



What factors drive the price–rent ratio for the housing market? A modified present-value analysis



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ABSTRACT

We consider which factors determined the price–rent ratio for the housing market in 18 U.S. metropolitan statistical areas (MSAs) and at the national level over the period of 1975–2014. Based on a present-value framework, our proposed empirical model separates the price–rent ratio for a given market into unobserved components related to the expected real rent growth and the expected housing return, but is modified from standard present-value analysis by also including a residual component that captures non-stationary deviations of the price–rent ratio from its present-value level. Estimates for the modified present-value model suggest that the present-value residual (PVR) component is always important and sometimes very large at the national and MSA levels, especially for MSAs that have experienced frequent booms and busts in the housing market. In further analysis, we find that house prices in MSAs that have larger PVR components are more sensitive to mortgage rate changes. These are also the MSAs with less elastic housing supply. Also, comparing our results with a recent statistical test for periodically-collapsing bubbles, we find that MSAs with large estimated PVR components are the same MSAs that test positively for explosive sub-periods in their price–rent ratios, especially during the 2005–2007 subsample. Our approach allows us to estimate the correlation between shocks to expected rent growth, the expected housing return, and the PVR component. We find that the expected housing return and movements in the PVR component are highly positively correlated implying an impact of the expected housing return on house prices that is amplified from what a standard present-value model would imply. Our results also show that most of the variation in the present-value component of the price–rent ratio arises due to the variation in the expected housing return.

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1. Introduction

The financial crisis of 2008–2009 had its roots in the boom and the bust of the U.S. housing market. The collapse of house prices led to the overall decline in financial and macroeconomic stability, starting with a big decline in the stock market.¹ The sustained increase in house prices prior to 2007 attracted widespread attention from the empirical researchers. A big portion of the housing market literature has focused on the price–rent ratio as a metric to measure the extent of

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¹ It has been estimated that the net worth of the U.S. households declined by \$13 trillion dollars between 2007 and 2009 (flow of funds data).

overvaluation in the housing market. Most of these empirical studies have used some version of the present-value model of house prices to examine the sources of variation in the price–rent ratio and have found a mixture of results depending on which market is considered.²

The present-value model of house prices is based on [Campbell and Shiller's \(1988\)](#) model for asset prices, which has been applied extensively in the finance and exchange rate literature. The housing market version of this model implies that the current price–rent ratio reflects households' expectations about future rent growth and future housing returns. In particular, the price–rent ratio according to the conventional present-value model has the following representation:

$$p_t - r_t = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t(\Delta r_{t+j} - h_{t+j}) \quad (1)$$

where $p_t - r_t$ is the log of the price–rent ratio, Δr_t is the rent growth, h_t is the housing return, ρ is the discount factor and κ is a constant. The above model suggests that the log of the price–rent ratio can be expressed as the expected discounted sum of future rent growth minus future housing returns. If expected rent growth and the expected housing return are both stationary, then the price–rent ratio should also be stationary. Intuitively, this implies that if there is any deviation from the long-run equilibrium value, the price–rent ratio should self-correct. An upward surprise in the price–rent ratio today must correspond to news that future housing returns will be higher or to a downward revision in expected rent growth. The conventional analysis of the housing market takes this approach and assumes that the price–rent ratio is stationary.

The evidence, however, clearly suggests that the price–rent ratio is non-stationary. [Table 1](#) shows results for unit root tests of the log of price–rent ratio for 18 U.S. metropolitan areas (MSAs) and the nation.³ The results overwhelmingly support the presence of a unit root in the price–rent ratio, even when allowing for a structural break in the mean. This finding is not driven by the presence of a unit root in either rent growth or the housing return, as these fundamental variables are stationary. One explanation of the non-stationarity of the price–rent ratio is that there are some other factors that drive the variation in price–rent ratio and these factors are non-stationary. This is consistent with the nature of U.S. housing markets. Unlike stock markets, the functioning of the housing markets in the U.S. is characterized by illiquidity, high transaction costs, differential tax regimes, and zoning laws. To take into account these features of the housing market, we propose a modified present-value model that decomposes the price–rent ratio into the present-value of expected house price growth, the present-value of expected rent growth, and a present-value residual (PVR) component that captures non-stationary factors.

We take an unobserved component approach to estimate a modified present-value model for the U.S. and each of the 18 MSAs. Our framework explicitly takes into account the fact that the price–rent ratio may move due to changes in expected return to housing, expected rent growth variation, and a PVR component that cannot be accounted for by a conventional model.⁴ We treat expected rent growth, expected housing return, and the PVR component as unobserved (to the econometrician) variables that follow exogenously specified time series processes. In particular, we assume a parsimonious AR(1) specification for both expected rent growth and expected housing return and a random walk specification for the PVR component. Because these latent variables are estimated using the Kalman filter, by construction we use information from the whole history of past realized rent growth, realized housing return, and the price–rent ratio when making inferences. We express realized variables as the sum of an expected component and an error term that is unforecastable.

Only a few other studies have used a present-value model to examine the determinants of the price–rent ratio for housing. Notably, [Campbell et al. \(2009\)](#) employ a reduced-form VAR approach to explain the movements in the price–rent ratio. They measure expectations by fixed coefficient VAR model. The VAR is used to directly compute expected future housing returns and then use an accounting identity to identify expected future rents as a residual given data on rent–price ratios. Recently, [Fairchild et al. \(2015\)](#) use a dynamic factor model in the present-value framework and estimate the relative share of national and local share in variation of the price–rent ratio. They also use data from the U.S. and 17 MSAs. They find that a large fraction of the variation is based on local factors. In their analysis, they allow for a pricing error in the present-value relationship, but their model also assumes that this pricing error is stationary in nature. In other related work, [Ambrose et al. \(2013\)](#) use 355 years of data for Amsterdam and find that deviations of house prices from their fundamentals are long-lasting and persistent.

Our approach has several advantages over more conventional analysis. First, we are not aware of any study that takes into account the non-stationarity of the price–rent ratio and modifies the present-value model accordingly. This also allows

² Using a present-value model, [Case and Shiller \(2003\)](#) argue that the U.S. housing market in 2004 was over-valued because the price–rent ratio was significantly above its historical average. However, [Himmelberg et al. \(2005\)](#) find no evidence of a bubble in 2004 in any of the regional markets. Using data from Northern California, [Meese and Wallace \(1994\)](#) reject both constant and time-varying discount rate versions of the housing price present-value relation in the short run. Long-run results are consistent with the housing price present-value relation when they adjust the discount factor for changes in both tax rates and borrowing costs for their 1970–1988 sample period.

³ The 18 MSAs in our study are Atlanta, Boston, Chicago, Cleveland, Dallas, Denver, Houston, Los Angeles, Miami, Milwaukee, Minneapolis, New York, Philadelphia, Pittsburgh, Portland, San Francisco, Seattle and St. Louis. Our MSA sample is based on data availability for rent from the BLS for our sample period (1975–2014).

⁴ Present-value models have also been studied extensively in finance and exchange rate literature to study the behavior of equity market and exchange rates. For example, [Balke and Wohar \(2002\)](#) apply a state-space/present-value model of stock prices to estimate what drives low-frequency movements in the price-dividend ratio. [Binsbergen and Koijen \(2010\)](#) follow a similar approach to estimate the expected stock returns, and apply it to predict stock returns. For application of present-value models to exchange rates, see [Engel and West \(2004, 2005\)](#) among others.

Table 1
Unit root tests.

City	Phillips–Perron P -value			Zivot–Andrews statistic
	$p_t - r_t$	Δr_t	Δh_t	$p_t - r_t$
USA	0.47	0.00	0.00	–4.29
Atlanta	0.22	0.00	0.00	–3.65
Boston	0.49	0.00	0.04	–3.61
Chicago	0.28	0.00	0.00	–4.13
Cleveland	0.73	0.00	0.00	–3.34
Dallas	0.53	0.00	0.00	–3.89
Denver	0.34	0.00	0.00	–3.65
Houston	0.49	0.00	0.00	–3.67
Los Angeles	0.17	0.00	0.02	–3.99
Miami	0.29	0.00	0.00	–4.19
Milwaukee	0.44	0.00	0.00	–2.46
Minneapolis	0.55	0.00	0.00	–3.70
New York City	0.43	0.00	0.04	–3.83
Philadelphia	0.48	0.00	0.00	–4.00
Pittsburgh	0.35	0.00	0.00	–3.01
Portland	0.53	0.00	0.00	–3.02
San Francisco	0.17	0.00	0.00	–3.99
Seattle	0.41	0.00	0.00	–3.71
St. Louis	0.47	0.00	0.00	–3.81

Null hypothesis implies a unit root. The test equation includes a constant. The results for the Phillips–Perron test are robust for other unit root tests. The critical values for Zivot–Andrews Test are –4.58, –4.93 and –5.34 for 10%, 5% and 1% significance level, respectively.

us to observe the time-variation in the deviation of the price–rent ratio from its present-value. Moreover, because future rent growth and housing return are unobserved to econometricians, an unobserved component model is more suitable to model the housing market than an approach that assumes they are observable. Third, as pointed out by [Cochrane \(2008\)](#), a structural state-space model is able to capture individually small but possibly important moving average error terms in the long run. Another important contribution of our work is that our approach allows us to estimate the correlation between expected rent growth, expected housing return and the PVR component. This allows us to examine the dynamic interaction between agents' expectation about the future rents and housing returns and the extent of deviation of the price–rent ratio from its present-ratio.

Our empirical estimates suggest that the PVR component is significant both at the national and regional levels. This is especially true for MSAs that have experienced frequent booms and busts in the housing market. Our results show that this deviation was biggest in Boston, Los Angeles, Miami, New York, San Francisco and Seattle, whereas the absolute value of this deviation was between 0 and 15% of the price–rent ratio in MSAs like Atlanta, Chicago, Dallas, Denver, Houston, Philadelphia, Pittsburgh, and St. Louis. We also show that MSAs with high PVR component are also the MSAs with high degree of regulation in land use and inelastic housing supply. In further analysis, we find a negative relationship between the PVR component and mortgage interest rates. This negative relationship is large and significant for all the MSAs for the 1991–2005 sample period. More importantly, we also find that the MSAs that display larger deviations from their present-value levels are more sensitive to mortgage rate changes. This reflects the heterogeneity in the response of housing markets across different MSAs to interest rates.

Our approach also allows us to estimate the correlation between expected rent growth, expected housing return, and the PVR component. We find that shocks to the expected housing return and shocks to the non-stationary PVR component are highly positively correlated. This positive correlation may imply that, if there is a positive shock to the PVR component, expected housing return may also increase, corresponding to a positive feedback effect. One could think of the shock to this PVR component arising from some regulatory changes or through monetary policy actions. For example, adoption of automated underwriting systems at Fannie/Freddie in the late-1990s and early 2000s, as well as the mid-2000s relaxation of underwriting standards are example of regulatory changes that can lead to positive correlation between expected housing return and the PVR component. Similarly, a reduction in transaction costs associated with housing market may enter as a positive shock to the PVR component that may also push expected housing return higher. We also find positive correlation between expected rent growth and expected housing return. This positive correlation is intuitive because a shock to expected housing return is also expected to lead to an increase in expected rent growth.

For comparison, we also consider the supADF test of [Phillips et al. \(2011\)](#). This test gives rise to a dating strategy which identifies points of origination and termination of possible bubbles that may reflect exuberation or herd behavior. Overall, the results from the supADF test and our model seem to indicate that the MSAs that had large PVR components are the same MSAs that witnessed explosive sub-periods in their price–rent ratios, especially during the 2005–2007 subsample.

The plan of this paper is as follows: [Section 2](#) proposes a modified present-value model. [Section 3](#) describes the data. [Section 4](#) discusses the empirical results. [Section 5](#) compares results from our model with [Phillips et al. \(2011\)](#) explosive bubble test. [Section 6](#) concludes.

2. Model specification

2.1. An unobserved component approach to estimating a modified present-value model of house prices

In this section, we present a modified present-value model of the price–rent ratio in the spirit of [Campbell and Shiller \(1988\)](#) and [Binsbergen and Kojien \(2010\)](#). In contrast to [Campbell et al. \(2009\)](#) who estimate expected housing return and expected rent growth from a VAR model, we assume that the expected house price return and expected real rent growth are latent variables and there is a non-stationary deviation from the long-run stationary value of the price–rent ratio represented by the conventional present-value model. Therefore, we can express the log price–rent ratio as the sum of three pieces: the future expected housing return,⁵ rent growth, and a non-stationary residual term

$$p_t - r_t = \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t(\Delta r_{t+j} - h_{t+j}) + pvr_t \quad (2)$$

We assume that expected rent growth and expected housing return are latent variables. We follow a parsimonious modeling strategy by modeling expected rent growth and expected housing return as AR(1) processes, while we assume that the PVR component follows a random walk process:

$$\Delta r_{t+1}^e = \gamma_0 + \gamma_1(\Delta r_t^e - \gamma_0) + \varepsilon_{t+1}^r, \varepsilon_{t+1}^r \sim iidN(0, \sigma_{r^e}^2) \quad (3)$$

$$h_{t+1}^e = \delta_0 + \delta_1(h_t^e - \delta_0) + \varepsilon_{t+1}^h, \varepsilon_{t+1}^h \sim iidN(0, \sigma_{h^e}^2) \quad (4)$$

$$pvr_{t+1} = pvr_t + \varepsilon_{t+1}^{pvr}, \varepsilon_{t+1}^{pvr} \sim iidN(0, \sigma_{pvr}^2) \quad (5)$$

where

$$h_t^e = E_t[h_{t+1}]$$

$$\Delta r_t^e = E_t[\Delta r_{t+1}]$$

The realized rent growth and realized housing return are equal to the expected rent growth and expected housing return plus an idiosyncratic shock:

$$\Delta r_{t+1} = \Delta r_t^e + \varepsilon_{t+1}^r \quad (6)$$

$$h_{t+1} = h_t^e + \varepsilon_{t+1}^h \quad (7)$$

Plugging Eqs. (3)–(7) in (2) and solving, we get

$$p_t - r_t = \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho} + \frac{\Delta r_t^e - \gamma_0}{1-\rho\gamma_1} - \frac{h_t^e - \delta_0}{1-\rho\delta_1} + pvr_t \quad (8)$$

which can be written as

$$p_t - r_t = A + B_1(\Delta r_t^e - \gamma_0) - B_2(h_t^e - \delta_0) + pvr_t \quad (9)$$

where $A = \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho}$, $B_1 = \frac{1}{1-\rho\gamma_1}$, $B_2 = \frac{1}{1-\rho\delta_1}$. The log price–rent ratio is linear in the expected rent growth r_t^e , and expected housing return h_t^e and the residual term pvr_t . The loadings (B_1 and B_2) depend on the persistence of rent growth and the housing return. There are five shocks in the model, a shock to expected rent growth (ε_t^r), a shock to expected housing return (ε_t^h), a shock to the PVR component (ε_t^{pvr}), a shock to realized rent growth (ε_t^r), and a shock to the realized housing return (ε_t^h).

2.2. State space representation

The present-value model of the house price–rent ratio has three latent variables: expected rent growth, Δr_t^e , expected housing return, h_t^e , and the residual term pvr_t . We define the demeaned state variables as

$$\Delta \widehat{r}_t^e = \Delta r_t^e - \gamma_0 \quad (10)$$

$$h_t^e = \delta_0 + \widehat{h}_t^e \quad (11)$$

There are three transition equations associated with above demeaned latent variables

$$\Delta \widehat{r}_{t+1}^e = \gamma_1 \widehat{\Delta r}_t^e + \varepsilon_{t+1}^r \quad (12)$$

⁵ Note that $h_{t+1} = \log\left(\frac{p_{t+1} + R_{t+1}}{p_t}\right)$.

$$\widehat{h}_{t+1}^e = \delta_1 \widehat{h}_t^e + \varepsilon_{t+1}^{h^e} \tag{13}$$

$$pvr_{t+1} = pvr_t + \varepsilon_{t+1}^{pvr} \tag{14}$$

and three measurement equations

$$\Delta r_{t+1} = \gamma_0 + \widehat{\Delta r}_t^e + \varepsilon_{t+1}^r \tag{15}$$

$$h_{t+1} = \delta_0 + \widehat{h}_t^e + \varepsilon_{t+1}^h \tag{16}$$

$$p_t - r_t = A + B_1(\Delta r_t^e - \gamma_0) - B_2(h_t^e - \delta_0) + pvr_t \tag{17}$$

The transition equation has the following variance–covariance matrix:

$$Q = \begin{bmatrix} \sigma_{r^e}^2 & \sigma_{r^e h^e} & \sigma_{r^e pvr} \\ \sigma_{r^e h^e} & \sigma_{h^e}^2 & \sigma_{h^e pvr} \\ \sigma_{r^e pvr} & \sigma_{h^e pvr} & \sigma_{pvr}^2 \end{bmatrix}$$

The measurement equation has the following variance–covariance matrix:

$$R = \begin{bmatrix} \sigma_r^2 & \sigma_{rh} \\ \sigma_{rh} & \sigma_h^2 \end{bmatrix}$$

As suggested by [Cochrane \(2008\)](#), we need to impose restrictions on the covariance structure in the above state space model to achieve identification.⁶ We follow [Binsbergen and Koijen \(2010\)](#) identification strategy and assume that shocks to realized variables are uncorrelated with shocks to unobserved state variables. Also, we assume that the shocks to realized rent growth and realized housing return are uncorrelated ($\sigma_{rh} = 0$). Our approach allows us to estimate the correlation between different shocks in the transition equation. We can estimate the above state space model using maximum likelihood via the Kalman filter.

2.3. Variance decomposition of the present-value level

The stationarity of the present-value components of the price–rent ratio allows us to perform a variance decomposition using Eq. (9). The variance decomposition of the present-value level of the price–rent ratio is defined as

$$\begin{aligned} var(p_t^* - r_t^*) &= B_1^2 var(\Delta r_t^e) + B_2^2 var(h_t^e) - 2B_1 B_2 cov(r_t^e, h_t^e) \\ var(p_t^* - r_t^*) &= \frac{(B_1 \sigma_{r^e})^2}{1 - \gamma_1^2} + \frac{(B_2 \sigma_{h^e})^2}{1 - \delta_1^2} - \frac{2B_1 B_2 \sigma_{r^e h^e}}{1 - \gamma_1 \delta_1} \end{aligned} \tag{18}$$

where $p_t^* - r_t^*$ corresponds to the present-value level of the price–rent ratio $p_t - r_t$. The above formula implies that proportion of variation of present-value level of the price–rent ratio explained by expected rent growth is $\frac{(B_1 \sigma_{r^e})^2}{1 - \gamma_1^2}$, and percentage of variation explained by housing return is $\frac{(B_2 \sigma_{h^e})^2}{1 - \delta_1^2}$. It may also be possible that the covariance explains some of the variation in the stationary component of the price–rent ratio.

3. Data description

We use quarterly data and our sample runs from 1975:Q1 through 2014:Q2. The data on house prices are from Freddie Mac. Rent data is the rent of primary residences from the BLS. Some researchers have used owner’s equivalent rent of residences as a measure of rent, but the sample period for that series only begins in 1982. We convert the nominal rent growth and house price growth to real growth rates by deflating nominal rents and house prices by CPI of each MSAs and the nation. The quarterly data for CPI and the rent have been computed by taking the monthly averages. The monthly data are not available for all the MSAs in our sample, so we take the average of available month within the quarter to calculate the quarterly estimate. For example, if data for only January and March are available, we take the average of January and March for the first quarter. In a few MSAs, only semi-annual data were available in the initial sample period. To convert these into quarterly frequency, we use linear match frequency converter method. We do not need to make this conversion for most of the MSAs.⁷ To use the price–rent ratio for the present-value model, we first normalize the data using the initial data for price and rent from [Davis et al. \(2008\)](#) and convert the data from indices to dollar values. Once we calculate the ratio and price and rent for the nation, we normalize the ratio for all the MSAs at the beginning of the sample to equal the ratio for the U.S. This also provides us a benchmark to compare the evolution of the price–rent ratio of different MSAs across time. The formula for calculating housing return is $h_{t+1} = \log\left(\frac{P_{t+1} + R_{t+1}}{P_t}\right)$ where P_{t+1} is the real house price and R_{t+1} is the real

⁶ Also, see [Morley et al. \(2003\)](#) on identification of unobserved components models with a general variance-covariance matrix.

⁷ In our sample, only Atlanta and Miami have missing data.

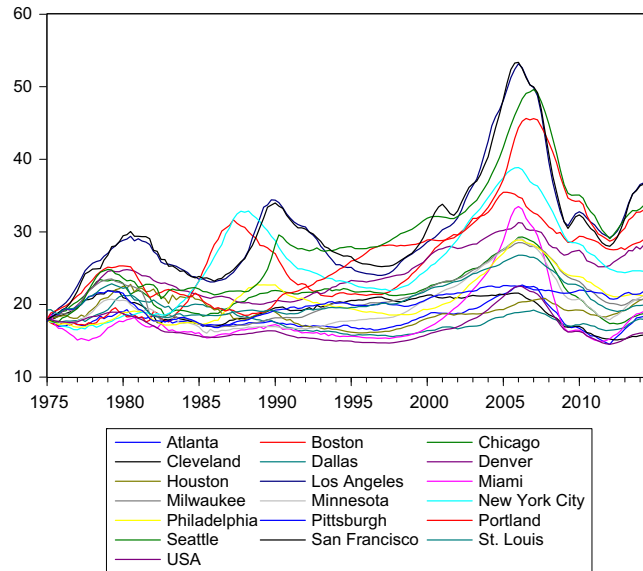


Fig. 1. Price-rent ratios.

rent. Our MSA sample is based on data availability for rent from the BLS for our sample period. The growth rate is calculated as the quarterly change of the log level and is annualized. The mortgage rate is 30-year conventional rate and has been obtained from the FRED data base.

4. Empirical results

As discussed in the Introduction of the paper, we find strong evidence in support of the presence of a unit root in the price–rent ratio. Our findings are not surprising as visual inspection of the price–rent ratios in the United States and the MSAs in Fig. 1 makes it clear that the ratio is extremely persistent in every case. To compare the relative variation in the price–rent ratio across different MSAs, we normalize the ratio for all the MSAs at the beginning of the sample to equal the ratio for the U.S. Fig. 1 clearly shows that the price–rent ratio in the coastal cities on average are higher and more volatile. San Francisco and Los Angeles have the highest and the most volatile price–rent ratios in our sample.

Our model assumes that rent growth and housing return are stationary. To test our assumption, we also perform a unit root test for these variables. The results are reported in the second and third columns of Table 1. The results clearly show that we can reject the null hypothesis of unit root at conventional significance levels for all the MSAs and the nation. Meanwhile, one issue that has attracted significant attention in macro and finance literature with regard to a unit root test is whether results are robust to allowing for structural breaks. For example, Lettau and Van Nieuwerburgh (2008) report evidence of structural breaks in the mean of the price–dividend ratio for the stock market. Therefore, we also examine whether our unit root test results for the price–rent ratio are robust to allowing for structural breaks. For this purpose, we use the Zivot and Andrews (1992) unit root test that takes into account a structural break in mean. The results are reported in Table 1. The results clearly show that the null hypothesis of unit root is not rejected, even at a 10% level of significance. To summarize then, our results clearly suggest the presence of a unit root in price–rent ratio, and this conclusion is robust to consideration of a structural break in mean and is despite the fact that rent growth and housing market returns are stationary for all the MSAs and the nation.

To estimate the modified present-value model, we cast Eqs. (3)–(9) into state space form and apply the Kalman filter to estimate the hyperparameters of the model.⁸ The estimated hyperparameters are shown in Table 2 and 3. The unconditional mean of expected rent growth and expected housing return (γ_0 and δ_0) vary across different MSAs. It can be clearly seen that the mean of expected real rent growth is much smaller in magnitude than the expected housing return implying that in all the MSAs and the United States, rents have grown roughly at the same rate as the overall inflation. The results suggest that the persistence parameter (AR coefficient δ_1) for the expected housing return is much higher than for the expected rent growth. The high persistence of expected housing return is similar to what researchers have found for expected financial asset returns in the finance literature.⁹

⁸ Measurement and transition equations for the state space model are provided in the Appendix.

⁹ For example, Binsbergen and Kojien (2010), Fama and French (1988), Pastor and Stambaugh (2009) among others have also found the expected return on stocks to be highly persistent.

Table 2
Parameter estimates.

	σ_{r^e}	σ_{h^e}	σ_{pvr}	σ_r	σ_h	γ_0	γ_1	δ_0	δ_1
USA	0.002 (0.001)	0.006 (0.001)	0.066 (0.012)	0.010 (0.001)	0.001 (0.002)	0.002 (0.002)	0.899 (0.030)	0.063 (0.002)	0.915 (0.015)
Atlanta	0.010 (0.001)	0.013 (0.001)	0.105 (0.011)	0.004 (0.001)	0.002 (0.001)	0.001 (0.002)	0.669 (0.035)	0.056 (0.003)	0.886 (0.013)
Boston	0.008 (0.001)	0.012 (0.001)	0.154 (0.018)	0.006 (0.001)	0.001 (0.001)	0.002 (0.001)	0.429 (0.051)	0.043 (0.002)	0.932 (0.009)
Chicago	0.008 (0.001)	0.012 (0.001)	0.096 (0.005)	0.004 (0.001)	0.001 (0.003)	0.001 (0.001)	0.525 (0.047)	0.047 (0.002)	0.888 (0.001)
Cleveland	0.009 (0.001)	0.012 (0.001)	0.072 (0.011)	0.004 (0.001)	0.002 (0.001)	−0.001 (0.002)	0.587 (0.051)	0.051 (0.003)	0.849 (0.025)
Dallas	0.009 (0.001)	0.012 (0.001)	0.100 (0.015)	0.004 (0.001)	0.002 (0.003)	0.001 (0.002)	0.663 (0.057)	0.057 (0.003)	0.897 (0.017)
Denver	0.007 (0.001)	0.009 (0.001)	0.101 (0.030)	0.001 (0.001)	0.005 (0.001)	0.001 (0.006)	0.924 (0.043)	0.043 (0.005)	0.896 (0.021)
Houston	0.016 (0.001)	0.014 (0.001)	0.138 (0.004)	0.004 (0.001)	0.001 (0.001)	0.001 (0.005)	0.743 (0.053)	0.053 (0.006)	0.907 (0.013)
Los Angeles	0.005 (0.001)	0.012 (0.001)	0.130 (0.005)	0.005 (0.001)	0.001 (0.001)	0.002 (0.001)	0.474 (0.036)	0.036 (0.002)	0.914 (0.006)
Miami	0.005 (0.001)	0.015 (0.001)	0.126 (0.007)	0.007 (0.001)	0.001 (0.001)	0.001 (0.001)	0.315 (0.081)	0.058 (0.002)	0.891 (0.016)
Milwaukee	0.006 (0.001)	0.015 (0.001)	0.106 (0.001)	0.001 (0.001)	0.007 (0.001)	−0.001 (0.001)	0.698 (0.039)	0.047 (0.002)	0.877 (0.018)
Minneapolis	0.005 (0.001)	0.011 (0.001)	0.138 (0.005)	0.005 (0.001)	0.001 (0.000)	−0.001 (0.001)	0.454 (0.052)	0.051 (0.001)	0.931 (0.009)
New York City	0.003 (0.001)	0.010 (0.001)	0.156 (0.020)	0.006 (0.001)	0.001 (0.002)	0.001 (0.001)	0.295 (0.086)	0.042 (0.010)	0.947 (0.008)
Philadelphia	0.006 (0.001)	0.009 (0.001)	0.095 (0.015)	0.005 (0.001)	0.001 (0.001)	0.001 (0.001)	0.529 (0.051)	0.049 (0.002)	0.922 (0.013)
Pittsburgh	0.005 (0.001)	0.008 (0.001)	0.065 (0.009)	0.003 (0.001)	0.001 (0.002)	−0.001 (0.001)	0.586 (0.042)	0.049 (0.001)	0.877 (0.017)
Portland	0.004 (0.001)	0.012 (0.001)	0.182 (0.005)	0.005 (0.001)	0.003 (0.001)	0.001 (0.001)	0.448 (0.069)	0.038 (0.001)	0.946 (0.008)
Seattle	0.004 (0.001)	0.014 (0.001)	0.141 (0.001)	0.001 (0.001)	0.008 (0.001)	0.001 (0.002)	0.834 (0.048)	0.037 (0.003)	0.906 (0.015)
San Francisco	0.010 (0.001)	0.014 (0.001)	0.170 (0.005)	0.005 (0.001)	0.002 (0.002)	0.002 (0.002)	0.600 (0.038)	0.036 (0.003)	0.927 (0.007)
St. Louis	0.003 (0.001)	0.008 (0.001)	0.089 (0.003)	0.003 (0.001)	0.002 (0.004)	−0.001 (0.001)	0.497 (0.054)	0.048 (0.001)	0.923 (0.010)

The labels r^e and h^e refer to expected real rent growth and expected housing return, while pvr refers to the deviation from the present-value components. The parameters γ_0 and δ_0 are the constants in autoregressive process for expected rent growth and housing return, while γ_1 and δ_1 refer to the corresponding estimated AR(1) coefficients. Standard errors are in parentheses.

The results presented in Table 2 show the estimated standard deviations of expected rent growth (σ_{r^e}), expected housing return (σ_{h^e}), the PVR component (σ_{pvr}), realized rent growth (σ_r) and realized housing return (σ_h). We find that the standard deviation of the shock to the PVR component is much larger than the standard deviation of the expected housing return, which in turn is higher than the standard deviation of the expected rent growth. The smaller magnitude of the shock to expected rent growth is not surprising as the rent series for all of the MSAs and the United States do not exhibit huge variation.

Our approach also allows us to estimate the correlation between state variables of the present-value model. The results are shown in Table 3. The results suggest that there is positive correlation ($\rho_{r^e h^e}$) between expected rent growth and expected housing return. This positive correlation is intuitive because a shock to expected housing return is also expected to lead to an increase in expected rent growth. This positive correlation between the rent growth and the housing return is consistent with what other researchers have found for the stock market. For example, Bernanke and Kuttner (2005) and Campbell and Ammer (1993) found that shocks to expected dividend growth and equity premia are positively correlated. The positive correlation between expected future rent growth and housing return that we document could simply indicate that rents do not increase by “enough” during periods of rising house price growth, which mechanically implies a contemporaneous increase in housing return. We find that the shock to the expected housing return and the shock to the PVR component are highly positively correlated ($\rho_{h^e pvr}$). Even though the correlation is more than 0.62 for all the MSAs and the United States, the degree of correlation is lowest for Chicago, Cleveland, Dallas, Denver, Houston and Pittsburgh. These are also the MSAs where the PVR component is low as compared to the other MSAs. The positive correlation between the expected return and the shock to the PVR component can be motivated by a regulatory change. For example, the changes in underwriting standards may represent a positive shock to the PVR component, which in turn also raises expected housing returns. The high positive correlation between the shock to expected rent growth and the shock to the PVR component implies that a positive shock to PVR increased expected rent growth.

Table 3
Correlation estimates.

	$\rho_{r^e h^e}$	$\rho_{r^e pvr}$	$\rho_{h^e pvr}$
USA	0.036 (0.191)	−0.288 (0.190)	0.947 (0.027)
Atlanta	0.358 (0.070)	0.001(0.084)	0.934 (0.015)
Boston	0.129 (0.085)	−0.045(0.087)	0.985 (0.004)
Chicago	0.566 (0.065)	0.364(0.081)	0.974 (0.005)
Cleveland	0.484 (0.073)	0.072(0.100)	0.908 (0.026)
Dallas	0.546 (0.067)	0.255 (0.092)	0.949 (0.014)
Denver	0.218 (0.106)	−0.627(0.173)	0.623 (0.143)
Houston	0.311 (0.077)	−0.227(0.092)	0.855 (0.034)
Los Angeles	0.404 (0.068)	0.284(0.059)	0.992 (0.002)
Miami	0.243 (0.113)	0.105 (0.116)	0.990 (0.003)
Milwaukee	0.627 (0.054)	0.477(0.071)	0.984 (0.005)
Minneapolis	0.307 (0.093)	0.213(0.095)	0.995 (0.001)
New York City	0.489 (0.117)	0.430(0.122)	0.998 (0.023)
Philadelphia	0.613 (0.074)	0.463(0.090)	0.984 (0.005)
Pittsburgh	0.387 (0.082)	0.144(0.089)	0.968 (0.008)
Portland	0.211 (0.114)	0.140(0.114)	0.997 (0.001)
Seattle	0.081 (0.095)	−0.128(0.109)	0.978 (0.011)
San Francisco	0.318 (0.088)	0.114(0.100)	0.978 (0.014)
St. Louis	0.033 (0.088)	−0.087(0.087)	0.993 (0.023)

The labels r^e and h^e refer to expected real rent growth and expected housing return, while pvr refers to the deviation from the present-value components. $\rho_{r^e h^e}$ is correlation between expected rent growth and expected housing return. Standard errors are in parentheses.

One question that naturally arises is what are the sources of fluctuations in the PVR component. Because we have motivated this component through the institutional, regulatory, and macroeconomic changes, one could think of the shock to this residual term as arising from some regulatory changes or through monetary policy actions which the past behavior of rent growth, housing return, and the price–rent ratio cannot predict. The historical evolution of the U.S. housing finance system is a clear example of regulatory change, the impact of which may not have been foreseen by the past price–rent ratios. Similarly, there were changes to the underwriting systems at the housing finance agencies. For example, Fannie/Freddie adopted automated underwriting systems in the late-1990s and early 2000s. There was also a relaxation of underwriting standards in the mid-2000s which was not specifically taken into account by present-value models.

Because the present-value level of the price–rent ratio is stationary, we can perform a variance decomposition of it to examine the relative importance of rents and housing return in driving the price–rent ratio. The left panel of Table 4 shows the results for this exercise. We find that most of the variation in the present-value component is explained by expected changes in the housing return. In fact, for all the MSAs as well as for the United States, the percentage of variation explained by expected rent growth is never higher than 6.5%. This is consistent with what Fairchild et al. (2015) find in their study. In fact, the share of expected housing return is higher than 100% for more than half of the MSAs. The negative share of the covariances dampen the overall variation in price–rent ratio for the MSAs where the share of expected housing return is higher than 100%. These results are consistent with Glaeser (2013), who argues that there are many similarities between the most recent boom and previous booms in the United States where rising prices reflected optimistic expectations. For comparison purposes, we also report the unconditional variance of the present-value component and the variance of the shock to the PVR component in the right panel in Table 4. The results suggest sizable differences in the relative variance of these components across different MSAs. Because the PVR component is non-stationary by construction, a typical variance decomposition is not meaningful for the level of price–rent ratio. It should also be noted that our model allows for correlation between the shock to the PVR component and the present-value component, so a typical comparison of the variances of these two components would not provide a complete decomposition even if the PVR component were stationary.

Once the state space model is estimated using maximum likelihood via the Kalman filter, we can also examine the extent of the deviation of the price–rent ratio from the level implied by the conventional present-value model. Fig. 2 show the estimated deviations from the present-value level for the 18 MSAs and for the U.S. from the corresponding unobserved component models. The vertical axis represents the percentage deviation from the level of price–rent ratio implied by the present-value model. To be consistent, we make the vertical scaling same for each MSA and the nation. This allows us to observe the relative variation in the PVR component across different MSAs and the nation. First, we find that there was a build up in the PVR component prior to 2006 and then a big decline that coincided with the housing market collapse. This result is uniform across all of the MSAs. The magnitude of the increase and decline, however, varies across different MSAs. On the one hand, there are MSAs like Boston, Los Angeles, Miami, New York, Seattle and San Francisco where the PVR component was higher than 20% during the boom. On the other hand, there are MSAs like Cleveland, Dallas, Houston, and Pittsburgh where the deviation was very small and ranged from 3 to 10%. We also have MSAs like Atlanta, Chicago, Denver,

Table 4
Variances of unobserved components.

	$\text{var}(r_t^e)$	$\text{var}(h_t^e)$	$\text{cov}(r_t^e, h_t^e)$	$\text{var}(p_t^* - r_t^*)$	σ_{pvr}^2
USA	0.065	0.804	0.131	0.027	0.004
Atlanta	0.031	0.969	0.000	0.046	0.011
Boston	0.002	0.997	0.002	0.159	0.024
Chicago	0.008	1.041	-0.050	0.042	0.009
Cleveland	0.041	0.983	-0.025	0.017	0.005
Dallas	0.023	1.041	-0.064	0.052	0.010
Denver	0.364	0.258	0.379	0.103	0.010
Houston	0.067	0.840	0.093	0.110	0.019
Los Angeles	0.001	1.012	-0.013	0.092	0.017
Miami	0.001	1.003	-0.004	0.064	0.016
Milwaukee	0.014	1.091	-0.105	0.044	0.011
Minneapolis	0.001	1.006	-0.006	0.128	0.019
New York City	0.000	1.004	-0.004	0.222	0.024
Philadelphia	0.003	1.031	-0.035	0.061	0.009
Pittsburgh	0.012	1.013	-0.026	0.016	0.004
Portland	0.000	1.002	-0.002	0.283	0.033
Seattle	0.022	0.943	0.035	0.096	0.020
San Francisco	0.005	1.006	-0.011	0.192	0.029
St. Louis	0.001	0.995	0.004	0.045	0.008

The left panel on this table presents the variance decomposition of the present-value components of the price-rent ratio. $\text{var}(r^e)$ represents the share explained by expected rent growth and $\text{var}(h^e)$ shows the share explained by expected housing return. The right panel reports the variance of the present-value component and the shock to the PVR component. $\text{var}(p_t^* - r_t^*)$ is the unconditional variance of the present-value component.

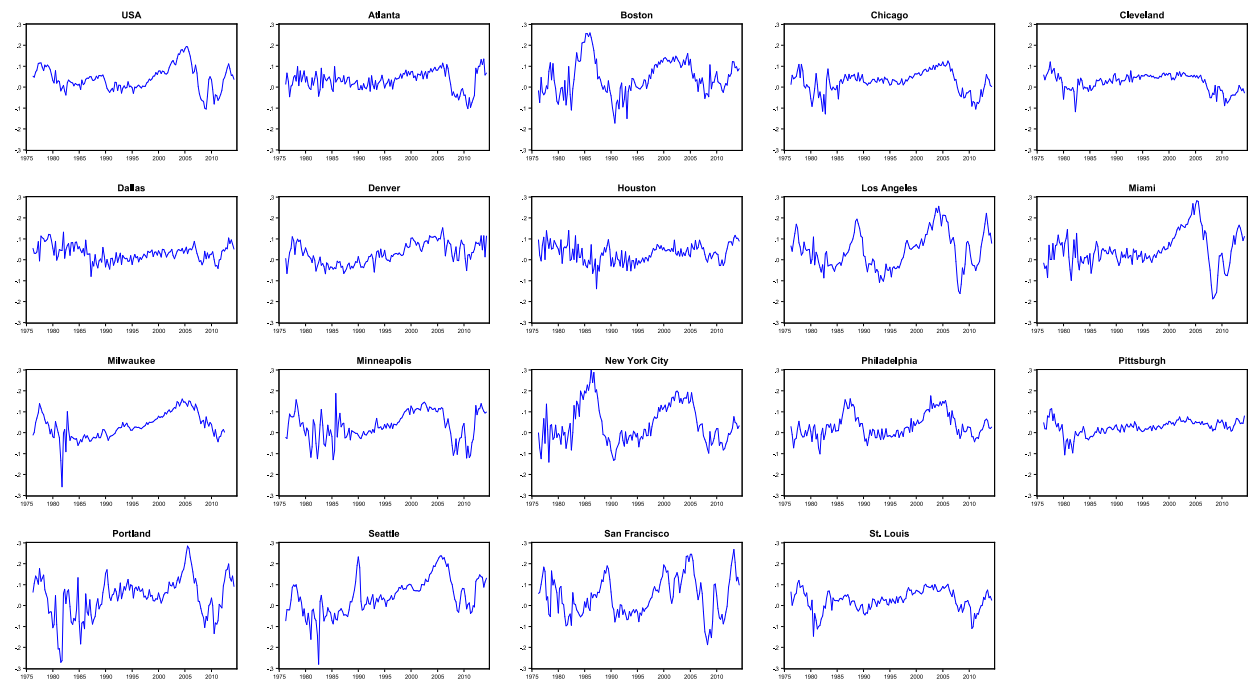


Fig. 2. Deviations from the present-value level.

Milwaukee, Minneapolis, Philadelphia, Portland, and St. Louis where the PVR component was somewhere in the middle. It should be noted that for some MSAs like Dallas, Houston, and San Francisco, the PVR component is almost the same or higher in 2013/2014 as compared to its corresponding value during the financial crisis. This is not surprising because the housing market in Texas, especially in Dallas and Houston, was fairly stable during the crisis period. The house price index fell less than 1% in Texas from its peak in 2007 to its trough in 2011, while it plunged 20% nationally. In San Francisco, even though there was a big drop in the housing market at the height of the crisis, this was also the MSA where recovery was one of the quickest. At the end of our sample period, house prices in San Francisco were very close to the peak that was attained in 2006:Q2.

The estimated deviation is consistent with observed volatility in the housing market in these MSAs. Historically, housing markets have not witnessed much volatility in MSAs like Atlanta, Cleveland, Chicago, Dallas, Denver, and Houston as compared to the other MSAs in our sample. Not surprisingly, we find that at the national level, the PVR component is somewhere in the middle. At the height of the boom in housing market, the price–rent ratio in our model was 15–20% higher than the level implied by the present-value components.

It should be noted that a deviation of 15–20% is not akin to saying that housing market was overvalued by 15–20% at the height of the housing market boom. This overvaluation may also arise from variation in the present-value component. What our results suggest is that a big portion of the overall variation in the price–rent ratio cannot be explained by the present-value model itself. In fact, [Campbell et al. \(2009\)](#) show that for the recent run-up in the housing market, the present-value model is not able to capture a big portion of the increase in price–rent ratio. Therefore, we need to modify the present-value model to allow for a deviation that takes into account shocks that can have permanent impact on the level of price–rent ratio. It may be tempting to consider this residual term as reflecting some form of ‘bubble’ in the housing market, although this is also not necessarily the case. Our paper is similar to the strand of literature in finance where the focus is on the empirical test of the present-value models and not necessarily on the existence of bubbles per se, although we consider tests for bubbles in [Section 5](#).

4.1. Deviations from the present-value level and housing supply elasticity

The literature on housing markets suggests that house prices are more volatile in areas where housing is less elastically supplied. Because our measure of PVR estimates the deviation of the price–rent ratio from its present-value level, we will also expect this component to be more volatile in MSAs where housing supply is inelastic. To examine the relationship between our estimated PVR measure and housing supply elasticity, we use two separate measures of supply elasticity: the primarily geography-based measure of [Saiz \(2010\)](#), and the regulation-based measure from the Wharton Regulation Index ([Gyourko et al., 2008](#)). [Saiz \(2010\)](#) processes satellite-generated data on terrain elevation and the presence of water bodies to precisely estimate the amount of developable land in U.S. metropolitan areas and estimate housing supply elasticity. [Gyourko et al. \(2008\)](#) use survey measure to calculate regulation based housing supply elasticity. They find that coastal markets tend to be more highly regulated, with communities in Northeast region of America being the most highly regulated on average, followed by those in the West region, especially California. Because there are no time-series data on these housing supply elasticities, we exploit the cross-sectional variation in the PVR component across different MSAs. We find that the correlation between the absolute mean of the PVR component and geography based supply elasticity is -0.642 , whereas the correlation between the estimated standard deviation of the shock to the PVR component and the geography based elasticity is -0.662 . This implies that MSAs with inelastic housing supply based on geography are also the MSAs with price–rent ratios that have higher deviations from the present-value level. Using the regulation based measure, we find that the correlation between the absolute mean and the regulation index is 0.615 , whereas this correlation is 0.652 for standard deviation of the shock to the PVR component and the regulation index. This result implies that MSAs with higher regulation index are also the MSAs that have a more volatile and higher absolute mean PVR component. Overall, our results are consistent with the strand of literature on the housing market in which researchers have found close relationship between housing supply elasticity and house price volatility.

4.2. Deviations from the present-value level and mortgage rates

Our method modifies the present-value model and allows us to estimate the deviation from the present-value level. In this section, we examine the relationship between interest rates and the PVR component. A significant amount of literature has suggested that a very accommodative stance of the monetary policy was responsible for the run-up in the housing prices (see [Taylor, 2007](#) among others). [Taylor \(2007\)](#) argues that monetary policy played a significant role in the run-up of the house prices. One drawback of this argument is that there was a big variation in the increase in house prices across different MSAs and states. Therefore, easy monetary policy cannot be the single factor that explains the housing market boom. Using a dynamic factor model, [Del Negro and Otrok \(2007\)](#) find that, historically, local factors have played a dominating role in driving the movement in house prices in different states. They find that between 2001 and 2005, the increase in house prices was a national phenomenon. However, they also find that monetary policy played only a small role in the house price fluctuations. Likewise, [Fairchild et al. \(2015\)](#) find that interest rate accounts for only about 17% of the variation in the aggregate log price–rent ratios. Most of these studies, however, look at the relationship between the housing market and the interest rate at the national level. It is clearly evident from the results presented above that there is significant heterogeneity in the behavior of the housing market across different MSAs. Therefore, we would like to examine this heterogeneity by looking at the relationship between the PVR component and mortgage rates.

To explore the relationship between the housing market and interest rates, we examine whether the deviation component in our model is also related to interest rates. This exercise is similar in flavor to [Brunnermeier and Julliard \(2008\)](#), who argue that inflation and nominal interest rates explain a larger share of mispricing in the housing market. They consider a behavioral approach to motivate the decomposition of the price–rent ratio into a rational component and a mispricing term and show that the mispricing term is highly correlated with nominal interest rate and inflation. They

Table 5

Relationships between the deviation of the price–rent ratio from its present-value level and the mortgage rate.

	1976:02–2014:02	1991:01–2005:04	1991:01–2014:02
USA	0.003 [0.25]	–0.048 [0.00]	–0.005 [0.42]
Atlanta	–0.001 [0.78]	–0.019 [0.00]	0.001 [0.93]
Boston	–0.002 [0.67]	–0.043 [0.00]	–0.010 [0.14]
Chicago	–0.003 [0.27]	–0.023 [0.00]	0.011 [0.06]
Cleveland	–0.001 [0.80]	–0.003 [0.09]	0.019 [0.00]
Dallas	0.002 [0.19]	–0.011 [0.00]	–0.006 [0.05]
Denver	–0.008 [0.00]	–0.028 [0.00]	–0.009 [0.02]
Houston	–0.001 [0.55]	–0.012 [0.00]	–0.009 [0.02]
Los Angeles	–0.008 [0.02]	–0.074 [0.00]	–0.023 [0.01]
Miami	–0.006 [0.11]	–0.063 [0.00]	–0.009 [0.25]
Milwaukee	–0.011 [0.00]	–0.040 [0.00]	–0.005 [0.52]
Minneapolis	–0.007 [0.01]	–0.033 [0.00]	–0.001 [0.95]
New York City	0.001 [0.82]	–0.060 [0.00]	–0.001 [0.90]
Philadelphia	–0.004 [0.09]	–0.045 [0.00]	–0.006 [0.19]
Pittsburgh	–0.007 [0.00]	–0.010 [0.00]	–0.004 [0.02]
Portland	–0.014 [0.06]	–0.030 [0.00]	–0.001 [0.92]
Seattle	–0.016 [0.00]	–0.047 [0.00]	–0.011 [0.10]
San Francisco	–0.007 [0.04]	–0.059 [0.00]	–0.017 [0.13]
St. Louis	–0.005 [0.07]	–0.023 [0.00]	0.004 [0.40]

This table presents the results for OLS regression of the pvr Component as a percentage of price–rent ratio on the nominal mortgage rate. The entries are estimates of the coefficient on the mortgage rate. Newey–West *P*-values are in square brackets.

attribute this behavior to ‘money illusion’. To examine the relationship between nominal interest rate and the PVR component, we run a simple OLS regression of the deviation component on the 30-year nominal mortgage rate.

We examine the relationship for three sample periods: 1976–2014, 1991–2005 and 1991–2014. In addition to the full sample period, we also break the sample in 1991 and 2005. We choose different subsample periods because it has been suggested in the literature that housing market responded differently to monetary policy in different periods. In particular, we are interested in examining the 1991–2005 period, when there was a boom in the U.S. housing market.

The results are presented in Table 5. Newey–West HAC *P*-values are in parentheses. We find substantial variation in the estimates across different sample periods. In most of the cases, we find negative coefficient on mortgage rate. This implies that a fall in mortgage rate is associated with an increase in the PVR component. For the full sample period, we find that the coefficient on mortgage rate is insignificant in half of the MSAs. The sample period 1991–2005 yields the most interesting results. As can be seen, there is economically meaningful and significant relationship between the magnitude of the deviation from the present-value model and its sensitivity to the mortgage rate. The results suggest that MSAs which have the highest deviation from the present-value level are also the MSAs that are most sensitive to mortgage rate changes. On the one hand, a one percentage point decline in the mortgage rate is associated with 7.4% increase in deviation from PV model in Los Angeles. On the other hand, this estimate is only 0.3% for Cleveland. The results presented here show that low interest rates may have played a role in the housing market boom after all, but only for those MSAs that were more sensitive to the changes in the interest rates. This result is consistent with studies that document possibly divergent sensitivities of disaggregate housing markets to a monetary-policy shock.¹⁰

4.3. Robustness checks

4.3.1. Alternative correlation structures

In the baseline specification of our model, we assume that the shocks to realized rent growth and realized housing return are uncorrelated, whereas the shocks to expected rent growth and expected housing return are allowed to have non-zero correlation. We also allow the shock to the PVR component to be correlated with the shocks to the expected return and rent growth. The zero correlation assumption was imposed for the purposes of identification. In this section, we compare our baseline model with the models based on alternative correlation structures. We first examine the case where we restrict all the correlations to zero. The log likelihood value of this restricted model for all the MSAs and the nation along with the likelihood values for alternative correlation structures are presented in Table 6. As can be seen, the fit of the model deteriorates significantly when imposing zero correlation for all shocks. The difference between the likelihood values for the baseline model and the zero correlation model is more than 100 for all of the MSAs and the nation, implying the rejection of null hypothesis of zero correlation.¹¹ Next, we allow a non-zero correlation between the shock to realized rent growth and the shock to the realized housing return and restrict other correlations to be equal to zero. This assumption can be

¹⁰ See Carlino and Defina (1998), Fratantoni and Schuh (2003), among others.

¹¹ A likelihood ratio test statistic for the hypothesis that the correlations are all zero will have a $\chi(3)^2$ distribution asymptotically. Based on this distribution, we can easily reject the null hypothesis at the 1% level in all cases.

Table 6
Log likelihood values for alternative model specifications.

	Baseline model	$\rho_{rh} \neq 0$	Zero correlation	$\sigma_{pvr} = 0$
USA	1995.41	1656.07	1654.83	1645.90
Atlanta	1830.55	1604.95	1595.07	1543.82
Boston	1765.57	1515.33	1514.79	1482.30
Chicago	1873.34	1616.55	1605.19	1592.29
Cleveland	1832.85	1642.69	1618.25	1555.57
Dallas	1874.63	1695.41	1672.79	1545.60
Denver	1930.10	1710.29	1697.73	1667.25
Houston	1764.47	1589.37	1571.01	1488.57
Los Angeles	1874.49	1521.72	1517.70	1491.60
Miami	1747.02	1466.85	1463.08	1455.74
Milwaukee	1874.43	1648.69	1629.12	1531.99
Minneapolis	1931.55	1661.41	1654.79	1594.27
New York City	1901.13	1646.69	1640.37	1616.94
Philadelphia	1925.21	1678.18	1659.44	1633.29
Pittsburgh	2034.16	1778.95	1763.53	1726.20
Portland	1897.83	1619.53	1618.36	1614.11
Seattle	1862.60	1598.29	1597.32	1544.91
San Francisco	1780.65	1430.55	1427.07	1423.56
St. Louis	2085.62	1804.32	1784.96	1768.79

This table reports the log likelihood values for alternative correlation structures of the variance–covariance matrix. The baseline case has $\rho_{rh} = 0$, but keeps $\rho_{rs}, \rho_{rv}, \rho_{rvr}, \rho_{rh^c}, \rho_{rh^c pvr}$ unrestricted. The case $\rho_{rh} \neq 0$ refers to other correlations being zero except ρ_{rh} , and zero correlation implies all pairwise correlations are zero. The restriction $\sigma_{pvr} = 0$ implies a model with only a stationary present-value component.

motivated by the idea that unexpectedly higher rent growth can generate an unexpectedly higher return. If we compare this model with our baseline model that assumes zero correlation between realized rent growth and realized return, we find that the baseline model fits much better.¹²

4.3.2. Significance of the non-stationary present-value residual component

Our model is motivated by the idea that persistent movement in price–rent ratio cannot be accounted for by persistent movements in rent growth and housing alone. In particular, the price–rent ratio also includes a PVR component that is non-stationary. Even though our results presented in Table 2 clearly suggest that shocks to the PVR component are large, a test for the hypothesis that $\sigma_{pvr} = 0$ is nonstandard. Specifically, it is a test for stationarity of the price–rent ratio.

As shown in Morley et al. (2015), one way to test stationarity in the context of an unobserved components model is to conduct a likelihood ratio test for the hypothesis that $\sigma_{pvr} = 0$, noting that the asymptotic critical value for this nonstandard test is 4.42 at the 1% significance level. Again looking at Table 6, we can see that the fit of our benchmark model is much better than a model without the PVR component and the difference in the log likelihood is highly significant in all cases. Thus, just as we were unable to reject a unit root in the price–rent ratio, we can reject stationarity of the price–rent ratio in every case.

5. Comparison with a test of explosive bubbles and date stamping

In a recent paper, Phillips et al. (2011) have developed a recursive method for testing for explosive bubbles. The method involves the recursive implementation of a right-side unit root test and a sup test. Right sided unit root tests, as shown in Phillips et al. (2011), are informative about mildly explosive or submartingale behavior in the data. This procedure gives rise to a date stamping strategy which identifies points of origination and termination of a bubble. This test procedure is shown to have discriminatory power in detecting periodically collapsing bubbles, thereby overcoming a weakness in earlier applications of unit root tests for economic bubbles. The explosive behavior may reflect exuberation and herd behavior. Even though our paper does not make a claim about existence of bubbles in the price–rent ratio, it is an interesting exercise to compare the estimated PVR component with these bubble tests. In particular, we are interested in examining whether periods of big PVR components coincide with the periods of explosive bubbles according to a test. For this purpose, we perform the supADF test as explained in Phillips et al. (2011).¹³ Table 7 reports the supADF statistic for all the MSAs, as well as for the United States. The results show that in 13 out of 19 cases, the supADF statistics exceed their respective 5% right-tail critical values giving strong evidence that price–rent ratio had explosive subperiods. The critical values are obtained from Monte Carlo simulation with 10,000 replications and sample size of 158. The smallest window size has 40 observations (10

¹² Because these two models are non-nested, we need to make a formal comparison based on a model selection criterion such as the Schwarz Information Criterion (SIC). Based on SIC, our baseline model is chosen in every case.

¹³ For very large sample sizes, Phillips et al. (2013) develop a generalized supADF test to test for multiple bubbles. Our study only has 39 years of data and therefore, we use the basic supADF test.

Table 7
The supADF test of the price–rent ratio.

City	supADF
USA	4.03
Atlanta	2.41
Boston	3.61
Chicago	1.10
Cleveland	1.00
Dallas	0.56
Denver	0.17
Houston	−0.08
Los Angeles	1.63
Miami	6.20
Milwaukee	1.52
Minneapolis	4.57
New York City	6.97
Philadelphia	3.43
Pittsburgh	−0.09
Portland	3.56
San Francisco	2.16
Seattle	1.46
St. Louis	2.26

99% cv is 1.74, 95% cv is 1.12 and 90% cv is 0.81. Critical values are obtained from Monte Carlo Simulation with 10,000 replications for a sample size of 158. The smallest window size has 40 observations.

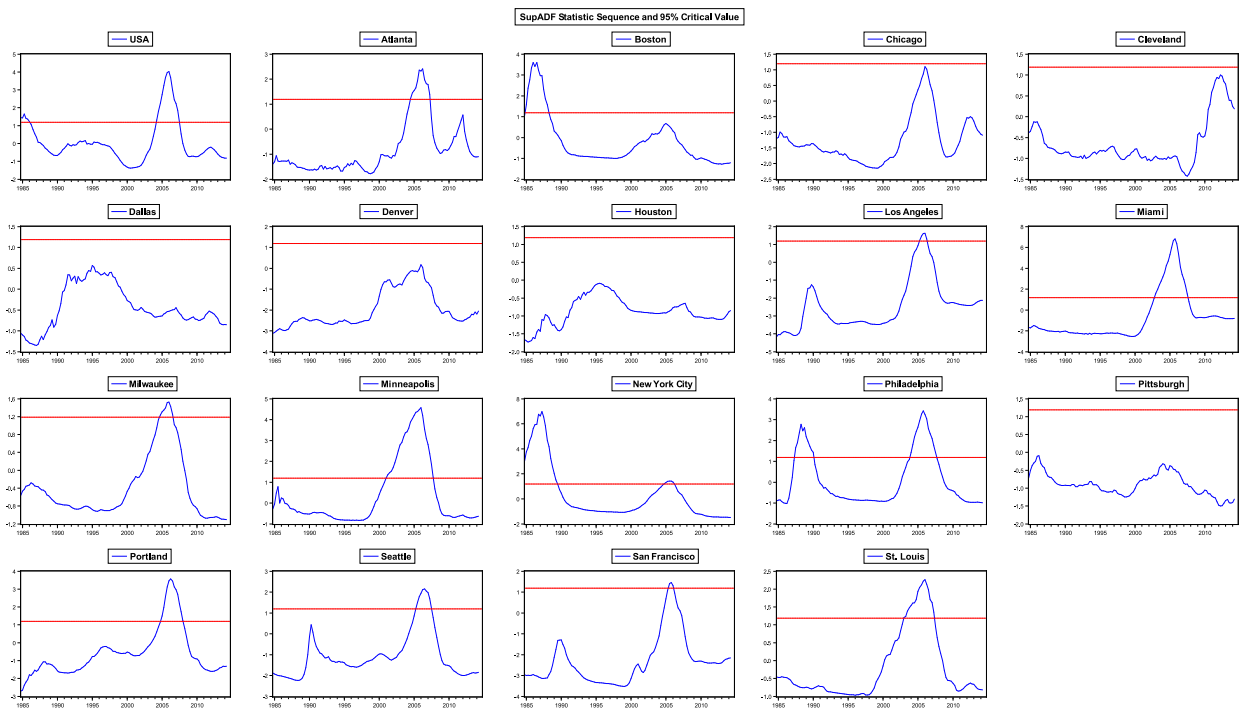


Fig. 3. Date-stamping bubble periods in the price–rent ratio using the supADF procedure.

years). The results clearly show that the MSAs that had high PVR components are also the cities that had exploding price–rent ratios. We find that MSAs like Dallas, Houston, Cleveland, and Pittsburgh that have small PVR components did not witness exploding price–rent ratios.

To locate specific bubble periods, we compare the backward supADF statistic sequence with a 95% supADF critical value sequence. The results are shown in Fig. 3. The findings suggest that for most of the MSAs with a high PVR component, the explosive bubble period started around 2005:Q1 and terminated at the end of 2007. This is also the period when the PVR

component was highest for these MSAs. For Boston, however, the supADF test shows that the explosive bubble was in the late 1980s. This is also consistent with the boom and the bust of the housing market in Boston in the late 1980s and the early 1990s. We also find that for New York City, Philadelphia, and the U.S., there were two periods of explosive sub-periods. In addition to the explosive subperiod in 2005–2007, there was also an explosive sub-period in the 1980s. Not surprisingly, we find that Chicago, Cleveland, Dallas, Denver, Houston, and Pittsburgh do not have the explosive sub-periods according to this recursive supADF test. Overall, the results from the supADF tests and our model seem to indicate that the MSAs that had large PVR components are also the MSAs that witnessed explosive sub-periods in their price–rent ratios, especially during the 2005–2007 sample period.

6. Conclusions

In this paper, we have proposed a modified present-value model that decomposes the price–rent ratio into expected real rent growth, expected housing return, and a residual term that represents the deviation of the price–rent ratio from its conventional present-value level and which we refer to as the present-value residual (PVR). This PVR term takes into account the fact that price–rent ratio at the national and the MSA levels is non-stationary, whereas the conventional present-value model approach assumes that this ratio is stationary. To estimate this modified present-value model, we take an unobserved component approach. We treat expected rent growth, expected housing return, and the PVR component as unobserved variables that follow exogenously specified time series processes.

Our findings suggest that the PVR term representing the deviation of the price–rent ratio from its present-value level is important both at the national and the regional levels. This is especially true for the MSAs that have experienced frequent booms and busts in the housing market. At the height of the housing boom, our estimated PVR components were almost 30% above the level implied by the present-value model for the MSAs like Los Angeles, Miami, New York, and San Francisco. Our findings show that the MSAs which display larger deviations from the present-value model are more sensitive to mortgage rate changes, implying significant heterogeneity in response to monetary policy actions. We also find a negative correlation between different measures of housing supply elasticity and the PVR component, implying that the MSAs with higher PVR component are also the MSAs with less elastic housing supply. Our approach also allows us to estimate the correlation between expected rent growth, expected housing return, and the shock to the PVR component. We find that a shock to the expected housing return and a shock to the PVR component are highly positively correlated. The variance decomposition of the stationary present-value components shows that most of the variation in the present-value level of the price–rent ratio arises due to the variation in the expected housing return. We also compare our results with a recent statistical test for periodically collapsing bubbles. Our results suggest that MSAs with large estimated PVR components are the same MSAs that test positively for explosive sub-periods in their price–rent ratios, especially during the 2005–2007 subsample.

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Appendix A. State space representation of the present-value model

Eqs. (3)–(9) can be represented in a state-space form. The measurement equation can be written as

$$\begin{bmatrix} \Delta r_{t+1} \\ h_{t+1} \\ p_t - r_t \end{bmatrix} = \begin{bmatrix} \gamma_0 \\ \delta_0 \\ A \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ B_1 & -B_2 & 1 \end{bmatrix} \begin{bmatrix} \widehat{\Delta r}_t^e \\ \widehat{h}_t^e \\ \widehat{pvr}_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1}^r \\ \varepsilon_{t+1}^h \\ 0 \end{bmatrix} \quad (\text{A.1})$$

The transition equation is represented as

$$\begin{bmatrix} \widehat{\Delta r}_t^e \\ \widehat{h}_t^e \\ \widehat{pvr}_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \widehat{\Delta r}_{t-1}^e \\ \widehat{h}_{t-1}^e \\ \widehat{pvr}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^h \\ \varepsilon_t^{pvr} \end{bmatrix} \quad (\text{A.2})$$

The variance–covariance matrix of the transition equation errors is

$$Q = \text{var} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^h \\ \varepsilon_t^{pvr} \end{bmatrix} = \begin{bmatrix} \sigma_{r^2} & \sigma_{r^e h^e} & \sigma_{r^e pvr} \\ \sigma_{r^e h^e} & \sigma_{h^e}^2 & \sigma_{h^e pvr} \\ \sigma_{r^e pvr} & \sigma_{h^e pvr} & \sigma_{pvr}^2 \end{bmatrix}$$

The variance–covariance matrix of the measurement equation errors is

$$R = \text{var} \begin{bmatrix} e_{t+1}^r \\ e_{t+1}^h \end{bmatrix} = \begin{bmatrix} \sigma_r^2 & \sigma_{rh} \\ \sigma_{rh} & \sigma_h^2 \end{bmatrix}$$

For identification purposes, we also assume that $\sigma_{rh} = 0$.

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