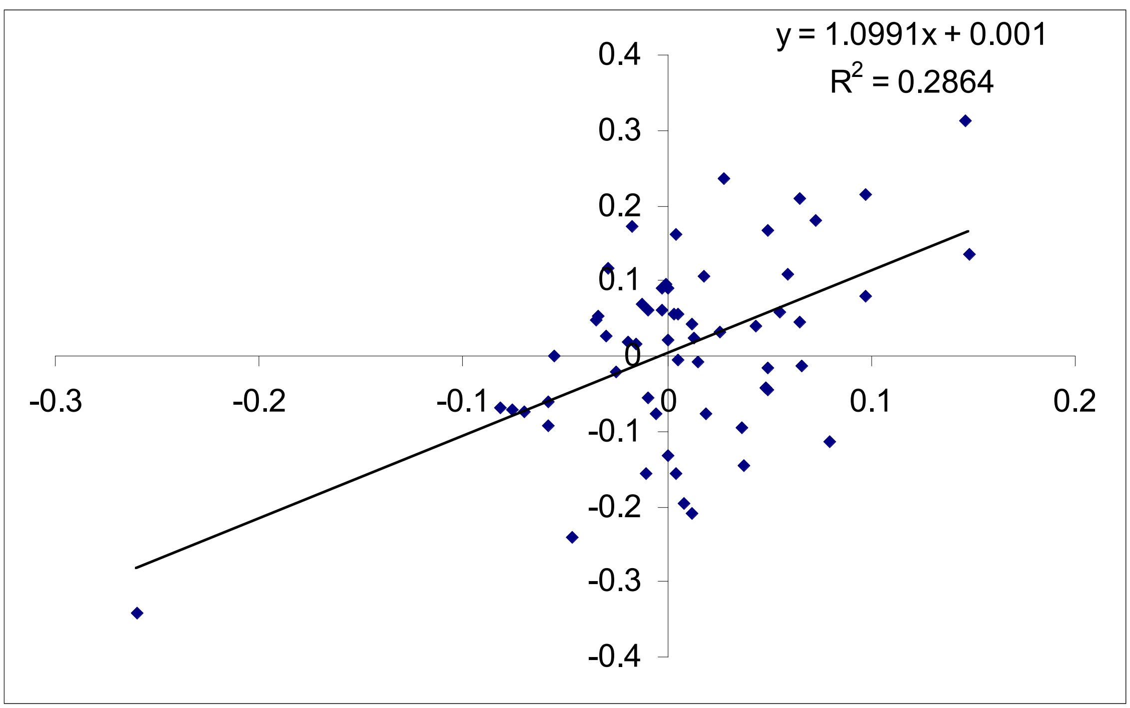


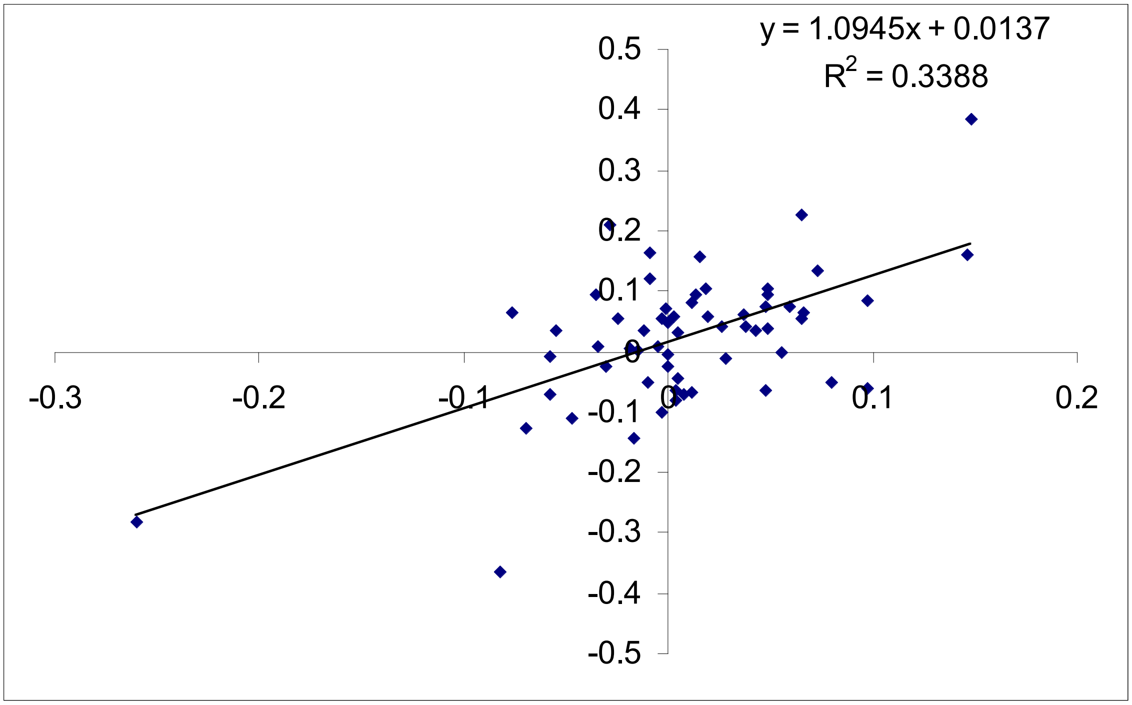
GENMIL: There is less of a distribution of returns about the line and R-squared is higher. Note the good fit of the 1987 crash.



GERBER: Again, the line fits better than before. There is less distribution about the line and the R-squared is higher.

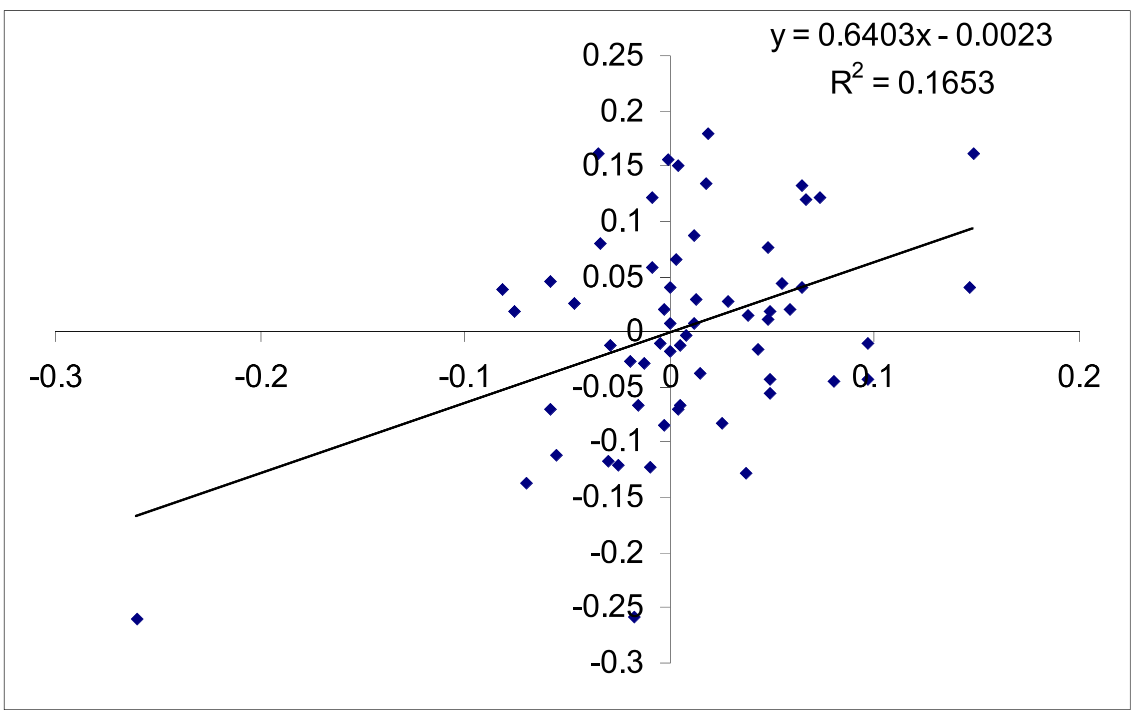
DATGEN: Same as above.

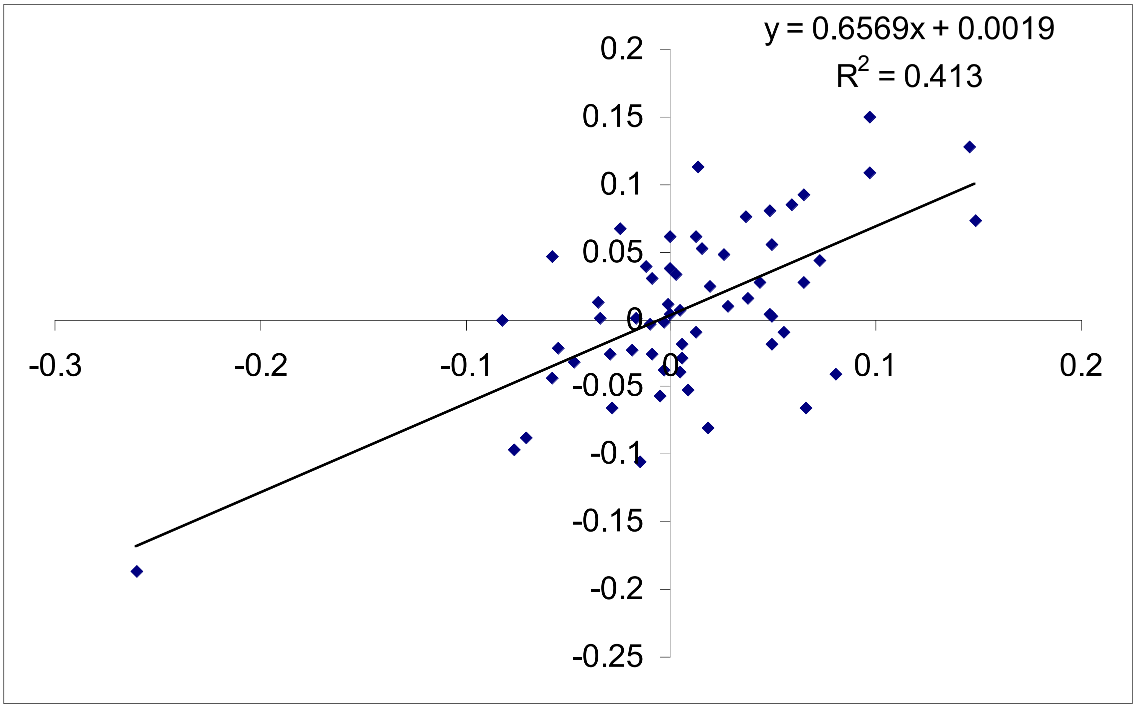




DEC: For the most part, the line fits pretty well. There are a few outliers, but the 1987 crash is fitted almost perfectly. The R-squared is high.

DELTA: Here there is a fairly large distribution of returns about the line, even though beta is relatively significant. Thus, R-squared is relatively small.





IBM: The fit of the line is quite good. There are few observations far away from it. This case has the highest R-squared.

1c.
CONED:
95% Interval

-0.1870	0.2369

	two-tailed t-test for beta	two-tailed t-test for beta
stat	-9.2083	-9.2083
p-value	0.0000	3.01001E-13

Can strongly reject.

PSNH:
95% Interval

-0.4628	0.8252

	two-tailed t-test for beta	one-tailed t-test for beta
stat	-2.5450	-2.5450
p-value	0.0136	0.006805293

Can reject.

GENMIL:
95% Interval

0.2974	0.8358

	t-test for beta	one-tailed t-test for beta
stat	-3.2228	-3.2228
p-value	0.0021	0.001042149

Can reject.

GERBER:
95% Interval

0.5735	1.2575

	t-test for beta	one-tailed t-test for beta
stat	-0.4947	-0.4947
p-value	0.6227	0.311342151

Cannot reject.

DATGEN:
95% Interval

0.6431	1.5551

	t-test for beta	one-tailed t-test for beta
stat	0.4350	0.4350
p-value	0.6652	0.332577312

Cannot reject.

DEC:
95% Interval

0.6926	1.4964

	t-test for beta	one-tailed t-test for beta
stat	0.4708	0.4708
p-value	0.6396	0.319788652

Cannot reject.

DELTA:
95% Interval

0.2621	1.0185

	t-test for beta	one-tailed t-test for beta
stat	-1.9038	-1.9038
p-value	0.0619	0.030948113

Cannot reject using two-tailed test, but can reject using one-tailed test. The question is whether you know that beta wasn't greater than 1 before looking at the data. Did you?

IBM:

95% Interval

0.4510	0.8627

	t-test for beta	one-tailed t-test for beta
stat	-3.3366	-3.3366
p-value	0.0015	0.000742147

Can reject.

No surprises.

1d.

CONED:

Proportion non-diversifiable (R-squared) < 1%

Proportion diversifiable (1 – R-squared) > 99%

Much less than typical stock.

PSNH:

Proportion non-diversifiable (R-squared) < 1%

Proportion diversifiable (1 – R-squared) > 99%

Much less than typical stock.

GENMIL:

Proportion non-diversifiable (R-squared) = 23%

Proportion diversifiable (1 – R-squared) = 77%

Less than typical stock.

GERBER:

Proportion non-diversifiable (R-squared) = 33%

Proportion diversifiable (1 – R-squared) = 67%

Close to typical stock.

DATGEN:

Proportion non-diversifiable (R-squared) = 29%

Proportion diversifiable (1 – R-squared) = 71%

Close to typical stock.

DEC:
Proportion non-diversifiable (R-squared) = 34%
Proportion diversifiable (1 – R-squared) = 66%
Close to typical stock.

DELTA:
Proportion non-diversifiable (R-squared) = 17%
Proportion diversifiable (1 – R-squared) = 83%
Less than typical stock.

IBM:
Proportion non-diversifiable (R-squared) = 41%
Proportion diversifiable (1 – R-squared) = 59%
More than typical stock.

Proportions are not surprising. IBM is a sizable part of total market and a well-diversified company, so there are fewer "company-specific" shocks that might affect it. Delta is much less diversified, so even though it moves with the market, it has a large proportion of "company-specific" risk.

1e.
In this sample, there is generally a positive relationship between R-squared and Beta. Given the following definition of R-squared:

$$R^2 = \frac{\beta^2 \sigma_m^2}{\beta^2 \sigma_m^2 + \sigma_\varepsilon^2},$$

we would expect the relationship to be exact if firms have roughly similar firm-specific risk. However, not all firms have the same level of firm-specific risk. Notably, IBM has half as much firm-specific risk as Delta. This explains why given very similar betas, IBM has a much higher R-squared than Delta.

2a.
The statistical assumption of the MM are:
i. (R_{it}, R_{jt}) *are jointly normally distributed for i,j=1,..,N and t=1,…,T.*
ii. $E[\varepsilon_{it}] = 0$
iii. $Var(\varepsilon_{it}) = \sigma_{\varepsilon i}^2$
iv. $\varepsilon_{it} \sim iidN(0, \sigma_{\varepsilon i}^2)$
v. ε_{it} *is independent of* R_{Mt}

The economic assumptions of the CAPM are:
i. *There are many investors who are price takers.*
ii. *All investors plan to invest over the same horizon.*
iii. *There are no taxes or transaction costs.*

iv. *Investors can borrow or lend at the same risk free rate.*

v. *Investors only care about expected return and variance. They like high expected return, but dislike high variance.*

vi. *All investors have the same information and beliefs about the distribution of returns.*

vii. *The market portfolio consists of all publicly traded assets.*

Relationship: $\alpha_i^* = \alpha_i - (1-\beta_{i,M})r_f$

CAPM restriction: $\alpha_i^* = 0 \Leftrightarrow \alpha_i = (1-\beta_{i,M})r_f$

2b.
Let $z_{it} \equiv r_{it} - r_{ft}$.

CONED:
$$\hat{z}_{it} = \underset{(0.0063)}{0.0135} + \underset{(0.1057)}{0.0200}\, z_{mt}, \quad R^2 = 0.0006$$

PSNH:
$$\hat{z}_{it} = -\underset{(0.0191)}{0.0235} + \underset{(0.3225)}{0.1805}\, z_{mt}, \quad R^2 = 0.0054$$

GENMIL:
$$\hat{z}_{it} = \underset{(0.0080)}{0.0124} + \underset{(0.1349)}{0.5670}\, z_{mt}, \quad R^2 = 0.2336$$

GERBER:
$$\hat{z}_{it} = \underset{(0.0101)}{0.0118} + \underset{(0.1712)}{0.9122}\, z_{mt}, \quad R^2 = 0.3286$$

DATGEN:
$$\hat{z}_{it} = \underset{(0.0135)}{0.0016} + \underset{(0.2284)}{1.0923}\, z_{mt}, \quad R^2 = 0.2829$$

DEC:
$$\hat{z}_{it} = \underset{(0.0119)}{0.0142} + \underset{(0.2012)}{1.0993}\, z_{mt}, \quad R^2 = 0.3398$$

DELTA:
$$\hat{z}_{it} = -\underset{(0.0112)}{0.0043} + \underset{(0.1894)}{0.63953}\, z_{mt}, \quad R^2 = 0.1643$$

IBM:

$$\hat{z}_{it} = 0.0000 + 0.6545 \; z_{mt}, \quad R^2 = 0.4106$$
$$(0.0061) \quad (0.1030)$$

2c.
CONED: t=2.152, so reject null
PSNH: t=-1.230, so do not reject
GENMIL: t=1.549, so do not reject
GERBER: t=1.159, so do not reject
DATGEN: t=0.1192, so do not reject
DEC: t=1.1932, so do not reject
DELTA: t=-0.3829, so do not reject
IBM: t=-0.0043, so do not reject

In the CONED case, the rejection may not reflect a failure of the CAPM, but of the market proxy (value-weighted returns on NYSE stocks).

3a.

$$\begin{aligned}
\beta_p &= \frac{\text{cov}(R_p, R_M)}{\text{var}(R_M)} \\
&= \frac{\text{cov}(x_A R_A + x_B R_B, R_M)}{\text{var}(R_M)} \\
&= \frac{\text{cov}(x_A R_A, R_M) + \text{cov}(x_B R_B, R_M)}{\text{var}(R_M)} \\
&= x_A \frac{\text{cov}(R_A, R_M)}{\text{var}(R_M)} + x_B \frac{\text{cov}(R_B, R_M)}{\text{var}(R_M)} \\
&= x_A \beta_A + x_B \beta_B, \qquad \textit{Q.E.D.}
\end{aligned}$$

3b.
EW portfolio:

$$\hat{r}_{pt} = 0.00517 + 0.6474 \; r_{mt}, \quad R^2 = 0.5474$$
$$(0.0046) \quad (0.0773)$$

beta=0.6474 is equal to the average of the individual Market Model betas.

3c.
Proportion non-diversifiable (R-squared) = 55%
Proportion diversifiable (1 – R-squared) = 45%
This proportion is greater than all of the proportions from question 1. This reflects the fact that much of the firm-specific risk is diversified away by constructing even a small portfolio of 8 firms using a equal weights.