11. Dietterich TG (2000) Ensemble methods in machine learning. In: Proc. of the 1st international workshop on multiple classifier systems (MCS '00). Springer, Berlin, pp 1–15
12. Dietterich TG, Bakiri G (1995) Solving multiclass learning problems via error-correcting output codes. J Artif Intell Res 2:263–286
13. Džeroski S, Ženko B (2004) Is combining classifiers with stacking better than selecting the best one? Mach Learn 54(3):255–273
14. Efron B (1979) Bootstrap methods: Another look at the jackknife. Ann Stat 7(1):1–26
15. Freund Y, Schapire RE (1996) Experiments with a new boosting algorithm. In: Saitta L (ed) Machine learning: Proc. of the 13th international conference (ICML '96). Morgan Kaufmann, San Francisco, pp 148–156
16. Friedman JH (2001) Greedy function approximation: a gradient boosting machine. Ann Stat 29(5):11189–1232
17. Friedman JH, Popescu BE (2005) Predictive learning via rule ensembles. Technical report, Stanford University, Department of Statistics
18. Friedman JH, Hastie T, Tibshirani RJ (1998) Additive logistic regression: a statistical view of boosting. Technical report, Stanford University, Department of Statistics
19. Hamming RW (1950) Error detecting and error correcting codes. Bell Syst Tech J 26(2):147–160
20. Hansen LK, Salamon P (1990) Neural network ensembles. IEEE Trans Pattern Anal Mach Intell 12(10):993–1001
21. Hastie T, Tibshirani RJ, Friedman JH (2001) The elements of statistical learning. Springer Series in Statistics. Springer, Berlin
22. Ho TK (1998) The random subspace method for constructing decision forests. IEEE Trans Pattern Anal Mach Intell 20(8):832–844
23. Ho TK (2000) Complexity of classification problems and comparative advantages of combined classifiers. In: Kittler J, Roli F (eds) Proc. of the 1st international workshop on multiple classifier systems (MCS '00), vol 1857. Springer, Berlin, pp 97–106
24. Jacobs RA (1995) Methods for combining experts' probability assessments. Neural Comput 7(5):867–888
25. Jordan MI, Jacobs RA (1992) Hierarchies of adaptive experts. In: Moody JE, Hanson S, Lippmann RP (eds) Advances in Neural Information Processing System (NIPS). Morgan Kaufmann, San Mateo, pp 985–992
26. Kearns MJ, Vazirani UV (1994) An introduction to computational learning theory. MIT Press, Cambridge
27. Kittler J, Hatef M, Duin RPW, Matas J (1998) On combining classifiers. IEEE Trans Pattern Anal Mach Intell 20(3):226–239
28. Kononenko I, Kukar M (2007) Machine learning and data mining: introduction to principles and algorithms. Horwood, Chichester
29. Kuncheva LI (2004) Combining pattern classifiers: methods and algorithms. Wiley-Interscience, Hoboken
30. Kuncheva LI, Whitaker CJ (2003) Measures of diversity in classifier ensembles and their relationship with the ensemble accuracy. Mach Learn 51(2):181–207
31. Mitchell T (1997) Machine Learning. McGraw-Hill, New York
32. Panov P, Džeroski S (2007) Combining bagging and random subspaces to create better ensembles. In: Proc. of 7th international symposium on intelligent data analysis (IDA '07), vol 4723. Lecture notes in computer science. Springer, Berlin, pp 118–129
33. Polikar R (2006) Ensemble based systems in decision making. IEEE Circuits Syst Mag 6(3):21–45
34. Rätsch G, Demiriz A, Bennett KP (2002) Sparse regression ensembles in infinite and finite hypothesis spaces. Mach Learn 48(1–3):189–218
35. Ridgeway G, Madigan D, Richardson T (1999) Boosting methodology for regression problems. In: Heckerman D, Whittaker J (eds) Proc. of the 7th international workshop on artificial intelligence and statistics. Morgan Kaufmann, San Francisco, pp 152–161
36. Rosenblatt F (1962) Principles of neurodynamics: perceptron and the theory of brain mechanisms. Spartan Books, Washington
37. Schapire RE (1990) The strength of weak learnability. Mach Learn 5(2):197–227
38. Schapire RE (1999) A brief introduction to boosting. In: Proc. of the 6th international joint conference on artificial intelligence. Morgan Kaufmann, San Francisco, pp 1401–1406
39. Schapire RE (2001) The boosting approach to machine learning: an overview. In: MSRI workshop on nonlinear estimation and classification, Berkeley, CA, 2001
40. Schapire RE, Singer Y (1999) Improved boosting using confidence-rated predictions. Mach Learn 37(3):297–336
41. Schapire RE, Freund Y, Bartlett P, Lee WS (1998) Boosting the margin: a new explanation for the effectiveness of voting methods. Ann Stat 26(5):1651–1686
42. Ting KM, Witten IH (1999) Issues in stacked generalization. J Artif Intell Res 10:271–289
43. Witten IH, Frank E (2005) Data mining: practical machine learning tools and techniques, 2nd edn. Morgan Kaufmann, San Francisco
44. Wolpert DH (1992) Stacked generalization. Neural Netw 5(2):241–259

Books and Reviews

Brown G Ensemble learning bibliography. http://www.cs.man.ac.uk/~gbrown/ensemblebib/index.php. Accessed 26 March 2008

Weka 3: Data mining software in Java. http://www.cs.waikato.ac.nz/ml/weka/. Accessed 26 March 2008

Macroeconomics, Non-linear Time Series in

JAMES MORLEY
Washington University, St. Louis, USA

Article Outline

Glossary

Nonlinear time series in macroeconomics A field of study in economics pertaining to the use of statistical analysis of data in order to make inferences about nonlinearities in the nature of aggregate phenomena in the economy.

Time series A collection of data corresponding to the values of a variable at different points of time.

Linear Refers to a class of models for which the dependence between two random variables can be completely described by a fixed correlation parameter.

Nonlinear Refers to the class of models for which the dependence between two random variables has a more general functional form than a linear equation and/or can change over time.

Structural change A change in the model describing a time series, with no expected reversal of the change.

Level Refers to a definition of the business cycle that links the cycle to alternation between phases of expansion and recession in the level of economic activity.

Deviations Refers to a definition of the business cycle that links the cycle to transitory deviations of economic activity from a trend level.

Fluctuations Refers to a definition of the business cycle that links the cycle to any short-run changes in economic activity.

Deepness A characteristic of a process with a skewed unconditional distribution.

Steepness A characteristic of a process with a skewed unconditional distribution for its first-differences.

Sharpness A characteristic of a process for which the probability of a peak when increasing is different than the probability of a trough when decreasing.

Time reversibility The ability to substitute $-t$ and t in the equations of motion for a process without changing the process.

Markov-switching models Models that assume the prevailing regime governing the conditional distribution of a variable or variables being modeled depends on an unobserved discrete Markov process.

Self-exciting threshold models Models that assume the prevailing regime governing the conditional distribution of a variable or variables being modeled is observable and depends on whether realized values of the time series being modeled exceed or fall below certain "threshold" values.

Nuisance parameters Parameters that are not of direct interest in a test, but influence the distribution of a test statistic.

Pivotal Refers to the invariance of the distribution of a test statistic with respect to values of parameters in the data generating process under the null hypothesis.

Size Probability of false rejection of a null hypothesis in repeated experiments.

Power Probability of correct rejection of a null hypothesis in repeated experiments.

Definition of the Subject

Nonlinear time series in macroeconomics is a broad field of study in economics. It refers to the use of statistical analysis of data to make inferences about nonlinearities in the nature of aggregate phenomena in the economy. This analysis is relevant for forecasting, the formulation of economic policy, and the development and testing of macroeconomic theories.

Introduction

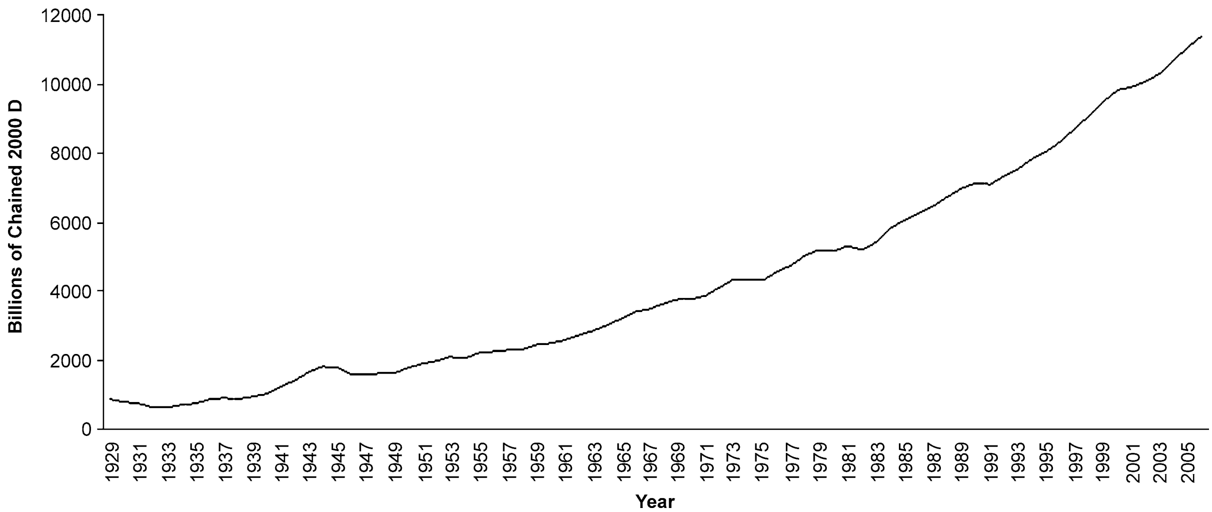
In macroeconomics, the primary aggregate phenomenon is the flow of total production for the entire economy over the course of a year, which is measured by real gross domestic product (GDP). A collection of data corresponding to the values of a variable such as real GDP at different points of time is referred to as a *time series*. Figure 1 presents the time series for US real GDP for each year from 1929 to 2006.

Time series analysis employs stochastic processes to explain and predict the evolution of a time series. In particular, a process captures the idea that different observations are in some way related to each other. The relationship can simply be that the observations behave as if they are drawn from random variables with the same distribution. Or the relationship can be that the distribution assumed to generate one observation depends on the values of other observations. Either way, a relationship implies that the observations can be used jointly to make inferences about the parameters describing the distributions (a.k.a. "estimation").

Within the context of time series in macroeconomics, the terms "linear" and "nonlinear" typically refer to classes of models for processes, although other meanings arise in the literature. For the purposes of this survey, a model that assumes the dependence between two random variables in a process can be completely captured by a fixed correlation parameter is said to be *linear*. A very basic example of a linear time series model is the workhorse first-order autoregressive (AR(1)) model:

$$y_t = c + \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{ i.i.d. } (0, \sigma^2) , \qquad (1)$$

where $|\phi| < 1$. In words, the random variable y_t that generates the observation in period t is a linear function of



Macroeconomics, Non-linear Time Series in, Figure 1
US real GDP 1929–2006 (Source: St. Louis Fed website)

the random variable y_{t-1} that generates the observation in period $t-1$. The process $\{y_t\}_{-\infty}^{\infty}$ is stochastic because it is driven by random "shocks", such as ε_t in period t. These shocks have the same distribution in every period, meaning that, unlike with y_t and y_{t-1}, the distribution of ε_t does not depend on the value of ε_{t-1} or, for that matter, any other shock in any other period (hence the "i.i.d." tag, which stands for "independently and identically distributed"). It is straightforward to show that the correlation between y_t and y_{t-1} is equal to ϕ and this correlation describes the entire dependence between the two random variables. Indeed, for the basic AR(1) model, the dependence and correlation between any two random variables y_t and y_{t-j}, for all t and j, depends only on the fixed parameter ϕ according to the simple function ϕ^j and, given $|\phi| < 1$, the process has finite memory in terms of past shocks. For other time series models, the functions relating parameters to correlations (i. e., "autocorrelation generating functions") are generally more complicated, as are the restrictions on the parameters to ensure finite memory of shocks. However, the models are still linear, as long as the parameters and correlations are fixed.

In contrast to the linear AR(1) model in (1) and other models with fixed correlations, any model that allows for a more general functional form and/or time variation in the dependence between random variables can be said to be *nonlinear*. This nomenclature is obviously extremely open-ended and examples are more revealing than general definitions. Fortunately, macroeconomics provides many examples, with "nonlinear" typically used to describe models that are closely related to linear models, such as the AR(1) model, but which relax one or two key assumptions in order to capture some aspect of the data that cannot be captured by a linear model. The focus of this survey is on these types of nonlinear models.

It should be mentioned at the outset that, in addition to nonlinear models, "nonlinear time series" evokes nonparametric and semiparametric methods (e. g., neural networks). These methods tend to be data intensive and so find more use in finance and other fields where sample sizes are larger than in macroeconomics. "Nonlinear time series" also evokes the development and application of tests for nonlinearity. However, these are the purview of econometrics, not macroeconomics. Thus, tests for nonlinearity will only be discussed in the context of applications that are particularly relevant to the choice of appropriate models for macroeconomic data.

Types of Nonlinear Models

Starting with the linear AR(1) model in (1), there are many ways to introduce nonlinearities. An obvious way is to consider a nonlinear specification for the relationship between the random variables in the model. For example, consider the simple bilinear model:

$$y_t = c + \phi y_{t-1} + \varepsilon_t + \theta(\varepsilon_{t-1} \cdot y_{t-1}) \,, \qquad \varepsilon_t \sim \text{ i.i.d. } (0, \sigma^2) \,. \quad (2)$$

See Granger and Andersen [57] and Rao and Gabr [139] on bilinear models. In macroeconomics at least, there are relatively few applications of bilinear models, although see Peel and Davidson [119], Rothman [128], and Hristova [71].

A more typical approach to introducing nonlinearities in macroeconomics is to allow one (or more) of the parameters in a linear model to be driven by its own process. For example, in a macroeconomics paper that was motivated in part by bilinear models, Engle [46] assumed the squares of shocks (i. e., ε_t^2) follow an AR process, with the implication that the conditional variance of y_t is no longer a constant parameter. Given an AR(1) assumption for ε_t^2, the conditional variance is

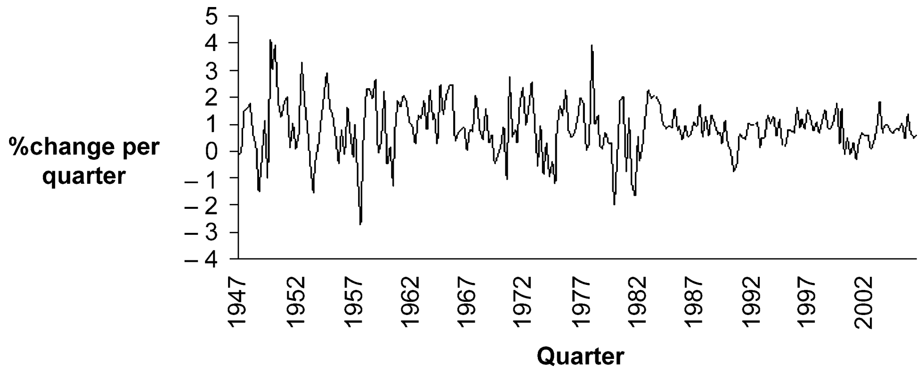
$$E_{t-1}\left[\sigma_t^2\right] = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \,, \tag{3}$$

where $E_{t-1}[\,]$ is the conditional expectations operator, with expectations formed using information available in period $t-1$. Engle [46] applied this "autoregressive conditional heteroskedasticity" (ARCH) model to U.K. inflation, although in subsequent research, it has mostly been applied to financial time series. In particular, asset returns tend to display little dependence in the mean, but high positive dependence in terms of the variance (a.k.a. "volatility clustering"), which is exactly what the ARCH model was designed to capture. Beyond Engle's original paper, ARCH models have found little use in macroeconomics, although Bansal and Yaron [4] have recently attempted to resolve the so-called "equity premium puzzle" in part by assuming that US aggregate consumption growth follows a GARCH(1,1) process that generalizes Engle's original ARCH process. However, Ma [104] shows that estimates supporting a GARCH(1,1) model for aggregate consumption growth are due to weak identification, with an appropriate confidence interval suggesting little or no conditional heteroskedasticity. Weak identification is also likely a problem for the earlier application of GARCH models to macroeconomic variables by French and Sichel [49]. In general, because most macroeconomic data series are highly aggregated, the central limit theorem is relevant, at least in terms of eliminating "fat tails" due to volatility clustering that may or may not be present at the microeconomic level or at higher frequencies than macroeconomic data are typically measured.

The ARCH model begs the question of why not consider a stochastic process directly for the variance, rather than for the squares of the shocks. The short answer is a practical one. A model with "stochastic volatility" is more difficult to estimate than an ARCH model. In particular, it can be classified as a state-space model with an unobserved non-Gaussian volatility process that has a nonlinear relationship to the observable time series being modeled. In the simple case of no serial correlation in the underlying series (e. g., no AR dynamics), a stochastic volatility model can be transformed into a linear state-space model for the squares of the series, although the model still has non-Gaussian errors. However, the lack of serial correlation means that this simple version of the model would be more appropriate for applications in finance than macroeconomics. In any event, while the Kalman filter can be employed to help estimate linear Gaussian state-space models, it is less suitable for non-Gaussian state-space models and not at all suitable for nonlinear state-space models. Recent advances in computing power have made simulation-based techniques (the Gibbs sampler and the so-called "particle filter") available to estimate such models, but these techniques are far from straightforward and are highly computationally intensive. See Kim, Shephard, and Chib [88] and Chib, Nardari, and Shephard [21] on estimation of stochastic volatility models via the Gibbs sampler and particle filtering. Meanwhile, such models have rarely been applied to macroeconomic data due to the lack of interesting volatility dynamics discussed above.

To the extent that stochastic volatility models have been applied in macroeconomics, the focus has been on capturing structural change (i. e., permanent variation) in volatility rather than volatility clustering. For example, Stock and Watson [138] investigate the so-called "Great Moderation" using a stochastic volatility model and confirm the findings reported in Kim and Nelson [77] and McConnell and Perez-Quiros [107] that there was a permanent reduction in the volatility of US real GDP growth in the mid-1980s (see also [82,116,132]). This change in volatility is fairly evident in Fig. 2, which presents the time series for US real GDP growth for each quarter from 1947:Q2 to 2006:Q4.

Yet, while it is sometimes merely a matter of semantics, it should be noted that "structural change" is a distinct concept from "nonlinearity". In particular, *structural change* can be thought of as a change in the model describing a time series, where the change is permanent in the sense that it is not expected to be reversed. Then, if the underlying structure of each model is linear, such as for the AR(1) model in (1), there is nothing particularly "nonlinear" about structural change. On the other hand, Bayesian analysis of structural change blurs the distinction between structural change and nonlinearity. In particular, it treats parameters as random variables for the purposes of making inferences about them. Thus, the distinction between a model that allows "parameters" to change according to a stochastic process and a collection of models with the same structure, but different parameters, is essentially a matter of taste, even if the former setup is clearly nonlinear, while the latter is not. For example, consider the classic time-varying parameter model (see, for example [29]).



Macroeconomics, Non-linear Time Series in, Figure 2
US real GDP growth 1947–2006 (Source: St. Louis Fed website)

Macroeconomics, Non-linear Time Series in, Figure 3
US inflation 1960–2006 (Source: St. Louis Fed website)

Like the stochastic volatility model, it assumes a stochastic process for the parameters in what would, otherwise, be a linear process. Again, starting with the AR(1) model in (1) and letting $\beta = (c, \phi)'$, a time-varying parameter model typically assumes that the parameter vector evolves according to a multivariate random walk process:

$$\beta_t = \beta_{t-1} + \nu_t , \quad \nu_t \sim \text{ i.i.d. } (0, \Sigma) . \tag{4}$$

Because the time-varying parameter model treats the evolution of parameters as a stochastic process, it is clearly a nonlinear model. At the same time, its application to data provides an inherently Bayesian investigation of structural change in the relationships between dependent and independent variables, where those relationships may, in fact, be linear. In general, then, analysis of structural change in linear relationships should be considered an example of nonlinear time series analysis when nonlinear models, such as stochastic volatility models or time-varying parameter models, are used in the analysis, but structural change should not be thought of as nonlinear in itself.

In terms of macroeconomics, time-varying parameter models have recently been used to consider structural change in vector autoregressive (VAR) models of the US economy. Cogley and Sargent [26] employ such a model to argue that US inflation dynamics have changed considerably in the postwar period. Based on Sims' [135] critique that evidence for structural change in time-varying parameters may be the spurious consequence of ignoring heteroskedasticity in the error processes for a VAR model, Cogley and Sargent [27] augment their time-varying parameter model with stochastic volatility and find that their results are robust. Primiceri [123] employs a structural VAR with time-varying parameters and stochastic volatility and also finds evidence of structural changes in inflation dynamics, although he questions the role of monetary policy in driving these changes. Whether these structural changes are evident in Fig. 3, which displays US consumer price inflation for each month from 1960:M1 to 2006:M12, is debatable. However, it is fairly clear that a basic AR process with constant parameters would be an inadequate model for inflation.

It is worth mentioning that there is a simpler time-varying parameter model that has seen considerable use in macroeconomics. It is the unobserved components (UC) model used for trend/cycle decomposition. A standard version of the model has the following form:

$$y_t = \tau_t + c_t \,, \tag{5}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t \,, \quad \eta_t \sim \text{i.i.d.N}\left(0, \sigma_\eta^2\right) \,, \tag{6}$$

$$\phi(L) c_t = \varepsilon_t \,, \quad \varepsilon_t \sim \text{i.i.d.N}\left(0, \sigma_\varepsilon^2\right) \,, \tag{7}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, the roots of $\phi(z) = 0$ lie outside the unit circle, and $\text{corr}\,(\eta_t, \varepsilon_t) = \rho_{\eta\varepsilon}$. It is possible to think of the UC model as a time-varying parameter model in which the unconditional mean of the process is equal to the trend τ_t, meaning that it undergoes structural change, rather than remaining constant, as it does for the AR(1) process described by (1). A glance at the upward trajectory of real GDP in Fig. 1 makes it clear that a basic AR process would be an extremely bad model for the time series. Indeed, Morley, Nelson, and Zivot [114] applied the model in (5)–(7) to 100 times the natural logarithms of US real GDP under the assumption that the lag order $p = 2$ and with no restrictions on the correlation between η_t and ε_t and found that most of the variation in log real GDP was due to the trend rather than the AR cycle c_t (note that natural logarithms are more appropriate for time series modeling than the raw data in Fig. 1 because the "typical" scale of variation for real GDP is more closely linked to percentage changes than to absolute changes). Yet, while the UC model can be thought of as a time-varying parameter model, it is not, in fact, nonlinear. In particular, the UC model for log real GDP is equivalent to an autoregressive moving-average (ARMA) model for the first differences of log real GDP. Likewise, the AR(1) model in (1) may be very sensible for real GDP growth in Fig. 2, even though it would be a bad model for real GDP in Fig. 1. In general, if it is possible to transform a time series, such as going from Fig. 1 to Fig. 2, and employ a linear model for the transformed series, then the time series analysis involved is linear. Likewise, under this formulation, the simple version of the stochastic volatility model for a series with no serial correlation also falls under the purview of linear time series analysis. Only time-varying parameter and stochastic volatility models that cannot be transformed into linear representations are nonlinear.

Of course, the semantics over "linear" and "nonlinear" are hardly important on their own. What is important is whether structural change is mistaken for recurring changes in parameters or vice versa. In terms of structural VAR models for the US economy, Sims and Zha [136] argue that when parameters are allowed to undergo large, infrequent changes, rather than the smaller, more continuous changes implied by a time-varying parameter model, there is no evidence for changes in dynamic structure of postwar macroeconomic data. Instead, there are only a few large, infrequent changes in the variance of shocks. Furthermore, among the models that assume some change in dynamics, their Bayesian model comparison favors a model in which only the monetary policy rule changes. Among other things, these findings have dramatic implications for the Lucas [100,101] critique, which suggests that correlations between macroeconomic variables should be highly sensitive to changes in policy, thus leaving successful forecasting to "structural" models that capture optimizing behavior of economic agents, rather than "reduced-form" models that rely on correlations between macroeconomic structures. The results in Sims and Zha [136] suggest that the Lucas critique, while an interesting theoretical proposition with the virtue of being empirically testable, is not, in fact, supported by the data.

From the point of view of time series analysis, an interesting aspect of the Sims and Zha [136] paper and earlier papers on structural change in the US economy by Kim and Nelson [77] and McConnell and Perez-Quiros [107] is that they consider nonlinear regime-switching models that allow for changes in parameters to be recurring. That is, while the models can capture structural change, they do not impose it. Using univariate regime-switching models of US real GDP growth, Kim and Nelson [77] and McConnell and Perez-Quiros [107] find a one-time permanent reduction in output growth volatility in 1984. However, using their regime-switching VAR model, Sims and Zha [136] find that a small number of volatility regimes recur multiple times in the postwar period. In terms of the earlier discussion about the lack of volatility dynamics in macroeconomic data, this finding suggests that there are some volatility dynamics after all, but these dynamics correspond to less frequent changes than would be implied by ARCH or a continuous stochastic volatility process. More generally, the allowance for recurring regime switches is relevant because time series models with regime switches have been the most successful form of nonlinear models in macroeconomics. However, for reasons discussed in the next section, regime-switching models are typically employed to capture changing dynamics in measures of economic activity over different phases of the business cycle, rather than structural change in inflation or recurring changes in shock variances.

To summarize this section, there are different types of nonlinear time series models employed in macroeconomics. While models that assume a nonlinear specification for the relationship between observable variables exist (e. g., the bilinear model), they are rarely used in practice. By contrast, models that allow some parameters to undergo changes over time are much more common in macroeconomics. The examples discussed here are ARCH models, stochastic volatility models, time-varying parameter models, and regime-switching models. When examining structural change, there is a conceptual question of whether the analysis is "linear" or "nonlinear". However, as long as the process of structural change is an explicit part of the model (e. g., the time-varying parameter model), and excluding cases where it is possible to transform the model to have a linear representation (e. g., the UC model to an ARMA model), the analysis can be thought of as nonlinear. Meanwhile, time series analysis of recurring regime switches is unambiguously nonlinear. As discussed in the next section, nonlinear regime-switching models come in many versions and have found wide use in macroeconomics modeling business cycle asymmetry.

Business Cycle Asymmetry

The topic of business cycle asymmetry is broad and the literature on it extensive. As a result, it is useful to divide the discussion in this section into four areas: i) concepts of business cycle asymmetry and their relationships to nonlinearity; ii) nonlinear models of business cycle asymmetry; iii) evidence for nonlinear forms of business cycle asymmetry; and iv) the relevance of nonlinear forms of business cycle asymmetry for macroeconomics.

Concepts

Notions of business cycle asymmetry have a long tradition in macroeconomics. Classic references to the idea that recessions are shorter, sharper, and generally more volatile than expansions are Mitchell [109], Keynes [72], and Burns and Mitchell [13]. For example, in his characteristic style, John Maynard Keynes writes, "…the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency." (see p. 314 in [72]). Similarly, albeit more tersely, Wesley Mitchell writes, "…the most violent declines exceed the most considerable advances. The abrupt declines usually occur in crises; the greatest gains occur in periods of revival… Business contractions appear to be a briefer and more violent process than business expansions." (see p. 290 in [109]). Milton Friedman also saw business cycle asymmetry in the form of a strong relationship between the depth of recession and the strength of a recovery, with no corresponding relationship between the strength of an expansion with the severity of the subsequent recession (see [50,51]).

The link between business cycle asymmetry and nonlinearity depends, in part, on the definition of "business cycle". Harding and Pagan [67] discuss three possible definitions that are presented here using slightly modified terminology. Based on the work of Burns and Mitchell [13], the first definition is that the business cycle is the alternation between phases of expansion and recession in the *level* of economic activity. The second definition, which is often left implicit when considered, is that the business cycle represents transitory *deviations* in economic activity from a permanent or "trend" level. The third definition, which is also often only implicitly considered, is that the business cycle corresponds to any short-run *fluctuations* in economic activity, regardless of whether they are permanent or transitory.

Under the "level" definition of the business cycle, there is nothing inherently nonlinear about asymmetry in terms of the duration of expansions and recessions. Positive drift in the level of economic activity implies longer expansions than recessions, even if the underlying process is linear. Even asymmetry in the form of relative sharpness and steepness of a recession alluded to in the above quote from Keynes does not necessarily indicate nonlinearity. Again, given positive drift, an outright decline in economic activity only occurs when there are large negative shocks to the underlying process, while an expansion occurs for all positive shocks and small negative shocks. Thus, a recession is likely to look like a relatively sharp reversal in the level. Furthermore, with positive serial correlation in growth, such as implied by a linear AR(1) process as in (1) with $\phi > 0$, recessions will appear steeper than expansions due to the dynamic effects of large negative shocks. On the other hand, as discussed in more detail later, nonlinear models are much more successful than linear models at reproducing business cycle asymmetry in the form of a strong link between recessions and their recoveries versus a weak link between expansions and subsequent recessions noted by Friedman [50].

Under the "deviations" definition of the business cycle, asymmetry is closely linked to nonlinearity. While it is possible for asymmetry in the independent and identical distribution of the underlying shocks to generate asymmetry in a linear process, any persistence in the process would severely dampen the asymmetries in the unconditional distribution. Thus, under the assumption that the transitory component of economic activity is at least somewhat persistent, asymmetries such as differences in the durations of positive and negative deviations from trend or relative sharpness and steepness in negative deviations compared to positive deviations are more suggestive of nonlinear dynamics (i. e., changing correlations) than underlying asymmetric shocks.

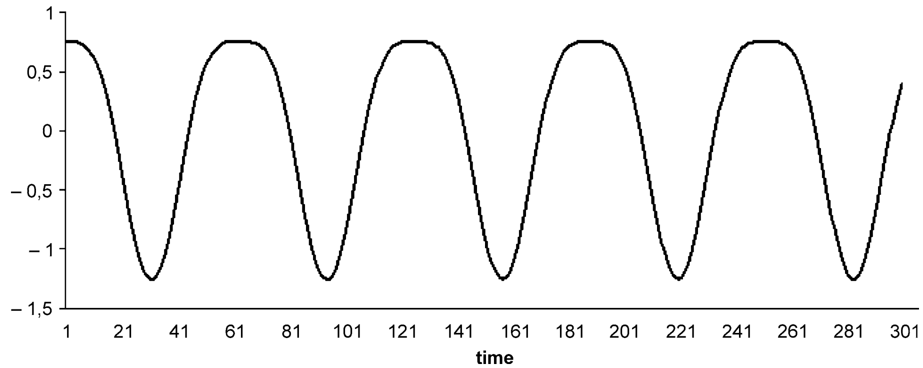
Under the "fluctuations" definition of the business cycle, the link between nonlinearity and asymmetry also depends on the relative roles of shocks and dynamics in generating asymmetries. However, because growth rates are less persistent than most measures of the transitory component of economic activity and because they mix together permanent and transitory shocks that may have different means and variances, it is quite plausible that asymmetry in the distribution of shocks is responsible for asymmetry in growth rates. Of course, nonlinear dynamics are also a plausible source of asymmetry for growth rates.

In terms of asymmetries, it is useful to consider the formal classifications developed and discussed in Sichel [133], McQueen and Thorley [108], Ramsey and Rothman [124], Clements and Krolzig [24], and Korenok, Mizrach, and Radchenko [95] of "deepness", "steepness", and "sharpness". Following Sichel [133], a process is said to have *deepness* if its unconditional distribution is skewed and *steepness* if the distribution of its first-differences is skewed. Following McQueen and Thorley [108], a process is said to have *sharpness* if the probability of a peak occurring when it has been increasing is different than the probability of a trough occurring when it has been decreasing. However, despite these definitions, the different types of asymmetries are most easily understood with visual examples.

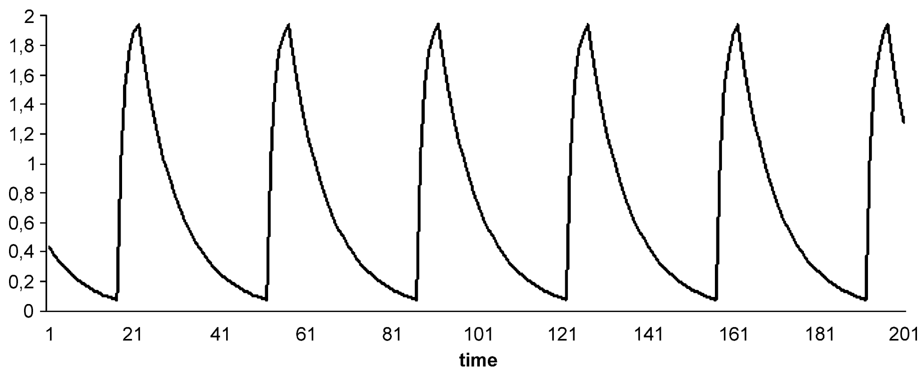
Figure 4 presents an example of a simulated time series with deepness, with the distance from peak of the cycle to the mean less than the distance from the mean to trough of the cycle (see [124], for the details of the process generating this time series). In addition to deepness, the series appears to display sharpness in recessions, with the peak of the cycle more rounded than the trough, although the fact that the simulated series is deterministic means it cannot be directly related to the definition of sharpness in McQueen and Thorley [108] mentioned above. Meanwhile, there is no steepness because the slope from peak to trough is the same magnitude as the slope from trough to peak.

As discussed in Ramsey and Rothman [124], these different types of asymmetry can be classified in two broader categories of "time reversible" and "time irreversible". *Time reversibility* means that the substitution of $-t$ for t in the equations of motion for a process leaves the process unchanged. The upward drift that is present in many macroeconomic time series (such as real GDP) is clearly time irreversible. More generally, the issue of time reversibility is relevant for determining whether business cycle asymmetry corresponds to deepness and sharpness, which are time reversible, or steepness, which is time irreversible. For example, the time series in Fig. 4 can be flipped on the vertical axis without any resulting change. Thus, it is time reversible. By contrast, consider the simulated time series with "steepness" in Fig. 5. The series is generated from a regime-switching process with asymmetric shocks across two regimes and different persistence for shocks in each regime. In this case, flipping the series on the vertical axis would produce flat inclines and steep declines. Thus, it is time irreversible.

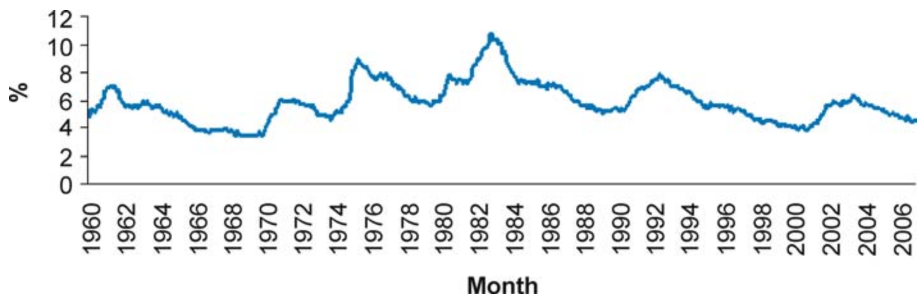
The relevance of the distinction between time reversible and time irreversible processes is obvious from Fig. 6, which presents the time series for the US civilian unemployment rate for each month from 1960:M1 to 2006:M12. The inclines are steep relative to the declines. Thus, there is a clear visual suggestion of the steepness form of asymmetry. Indeed, the modern literature on business cycle asymmetry begins with Neftçi's [115] investi-



Macroeconomics, Non-linear Time Series in, Figure 4
A "deep" cycle (Source: Author's calculations based on Ramsey and Rothman [124])



Macroeconomics, Non-linear Time Series in, Figure 5
A "steep" cycle (Source: Author's calculations)



Macroeconomics, Non-linear Time Series in, Figure 6
US civilian unemployment rate 1960–2006 (Source: St. Louis Fed website)

gation of this issue using a nonlinear regime-switching model in which the prevailing "business cycle" regime in a given period is assumed to depend on a discrete Markov process driven by whether the US unemployment rate is rising or falling in that period. Given the link to the first differences of the unemployment rate, his finding that the continuation probabilities for the two regimes are different, with declines more likely to persist than increases, provides formal support for the presence of the steepness forms of asymmetry in the unemployment rate (also, see [127]). It should also be noted that, while not related to time irreversibility, the different continuation probabilities also directly imply sharpness.

Models

The subsequent literature on regime-switching models in macroeconomics can be usefully divided into two categories that are both related to Neftçi's [115] model. First, *Markov-switching models* assume that the prevailing regime depends on an unobserved discrete Markov process. The main distinction from Neftçi [115] is that the Markov process is unobserved (hence, these models are sometimes referred to as a "hidden Markov models"). Second, *self-exciting threshold models* assume that the prevailing regime is observable and depends on whether realized values of the time series being modeled exceed or fall below certain "threshold" values, much like the regime in Neftçi's [115] model depends on whether the change in the unemployment rate was positive or negative.

Hamilton [59] is the seminal paper in terms of Markov-switching models. His model has a basic AR structure, like in (1), but for the first-differences of the time series of interest:

$$\phi(L)\left(\Delta y_t - \mu_t\right) = \varepsilon_t, \quad \varepsilon_t \sim \text{ i.i.d. } (0, \sigma^2)\,, \tag{8}$$

where Δy_t is 100 times the change in the natural logarithm of real Gross National Product (GNP). The only difference from a linear AR model is that the mean follows a stochastic process:

$$\mu_t = \mu_1 \cdot I(S_t = 1) + \mu_2 \cdot I(S_t = 2)\,, \tag{9}$$

with the indicator function $I(S_t = j)$ equal to 1 if $S_t = j$ and 0 otherwise and $S_t = \{1, 2\}$ following an unobserved discrete Markov state variable that evolves according to the following fixed transition matrix:

$$\begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix},$$

where $p_{ij} \equiv \Pr[S_t = j | S_{t-1} = i]$ and the columns sum to one.

There are two aspects of Hamilton's [59] model that should be mentioned. First, while the demeaned specification is equivalent to a regression model specification (e. g., (1)) in the linear setting, with $\mu = c/(1-\phi)$, the two specifications are no longer equivalent in the nonlinear setting. In particular, if the intercept c were switching instead of the mean μ, then past regime switches would be propagated by the AR dynamics (see [61], for an example of such a model). By contrast, with μ switching, there is a clear separation between the "nonlinear" dynamics due to the evolution of the state variable (which does alter the correlations between Δy_t and its lags) and the "linear" dynamics due to the ε_t shocks and the AR parameters. Second, in order to eliminate arbitrariness in the labeling of states, it is necessary to impose a restriction such as $\mu_1 > \mu_2$, which corresponds to higher mean growth in state 1 than in state 2. Furthermore, given the application to output growth, if $\mu_1 > 0$ and $\mu_2 < 0$, the states 1 and 2 can be labeled "expansion" and "recession", respectively.

Hamilton's [59] paper had a big impact on econometrics and macroeconomics for two reasons. First, it included an elegant filter that could be used to help estimate Markov-switching models via maximum likelihood and, along with a smoother, calculate the posterior distribution of the unobserved state variable (filters and smoothers are recursive algorithms that make inferences about unobserved state variables, with filters considering only information available at the time the state variable is realized and smoothers incorporating any subsequent available information). Second, the resulting posterior probability of the "recession" regime corresponded closely to the National Bureau of Economic Research (NBER) dating of recessions. The NBER dating is based on non-structural and subjective analysis of a wide variety of indicators. The official line from its website is "The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales." (www.nber.org/cycles/cyclesmain.html). Thus, it is, perhaps, remarkable that a simple time series model using only information in real GNP could find such similar dates for recessions. Of course, as emphasized by Harding and Pagan [66], a simple rule like "two consecutive quarters of decline in real GDP" also does extremely well in matching the NBER recessions, regardless of NBER claims that it is not following such a rule. Yet, more important is the notion implied by Hamilton's [59] results that the NBER is identifying a meaningful structure in the economy, rather than simply reporting (sometimes with considerable lag) that the economy had an episode of prolonged negative growth. Specifically, "recession" appears to be an indicator of a different state for the dynamics of the economy, rather than a label for particular realizations of linear process. (As an aside, the fact that the popular press pays so much attention to NBER pronouncements on recessions also supports the idea that it is identifying a meaningful macroeconomic structure).

Numerous modifications and extensions of Hamilton's [59] model have been applied to macroeconomic data. For example, while estimates for Hamilton's [59] model imply that the linear ε_t shocks have large permanent effects on the level of real GDP, Lam [96] considers a model in which the only permanent shocks to real GNP are due to regime switches. Despite this very different assumption, he also finds that the regime probabilities implied by his model correspond closely to NBER dating of expansions and recessions. Kim [74] develops a filter that can be used for maximum likelihood estimation of state-space models with Markov-switching parameters and confirms the results for Lam's [96] model. Motivated by Diebold and Rudebusch's [38] application of Hamilton's [59] model to the Commerce Department's coincident index of economic activity instead of measures of aggregate output such as real GNP or real GDP, Chauvet [19] employs Kim's [74] filter to estimate an unobserved components model of a coincident index using Hamilton's [59] model as the specification for its first differences. Other multivariate extensions include Kim and Yoo [87], Ravn and Sola [125], Kim and Nelson [76], Kim and Murray [75], Kim and Piger [81], Leamer and Potter [97], Camacho [14], and Kim, Piger, and Startz [84]. The general theme of these studies is that the multivariate information, such as coincident indicators or aggregate consumption and investment, helps to strongly identify the nonlinearity in economic activity, with regimes corresponding even more closely to NBER dates than for univariate analysis based on real GNP or real GDP.

In terms of business cycle asymmetry, an important extension of Hamilton's [59] model involves allowing for three regimes to capture three phases of the business cycle: "expansion", "recession", and "recovery" (see [134]). Papers with three-regime models include Boldin [8], Clements and Krolzig [23], and Leyton and Smith [98].

The specification in Boldin [8] modifies the time-varying mean in Hamilton's [59] model as follows:

$$\mu_t = \mu_1 \cdot I(S_t = 1) + \mu_2 \cdot I(S_t = 2) + \mu_3 \cdot I(S_t = 3)\,, \quad (10)$$

where $S_t = \{1, 2, 3\}$ has fixed transition matrix:

$$\begin{bmatrix} p_{11} & 0 & p_{31} \\ p_{12} & p_{22} & 0 \\ 0 & p_{23} & p_{33} \end{bmatrix} .$$

The zeros in the transition matrix restrict the state sequence to follow the pattern of $\{S_t\} = \cdots 1 \to 2 \to 3 \to 1 \cdots$. Given the normalization $\mu_1 > \mu_2$, the restriction on the transitional matrix implies that the economy goes from expansion to recession to recovery and back to expansion. While there is no restriction on μ_3, Boldin [8] finds it is greater than μ_1, which means that the third regime corresponds to a high-growth recovery. As discussed in Clements and Krolzig [24], this third regime allows for steepness in output growth, while the basic two-regime Hamilton [59] model can only capture deepness and sharpness (the two are inextricably linked for a two-regime model) in growth. Note, however, from the definitions presented earlier, deepness in growth implies steepness the level of output.

It is possible to capture high-growth recoveries without resorting to three regimes. For example, Kim and Nelson [79] develop an unobserved components model that assumes two regimes in the transitory component of US real GDP. A slightly simplified version of their model is given as follows:

$$y_t = \tau_t + c_t\,, \quad (11)$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t, \quad \eta_t \sim \text{i.i.d.N}\left(0, \sigma_\eta^2\right)\,, \quad (12)$$

$$\phi(L)c_t = \lambda \cdot I(S_t = 2) + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.N}\left(0, \sigma_\varepsilon^2\right)\,, \quad (13)$$

where y_t is 100 times log real GDP, $S_t = \{1, 2\}$ is specified as in Hamilton's [59] model, and state 2 is identified as the recession regime by the restriction $\lambda < 0$ (see [112,113], on the need for and implications of this restriction). Unlike Morley, Nelson, and Zivot [114], Kim and Nelson [79] impose the restriction that $\rho_{\eta\varepsilon} = 0$ in estimation, which they conduct via approximate maximum likelihood using the Kim [74] filter. As with Hamilton [59] and Lam [96], the regimes correspond closely to NBER-dated expansions and recessions. However, because the regime switching is in the transitory component only, the transition from state 1 to state 2 corresponds to a downward "pluck" in economic activity that is followed by a full recovery to trend after the transition from state 2 to state 1. Kim and Nelson [79] motivate their model as nesting Friedman's [50,51] plucking model, which assumes output cannot exceed a ceiling level, but is occasionally plucked below full capacity by recessionary shocks resulting from activist monetary policy. In line with Friedman's observations, Kim and Nelson's [79] model relates the strength of a recovery to the severity of the preceding recession, with no corresponding link between the strength of an expansion and the severity of a recession (see also [2,134,150]). Notably, the transitory component for their estimated model achieves the trifecta of business cycle asymmetries in the form of deepness, steepness, and sharpness.

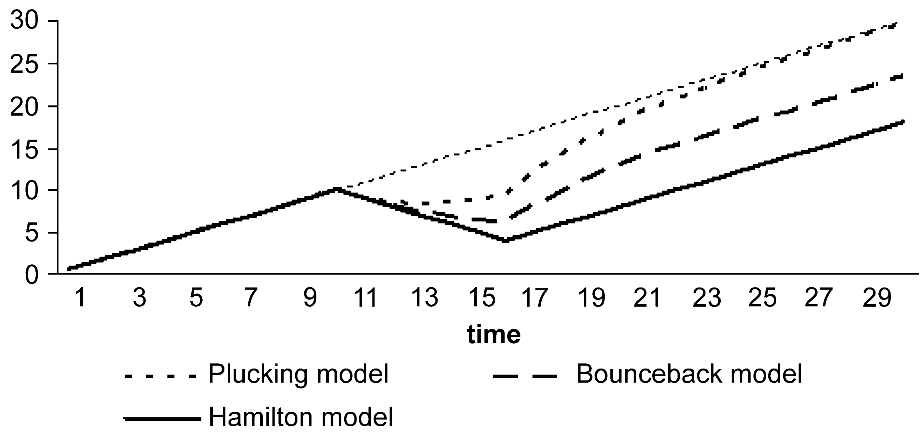
Another model that captures three phases of the business cycle with only two underlying regimes is the "bounceback" model of Kim, Morley, and Piger [83]. The model modifies the time-varying mean in Hamilton's [59] model as follows:

$$\mu_t = \mu_1 \cdot I(S_t = 1) + \mu_2 \cdot I(S_t = 2) + \lambda \cdot \sum_{j=1}^{m} I\left(S_{t-j} = 2\right), \quad (14)$$

where the number of lagged regimes to consider in the third term on the right hand side of (14) is determined by the discrete "memory" parameter m, which is estimated to be six quarters for US postwar quarterly real GDP. Given the restriction $\mu_1 > \mu_2$, the third term can be interpreted as a pressure variable that builds up the longer a recession persists (up to m periods, where $m = 6$ quarters is long enough to capture all postwar recessions) and is motivated by the "current depth of recession" variable of Beaudry and Koop [6] discussed later. Then, if $\lambda > 0$, growth will be above μ_1 for up to the first six quarters of an expansion. That is, there is a post-recession "bounceback" effect, as in Kim and Nelson's [79] plucking model. Meanwhile, the specification in (14) can be thought of as a "u-shaped recession" version of the model because the pressure variable starts mitigating the effects of a recession the longer the regime persists. Morley and Piger [111] consider a slightly modified "v-shaped recession" version of the model that assumes the pressure variable only affects growth after the recession ends, thus producing a sharper trough:

$$\mu_t = \mu_1 \cdot I(S_t = 1) + \mu_2 \cdot I(S_t = 2) + \lambda \cdot \sum_{j=1}^{m} I(S_t = 1) \cdot I\left(S_{t-j} = 2\right)\,. \quad (15)$$

This version of the model is identical to Hamilton's [59] model in all but the first m periods of an expansion. Finally, Morley and Piger [113] consider a "depth" version



Macroeconomics, Non-linear Time Series in, Figure 7
Simulated paths for "Output" (Source: Author's calculations)

of the model that relates the pressure variable to both the length and severity of a recession:

$$\mu_t = \mu_1 \cdot I(S_t = 1) + \mu_2 \cdot I(S_t = 2) + \lambda \cdot \sum_{j=1}^{m} (\mu_1 - \mu_2 - \Delta y_{t-j}) \cdot I\left(S_{t-j} = 2\right) . \quad (16)$$

In this case, the post-recession bounceback effect depends on the relative severity of a recession. Regardless of the specification, the estimated bounceback effect for US real GDP based on maximum likelihood estimation via the Hamilton [59] filter is large (see [83,111,113]).

While Kim, Morley, and Piger's [83] bounceback model can capture "plucking" dynamics, there is no restriction that regime switches have only transitory effects. Instead, the model nests both the Hamilton [59] model assumption that recessions have large permanent effects in the case that $\lambda = 0$ and Kim and Nelson's [79] "plucking" model assumption that recessions have no permanent effects in the case that $\lambda = (\mu_1 - \mu_2)/m$ (for the specification in (14)). Figure 7 presents examples of simulated time series for the plucking model, the bounceback model, and the Hamilton model. In each case, "output" is subject to a recession regime that lasts for six periods. For the plucking model, output returns to the level it would have been in the absence of the recession. For the Hamilton model, output is permanently lower as a result of the recession. For the bounceback model, recessions can have permanent effects, but they will be less than for the Hamilton model if $\lambda > 0$ (indeed, if $\lambda > (\mu_1 - \mu_2)/m$, the long-run path of the economy can be increased by recessions, a notion related to the "creative destruction" hypothesis of Schumpeter [131]). In practice, Kim, Morley, and Piger [83] find a very small negative long-run impact of US recessions, providing support for the plucking model dynamics and implying considerably lower economic costs of recessions than the Hamilton model.

Another extension of Hamilton's [59] model involves relaxing the assumption that the transition probabilities for the unobserved state variable are fixed over time (see [39]). Filardo [48] considers time-varying transition probabilities for a regime-switching model of industrial production growth where the transition probabilities depend on leading indicators of economic activity. Durland and McCurdy [40] allow the transition probabilities for real GNP growth to depend on the duration of the prevailing regime. DeJong, Liesenfeld, and Richard [34] allow the transition probabilities for real GDP growth depend on an observed "tension index" that is determined by the difference between recent growth and a "sustainable" rate that corresponds to growth in potential output. Kim, Piger, and Startz [84] allow for a dynamic relationship between multiple unobserved discrete state variables in a multivariate setting and find that regime-switches in the permanent component of economic activity tend to lead regime-switches in the transitory component when the economy heads into recessions.

The distinction between Markov-switching models and threshold models is blurred somewhat by time-varying transition probabilities. A standard demarcation is that Markov-switching models typically assume the discrete state variables driving changes in regimes are exogenous, while threshold models allow for endogenous switching. However, this exogenous/endogenous demarcation is less useful than it may at first appear. First, as is always the problem in macroeconomics, it is unlikely that the variables affecting time-varying transition probabilities are actually strictly exogenous, even if they are predetermined. Second, Kim, Piger and Startz [85] have developed an approach for maximum likelihood estimation of Markov-switching models that explicitly allow for endogenous switching. In terms of macroeconomics, Sinclair [137] applies their approach to estimate a version of the regime-switching UC model in (11)–(13) for US real GDP that allows for a non-zero correlation between the regular shocks η_t and ε_t, as in Morley, Nelson, and Zivot [114], as well as dependence between these shocks and the unobserved state variable S_t that generates downward plucks in output. She finds that permanent shocks are more important than suggested by Kim and Nelson's [79] estimates. However, she confirms the importance of the plucking dynamic, with a test supporting the standard exogeneity assumption for the discrete Markov-switching state variable.

Another demarcation that would seem to provide a possible means of distinguishing between Markov-switching models and threshold models arises from the fact that, starting from an AR specification, threshold models typically extend the basic model by allowing for changes in AR parameters, while, as discussed earlier, Markov-switching models typically extend the model by allowing for changes in the mean. However, this demarcation is also less useful than it may at first appear since Markov-switching models have alternative representations as autoregressive processes (see [59]). Furthermore, some threshold models assume constant AR parameters (e. g., [120]). In particular, regardless of presentation, both types of models capture nonlinear dynamics in the conditional mean.

The more general and useful demarcation between Markov-switching models and threshold models is that the prevailing regime is unobservable in the former, while it is observed in the latter. Meanwhile, the observable regimes in threshold models make it feasible to consider more complicated transitions between regimes than Markov switching models. In particular, it is possible with a threshold model to allow a mixture of regimes to prevail in a given time period.

Tong [145] introduced the basic threshold autoregressive (TAR) model. In a "self-exciting" TAR model, the autoregressive coefficient depends on lagged values of the time series. For example, a simple two-regime AR(1) TAR model is given as follows:

$$y_t = c + \phi^{(1)} \cdot I\left(y_{t-m} < \tau\right) \cdot y_{t-1} + \phi^{(2)} \cdot I\left(y_{t-m} \geqslant \tau\right) \cdot y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}(0, \sigma^2), \tag{17}$$

where $\phi^{(1)}$ and $\phi^{(2)}$ are the AR(1) parameters associated with the two regimes, τ is the threshold, and m is the discrete delay parameter. A variant of the basic TAR model that allows multiple regimes to prevail to different degrees is the smooth transition autoregressive (STAR) model (see [18,58,140,142]). For STAR models, the indicator function is replaced by transition functions bounded between zero and one. The STAR model corresponding to (17) is

$$\begin{aligned} y_t = c &+ \phi^{(1)} \cdot F_1\left(y_{t-m}|\tau, \gamma\right) \cdot y_{t-1} \\ &+ \phi^{(2)} \cdot F_2\left(y_{t-m}|\tau, \gamma\right) \cdot y_{t-1} + \varepsilon_t, \\ &\varepsilon_t \sim \text{ i.i.d. } (0, \sigma^2)\,, \end{aligned} \tag{18}$$

where $F_2(y_{t-m}|\tau, \gamma) = 1 - F_1(y_{t-m}|\tau, \gamma)$ and γ is a parameter that determines the shape of the transition function (in general, the larger γ, the closer the STAR model is to the TAR model). The two most popular transition functions are exponential (ESTAR) and logistic (LSTAR). The exponential transition function is

$$F_1^e = 1 - \exp\left(-\gamma(y_{t-m} - \tau)^2\right), \quad \gamma > 0\,, \tag{19}$$

while the logistic transition function is

$$F_1^l = \left[1 + \exp\left(-\gamma(y_{t-m} - \tau)\right)\right]^{-1}, \quad \gamma > 0\,. \tag{20}$$

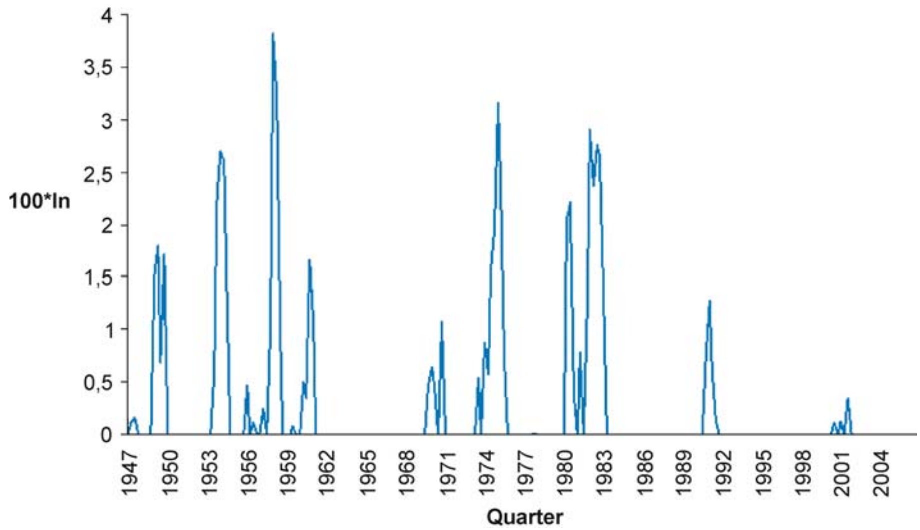
For STAR models the transition functions are such that the prevailing autoregressive dynamics are based on a weighted average of the autoregressive parameters for each regime, rather than reflecting only one or the other, as in TAR models.

In terms of macroeconomics, both TAR and STAR models have been employed to capture business cycle asymmetry. A key question is what observed threshold might be relevant. On this issue, a highly influential paper is Beaudry and Koop [6]. Related to the notion discussed above that recessions represent a meaningful macroeconomic structure, they consider whether real GDP falls below a threshold defined by its historical maximum. Specifically, they define a "current depth of recession" (CDR) variable as follows:

$$\text{CDR}_t = \max\left\{y_{t-j}\right\}_{j \geqslant 0} - y_t\,. \tag{21}$$

Figure 8 presents the current depth of recession using US real GDP for each quarter from 1947:Q1 to 2006:Q4.

Beaudry and Koop [6] augment a basic linear ARMA model of US real GNP growth with lags of the CDR variable. They find that the inclusion of the CDR variable implies much less persistence for large negative shocks than



Macroeconomics, Non-linear Time Series in, Figure 8
Current depth of recession 1947–2006 (Source: Author's calculations based on Beaudry and Koop [6])

for small negative shocks or positive shocks. The asymmetry in terms of the response of the economy to shocks corresponds closely to the idea discussed earlier that deep recessions produce strong recoveries. Indeed, the Beaudry and Koop [6] paper provided a major motivation for most of the extensions of Hamilton's [59] model discussed earlier that allow for high-growth recoveries.

In terms of threshold models in macroeconomics, Beaudry and Koop [6] initiated a large literature. Tiao and Tsay [144], Potter [121], and Clements and Krolzig [23] consider two-regime TAR models with the threshold either fixed at zero or estimated to be close to zero. Pesaran and Potter [120] and Koop and Potter [91] consider a three-regime TAR model (with many restrictions for tractability) that incorporates the CDR variable and an "overheating" (OH) variable reflecting cumulated growth following large positive shocks. Specifically, a simple homoskedastic, AR(1) version of Pesaran and Potter's [120] "floor and ceiling" model is given as follows:

$$\Delta y_t = c + \phi \Delta y_{t-1} + \lambda_1 \mathrm{CDR}_{t-1} + \lambda_2 \mathrm{OH}_{t-1} + \varepsilon_t , \quad \varepsilon_t \sim N(0, \sigma^2) , \qquad (22)$$

where

$$\mathrm{CDR}_t = -(\Delta y_t - \tau_F) \cdot F_t \cdot (1 - F_{t-1}) + (\mathrm{CDR}_{t-1} - \Delta y_t) \cdot F_t \cdot F_{t-1} , \qquad (23)$$

$$F_t = I\left(\Delta y_t < \tau_F\right) \cdot (1 - F_{t-1}) + I\left(\mathrm{CDR}_{t-1} - \Delta y_t > 0\right) \cdot F_{t-1} , \qquad (24)$$

$$\mathrm{OH}_t = (\mathrm{OH}_{t-1} + \Delta y_t - \tau_C) \cdot C_t , \qquad (25)$$

$$C_t = (1 - F_t) \cdot I\left(\Delta y_t > \tau_C\right) \cdot I\left(\Delta y_{t-1} > \tau_C\right) . \qquad (26)$$

The indicator variable $F_t = \{0, 1\}$ denotes whether the economy is in the "floor" regime, while $C_t = \{0, 1\}$ denotes whether the economy is in the "ceiling" regime. The CDR variable is the same as in (20) if the threshold $\tau_F = 0$. Thus, a high-growth post-recession recovery is implied by $\lambda_1 > 0$. In particular, with $\tau_F = 0$, the "floor" regime is activated when real GDP falls below its historical maximum at the onset of a recession and remains activated until output recovers back to its pre-recession level. The OH variable captures whether real GDP is above a sustainable level based on the threshold level τ_C of growth. A capacity-constraint effect is implied by $\lambda_2 < 0$. Note, however, that the "ceiling" regime that underlies the OH variable can be activated only when the "floor" regime is off, ruling out the possibility that a high-growth recovery from the trough of a recession is labeled as "overheating". There is also a requirement of two consecutive quarters of fast growth above the threshold level τ_C in order to avoid labeling a single quarter of fast growth as "overheating". Meanwhile, a heteroskedastic version of the model allows the variance of the shocks to evolve as follows:

$$\sigma_t^2 = \sigma_1^2 \cdot I(F_{t-1} + C_{t-1} = 0) + \sigma_2^2 F_{t-1} + \sigma_3^2 C_{t-1} . \qquad (27)$$

Also, in a triumph of controlled complexity, Koop and Potter [92] develop a multivariate version of this model, discussed later.

A related literature on STAR models of business cycle asymmetry includes Teräsvirta and Anderson [143], Teräsvirta [141], van Dijk and Franses [148], and Öcal and Osborn [117]. Similar to the development of Markov-switching models and TAR models, van Dijk and Franses [148] develop a multi-regime STAR model and find evidence for more than two regimes in economic activity. Likewise, using U.K. data on industrial production, Öcal and Osborn [117] find evidence for three regimes corresponding to recessions, normal growth, and high growth. Rothman, van Dijk, and Franses [130] develop a multivariate STAR model to examine nonlinearities in the relationship between money and output.

While there are many different nonlinear models of economic activity, it should be noted that, in a general sense, Markov-switching models and threshold models are close substitutes for each other in terms of their abilities to forecast (see [23]) and their abilities to capture business cycle asymmetries such as deepness, steepness, and sharpness (see [24]). On the other hand, specific models are particularly useful for capturing specific asymmetries and, as discussed next, for testing the presence of nonlinear dynamics in macroeconomic time series.

Evidence

While estimates for regime-switching models often imply the presence of business cycle asymmetries, it must be acknowledged that the estimates may be more the consequence of the flexibility of nonlinear models in fitting the data than any underlying nonlinear dynamics. In the regime-switching model context, an extreme example of over-fitting comes from a basic i.i.d. mixture model. If the mean and variance are allowed to be different across regimes, the sample likelihood will approach infinity as the estimated variance approaches zero in a regime for which the estimated mean is equal to a sample observation. (It should be noted, however, that the highest local maximum of the sample likelihood for this model produces consistent estimates of the model parameters. See [73]). Thus, it is wise to be skeptical of estimates from nonlinear models and to seek out a correct sense of their precision. Having said this, the case for nonlinear dynamics that correspond to business cycle asymmetries is much stronger than it is often made out to be, although it would be a mistake to claim the issue is settled.

From the classical perspective, the formal problem of testing for nonlinearity with regime-switching models is that the models involve *nuisance parameters* that are not identified under the null hypothesis of linearity, but influence the distributions of test statistics. For example, Hamilton's [59] model outlined in (8)–(9) collapses to a linear AR model if $\mu_1 = \mu_2$. However, under this null hypothesis, the two independent transition probabilities p_{11} and p_{22} in the transition matrix will no longer be identified (i. e., they can take on different values without changing the fit of the model). The lack of identification of these nuisance parameters is referred to as the Davies [32] problem and it means that test statistics of the null hypothesis such as a t-statistic or a likelihood ratio (LR) statistic will not have their standard distributions, even asymptotically. An additional problem for Markov-switching models is that the null hypothesis of linearity often corresponds to a local maximum for the likelihood, meaning that the score is identically zero for some parameters, thus violating a standard assumption in classical testing theory. The problem of an identically zero score is easily seen by noting that one of the fundamental tests in classical statistics, the Lagrange multiplier (LM) test, is based on determining whether the score is significantly different than zero when imposing the null hypothesis in a more general model. For Hamilton's [59] model, the scores are zero for $\mu_d = \mu_2 - \mu_1, p_{11}$, and p_{22}. Again, identically zero scores imply nonstandard distributions for a t-statistic or an LR statistic. In practice, these nonstandard distributions mean that, if researchers were to apply standard critical values, they would over-reject linearity.

Hansen [61] derives a bound for the asymptotic distribution of a likelihood ratio statistic in the setting of unidentified nuisance parameters and identically zero scores. The bound is application-specific as it depends on the covariance function of an empirical process associated with the likelihood surface in a given setting (i. e., it is model and data dependent). The distribution of the empirical process can be obtained via simulation. In his application, Hansen [61] tests linearity in US real GNP using Hamilton's [59] model. His upper bound for the p-value of the likelihood ratio test statistic is far higher than conventional levels of significance. Thus, he is unable to reject linearity with Hamilton's [59] model. However, when he proposes an extended version of the model that assumes switching in the intercept and AR coefficients, rather than the mean as in (8)–(9), he is able to reject linearity with an upper bound for the p-value of 0.02.

In a subsequent paper, Hansen [62] develops a different method for testing in the presence of unidentified nuisance parameters that yields an exact critical value rather than an upper bound for a p-value. Again, the method requires simulation, as the critical value is model and data dependent. However, this approach assumes nonzero scores and is, therefore, more appropriate for testing threshold models than Markov-switching models. In his application for this approach, Hansen [62] tests linearity in US real GNP using Potter's [121] TAR model mentioned earlier (see also [17,63,146,147], on testing TAR models and [140], on testing STAR models). Referring back to the TAR model in (17), the threshold τ and the delay parameter m are unidentified nuisance parameters under the null of linearity (i. e., the case where the AR parameters and any other parameters that are allowed to switch in the model are actually the same across regimes). Hansen [62] finds that the p-values for a variety of test statistics are above conventional levels of significance, although the p-value for the supLM (i. e., the largest LM statistic for different values of the nuisance parameters) under the hypothesis of homoskedastic errors is 0.04, thus providing some support for nonlinearity.

Garcia [53] reformulates the problem of testing for Markov-switching considered in Hansen [61] by proceeding as if the score with respect to the change in Markov-switching parameters (e. g., $\mu_d = \mu_2 - \mu_1$ for Hamilton's [59], model) is not identically zero and examining whether the resulting asymptotic distribution for a likelihood ratio test statistic is approximately correct. The big advantage of this approach over Hansen [61] is that the distribution is no longer sample-dependent, although it is still model-dependent. Also, it yields an exact critical value instead of an upper bound for the p-value. Garcia [53] reports asymptotic critical values for some basic Markov-switching models with either no linear dynamics or a mild degree of AR(1) linear dynamics ($\phi = 0.337$) and compares these to critical values based on a simulated distribution of the LR statistic under the null of linearity and a sample size of 100. He finds that his asymptotic critical values are similar to the simulated critical values for the simple models, suggesting that they may be approximately correct despite the problem of an identically zero score. The asymptotic critical values are considerably smaller than the simulated critical values in the case of Hamilton's [59] model with an AR(4) specification, although this is perhaps due to small sample issues rather than approximation error for the asymptotic distribution. Regardless, even with the asymptotic critical values, Garcia [53] is unable to reject linearity for US real GNP using Hamilton's [59] model at standard levels of significance, although the p-value is around 0.3 instead of the upper bound of around 0.7 for Hansen [61].

It is worth mentioning that the simulated critical values in Garcia's [53] study depend on the values of parameters used to simulate data under the null hypothesis. That is, the LR statistic is not *pivotal*. Thus, the approach of using the simulated critical values to test linearity would correspond to a parametric bootstrap test (see [105,106], for excellent overviews of bootstrap methods). The use of bootstrap tests (sometimes referred to as Monte Carlo tests, although see MacKinnon [105,106], for the distinction) for Markov-switching models has been limited (although see [96], for an early example) for a couple of reasons. First, the local maximum at the null hypothesis that is so problematic for asymptotic theory is also problematic for estimation. While a researcher is likely to re-estimate a nonlinear model using different starting values for the parameters when an optimization routine converges to this or another local maximum in an application, it is harder to do an exhaustive search for the global maximum for each bootstrap sample. Thus, the bootstrapped critical value may be much lower than the true critical value (note, however, that Garcia's [53], bootstrapped critical values were considerably higher than his asymptotic critical values). Second, given the unidentified nuisance parameters, the test statistic may not even be asymptotically pivotal. Thus, it is unclear how well the bootstrapped distribution approximates the true finite sample distribution. Despite this, bootstrap tests have often performed better in terms of *size* (the probability of false rejection of the null hypothesis in repeated experiments) than asymptotic tests in the presence of unidentified nuisance parameters. For example, Diebold and Chen [37] consider Monte Carlo analysis of bootstrap and asymptotic tests for structural change with an unknown breakpoint that is a nuisance parameter and find that the bootstrap tests perform well in terms of size and better than the asymptotic tests. Enders, Falk, and Siklos [44] find that bootstrap and asymptotic tests both have size problems for TAR models, although bootstrap LR tests perform better than the asymptotic tests or other bootstrap tests. In terms of testing for nonlinearity with Markov-switching models, Kim, Morley, and Piger [83] bootstrap the distribution of the LR statistic testing linearity for the bounceback model discussed above and reject linearity with a p-value of less than 0.01. The local maximum problem is addressed by conducting a grid search across transition probabilities.

In a recent paper, Carrasco, Hu, and Ploberger [15] develop an information matrix-type test for Markov-switching that is asymptotically optimal and only requires estimation under the null of no Markov-switching (their null allows for other forms of nonlinearity such as ARCH). At this point, there is little known about the finite sample properties of the test. However, Carrasco, Hu, and Ploberger [15] show that it has higher *power* (probability of correct rejection of the null hypothesis in repeated experiments) than Garcia's [53] approach for a basic Markov-switching model with no autoregressive dynamics. Hamilton [60] applies Carrasco, Hu, and

Ploberger's [15] method to test for Markov switching in the US unemployment rate (he also provides a very helpful appendix describing how to conduct the test). The null hypothesis is a linear AR(4) model with student t errors. The alternative is an AR(4) with student t errors where the intercept is Markov-switching with three regimes. The test statistic is 26.02, while the 5 percent critical value is 4.01. Thus, linearity can be rejected for the unemployment rate. Meanwhile, the estimated Markov-switching model implies asymmetry in the form of steepness (the unemployment rate rises above its average more quickly than it falls below its average rate).

In contrast to Markov-switching models or threshold models, Beaudry and Koop's [6] ARMA model with the CDR variable provides a very simple test of nonlinearity. In particular, for their preferred specification, Beaudry and Koop [6] find support for nonlinearity with a t-statistic of 3.39 for the CDR variable. Hess and Iwata [68] question the significance of this statistic on the basis of Monte Carlo analysis. However, the data generating process in their Monte Carlo study assumed no drift in the simulated "output" series, meaning that the simulated CDR variable behaves much like a unit root process. By contrast, given drift, the CDR variable can be expected to revert to zero over a fairly short horizon, as it does in the real world (see Fig. 8). Elwood [43] develops an unobserved components model with a threshold process for the transitory component and argues that there is no evidence for asymmetry in the responses to positive and negative shocks. However, his model does not confront the key distinction between large negative shocks versus other shocks that Beaudry and Koop [6] address directly with the inclusion of the CDR variable in their model. A more fundamental issue is whether the CDR variable is merely a proxy for another variable such as the unemployment rate or interest rates and the apparent nonlinearity is simply the result of an omitted variable. However, as discussed in more detail later, the results in Clarida and Taylor [22] and Morley and Piger [113] suggest that Beaudry and Koop's [6] model is capturing a nonlinear dynamic that is fundamentally different than what would be implied by any linear model.

Hess and Iwata [69] provide a more formidable challenge to Beaudry and Koop's [6] model, and, indeed, to many of the regime-switching models discussed earlier, by examining the relative abilities of linear and nonlinear models to reproduce particular features of US real GDP. This alternative form of model evaluation is related to encompassing tests for non-nested models (see [110], on encompassing tests and [9], on the use of encompassing tests to evaluate Markov-switching models). In particular, Hess and Iwata [69] simulate data from a variety of models of output growth, including an AR(1) model, an ARMA(2,2) model, Beaudry and Koop's [6] model, Potter's [121] two-regime TAR model, Pesaran and Potter's [120] "floor and ceiling" model, Hamilton's [59] two-regime Markov-switching model, and a three-regime Markov-switching model with restrictions on the transition matrix as in Boldin [8]. They then consider whether the simulated data for each model can successfully reproduce "business cycle" features in terms of the duration and amplitude of expansions and recessions. Their definition of the business cycle is related to the level of real GDP. However, they label any switch between positive and negative growth, no matter how short-lived, to be a business cycle turning point. For US real GDP, their approach identifies twice as many turning points as reported by the NBER. Under this definition, Hess and Iwata [69] find that the linear AR(1) model is better than the nonlinear models at reproducing the duration and amplitude of "expansions" and "recessions" in US real GDP.

Harding and Pagan [65] and Engel, Haugh, and Pagan [45] confirm Hess and Iwata's [69] findings of little or no "value-added" for nonlinear models over linear models using a business cycle dating procedure that more closely matches NBER dates. The procedure is a quarterly version of an algorithm by Bry and Broschan [12] and identifies recessions as being related to two consecutive quarters of decline in real GDP. In terms of nonlinear models, Engel, Haugh, and Pagan [45] move beyond Hess and Iwata [69] by considering van Dijk and Franses' [149] version of the floor and ceiling model with ARCH errors, Kim, Morley, and Piger's [83] bounceback model, and DeJong, Liesenfeld, and Richard's [34] tension index model. Meanwhile, Clements and Krolzig [25] find that multivariate two-regime Markov-switching models provide little improvement over linear models in capturing business cycle features.

However, beyond the issue of how to define a business cycle, the major question in the literature on reproducing business cycle features is which features to consider. Galvão [52], Kim, Morley, and Piger [83], and Morley and Piger [111] examine the ability of linear and nonlinear models to capture high-growth recoveries that are related to the severity of the preceding recessions, which is the asymmetry emphasized by Friedman [50], Wynne and Balke [150], Sichel [134], and Balke and Wynne [2]. When considering this feature, there is strong support for Kim and Nelson's [79] plucking model and Kim, Morley, and Piger's [83] bounceback model over linear models. Interestingly, the three-regime Markov-switching model does not reproduce this feature. In particular, even though it implies high-growth recoveries, the fixed transition probabilities mean that the strength of the recovery is independent of the severity of the preceding recession. However, the strong support for the plucking model and bounceback model over linear models when considering the relationship between recessions and their recoveries represents a major reversal of the earlier findings for linear models by Hess and Iwata [69] and others.

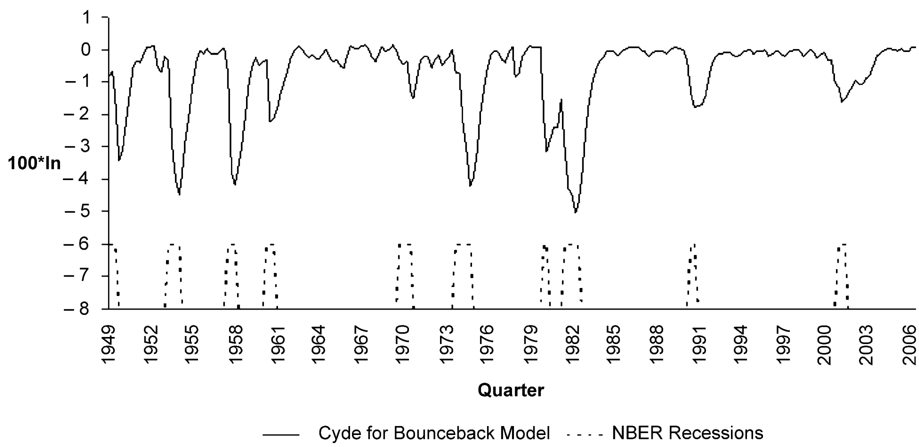
In terms of directly testing business cycle asymmetries, DeLong and Summers [35] consider a nonparametric test for steepness in real GNP and unemployment rates for eight countries (including the US). In particular, they test for skewness in output growth rates and changes in unemployment rates. With the exception of changes in the US unemployment rate, the measures of economic activity produce no statistically significant evidence of skewness, although the point estimates are generally large and negative for output growth and large and positive for the unemployment rates. Of course, given that the nonparametric test of skewness is unlikely to have much power for the relatively small sample sizes available in macroeconomics, it is hard to treat the non-rejections as particularly decisive. In a more parametric setting, Goodwin [56] considers a likelihood ratio test for sharpness using Hamilton's [59] model. Applying the model and test to real GNP for eight countries (including the US), he is able to reject nonsharpness in every country except Germany. In a more general setting, Clements and Krolzig [24] develop tests of deepness, steepness, and sharpness that are conditional on the number of regimes. For a three-regime model, they are able to reject the null hypotheses of no steepness and no sharpness in US real GDP growth, although the test results are somewhat sensitive to the sample period considered. Meanwhile, Ramsey and Rothman [124] develop a test of time reversibility and find that many measures of economic activity are irreversible and asymmetric, although the nature of the irreversibility does not always provide evidence for nonlinearity.

In addition to classical tests of nonlinear models and the encompassing-style approach discussed above, there are two other approaches to testing nonlinearity that should be briefly mentioned: nonparametric tests and Bayesian model comparison. In terms of nonparametric tests, there is some evidence for nonlinearity in macroeconomic time series. For example, Brock and Sayers [11] apply the nonparametric test for independence (of "prewhitened" residuals using a linear AR model) developed by Brock, Dechert, and Schienkman [10] and are able to reject linearity for the US unemployment rate and industrial production. However, as is always the case with such general tests, it is not clear what alternative is being supported (i. e., is it nonlinearity in the conditional mean or time-variation in the conditional variance?). Also, again, the nonparametric approach is hampered in macroeconomics by relatively small sample sizes. In terms of Bayesian analysis, there is some support for nonlinearity related to business cycle asymmetry using Bayes factors for multivariate models (see [80]). Bayes factors correspond to the posterior odds of one model versus another given equal prior odds. In essence, they compare the relative abilities of two models to predict the data given the stated priors for the model parameters. Obviously, Bayes factors can be sensitive to these priors. However, given diffuse priors, they have a tendency to favor more tightly parametrized models, as some of the prior predictions from the more complicated models can be wildly at odds with the data. Thus, because the findings in favor of nonlinear models correspond to relatively more complicated models, evidence for nonlinearity using Bayes factors is fairly compelling.

Relevance

Even accepting the presence of nonlinear dynamics related to business cycle asymmetry, there is still a question of economic relevance. Following the literature, the case can be made for relevance in three broad, but related areas: forecasting, macroeconomic policy, and macroeconomic theory.

In terms of forecasting, the nonlinear time series models discussed earlier directly imply different conditional forecasts than linear models. Beaudry and Koop's [6] model provides a simple example with a different implied persistence for large negative shocks than for other shocks. By contrast, linear models imply that the persistence of shocks is invariant to their sign or size. Koop, Pesaran, and Potter [94] develop "generalized impulse response functions" to examine shock dynamics for nonlinear models. Their approach involves simulating artificial time series both in the presence of the shock and in the absence of the shock, holding all else (e. g., other shocks) equal, and comparing the paths of the two simulated time series. This simulation can be done repeatedly for different values of other shocks in order to integrate out their impact on the difference in conditional expectations of the time series implied by presence and absence of a shock. Clarida and Taylor [22] use related simulated forecasts to carry out the Beveridge–Nelson (BN) decomposition (see [7]) for US real GNP using Beaudry and Koop's [6] model. The BN decomposition produces estimates of the permanent and transitory components of a time series based on long-horizon conditional forecasts. Importantly, the estimated cycle (under the "deviations" definition of the business cycle)



Macroeconomics, Non-linear Time Series in, Figure 9
"Bounceback" cycle and NBER recessions (Source: Author's calculations based on Morley and Piger [113], and NBER website)

for Beaudry and Koop's [6] model displays deepness that would be difficult to replicate with any linear forecasting model, even with multivariate information. Thus, there is a direct sense in which Beaudry and Koop's [6] model is not just approximating a linear multivariate model.

In a recent paper, Morley and Piger [112] develop an extension of the BN decomposition that produces optimal (in a "minimum mean squared error" sense) estimates of the cyclical component of an integrated time series when the series can be characterized by a regime-switching process such as for a Markov-switching model with fixed transition probabilities. The approach, which is labeled the "regime-dependent steady-state" (RDSS) decomposition, extracts the trend by constructing a long-horizon forecast conditional on remaining in a particular regime (hence, "regime-dependent"). In Morley and Piger [113], the RDSS decomposition is applied to US real GDP using the "depth" version of Kim, Morley, and Piger's [83] bounceback model given by (8) and (16). Figure 9 presents the estimated cycle for a version of the model with a structural break in σ^2, μ_1, and μ_2 in 1984:Q2 to account for the Great Moderation. The figure also displays an indicator variable for NBER-dated recessions for each quarter from 1949:Q2 to 2006:4. (For visual ease, the indicator variable is -8 in expansions and -6 in recessions).

There are three particularly notable features of the cycle in Fig. 9. First, there is a close correspondence between the big negative movements in it and the NBER-dated periods of recession. Thus, in practice, there is a direct relationship between the level and deviations definitions of the business cycle discussed earlier. Also, this correspondence directly implies that the NBER is identifying a meaningful macroeconomic structure (i. e., it is capturing a phase that is closely related to large movements in the transitory component of economic activity), rather than merely noting negative movements in economic activity. Second, it is fairly evident from Fig. 9 that the cycle displays all three business cycle asymmetries in the form of deepness, steepness, and sharpness. Third, the unconditional mean of the cycle is negative. As discussed in Morley and Piger [113], this finding stands in contrast to cyclical estimates for all linear models, whether univariate or multivariate.

The negative mean of the cycle in US real GDP has strong implications for the potential benefits of macroeconomic stabilization policy. Lucas [102,103] famously argued that the elimination of all business cycle fluctuations would produce a benefit equivalent to less than one-tenth of one percent of lifetime consumption. One reason for this extraordinarily low estimate is that his calculation assumes business cycle fluctuations are symmetric. However, as discussed in DeLong and Summers [36], Cohen [28], Barlevy [5], and Yellen and Akerlof [151], a nonzero mean cyclical component of economic activity directly implies that stabilization policies, if effective, could raise the average level of output and lower the average level of the unemployment rate. In this setting, the potential benefits of stabilization policy are much larger than calculated by Lucas [102,103]. (In deference to Milton Friedman and his plucking model, it is worth mentioning that the optimal "stabilization" policy might be a passive rule that prevents policymakers from generating recessionary shocks in the first place. Regardless, the point is that, given a negative mean for the cycle in real GDP, the costs of business cycles are high and can be affected by policy).

A related issue is asymmetry in terms of the effects of macroeconomic policy on economic activity. For example, DeLong and Summers [36] and Cover [31] find that negative monetary policy shocks have a larger effect on output than positive shocks of the same size (the so-called "pushing on a string" hypothesis). This form of asymmetry represents a third type of nonlinearity in macroeconomics beyond structural change and business cycle asymmetry, although it is clearly related to business cycle asymmetry. Indeed, Garcia and Schaller [54] and Lo and Piger [99] consider Markov-switching models and find that asymmetry in the effects of monetary policy shocks is more closely related to whether the economy is in an expansion or a recession, rather than whether the shock was positive or negative. In particular, positive shocks can have large effects on output, but only in recessions. There is an obvious link between this result, which is suggestive of a convex short-run aggregate supply curve rather than the "pushing on a string" hypothesis, and the business cycle displayed in Fig. 9, which is also highly suggestive of a convex short-run aggregate supply curve.

In addition to the implications for more traditional theoretical notions in macroeconomics such as the shape of the short-run aggregate supply curve, the findings for business cycle asymmetry are important for modern macroeconomic theory because dynamic stochastic general equilibrium (DSGE) models are often evaluated and compared based on their ability to generate internal propagation that matches what would be implied by linear AR and VAR models of US real GDP (see, for example [126]). These linear models imply a time-invariant propagation structure for shocks, while the business cycle presented in Fig. 9 suggests that theory-based models should instead be evaluated on their ability to generate levels of propagation that vary over business cycle regimes, at least if they are claimed to be "business cycle" models.

Future Directions

There are several interesting avenues for future research in nonlinear time series in macroeconomics. However, two follow directly from the findings on nonlinearities summarized in this survey. First, in terms of structural change, it would be useful to determine whether the process of change is gradual or abrupt and the extent to which it is predictable. Second, in terms of business cycle asymmetries, it would be useful to pin down the extent to which they reflect nonlinearities in conditional mean dynamics, conditional variance dynamics, and/or the contemporaneous relationship between macroeconomic variables.

The issue of whether structural change is gradual or abrupt is only meaningful when structural change is thought of as a form of nonlinearity in a time series model. In particular, formal classical tests of structural change based on asymptotic theory make no distinction between whether there are many small change or a few large changes. All that matters is the cumulative magnitude of changes over the long horizon (see [42], on this point). Of course, a time-varying parameter model and a regime-switching model with permanent changes in regimes can fit the data in very different ways in finite samples. Thus, it is possible to use finite-sample model comparison (e. g., Bayes factors) to discriminate between these two behaviors. It is even possible to use a particle filter to estimate a nonlinear state-space model that nests large, infrequent changes and small, frequent changes (see [90]). In terms of predicting structural change, Koop and Potter [93] develop a flexible model that allows the number of structural breaks in a given sample and the duration of structural regimes to be stochastic processes and discuss estimation of the model via Bayesian methods.

The issue of the relative importance of different types of recurring nonlinearities is brought up by the findings in Sims and Zha [136], discussed earlier, that there are no changes in the conditional mean dynamics, but only changes in the conditional variance of shocks for a structural VAR model of the US economy. Likewise, in their multivariate three-regime TAR model, Koop and Potter [92] consider a VAR structure, and find that a linear VAR structure with heteroskedastic errors is preferred over a "vector floor and ceiling" structure for the conditional mean dynamics. The question is how to reconcile these results with the large body of evidence supporting nonlinearity in conditional mean dynamics discussed at length in this survey. A short answer is that VAR models are highly parametrized in terms of the conditional mean. Thus, it may be hard to identify regime shifts or nonlinear forms of time-variation in conditional means using a VAR model, even if they are present. On the other hand, even for their nonlinear model, Koop and Potter [92] find stronger evidence for nonlinearity in the contemporaneous relationship between variables than in the conditional mean dynamics. Meanwhile, in terms of multivariate analysis, consideration of more parsimonious factor models has typically increased the support for nonlinear models over linear models (e. g. [80]). Thus, a full comparison of different types of nonlinearity in the context of a parsimonious nonlinear model would be useful.

Another important avenue for future research in macroeconomics is an increased integration of the findings in nonlinear time series into macroeconomic theory. In terms of structural change, there has been considerable progress in recent years. In particular, some of the papers on changes in policy regimes discussed earlier (e. g. [123, 136]) can be classified as "theory-oriented" given their consideration of structural VAR models. Another nonlinear time series paper on changing policy regimes with a structural model is Owyang and Ramey [118], which considers the interaction between regime switching in the Phillips curve and the policy rule. Meanwhile, Fernández-Villaverde and Rubio-Ramírez [47] and King [89] directly incorporate structural change (of the gradual form) in theory-based DSGE models, which they proceed to estimate with the aid of particle filters. In terms of Bayesian analysis of the sources of the Great Moderation, Chauvet and Potter [20] and Kim, Nelson, and Piger [82] consider disaggregated data (in a joint model and separately, respectively) and find that the decline in volatility of economic activity is a broadly-based phenomenon, rather than corresponding to particular sectors, while Kim, Morley, and Piger [86] employ structural VAR models and find that the decline in volatility cannot be explained by changes in aggregate demand shocks, monetary policy shocks, or the response of the private sector or policymakers to shocks.

In terms of the integration of business cycle asymmetries into macroeconomic theory, there has been less progress in recent years, perhaps due the obviously greater difficulty in modeling endogenous regime switching than in simply assuming exogenous structural change. However, the theoretical literature contains some work on asymmetries. In particular, mechanisms for regime switching in the aggregate data that have been considered in the past include spillovers and strategic complementarities [41], animal spirits [70], a history-dependent selection criterion in an economy with multiple Nash equilibria corresponding to different levels of productivity [30], and intertemporal increasing returns [1]. However, Potter [122] notes that, while these mechanisms can generate regime switching in the aggregate data, they cannot explain asymmetry in the form of high-growth recoveries following large negative shocks. He proposes a model with Bayesian learning and an information externality (see [16]) that can generate such dynamics. Meanwhile, in terms of business cycle asymmetry more generally, obvious mechanisms are investment irreversibilities [55] and capacity constraints [64]. More promisingly for future developments in macroeconomic theory, there is a growing empirical literature on the sources of business cycle asymmetries. For example, Korenok, Mizrach, and Radchenko [95] use disaggregated data and find that asymmetries are more pronounced in durable goods manufacturing sectors than nondurable goods manufacturing sectors (also see [129]) and appear to be related to variation across sectors in credit conditions and reliance on raw material inventories, while they do not appear to be related to oil price shocks [33] or adjustment costs [3].

Bibliography

Primary Literature

1. Acemoglu D, Scott A (1997) Asymmetric business cycles: Theory and time-series evidence. J Monet Econ 40:501–533
2. Balke NS, Wynne MA (1996) Are deep recessions followed by strong recoveries? Results for the G-7 countries. Appl Econ 28:889–897
3. Ball L, Mankiw NG (1995) Relative price changes as aggregate supply shocks. Q J Econ 110:161–193
4. Bansal R, Yaron A (2004) Risks for the long run: A potential resolution of asset pricing puzzles. J Financ 59:1481–1509
5. Barlevy G (2005) The cost of business cycles and the benefits of stabilization. Econ Perspect 29:32–49
6. Beaudry P, Koop G (1993) Do recessions permanently change output? J Monet Econ 31:149–163
7. Beveridge S, Nelson CR (1981) A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle. J Monet Econ 7:151–174
8. Boldin MD (1996) A check on the robustness of Hamilton's Markov switching model approach to the economic analysis of the business cycle. Stud Nonlinear Dyn Econom 1:35–46
9. Breunig R, Najarian S, Pagan A (2003) Specification testing of Markov-switching models. Oxf Bull Econ Stat 65:703–725
10. Brock WA, Dechert WD, Scheinkman JA (1996) A test of independence based on the correlation dimension. Econom Rev 15:197–235
11. Brock WA, Sayers C (1988) Is the business cycle characterized by deterministic chaos? J Monet Econ 22:71–90
12. Bry G, Boschan C (1971) Cyclical analysis of time series: Selected procedures and computer programs. NBER, New York
13. Burns AF, Mitchell WA (1946) Measuring Business Cycles. NBER, New York
14. Camacho M (2005) Markov-switching stochastic trends and economic fluctuations. J Econ Dyn Control 29:135–158
15. Carrasco M, Hu L, Ploberger W (2007) Optimal test for Markov switching. Working Paper
16. Chalkley M, Lee IH (1998) Asymmetric business cycles. Rev Econ Dyn 1:623–645
17. Chan KS (1991) Percentage points of likelihood ratio tests for threshold autoregression. J Royal Stat Soc Ser B 53:691–696
18. Chan KS, Tong H (1986) On estimating thresholds in autoregressive models. J Tim Ser Analysis 7:179–190
19. Chauvet M (1998) An econometric characterization of business cycle dynamics with factor structure and regime switches. Int Econ Rev 39:969–996
20. Chauvet M, Potter S (2001) Recent changes in the US business cycle. Manch Sch 69:481–508
21. Chib S, Nardari F, Shephard N (2002) Markov chain Monte Carlo methods for stochastic volatility models. J Econom 108:281–316
22. Clarida RH, Taylor MP (2003) Nonlinear permanent-temporary decompositions in macroeconomics and finance. Econ J 113:C125–C139
23. Clements MP, Krolzig HM (1998) A comparison of the forecast

performance of Markov-switching and threshold autoregressive models of US GNP. Econ J 1:C47–C75
24. Clements MP, Krolzig HM (2003). Business cycle asymmetries: Characterization and testing based on Markov-switching autoregressions. J Bus Econ Stat 21:196–211
25. Clements MP, Krolzig HM (2004) Can regime-switching models reproduce the business cycle features of US aggregate consumption, investment and output? Int J Financ Econ 9:1–14
26. Cogley T, Sargent TJ (2001) Evolving post-World War II US inflation dynamics. In: Bernanke BS, Rogoff K (eds) NBER Macroeconomics Annual 2001. MIT Press, Cambridge, pp 331–373
27. Cogley T, Sargent TJ (2005) Drift and volatilities: Monetary policies and outcomes in the post WW II US. Rev Econ Dyn 8:262–302
28. Cohen D (2000) A quantitative defense of stabilization policy. Federal Reserve Board Finance and Economics Discussion Series. Paper 2000-34
29. Cooley TF, Prescott EC (1976) Estimation in the presence of stochastic parameter variation. Econometrica 44:167–184
30. Cooper R (1994) Equilibrium selection in imperfectly competitive economies with multiple equilibria. Econ J 104:1106–1122
31. Cover JP (1992) Asymmetric effects of positive and negative money-supply shocks. Q J Econ 107:1261–1282
32. Davies RB (1977) Hypothesis testing when a nuisance parameter is present only under the alternative. Biometrika 64:247–254
33. Davis SJ, Haltiwanger J (2001) Sectoral job creation and destruction responses to oil price changes. J Monet Econ 48:468–512
34. DeJong DN, Liesenfeld R, Richard JF (2005) A nonlinear forecasting model of GDP growth. Rev Econ Stat 87:697–708
35. DeLong JB, Summers LH (1986) Are business cycles symmetrical? In: Gordon RJ (ed) The American Business Cycle. University of Chicago Press, Chicago, pp 166–179
36. DeLong B, Summers L (1988) How does macroeconomic policy affect output? Brook Papers Econ Activity 2:433–480
37. Diebold FX, Chen C (1996) Testing structural stability with endogenous breakpoint: A size comparison of analytic and bootstrap procedures. J Econ 70:221–241
38. Diebold FX, Rudebusch GD (1996) Measuring business cycles: A modern perspective. Rev Econ Stat 78:67–77
39. Diebold FX, Lee JH, Weinbach G (1994) Regime switching with time-varying transition probabilities. In: Hargreaves C (ed) Nonstationary Time Series Analysis and Cointegration. Oxford University Press, Oxford, pp 283–302
40. Durland JM, McCurdy TH (1994) Duration-dependent transitions in a Markov model of US GNP growth. J Bus Econ Stat 12:279–288
41. Durlauf SN (1991) Multiple equilibria and persistence in aggregate fluctuations. Am Econ Rev Pap Proc 81:70–74
42. Elliott G, Müller U (2006) Efficient tests for general persistent time variation in regression coefficients. Rev Econ Stud 73:907–940
43. Elwood SK (1998) Is the persistence of shocks to output asymmetric? J Monet Econ 41:411–426
44. Enders W, Falk BL, Siklos P (2007) A threshold model of real US GDP and the problem of constructing confidence intervals in TAR models. Stud Nonlinear Dyn Econ 11(3):4
45. Engel J, Haugh D, Pagan A (2005) Some methods for assessing the need for non-linear models in business cycles. Int J Forecast 21:651–662
46. Engle RF (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50:987–1007
47. Fernández-Villaverde J, Rubio-Ramírez JF (2007) Estimating macroeconomic models: A likelihood approach. Rev Econ Stud 54:1059–1087
48. Filardo AJ (1994) Business-cycle phases and their transitional dynamics. J Bus Econ Stat 12:299–308
49. French MW, Sichel DE (1993) Cyclical patterns in the variance of economic activity. J Bus Econ Stat 11:113–119
50. Friedman M (1964) Monetary Studies of the National Bureau, the National Bureau Enters Its 45th Year. 44th Annual Report. NBER, New York, pp 7–25; Reprinted in: Friedman M (1969) The Optimum Quantity of Money and Other Essays. Aldine, Chicago, pp 261–284
51. Friedman M (1993) The "plucking model" of business fluctuations revisited. Econ Inq 31:171–177
52. Galvão AB (2002) Can non-linear time series models generate US business cycle asymmetric shape? Econ Lett 77:187–194
53. Garcia R (1998) Asymptotic null distribution of the likelihood ratio test in Markov switching models. Int Econ Rev 39:763–788
54. Garcia R, Schaller H (2002) Are the effects of interest rate changes asymmetric? Econ Inq 40:102–119
55. Gilchrist S, Williams JC (2000) Putty-clay and investment: A business cycle analysis. J Political Econ 108:928–960
56. Goodwin TH (1993) Business-cycle analysis with a Markov-switching model. J Bus Econ Stat 11:331–339
57. Granger CWJ, Andersen AP (1978) An Introduction to Bilinear Time Series Models. Vandenhoek and Ruprecht, Göttingen
58. Granger CWJ, Teräsvirta T (1993) Modelling Nonlinear Economic Relationships. Oxford University Press, Oxford
59. Hamilton JD (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57:357–384
60. Hamilton JD (2005) What's real about the business cycle? Fed Reserve Bank St. Louis Rev 87:435–452
61. Hansen BE (1992) The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP. J Appl Econ 7:S61–S82
62. Hansen BE (1996) Inference when a nuisance parameter is not identified under the null hypothesis. Econometrica 64:413–430
63. Hansen BE (1997) Inference in TAR models. Stud Nonlinear Dyn Econom 2:1–14
64. Hansen GD, Prescott EC (2005) Capacity constraints, asymmetries, and the business cycle. Rev Econ Dyn 8:850–865
65. Harding D, Pagan AR (2002) Dissecting the cycle: A methodological investigation. J Monet Econ 49:365–381
66. Harding D, Pagan AR (2003) A Comparison of Two Business Cycle Dating Methods. J Econ Dyn Control 27:1681–1690
67. Harding D, Pagan AR (2005) A suggested framework for classifying the modes of cycle research. J Appl Econom 20:151–159
68. Hess GD, Iwata S (1997) Asymmetric persistence in GDP? A deeper look at depth. J Monet Econ 40:535–554
69. Hess GD, Iwata S (1997) Measuring and comparing business-cycle features. J Bus Econ Stat 15:432–444

70. Howitt P, McAfee RP (1992) Animal spirits. Am Econ Rev 82:493–507
71. Hristova D (2005) Maximum likelihood estimation of a unit root bilinear model with an application to prices. Stud Nonlinear Dyn Econom 9(1):4
72. Keynes JM (1936) The General Theory of Employment, Interest, and Money. Macmillan, London
73. Kiefer NM (1978) Discrete parameter variation: Efficient estimation of a switching regression model. Econometrica 46:413–430
74. Kim CJ (1994) Dynamic linear models with Markov switching. J Econom 60:1–22
75. Kim CJ, Murray CJ (2002) Permanent and transitory components of recessions. Empir Econ 27:163–183
76. Kim CJ, Nelson CR (1998) Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching. Rev Econ Stat 80:188–201
77. Kim CJ, Nelson CR (1999) State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications. MIT Press, Cambridge
78. Kim CJ, Nelson CR (1999) Has the US economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. Rev Econ Stat 81:608–616
79. Kim CJ, Nelson CR (1999) Friedman's plucking model of business fluctuations: Tests and estimates of permanent and transitory components. J Money Credit Bank 31:317–34
80. Kim CJ, Nelson CR (2001) A Bayesian approach to testing for Markov-switching in univariate and dynamic factor models. Int Econ Rev 42:989–1013
81. Kim CJ, Piger JM (2002) Common stochastic trends, common cycles, and asymmetry in economic fluctuations. J Monet Econ 49:1181–1211
82. Kim CJ, Nelson CR, Piger J (2004) The less-volatile US economy: A Bayesian investigation of timing, breadth, and potential explanations. J Bus Econ Stat 22:80–93
83. Kim CJ, Morley J, Piger J (2005) Nonlinearity and the permanent effects of recessions. J Appl Econom 20:291–309
84. Kim CJ, Piger J, Startz R (2007) The dynamic relationship between permanent and transitory components of US business cycles. J Money Credit Bank 39:187–204
85. Kim CJ, Piger J, Startz R (2008) Estimation of Markov regime-switching regression models with endogenous switching. J Econom 143:263–273
86. Kim CJ, Morley J, Piger J (2008) Bayesian Counterfactual Analysis of the Sources of the Great Moderation. J Appl Econom 23:173–191
87. Kim M-J, Yoo J-S (1995) New index of coincident indicators: A multivariate Markov switching factor model approach. J Monet Econ 36:607–630
88. Kim S, Shephard N, Chib S (1998) Stochastic volatility: Likelihood inference and comparison with ARCH models. Rev Econ Stud 65:361–393
89. King TB (2006) Dynamic equilibrium models with time-varying structural parameters. Working Paper
90. King TB, Morley J (2007) Maximum likelihood estimation of nonlinear, non-Gaussian state-space models using a multistage adaptive particle filter. Working Paper
91. Koop G, Potter S (2003) Bayesian analysis of endogenous delay threshold models. J Bus Econ Stat 21:93–103
92. Koop G, Potter S (2006) The vector floor and ceiling model. In: Milas C, Rothman P, Van Dijk D (eds) Nonlinear Time Series Analysis of Business Cycles. Elsevier, Amsterdam, pp 97–131
93. Koop G, Potter S (2007) Estimation and forecasting in models with multiple breaks. Rev Econ Stud 74:763–789
94. Koop G, Pesaran MH, Potter S (1996) Impulse response analysis in nonlinear multivariate models. J Econometrics 74:119–148
95. Korenok O, Mizrach B, Radchenko S (2009) A note on demand and supply factors in manufacturing output asymmetries. Macroecon Dyn (forthcoming)
96. Lam PS (1990) The Hamilton model with a general autoregressive component: Estimation and comparison with other models of economic time series. J Monet Econ 26:409–432
97. Leamer EE, Potter SM (2004) A nonlinear model of the business cycle. Working Paper
98. Leyton AP, Smith D (2000) A further note of the three phases of the US business cycle. Appl Econ 32:1133–1143
99. Lo MC, Piger J (2005) Is the response of output to monetary policy asymmetric? Evidence from a regime-switching coefficients model. J Money Credit Bank 37:865–887
100. Lucas RE (1972) Econometric testing of the natural rate hypothesis. In: Eckstein O (ed) Econometrics of Price Determination. US Federal Reserve Board, Washington DC, pp 50–59
101. Lucas RE (1976) Econometric policy evaluation: A critique. In: Brunner K, Meltzer A (eds) The Phillips Curve and Labor Markets, vol 1. Carnegie-Rochester Ser Public Policy, pp 19–46
102. Lucas RE (1987) Models of Business Cycles. Basil Blackwell, Oxford
103. Lucas RE (2003) Macroeconomic Priorities. Am Econ Rev 93:1–14
104. Ma J (2007) Consumption persistence and the equity premium puzzle: New evidence based on improved inference. Working paper
105. MacKinnon J (2002) Bootstrap inference in econometrics. Can J Econ 35:615–645
106. MacKinnon J (2006) Bootstrap methods in econometrics. Econ Rec 82:S2–S18
107. McConnell MM, Quiros GP (2000) Output fluctuations in the United States: What has changed since the early 1980s? Am Econ Rev 90:1464–1476
108. McQueen G, Thorley SR (1993) Asymmetric business cycle turning points. J Monet Econ 31:341–362
109. Mitchell WA (1927) Business Cycles: The Problem and Its Setting. NBER, New York
110. Mizon GE, Richard JF (1986) The encompassing principle and its application to non-nested hypotheses. Econometrica 54:657–678
111. Morley J, Piger J (2006) The Importance of Nonlinearity in Reproducing Business Cycle Features. In: Milas C, Rothman P, Van Dijk D (eds) Nonlinear Time Series Analysis of Business Cycles. Elsevier, Amsterdam, pp 75–95
112. Morley J, Piger J (2008) Trend/cycle decomposition of regime-switching processes. J Econom (forthcoming)
113. Morley J, Piger J (2008) The asymmetric business cycle. Working Paper
114. Morley JC, Nelson CR, Zivot E (2003) Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different? Rev Econ Stat 85:235–243
115. Neftçi SH (1984) Are economic time series asymmetric over the business cycle? J Political Econ 92:307–328

116. Niemira MP, Klein PA (1994) Forecasting Financial and Economic Cycles. Wiley, New York
117. Öcal N, Osborn DR (2000) Business cycle non-linearities in UK consumption and production. J Appl Econom 15:27–44
118. Owyang MT, Ramey G (2004) Regime switching and monetary policy measurement. J Monet Econ 51:1577–1198
119. Peel D, Davidson J (1998) A non-linear error correction mechanism based on the bilinear model. Econ Lett 58:165–170
120. Pesaran MH, Potter SM (1997) A floor and ceiling model of US output. J Econ Dyn Control 21:661–695
121. Potter SM (1995) A nonlinear approach to US GNP. J Appl Econ 10:109–125
122. Potter SM (2000) A nonlinear model of the business cycle. Stud Nonlinear Dyn Econom 4:85–93
123. Primiceri GE (2005) Time varying structural vector autoregressions and monetary policy. Rev Econ Stud 72:821–852
124. Ramsey JB, Rothman P (1996) Time irreversibility and business cycle asymmetry. J Money Credit Bank 28:1–21
125. Ravn MO, Sola M (1995) Stylized facts and regime changes: Are prices procyclical? J Monet Econ 36:497–526
126. Rotemberg JJ, Woodford M (1996) Real-business-cycle Models and the forecastable movements in output, hours, and consumption. Am Econ Rev 86:71–89
127. Rothman P (1991) Further Evidence on the Asymmetric Behavior of Unemployment Rates Over the Business Cycle. J Macroeconom 13:291–298
128. Rothman P (1998) Forecasting asymmetric unemployment rates. Rev Econ Stat 80:164–168
129. Rothman P (2008) Reconsideration of Markov chain evidence on unemployment rate asymmetry. Stud Nonlinear Dyn Econo 12(3):6
130. Rothman P, van Dijk D, Franses PH (2001) A multivariate STAR analysis of the relationship between money and output. Macroeconom Dyn 5:506–532
131. Schumpeter J (1942) Capitalism, socialism, and democracy. Harper, New York
132. Sensier M, van Dijk D (2004) Testing for volatility changes in US macroeconomic time series. Rev Econ Stat 86:833–839
133. Sichel DE (1993) Business cycle asymmetry: A deeper look. Econ Inq 31:224–236
134. Sichel DE (1994) Inventories and the three phases of the business cycle. J Bus Econ Stat 12:269–277
135. Sims CA (2001) Comment on Sargent and Cogley's: Evolving Post-World War II US Inflation Dynamics. In: Bernanke BS, Rogoff K (eds) NBER Macroeconomics Annual 2001. MIT Press, Cambridge, pp 373–379
136. Sims CA, Zha T (2006) Were there regime switches in US monetary policy? Am Econ Rev 96:54–81
137. Sinclair TM (2008) Asymmetry in the business cycle: Friedman's plucking model with correlated innovations. Working Paper
138. Stock JH, Watson MW (2002) Has the business cycle changed and why? In: Gertler M, Rogoff K (eds) NBER Macroeconomics Annual 2002. MIT Press, Cambridge, pp 159–218
139. Subba Rao T, Gabr MM (1984) An Introduction to Bispectral Analysis and Bilinear Time Series Models. Lecture Notes in Statistics, vol 24. Springer, New York
140. Teräsvirta T (1994) Specification, estimation, and evaluation of smooth transition autoregressive models. J Am Stat Assoc 89:208–218
141. Teräsvirta T (1995) Modeling nonlinearity in US Gross National Product 1889–1987. Empir Econ 20:577–598
142. Teräsvirta T (1998) Modelling economic relationships with smooth transition regressions. In: Ullah A, Giles DEA (eds) Handbook of Applied Economic Statistics. Marcel Dekker, New York, pp 507–552
143. Teräsvirta T, Anderson HM (1992) Characterizing nonlinearities in business cycles using smooth transition autoregressive models. J Appl Econom 7:S119–S136
144. Tiao GC, Tsay RS (1994) Some advances in non-linear and adaptive modeling in time- series analysis. J Forecast 13:109–131
145. Tong H (1978) On a threshold model. In: Chen CH (ed) Pattern Recognition and Signal Processing. Sijhoff and Noordhoff, Amsterdam, pp 575–586
146. Tsay RS (1989) Testing and modeling threshold autoregressive processes. J Am Stat Assoc 84:231–240
147. Tsay RS (1998) Testing and modeling multivariate threshold processes. J Am Stat Assoc 93:1188–1202
148. van Dijk D, Franses PH (1999) Modeling multiple regimes in the business cycle. Macroeconom Dyn 3:311–340
149. van Dijk D, Franses PH (2003) Selecting a nonlinear time series model using weighted tests of equal forecast accuracy. Oxf Bull Econ Stat 65:727–744
150. Wynne MA, Balke NS (1992) Are deep recessions followed by strong recoveries? Econ Lett 39:183–189
151. Yellen JL, Akerlof GA (2006) Stabilization policy: A reconsideration. Econ Inq, pp 44:1–22

Books and Reviews

Davidson R, MacKinnon JG (2004) Econometric Theory and Methods. Oxford University Press, Oxford

Diebold FX (1998) The past, present, and future of macroeconomic forecasting. J Econ Perspectives 12:175–192

Engle R (2001) GARCH 101: The use of ARCH/GARCH models in applied econometrics. J Econc Perspectives 15:157–168

Franses PH (1998) Time Series Models for Business and Economic Forecasting. Cambridge University Press, Cambridge

Hamilton JD (1994) State-space models. In: Engle RF, McFadden DL (eds) Handbook of Econometrics, vol 4. Elsevier, Amsterdam, pp 041–3080

Hamilton JD (1994) Time Series Analysis. Princeton University Press, Princeton

Koop G (2003) Bayesian Econometrics. Wiley, Chichester

Teräsvirta T, Tjøstheim D, Granger CWJ (1994) Aspects of modeling nonlinear time series. In: Engle RF, McFadden DL (eds) Handbook of Econometrics, vol 4. Elsevier, Amsterdam, pp 2919–2957

Tsay RS (2005) Analysis of Financial Time Series. Wiley, Hoboken

Manipulating Data and Dimension Reduction Methods: Feature Selection

HUAN LIU, ZHENG ZHAO
Computer Science and Engineering, Arizona State University, Tempe, USA