

Notes on using Excel to solve Portfolio Theory Questions:

The basic input necessary to solve the portfolio question is an assumption about the joint distribution of the underlying assets that can be combined to form a portfolio. Following the CER model, we assume a joint normal distribution. This means the behaviour of the assets can be summarized completely by the means, variances, and covariance.

	R_A	R_B
E[R]	0.01	0.03
Var[R]	0.001	0.005
Std. Dev.	0.031623	0.070711
Cov(A,B)	-0.001	
Corr(A,B)	-0.44721	

Answering the portfolio question can be done by performing two sequential tasks. The first task is to find the investment possibilities set (a.k.a. the portfolio frontier). The second task is to find a specific point or set of points on the frontier.

The portfolio frontier can be represented graphically by plotting the mean vs. standard deviation of a portfolio for all possible portfolio weights. In a world with short-selling, consider negative weights. We can find the frontier by setting up a table.

To set up a table, start with an example of one of the entries. In this case, start with an example portfolio. An obvious example is a portfolio that is 100% in asset A and 0% in asset B.

x_A	sigma_p	mu_p
1	0.031623	0.01

In this case the formula under sigma_p is

`"=SQRT((F8^2)*B3+((1-F8)^2)*C3+2*F8*(1-F8)*B5)"`

and under mu_p is

`"=F8*B2+(1-F8)*C2",`

where F8 refers to the cell under x_A, \$B\$3 refers to the variance for asset A, \$C\$3 refers to the variance for asset B, \$B\$5 refers to the covariance for A and B, \$B\$2 refers to the mean for asset A, and \$C\$2 refers to the mean for asset B. Next, fill in below the 1 for all of the portfolio weights under considerations:

x_A	sigma_p	mu_p
1	0.031623	0.01
-1		
-0.9		
-0.8		

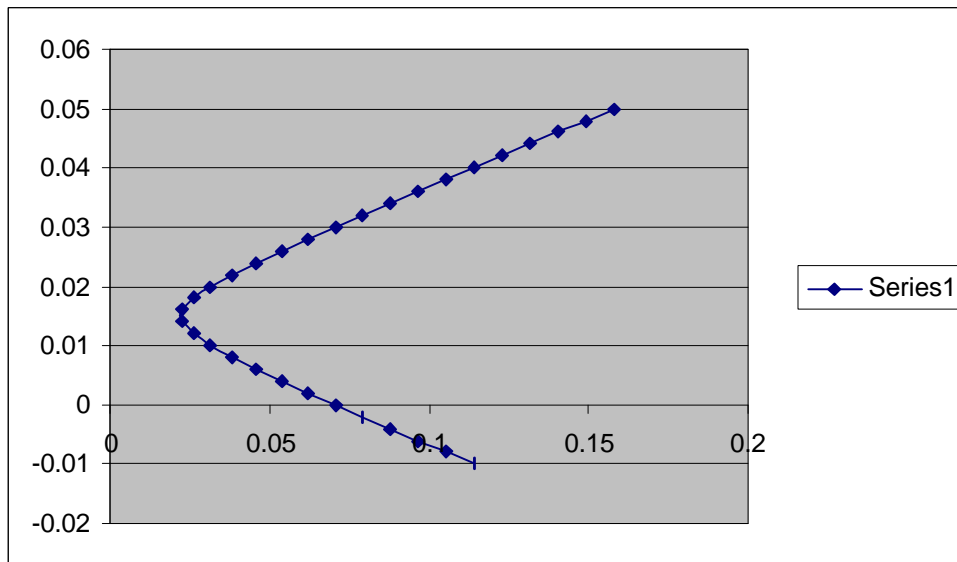
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Highlight the table (including the first example row) and select “Table...” from the Data menu. For the column input cell select the sample weight cell (e.g., the 1 below the x_A). Click Okay. The table should fill in as follows:

x_A	sigma_p	mu_p
1	0.031623	0.01
-1	0.158114	0.05
-0.9	0.149265	0.048
-0.8	0.140428	0.046
-0.7	0.131605	0.044
-0.6	0.122801	0.042
-0.5	0.114018	0.04
-0.4	0.105262	0.038
-0.3	0.09654	0.036
-0.2	0.087864	0.034
-0.1	0.079246	0.032
0	0.070711	0.03
0.1	0.06229	0.028
0.2	0.054037	0.026
0.3	0.046043	0.024
0.4	0.038471	0.022

0.5	0.031623	0.02
0.6	0.026077	0.018
0.7	0.022804	0.016
0.8	0.022804	0.014
0.9	0.026077	0.012
1	0.031623	0.01
1.1	0.038471	0.008
1.2	0.046043	0.006
1.3	0.054037	0.004
1.4	0.06229	0.002
1.5	0.070711	0
1.6	0.079246	-0.002
1.7	0.087864	-0.004
1.8	0.09654	-0.006
1.9	0.105262	-0.008
2	0.114018	-0.01

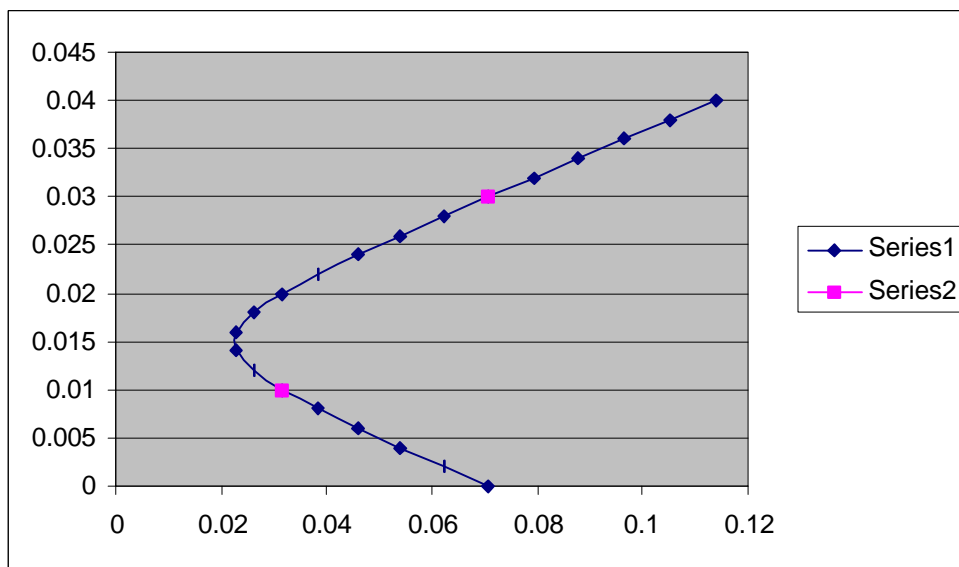
The second and third columns of this table can be used to plot the portfolio frontier using the XY (Scatter) plot with data points connected by a smooth line.



Excel tip: If there is an additional point (or points) on the line that you want to highlight, put its mean in an additional column and select it as well when forming the plot.

-0.5	0.114018	0.04	
-0.4	0.105262	0.038	
-0.3	0.09654	0.036	
-0.2	0.087864	0.034	
-0.1	0.079246	0.032	
0	0.070711	0.03	0.03
0.1	0.06229	0.028	
0.2	0.054037	0.026	

0.3	0.046043	0.024	
0.4	0.038471	0.022	
0.5	0.031623	0.02	
0.6	0.026077	0.018	
0.7	0.022804	0.016	
0.8	0.022804	0.014	
0.9	0.026077	0.012	
1	0.031623	0.01	0.01
1.1	0.038471	0.008	
1.2	0.046043	0.006	
1.3	0.054037	0.004	
1.4	0.06229	0.002	
1.5	0.070711	0	



Again, the second task in answering the portfolio question is to find specific points on the portfolio frontier. Suppose we want to find the tangency portfolio, we need to solve a maximization problem in which we maximize the Sharpe ratio.

Set up cells as below. The μ_p and σ_p cells are based on the weights given below x_a and x_b and the parameter values given at the top of this document. The risk free rate is set to 0.003. The sharpe ratio is calculated as $(\mu_p - r_f) / \sigma_p$. The cell below constraint is equal to the sum of the weights, which is, of course, 1.

μ_p	σ_p	r_f		
0.02	0.031623	0.003		
x_a	x_b	constraint	sharpe ratio	
0.5	0.5	1	0.537587	

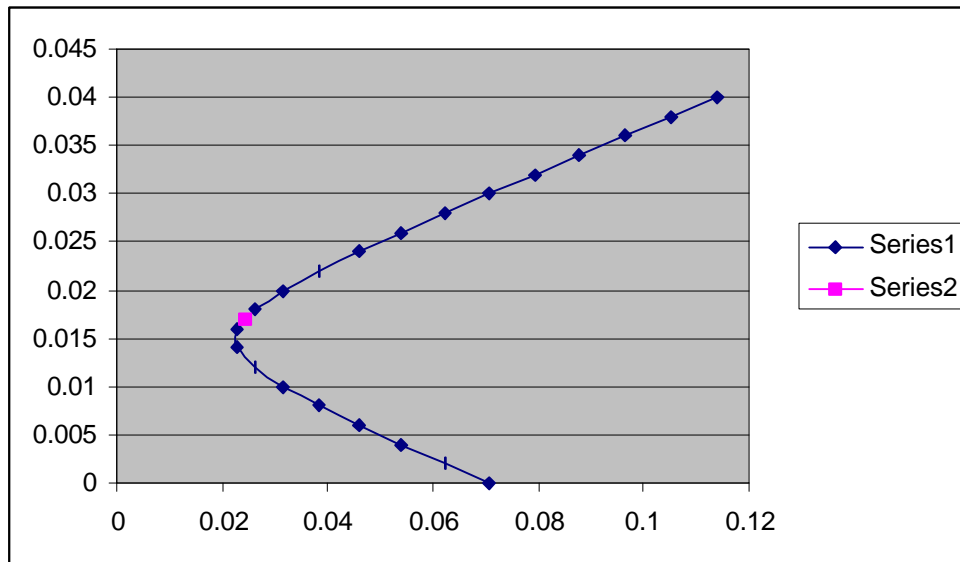
To use Excel solver, highlight the target cell (the one below the sharpe ratio). Select solver from the Tools menu. This is a maximization problem. The cells that are allowed

to vary are the two weight cells. The maximization problem is subject to the constraint that the cell below constraint is equal to 1. (Set the constraint box to show the following three things: "C20" "=" "=1", where C20 is the cell below constraint. Note that an equal sign must be in front of the 1.)

The solution should look as follows:

mu_p	sigma_p	r_f		
0.017083	0.024224	0.003		
x_a	x_b	constraint	sharpe	
			ratio	
0.645833	0.354167	1	0.581378	

We can plot this point on the frontier. This is the Tangency portfolio:

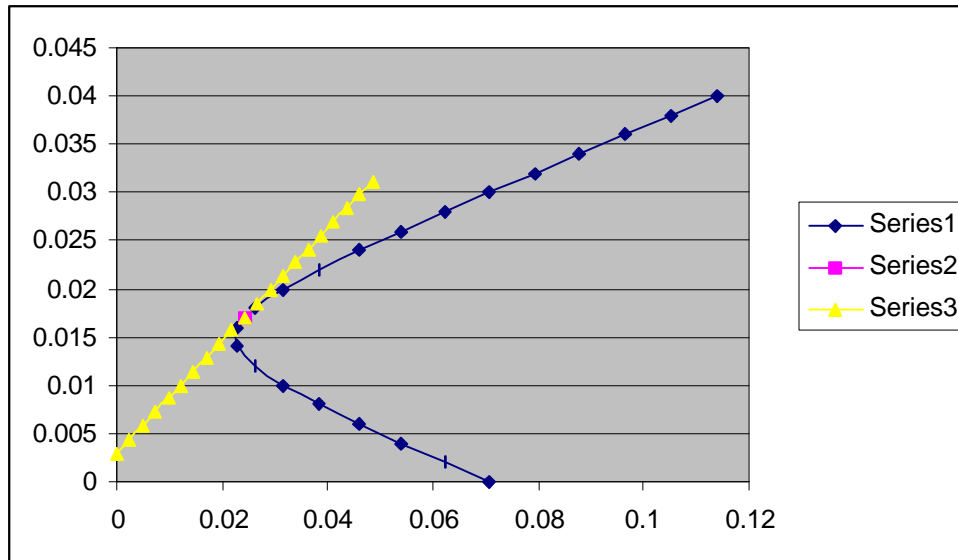


Even though the Tangency portfolio given above was calculated under the assumption of a risk free rate, the portfolio frontier assumes the existence of only two risky assets and no risk free asset. If there is also a risk free asset, then the portfolio frontier will look different. Indeed the only point above on the portfolio frontier in a world with 2 risky assets and a risk-free asset will be the tangency portfolio which is the special case of the weight on the risk free asset being zero.

Form a new table of portfolio means and standard deviations

x_f	sigma_p	mu_p
0	0.024224	0.017083

The formula below sigma_p is “=(1-F42)*B18” and below mu_p is “=0.003+(1-F42)*(A18-0.003)”, where F42 is the cell below x_f, B18 is the tangency portfolio standard deviation and A18 is the tangency portfolio mean calculated above.



Portfolio Theory using Matrix notation:

The procedures are much the same as before. The key is to name collections of cells after their matrix names. E.g., consider the following 3x3 variance covariance matrix

Sigma

0.001	-0.001	0.002
-0.001	0.005	0.001
0.002	0.001	0.009

The cells of the matrix should be highlighted and the whole group should be named, say, "Sigma". Then, using solver to find an efficient portfolio with a target expected return can work as follows:

Solving Using Solver

x	mu	ones	Constraint 1	Constraint 2	Objective
0	0.01	1	1	0.03	0.005
1	0.03	1			
0	0.05	1			

The three cells below x are named "x". The three cells below mu are named "mu". The three cells below ones are named "ones". Then, constraint 1 is the portfolio definition constraint and the cell below it reads "= $\text{MMULT}(\text{TRANSPOSE}(x), \text{ones})$ ". That is, it is $x'1_3$. Given the initial guesses for x, this cell satisfies the portfolio definition constraint.

Note: to calculate matrix multiplications using the "= $\text{mmult}(a,b)$ " command, one cannot simply type <enter> after entering the operation. Instead, one must type <ctrl><shift><enter>.

Constraint 2 is the target expected return constraint. In this case, the target expected return is the expected return on asset B=0.03. In the cell below is

“=MMULT(TRANSPOSE(x),mu)”. That is, $\mathbf{m}_{p,x} \equiv x' \mathbf{m}$.

Finally, in the objective function cell is the variance of portfolio x. It reads

“=MMULT(MMULT(TRANSPOSE(x),sigma),x)”. That is, $\mathbf{s}_{p,x}^2 \equiv x' \Sigma x$.

Solver works the same as before.