Washington University
Department of Economics

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Economics 518B

Homework #3
Exact MLE Using the Kalman Filter
Due: Thursday, 9/25

For this homework exercise you will estimate ARMA models of GDP growth using the Kalman filter. While you can easily do exact MLE using the conditional and the marginal densities for AR models, using the prediction error decomposition and the Kalman filter is the best way to do exact MLE for MA models and ARMA models. Meanwhile, it is important to remember that comparison of AIC and BIC statistics is best using the same sample period be used for each model considered. With exact MLE, it is straightforward to satisfy this requirement since the sample period is always the full sample (with conditional MLE the adjusted sample depends on the number of AR terms).

1. GAUSS Exercise

a. Save “hw3.prg” and “gdpq.prn” from the class website.

b. Open GAUSS. Select Edit from the file menu and open “hw3.prg”. This program allows you to calculate exact maximum likelihood estimates for ARMA(2,2) and lower order models. The program is initially set to estimate an ARMA(2,2) model. Run the program for the ARMA(2,2) case and copy your output to a file. Calculate the AIC and BIC by hand:

$$AIC = -\frac{2}{T} l_i(\hat{\theta}_i) + \frac{2}{T} k_i$$

$$BIC = -\frac{2}{T} l_i(\hat{\theta}_i) + \frac{\ln(T)}{T} k_i$$

Note: $l_i(\hat{\theta}_i)$ refers to the log likelihood and k_i refers to the number of parameters for model i. Also, in Hamilton, EViews, and throughout the course, assume “log” refers to natural logarithms unless otherwise specified. I will try to write “ln” when referring to this operator.

c. It is straightforward to change the program in order to estimate lower order models. Run the program for each case mentioned below. Append your output to same file and calculate the AIC and BIC.

Suppose you want to estimate an ARMA(2,1). On line 21 (initial values for estimation), you should remove the last parameter. On line 62, you should change “theta2=prmtr[6];” to “theta2=0;”. On lines 148 and 156, you should remove “/*” and “*/” to constrain estimation of the MA(1) parameter to invertible case. On lines 137 and 145 you should add "/*" and "*/", respectively. Run the program, record output, and calculate the AIC and BIC.

I'll let you figure out how to estimate the other cases. Don't forget the ARMA(0,0) case when considering AIC and BIC statistics.

d. Did you pick the same model(s) using the AIC and BIC as you did for homework #2? Compare results for the same set of models.

e. Modify the initial program to estimate the ARMA(2,2) model using the following alternative state-space structure for the model:

$$\beta_t = \begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix}, \; F = \begin{bmatrix} \phi_1 & \phi_2 & \theta_1 & \theta_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; Q = \begin{bmatrix} \sigma^2 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma^2 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; H' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Hint: Remember that your state vector is 4x1 instead of 3x1 when setting up the initial state vector.

Run the program for the ARMA(2,2) case only. Append your output to the same file. Also, append the part of the likelihood procedure that you changed in order to estimate this alternative state-space structure. Are your estimates the same as before?