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THE TWO INTERPRETATIONS OF THE BEVERIDGE–NELSON DECOMPOSITION

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The Beveridge–Nelson decomposition calculates trend and cycle for an integrated time series. However, there are two ways to interpret the results from the decomposition. One interpretation is that the optimal long-run forecast (minus any deterministic drift) used to calculate the Beveridge–Nelson trend corresponds to an *estimate* of an unobserved permanent component. The other interpretation is that the optimal long-run forecast *defines* an observable permanent component. This paper examines some issues surrounding these two interpretations and provides empirical support for interpreting the Beveridge–Nelson trend as an estimate when considering macroeconomic data.

Keywords: Beveridge–Nelson Decomposition, Trend/cycle Decomposition, State-Space Models, Unobserved-Components Models

> *The Beveridge–Nelson decomposition can be seen as an ingenious decomposition of an I(1) variable, but it does not properly fit into the unobserved components framework, since the components are, in fact, observable.... The assumption ... that the permanent and transitory component share, at every period, the same innovation is a strong assumption, of limited appeal.*
>
> —Maravall (1995)

1. INTRODUCTION

In the literature on trend/cycle decomposition, the Beveridge–Nelson decomposition has been subject to two very distinct interpretations. One interpretation, emphasized by Watson (1986) and Morley et al. (2003), is that the long-horizon conditional forecast used to calculate the Beveridge–Nelson trend corresponds to an estimate of the permanent component of an integrated time series. The Beveridge–Nelson decomposition provides a sensible estimate because, under the

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assumption that the permanent component follows a random walk (with drift) and the transitory component is stationary with an unconditional mean of zero, the optimal long-horizon forecast (in a minimum–mean squared error sense) will be equal to the conditional expectation of the permanent component. A second interpretation, emphasized in the original paper by Beveridge and Nelson (1981), is that the Beveridge–Nelson trend provides a definition of the permanent component of an integrated time series. In this case, given the optimal long-horizon forecast, the permanent component does not need to be estimated, but instead is directly observable by an econometrician for the simple reason that it is the Beveridge–Nelson trend.

In this paper, I compare the two interpretations of the Beveridge–Nelson decomposition. Although many other studies, including Watson (1986), Maravall (1995), Harvey and Koopman (2000), Proietti and Harvey (2000), Proietti (2006), and Oh et al. (2008), have alluded to two interpretations of the Beveridge–Nelson decomposition, I consider exactly how the interpretations are related to each other and whether there is any empirical reason to prefer one interpretation over the other. In particular, I demonstrate that, even though the two interpretations can be observationally equivalent in a univariate setting for an econometrician, they are empirically distinguishable in a multivariate setting. Notably, this empirical distinction is possible even if the autocovariance structures of the series under examination can be completely captured by univariate time series models. In an application to U.S. macroeconomic data, I find support for interpreting the Beveridge–Nelson decomposition as providing estimates of trend and cycle. This finding is important because the view that the Beveridge–Nelson decomposition defines the trend persists in many applied studies [e.g., Clarida and Taylor (2003) and Anderson et al. (2006)] and forms the basis for skepticism about its general relevance (see, for example, the quotation from Maravall at the beginning of this paper). In contrast, the view that the Beveridge–Nelson decomposition provides estimates of trend and cycle suggests that it is a highly general and practical method for trend/cycle decomposition.

2. THE BEVERIDGE–NELSON DECOMPOSITION

The Beveridge–Nelson (BN) trend of an integrated time series y_t is given as follows:

$$\mathrm{BN}_t = \lim_{M\to\infty} E[y_{t+M} - M\mu \mid \Omega_t], \tag{1}$$

where $\mu = E[\Delta y_t]$ is the deterministic drift and Ω_t is the information set used to calculate the conditional expectation. In words, the BN trend is the optimal long-horizon conditional point forecast of the time series process $\{y_t\}$, with any future drift removed. Meanwhile, the BN cycle is simply the difference between the series and the BN trend.

In practice, the BN decomposition is often calculated using an autoregressive moving-average (ARMA) model that is designed to capture the autocovariance structure of $\{\Delta y_t\}$. For example, assuming an AR(1) model

$\Delta y_t = \mu + \phi(\Delta y_t - \mu) + e_t$, where $|\phi| < 1$ and $e_t \sim \text{i.i.d.}N(0, \sigma^2)$, the BN trend is $\text{BN}_t = y_t + \frac{\phi}{1-\phi}(\Delta y_t - \mu)$ [see Morley (2002)]. Note that, given known parameter values (ϕ and μ) and the data (y_t and Δy_t), the BN trend for y_t is observed by an econometrician at time t.

To compare the two interpretations of the BN decomposition within a unified framework, I consider a state-space representation for y_t. In particular, assuming a known Gaussian ARMA structure for the first differences $\{\Delta y_t\}$, the level y_t can be thought of as made up of a permanent component, denoted τ_t, and a transitory component, denoted c_t:

$$y_t = \tau_t + c_t, \tag{2a}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t, \quad \eta_t \sim \text{i.i.d.}N\left(0, \sigma_\eta^2\right), \tag{2b}$$

$$\phi(L)c_t = \theta(L)\varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}N\left(0, \sigma_\varepsilon^2\right), \tag{2c}$$

$$\text{Corr}(\eta_t, \varepsilon_t) = \rho_{\eta\varepsilon}. \tag{2d}$$

According to (2b), the permanent component follows a random walk with drift and, according to (2c), the transitory component follows a stationary ARMA process with a mean of zero.

Within the context of the state-space representation in (2), the two interpretations of the BN decomposition can be understood as follows:

INTERPRETATION 1 (BN-as-estimate). *Under this interpretation, the permanent component of a time series is unobservable because of the assumed presence of transitory shocks that have no impact on the permanent component. In particular, the serially uncorrelated innovation to c_t in* (2c) *can be rewritten in the following way:*

$$\varepsilon_t = \alpha\eta_t + \varepsilon_t^*, \quad \varepsilon_t^* \sim \text{i.i.d.}N\left(0, \sigma_{\varepsilon^*}^2\right). \tag{3a}$$

Then, as long as the variance of the transitory shocks is positive (i.e., $\sigma_{\varepsilon^*}^2 > 0$), there will be imperfect correlation between the innovations to the permanent and transitory components:

$$|\rho_{\eta\varepsilon}| < 1. \tag{3b}$$

With imperfect correlation, (2) becomes an unobserved-components (UC) representation and, following the analysis in Watson (1986) and Morley et al. (2003), the BN trend provides an optimal estimate of the permanent component under the assumption that it follows a random walk and the unconditional expectation of $\{c_t\}$ is zero.

INTERPRETATION 2 (BN-as-definition). *Under this interpretation, the BN trend is the permanent component of a time series because there is assumed to be only one type of shock driving $\{y_t\}$. Specifically, the serially uncorrelated innovation to c_t in* (2c) *can be rewritten in the following way:*

$$\varepsilon_t = \alpha\eta_t, \tag{4a}$$

where α is a scalar that allows permanent and transitory innovations to have different signs and variances despite being driven by the same underlying shock. With only one underlying shock, the innovations to the permanent and transitory components will be perfectly correlated:

$$|\rho_{\eta\varepsilon}| = 1. \tag{4b}$$

In this case, the permanent and transitory innovations are observable and can be measured using the forecast error from the reduced-form ARMA representation for $\{\Delta y_t\}$, with the resulting permanent component equal to the BN trend.[1]

It should be noted that these two interpretations still apply even if the structure of $\{\Delta y_t\}$ is better captured by a multivariate time series model, rather than a univariate ARMA model. In such a case, the *BN-as-estimate* interpretation would correspond to the idea that the permanent and transitory innovations are imperfectly correlated across series, as well as with each other. Meanwhile, the *BN-as-definition* interpretation would correspond to the idea there are as many underlying shocks as there are series under examination, with the shocks being observable and proportional to the forecast errors from the multivariate model. In particular, for a given series, the permanent and transitory innovations would be proportional to the same linear combination of forecast errors and, therefore, would remain perfectly correlated.

3. UNIVARIATE OBSERVATIONAL EQUIVALENCE

One problem in distinguishing between the two interpretations of the BN decomposition presented in the preceding section is that, despite the apparent restrictiveness of the correlation in (4b) compared to (3b), the two interpretations can be observationally equivalent in terms of their implied univariate autocovariance structure for $\{\Delta y_t\}$.

An empirical example may help clarify the distinction between the two interpretations and illustrate the problem of observational equivalence in the univariate setting. Based on the state-space representation in (2), Morley et al. (2003) estimate an UC model for 100 times the natural logarithms of quarterly U.S. real GDP under the assumption of an AR(2) structure for the transitory component and with no restriction on the correlation between the innovations to the permanent and transitory components. Using data from the St. Louis Fed database for the updated sample period of 1947Q1 to 2009Q3, the maximum-likelihood estimates for this UC model are as follows:

$$y_t = \tau_t + c_t, \tag{5a}$$

$$\tau_t = \underset{(0.08)}{0.80} + \tau_{t-1} + \eta_t, \quad \eta_t \sim N(0, \underset{(0.06)}{1.19}^2), \tag{5b}$$

$$c_t = \underset{(0.05)}{1.31}\, c_{t-1} - \underset{(0.04)}{0.70}\, c_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \underset{(0.02)}{0.74}^2), \tag{5c}$$

$$\mathrm{Corr}(\eta_t, \varepsilon_t) = \underset{(0.04)}{-0.93}, \tag{5d}$$

where standard errors are in parentheses and the log likelihood value is −331.30. Because of the assumed UC structure, the permanent component is not observable and must be estimated. Given the parameter estimates, this can be done with the Kalman filter, which calculates $E[\tau_t \mid \Omega_t]$, where $\Omega_t = (y_1, \ldots, y_t)$.

The estimated UC model in (5) places no binding restrictions on the autocovariance structure of $\{\Delta y_t\}$ beyond those implied by an estimated reduced-form ARMA(2,2) model. As shown in Morley et al. (2003), the equivalent autocovariance structure implies that $E[\tau_t \mid \Omega_t]$ from the Kalman filter is identical to the BN trend given the ARMA(2,2) model. Meanwhile, any restriction on the correlation parameter in (5d) would place implicit restrictions on the parameters for the reduced-form ARMA(2,2) model, resulting in a BN trend different from that implied by the unrestricted ARMA(2,2) model. For example, it is possible to consider a restricted version of the state-space model in which the correlation between permanent and transitory innovations is restricted to be −1 instead of its estimated value of −0.91 in (5d). Although it might seem that this version of the state-space model corresponds to the *BN-as-definition* interpretation, it would actually produce a different permanent component than the unrestricted UC model in (5).

Given that imposing a perfect negative correlation would meaningfully restrict the model in (5), the obvious question is why there is an observational equivalence between the two interpretations of the BN decomposition. The answer lies in Anderson et al.'s (2006) insight that reduced-form ARMA models are equivalent to state-space models with only one type of shock, but comparatively more complicated dynamics. For example, they show that an unrestricted ARMA(2,2) model for $\{\Delta y_t\}$ is equivalent to a state-space model for $\{y_t\}$ in which innovations between permanent and transitory components are perfectly correlated and the transitory component follows an ARMA(2,1) process, instead of the AR(2) process in (5c). Using the same data as before, the maximum-likelihood estimates for this alternative state-space model are given as follows:

$$y_t = \tau_t + c_t, \tag{6a}$$

$$\tau_t = \underset{(0.08)}{0.80} + \tau_{t-1} + \eta_t, \quad \eta_t \sim N(0, \underset{(0.06)}{1.19^2}), \tag{6b}$$

$$c_t = \underset{(0.05)}{1.31}\, c_{t-1} - \underset{(0.04)}{0.70}\, c_{t-2} - \underset{(0.01)}{0.24}\, \eta_t + \underset{(0.03)}{0.31}\, \eta_{t-1}, \tag{6c}$$

where the log likelihood value is −331.30. Note that the log likelihood is the same as for the model in (5), meaning that the models in (5) and (6) are observationally equivalent in terms of fitting the sample data.[2]

Regarding the observation equivalence, a few issues should be mentioned. First, given the same ARMA model for $\{y_t\}$, the BN trend is the same under either interpretation. Thus, inferences about the variability of the permanent component are not sensitive to the interpretation. This robustness to interpretation stands in contrast to the sensitivity of inferences to different assumptions about the correlation between permanent and transitory innovations for a given UC model. Second, despite implying the same variability of the permanent component, the observational equivalence does not mean that deciding between the two interpretations is merely a matter of normalization. The two interpretations have very different implications in terms of subsequent econometric analysis. Under the *BN-as-definition* interpretation, the BN trend and the implied transitory component are observable and, therefore, can be treated as regular data in any regression analysis, at least assuming a reasonable model of the autocovariance structure and precise parameter estimates. However, under the *BN-as-estimate* interpretation, the estimated components will contain a large degree of measurement error, even given the correct model of the autocovariance structure and as parameter uncertainty goes away asymptotically.

4. MEASUREMENT ERROR IN THE MULTIVARIATE SETTING

The different implications for the presence of measurement error suggest that it should be possible to distinguish between the two interpretations in a multivariate setting. Specifically, if there is a sizable amount of measurement error, subsequent estimates of relationships between BN trends and/or cycles for different variables will be strongly biased and inconsistent due to the endogeneity that arises from measurement error. Thus, in principle, the two interpretations can be compared using a Hausman (1978) test, with evidence for endogeneity supporting the *BN-as-estimate* interpretation.[3] On the other hand, the practical applicability of an endogeneity test is not entirely obvious. First, any apparent evidence of measurement error based on a particular set of instruments may, in fact, be due to a mistaken exclusion of these variables from the forecasting model used to construct the BN trend and cycle. Thus, one purpose of this section is to demonstrate that it is possible to test for measurement error even if the forecasting model used for the BN decomposition includes an instrument used in the subsequent Hausman test for endogeneity. Second, parameter uncertainty potentially mitigates the ability to use measurement error to test the *BN-as-definition* interpretation. Thus, another purpose of this section is to demonstrate that the Hausman test can still be informative about the two interpretations in finite samples.

Monte Carlo analysis provides an ideal means of illustrating the key issues related to the BN decomposition and measurement error because it allows direct consideration of what would happen when one of the two interpretations is true, but the econometrician does not know which one. For the analysis, I consider two stylized data-generating processes (DGPs) corresponding to the two interpretations of the BN decomposition. For each DGP, I consider two series that are related to each other and examine the ability to empirically detect the true relationship between the series in different circumstances. The DGPs are given as follows:

DGP 1 (Permanent and transitory components are unobservable). *For this DGP, the two underlying series have the same general structure as the model in* (5)*:*

$$y_{it} = \tau_{it} + c_{it}, \tag{7a}$$

$$\tau_{it} = 1 + \tau_{i,t-1} + \eta_{it}, \quad \eta_{it} \sim N(0, 1), \tag{7b}$$

$$c_{it} = 1.25c_{i,t-1} - 0.75c_{i,t-2} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, 0.5^2), \tag{7c}$$

$$\mathrm{Corr}(\eta_{it}, \varepsilon_{it}) = 0, \tag{7d}$$

where $i = 1, 2$. *In this case, the two series are related to each other through the following correlation:*

$$\mathrm{Corr}(\varepsilon_{1t}, \varepsilon_{2t}) = 0.5. \tag{7e}$$

That is, only the transitory innovations are positively correlated across the two series.

DGP 2 (Permanent and transitory components are BN trend/cycle). *For this DGP, the two underlying series have the same general structure as the model in* (6)*:*

$$y_{it} = \tau_{it} + c_{it}, \tag{8a}$$

$$\tau_{it} = 1 + \tau_{i,t-1} + \eta_{it}, \quad \eta_{it} \sim N(0, 1), \tag{8b}$$

$$c_{it} = 1.25c_{i,t-1} - 0.75c_{i,t-2} - 0.2\eta_{it} + 0.3\eta_{i,t-1}, \tag{8c}$$

where $i = 1, 2$. *Meanwhile, the two series are related to each other through the following correlation:*

$$\mathrm{Corr}(\eta_{1t}, \eta_{2t}) = 0.5. \tag{8d}$$

That is, permanent and transitory innovations are positively correlated across the two series.

For each DGP, I replicate a large number of samples of simulated observations and then calculate the BN trends and cycles for each sample. Initially, the BN calculations are made under the assumption of known parameters in order to illustrate the key role of the two interpretations when looking at characteristics of the BN trends and cycles, although I also consider the role of parameter uncertainty below.

Table 1 reports results for Monte Carlo experiments about various estimators of interest assuming known parameter values and given 10,000 replications and sample sizes of 200 and 1,000 observations. Starting with inferences about variation in permanent and transitory components and, for the time being, focusing on the results given BN calculations based on univariate forecasting models, the first thing to notice is that, even when the BN trend is only an estimate, the OLS estimator of the standard deviation of the BN trend innovations appears to provide an unbiased estimator of the standard deviation of the true permanent

TABLE 1. Monte Carlo analysis of estimators

	True value	Sample size	Estimator	Mean (std. dev.) of estimator for univariate BN	Mean (std. dev.) of estimator for multivariate BN
			A. DGP # 1—BN is estimate		
Std. dev. of permanent innovations	1	200	OLS	1.00 (0.05)	1.00 (0.05)
		1,000	OLS	1.00 (0.02)	1.00 (0.02)
Std. dev. of transitory components	2.15	200	OLS	1.71 (0.17)	1.74/1.81 (0.18)
		1,000	OLS	1.72 (0.08)	1.75/1.82 (0.08)
Slope of relationship between transitory components	0.5	200	OLS	0.39 (0.13)	0.57 (0.11)
			IV	0.50 (0.23)	0.50 (0.12)
		1,000	OLS	0.39 (0.06)	0.57 (0.05)
			IV	0.50 (0.10)	0.50 (0.05)
			B. DGP # 2—BN is true value		
Std. dev. of permanent innovations	1	200	OLS	1.00 (0.05)	1.00 (0.05)
		1,000	OLS	1.00 (0.02)	1.00 (0.02)
Std. dev. of transitory components	0.45	200	OLS	0.45 (0.04)	0.45 (0.04)
		1,000	OLS	0.45 (0.02)	0.45 (0.02)
Slope of relationship between transitory components	0.5	200	OLS	0.50 (0.12)	0.50 (0.12)
			IV	0.50 (0.16)	0.05 (0.16)
		1,000	OLS	0.50 (0.05)	0.50 (0.05)
			IV	0.50 (0.07)	0.50 (0.07)

Notes: Each Monte Carlo experiment consists of 10,000 replications. For each experiment, three series were generated for the specified sample sizes, with the first two series generated from either (7) or (8) and the third series generated from (9). Then the BN decompositions were calculated for the first and second series given known parameters. "Univariate BN" and "multivariate BN" denote whether the BN decomposition was calculated given a univariate forecasting model or a multivariate forecasting model that includes all three variables. Experiment results separated by a slash ("/") correspond to different means/standard deviations for the first and the second series.

innovations.[4] Of course, the i.i.d. structure of the trend innovations means that the OLS estimator of the standard deviation is unbiased and consistent when the BN trend is the true permanent component. However, it is interesting to note that the estimator appears to be equally precise for both DGPs. The second thing to notice is that, when the BN cycle is only an estimate, the OLS estimator of its standard deviation appears to provide a biased estimator of the true standard deviation of the transitory component, with the BN cycle understating the variability of the true transitory component. This result presents the first instance of why it matters which interpretation of the BN decomposition is considered in practice. Although the estimated trend behaves like the true permanent component, meaning that inferences about it are robust to interpretation, the estimated cycle does not, meaning that inferences depend on the interpretation, with estimated cycles displaying variation different from that of the true transitory components because of the presence of measurement error.

The next thing to notice is the OLS inferences about the relationship between transitory components for two related series. As with inferences about variation of the transitory components, the OLS estimator appears biased when the BN cycles are estimates rather than true values. This result is a simple example of the classic errors-in-variables problem. For BN calculations based on univariate forecasting models, the measurement error in the estimated cycles produces downward-biased estimates of the true relationship between the transitory components. Meanwhile, not surprisingly, the OLS estimator appears unbiased when the BN cycles are the true transitory components.

The results thus far suggest that the reliability of OLS inferences about the variation of the transitory components and their relationship across series depends crucially on which interpretation of the BN decomposition is appropriate. As mentioned above, models corresponding to the two interpretations have the same in-sample fit, so it might appear to be completely a matter of identification regarding the nature of the permanent and transitory components of a time series process. However, the errors-in-variables problem for the estimated trend and cycle directly implies a way to move beyond the problem of observational equivalence in the univariate setting, although it requires the existence of an additional time series that can serve as an instrument for the transitory component in one of the original two series. For the errors-in-variables problem, a good instrument will be correlated with the transitory component, but uncorrelated with the measurement error in the BN cycle. For the Monte Carlo analysis, I add a third series to each DGP that is stationary and imperfectly correlated with the transitory component of the second series according to the structure

$$y_{3t} = c_{2t} + u_t, \quad u_t \sim N\left(0, \sigma_u^2\right), \tag{9}$$

where the variance of the error u_t is calibrated to produce a correlation of 50% between the instrument y_{3t} and the transitory component c_{2t} (the variance is different for the two DGPs).

The results for the IV inferences suggest that, given a good instrument, IV estimation of the relationship between transitory components works well. The IV estimator appears unbiased, even for the DGP in which the permanent and transitory components are unobservable. Thus, in principle, it should be possible to compare OLS and IV inferences in order to determine which interpretation of the BN decomposition is more appropriate for a given set of time series. If the OLS and IV inferences are significantly different, it supports the *BN-as-estimate* interpretation. If the OLS and IV inferences are essentially the same, with the

IV estimates only being somewhat less precise, it supports the *BN-as-definition* interpretation.

The results above do not merely reflect implicitly different DGPs when a third variable is introduced into the system via IV analysis. Even given BN decompositions based on multivariate forecasting models that take into account the true joint autocovariance structure of all three series, the Monte Carlo results display the same overall pattern as for the univariate forecasting models.[5] To be sure, there are some quantitative differences in the findings for the multivariate case when the BN trends and cycles are estimates. The apparent bias in the inferences about the standard deviation of the transitory components is smaller than in the univariate case, especially for the transitory component of the second series, which is estimated more precisely because of the information in the third series. There is also less apparent bias in the OLS inferences about the relationship between the transitory components, with the direction of the bias switching from before.[6] However, the important result is that there is still a difference between the OLS and IV inferences when the BN trends and cycles are estimates, with only the IV estimator appearing to be unbiased.

The difference between OLS and IV inferences is the key because, in practice, whether or not there is a change in certain inferences (e.g., the variability of the cycle) as a result of switching between univariate and multivariate forecasting models is not sufficient to distinguish between the two interpretations of the BN decomposition. It only raises possible doubts about the relevance of the univariate forecasting model (or, on the contrary, whether the multivariate model is overfitting the data). For example, for the DGP where the BN trends and cycles are only estimates, the multivariate BN decomposition is different from the univariate BN decomposition because the third series Granger-causes the second series (i.e., it contains marginal predictive information) and it requires a multivariate model to fully capture the joint autocovariance structure of the three series. However, the measurement error was still evident when the less efficient univariate models were considered. Meanwhile, for the DGP where the BN trends and cycles are the true permanent and transitory components, the multivariate BN decomposition is actually the same as the univariate BN decomposition because there is no marginal predictive information in the third series, meaning that the univariate model is sufficient for summarizing the autocovariance structures of the first and second series. However, as discussed in Section 2, it would always be possible to consider a DGP for which observable shocks to the permanent and transitory components are proportional to the forecast errors from a multivariate model. Thus, although consideration of univariate versus multivariate models is important for capturing the autocovariance structure of the time series under examination, a test of one model versus another has no direct implications for the correct interpretation of the BN decomposition.

Up to this point, the Monte Carlo analysis has abstracted from parameter uncertainty in order to illustrate that the two interpretations of the BN decomposition are no longer observationally equivalent in a multivariate setting. However, the

TABLE 2. Monte Carlo analysis of rejection frequencies for the Hausman test

Sample size	Known parameters	Estimated parameters
	A. DGP # 1: BN is estimate	
100	11%	12%
200	13%	14%
500	23%	24%
1,000	44%	44%
	B. DGP # 2: BN is true value	
100	5%	14%
200	5%	13%
500	5%	11%
1,000	5%	8%

Notes: Each Monte Carlo experiment consists of 1,000 replications. For each experiment, three series were generated for the specified sample sizes, with the first two series generated from either (7) or (8) and the third series generated from (9). Then the BN decompositions were calculated for the first and second series based on known and estimated parameters, respectively. The rejection frequencies correspond to the fraction of Monte Carlo replications for which the Hausman test rejects based on a 5% critical value.

practical relevance of using a Hausman test to distinguish between the two interpretations might seem questionable given that there will always be some measurement error in finite samples due to model and parameter uncertainty. I address the effects of parameter uncertainty by repeating the above Monte Carlo analysis, but estimating parameters prior to calculating the BN decompositions.[7]

Table 2 reports results for Monte Carlo experiments about the Hausman test assuming unknown parameter values and given 1,000 replications and sample sizes of 100, 200, 500, and 1,000 observations. I focus on results for BN calculations based on univariate forecasting models and calculate the rejection frequencies for the Hausman test with a 5% level, comparing them to the rejection frequencies for the same simulated data given known parameter values.[8] First, under the *BN-as-estimate* interpretation, the rejection frequency increases with the sample size, corresponding to increasing power of the test. For the smaller sample sizes, the rejection frequency is slightly higher when parameter uncertainty is taken into account. However, it is striking that most of the Hausman test's power appears to arise from measurement error due to the fact that the transitory components are unobserved rather than parameter uncertainty. Second, under the *BN-as-definition* interpretation, parameter uncertainty generates somewhat higher rejection frequencies than the 5% level of the test. However, in contrast to the *BN-as-estimate* case, the rejection frequencies become smaller as the sample size increases. Thus, the two interpretations are still distinguishable in the sense that, as the sample size increases, the evidence for measurement error should typically get stronger under the *BN-as-estimate* interpretation, whereas it should typically get weaker under the *BN-as-definition* interpretation.

5. APPLICATION TO MACROECONOMIC DATA

Given that it is possible to distinguish empirically between the two interpretations of the BN decomposition, an immediate question is which interpretation is more appropriate in practice when dealing with macroeconomic data. In this section, I consider an application of the OLS and IV analysis in the preceding section to U.S. real GDP and the Industrial Production Index, with capacity utilization for manufacturing serving as an instrument. The raw data were obtained from the St. Louis Fed database for the sample period of 1972Q1 to 2009Q3 and converted into 100 times natural logarithms.[9]

As a first step in motivating the analysis, I test for the presence of stochastic trends in the data series using the basic augmented Dickey–Fuller unit root test with lag selection based on the Schwarz information criterion (BIC). At the 5% significance level, I am unable to reject the null of a unit root against the trend stationary alternative for either log real GDP ($t = -2.10$ for one lag of difference terms, with a p-value of 0.54) or log industrial production ($t = -2.89$ for one lag of difference terms, with a p-value of 0.17). For log capacity utilization, I am able to reject the null of a unit root in favor of a level stationary alternative ($t = -3.68$ for one lag of difference terms, with a p-value of <0.01). Thus, I consider applying the BN decomposition to the real GDP and industrial production series, but not to the capacity utilization series. At the same time, the fact that capacity utilization appears to be stationary suggests that it could be a good instrument for the transitory component of industrial production.

The main practical issue in applying the BN decomposition is determining which forecasting model to use. In this paper, I attempt to remain agnostic about which model is most appropriate for the variables under consideration and check the robustness of my findings to different modeling assumptions. The most important difference between models is in terms of their implied long-horizon predictability of real economic activity. For example, estimates for low-order ARMA models of U.S. output growth imply positive serial correlation at short horizons, but little long-horizon predictability. Meanwhile, estimates for higher-order ARMA models and some multivariate models imply a large degree of negative serial correlation at long horizons. In terms of the BN decomposition, these two competing views about long-horizon predictability produce very different looking cycles. Thus, I consider models that accommodate both of these views.

I first consider AR(12) models for the growth rates of real GDP and industrial production.[10] Figure 1 displays the implied BN cycles, along with capacity utilization. These BN cycles reflect negative serial correlation in the growth rates at long horizons and display a strong correspondence to the NBER reference cycle. The two BN cycles also appear to be related to each other. This relationship is not particularly surprising, given the large role manufacturing plays in overall economic activity. However, it is an interesting empirical question how closely the manufacturing sector moves with the overall economy. To quantify the relationship, I regress the real GDP BN cycle on the industrial production BN cycle using

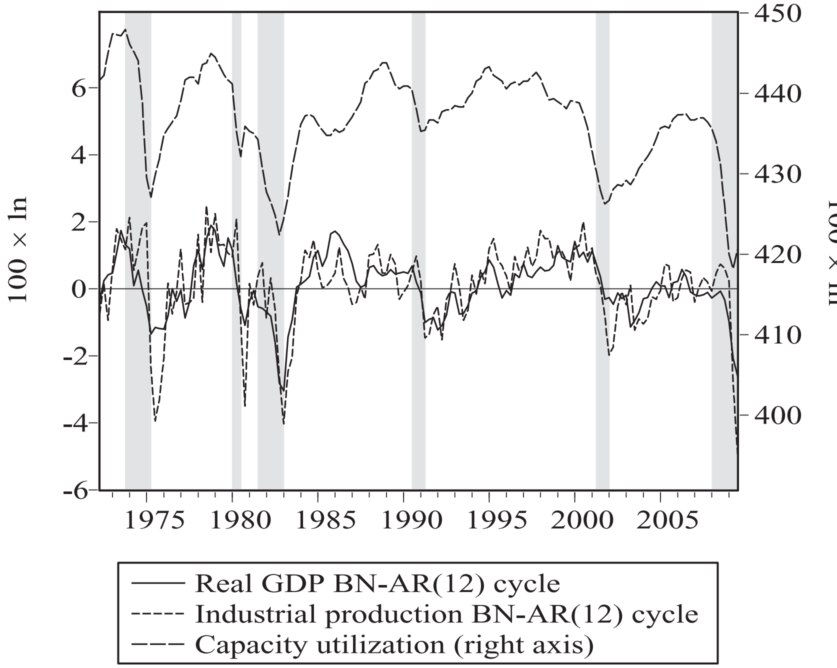

FIGURE 1. Beveridge–Nelson cycles for U.S. real GDP and industrial production based on AR(12) models and capacity utilization (NBER recessions shaded).

OLS:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.08)}{0.04} + \underset{(0.03)}{0.53}\,(ip_t - \mathrm{BN}_{ip,t}) + e_t, \quad R^2 = 0.58. \qquad (\mathbf{10})$$

Given the standard errors reported in parentheses, the estimates suggest a strong relationship, but it is clearly less than one-for-one.[11]

The main question raised in this paper is whether estimates such as those in (10) are reliable, even assuming that the forecasting models used in the BN decompositions are reasonable approximations of the true autocovariance structures of the time series under examination. In particular, if the BN trend is an estimate for these series, then the true transitory components will be measured with error and the OLS estimate of their relationship will be biased and inconsistent. To examine this issue, I consider capacity utilization as an instrument for the transitory component of industrial production. Economic considerations suggest that capacity utilization should be strongly correlated with the true transitory component, but presumably it should be unrelated to any measurement error in the BN cycle. Meanwhile, if the BN cycle is the true transitory component, IV estimates should not be significantly different from the OLS estimates reported in (10).[12] Figure 1 suggests that there is a strong relationship between the transitory component in industrial production and the capacity utilization series. Indeed, the sample correlation between the BN cycle for industrial production and capacity utilization is 58%. Thus, there is little concern about a weak instrument.

The IV regression results for the BN cycles based on AR(12) models and capacity utilization as an instrument are given as follows:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.09)}{0.01} + \underset{(0.09)}{0.78}\,(ip_t - \mathrm{BN}_{ip,t}) - \underset{(0.10)}{0.37}\,\hat{u}_t + e_t, \quad R^2 = 0.65, \qquad (\mathbf{11})$$

where $\hat{u}_t = ip_t - \mathrm{BN}_{ip,t} + 54.74_{(9.34)} - 0.13_{(0.02)}\mathrm{capu}_t$ is the residual based on a first-stage regression of the BN cycle for industrial production on the capacity utilization series. The first thing to notice is the coefficient on $\hat{u}_t$. A t-test for this coefficient is equivalent to the Hausman test for endogeneity. Thus, the t-statistic of -3.85 corresponds to a strong rejection of the null hypothesis of no endogeneity at the 1% level, which suggests that measurement error is a problem for the BN cycles. That is, this test supports the *BN-as-estimate* interpretation. The second thing to notice is that the measurement error has meaningful and predictable effects on estimates of relationship between the transitory components. In particular, the larger coefficient on the BN cycle for industrial production in (11) than in (10) suggests a stronger relationship between the true transitory components than between the BN cycles.

As discussed in the preceding section, evidence of measurement error could simply reflect sampling variability for the parameters used to construct the BN decomposition. However, if this is the case, the evidence should weaken as the sample size increases and the parameter estimates become more precise. Figure 2 plots the expanding-sample Hausman test statistics for the BN cycles based on AR(12) models and the sample periods of 1972Q2–1983Q4 through 1972Q2–2009Q3. It is clear from the figure that the evidence generally strengthens as the sample size increases. Furthermore, the changes in the t-statistics appear to reflect information from variation in the BN cycles and capacity utilization (see Figure 1) rather than parameter uncertainty.[13]

Typically, model selection procedures favor low-order ARMA models for growth rates of real economic activity. Thus, to check on the robustness of the above results, I also consider AR(2) models for the growth rates of real GDP and industrial production.[14] Figure 3 displays the implied BN cycles, along with the change in the capacity utilization series. These BN cycles display much less persistence and amplitude than those in Figure 1. Also, consistent with the findings in the original paper by Beveridge and Nelson (1981), the cycles, when defined as the difference between the series and the BN trend, are typically positive during NBER recessions, reflecting the positive momentum structure implicit in the AR(2) models. In this case, the OLS results for the relationship between cycles are given as follows:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.05)}{0.00} + \underset{(0.03)}{0.41}(ip_t - \mathrm{BN}_{ip,t}) + e_t, \quad R^2 = 0.57. \qquad (\mathbf{12})$$

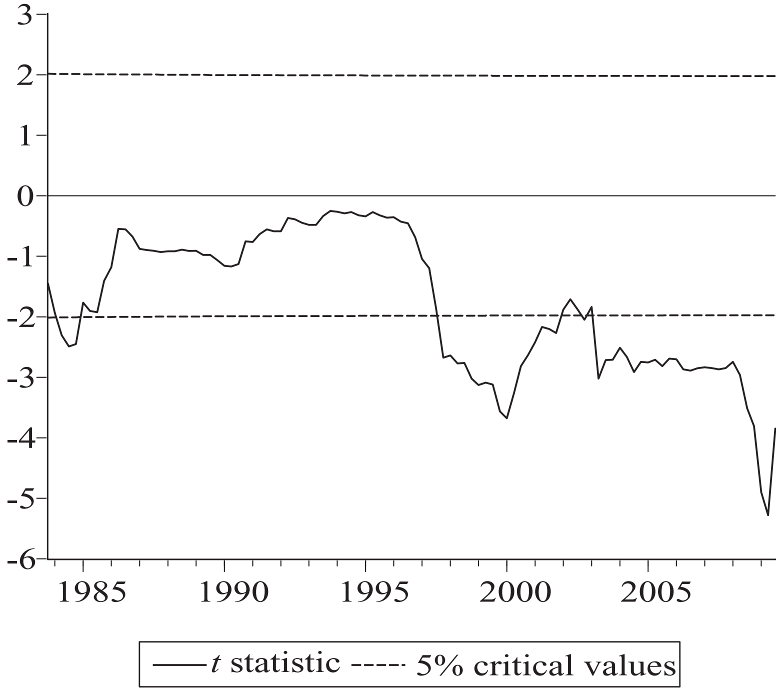

FIGURE 2. Expanding-sample Hausman test statistics from 1972Q2–1983Q4 to 1972Q2–2009Q3 for cycles based on AR(12) models (*x*-axis denotes end of sample).

As before, the results suggest a positive relationship between the transitory components and the relationship is far below one-for-one.

The IV results for the cycles based on AR(2) models and using the capacity utilization series as an instrument are given as follows:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.05)}{0.00} + \underset{(0.13)}{0.65}(ip_t - \mathrm{BN}_{ip,t}) - \underset{(0.13)}{0.25}\,\hat{u}_t + e_t, \quad R^2 = 0.59, \qquad (\mathbf{13})$$

where $\hat{u}_t = ip_t - \mathrm{BN}_{ip}, t - 29.85_{(11.37)} + 0.07_{(0.03)}\mathrm{capu}_t$ is the residual based on a first-stage regression of the BN cycle for industrial production on the capacity utilization series. The *t*-statistic of -1.95 for the Hausman test is significant at the 10% level, although it is just shy of the 5% level. As in the AR(12) case, the IV estimates imply a stronger relationship between the transitory components than do the OLS estimates.

Figure 4 plots the expanding-sample Hausman test statistics for the BN cycles based on AR(2) models and the sample periods of 1972Q2–1983Q4 through 1972Q2–2009Q3. Contrary to the idea that the evidence of measurement error is driven by parameter uncertainty, the test statistics are insignificant for smaller subsamples and only become significant when the sample is extended to include

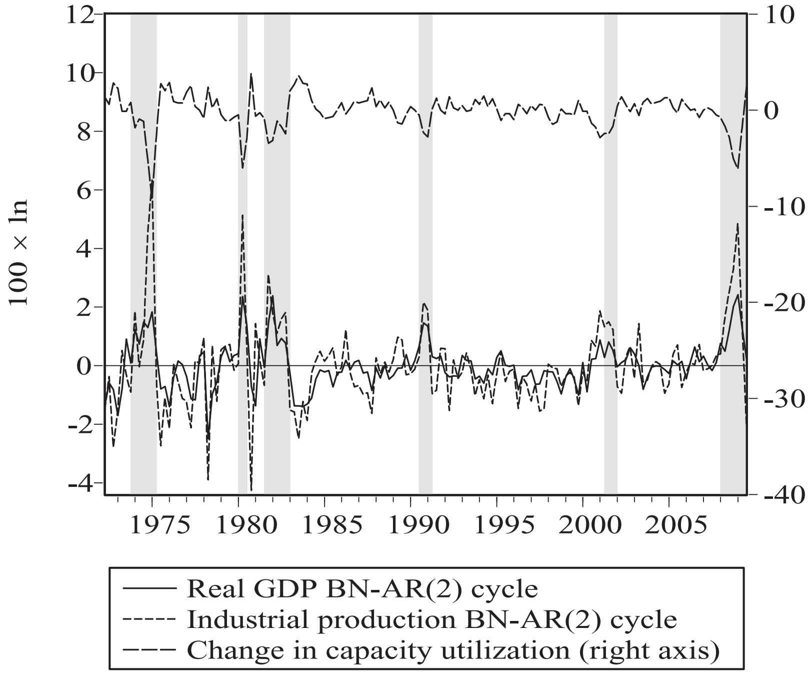

FIGURE 3. Beveridge–Nelson cycles for U.S. real GDP and industrial production based on AR(2) models and the change in capacity utilization (NBER recessions shaded).

the last couple of years of data. As in the AR(12) case, the changes in the t-statistics appear to reflect information from variation in the BN cycles and capacity utilization (see Figure 3), rather than parameter uncertainty.[15]

One possible concern is that capacity utilization may be a weak instrument for the transitory component in industrial production. Its sample correlation with the BN cycle based on the AR(2) model is only -14%. Meanwhile, if the AR(2) model is more appropriate for the growth rate of industrial production than an AR(12) model, it may not make sense to think of the capacity utilization series as stationary, despite the unit root test result reported above. From Figure 3, it is clear that the change in the capacity utilization series is stationary and highly (negatively) correlated with the BN cycle for industrial production. The sample correlation is -93%. Thus, I also consider the change in the capacity utilization series as an instrument, with the following results:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.05)}{0.00} + \underset{(0.03)}{0.46}(ip_t - \mathrm{BN}_{ip,t}) - \underset{(0.08)}{0.32}\,\hat{u}_t + e_t, \quad R^2 = 0.62, \qquad (\mathbf{14})$$

where $\hat{u}_t = ip_t - \mathrm{BN}_{ip,t} + 0.01_{(0.06)} + 0.73_{(0.03)}\,\Delta\mathrm{capu}_t$. In this case, the t-statistic of -3.89 for the Hausman test is significant at the 1% level. As before, accounting for endogeneity produces a larger estimate of the relationship between transitory

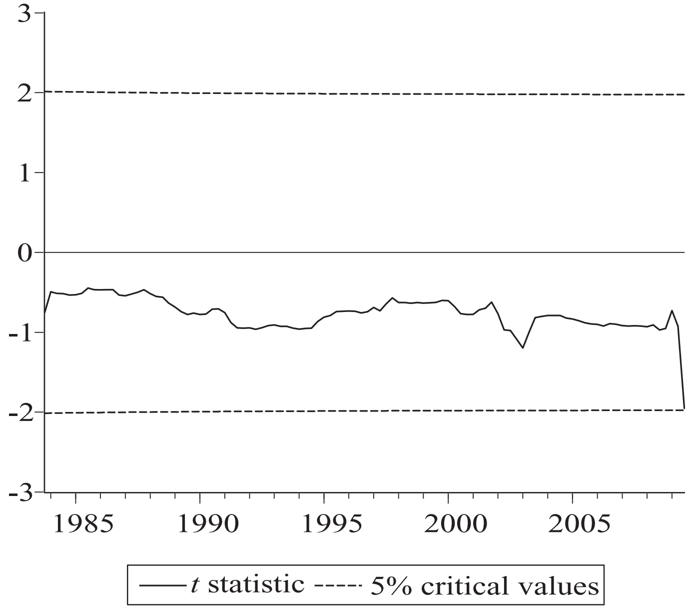

FIGURE 4. Expanding-sample Hausman test statistics from 1972Q2–1983Q4 to 1972Q2–2009Q3 for cycles based on AR(2) models (x-axis denotes end of sample).

components than OLS, although the magnitude of the increase is not as large in this case.

Another possible concern is that the univariate results are driven by the omission of useful multivariate forecasting information, including that from the instrument, in calculating the BN cycles. Thus, I also consider a vector autoregressive (VAR) model that includes information from real GDP, industrial production, and capacity utilization. Specifically, I consider a VAR(2) model for the growth rates for real GDP, industrial production, and log capacity utilization.[16] The OLS regression results for the relationship between the cycles are given as follows:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.05)}{0.00} + \underset{(0.01)}{0.52}(ip_t - \mathrm{BN}_{ip,t}) + e_t, \quad R^2 = 0.94. \qquad (\mathbf{15})$$

As before, the estimated relationship is positive, but clearly less than one-for-one.

Given that the VAR(2) model nests the univariate AR(2) models and includes capacity utilization, the main question is whether there is still any evidence for endogeneity using the BN cycles based on the VAR(2) model and using capacity utilization as an instrument. The IV regression results are given as follows:

$$y_t - \mathrm{BN}_{y,t} = \underset{(0.02)}{0.01} + \underset{(0.01)}{0.58}(ip_t - \mathrm{BN}_{ip,t}) - \underset{(0.01)}{0.29}\,\hat{u}_t + e_t, \quad R^2 = 0.99, \qquad (\mathbf{16})$$

where $\hat{u}_t = ip_t - \mathrm{BN}_{ip,t} + 167.58_{(11.47)} + 0.38_{(0.03)}\mathrm{capu}_t$. The increase in the estimated relationship is less than that in the univariate case. However, the t-statistic for the Hausman test of endogeneity is a huge -31.31, which is significant at much better than the 1% level. Thus, consistent with the Monte Carlo analysis in the preceding section, the exclusion of forecasting information from the instrument does not appear to explain univariate evidence of measurement error. Instead, inclusion of this information mostly appears to increase the power to detect the measurement error.

As a last robustness check, I consider whether the inclusion of information inherent in the capacity utilization series is actually identifying a structural relationship, instead of accounting for measurement error in estimating a reduced-form relationship between transitory components. To examine this possibility, I consider the reduced-form relationship between the growth rates of real GDP and industrial production.[17] The OLS results for this relationship are given as follows:

$$\Delta y_t = \underset{(0.04)}{0.48} + \underset{(0.03)}{0.42}\,\Delta ip_t + e_t, \quad R^2 = 0.62. \tag{17}$$

Because the growth rates behave similarly to the BN cycles for the AR(2) case, I consider the change in the capacity utilization series as an instrument (the correlation with industrial production growth is 94%) and obtain the following results for IV estimation:

$$\Delta y_t = \underset{(0.04)}{0.48} + \underset{(0.03)}{0.43}\,\Delta ip_t - \underset{(0.08)}{0.05}\,\hat{u}_t + e_t, \quad R^2 = 0.62, \tag{18}$$

where $\hat{u}_t = \Delta ip_t - 0.62_{(0.12)} - 0.82_{(0.02)}\Delta\mathrm{capu}_t$. In this case, the t-statistic of -0.62 for the Hausman test means that I cannot reject the null of no endogeneity at even the 10% level. This result is telling because, if the rejection in (14) for BN cycles based on AR(2) models and using the change in capacity utilization as an instrument were driven by structural identification instead of measurement error, this would presumably show up as a rejection in (18), given that the regressions are very similar in every dimension, except that there is presumably much less measurement error in the real GDP and industrial production data than in their BN cycles.

6. CONCLUSIONS

Given a particular autocovariance structure for the growth rate of an integrated time series, there can be multiple state-space representations for the permanent and transitory components of the level of the series.[18] In one case, the time series is subject to permanent and transitory innovations with imperfect correlation and the permanent and transitory components are unobservable. In another case, the time series is subject to observable shocks only, directly implying that the permanent and transitory components are observable. In a univariate setting, which representation should be assumed is a matter of identification. However, although both representations correspond to identical inferences about the variability of the permanent component, they have very different implications in terms of the uncertainty about the measure of the permanent component. Meanwhile, the possibility of measurement error suggests that the two interpretations are testable in a multivariate setting. In particular, Monte Carlo analysis suggests that instrumental variables analysis can be used to detect the presence or absence of errors in variables, even if the instrument is included in the forecasting model used to calculate the Beveridge–Nelson decomposition. An application of the instrumental variables analysis to U.S. real GDP, industrial production, and capacity utilization in manufacturing provides support for the practical relevance of the *BN-as-estimate* interpretation for macroeconomic data.

NOTES

1. The notion that the permanent and transitory components for the state-space representation are either unobservable or observable can be related to the formal literature on "observability" and state-space models [see, for example, Harvey (1989, pp. 113–115)]. In particular, when (2) is cast into the standard state-space form used in control engineering, the transition and loading matrices under both interpretations satisfy the necessary and sufficient rank conditions for observability. However, the values of the "control vector" (i.e., the shocks to the state variables) are unknown to the econometrician under the first interpretation, but known under the second interpretation (because they are proportional to the forecast error from the reduced-form ARMA representation).

2. Proietti (2006) and Oh et al. (2008) also consider the observational equivalence between different state-space models related to a reduced-form model ARMA(2,2) for the first differences. Oh et al. specifically discuss how the reduced-form model is consistent with a continuum of state-space models with ARMA(2,1) transitory dynamics and imperfect correlation. However, the two extreme cases from that continuum considered here are of particular interest because the model in Morley et al. (2003) is fundamentally distinct from the other state-space models in terms of what, in principle, should be a testable assumption about the dynamic structure of the transitory component, whereas the model with perfect correlation is the only one that is consistent with the *BN-as-definition* interpretation.

3. In the related setting of trend/cycle decomposition based on UC models, Watson (1986) compared ordinary least squares (OLS) and instrumental variable (IV) estimates for a test of the permanent income hypothesis using filtered and smoothed estimates of transitory components from UC models, with lagged data serving as instruments. He found that filtered estimates, which condition on data up to and including the period in which the inferences about trend and cycle are being made, were subject to measurement error in the sense that they were different from smoothed estimates, which condition on the full available sample of data. Thus, given the link between the BN decomposition and filtered estimates discussed in Morley et al. (2003), there is a direct suggestion from Watson's analysis that the *BN-as-estimate* interpretation is more appropriate in terms of thinking about "permanent income" as a structural quantity that has a relationship with other macroeconomic variables.

4. For the Monte Carlo analysis with the univariate BN decompositions, the BN trend and cycle are calculated by directly applying the Kalman filter to separate state-space models for each series in (7) and (8). Given the DGPs, this approach is equivalent to solving for the reduced-form forecasting model for each series, casting it into state-space form, and applying the state-space approach to the BN decomposition presented in Morley (2002) to obtain the corresponding BN trend and cycle. Also, because the two series for each DGP have the same variance parameters, I make no distinction between the two series for a given DGP when discussing inferences about variation in their permanent and transitory components.

5. For the Monte Carlo analysis with the multivariate BN decompositions, the BN trend and cycle are calculated by applying the Kalman filter to a multivariate state-space model that incorporates all three series for each DGP. Again, this approach is equivalent to solving for the reduced-form multivariate forecasting model, casting it into state-space form, and applying the state-space approach to the BN decomposition.

6. The change in the direction of the bias reflects taking the correlation between transitory components into account when calculating the BN decompositions by applying the Kalman filter to a multivariate state-space model. Specifically, the filtered inferences for the transitory components have a stronger correlation than the true correlation between the transitory components. This is analogous to the perfect negative correlation between filtered inferences about permanent and transitory innovations for a UC model, even when the true innovations are imperfectly correlated [see Morley et al. (2003)].

7. Model uncertainty is related to parameter uncertainty as long as an econometrician considers a broad enough class of models to nest the true DGP and follows consistent testing and estimation procedures.

8. The focus on univariate models and the consideration of 1,000 Monte Carlo replications per experiment reflect the computational burden of maximum-likelihood estimation of the state-space models via numerical optimization given many different initial values for the parameters.

9. The sample period is determined by the availability of the capacity utilization series. The industrial production series is monthly and is converted to a quarterly series by taking averages. Growth rates are measured using first-differences of log values.

10. For the AR(12) models, the 12th lag is significant at the 1% level for real GDP and at the 5% level for industrial production. Also, for both series, the AR(12) model is chosen by the Akaike information criterion (AIC) among AR models up to AR(14) for the common adjusted evaluation sample period of 1975Q4–2009Q3.

11. Because of some possible autoregressive correlation in the residuals, the standard errors are based on a first-order Cochrane–Orcutt correction throughout this section, although the OLS standard errors are generally similar.

12. There are two other possible sources of endogeneity in a regression of one transitory component on the other: omitted variables and simultaneity. However, these sources have to do with a failure to identify a structural relationship in which one transitory component causes the other. In contrast, I am interested in measuring the reduced-form correlation between the transitory components rather than any structural relationships. Capacity utilization is a good instrument for this purpose, not only because it should be uncorrelated with measurement error in the BN cycles, but also because it would not be a good instrument or omitted variable for identifying a structural relationship between the two transitory components. In particular, it should be subject to all of the same structural shocks as the two transitory components. I provide a test of this assumption below.

13. There is little evidence of parameter instability for the AR(12) models used to calculate the BN cycles. For both series, standard tests for a structural break with an unknown break date [e.g., Andrews (1993) and Andrews and Ploberger (1994)] are highly insignificant for a break in all of the AR(12) model parameters. Meanwhile, in terms of individual parameters, out of 26 parameters for the two series, only the AR(1) coefficient for industrial production growth tested positively for a structural break with an unknown break at better than a 10% level. In this case, the estimated break date that maximized the "sup"-versions of test statistics was 1982Q1, which does not correspond to the timing of the increasing evidence of measurement error. Of course, a test rejection for only one of 26 parameters at the 10% level does not really suggest there was any parameter instability in the first place.

14. For the AR(2) models, the second lag is close to, but not quite significant at the 10% level for real GDP and at the 5% level for industrial production. For both series, an AR(1) model is chosen by BIC among AR models up to AR(14) for the common adjusted evaluation sample period of 1975Q4–2009Q3. However, the AR(2) model is a close second in both cases and inferences about the BN cycles are very similar whether an AR(1) or AR(2) model is considered. Given the borderline evidence for the significance of the second lags and a preference for parameter consistency over efficiency, the results for AR(2) models are reported.

15. Again, there is little evidence of parameter instability for the AR(2) models used to calculate the BN cycles. Indeed, for both series, standard tests for a structural break with an unknown break date in the AR(2) model parameters, including in individual parameters, are all insignificant.

16. For the VAR(2) model, the second lag of real GDP growth is significant at the 1% level for industrial production growth and vice versa. The VAR(2) model is chosen by both AIC and BIC among VAR models up to VAR(14) for the common adjusted evaluation sample period of 1975Q4–2009Q3.

Also, the results are generally robust, including for the Hausman test, to the consideration of the change in the capacity utilization series instead of the level.

17. The relationship between the growth rates mixes the underlying relationships between changes in permanent components and changes in transitory components. However, as long as there is a stable relationship between the relative importance of permanent and transitory components, there should be a stable relationship between the growth rates.

18. Beyond the two representations considered here, there can be additional representations if more general specifications for the permanent component are considered. See Blanchard and Quah (1989), Quah (1992), and Proietti (1995, 2006).

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