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## The Slow Adjustment of Aggregate Consumption to Permanent Income

This paper investigates the relationship between aggregate consumption and permanent income using a new approach to the estimation of cointegrated systems that builds on Stock and Watson's common stochastic trends representation. The permanent and transitory movements in aggregate income and consumption are estimated directly using the Kalman filter and are allowed to be correlated. This approach avoids any implicit restriction that permanent income be as smooth as consumption. Instead, permanent income appears to be relatively volatile, with consumption adjusting toward it only slowly over time. These results provide a clear rejection of the standard version of the permanent income hypothesis and are suggestive of alternative theories of consumption behavior such as habit formation or precautionary savings.

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WHY IS AGGREGATE consumption so smooth? The permanent income hypothesis (PIH) provides the usual economic explanation. In the standard version of the PIH due to Hall (1978), the representative economic agent alters consumption only when faced with unpredictable changes in income. Predictable changes are already taken into account, leaving consumption to follow a random walk. Yet, as Campbell and Deaton (1989) point out, the PIH can explain the smoothness of

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consumption only if changes in income are largely predictable. They note that a number of univariate time series studies of aggregate income, including Nelson and Plosser (1982) and Campbell and Mankiw (1987), find that the predictable component of income is small. Thus, they argue that permanent income is much more volatile than consumption and, therefore, does not explain its smoothness. On the other hand, Cochrane (1994) employs bivariate cointegration analysis of aggregate income and consumption and finds that aggregate income is largely predictable given past consumption. Thus, he argues that the PIH is a reasonable description of aggregate consumption behavior. It remains an open question as to which interpretation of the data is correct.

In this paper, I investigate the relationship between aggregate consumption and permanent income using a new approach to the estimation of cointegrated systems. Most cointegration studies, including Cochrane (1994), assume that the time series of interest have a finite-order vector autoregression (VAR) representation in levels or, equivalently, a vector error correction model (VECM) representation in differences. Inspired by Stock and Watson's (1988a) common stochastic trends representation of cointegration, I develop a correlated unobserved components (UC) model of cointegration between aggregate income and consumption.<sup>1</sup> Importantly, the correlated UC model allows the cointegrated variables to have different speeds of adjustment in terms of restoring their long-run equilibrium relationship, while the error correction mechanism from a VECM allows different expected magnitudes of adjustment, but generally implies the same speed of adjustment.<sup>2</sup>

A crucial aspect of the modeling approach developed in this paper is the allowance for correlation between movements in the different components for the time series. This innovation to the standard UC approach is motivated by the insight in Morley, Nelson, and Zivot (2003) that correlations can be identified given sufficiently rich model dynamics. In that paper, the innovations to the stochastic trend in U.S. real GDP are allowed to be correlated with the innovations to an AR(2) cycle. As discussed in Stock and Watson (1988b), economic theory does not rule out the presence of correlation between permanent and transitory movements in a given time series. For

1. There are a few other recent papers that consider versions of the multivariate correlated unobserved components model developed here. Engel and Morley (2001) estimate a correlated unobserved component model of exchange rates and price levels assuming long-run equilibrium relationships defined by purchasing power parity. Schleicher (2003) presents a general discussion of a number of technical issues involved in the identification and estimation of multivariate correlated unobserved components models of common trends and common cycles. Sinclair (2007) also discusses technical issues related to identification and estimates a correlated unobserved components model of output and unemployment that does not impose cointegration. Finally, Corradini (2005) applies the model in this paper to Italian data.

2. In this paper, "speed of adjustment" is measured in terms of the half-life response of a variable to a shock. This measure can also be thought of as related to the "period" of adjustment. That is, "speed" is measured in terms of time (fast versus slow), not magnitude (large versus small). The point in this paper is to have a measure that is independent of the magnitude of adjustment, which is already known to be small for aggregate consumption, given its relative smoothness. While the model parameters in the UC approach provide a simple way to measure half-life responses to a shock, the error correction coefficients in the VECM approach provide a measure of expected adjustment to the long-run equilibrium that is related to magnitude. The implied speed of adjustment from the error correction mechanism is the same for all variables, except those that are assumed to be weakly exogenous with respect to long-run relationships and, therefore, adjust instantaneously. This issue is discussed in greater detail in Section 2.

example, a productivity shock might permanently increase the level of real output, but if it takes a few quarters for the effects of the shock to fully propagate due to “time-to-build” effects, the shock will also imply predictable transitory movements in the series. In particular, given such productivity shocks, movements in the permanent and transitory components of real GDP should be negatively correlated, as is found in Morley, Nelson, and Zivot (2003).

In terms of cointegration analysis, I show that allowing for correlation between innovations to the common stochastic trend and the transitory components of the series under examination avoids an arbitrary smoothness restriction on the common stochastic trend component. In particular, given balanced growth among cointegrated series, the standard UC assumption of uncorrelated components restricts the stochastic trend to be at least as smooth as the smoothest series under examination. In practice, such a restriction is far from innocuous. In particular, it would rule out the possibility that permanent income, as measured by the common stochastic trend in aggregate income and consumption, is more volatile than consumption. That is, it would not allow investigation in the UC framework of Campbell and Deaton’s (1989) conjecture about the failure of the PIH to explain the smoothness of aggregate consumption.

Whenever there is correlation, there is a question of structural interpretation. In a fully specified economic model, the underlying structural shocks should be uncorrelated. Thus, any correlation between different components reflects some form of unmodeled causality. If the goal is to make inferences about the dynamic behavior of a variable or component, it may be possible to remain agnostic about the source of a correlation. However, economic theory often conveniently implies a structural interpretation of the relationship between different components. For example, long-run neutrality propositions suggest that any correlation between permanent and transitory movements in real GDP reflects the effects of “real” (e.g., productivity or preference) shocks on the transitory component, rather than the effects of “nominal” (e.g., monetary) shocks on the permanent component.<sup>3</sup> In terms of investigating aggregate consumption behavior, there is also a natural structural interpretation. Because the structural conception of permanent income (a fixed portion of the expected discounted sum of future income) follows a random walk under rational expectations, one can directly interpret the common stochastic trend (i.e., the common unobserved random walk component) of aggregate income and consumption as permanent income.<sup>4</sup> Then, long-run neutrality can be invoked to interpret any correlation between innovations to the common stochastic trend and the transitory components of the series under examination as reflecting the causal effects of shocks to permanent income on these transitory components.

3. Notably, Blanchard and Quah (1989) show that this implication of long-run neutrality can be used to identify nominal shocks in a bivariate structural VAR system.

4. Quah (1990) makes the insight that individuals may be better able than econometricians to evaluate permanent income, implying that permanent income is an unobserved component of income, rather than the observed long-run forecast from an econometric model. He also argues that the “permanent” component of income need not follow a random walk and, therefore, could be smooth even if income growth is highly unpredictable. However, in the context of the PIH, the structural concept of permanent income, while unobservable to econometricians, should follow a random walk under rational expectations.

My findings can be summarized as follows. First, permanent income appears to be much more volatile than consumption. Second, innovations to permanent income are negatively correlated with the transitory components of income and consumption, with consumption adjusting more slowly than income toward permanent income. The different speeds of adjustment suggest that the correlated UC approach to cointegration analysis developed in this paper is more appropriate than simply looking at the error correction mechanism from a VECM for the data under examination. Third, restrictions on the correlation parameters based on the PIH can be strongly rejected, as can the standard UC assumption of uncorrelated components. These findings support Campbell and Deaton's (1989) conjecture about the failure of the standard version of the PIH to explain the smoothness of aggregate consumption and are suggestive of alternative theories of consumption behavior such as habit formation in consumer preferences or a precautionary savings motive under uncertainty about future income.<sup>5</sup>

## 1. MODEL

There are multiple representations of a cointegrated system. Stock and Watson (1988a) provide a fundamental representation that decomposes time series into common stochastic trends (random walks) and idiosyncratic stationary components. I follow this approach in developing an empirical model for the logarithms of aggregate income and consumption:

$$y_t = \tau_t + u_{yt}, \quad (1)$$

$$c_t = \bar{c} + \gamma \tau_t + u_{ct}, \quad (2)$$

where  $\tau_t$  is the common stochastic trend or "permanent income,"  $u_{yt}$  is the transitory component of income, and  $u_{ct}$  is the transitory component of consumption. The parameter  $\bar{c}$  reflects the long-run impact of taxes and private saving on consumption and the parameter  $\gamma$  is the marginal propensity to consume out of permanent income. Permanent income follows an unobservable random walk and is allowed to have deterministic drift  $\mu$  to capture a positive long-run growth rate in the economy:

$$\tau_t = \mu + \tau_{t-1} + v_t, \quad (3)$$

where  $v_t \sim \text{iid } N(0, \sigma_v^2)$ . To make the model tractable, I assume that the transitory components of income and consumption follow unobservable finite-order autoregressive (AR) processes:

5. On habit formation, see Deaton (1987), Abel (1990), Constantinides (1990), Carroll and Weil (1994), Heaton (1995), Campbell and Cochrane (1999), Carroll, Overland, and Weil (2000), Dynan (2000), Fuhrer (2000), Boldrin, Christiano, and Fisher (2001), and references therein. On precautionary savings, see Leland (1968), Caballero (1990), Carroll (1994), Hubbard, Skinner, and Zeldes (1994), Normandin (1994), Carroll and Samwick (1998), Carroll, Dynan, and Krane (2003), and references therein.

$$\phi_y(L)u_{yt} = \varepsilon_{yt}, \tag{4}$$

$$\phi_c(L)u_{ct} = \varepsilon_{ct}, \tag{5}$$

where  $\varepsilon_{yt} \sim \text{iid } N(0, \sigma_y^2)$ ,  $\varepsilon_{ct} \sim \text{iid } N(0, \sigma_c^2)$ , and the lag polynomials are normalized by setting  $\phi_{i,0} = 1$ , where  $i = y, c$ .<sup>6</sup> To complete the model, I assume that the innovations to the UCs are correlated:

$$\rho_{vy} = \text{corr}(v_t, \varepsilon_{yt}), \tag{6}$$

$$\rho_{vc} = \text{corr}(v_t, \varepsilon_{ct}), \tag{7}$$

$$\rho_{yc} = \text{corr}(\varepsilon_{yt}, \varepsilon_{ct}). \tag{8}$$

In order to nest different theories of aggregate consumption behavior, it is crucial that there be no zero restrictions on any of the variances or correlations in the model given by (1)–(8). For example, a classic Keynesian consumption function and multiplier process due to either liquidity-constrained or “rule-of-thumb” consumers implies a sizable positive correlation between the transitory components of income and consumption ( $\rho_{yc} > 0$ ). Meanwhile, given a preference for smooth consumption due to habit formation or precautionary savings, consumers may only partially adjust consumption each period to changes in permanent income, implying a negative correlation between permanent and transitory movements in consumption ( $\rho_{vc} < 0$ ). Finally, while the PIH implies that the transitory component of consumption will have no variance ( $\sigma_c^2 = 0$ ) and, therefore, no correlation ( $\rho_{vc} = \rho_{yc} = 0$ ), there is no reason that permanent and transitory movements in income cannot have a negative correlation ( $\rho_{vy} < 0$ ), such as might occur given “time-to-build” stories of real business cycles.

A particularly strong and undesirable restriction on the model would be the standard UC assumption of uncorrelated components. Under this restriction, all three correlations in (6)–(8) would be set to zero and the following two equalities would hold for the variances of the first differences of income and consumption:

$$\text{var}(\Delta y_t) = \text{var}(\Delta \tau_t) + \text{var}(\Delta u_{yt}), \tag{9}$$

$$\text{var}(\Delta c_t) = \gamma^2 \text{var}(\Delta \tau_t) + \text{var}(\Delta u_{ct}). \tag{10}$$

6. While allowing the speeds of adjustment to be different for different series, the autoregressive structure assumed in (4) and (5) imposes that a given series has the same speed of adjustment to all shocks. A more general vector autoregressive (VAR) structure for the transitory components would allow for different speeds of adjustment to different shocks. However, estimation of the speeds of adjustment for this more general model would require identification of the underlying orthogonal structural shocks driving the transitory components.

Empirically, consumption is less volatile than income (i.e.,  $\text{var}(\Delta c_t) < \text{var}(\Delta y_t)$ ) and, as will be shown in Section 3,  $\gamma$  is close to one, corresponding to balanced growth. Thus, noting that variances are non-negative (e.g.,  $\text{var}(\Delta u_{ct}) \geq 0$ ) and setting  $\gamma = 1$ , the equalities in (9) and (10) would imply that permanent income is at least as smooth as consumption and smoother than income:

$$\text{var}(\Delta \tau_t) \leq \text{var}(\Delta c_t) < \text{var}(\Delta y_t). \quad (11)$$

The restrictions in (11) would be highly undesirable because they would preclude, *a priori*, any investigation of Campbell and Deaton's (1989) conjecture that permanent income is more volatile than consumption.

It may not be immediately obvious that the correlated UC model given by (1)–(8) is identified. However, a reduced-form representation of the model can be used to show that UC parameters are identified given sufficient autoregressive dynamics in the transitory components. For simplicity, consider a version of the above model in which the intercept and drift parameters are zero ( $\bar{c} = \mu = 0$ ) and the marginal propensity to consume out of permanent income is unity ( $\gamma = 1$ ). Solving (1)–(5) for  $\Delta y_t$ ,  $\Delta c_t$ , and  $y_t - c_t$ , which captures the cointegrating relationship, gives the following:

$$\phi_y(L)\Delta y_t = \phi_y(L)v_t + (1 - L)\varepsilon_{yt}, \quad (12)$$

$$\phi_c(L)\Delta c_t = \phi_c(L)v_t + (1 - L)\varepsilon_{ct}, \quad (13)$$

$$\phi_y(L)\phi_c(L)(y_t - c_t) = \phi_c(L)\varepsilon_{yt} - \phi_y(L)\varepsilon_{ct}. \quad (14)$$

Then, by Granger's lemma (Granger and Newbold 1986), the system in (12)–(14) has the following reduced-form vector autoregressive moving-average (VARMA) representation:

$$\begin{aligned} & \begin{bmatrix} \phi_y(L) & 0 & 0 \\ 0 & \phi_c(L) & 0 \\ 0 & 0 & \phi_y(L)\phi_c(L) \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta c_t \\ y_t - c_t \end{bmatrix} \\ &= \begin{bmatrix} \theta_{yy}(L) & \theta_{yc}(L) \\ \theta_{cy}(L) & \theta_{cc}(L) \\ \theta_{yy}(L) - \theta_{yc}(L) & \theta_{cy}(L) - \theta_{cc}(L) \end{bmatrix} \begin{bmatrix} e_{yt} \\ e_{ct} \end{bmatrix}, \quad (15) \end{aligned}$$

where  $e_t \sim N(0, \Omega)$  and the lag polynomials for the moving average (MA) coefficients are normalized by setting  $\theta_{ij,0} = 1$  for  $i = j$  and  $\theta_{ij,0} = 0$  for  $i \neq j$ , where  $i, j = y, c$ . Then, it is straightforward to use (15) to determine whether a given UC model is identified. For example, suppose the transitory components of income and consumption both follow AR(1) processes. In this case, the UC model will have eight parameters corresponding to two autoregressive parameters ( $\phi_y$  and  $\phi_c$ ) and

six variance–covariance parameters (three variances,  $\sigma_v^2$ ,  $\sigma_y^2$ ,  $\sigma_c^2$ , and three correlations,  $\rho_{vy}$ ,  $\rho_{vc}$ ,  $\rho_{yc}$ ). Meanwhile, there will be nine parameters in the reduced-form representation in (15) corresponding to two autoregressive parameters ( $\phi_y$  and  $\phi_c$ ), three variance–covariance parameters for the forecast errors (the three independent elements of  $\Omega$ ), and four parameters for one lag of the vector MA process ( $\theta_{yy,1}$ ,  $\theta_{yc,1}$ ,  $\theta_{cy,1}$ , and  $\theta_{cc,1}$ ). The non-zero lags of the vector MA process reflect the number of autoregressive lags. Thus, for higher order autoregressive processes, the UC model will still be identified. See Morley, Nelson, and Zivot (2003), Schleicher (2003), and Sinclair (2007) on identification of correlated UC models.

It should be noted that, even though the correlated UC model is identified, weak identification could still be an issue in practice. For instance, weak identification could occur if some of the MA parameters in the reduced-form model in (15) are close to zero. Weak identification is a problem because it can lead to distorted inferences using estimated standard errors. Nelson and Startz (Forthcoming) provide an analysis of the phenomenon of weak identification with the intuition that, as the true variance of an estimator goes to infinity in the limit of no identification, the sample variance remains finite. They suggest using likelihood ratio statistics instead of Wald statistics for hypothesis testing when weak identification is a potential problem. Thus, in this paper, I consider likelihood ratio statistics for the main hypotheses of interest.

For estimation, I cast the correlated UC model given by (1)–(8) into state–space form and apply the Kalman filter and maximum likelihood based upon the prediction error decomposition (Harvey 1993). See the Appendix for details.

## 2. MOTIVATION

Before presenting the results for the correlated UC model of aggregate income and consumption, I motivate the approach by investigating the basic time series properties of the data and estimating the VECM used in Cochrane (1994). This analysis provides justification for the cointegration framework and illustrates key limitations of the VECM approach in discriminating between different theories of aggregate consumption behavior. It also provides an important benchmark from which to compare the results for the correlated UC model.

The data used in this paper are 100 times the natural logarithms of U.S. per capita real GDP and U.S. per capita real consumption of non-durables and services for the sample period of 1954:Q1–2005:Q2.<sup>7</sup> It is an open question as to which series should be used in a study of aggregate consumption behavior. The PIH is often considered to apply to consumption and either disposable income or labor income only. However, unlike real GDP, disposable income and labor income do not appear to be cointegrated with aggregate consumption on their own. Thus, there is a sense that consumption never fully adjusts to a permanent shock to disposable income or labor income. To

7. The underlying data are seasonally adjusted and are available from the St. Louis Fed website (<http://www.stls.frb.org/fred/>).

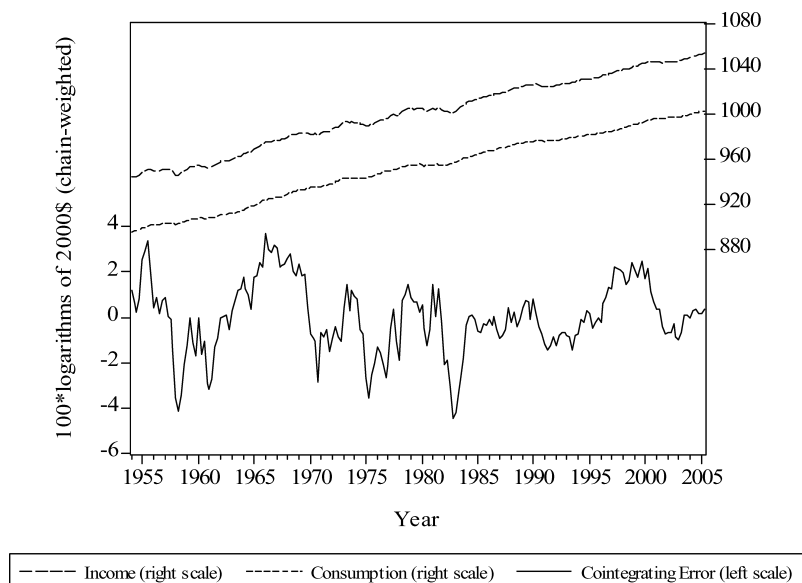


FIG. 1. U.S. per Capita Real GDP, U.S. per Capita Real Consumption of Non-Durables and Services, and Deviations from an Estimated Long-Run Equilibrium Relationship.

allow for full adjustment of consumption to permanent income, it is necessary to consider total income, as in Cochrane (1994) and this paper, or to include additional variables, such as aggregate wealth when considering cointegration between labor income and consumption, as in Lettau and Ludvigson (2001).<sup>8</sup>

In terms of the consumption data, there are two additional technical issues that should be mentioned. First, as discussed in Whelan (2000), there is problem in combining two chain-weighted series such as consumption of non-durables and consumption of services. In this paper, I follow Whelan's suggestion of using the Tornqvist approximation to the ideal Fisher index. Second, consumption data include service flow measures that are sometimes interpolated from annual data, thus inducing a false predictability in quarterly data. I address these issues by showing that, despite any imperfections, the data employed in this paper can be used to closely replicate Cochrane's (1994) main results, which appear to support the PIH. That is, the differences in conclusions regarding the PIH more closely reflect what the UC approach reveals about the data rather than particular idiosyncracies in the data.

Figure 1 plots the income and consumption data. Individually, both series appear to be non-stationary, yet the gap between the two series appears to be more stable. This

8. A correlated UC model of labor income, consumption, and wealth would provide an interesting extension to the model in this paper. However, it would be necessary to address the severe heteroskedasticity in aggregate wealth in order to apply the Kalman filter to estimate the model.



TABLE 1  
UNIT ROOT AND COINTEGRATION TESTS

	Lags of differences <sup>a</sup>	t-statistic <sup>b</sup>	5% critical value <sup>c</sup>
$y_t$	2	-3.12	-3.43
$c_t$	13	-1.63	-3.43
$\hat{z}_t$	11	-3.78	-3.42

<sup>a</sup>Chosen by a general-to-specific rule, starting with a maximum of 14 lags and reducing if the last lag is not significant using the 10% critical value of 1.645 for a standard *t*-test.

<sup>b</sup>For "unit root" null that largest root of autoregressive lag polynomial for levels is equal to 1.

<sup>c</sup>The test regressions for the first two variables include a constant and a time trend. The reported critical value for these tests is from MacKinnon (1991). The test regression for the third variable does not include any deterministic terms. However, because the variable is generated from a first-stage OLS cointegrating regression, the distribution of the test statistic is affected by the constant term in the first-stage regression and the fact that the right-hand side variable in the first-stage regression contains deterministic drift. The reported critical value for this test is from Phillips and Ouliaris (1990).

stable relationship is captured by the third series in the figure, which is the following error term from a linear regression of income on consumption:

$$\hat{z}_t = y_t - \hat{\alpha} - \hat{\beta}c_t, \tag{16}$$

where  $\hat{\alpha}$  and  $\hat{\beta}$  are calculated using OLS. If income and consumption are non-stationary, but the error term is stationary, then income and consumption are cointegrated. I investigate the basic time series properties of the data formally by using ADF tests for unit roots and cointegration. The results are reported in Table 1. I cannot reject the unit root for income and consumption at the 5% level. However, I can reject the unit root for  $\hat{z}_t = y_t - 20.39 - 1.03c_t$ , implying cointegration between income and consumption.<sup>9</sup>

One way to think about cointegration is in terms of forecasting. In particular, under stationarity, the cointegrating error term should be useful for forecasting future movements in income and/or consumption. A VECM provides a straightforward way to examine this issue. Under the assumption that income and consumption have a finite-order VAR(*p*) representation, the VECM is given by

$$\Delta y_t = \delta_y + \pi_y(y_{t-1} - \alpha - \beta c_{t-1}) + \sum_{j=1}^{p-1} (\zeta_{yc,j} \Delta c_{t-j} + \zeta_{yy,j} \Delta y_{t-j}) + e_{yt}, \tag{17}$$

$$\Delta c_t = \delta_c + \pi_c(y_{t-1} - \alpha - \beta c_{t-1}) + \sum_{j=1}^{p-1} (\zeta_{cc,j} \Delta c_{t-j} + \zeta_{cy,j} \Delta y_{t-j}) + e_{ct}, \tag{18}$$

9. I can also reject a unit root for the ratio of income to consumption, implying cointegration for a prespecified "balanced-growth" relationship. In this paper, I consider the more general case of an estimated cointegrating relationship because there is some evidence that the marginal propensity to consume out of permanent income is less than one for the measures of income and consumption under consideration. However, I note that all of the main results about the adjustment behavior of aggregate consumption are robust to the more restrictive assumption of balanced growth.

TABLE 2  
VECM ESTIMATES<sup>a</sup>

Panel A. Income growth						
$\delta_y$	$\pi_y$	$\zeta_{yy,1}$	$\zeta_{yy,2}$	$\zeta_{yc,1}$	$\zeta_{yc,2}$	$R^2$
-0.022 (0.096)	-0.163 (0.038)	0.134 (0.076)	0.063 (0.073)	0.667 (0.143)	0.203 (0.149)	0.274
Panel B. Consumption growth						
$\delta_c$	$\pi_c$	$\zeta_{cy,1}$	$\zeta_{cy,2}$	$\zeta_{cc,1}$	$\zeta_{cc,2}$	$R^2$
0.341 (0.052)	0.001 (0.021)	0.089 (0.041)	0.023 (0.040)	0.233 (0.078)	-0.001 (0.081)	0.143

<sup>a</sup>Calculated using OLS. Standard errors reported in parentheses.

where the error correction coefficients  $\pi_y$  and  $\pi_c$  reflect the predicted adjustment of income and consumption each quarter based on the deviation from their long-run equilibrium relationship.

Cochrane (1994) estimates the bivariate system in (17) and (18) by OLS given  $p = 3$  and a “balanced-growth” assumption for the cointegration relationship. Table 2 updates his results for the 1954–2005 sample period using chain-weighted data, GDP instead of GNP, and OLS estimates for the cointegrating error instead of imposing the assumption that  $\beta = 1$ . The reported estimates are close to Cochrane’s, although the  $R^2$  for the consumption regression is somewhat higher (14% versus 6% in Cochrane). Importantly, the error correction coefficients are very similar to Cochrane’s, with the coefficient for consumption,  $\pi_c$ , being extremely small. The point estimate for consumption suggests that the partial effect of the cointegrating error is an expected adjustment of consumption by only 0.1% of the deviation from the long-run equilibrium each quarter. The corresponding point estimate for income is 16%.

Figure 2 presents the error correction mechanism for the estimated VECM. In particular, the figure plots the responses of income and consumption implied by one unit cointegrating error and the estimated error correction coefficients.<sup>10</sup> Noting the different scales, income clearly does almost all of the adjustment in terms of restoring the cointegrating equilibrium, while consumption barely moves at all. However, while income adjusts by more than consumption, both have the same speed of adjustment. In particular, the error correction mechanism directly links the speed of adjustment of both income and consumption to the cointegrating error term, which follows an implied first-order autoregressive process:  $z_t = (1 + \pi_y - \beta\pi_c)z_{t-1} + e_{yt} - \beta e_{ct}$ ,

10. The error correction mechanism abstracts from the VAR coefficients associated with the lagged changes in income and consumption. These coefficients capture the short-run dynamics of income and consumption holding the cointegrating error constant. Meanwhile, the overall responses of income and consumption implied by the error correction mechanism and the short-run dynamics are not identified by the VECM, but depend on the configuration of the underlying structural shocks.

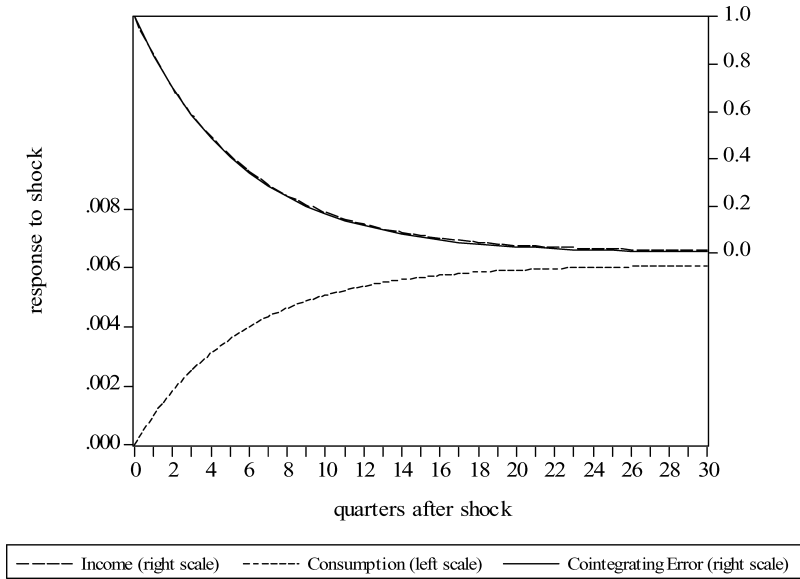


FIG. 2. The Error Correction Mechanism for the Estimated VECM.

where the results in Table 2 imply an autoregressive coefficient of  $1 + \hat{\pi}_y - \hat{\beta}\hat{\pi}_c = 0.836$ .

There is an important exception to the rule that an error correction mechanism implies the same speed of adjustment for the variables in terms of restoring their long-run equilibrium relationship. If one of the error correction coefficients is exactly equal to zero, the corresponding variable will be weakly exogenous with respect to the cointegrating relationship, which means that it adjusts instantaneously to its long-run equilibrium level. This exception is relevant in this case because, under the PIH, consumption should be weakly exogenous. An obvious test of the PIH, then, is whether the error correction coefficient for consumption is equal to zero ( $H_0: \pi_c = 0$ ). Based on the results in Table 2, the  $t$ -statistic for this test is 0.05. That is, according to the VECM results, the data are highly consistent with the PIH.

However, the ability of the VECM results to discriminate between different theories of aggregate consumption behavior is severely limited both by the restriction on speeds of adjustment and by the low power of the weak exogeneity test against relevant alternatives. The problem is that most theories of aggregate consumption behavior differ very little in terms of the implied amount that consumption adjusts to shocks in any given period (i.e., most theories are consistent with the idea that consumption is smooth). Where theories differ is in terms of the implied speeds of adjustment. For example, given habit formation in consumer preferences, consumption will adjust slowly in response to an increase in permanent income because the habit stock takes time to adjust. Similarly, in the buffer-stock/precautionary savings

theory, consumption adjusts slowly because agents are reluctant to consume out of changes in permanent income when those changes are highly volatile and, therefore, uncertain. In both cases, the small amount of adjustment each period will correspond to an extremely small, but non-zero, error correction coefficient,  $\pi_c$ . Thus, for small enough adjustment, the weak exogeneity test will have little power against the habit formation or precautionary savings hypotheses. Because the PIH implies an instantaneous adjustment of consumption, a more powerful way to test the PIH against habit formation or precautionary savings is to determine whether consumption adjustment is fast or slow. The correlated UC model of income and consumption in (1)–(8) allows for different speeds of adjustment of income and consumption to their long-run equilibrium levels, thus nesting different theories of aggregate consumption behavior and providing a simple means of discriminating between the PIH and other theories empirically.

### 3. RESULTS

Table 3 reports parameter estimates for the correlated UC model of income and consumption. An immediately striking result is that permanent income appears to be quite volatile. Compared to sample standard deviations of 0.9% for the first differences of log income and 0.5% for the first differences of log consumption, the estimated standard deviation of innovations to permanent income is larger ( $\hat{\sigma}_v = 1.6\%$ ). This result is similar to the estimated standard deviation for permanent GDP shocks of 1.2% that is reported in Morley, Nelson, and Zivot (2003) and is particularly notable because it arises from a multivariate model. In particular, Cochrane (1994) argues that univariate studies of aggregate income overstate the variability of permanent shocks because they do not account for the long-horizon predictability of income implied by cointegration between income and consumption. Yet, the model in (1)–(8) conditions on information inherent in the consumption data and takes cointegration into account. While it should be noted that sampling uncertainty leaves some question as to whether permanent income is actually more volatile than aggregate income, there is little doubt that it is more volatile than consumption. The  $t$ -statistic for  $H_0: \sigma_v \leq 0.5\%$  is 5.7, supporting Campbell and Deaton's (1988) conjecture that the PIH fails to explain why consumption is so smooth.<sup>11</sup>

There are two things to notice about the implied behavior of aggregate consumption in Table 3. First, the estimated marginal propensity to consume out of permanent income is 0.963, with a  $t$ -statistic for  $H_0: \gamma = 1$  of  $-6.2$ . Thus, it appears to be more

11. This  $t$ -statistic is constructed under the assumption that the true standard deviation of the first differences of consumption is 0.5%, even though this is only an estimate. The estimated standard error of the estimator of the standard deviation of the first differences of consumption is only 0.02. Thus, regardless of the covariance of the estimator of the standard deviation of the first differences of consumption and the estimator of the standard deviation of innovations to permanent income, a  $t$ -statistic which accounted for this sampling uncertainty would be very similar to the reported  $t$ -statistic.

TABLE 3  
CORRELATED UC MODEL ESTIMATES<sup>a</sup>

Panel A. Permanent income			
$\bar{c}$	$\gamma$	$\mu$	$\sigma_y$
-11.340 (6.480)	0.963 (0.006)	0.530 (0.108)	1.556 (0.184)
Panel B. Transitory income			
$\phi_{y,1}$	$\phi_{y,2}$	$\sigma_y$	
0.681 (0.056)	0.032 (0.044)	1.134 (0.172)	
Panel C. Transitory consumption			
$\phi_{c,1}$	$\phi_{c,2}$	$\sigma_c$	
0.851 (0.030)	0.057 (0.030)	1.098 (0.180)	
Panel D. correlations			
$\rho_{vy}$	$\rho_{vc}$	$\rho_{yc}$	
-0.875 (0.040)	-0.992 (0.005)	0.807 (0.060)	

<sup>a</sup>Calculated using MLE. Standard errors reported in parentheses.

appropriate to estimate a cointegrating relationship rather than impose a “balanced-growth” restriction. At the same time, the estimate is close enough to one that the analysis in Section 1 about the undesirable properties of the standard restriction of uncorrelated components is still relevant. Second, contrary to the PIH, consumption appears to adjust slowly toward permanent income. Figure 3 displays the impulse response functions for the transitory components of income and consumption. Using a standard measure for “speed of adjustment,” the half-life of a shock to the transitory component of income is less than a year. By contrast, the half-life of a shock to the transitory component of consumption is about 2 years. Evidently, the error correction mechanism restriction on speeds of adjustment would not be appropriate for these data. Indeed, the likelihood ratio statistic for  $H_0: \phi_{y,1} = \phi_{c,1}, \phi_{y,2} = \phi_{c,2}$  is 25.15, which far exceeds the 5% critical value of 5.99 for the test.<sup>12</sup> Meanwhile, the results are robust to different lag order specifications. In particular, the dynamics are driven mostly by the first lag of the autoregressive processes, with a likelihood ratio test

12. Note, however, that the restricted model is not exactly the same as the VECM model in (17) and (18). In particular, while the error correction mechanism of a VECM has a more restrictive structure than the UC model, the VAR coefficients associated with the lagged changes in income and consumption in the VECM allow for more general transitory dynamics for different underlying structural shocks than the UC model, which imposes the same transitory dynamics for all shocks. Thus, the correlated UC model and a VECM are non-nested and cannot be directly compared using a likelihood ratio test.

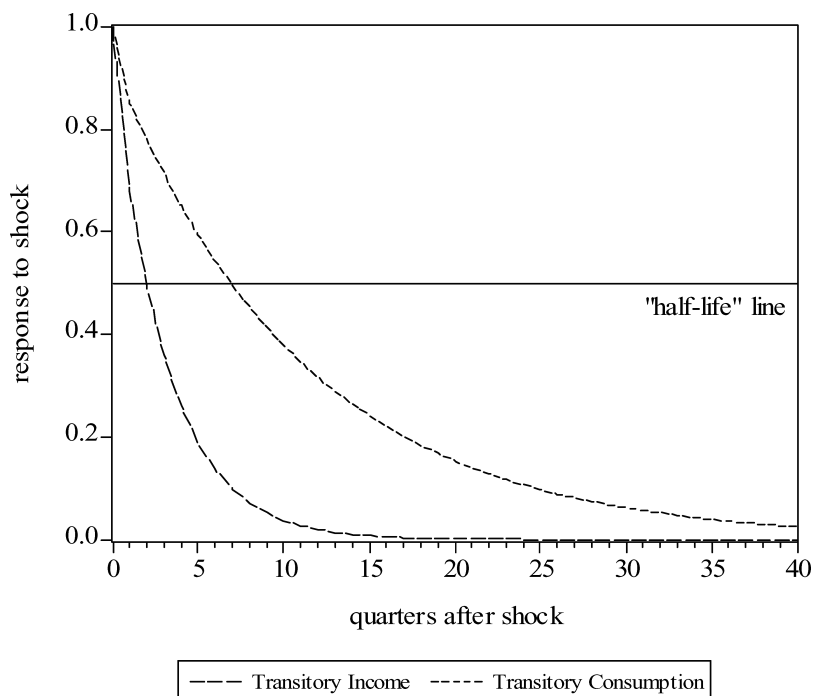


FIG. 3. Impulse Response Functions for the Transitory Components of Income and Consumption from the Estimated Correlated UC Model.

statistic for  $H_0: \phi_{y,2} = \phi_{c,2} = 0$  of 3.91, which has a  $p$ -value of 0.14. I report the AR(2) estimates instead of AR(1) estimates to make it clear that the lack of any hump shape or periodic behavior in the impulse response functions is the result of the data, not the model specification.

Perhaps the most interesting results in Table 3 relate to the correlation parameters defined in (6)–(8). In order for permanent income to be more volatile than consumption and income, innovations to permanent income must be negatively correlated with the transitory components of income and consumption, which is what I find. The estimate for the correlation between permanent and transitory movements in income ( $\hat{\rho}_{vy} = -0.875$ ) is very similar to the  $-0.906$  estimate for the univariate UC model of GDP reported in Morley, Nelson, and Zivot (2003). The natural structural interpretation for this correlation is that it reflects the effects of real productivity shocks on the transitory component of aggregate income.<sup>13</sup> That is, income does not immediately adjust fully to a change in permanent income, perhaps due to “time-to-build” effects. Thus, the transitory component of income will initially be negative when permanent

13. Proietti (2006) points out that the negative correlation could also result from a reverse hysteresis in which positive cyclical (e.g., fiscal) shocks lower the trend and vice versa.

income increases and income lags behind.<sup>14</sup> Meanwhile, innovations to permanent income and the transitory component of consumption are almost perfectly negatively correlated ( $\hat{\rho}_{vc} = -0.992$ ). The natural structural interpretation of this correlation is that it reflects the effects of shocks to permanent income on the transitory component of consumption. That is, combined with the adjustment dynamics displayed in Figure 3, the near perfect negative correlation suggests that, while consumption is driven almost entirely by shocks to permanent income, it does not adjust to those shocks immediately, but only slowly over time. Again, it may be useful to think about this partial adjustment behavior in terms of a representative agent who prefers smooth consumption due to habit formation or a precautionary savings motive.<sup>15</sup> The monotonic adjustment of transitory consumption in Figure 3, which is not imposed by the AR(2) specification, is consistent with this interpretation of an underlying preference for smooth consumption. Finally, the positive correlation between the transitory components of income and consumption ( $\hat{\rho}_{yc} = 0.807$ ) appears at first glance to be consistent with the Keynesian view of aggregate consumption behavior. However, it is also the simple counterpart to the fact that the transitory components of income and consumption are both highly negatively correlated with innovations to permanent income. Indeed, the fact that consumption takes significantly longer to adjust than income, as displayed in Figure 3, argues against liquidity-constrained or “rule-of-thumb” consumers stimulating income through the multiplier process.<sup>16</sup> Meanwhile, the apparent lack of influence of nominal shocks on transitory consumption that is implied by the near perfect negative correlation between the transitory component of consumption and innovations to permanent income also argues against the Keynesian view.<sup>17</sup>

An important issue is the significance of the correlation parameters  $\rho_{vy}$ ,  $\rho_{vc}$ , and  $\rho_{yc}$ . Individually, they appear to be highly significant with  $t$ -statistics of  $-21.9$ ,  $-198.4$ , and  $13.5$ , respectively. However, given concerns about weak identification, Table 4

14. Note that, consistent with univariate studies of U.S. real GDP, the negative correlation between permanent and transitory movements and the partial adjustment of the transitory component over time correspond to positive serial correlation in the growth rates. In particular, consistent with first-order autoregressive dynamics, a positive shock implies a number of periods of above-average growth and vice versa.

15. Christiano (1987) shows that slow adjustment of consumption could also be consistent with standard real business cycle models in which the interest rate is allowed to change following shocks to permanent income. However, Boldrin, Christiano, and Fisher (2001) argue that other aspects of the data, such as the positive serial correlation of income growth, which is implied by the estimated UC model in Table 3, are more consistent with a real business cycle model augmented with habit formation in consumer preferences.

16. However, Zeldes (1989) points out that liquidity constraints can affect consumption behavior even if they are not binding. The possibility of liquidity constraints binding at some point in the future can cause consumers to save today to provide insurance against possible future falls in income. In this sense, the possibility of liquidity constraints binding in the future can produce behavior that is similar to consumption under a precautionary savings motive.

17. Indeed, perfect negative correlation would correspond to the idea that consumption is driven entirely by permanent income shocks. Note, however, that a restricted version of the model imposing this perfect correlation can be rejected at the 5% level. The likelihood ratio statistic is 6.52, which has a  $p$ -value 0.04 given two parameter restrictions (in addition to the restriction of perfect negative correlation between permanent and transitory shocks to consumption, the correlation between the transitory components of income and consumption is no longer an independent parameter for the restricted model).

TABLE 4  
 LIKELIHOOD RATIO TESTS

	LR statistic	5% critical value <sup>a</sup>	Nuisance parameters
$H_0: \rho_{vy} = \rho_{vc} = \rho_{yc} = 0$	53.60	7.81	None
$H_0: \rho_{vy} = \rho_{vc} = 0$	48.07	5.99	None
$H_0: \sigma_c = 0$	58.06	11.07 <sup>b</sup>	$\phi_{c,1}, \phi_{c,2}, \rho_{vc}, \rho_{yc}$

<sup>a</sup>Based on chi-square distributions with degrees of freedom equal to the number of restricted parameters under the null hypothesis.

<sup>b</sup>Due to the presence of nuisance parameters, this critical value likely overstates the significance level of the test statistic. It is reported for illustrative purposes only.

reports results for a likelihood ratio test of their joint significance, as well as two other likelihood ratio tests. The hypothesis of uncorrelated components ( $H_0: \rho_{vy} = \rho_{vc} = \rho_{yc} = 0$ ), which corresponds to the standard UC assumption, can be strongly rejected.<sup>18</sup> The second test considers a slightly less restrictive hypothesis that allows the transitory components of income and consumption to be correlated, but maintains an assumption that permanent and transitory movements are uncorrelated ( $H_0: \rho_{vy} = \rho_{vc} = 0$ ). The restriction in this case is that permanent income is essentially as smooth as consumption because permanent and transitory movements in consumption will be uncorrelated and, given  $\gamma \approx 1$ , the implications of (9)–(11) in Section 1 are relevant. This restriction, which captures the idea emphasized by Cochrane (1994) that consumption is nearly, but not exactly, a random walk, is strongly rejected. Instead, permanent income appears to be more volatile than consumption, implying that consumption is smooth in spite of, not because of, permanent income. The third test in Table 4 is of the strict version of the PIH, under which consumption follows a random walk and is the stochastic trend in income. Not surprisingly, given the other results, the test statistic is quite large, although the level of significance is somewhat clouded by the presence of nuisance parameters under the alternative hypothesis. Taken together, however, the results of the likelihood ratio tests provide strong evidence against the PIH.

#### 4. INTERPRETATION

To help with the interpretation of the results, I present two simulations based on the estimated correlated UC model. In the first simulation, I consider a one-time shock to permanent income. In the second simulation, I generate an artificial sample of data from the model.

18. The standard UC assumption of uncorrelated components is often referred to as an “identifying” assumption. However, if the uncorrelated components assumption really were really necessary to identify the model, the likelihood ratio statistic would be zero.



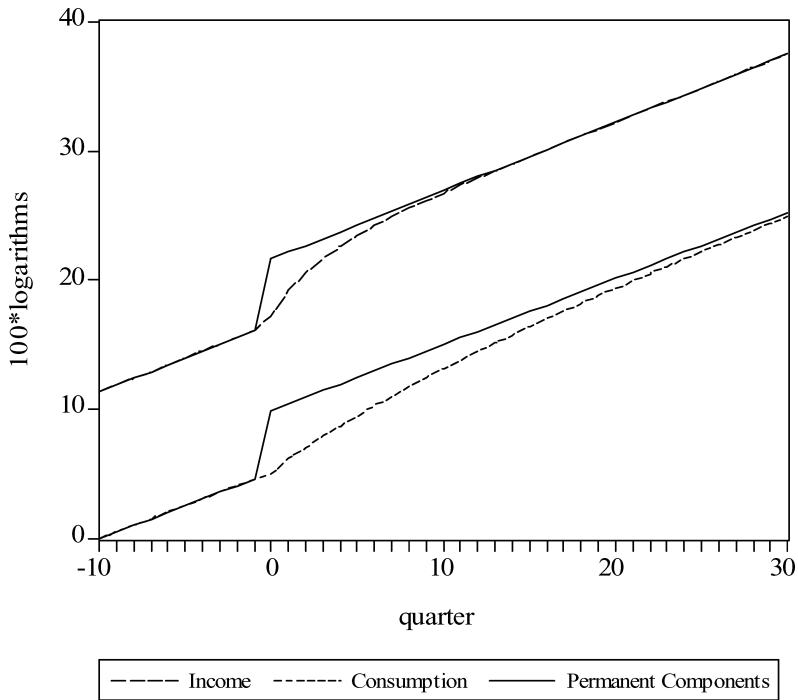


FIG. 4. The Paths of Income and Consumption Given a One-Time Shock to Permanent Income Based on the Estimated Correlated UC Model.

Figure 4 presents the results for the first simulation. Taking the negative correlations in Table 3 as reflecting the causal effects of shocks to permanent income on the transitory components of income and consumption, I find that both income and consumption take many quarters to fully adjust to a one-time increase in permanent income. Of the two series, income adjusts relatively quickly, although the lack of complete immediate adjustment is suggestive of “time-to-build” dynamics. Consumption eventually responds on a one-to-one basis to the change, but it undergoes a slow and monotonic adjustment that is consistent with a slowly adjusting habit stock or precautionary savings given high uncertainty over whether the shock to permanent income will be reversed in the future. The different speeds of adjustment of income and consumption are simply the counterparts to the dynamics for the transitory components presented in Figure 3. However, the simulation in Figure 4 makes the typical context of those dynamics clearer, especially in terms of how they relate to the negative correlations between permanent and transitory movements. Specifically, it is the fact that income and consumption remain temporarily below their new permanent levels that generates negative innovations to their transitory components following a positive shock to permanent income.

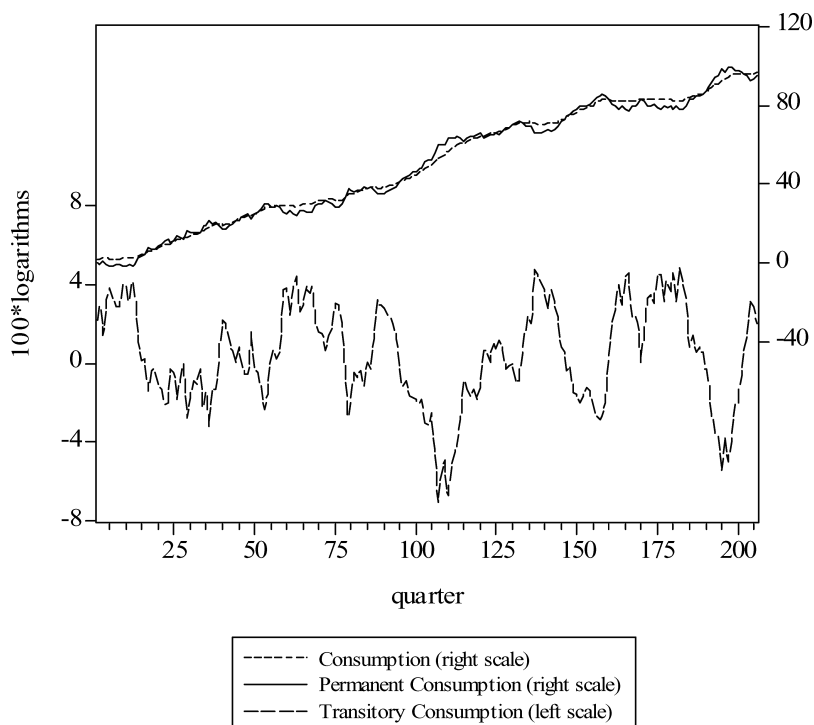


Fig. 5. An Artificial Sample of Consumption and Its Components Based on the Estimated Correlated UC Model.

While Figure 4 is somewhat revealing about the dynamics of income and consumption implied by the estimated UC model, it is highly deceptive in one key respect. In particular, the simulation abstracts from the implication of the estimated model that permanent income is highly volatile from period to period. Figure 5 presents results from a simulation that captures this implication. Specifically, I generate an artificial sample of consumption and its components based on the estimates in Table 3. As can be seen, consumption is much smoother than its permanent component. It is also easy to see the negative correlation between the permanent and transitory movements in consumption. When the permanent component moves below consumption, the transitory component is positive and vice versa. Meanwhile, it might appear that consumption traces out a meaningful trend for the permanent component, but it is only an illusion. By construction of the simulation, the permanent component of consumption follows a random walk and does not predictably revert back to consumption. Instead, at any given point of time, consumption is slowly adjusting toward the permanent component. While the volatility of the permanent component means that it sometimes crosses over consumption *ex post*, it is not expected to do so

*ex ante*.<sup>19</sup> Thus, it is clear from this figure that permanent income can be volatile even if income and consumption are cointegrated and consumption is smooth.

## 5. CONCLUSION

According to a correlated UC model of cointegration, the common stochastic trend of income and consumption, which can be interpreted as permanent income, is relatively volatile. This finding stands in contrast to Cochrane's (1994) findings from a VECM, but is consistent with findings in a number of univariate studies, including Morley, Nelson, and Zivot (2003). In terms of aggregate consumption behavior, the volatility of permanent income confirms Campbell and Deaton's (1989) conjecture that the PIH does not explain the smoothness of consumption over time. Instead, the slow adjustment of consumption toward permanent income is suggestive of habit formation in consumer preferences or the presence of a precautionary savings motive. The volatility of permanent income is particularly suggestive of precautionary savings under uncertainty. Meanwhile, it should be emphasized that these findings are obtained from a relatively simple and unrestrictive time series model that "let the data speak for themselves" on the issue of the speed of adjustment of consumption to permanent income.<sup>20</sup>

I conclude by noting that the suggestion of habit formation in consumer preferences is intriguing because a number of studies have shown that it can help explain otherwise puzzling economic phenomena, including the asset-pricing anomalies of the "equity premium puzzle" and the "risk-free rate puzzle" (Abel 1990, Constantinides 1990, Heaton 1995, Campbell and Cochrane 1999, Boldrin, Christiano, and Fisher 2001),<sup>21</sup> the finding in the growth literature that growth Granger-causes savings and not vice versa (Carroll and Weil 1994, Carroll, Overland, and Weil 2000), the gradual response of consumption and inflation to monetary policy (Fuhrer 2000), and the behavior of consumption in periods surrounding exchange-rate stabilization programs (Uribe

19. This *ex post* versus *ex ante* distinction is related to Marsh and Merton's (1986) criticism of variance bounds tests of stock market rationality. While stock market prices are much more volatile than dividend payments, prices could still follow a random walk that also drives dividends as long as managers smooth dividends over time (i.e., in terms of Figure 5, stock prices would be like permanent consumption and dividends would be like consumption). Variance bounds tests are misleading because they ignore the possibility that movements in the permanent component of dividends (i.e., stock market prices under traditional notions of market efficiency) could be negatively correlated with the transitory component of dividends, as they would be under a policy of dividend smoothing in the face of a volatile intrinsic value of the firm.

20. While the high volatility of permanent income is an important result from the analysis, it is interesting to note that even if income and consumption are modeled as trend stationary AR(2) processes, implying no stochastic variation in permanent income, the estimated adjustment to permanent income is much slower for consumption than for income. Given OLS estimates, the half-life of a shock to income is about four years, while the half-life of a shock to consumption is over nine years. Thus, the slower adjustment of aggregate consumption is robust to the assumption that there is no unit root in aggregate income and consumption.

21. However, Otrok, Ravikumar, and Whiteman (2002) provide a somewhat more mixed view on the ability of habit formation to explain the asset pricing anomalies.

2000). As for the suggestion of a precautionary savings motive in consumer behavior, Carroll and Weil (1994) cite it as an important additional explanation for the growth literature Granger-causation result and Hubbard, Skinner, and Zeldes (1994) argue that it can help explain a number of puzzles in the household-level consumption data including different savings behavior between rich and poor households.

## APPENDIX

There are usually multiple ways to represent a time series model in state-space form. However, in order to allow for correlation between innovations to different components, it is most convenient to treat the observation equation as an identity and place all of the components in the state vector. Specifically, I consider the following structure for the state equation:

$$\beta_t = \tilde{\mu} + F\beta_{t-1} + G\tilde{v}_t, \quad (\text{A1})$$

where, based on (3)–(5) and AR(2) specifications for the transitory components,

$$\beta_t = \begin{bmatrix} \tau_t \\ u_{yt} \\ u_{y,t-1} \\ u_{ct} \\ u_{c,t-1} \end{bmatrix}, \quad \tilde{v}_t = \begin{bmatrix} v_t \\ \varepsilon_{yt} \\ \varepsilon_{ct} \end{bmatrix},$$

$$\tilde{\mu} = \begin{bmatrix} \mu \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_{y,1} & \phi_{y,2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{c,1} & \phi_{c,2} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \text{and} \quad G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, based on (6)–(8), the covariance matrix for  $\tilde{v}_t$ , denoted  $Q$ , is

$$Q = \begin{bmatrix} \sigma_v^2 & \rho_{vy}\sigma_v\sigma_y & \rho_{vc}\sigma_v\sigma_c \\ \rho_{vy}\sigma_v\sigma_y & \sigma_y^2 & \rho_{yc}\sigma_y\sigma_c \\ \rho_{vc}\sigma_v\sigma_c & \rho_{yc}\sigma_y\sigma_c & \sigma_c^2 \end{bmatrix}.$$

Meanwhile, the observation equation is

$$\tilde{y}_t = A + H\beta_t, \quad (\text{A2})$$

where, based on (1) and (2),

$$\tilde{y}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix}, \quad A = \begin{bmatrix} 0 \\ \bar{c} \end{bmatrix}, \quad \text{and} \quad H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ \gamma & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The Kalman filter for this state-space model is given by the following six equations:

$$\beta_{t|t-1} = \tilde{\mu} + F\beta_{t-1|t-1}, \tag{A3}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + GQG', \tag{A4}$$

$$\eta_{t|t-1} = \tilde{y}_t - H\beta_{t|t-1}, \tag{A5}$$

$$f_{t|t-1} = HP_{t|t-1}H', \tag{A6}$$

$$\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1}, \tag{A7}$$

$$P_{t|t} = P_{t|t-1} - K_tHP_{t|t-1}, \tag{A8}$$

where  $\beta_{t|t-1} \equiv E_{t-1}[\beta_t]$ , for example, denotes the expectation of  $\beta_t$  conditional on information up to time  $t - 1$ ;  $P_{t|t-1}$  is the variance–covariance of  $\beta_{t|t-1}$ ;  $\eta_{t|t-1}$  is a vector of the conditional forecast errors of the observed series;  $f_{t|t-1}$  is the variance–covariance of  $\eta_{t|t-1}$ ; and  $K_t \equiv P_{t|t-1}H'f_{t|t-1}^{-1}$  is the Kalman gain.

Given any set of parameter values and initial values  $\beta_{0|0}$  and  $P_{0|0}$ , (A3)–(A8) can be solved recursively for  $t = 1, \dots, T$  to obtain  $\eta_{t|t-1}$  and  $f_{t|t-1}$ . Based on 1952:Q1–1953:Q4 data for U.S. per capita real GDP, I set  $\tau_{0|0} = 947.5$ , which is the value of permanent income in 1953:Q4 implied by a linear time trend for the pre-1954 data. Then, to account for the large amount of uncertainty surrounding this admittedly arbitrary estimate, I set  $\text{var}(\tau_{0|0}) = 100$ , corresponding to a relatively diffuse prior for permanent income, with the 95% confidence bands far exceeding the range of the realized data over that short sample period. The estimation results are highly robust to other priors. Meanwhile, the other elements of  $\beta_{0|0}$  and  $P_{0|0}$  are set to the unconditional means and variances of transitory income and consumption implied by a given set of parameter values. Maximum likelihood estimates are calculated using the following prediction error decomposition of the log likelihood function (see Harvey 1993):

$$l(\theta) = -\frac{1}{2} \sum_{t=1}^T \ln [(2\pi)^2 |f_{t|t-1}|] - \frac{1}{2} \sum_{t=1}^T \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1}, \tag{A9}$$

where  $\theta$  is the vector of parameters.<sup>22</sup> Because  $l(\theta)$  is a non-linear function of the model parameters, estimation requires numerical optimization. Parameters are

22. In practice, I drop the first prediction error from evaluation of the likelihood function to make the sample period conform with that of the VECM for the first differences of log income and consumption reported in Table 2.

constrained to lie in feasible regions (e.g., the variance–covariance matrix for the shocks is restricted to be positive definite and the autoregressive parameters are restricted to imply stationarity for the transitory components). Standard errors of the maximum likelihood estimates are based on numerical second derivatives.

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