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Reproducing Business Cycle Features: Are Nonlinear Dynamics a Proxy for Multivariate Information?

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Reproducing Business Cycle Features: Are Nonlinear Dynamics a Proxy for Multivariate Information?

James Morley, Jeremy Piger, and Pao-Lin Tien

Abstract

We consider the extent to which different time-series models can generate simulated data with the same business cycle features that are evident in U.S. real GDP. We focus our analysis on whether multivariate linear models can improve on the previously documented failure of univariate linear models to replicate certain key business cycle features. We find that a particular nonlinear Markov-switching specification with an explicit "bounceback" effect continues to outperform linear models, even when the models incorporate variables such as the unemployment rate, inflation, interest rates, and the components of GDP. These results are robust to simulated data generated either using Normal disturbances or bootstrapped disturbances, as well as to allowing for a one-time structural break in the variance of shocks to real GDP growth.

1. Introduction

A large literature in macroeconomics focuses on the development and evaluation of nonlinear time-series models to describe U.S. real gross domestic product (GDP). Much of this literature builds on the threshold model of Tong (1983) or the Markov-switching model of Hamilton (1989), which are extensions of linear ARIMA models to allow for regime-switching parameters. Of crucial interest in the literature is the statistical evidence for nonlinear dynamics in univariate models of real GDP. One popular approach to addressing this issue is to test the statistical significance of the nonlinear model against its nested linear counterpart. Such tests, which are complicated by non-standard asymptotic distributions for test statistics, have yielded somewhat mixed conclusions that depend primarily on the particular nonlinear model being evaluated.[1]

An alternative approach to assessing the statistical evidence for nonlinear dynamics in real GDP is to evaluate the ability of linear and nonlinear models to produce simulated data that display certain business cycle features evident in actual real GDP data.[2] This "features" approach is well suited for comparing nonlinear vs. linear models for at least two reasons. First, the relevant comparison is often between non-nested models, such as univariate nonlinear models and multivariate linear models. This consideration of non-nested models greatly complicates the use of hypothesis tests, but poses no particular problem for considering the ability of a model to replicate features. Second, the features approach allows researchers to concentrate model comparison on features of the data that are directly related to business cycles. This provides a natural way to assess the benefits of introducing nonlinearity into time-series models of GDP, since many of the nonlinearities explored for GDP have been motivated as being related to the business cycle.

The features approach to assessing the importance of nonlinear dynamics for real GDP has been taken in a number of previous studies, including Hess and Iwata (1997), Harding and Pagan (2002), Galvão (2002), Clements and Krolzig (2004), and Morley and Piger (2006) for U.S. data, and Demers and Macdonald (2007) for Canadian data. For a range of linear and nonlinear models, Hess and Iwata (1997), Harding and Pagan (2002), and Clements and Krolzig (2004) find that simple linear ARIMA(1,1,0) or ARIMA(2,1,0) models reproduce business cycle features of actual real GDP just as well as, if not better than, their more

[1] For example, Garcia (1998) and Hansen (1992) are unable to reject a nested linear model in favor of Hamilton's original model of real GNP growth. However, Hansen (1992), Kim, Morley and Piger (2005), and Morley and Piger (2012) reject linearity in favor of extended versions of Hamilton's model.

[2] This approach can be viewed as related to a broader approach to model comparison and evaluation based on encompassing tests (see, for example, Breunig, Najarian, and Pagan, 2003).

complicated counterparts. Following the principle of parsimony, all three studies draw the conclusion that researchers should pick the simpler linear models over more complicated models. However, Galvão (2002), Morley and Piger (2006), and Demers and Macdonald (2007) find that there are some important features that certain nonlinear models are substantially better able to replicate than linear models, while there are no features for which linear models dominate. These studies conclude that certain nonlinear models provide an improvement for modeling business cycle features over linear models.

This existing literature on business cycle features has largely focused on comparing univariate linear models to univariate nonlinear models, while generally ignoring multivariate linear models that include other macroeconomic variables, such as the unemployment rate, inflation, interest rates, and the components of GDP, that are widely-believed to help explain important dynamics in real GDP.[3] The omission of multivariate models is important because, if the true data generating process (DGP) is a multivariate linear process, the apparent nonlinearity suggested by univariate models could simply be proxying for omitted variables. As evidence of this possibility, it is helpful to consider two simple simulation experiments. In the first, the true DGP for real GDP is taken to be the linear vector error correction model (VECM) of output, consumption, and investment in King, Plosser, Stock, and Watson (1991), while in the second it is taken to be a linear AR(2) model of output growth. In both cases the simulations are based on parameter estimates from fitting the models to U.S data, as discussed in more detail in Section 4. For each of 100 simulated real GDP series from these two DGPs, we compute the likelihood ratio (LR) statistic comparing the null hypothesis of a univariate linear AR(2) model to the alternative hypothesis of the univariate Markov-switching model presented in Morley and Piger (2005), which has found some support in the existing business cycle features literature. We find that the resulting test statistic when the DGP is multivariate is, on average, more than four times as large as when the DGP is univariate, which suggests that the univariate nonlinear model is proxying for omitted information from the multivariate linear DGP that is missed by the univariate linear model. This result

[3] Of the studies mentioned above, only Clements and Krolzig (2004) have systematically compared univariate models against multivariate models. Our analysis here differs from Clements and Krolzig (2004) in three important ways. First, we consider a different set of business cycle features, including a feature that captures the relationship between the severity of recession and robustness of recoveries. This is a feature that a large class of nonlinear models were designed to capture, and thus is important for the evaluation of the value added of nonlinear models. Second, Clements and Krolzig (2004) use an algorithm to date business cycle episodes in their simulated data that does not impose a minimum length requirement for business cycle phases, an important element of algorithms that are successful at matching the National Bureau of Economic Research's business cycle dates. Third, we allow for greater flexibility in terms of the distribution of disturbances for data simulation purposes than considered in the previous literature.

raises possible doubts about why certain univariate nonlinear models have outperformed univariate linear models in the previous literature, as such superior performance may simply have been a reflection of omitted multivariate linear dynamics.

In this paper, we directly address the question of what drives the previous results for univariate time series models by evaluating the relative abilities of univariate linear, multivariate linear, and univariate nonlinear models to simulate data that display the same business cycle features found in actual U.S. real GDP. Including multivariate linear models in the evaluation allows us to assess whether nonlinear dynamics are truly inherent in U.S. GDP or are simply a proxy for unmodeled multivariate dynamics. To circumvent problems with non-nested models, we employ the business cycle features approach, allowing us to compare the preferred univariate linear and nonlinear models in Morley and Piger (2006) with three popular multivariate linear models: the two-variable (VAR) model of Blanchard and Quah (1989); the four-variable VAR model in Ahmed, Levin, and Wilson (2004); and the three-variable vector error correction model (VECM) in King, Plosser, Stock, and Watson (1991) used in the simulation above.

Contrary to the idea that univariate nonlinear dynamics are a proxy for multivariate linear dynamics, we find that the multivariate linear models do not reproduce business cycle features as well as a univariate nonlinear model that incorporates an explicit "bounceback" effect in which the strength of economic recovery is allowed to depend on the severity of the preceding recession. Our results are robust to allowing for a structural break in the variance of U.S. real GDP growth corresponding to the so-called "Great Moderation" in 1984. We also find no benefit in terms of reproducing business cycle features when considering simulated data based on bootstrapped disturbances instead of drawing from a Normal distribution, suggesting that nonlinear models are doing more than simply capturing fat tails or skewness in the unconditional distribution of output growth.

The remainder of this paper proceeds as follows: Section 2 details the algorithm used to measure the business cycle in U.S. real GDP and in simulated data, and defines the business cycle features that we consider, while Section 3 documents these features for U.S. real GDP. Section 4 specifies the time-series models under consideration and evaluates the ability of the competing univariate and multivariate models to reproduce the business cycle features exhibited by U.S. real GDP. Section 5 concludes.

2. Definition of Business Cycle Features

2.1 Business Cycle Dating Algorithm

The business cycle features approach to model comparison requires a measure of the business cycle for both actual and simulated data. In the business cycle features literature, the business cycle is defined as the *classical cycle* (or *reference cycle*) as described by Burns and Mitchell (1946) rather than the *cyclical component* of a series obtained after detrending, although the two concepts may be closely related (see Morley and Piger, 2012). The classical cycle defines the business cycle as a series of distinct phases in economic activity, with the phases corresponding to recession and expansion and the turning points between phases indicated as peaks and troughs. The de facto business cycle peak and trough dates in the United States are determined by the National Bureau of Economic Research (NBER) Business Cycle Dating Committee, which reviews a variety of economic statistics and indicators of U.S. business conditions before dating turning points in the economy.

The NBER business cycle dates are widely used in economic research requiring business cycle peak and trough dates, and it seems natural to use them for calculating business cycle features in actual U.S. real GDP data. To establish turning points in data simulated from models, we then require a formal procedure capable of mimicking the NBER decision-making process. The standard algorithm to establish business cycle turning points in the literature is the Bry-Boschan Quarterly (BBQ) algorithm developed by Harding and Pagan (2002), which is a quarterly version of the Bry and Boschan (1971) algorithm. The specifics of the BBQ algorithm can be summarized as follows:

Step 1: Using the log level of U.S. quarterly real GDP (y_t), establish candidate dates of peaks and troughs as local maxima and minima in the data such that a peak occurs at time t if:
$y_{t-2} - y_t < 0$; $y_{t-1} - y_t < 0$; $y_{t+1} - y_t < 0$; $y_{t+2} - y_t < 0$,
and a trough occurs at time t if:
$y_{t-2} - y_t > 0$; $y_{t-1} - y_t > 0$; $y_{t+1} - y_t > 0$; $y_{t+2} - y_t > 0$.

Step 2: Censor the turning points to ensure that peaks and troughs alternate. In the case of two consecutive peaks (troughs), eliminate the peak (trough) with the lower (higher) value of y_t.

Step 3: Censor the turning points to ensure that each business cycle phase (peak-to-trough and trough-to-peak) lasts a minimum of two quarters, while each complete business cycle (peak-to-peak and trough-to-trough) lasts a minimum of five quarters.

The peak and trough dates established by the NBER for the sample period 1948Q4 to 2007Q4, along with the dates established by the BBQ algorithm applied to quarterly U.S. real GDP are reported in Table 1.[4] The BBQ algorithm does a reasonable job of matching the NBER peak and trough dates. It identifies eight of the nine peaks and nine of the ten troughs reported by the NBER. Just two of the peak dates differ from the corresponding NBER peak dates, each by a single quarter, while five of the trough dates differ from the corresponding NBER trough dates, with the differences ranging from one to three quarters.

Table 1 – Peak and Trough Dates from NBER Business Cycle Dating Committee and the BBQ and MBBQ Algorithms Applied to U.S. Real GDP (1948Q4 – 2007Q4)

Business Cycle Peaks			Business Cycle Troughs		
NBER	BBQ	MBBQ	NBER	BBQ	MBBQ
1948Q4	-	-	1949Q4	**1949Q2**	1949Q4
1953Q2	1953Q2	1953Q2	1954Q2	**1954Q1**	1954Q2
1957Q3	1957Q3	1957Q3	1958Q2	**1958Q1**	**1958Q1**
1960Q2	**1960Q1**	**1960Q1**	1961Q1	**1960Q4**	**1960Q4**
1969Q4	**1969Q3**	**1969Q3**	1970Q4	1970Q4	1970Q4
1973Q4	1973Q4	1973Q4	1975Q1	1975Q1	1975Q1
1980Q1	1980Q1	1980Q1	1980Q3	1980Q3	1980Q3
1981Q3	1981Q3	1981Q3	1982Q4	**1982Q1**	1982Q4
1990Q3	1990Q3	1990Q3	1991Q1	1991Q1	1991Q1
2001Q1	-	-	2001Q4	-	-

Notes: Bold indicate that the identified turning points differ from the NBER dates. We ignore the first NBER peak date in our evaluation of the BBQ and MBBQ algorithm because given our sample period, the earliest date at which the algorithms can identify a turning point is 1949Q2.

It is interesting that all the errors made by the BBQ algorithm shift the turning points forward in time relative to the NBER dates. This systematic error suggests that Step 1 of the BBQ algorithm can be modified to correct for it. Morley and Piger (2006) modified the BBQ algorithm by optimizing the threshold values that indicate turning points. We refer to this modified BBQ algorithm as MBBQ. Specifically, MBBQ restates Step 1 of the BBQ algorithm as follows:

[4] The quarterly U.S. real GDP series begins in 1947Q1. However, to avoid any ambiguity in measuring features such as the average length of phases, we start the sample period with the first turning point (i.e., a peak in 1948Q4).

Step 1: Using the log level of U.S. quarterly real GDP (y_t), establish candidate dates of peaks and troughs as local maxima and minima in the data such that a peak occurs at time t if:

$y_{t-2} - y_t < \alpha_1$; $y_{t-1} - y_t < \alpha_1$; $y_{t+1} - y_t < \alpha_2$; $y_{t+2} - y_t < \alpha_2$,

and a trough occurs at time t if:

$y_{t-2} - y_t > \alpha_3$; $y_{t-1} - y_t > \alpha_3$; $y_{t+1} - y_t > \alpha_4$; $y_{t+2} - y_t > \alpha_4$.

MBBQ differs from BBQ in that the threshold parameters that signal turning points are allowed to deviate from 0. The thresholds are also allowed to vary from peak to trough and on different sides of the turning points. To determine the values of the α_i's, i = 1, 2, 3, 4, a grid search is conducted for values between –0.005 and 0.005, i.e. $\alpha_i \in (-0.005, 0.005)$. For each possible combination of the α_i's in the grid, the root mean squared error (RMSE) is calculated as:

$$RMSE(\alpha_i) = \sqrt{\frac{\sum_{t=1}^{T}[MBBQ_t(\alpha_i) - NBER_t]^2}{T}} \ ,$$

where $NBER_t$ = 1 if quarter t is an NBER recession quarter and $NBER_t$ = 0 otherwise, while $MBBQ_t(\alpha_i)$ = 1 if quarter t is a recession quarter according to the MBBQ algorithm with threshold values α_i, and $MBBQ_t(\alpha_i)$ = 0 otherwise. The α_i's that minimize RMSE(α_i) are chosen to be the final threshold values for the algorithm. In the case of ties, α_i's that are closest to 0, as measured by $\sum_{i=1}^{4}|\alpha_i|$, are chosen.

The turning point dates established by the MBBQ algorithm are reported in Table 1 as well. Threshold values chosen for this sample period are: α_1 = 0, α_2 = 0, α_3 = 0.001, α_4 = –0.002. It is clear from Table 1 that the MBBQ algorithm offers substantial improvement over the BBQ algorithm, especially for trough dates. It identifies the same number of peaks and troughs as the BBQ algorithm, though only two of the peak dates and two of the trough dates deviate from their corresponding NBER dates, each by a single quarter. Given this improvement, we will use the MBBQ algorithm when establishing peak and trough dates in simulated real GDP data.

Note that both the BBQ and MBBQ algorithms miss the 2001 NBER recession. This is because real GDP growth in 2001Q2, the middle quarter of this three quarter recession, is positive.[5] As both dating algorithms require two

[5] Real GDP growth in 2001Q2 was negative when the NBER initially identified the 2001 recession, and was revised to be positive in a 2004 benchmark data revision. Despite this revision, there is ample evidence that 2001 remains a recession phase, and the NBER Business Cycle

quarters of decline following a peak, this implies that neither algorithm is able to pick up the beginning of the 2001 recession. However, given that both BBQ and MBBQ do fairly well at matching NBER dates prior to 2001, we do not believe that this problem is serious enough for us to abandon the use of these algorithms altogether.

2.2 Business Cycle Features

Given a set of peak and trough dates, we identify four business cycle phases over which to compute features, defined as follows: (1) Recession – the quarter following a peak date to the subsequent trough date, (2) Expansion – the quarter following a trough date to the subsequent peak date, (3) Recovery – the first four quarters of the expansion phase, and (4) Mature Expansion – the remaining quarters of an Expansion phase following the Recovery phase.[6] Given this definition of phases, we consider the following business cycle features for any given realization of data:

- Number of business cycle peaks
- Average and standard deviation of the lengths of Recession and Expansion phases.
- Average and standard deviation of annualized quarterly growth rates in Recession, Expansion, Recovery, and Mature Expansion phases
- Correlation between the cumulative decline during a Recession and the cumulative growth in the subsequent Recovery phase.

These features are the focus of much of the previous literature on business cycle features discussed in the introduction or are closely related to features previously considered in that literature. Of particular importance is the correlation feature, which was considered as long ago as Friedman (1964) and is also related to the

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Dating Committee has never raised the possibility of revising the 2001 peak and trough dates. Even though the 2001 recession is no longer obvious from the level of the GDP series alone, it is still apparent in other series such as payroll employment. In addition, nonlinear Markov-switching type models still identify 2001 as a recession episode with the updated GDP data. Another interesting anecdote is that if we feed real gross domestic income (real GDI) into the algorithms rather than real GDP, both BBQ and MBBQ pick up the 2001 peak and trough dates, although they miss the 1980 peak and trough instead. Hence, despite the recent attention paid to GDI by the Business Cycle Dating Committee in their report on the determination of the December 2007 peak in economic activity, using GDI does not offer an absolute improvement to using GDP in terms of producing peak and trough dates that match the NBER dates.

[6] We also consider a recovery phase defined as the first three or five quarters of the expansion phase. As all reported results are robust to these alternative definitions, we only report the results for the four-quarter window here.

"excess cumulated movements" statistic for troughs to peaks considered in Harding and Pagan (2002) and some other studies.

Table 2 – Business Cycle Features for U.S. Real GDP (1948Q4 – 2007Q4) Using NBER Turning Point Dates

Features	
Average quarterly growth rates	
Recession	–1.92
Expansion	4.59
Recovery	7.10
Mature expansion	3.94
Std. deviation of quarterly growth rates	
Recession	3.33
Expansion	3.54
Recovery	4.18
Mature expansion	3.05
Number of phases	
Number of peaks	9
Average length of phases	
Recession	3.44
Expansion	19.67
Std. deviation of length of phases	
Recession	1.13
Expansion	12.72
Correlation between growth rates	
Recession/Recovery	–0.66

3. Business Cycle Features in U.S. Real GDP Data

Table 2 presents the values of the business cycle features for quarterly U.S. real GDP data from 1948Q4 to 2007Q4 using the NBER turning point dates. First, as one would expect, average growth differs substantially between the Recession and Expansion phases. Recessions are associated with negative growth rates, averaging around an annualized –1.9% in each quarter, while Expansions are associated with positive growth rates close to an annualized 4.6% in each quarter. Second, when the Expansion phase is divided into Recovery and Mature Expansion phases, average growth in the Recovery phase is almost twice as large as in the Mature Expansion phase. Third, there is a large difference between the average length of the Recession and Expansion phases, with the Expansions phase lasting nearly six times as long as the Recession phase. Fourth, the variability of growth rates associated with the Recovery phase is much higher than for other phases. This relatively high variability also applies to the average length of the Expansion phase. Finally, there is a strong negative correlation between the cumulative growth in a Recession phase and the cumulative growth in the subsequent Recovery phase. This corroborates the observations made in Friedman (1964, 1993).

4. Business Cycle Features in Simulated Data from Time-Series Models

4.1 Description of Univariate Models

We consider two different univariate models. The first is the linear AR(2) model that has been found to do quite well in terms of matching business cycle features in the previous literature, and is the preferred model in Clements and Krolzig (2004). The second is the Kim, Morley, and Piger (2005) bounceback model, which is a nonlinear model with Markov-switching parameters. This version of the bounceback model is termed BBV indicating that its dynamic specification is capable of producing V-shaped recessions.[7] The key difference between the bounceback model and the standard Hamilton (1989) two-state Markov-switching model is that it allows for the possibility of a high-growth recovery phase following the end of recessions. Furthermore, unlike a three-state Markov-switching model, the strength of the high-growth recovery phase is related to the severity of the previous recession, as measured by its length for the BBV model. The BBV model was the best performing time-series model in Morley and Piger (2006), beating the three-state Markov-switching model of Boldin (1996), which was also designed to capture high-growth recovery business cycle phases. The specification and estimates of the two univariate time series models for quarterly U.S. real GDP are presented in the appendix.

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[7] V-shaped recession refers to recessions exhibiting “sharpness,” a term introduced by McQueen and Thorley (1993). A sharp series has the transition from contraction to expansion occurring more rapidly than the transition from expansion to contraction. This feature results in the level series being more rounded at peaks than at troughs.

4.2 Description of Multivariate Models

We consider three different multivariate models: the two-variable VAR model of Blanchard and Quah (1989) (B&Q); the four-variable VAR model in Ahmed, Levin, and Wilson (2004) (ALW); and the three-variable VECM in King, Plosser, Stock, and Watson (1991) (KPSW). These three models are of particular interest to us because they are widely cited multivariate models in the economics literature and they are specifically designed to explain aggregate economic fluctuations.

Blanchard and Quah (1989) examine the dynamic effects of aggregate demand and supply disturbances by considering GNP growth and the unemployment rate in their VAR model. Ahmed et al. (2004) investigate the source of the reduction in the volatility of GDP growth since 1984, and their VAR model includes GDP growth, inflation, commodity price inflation and the federal funds rate. Stock and Watson (2002) and Boivin and Giannoni (2006) also considered similar VAR models in their influential studies of the so-called "Great Moderation". King et al. (1991) examine the importance of productivity shocks on economic fluctuations using a VECM. Their model includes private GNP (y), consumption (c), and investment (i), with ($c - y$) and ($i - y$) as theory-based error-correction terms.[8] As with the univariate models, the specifications and estimates of each of these models applied to quarterly U.S. data are also presented in the appendix.[9]

4.3 Calculation of Business Cycle Features for Time-Series Models

To calculate business cycle features for the models under consideration, we use the estimated parameters reported in the appendix to simulate artificial real GDP series from 1948Q4 to 2007Q4, with the actual value of real GDP in 1948Q4 serving to normalize an initial value. We simulate 10,000 artificial data

[8] In the structural VAR literature, the type of identification method used is of vital importance. Blanchard and Quah (1989) and King et al. (1991) implemented long-run restrictions while Ahmed et al. (2004) used short-run restrictions. However, for the purpose of simulating data and calculating the business cycle features considered here, identification of structural shocks is irrelevant. What matters are the variables included in each VAR or VECM and the reduced-form dynamics of the models.

[9] The data used for estimation of the multivariate models vary in some cases from what was considered in the original studies. If the original study considered an output variable that was not real GDP (for example, Blanchard and Quah, 1989, used real gross national product), we replace it with real GDP in our estimation. As for the other variables in the models, we try to stay as close to those in the original studies as possible. Due to data availability and the number of lags required in estimation, the estimation sample periods for the multivariate models all start somewhat later than 1948Q4 (i.e., the B&Q sample starts in 1950Q1, the ALW sample starts in 1955Q3, and the KPSW sample starts in 1949Q2.)

series for each model, computing business cycle features from each simulation based on peak and trough dates established using the MBBQ algorithm. Following the convention in the literature, we neglect parameter uncertainty in our simulations. Thus, the only source of variation across simulations arises from the model disturbances. In our simulations, we take two approaches to generating realizations of these disturbances. The first approach follows the convention of most of the business cycle features literature and draws disturbances from an i.i.d. Normal distribution with mean of zero and variance equal to the estimate obtained from application of the model to actual data. The second approach bootstraps realizations of the disturbances by sampling with replacement from the estimated residuals for a given model. If the true disturbances do not have a Normal distribution, this bootstrapping approach should improve the performance of a model in terms of its ability to reproduce business cycle features. This bootstrapping approach also helps address any concerns that nonlinear models might be better than linear models at replicating business cycle features merely because they can capture fat tails or skewness in the unconditional distribution of output growth rather than any inherent nonlinear dynamics. In the following discussion we refer to simulations produced by generating disturbances from a Normal distribution as "Gaussian simulations" and to simulations produced by drawing from estimated residuals as "bootstrap simulations."

4.4 Business Cycle Features of Univariate Models

Table 3 reports the median of the 10,000 simulations for each business cycle feature from the univariate models. The median value for each feature is followed (in parentheses) by the proportion of simulations that fall below the corresponding sample feature (reproduced in column 1 of Table 3). These percentiles provide a sense of how likely the univariate models is to have produced a sample value for a particular business cycle feature that is as large or small as that exhibited by the actual GDP data. Percentiles less than 0.10 or greater than 0.90 are bolded to denote that it is unlikely that a given univariate time-series model would have produced data with that particular sample feature. The reported median provides a sense of whether a percentile is driven by closeness of the distribution in matching the sample feature or by a large dispersion of the simulated distribution. To succinctly summarize the results across features, we also report a root mean squared deviation measure in the last row of Table 3, defined as:

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(p_{im}-0.5\right)^2},$$

where p_{im} is the proportion of simulations for feature i and model m that fall below the corresponding sample feature (i.e. the numbers in parentheses in each of the columns of the table), and n = 14 features.

Table 3 – Business Cycle Features for Univariate Models (1948Q4 – 2007Q4)

Features	Real GDP	AR(2) (Gaussian)	AR(2) (Bootstrap)	BBV (Gaussian)	BBV (Bootstrap)
Average quarterly growth rates					
Recession	-1.92	-2.06 (0.63)	-2.19 (0.71)	-2.12 (0.69)	**-2.67 (0.93)**
Expansion	4.59	4.29 (0.80)	4.11 (0.89)	4.19 (0.89)	**4.16 (0.90)**
Recovery	7.10	**4.16 (1.00)**	**3.98 (1.00)**	5.87 (0.90)	6.16 (0.83)
Mature expansion	3.94	4.31 (0.18)	4.13 (0.33)	3.83 (0.66)	3.74 (0.76)
Std. deviation of quarterly growth rates					
Recession	3.33	**2.27 (0.99)**	**2.54 (0.96)**	**2.34 (0.98)**	2.81 (0.86)
Expansion	3.54	3.56 (0.46)	3.59 (0.43)	3.56 (0.46)	3.71 (0.27)
Recovery	4.18	**3.23 (0.97)**	3.19 (0.87)	4.02 (0.61)	4.09 (0.56)
Mature expansion	3.05	**3.62 (0.01)**	**3.63 (0.04)**	3.33 (0.13)	**3.45 (0.09)**
Number of phases					
Number of peaks	9	9 (0.40)	8 (0.61)	9 (0.50)	8 (0.55)
Average length of phases					
Recession	3.44	3.27 (0.60)	3.29 (0.60)	3.45 (0.49)	3.75 (0.36)
Expansion	19.67	21.00 (0.42)	24.43 (0.24)	22.33 (0.33)	22.88 (0.31)
Std. deviation of length of phases					
Recession	1.13	1.56 (0.27)	1.51 (0.29)	1.83 (0.20)	2.07 (0.14)
Expansion	12.72	16.30 (0.27)	19.08 (0.17)	17.14 (0.24)	17.70 (0.21)
Correlation between growth rates					
Recession/Recovery	-0.66	**0.07 (0.04)**	**0.06 (0.06)**	-0.44 (0.24)	-0.49 (0.29)
Root mean squared deviation	0.00	0.33	0.32	0.27	0.29

As can be seen from Table 3, Gaussian simulations from the AR(2) model do reasonably well in matching many of the sample features. The AR(2) model with Normal disturbances is particularly good at replicating the features related to the number or length of phases. However, the large difference between the median value in the simulated data and the sample value for the average and standard deviation of the length of Expansion phases shows there is sometimes substantial dispersion in the simulated distribution. Also, the AR(2) model fails to reproduce the high average growth rate exhibited by real GDP in the Recovery phase, and the standard deviation of growth rates for most phases are far from the sample values. Finally, the AR(2) model does a very poor job at replicating the strong negative correlation between the cumulative growth rates of the Recession and Recovery phases exhibited by actual GDP. Similar results are obtained for bootstrap simulations from the AR(2) model.

Turning to the results for the nonlinear BBV model, it is clear that it improves upon the AR(2) model. Column 4 of Table 3 shows that Gaussian simulations from the BBV model can match all features reasonably well except for the standard deviation of growth rates in the Recession phase. It is especially notable that the BBV model can capture the high average growth rate during the Recovery phase as well as the strong negative correlation between the cumulative growth rate in the Recession phase and the cumulative growth rate in the subsequent Recovery phase. Bootstrap simulations in this case do not lead to an improvement in the performance of the BBV model, reporting percentiles in excess of 0.9 for the average growth rate of Recession and Expansion phases. However, bootstrap simulations do allow the BBV model to generate a slightly stronger negative correlation between the cumulative growth during Recession and Recovery phases.[10]

The root mean squared deviation statistics reported for each model summarize the results across the different features. The model with the lowest average deviation is the BBV model with Gaussian simulations, and the BBV model, in general, outperforms the AR(2) model in terms of having lower average deviations. Overall, the results are consistent with the findings in Galvão (2002), Morley and Piger (2006), and Demers and Macdonald (2007) that certain univariate nonlinear models do a better job at capturing important asymmetries in the business cycle than univariate linear models.

[10] The generally weaker performance of the BBV model with bootstrap simulations could be due to the problem of measuring estimated residuals for this model. For Markov-switching models, the residuals are state (recession or expansion) dependent, and which state is observed depends on the probability of switching or staying in that state. To get around this problem, we assume the state is observable by imposing the NBER peak and trough dates. Then, with the estimated model parameters, we calculate a set of residuals based on these states. This allows us to simulate data by drawing bootstrap samples of estimated residuals conditional on which state is operational in the simulated data. However, to the extent that a Normality assumption is appropriate and the estimated residuals are measured with error, we might expect the bootstrap simulations to underperform the Gaussian simulations.

4.5 Business Cycle Features of Multivariate Models

Table 4 reports the results on business cycle features for the multivariate models. A brief glance at the table reveals that the three different multivariate models produce more or less the same results. All three models do well in terms of matching the number of peaks and the average and standard deviation of the length of Recession and Expansion phases. However, as with the linear AR(2) model discussed above, they fail to generate a high enough average growth rate for the Recovery phase or a strong enough negative correlation between the cumulative growth rates of Recession and Recovery phases. The ALW four-variable VAR model even has trouble with the average growth rates in the Expansion phase. The multivariate models also cannot replicate the standard deviation of growth rates in most of the business cycle phases. Although there are small improvements from using bootstrap simulations over Gaussian simulations, this choice does not greatly affect the performance of the multivariate models.

Based on the results reported in Table 4, we conclude that multivariate information does not improve the performance of linear models at replicating business cycle features of U.S. real GDP. In the best case, the B&Q model with bootstrap simulations replicates sample features about as well as the AR(2) with bootstrap simulations. In terms of the root mean squared deviation measure, the model with the lowest root mean squared deviation in Table 4 - the KPSW model with bootstrap simulations - is only marginally better than the AR(2) with bootstrap simulations.

The results so far suggest that the nonlinear BBV model is the best performing model in terms of replicating business cycle features. However, it is important to emphasize that not all nonlinear time-series models improve on linear models. For example, Morley and Piger (2006) found that the two-regime Markov-switching model of Hamilton (1989) performs about the same as the linear models. A key reason why the nonlinear BBV model does a superior job in reproducing business cycle features is that there is a mechanism embedded in the model to allow for high growth recoveries. Galvão (2002) also found this mechanism was essential when considering related models. Among the fifteen univariate nonlinear models she investigated, only two (a three-regime Markov-switching model and an unobserved components model with Markov-switching in the transitory component) were able to account for the asymmetries in the shape of the U.S. business cycle, and those two models are both characterized by mechanisms that capture high growth recoveries.

Table 4 – Business Cycle Features for Multivariate Models (1948Q4 – 2007Q4)

Features	Real GDP	B&Q (Gaussian)	B&Q (Bootstrap)	ALW (Gaussian)	ALW (Bootstrap)	KPSW (Gaussian)	KPSW (Bootstrap)
Average quarterly growth rates							
Recession	-1.92	-2.07 (0.65)	-2.12 (0.68)	-1.85 (0.42)	-1.88 (0.46)	-2.13 (0.71)	-2.14 (0.68)
Expansion	4.59	4.35 (0.78)	4.22 (0.90)	**3.97 (0.97)**	**3.81 (0.99)**	4.24 (0.84)	**4.01 (0.95)**
Recovery	7.10	**4.71 (1.00)**	**4.69 (1.00)**	**4.11 (1.00)**	**3.96 (1.00)**	**4.59 (1.00)**	**4.33 (1.00)**
Mature expansion	3.94	4.26 (0.17)	4.10 (0.31)	3.93 (0.51)	3.77 (0.69)	4.13 (0.32)	3.92 (0.53)
Std. deviation of quarterly growth rates							
Recession	3.33	**2.13 (1.00)**	**2.25 (1.00)**	**2.01 (1.00)**	**2.21 (0.97)**	**2.25 (1.00)**	**2.45 (0.95)**
Expansion	3.54	3.66 (0.28)	3.55 (0.48)	3.41 (0.73)	3.29 (0.80)	3.70 (0.23)	3.49 (0.56)
Recovery	4.18	**3.43 (0.94)**	3.36 (0.89)	**3.14 (0.98)**	**2.98 (0.94)**	**3.47 (0.93)**	**3.26 (0.93)**
Mature expansion	3.05	**3.70 (0.00)**	**3.56 (0.04)**	**3.46 (0.03)**	3.31 (0.19)	**3.74 (0.00)**	**3.52 (0.05)**
Number of phases							
Number of peaks	9	10 (0.24)	9 (0.40)	9 (0.34)	8 (0.59)	11 (0.14)	9 (0.37)
Average length of phases							
Recession	3.44	3.13 (0.72)	3.00 (0.80)	3.11 (0.72)	3.00 (0.77)	3.30 (0.60)	3.20 (0.67)
Expansion	19.67	19.10 (0.55)	21.63 (0.36)	20.22 (0.46)	24.43 (0.24)	17.18 (0.70)	20.89 (0.42)
Std. deviation of length of phases							
Recession	1.13	1.26 (0.39)	1.15 (0.48)	1.32 (0.37)	1.21 (0.45)	1.41 (0.27)	1.30 (0.36)
Expansion	12.72	13.59 (0.43)	15.89 (0.30)	15.43 (0.31)	18.85 (0.19)	12.08 (0.55)	14.94 (0.35)
Correlation between growth rates							
Recession/Recovery	-0.66	**-0.07 (0.04)**	**-0.14 (0.08)**	**0.01 (0.04)**	**-0.01 (0.07)**	**-0.09 (0.04)**	**-0.12 (0.07)**
Root mean squared deviation	0.00	0.33	0.32	0.34	0.33	0.34	0.31

4.6 Business Cycle Features and the "Great Moderation"

Numerous empirical studies have documented evidence for a marked decline in the volatility of U.S. real GDP growth since the mid 1980s, a stylized fact that has become known as the "Great Moderation."[11] As a primary feature of U.S. real GDP data, it should be taken into account in assessing the robustness of our results. One concern with not addressing the Great Moderation is that the linear models might be at a disadvantage in our analysis because linear models cannot "automatically" pick up a reduction in variance, while nonlinear models can potentially proxy for a structural break in variance or other forms of heteroskedasticity, especially given a Markov-switching structure. So the superior performance of the bounceback model reported above could potentially be due to its ability to capture the Great Moderation rather than nonlinearities related to the business cycle. To evaluate this possibility, we consider a break in the variance of real GDP growth in 1984Q1 for all five time-series models presented earlier.

To implement the structural break, we consider bootstrap simulations for all of the linear models. Specifically, we sample the estimated residuals for each of the time-series models with replacement from two separate bins corresponding to the pre-structural break sample (1948Q4 to 1984Q1) and post-structural break sample (1984Q2 to 2007Q4), with the bin chosen based on the quarter being simulated. For the BBV model, we simulate data from an estimated version of the model that allows for a structural break in the residual variance in 1984Q1.

Table 5 reports the results for business cycle features when taking into account the Great Moderation.[12] Looking at the univariate models first, one can see that the basic findings are very similar to those reported in Table 3. The AR(2) model fails to reproduce the exact same features as it did before taking the structural break into account (average growth in the Recovery phase, standard deviation of growth in the Recession and Mature expansion phases, and correlation between cumulative growth rates of Recession and Recovery phases). There is also very little change for the performance of the BBV model from allowing for the structural break in variance.[13]

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[11] See, for example, Kim and Nelson (1999) and McConnell and Perez-Quiros (2000).

[12] Accounting for the Great Moderation in the time-series models raises a question of whether some of the business cycle features themselves might have changed significantly over the sample period. Unfortunately, the number of business cycle episodes is too few to reliably detect structural breaks in features. For example, in a regression of the cumulative growth in a Recovery phase on a constant and the cumulative decline in the preceding Recession, the *t* statistic for a structural break in the regression coefficient on the dependent variable after the Great Moderation is insignificant with a *p*-value of 0.34.

[13] There is some evidence that the difference between the mean growth rates across regimes for the BBV model also changed with the Great Moderation, as was found in Kim and Nelson (1999) for the basic Hamilton (1989) model. Also, the bounceback parameter for the BBV model appears to

The most intriguing results in Table 5 relate to the multivariate models, where there appears to be some improvements in the performance of all the multivariate models, especially the KPSW VECM. This can be seen clearly in the reduction of the root mean squared deviations compared with those reported in Table 4. The models are now better at matching the standard deviation in the growth rates of business cycle phases. But perhaps the most notable change is in terms of matching the correlation feature. The multivariate models are now able to generate a more negative correlation between the cumulative growth rates of Recession and Recovery phases such that the proportion of simulated features below the corresponding NBER sample feature value is just above 10%. However, even with this improvement, the median correlation from the simulations is still far from the sample correlation and from the median correlation produced by the BBV model. Furthermore, the fact that the multivariate linear models cannot produce a strong enough negative correlation before taking into account the structural break in variance suggests that there is something about the volatility reduction in 1984 that helped generate it, rather than something inherent in the dynamics of the linear models. To investigate this conjecture, we conduct two experiments, one involving a counterfactual simulation and one using an "asymptotic" simulation.

To motivate the counterfactual experiment, we note that if there is something about the linear dynamics in the multivariate models that allow them to capture the strong negative correlation between growth in recessions and growth in recoveries exhibited by real GDP, it should be a recurring feature of the simulated data both prior to the structural break date of 1984Q1 and after it as well. This leads us to a simple counterfactual experiment in which we estimate each of the multivariate models using pre-1984Q1 data and post-1984Q1 data separately. We then assume that the pre or post break date parameters apply over the whole sample period and simulate corresponding counterfactual data to calculate the implied correlation between the cumulative growth rate of the Recession phase and the Recovery phase. We consider both Gaussian and bootstrap simulations, although the results are very similar.

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have decreased with the Great Moderation, although the evidence is weaker for a related specification considered in Morley and Piger (2012) that links the bounceback effect to the depth of the recession instead of its length. However, to make it clear that the superior performance of the BBV model compared to the multivariate models is not due to allowing more flexibility for the BBV model with a structural break to fit the data, we focus on only allowing the distribution of the disturbances to change with the Great Moderation for the various models.

Table 5 – Business Cycle Features for All Models with Structural Break (1948Q4 – 2007Q4 with Structural Break in Variance in 1984Q1)

Features	Real GDP	AR(2)	BBV	B&Q	ALW	KPSW
Average quarterly growth rates						
Recession	-1.92	-2.36 (0.79)	-2.18 (0.69)	-2.18 (0.71)	-2.10 (0.63)	-2.29 (0.76)
Expansion	4.59	4.31 (0.70)	**3.94 (0.96)**	4.29 (0.81)	**3.94 (0.95)**	4.24 (0.80)
Recovery	7.10	**4.55 (0.99)**	5.69 (0.88)	**5.00 (0.98)**	**4.33 (1.00)**	**4.82 (0.99)**
Mature expansion	3.94	4.26 (0.27)	3.62 (0.86)	4.13 (0.29)	3.84 (0.60)	4.10 (0.35)
Std. deviation of quarterly growth rates						
Recession	3.33	**2.67 (0.90)**	**2.39 (0.95)**	**2.33 (0.99)**	**2.47 (0.92)**	2.61 (0.89)
Expansion	3.54	3.81 (0.25)	3.66 (0.34)	3.70 (0.31)	3.47 (0.58)	3.64 (0.38)
Recovery	4.18	3.76 (0.69)	4.30 (0.44)	3.75 (0.72)	3.41 (0.83)	3.70 (0.76)
Mature expansion	3.05	**3.79 (0.05)**	3.42 (0.11)	**3.63 (0.04)**	3.43 (0.14)	**3.59 (0.06)**
Number of phases						
Number of peaks	9	7 (0.70)	7 (0.72)	8 (0.52)	8 (0.62)	8 (0.57)
Average length of phases						
Recession	3.44	3.33 (0.56)	3.43 (0.51)	3.00 (0.77)	3.10 (0.73)	3.20 (0.67)
Expansion	19.67	22.22 (0.36)	26.00 (0.19)	22.00 (0.34)	22.57 (0.34)	21.88 (0.37)
Std. deviation of length of phases						
Recession	1.13	1.60 (0.28)	1.83 (0.24)	1.17 (0.46)	1.28 (0.40)	1.30 (0.37)
Expansion	12.72	18.89 (0.23)	20.27 (0.15)	17.28 (0.23)	18.21 (0.22)	17.67 (0.25)
Correlation between growth rates						
Recession/Recovery	-0.66	**-0.07 (0.10)**	-0.53 (0.35)	-0.24 (0.13)	**-0.13 (0.10)**	-0.20 (0.12)
Root mean squared deviation	0.00	0.30	0.30	0.30	0.30	0.28

Table 6 details the outcome of the counterfactual experiment. It is clear from the results that a strong negative correlation between growth rates in recession and recovery phases is not a recurring feature using either pre or post break date parameters for any of the multivariate linear models. Under counterfactual 1 (pre-1984Q1 parameters), the median correlations for the simulations are only slightly negative or zero. With low corresponding percentiles, these results show that it is very unlikely that the sample value could have arisen from such models. Under counterfactual 2 (post-1984Q1 parameters), the median correlations for the simulations for all of the multivariate linear models are actually positive, although the corresponding percentiles are within the 0.1 to 0.9 range.

Table 6 – Counterfactual Experiment for Multivariate Models

	Correlation between Cumulative Growth in Recession Phase and Cumulative Growth in Recovery Phase	
	Pre-structural break Parameters (Counterfactual 1)	Post-structural break Parameters (Counterfactual 2)
Real GDP	-0.66	-0.66
B&Q		
Gaussian	**-0.11 (0.03)**	0.00 (0.28)
Bootstrap	**-0.14 (0.04)**	0.00 (0.34)
ALW		
Gaussian	**0.00 (0.02)**	0.04 (0.27)
Bootstrap	**0.00 (0.02)**	0.00 (0.32)
KPSW		
Gaussian	**-0.12 (0.03)**	0.14 (0.18)
Boostrap	**-0.15 (0.05)**	0.09 (0.27)

Notes: First row reports the U.S. real GDP growth sample features using NBER peak and trough dates. Following rows report simulated median feature for the multivariate models based on 10,000 simulations, with the proportion of simulated features that fall below the sample feature reported in row 1 in brackets. Bold indicates a percentile that is less than 0.1 or greater than 0.9, implying that it was unlikely that the particular time-series model could simulate data that replicates the behavior of actual GDP for that particular feature. The structural break date is 1984Q1.

To further investigate the negative correlation feature for the multivariate linear models, we also conducted an “asymptotic” simulation experiment. If the strong negative correlations produced by the multivariate linear models are driven by the one-time structural break in variance, we should see the effect of the structural break weaken as we increase the sample size for the simulated data. Table 7 reports the correlation between the cumulative growth in Recession phase and the cumulative growth in Recovery phase for the bounceback model, as well as the three multivariate linear models for an extended simulation sample period from 1884Q1 to 2084Q1 (100 years before the structural break date of 1984Q1 to 100 years after). The results show that, even though the median simulated correlation remains negative for the multivariate linear models, the proportion of the 10,000 simulated features falling below that reported for the actual real GDP growth data (–0.66) is now close to zero. However, for the bounceback model, the median correlation remains neative, and the percentile stays above the 10% cutoff point.

Table 7 – Asymptotic Simulation Experiment

	Correlation between Cumulative Growth in Recession Phase and Cumulative Growth in Recovery Phase
Real GDP	-0.66
BBV	-0.46 (0.11)
B&Q	**-0.26 (0.01)**
ALW	**-0.16 (0.01)**
KPSW	**-0.24 (0.01)**

Notes: First row reports the U.S. real GDP growth sample features using NBER peak and trough dates. Following rows report simulated median feature for the bounceback and multivariate linear models based on 10,000 simulations of length 200 years, with the proportion of simulated features that fall below the sample feature reported in row 1 in brackets. Bold indicates a percentile that is less than 0.1 or greater than 0.9, implying that it was unlikely that the particular time-series model could simulate data that replicates the behavior of actual GDP for that particular feature. The structural break date is 1984Q1.

5. Conclusion

In this paper we have assessed the ability of various time-series models to reproduce business cycle features exhibited by U.S. real GDP. Our primary interest has been to evaluate whether multivariate linear models can improve on the previously documented failure of univariate linear models to replicate certain key business cycle features. The results of our simulation experiments answer this question decidedly in the negative, demonstrating that multivariate linear models do not provide a substantial improvement over univariate linear models for reproducing business cycle features. Furthermore, a univariate nonlinear Markov-switching model with a mechanism for capturing a "bounceback" effect in output following recessions outperformed both univariate and multivariate linear models. These results are robust to simulated data generated either using Normal disturbances or bootstrapped disturbances, as well as to allowing for a one-time structural break in the variance of shocks to real GDP growth.

Overall, our results provide further evidence of an essential nonlinearity present in the U.S. business cycle. Specifically, the results demonstrate that the apparent nonlinearity found necessary for univariate models of real GDP is not simply proxying for relevant variables omitted from these models. Instead, there is something fundamentally different about the dynamics of real GDP across business cycle phases that linear models are not able to replicate.

Appendix

Here we present the estimates for quarterly U.S. GDP for the five time-series models under consideration. The reported estimates are used to calibrate the data generating process used in our Monte Carlo simulations. The AR(2) and the Kim, et. al. (2005) bounceback model are univariate, while the Blanchard and Quah (1989) VAR, the Ahmed et. al. (2004) VAR, and the King et. al. (1991) VECM are multivariate. For the univariate models, Δy_t is defined as annualized growth rate of output to be compatible with the specification in Morley and Piger (2006). For the multivariate models, Δy_t is defined as natural log difference of output to be compatible with their original specifications. Note that the full VAR/VECM models are simulated, but we only report the relevant output estimates here for brevity.

<u>The AR(2) model:</u>

Estimation period 1948Q4 to 2007Q4.

$$\Delta y_t = 0.0214 + 0.2976\Delta y_{t-1} + 0.0858\Delta y_{t-2} + \varepsilon_t ,$$
$$\sigma_\varepsilon = 0.0383 .$$

<u>The Kim, Morley, and Piger (2005) bounceback model (BBV):</u>

Estimation period 1948Q4 to 2007Q4.

$$\Delta y_t = 3.3521 - 4.4383 S_t + 1.3052(1 - S_t)\sum_{j=1}^{6} S_{t-j} + \varepsilon_t ,$$
$$\sigma_\varepsilon = 3.1122 ,\ P(S_t = 1 \mid S_{t-1} = 1) = 0.7321 ,\ P(S_t = 0 \mid S_{t-1} = 0) = 0.9450 ,$$

where $S_t = 1$ corresponds to recessions and $S_t = 0$ corresponds to expansions.

<u>The Kim, Morley, and Piger (2005) bounceback model with break in variance (BBV):</u>

Estimation period 1948Q4 to 2007Q4.

$$\Delta y_t = 3.1464 - 4.4459 S_t + 1.5110(1 - S_t)\sum_{j=1}^{6} S_{t-j} + \varepsilon_t ,$$
$\sigma_\varepsilon = 4.0732$ for t = 1948Q4 to 1984Q1,
$\sigma_\varepsilon = 1.9881$ for t = 1984Q2 to 2007Q4,

$$P(S_t = 1 | S_{t-1} = 1) = 0.7630,\ P(S_t = 0 | S_{t-1} = 0) = 0.9716,$$

where $S_t = 1$ corresponds to recessions and $S_t = 0$ corresponds to expansions.

Blanchard & Quah (1989) two-variable VAR model (B&Q):

Estimation period 1950Q1 to 2007Q4.

$$\begin{aligned}\Delta y_t = {} & 0.0022 + 0.1254\Delta y_{t-1} + 0.1682\Delta y_{t-2} + 0.0532\Delta y_{t-3} + 0.1426\Delta y_{t-4} + 0.06208\Delta y_{t-5} \\ & + 0.1596\Delta y_{t-6} - 0.0158\Delta y_{t-7} + 0.0231\Delta y_{t-8} - 0.7470u_{t-1} + 1.5542u_{t-2} - 0.5442u_{t-3} \\ & + 0.5880u_{t-4} - 0.8945u_{t-5} + 0.3827u_{t-6} - 0.2552u_{t-7} - 0.0012u_{t-8} + \varepsilon_t,\end{aligned}$$

$$\Sigma_\varepsilon = \begin{bmatrix} 0.0000762302 & -0.0000148006 \\ -0.0000148006 & 0.0000070473 \end{bmatrix},$$

where u_t denotes the civilian unemployment rate and the order of the variables in the VAR is $[\Delta y_t \ \ u_t]'$. The quarterly unemployment rate is the average of the monthly unemployment rate series.

Ahmed, Levin, and Wilson (2004) four-variable VAR model (ALW):

Estimation period 1955Q3 to 2007Q4.

$$\begin{aligned}\Delta y_t = {} & 0.0076 + 0.2145\Delta y_{t-1} + 0.1660\Delta y_{t-2} + 0.0021\Delta y_{t-3} - 0.0328\Delta y_{t-4} + 0.0673\Delta cpi_{t-1} \\ & - 0.0316\Delta cpi_{t-2} + 0.0214\Delta cpi_{t-3} - 0.1648\Delta cpi_{t-4} - 0.0144\Delta ppi_{t-1} + 0.0263\Delta ppi_{t-2} \\ & - 0.0238\Delta ppi_{t-3} + 0.0139\Delta ppi_{t-4} + 0.0160 ffr_{t-1} - 0.2538 ffr_{t-2} + 0.1140 ffr_{t-3} + 0.0998 ffr_{t-4} \\ & + \varepsilon_t,\end{aligned}$$

$$\Sigma_\varepsilon = \begin{bmatrix} 0.000066 & -0.000001 & 0.000006 & 0.000023 \\ -0.000001 & 0.000021 & 0.000039 & 0.000012 \\ 0.000006 & 0.000039 & 0.000165 & 0.000039 \\ 0.000023 & 0.000012 & 0.000039 & 0.000120 \end{bmatrix},$$

where Δcpi_t denotes the consumer price inflation rate, Δppi_t is the inflation rate of the producer price index: all commodities, and ffr_t is the federal funds rate. The order of the variables in the VAR is $[\Delta y_t \ \ \Delta cpi_t \ \ \Delta ppi_t \ ffr_t]'$. The quarterly cpi, ppi, and *ffr* are all constructed by picking the end of quarter value of the equivalent monthly series.

<u>King, Plosser, Stock, and Watson (1991) three-variable VECM (KPSW):</u>

Estimation period 1949Q2 to 2007Q4.

$$\begin{aligned}\Delta y_t &= 0.0008 + 0.0895\left(c_{t-1} - y_{t-1} + 0.4178\right) - 0.0265(i_{t-1} - y_{t-1} + 2.0545) + 0.1462\Delta y_{t-1} \\ &\quad + 0.0526\Delta y_{t-2} + 0.0276\Delta y_{t-3} - 0.0636\Delta y_{t-4} + 0.1650\Delta y_{t-5} + 0.0752\Delta y_{t-6} - 0.0865\Delta y_{t-7} \\ &\quad + 0.0057\Delta y_{t-8} + 0.2790\Delta c_{t-1} + 0.1360\Delta c_{t-2} - 0.0079\Delta c_{t-3} + 0.1432\Delta c_{t-4} - 0.1311\Delta c_{t-5} \\ &\quad - 0.0009\Delta c_{t-6} + 0.1878\Delta c_{t-7} - 0.0724\Delta c_{t-8} + 0.0134\Delta i_{t-1} + 0.0202\Delta i_{t-2} - 0.0084\Delta i_{t-3} \\ &\quad + 0.0138\Delta i_{t-4} - 0.0333\Delta i_{t-5} - 0.0026\Delta i_{t-6} + 0.0121\Delta i_{t-7} + 0.0100\Delta i_{t-8} + \varepsilon_t,\end{aligned}$$

$$\Sigma_\varepsilon = \begin{bmatrix} 0.000078 & 0.000039 & 0.000270 \\ 0.000039 & 0.000057 & 0.000053 \\ 0.000270 & 0.000053 & 0.001631 \end{bmatrix},$$

where c_t denotes real personal consumption expenditure and i_t is the real gross private domestic investment. The order of the variables in the VECM is $[y_t \;\; c_t \;\; i_t]'$ and the two cointegrating relationships based on the balance growth theory are $(c_t - y_t)$ and $(i_t - y_t)$.

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