JAMES MORLEY
AARTI SINGH

Inventory Shocks and the Great Moderation

Why did the volatility of U.S. real GDP decline by more than the volatility of final sales with the Great Moderation in the mid-1980s? One explanation is that firms shifted their inventory behavior toward a greater emphasis on production smoothing. We investigate the role of inventories in the Great Moderation by estimating an unobserved components model that identifies inventory and sales shocks and their propagation in the aggregate data. Our estimates provide no support for increased production smoothing. Instead, smaller transitory inventory shocks are responsible for the excess volatility reduction in output compared to sales. These shocks behave like informational errors related to production that must be set in advance and their reduction also helps explain the changed forecasting role of inventories since the mid-1980s. Our findings provide an optimistic prognosis for a continuation of the Great Moderation, despite the dramatic movements in output during the recent economic crisis.

JEL codes: E22, E32, C32
Keywords: Great Moderation, inventories, production smoothing, unobserved components model.

THE LOWER VOLATILITY OF U.S. real GDP since the mid-1980s, first documented by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), has spurred extensive research into its causes. A change in inventory behavior is often put forward as a leading explanation for this so-called Great Moderation, along with improved monetary policy practices ("better policy") and fortuitously smaller

We thank the editor, three anonymous referees, Steve Fazzari, Ben Herzon, Jun Ma, David Papell, Krishna Pendakur, Herman Stekler, Rodney Strachen, Shaun Vahey, seminar, and conference participants at George Washington University, Simon Fraser University, Australian National University, University of Adelaide, University of Cincinnati, University of Houston, University of Melbourne, University of New South Wales, University of Sydney, University of Wisconsin-Milwaukee, the SCE Meetings, the SNDE Meetings, the Australian Conference on Quantitative Macroeconomics, and the Australiasian Macroeconomics Workshop for helpful comments. The paper was written in part while Morley was a Visiting Scholar at the University of Sydney. The usual disclaimers apply.

JAMES MORLEY *is a professor in the School of Economics, UNSW Business School, University of New South Wales, Australia* (*E-mail:* james.morley@unsw.edu.au). AARTI SINGH *is a senior lecturer in the School of Economics, University of Sydney, Australia* (*E-mail:* aarti.singh@sydney.edu.au).

Received September 11, 2013; and accepted in revised form July 24, 2015.

Journal of Money, Credit and Banking, Vol. 48, No. 4 (June 2016)

macroeconomic shocks ("good luck").[1] The focus on inventories in particular is motivated by a striking feature of the aggregate data—output was more volatile than sales prior to the mid-1980s, but since then both variables have shared a common lower level of volatility. Given the accounting relationship between output, sales, and inventory investment, this excess volatility reduction in output compared to sales implies some role for inventories in the Great Moderation.

But what is it about inventory behavior that has changed? One possible answer is that firms shifted toward a greater emphasis on production smoothing. Golob (2000) finds that the stylized facts in the aggregate data emphasized by Blinder and Maccini (1991) as being so challenging to the relevance of production smoothing have shifted in a more favorable direction in recent years. Kahn, McConnell, and Perez-Quiros (2002) focus on the durable goods sector and find evidence of an improved ability of inventories to forecast future sales, leading them to argue that better information has facilitated increased production smoothing. By contrast, Herrera and Pesavento (2005) consider industry-level manufacturing and trade data and find little evidence of a change in the relationship between inventories and sales. [2]

In this paper, we estimate an unobserved components (UC) model using aggregate data to further investigate the role of inventories in explaining the decline in the volatility of U.S. real GDP. We find that a change in the sales process explains about half of the overall decline in output volatility, possibly reflecting the other leading explanations of better policy and good luck. Meanwhile, in terms of the excess volatility reduction in output compared to sales, we find that it reflects smaller transitory inventory shocks rather than a shift toward greater production smoothing. These inventory shocks behave like informational errors made by firms in response to noisy signals when setting production in advance of sales, and their reduction also helps to explain the apparent changed forecasting role of inventories with the Great Moderation.

Our findings have important implications for the much-questioned continuation of the Great Moderation. Although inventory shocks due to informational errors are likely to continue buffeting the economy, their reduction in size should reflect structural changes such as improved informational flows and the rise of "just-in-time" production. Thus, even if the Great Moderation has been primarily driven by smaller macroeconomic shocks rather than changes in their propagation, as emphasized by Stock and Watson (2003), Ahmed, Levin, and Wilson (2004), and many others, the shocks are not just be those that fit under the ephemeral-sounding "good luck" hypothesis. In particular, despite the dramatic movements in output during the recent economic crisis, the likely technological and structural reasons behind smaller

1. On these explanations, see McConnell and Perez-Quiros (2000), Clarida, Gali, and Gertler (2000), Stock and Watson (2003), and Ahmed, Levin, and Wilson (2004), among many others.

2. More consistent with increased production smoothing, Irvine and Schuh (2005) find declining covariances for sales and inventories within and across industrial sectors. However, Herrera, Murtazashvili, and Pesavento (2009) find rising, not declining, correlations across sectors when controlling for changing dynamics. Meanwhile, based on a vector autoregressive (AR) model of aggregate and industry-level data, McCarthy and Zakrajsek (2007) find some support for increased production smoothing, but that other factors are more important.

inventory shocks suggest that we should not expect a return to the persistent high levels of output volatility experienced during the 1970s and earlier.[3]

The rest of this paper is organized as follows. Section 2 provides background in terms of stylized facts motivating our analysis and a cost-minimization problem that sets the theoretical context for interpreting our empirical results. Section 3 develops the UC model that we use to disentangle the roles of inventory and sales shocks and their propagation in explaining the Great Moderation. Section 4 reports the empirical results for our model. Section 5 concludes.

1. BACKGROUND

1.1 Output Volatility and Its Components

In our empirical analysis, we relate output, sales, and inventories by the following identity:

$$y_t \equiv s_t + \Delta i_t, \tag{1}$$

where y_t is the natural logarithm of output, s_t is the natural logarithm of sales, and Δi_t is a residual-based measure of inventory investment. The true accounting identity is between the levels of output, sales, and inventory investment, whereas the residual measure is approximately equal to actual inventory investment as a percentage of sales (i.e., $\Delta i_t = ln(1 + \Delta I_t/S_t) \approx \Delta I_t/S_t$, where ΔI_t is the level of inventory investment and S_t is the level of sales). We consider the residual-based measure because it allows us to relate changes in the estimated sales and inventory processes from our UC model directly to the change in output volatility, which is the primary aim of our analysis.[4]

We use quarterly data for the sample period of 1960Q1–2014Q1 from the Bureau of Economic Analysis (BEA) on U.S. real GDP and final sales (NIPA Table 1.2.6) to measure the variables in equation (1). Corresponding to the timing of the Great Moderation, we split the data into pre- and postmoderation subsamples of 1960Q1–1984Q1 and 1984Q2–2014Q1.[5]

Table 1 reports sample statistics related to the volatility of the first differences of the variables in equation (1). The first stylized fact to emerge from these sample statistics is that U.S. real GDP growth stabilized dramatically in recent years, as

3. Somewhat related, Clark (2009) considers whether the increase in volatility during the Great Recession was as widespread across different sectors of the economy as with the decline in volatility for the Great Moderation. He finds that the increased volatility was driven by large oil price and financial shocks. Thus, he argues the Great Moderation will continue as the effects of these particular shocks dissipate.

4. This measure was also considered in Kahn, McConnell, and Perez-Quiros (2002). In Section 4.6, we consider the robustness of our results to using a direct measure of inventory investment based on the first differences of the log stock of inventories (i.e., inventory investment as a percentage of the lagged stock of inventories).

5. Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) estimate the structural break in the variance of U.S. real GDP growth to have occurred in 1984Q1. To keep our analysis focused, we treat this break date as known for the purposes of estimation, though we note that there is some degree of uncertainty about its exact timing (see, e.g., Stock and Watson 2003, Eo and Morley 2015).

TABLE 1
SAMPLE STATISTICS

	Premoderation (1960Q1–1984Q1)	Postmoderation (1984Q2–2014Q1)	Change across subsamples
$SD\ (\Delta y_t)$	1.06	0.61	−0.45
$SD\ (\Delta s_t)$	0.83	0.57	−0.26
$SD\ (\Delta^2 i_t)$	0.67	0.38	−0.29
corr.$(\Delta s_t, \Delta^2 i_t)$	−0.01	−0.23	−0.22

NOTES: Sample standard deviation (*SD*) and correlation (corr.) statistics are reported for the first differences of log output, log sales, and a residual measure of inventory investment based on the difference between log output and log sales. All series are multiplied by 100.

has been widely reported in the literature. The second stylized fact is that output was more volatile than sales in the premoderation period, but both variables have a similar lower level of volatility in the postmoderation period, which has also been discussed previously (see, e.g., Kahn, McConnell, and Perez-Quiros 2002, Golob 2000).

One possible explanation for the excess volatility reduction in output compared to sales is an increased emphasis on production smoothing by firms. Yet the sample statistics in Table 1 provide mixed signals about the overall relevance of production smoothing. In the premoderation period, the higher volatility of output compared to sales and the lack of a large negative contemporaneous correlation between sales and inventories directly undermine the idea that firms use inventories to buffer production from fluctuations in sales, a point emphasized in the survey article by Blinder and Maccini (1991). By contrast, the shift to more similar levels of volatility and a negative contemporaneous correlation between sales and inventories in the postmoderation period is more consistent with production smoothing, as pointed out by Golob (2000). However, the fact that sales and inventories also became less volatile in the postmoderation period clearly argues against production smoothing as the sole explanation for the Great Moderation. In addition, the fact that output is still no less volatile than sales in the postmoderation period continues to argue against production smoothing as the primary motive for holding inventories.[6]

These mixed signals from the sample statistics in Table 1 motivate our development of a UC model of the aggregate data to help disentangle the role of increased production smoothing from other factors in explaining the Great Moderation.

1.2 Inventories and Forecasting

Beyond the well-known reduction in volatility, the Great Moderation also corresponded to a change in the forecasting role of inventories (see Kahn, McConnell,

6. Also, as emphasized by Blinder and Maccini (1991), changes in finished goods inventories, which can be most directly related to the production smoothing motive, are neither the largest nor most volatile component of inventory investment.

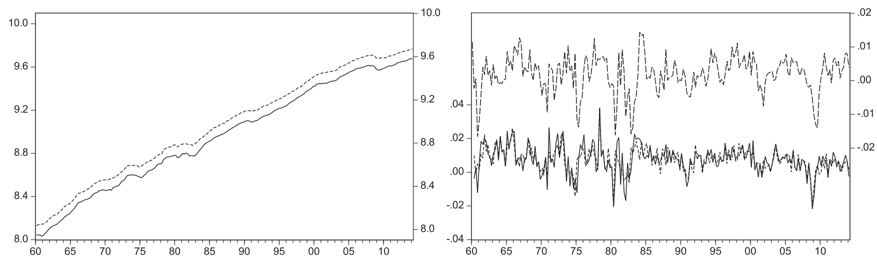

FIG. 1. Aggregate Output and Sales.

NOTES: The left panel plots real GDP (solid line, left vertical axis) and final sales (dashed line, right vertical axis), both expressed in natural logarithms. The first differences of the two series (solid line and dashed line, right vertical axis) and the difference between the two series (thick dashed line, left vertical axis) are plotted in the right panel. The sample period is 1960Q1–2014Q1.

and Perez-Quiros 2002). Figure 1 motivates why inventory investment is particularly useful for forecasting output and sales. The left panel plots log output and log sales using the BEA data discussed above. Both series are nonstationary, which is easily confirmed by standard unit root and stationarity tests. However, the two series appear to share the same stochastic trend. The right panel plots the first differences of both series and the difference between the two series, which is the residual measure of inventory investment defined in equation (1). All of these series are stationary, which again is easily confirmed by standard tests.

More formally, the idea that the residual measure of inventory investment is stationary corresponds to cointegration between log output and log sales, with a cointegrating vector of $(1, -1)'$.[7] Cointegration means that output and sales share the same stochastic trend, which is important because it implies that the cointegrating error term (i.e., the residual measure of inventory investment) must predict future movements in either output or sales (or both) in order for the long-run cointegrating relationship to be restored over time.

We demonstrate the change in the forecasting role of inventories with a simple vector error correction model (VECM) given as follows:

$$\Delta y_t = \gamma_{y,0} + \alpha_y \Delta i_{t-1} + \sum_{j=1}^{p} \gamma_{yy,j} \Delta y_{t-j} + \sum_{j=1}^{p} \gamma_{ys,j} \Delta s_{t-j} + \nu_{y,t}, \tag{2}$$

$$\Delta s_t = \gamma_{s,0} + \alpha_s \Delta i_{t-1} + \sum_{j=1}^{p} \gamma_{ss,j} \Delta s_{t-j} + \sum_{j=1}^{p} \gamma_{sy,j} \Delta y_{t-j} + \nu_{s,t}, \tag{3}$$

7. Granger and Lee (1989) find "multicointegration" between output and sales, with vector $(1, -1)'$, and between inventories and sales, with an estimated vector. However, their analysis is in terms of levels rather than logarithms and they consider sectoral data. For the aggregate data considered here, we find stronger adjustment to a long-run relationship for log output and log sales than for the levels, whereas we find no evidence of cointegration between the accumulation of the residual measure of inventory investment and log sales.

TABLE 2

Error-Correction Coefficient Estimates

	Premoderation (1960Q1–1984Q1)	Postmoderation (1984Q2–2014Q1)
α_y	−0.68 (0.18)	−0.21 (0.15)
α_s	−0.11 (0.16)	0.52 (0.14)

Notes: Ordinary least-squares estimates are reported, with standard errors in parentheses. Results are qualitatively robust for different numbers of lags and are reported for $p = 2$, with estimates of the other parameters omitted for simplicity.

where the α parameters are error-correction coefficients given that $\Delta i_t = y_t - s_t$, the p lagged differences of output and sales capture short-run dynamics, and the ν shocks are assumed to be serially uncorrelated.

Table 2 reports estimates for the error-correction coefficients. In the premoderation period, a positive change in inventories predicts a large decline in future output, all else equal, whereas inventory investment has no significant relationship with future sales. The results for the postmoderation period are strikingly different. First, even though the point estimate still suggest that a positive change in inventories predicts a decline in future output, the magnitude of the effect is much smaller and is no longer statistically significant at the 5% level. Second, a positive change in inventories predicts an increase in future sales, all else equal. Therefore, inventories appeared to have a strong negative relationship with future output prior to the Great Moderation, but since then inventories have had a strong positive relationship with future sales.

At first glance, the finding that inventories predict future sales in the postmoderation period might seem highly supportive of increased production smoothing. For example, Kahn, McConnell, and Perez-Quiros (2002) hypothesize that improvements in information technology have helped firms anticipate future sales, with inventories being more reflective of intentional production smoothing toward these future sales. However, as discussed in the next subsection, the forecasting role of inventories might have changed due to a different composition of the underlying shocks driving inventory investment rather than an increase in production smoothing. Unfortunately, the role of production smoothing versus a change in the composition of shocks cannot be disentangled from the VECM results alone. Again, as with the stylized facts in Table 1, we are motivated by these competing explanations to develop a UC model that is designed to identify the composition of shocks driving inventory investment.[8]

1.3 Cost Minimization Problem

To be more formal about the motives for holding inventories and set the theoretical context for interpreting our empirical results, we consider a linear-quadratic cost minimization problem that solves for optimal inventory management, similar to

8. A UC model can also be motivated by the finding for the error-correction coefficients that output and sales have played a role in restoring their long-run relationship. This directly implies the presence of a common unobserved trend, rather than one of the variables always acting as a *de facto* trend.

Blanchard (1983), West (1986, 1990), Ramey and West (1999), and Hamilton (2002), among many others.[9]

Letting C_t denote costs and assuming a discount factor $0 < \beta < 1$, the representative firm chooses a path for inventories to minimize its expected discounted costs over the infinite horizon:

$$\min_{\{I_{t+j}\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \beta^j C_{t+j}, \tag{4}$$

with the cost function given by

$$\begin{aligned} C_t = {} & 0.5a_1(\Delta Y_t)^2 + 0.5a_2(Y_t - S_t^*)^2 + 0.5a_3(\Delta I_t)^2 \\ & + 0.5a_4(I_{t-1} - a_5 S_t^*)^2 + u_{c,t} Y_t, \end{aligned} \tag{5}$$

where Y_t is the level of output, S_t^* is the long-run level of sales (and the permanent component of the marginal cost of production, as in Hamilton, 2002), $u_{c,t}$ is a transitory marginal cost shock, and the cost coefficients $a_i \geq 0$ for $i = 1, ..., 5$.

The costs motivating production smoothing are given by the first two terms in equation (5). Specifically, $a_1 > 0$ captures the idea that it is costly to change production in the short run and $a_2 > 0$ captures the idea that it is costly to have output too different from the long-run level of sales S_t^*. The costs motivating stockout avoidance are given by the third and fourth terms, where $a_3 > 0$ captures the idea that it is costly to draw down from or add to the stock of inventories in the short run and $a_4 > 0$ and $a_5 > 0$ capture the idea that it is costly to have inventories too different from a long-run level that depends positively on the long-run level of sales.[10]

For our partial equilibrium analysis, we treat prices and sales as exogenous. Thus, given the level of sales, the representative firm's inventory choices determine output according to the inventory identity:

$$Y_t \equiv S_t + \Delta I_t. \tag{6}$$

To complete the model, we need to specify the cost shock and the sales process, including its long-run level. First, we assume that the cost shock, $u_{c,t}$, is white noise and independent of sales.[11] Second, we assume that the level of sales has permanent

9. The cost-minimization problem is a version of the Holt et al.'s (1960) partial equilibrium linear-quadratic framework. See Wen (2005) for general equilibrium analysis of production smoothing and stockout avoidance motives for holding inventories.

10. For simplicity, we consider a continuous and symmetric version of the stockout avoidance motive. Instead of just being concerned with a literal "stockout" (i.e., having insufficient inventories to satisfy a large positive sales shock), which would correspond to a discrete and asymmetric specification for the cost, we assume that the representative firm implicitly has a large enough stock of inventories to satisfy any given sales shock, but the cost of doing so increases exponentially with the size of the shock.

11. Because the cost shock is multiplied by output in the cost function, its impact will be related to the scale of the economy. Meanwhile, its independence does not mean that all changes in marginal costs are independent of sales. Following Hamilton (2002), we assume that changes in costs impacting sales are reflected in sales shocks, though we acknowledge that this abstracts from any asymmetry whereby a

and transitory components, $S_t = S_t^* + e_{s,t}$, with $S_t^* = S_{t-1}^* + e_{p,t}$ corresponding to the stochastic trend in sales, where $e_{p,t}$ denotes a permanent sales shock, and transitory sales are driven by the transitory sales shock, $e_{s,t}$. As with the cost shock, the sales shocks are assumed to be white noise and independent of each other.

In Appendix A, we solve the cost minimization problem given in this section and derive some key theoretical results to inform our empirical analysis. These results and their main implications are summarized here.

RESULT 1. *Inventories and sales are cointegrated, with vector* $(1, -a_5)'$*, with deviations from the long-run relationship driven by permanent and transitory shocks.*

Based on this result, it is straightforward to show that log inventories and log sales will also be cointegrated, with vector $(1, -1)'$, an implication that informs the specification of our empirical model in the next section. Furthermore, the fact that deviations from the long-run relationship depend on permanent sales shocks in addition to transitory cost and sales shocks informs our allowance of permanent shocks to affect transitory deviations from trend in our empirical model.

RESULT 2. *Inventory investment is stationary and its persistence depends on the relative costs associated with production smoothing versus stockout avoidance, with the persistence increasing in* a_2*, the long-run cost motivating production smoothing, and decreasing* a_4*, the long-run cost motivating stockout avoidance.*

Based on this result and consistent with the VECM analysis in Section 2.2 and the specification of our empirical model in the next section, log output and log sales will also be cointegrated, with vector $(1, -1)'$. Our empirical model also implicitly allows inventory investment to be persistent.

RESULT 3. *The relative volatilities of output and sales and the correlation between sales and inventories depend on the prevailing shocks:*

i. *For permanent sales shocks,* $var(\Delta Y_t) > var(\Delta S_t)$ *and* $cov(\Delta S_t, \Delta^2 I_t) > 0$.
ii. *For transitory sales shocks,* $var(\Delta Y_t) < var(\Delta S_t)$ *and* $cov(\Delta S_t, \Delta^2 I_t) < 0$.
iii. *For cost shocks,* $var(\Delta Y_t) > var(\Delta S_t)$ *and* $cov(\Delta S_t, \Delta^2 I_t) = 0$.

Based on this result, a large role for permanent sales shocks could explain the higher volatility of output compared to sales reported in Table 1, though it does not explain the correlation results in either of the pre- or postmoderation periods.[12] A large role for transitory sales shocks could explain the negative correlation in the postmoderation period, though it cannot explain the relative volatilities in either of the pre- or postmoderation periods. Finally, a large role for cost shocks could explain the relatively high volatility of output and the lack of correlation between sales and

negative sales shock, such as an event like a spike in oil prices, affects costs more than an equivalent-sized positive shock.

12. Because output and sales share the same stochastic trend, it is straightforward to show using a first-order Taylor-series approximation evaluated at the common trend that a decrease in the ratio of variances of the first differences of the levels of output and sales is equivalent to an excess volatility reduction for continuously compounded output growth compared to continuously compounded sales growth.

inventories in the premoderation period, though it cannot explain the results in the postmoderation period.

RESULT 4. *The implied forecasting role of inventories depends on the prevailing shock:*

i. For a permanent sales shock, inventory investment has a positive relationship with future output growth but no relationship with future sales growth.
ii. For a transitory sales shock, inventory investment has a positive relationship with future output growth and future sales growth, with a larger effect on future sales growth.
iii. For a cost shock, inventory investment has a negative relationship with future output growth but no relationship with future sales growth.

Based on this result, a large role for permanent sales shocks is not consistent with the any of the VECM results reported in Table 2, except perhaps the insignificant relationship of inventory investment with future sales growth in the premoderation period. A large role for transitory sales shocks is consistent with the postmoderation error-correction coefficient estimates. Finally, a large role for cost shocks is consistent with the premoderation error-correction coefficient estimates for output and sales growth.

According to these theoretical results, the changes in the behavior of inventories and their forecasting properties with the Great Moderation could reflect a change in the relative costs motivating production smoothing versus stockout avoidance and a change in the sales process. In particular, the excess volatility reduction in output compared to sales reported in Table 1 could be due to a relative decrease in the costs motivating stockout avoidance (i.e., less of a cost to accessing inventory stocks compared to the cost of changing production plans). Likewise, the change in the relationship of inventory investment with future sales reported in Table 2 could be due to a change in composition of sales shocks, with permanent shocks becoming relatively more important than transitory shocks.

Meanwhile, as discussed in Blinder and Maccini (1991) and Kahn, McConnell, and Perez-Quiros (2002), the nature of informational flows in the production process is such that some changes in inventories could be unintentional and unrelated to actual sales rather than optimal responses to sales shocks—that is, they may correspond to informational errors or "inventory mistakes." Beyond the timing for the long-run stockout avoidance term in the cost function, it might seem that our cost minimization analysis abstracts from the fact that some production must be set in advance based on noisy signals about sales.[13] However, inventory mistakes provide a leading example of cost shocks that should have no effect on the sales process. Specifically, the mistakes are costly to make, but they should not alter the path of sales (if they are truly

13. The trade-off between production smoothing and stockout avoidance in the cost minimization analysis implicitly captures the idea that it is less costly to set production in advance. Specifically, the costs associated with accessing inventories only need to be borne if a firm also finds it costly to immediately change production when sales are realized. Consequently, the key abstraction in our cost minimization analysis is in terms of information flows about sales, rather than a need to set production in advance.

mistakes), and under rational expectations, the path of sales should not imply any predictability in the informational errors. Thus, inventory mistakes should have the same effects as the cost shocks in our cost minimization analysis, including possibly explaining the excess volatility of output relative to sales and the negative relationship between inventories and future output in the premoderation data, as reported in Tables 1 and 2.

The key question addressed in this paper then is to what extent the Great Moderation reflected an increase in production smoothing and a change in the sales process or a decline in the importance of transitory inventory shocks that capture inventory mistakes and other changes in marginal costs that do not affect sales. Again, to help sort out these competing explanations for the sample statistics and the VECM results in Tables 1 and 2 and to answer this key question, we develop a UC model in the next section.

2. UNOBSERVED COMPONENTS MODEL

2.1 Model Specification

Our UC model separates each of the observable series for log output, log sales, and a measure of log inventories (derived as an accumulation of the residual measure of inventory investment) into a permanent component and a transitory deviation from the permanent component:

$$y_t \equiv \tau_t^* + \left(y_t - \tau_t^*\right), \tag{7}$$

$$s_t \equiv \tau_t^* + \left(s_t - \tau_t^*\right), \tag{8}$$

$$i_t \equiv i_t^* + \left(i_t - i_t^*\right). \tag{9}$$

The permanent components are specified as follows:

$$i_t^* = \tau_t^* + \kappa_t, \tag{10}$$

$$\tau_t^* = \mu_\tau + \tau_{t-1}^* + \eta_t, \qquad \eta_t \sim i.i.d.N(0, \sigma_\eta), \tag{11}$$

$$\kappa_t = \mu_\kappa + \kappa_{t-1} + \lambda_{\kappa\eta}\eta_t + \omega_t \qquad \omega_t \sim i.i.d.N(0, \sigma_\omega), \tag{12}$$

where i_t^* is the trend for inventories, τ_t^* is the common trend for output and sales, which also affects inventories, and κ_t is an additional trend that allows for permanent changes in the inventory–sales ratio. The trends have deterministic drifts μ_τ and μ_κ, respectively, and they are driven by η_t, the permanent sales shock, and ω_t, the permanent shock to the inventory–sales ratio, respectively.

The specification of a common trend for output and sales is based on the empirical and theoretical results in Sections 2.2 and 2.3 that log output and log sales are cointegrated with vector $(1\ -1)'$. The additional trend, κ_t, captures the empirical result that our measure of inventories is not cointegrated with output or sales, perhaps reflecting time variation in the accelerator parameter a_5 in the cost minimization setting. Also, because the residual-based measure of inventories approximately corresponds to an accumulation of the inventory-investment-to-sales ratio, we allow the permanent sales shock to affect the additional trend via the impact coefficient $\lambda_{\kappa\eta}$ in equation (12), thus capturing the implied correlation between sales growth and the accumulated inventory–sales ratio.[14]

The transitory components follow stationary processes:

$$\psi_y(L)^{-1}\left(y_t - \tau_t^*\right) = \lambda_{y\eta}\eta_t + \lambda_{y\omega}\omega_t + \lambda_{y\epsilon}\epsilon_t + \upsilon_t, \tag{13}$$

$$\psi_s(L)^{-1}\left(s_t - \tau_t^*\right) = \lambda_{s\eta}\eta_t + \epsilon_t, \tag{14}$$

$$\psi_i(L)^{-1}\left(i_t - i_t^*\right) = \lambda_{i\eta}\eta_t + \lambda_{i\omega}\omega_t + \lambda_{i\epsilon}\epsilon_t + \upsilon_t, \tag{15}$$

where the $\psi(L)$ lag polynomials capture invertible Wold coefficients and $\lambda_{y\eta}$, $\lambda_{y\omega}$, $\lambda_{y\epsilon}$, $\lambda_{s\eta}$, $\lambda_{i\eta}$, $\lambda_{i\omega}$, and $\lambda_{i\epsilon}$ are the impact coefficients for transitory output, sales, and inventories in response to the shocks. The transitory shocks are $\epsilon_t \sim i.i.d.N(0, \sigma_\epsilon)$, and $\upsilon_t \sim i.i.d.N(0, \sigma_\upsilon)$, where ϵ_t is a transitory sales shock and υ_t is a transitory inventory shock, which, as discussed in more detail in the next subsection, could reflect informational errors.

Because output, sales, and inventory investment are linked together by equation (1), the impact coefficients are related as follows:

$$\lambda_{y\eta} = 1 + \lambda_{\kappa\eta} + \lambda_{i\eta} + \lambda_{s\eta}, \tag{16}$$

$$\lambda_{y\epsilon} = 1 + \lambda_{i\epsilon}, \tag{17}$$

$$\lambda_{y\omega} = 1 + \lambda_{i\omega}. \tag{18}$$

Therefore, only five of the eight impact coefficients in the model are independently determined. We also place bounds on these coefficients by relating them to the different terms in the cost function in the cost minimization problem in Section 2.3. In particular, an extreme focus on production smoothing corresponds to $\Delta y_t = 0$ for the short-run motive and $y_t - \tau_t^* = 0$ for the long-run motive, whereas an extreme focus on stockout avoidance corresponds to $\Delta i_t = 0$ for the short-run motive

14. The impact of transitory sales on the additional trend was not significant in a more general version of the model and so was dropped for simplicity.

and $i_t - i_t^* = 0$ for the long-run motive. Based on equations (13)–(18), it is reasonably straightforward to show that these extremes imply the following bounds: $\lambda_{y\eta} \in [\min(-1, \lambda_{\kappa\eta}), \max(0, 1 + \lambda_{\kappa\eta})]$ given $\lambda_{s\eta} \in [-1, 0]$, which depends on the extent to which sales adjust to a permanent shock on impact, with the extremes corresponding to $\Delta s_t = 0$ and $s_t - \tau_t^* = 0$; $\lambda_{y\epsilon} \in [0, 1]$; and $\lambda_{i\omega} \in [-1, 0]$.[15]

In this model, the transitory deviations from trend are driven not only by transitory shocks, but also by adjustments to permanent shocks, which is consistent with the first theoretical result reported in Section 2.3. By assuming this flexible structure, permanent and transitory movements are allowed to be correlated, even though the underlying shocks are specified to be mutually uncorrelated. As discussed in Morley, Nelson, and Zivot (2003), a UC model with correlated components is identified given sufficiently rich dynamics. For our application, we estimate the model for sales and inventories assuming AR(2) dynamics for their transitory components (i.e., $\psi_s(L)^{-1} = 1 - \phi_{s,1}L - \phi_{s,2}L^2$ and $\psi_i(L)^{-1} = 1 - \phi_{i,1}L - \phi_{i,2}L^2$, with roots of the AR polynomials lying strictly outside the unit circle to ensure stationarity) and leaving the process for output implicit. The two-variable UC model has 15 independent parameters and corresponds to a reduced-form vector autoregressive moving-average (VARMA) process with 17 parameters.[16] As a result, the model is identified, though weak identification is still a potential problem, as discussed in more detail in Section 4.1. A state-space representation of the UC model is presented in Appendix B.

2.2 *Interpretation of Shocks*

Permanent and transitory sales shocks, η_t and ϵ_t, should capture technology and demand factors in the aggregate economy. The permanent inventory shocks, ω_t, should capture any permanent changes in the inventory–sales ratio due to evolving inventory management practices, shifts in production from goods to services, and changes in the costs of accessing and holding inventories, whereas the transitory inventory shocks, υ_t, should capture informational errors that arise due to noisy signals firms receive about sales in conjunction with the fact that some production must be set in advance of sales.[17]

15. The bounds for the transitory sales shock and the permanent inventory shock are easy to derive given that $\Delta y_t \neq 0$ unless $\lambda_{y\epsilon} = 0$ and $\lambda_{i\omega} = 0$, whereas $\Delta i_t \neq 0$ unless $\lambda_{y\epsilon} = 1$ and $y_t - \tau_t^* \neq 0$ unless $\lambda_{i\omega} = -1$. The bounds for the permanent sales shock are a bit more complicated to derive, but follow given that $\Delta s_t \neq 0$ unless $\lambda_{s\eta} = -1$ and $s_t - \tau_t^* \neq 0$ unless $\lambda_{s\eta} = 0$, whereas $\Delta i_t \neq 0$ unless $\lambda_{y\eta} = 0$ when $\lambda_{s\eta} = 0$ and $i_t - i_t^* \neq 0$ unless $\lambda_{y\eta} = 1 + \lambda_{\kappa\eta}$ when $\lambda_{s\eta} = 0$ or $\lambda_{y\eta} = \lambda_{\kappa\eta}$ when $\lambda_{s\eta} = -1$.

16. There are four AR parameters and two drift terms that are common to both specifications. In addition, the two-variable UC model has four variance parameters and five independent impact coefficients, whereas the VARMA model has three variance–covariance parameters and eight MA parameters associated with two lags of vector MA terms. Note that because sales and inventories are not restricted to be cointegrated, our multivariate UC model is more analogous to Sinclair (2009) than to Morley (2007).

17. Kahn, McConnell, and Perez-Quiros (2002) consider similar unintended inventory shocks and note that their magnitude reflects the flow of information about future sales and the extent to which production needs to be set in advance. For example, if a firm increases production based on an advance order, a cancellation of the order would not be predicted and the resulting inventory accumulation would be a mistake. However, if production could be held off closer to the date of sale, fewer mistakes would be made.

The key distinction between sales shocks and inventory shocks in the UC model is that inventory shocks are assumed to have no direct impact on current or future sales—that is, this is analogous to a structural vector autoregressive (SVAR) model in which identification is achieved in part by assuming sales are exogenous, much like a foreign block is typically assumed to be exogenous in SVAR models for small open economies. Meanwhile, unexpected changes in inventories that do affect aggregate demand will be classified as transitory sales shocks, as will temporary cost shocks that have aggregate effects, including temporary shocks to productivity (e.g., Miron and Zeldes 1989, Hamilton 2002) or input cost shocks (e.g., Maccini, Moore, and Schaller 2015). Conversely, any temporary cost shocks that do not affect aggregate sales will be categorized as transitory inventory shocks. However, our conjecture is that at the aggregate level, most other cost shocks (e.g., oil price shocks) should have an impact on sales and would be captured by the sales shocks in our UC model.

2.3 *Implied Forecast Errors and Forecasting*

To the extent that the transitory inventory shocks reflect informational errors, they should be related to forecast errors for inventories. However, there is an important distinction between inventory mistakes, as captured by transitory inventory shocks in the UC model, and the overall forecast error for our residual measure of inventory investment. Indeed, this distinction explains why the UC model is particularly helpful in examining the role of inventories in the Great Moderation and the changed forecasting role of inventories.

The forecast error for our residual measure of inventory investment is defined as

$$\Delta i_t^{fe} \equiv \Delta i_t - E_{t-1}[\Delta i_t], \tag{19}$$

where Δi_t is the actual change in inventories and $E_{t-1}[\Delta i_t]$ is the expected change in inventories. Assuming firms observe the underlying shocks hitting the economy and have rational expectations, the UC model implies the following structure for the forecast error:

$$\Delta i_t^{fe} = y_t - s_t - E_{t-1}[y_t - s_t] = (\lambda_{y\eta} - \lambda_{s\eta})\eta_t + (\lambda_{y\epsilon} - 1)\epsilon_t + \lambda_{y\omega}\omega_t + \upsilon_t. \tag{20}$$

The forecast error reflects all sales and inventory shocks at date t, with only part of the forecast error due to informational errors based on noisy signals, as captured by the transitory inventory shock υ_t. For the other shocks, firms implicitly choose how to respond via the impact coefficients, where these coefficients can be related to their desire to smooth production versus a fear of stockouts, as mentioned when discussing bounds on these coefficients in Section 3.1. For instance, depending on firms' motives, along with how much sales immediately adjust to a permanent shock, there will be accumulation of inventories in the current period by a factor of $(\lambda_{y\eta} - \lambda_{s\eta})$, and this factor is what makes the accumulation intentional.

TABLE 3

PARTIAL EFFECTS OF FORECAST ERRORS

	Permanent shocks		Transitory shocks	
	η_t	ω_t	ϵ_t	υ_t
$\frac{\partial \Delta y_{t+1}}{\partial \Delta i_t^{fe}}$	$\frac{\lambda_{s\eta}(\phi_{s,1}-1)+\lambda_{i\eta}(\phi_{i,1}-2)-1-\lambda_{\kappa\eta}}{1+\lambda_{i\eta}+\lambda_{\kappa\eta}}$	$\frac{\lambda_{i\omega}(\phi_{i,1}-2)-1}{1+\lambda_{i\omega}}$	$\frac{(\phi_{s,1}-1)+\lambda_{i\epsilon}(\phi_{i,1}-2)}{\lambda_{i\epsilon}}$	$\phi_{i,1}-1$
$\frac{\partial \Delta s_{t+1}}{\partial \Delta i_t^{fe}}$	$\frac{\lambda_{s\eta}(\phi_{s,1}-1)}{1+\lambda_{i\eta}+\lambda_{\kappa\eta}}$	0	$\frac{(\phi_{s,1}-1)}{\lambda_{i\epsilon}}$	0

NOTES: Partial effects are based on marginal effects of shocks on the inventory forecast error and future output and sales growth.

How then does the UC model help in understanding the changed forecasting role of inventories captured by the VECM results in Table 2? One explanation for the VECM results is that inventory changes became more predictable and provide a better signal of future sales. We consider this possibility by calculating and comparing the variances of the inventory forecast error and expected inventory investment (i.e., $\Delta i_t^e = \Delta i_t - \Delta i_t^{fe} = E_{t-1}[\Delta i_t]$). Appendix C describes how we calculate these variances for our UC model.

An alternative explanation for the changed forecasting role is that the composition of underlying shocks in an inventory forecast error has changed, with inventory mistakes, as captured by transitory inventory shocks, playing a smaller role. To investigate the effects of a change in the composition of shocks, and therefore, relate the UC model to the VECM results, we solve for the partial effects of an inventory forecast error on future output growth and future sales growth. To do this, we first analytically compute the following marginal effects: (i) the impact of each shock on future output and sales growth and (ii) the impact of each shock on an inventory forecast error. Taking the ratio of these marginal effects, we are able to calculate the impact of an inventory forecast error on output growth and sales growth due to a particular shock, holding all else equal. This is similar to the results in Section 2.3 on the forecasting implications of different shocks in the cost minimization problem. Table 3 presents the implied partial effects of a forecast error, which are clearly different for the various underlying shocks. Thus, a change in the relative importance of these shocks directly implies a change in the reduced-form forecasting role of inventories.

3. EMPIRICAL RESULTS

3.1 Data and Estimation Methods

As considered in Section 2, the raw data are quarterly U.S. real GDP and final sales from the BEA for the subsample periods of 1960Q1–1984Q1 and 1984Q2–2014Q1. We estimate the UC model for sales and inventories, leaving the estimated process for output implicit. Our measure for sales is 100 times the natural logarithms of real sales, and our measure for inventories is calculated by first constructing a residual measure of the change in inventories based on the identity given in equation (1) for 100 times log output and 100 times log sales and then accumulating changes given an arbitrary initial level of log inventories.

We estimate the UC model using Bayesian posterior simulation based on Markov-chain Monte Carlo (MCMC) methods. Specifically, we consider a multiblock random-walk chain version of the Metropolis-Hastings (MH) algorithm with 500,000 draws after a burn-in of 20,000 draws. We check the robustness of our posterior moments to different runs of the chain and for different starting values. The multiblock setup allows us to obtain relatively low correlations between parameter draws, suggesting the sampler is working well. See Chib and Greenberg (1995) for more details on the MH algorithm.

There are two reasons why we consider Bayesian estimation. First, UC models can suffer from weak identification. In particular, UC models are closely related to VARMA models, which are notoriously difficult to estimate due to the problem of near cancellation of AR and MA terms and multiple modes for the likelihood surface. A particularly troublesome estimation difficulty is the so-called pile-up problem whereby maximum likelihood estimates tend to hit boundaries even when true parameters are not equal to boundary values. Preliminary analysis via maximum likelihood estimation (MLE) confirmed multiple modes and possible pile-up problems. By contrast, Bayesian estimation with relatively uninformative priors reveals a clear interior mode for the posterior. Our main inferences about the Great Moderation turn out to be robust to consideration of the MLE results or the interior mode. However, Bayesian estimation provides a sense of parameter uncertainty that we cannot obtain for the MLE results given that some parameters hit boundaries. The second reason why we consider Bayesian estimation is that it provides posterior moments not only for the model parameters, but also for complicated functions of the model parameters that are of particular interest such as counterfactual standard deviations of output growth and implied error-correction parameters.

Our priors are specified as follows: (1) AR coefficients have standard normal distributions (i.e., $N(0, 1)$), truncated to ensure stationarity (i.e., the roots of the characteristic equations for the AR lag polynomials lie outside the unit circle); (2) the drift for the additional trend in gross inventories has a diffuse $N(0, 100)$ distribution, whereas the drift for long-run sales (and output) is concentrated out of the likelihood by recentering the growth rate data; (3) the precisions (inverse variances) have $\Gamma(0.01, 0.01)$ distributions, corresponding to highly diffuse priors for the variances; (4) the impact coefficients have standard normal distributions with means recentered to the midpoints of the bounds given in Section 3.1 assuming $\lambda_{\kappa\eta} = 0$ and truncation to ensure that the coefficients lie within or on the bounds for any value of $\lambda_{\kappa\eta}$; and (5) the initial values for the permanent levels of sales and inventories in the pre-moderation period have diffuse normal distributions centered at initial observations (minus one-period drifts) and variances of $10,000$. All of the priors are relatively uninformative in the sense that the posteriors are dominated by the likelihood and

TABLE 4

Parameters and Implied Moments for the UC Model

	Premoderation (1960Q1–1984Q1)	Postmoderation (1984Q2–2014Q1)	Change across subsamples
Sales process			
σ_η	2.68 (1.63)	1.29 (0.40)	−1.39 (1.47)
σ_ϵ	0.56 (0.09)	0.31 (0.06)	−0.24 (0.11)
ϕ_s^*	0.81 (0.12)	0.81 (0.07)	0.00 (0.12)
$\lambda_{s\eta}$	−0.82 (0.12)	−0.74 (0.11)	0.08 (0.14)
Inventory process			
σ_ω	1.56 (1.14)	0.68 (0.22)	−0.88 (1.13)
σ_υ	0.37 (0.07)	0.17 (0.03)	−0.21 (0.08)
ϕ_i^*	0.88 (0.07)	0.70 (0.09)	−0.18 (0.09)
μ_κ	−0.68 (0.17)	−0.44 (0.07)	0.24 (0.18)
$\lambda_{\kappa\eta}$	0.04 (0.55)	−0.41 (0.18)	−0.44 (0.56)
$\lambda_{y\eta}$	−0.87 (0.12)	−0.75 (0.10)	0.13 (0.14)
$\lambda_{y\epsilon}$	0.76 (0.15)	0.73 (0.14)	−0.03 (0.20)
$\lambda_{i\omega}$	−0.86 (0.14)	−0.87 (0.09)	−0.01 (0.16)
Unconditional volatilities and correlations			
$SD\ (\Delta y_t)$	1.17 (0.14)	0.65 (0.05)	−0.53 (0.14)
$SD\ (\Delta s_t)$	0.97 (0.14)	0.57 (0.05)	−0.39 (0.14)
$SD\ (\Delta^2 i_t)$	0.73 (0.06)	0.40 (0.03)	−0.34 (0.07)
corr. $(\Delta s_t, \Delta^2 i_t)$	−0.08 (0.08)	−0.15 (0.07)	−0.08 (0.11)
$SD\ (\Delta i_t)$	0.85 (0.19)	0.48 0.06)	−0.38 (0.19)
$SD\ (\Delta i_t^{fe})$	0.47 (0.06)	0.22 (0.03)	−0.24 (0.06)
$SD\ (\Delta i_t^{e})$	0.70 (0.22)	0.42 (0.07)	−0.28 (0.22)
corr. $(\Delta s_t, \Delta i_t^{fe})$	−0.26 (0.11)	−0.25 (0.13)	0.00 (0.16)

Notes: Posterior means of the parameters and implied moments for the UC model are reported, with posterior standard deviations in parentheses. The ϕ^* parameters refer to sums of autoregressive coefficients for the AR(2) processes.

our main qualitative inferences are robust to a range of different priors, including the flat/improper priors implicit in the consideration of MLE.

3.2 *Estimates*

Table 4 reports posterior means and standard deviations for the parameters of the UC model and their changes from the premoderation period to the postmoderation period. The estimated shock volatilities become smaller in all cases, though the declines are only significant for the transitory shocks. The persistence of transitory sales is stable across the two periods, whereas the persistence of transitory inventories has declined. The estimated drift for the additional trend in inventories is negative, reflecting the fact that the unconditional expectation of inventory investment implied by the UC model is $E[\Delta i_t] = \mu_\tau + \mu_\kappa$ based on equations (9), (10), and (15) and the fact that inventory investment is close to zero on average (see Figure 1). In particular, if $E[\Delta i_t] \approx 0$, the positive drift for output and sales captured by μ_τ must be offset by a negative value of μ_κ of similar magnitude (i.e., $\mu_\tau \approx -\mu_\kappa$).[18] Correspondingly,

18. There are small differences in the magnitudes of the drifts that could reflect secular changes in the inventory–sales ratio, perhaps due to the shift in aggregate production from manufacturing to services and to improvements in inventory and supply chain management (see Davis and Kahn 2008).

the change in magnitude in the drift for the additional trend in inventories from the premoderation to postmoderation period is very similar to (but opposite sign of) the change in the average growth rate of sales, though it is not significant. Meanwhile, the estimated impact coefficients are generally similar in the two periods and their changes are not significant.

Given the mixed significance of parameter changes and the complicated relationships between the UC model parameters and unconditional volatilities and correlations, such as those reported in Table 1, we also report posterior means and standard deviations for certain implied moments for the UC model. The estimated unconditional volatilities decline with the Great Moderation in all cases and the comparable estimates are qualitatively similar to the sample statistics reported in Table 1.[19] The changes are significant, except for the volatility of expected inventory investment and the correlations. Consistent with a countercyclical relationship between inventories and sales, the correlation between unexpected inventories and sales is negative. Meanwhile, the volatility estimates suggest an increase in the relative importance of expected inventories in overall inventory investment. At first glance, this change appears consistent with increased production smoothing and potentially explains the changed forecasting role of inventories in the recent sample. We investigate these possibilities in the next few subsections.

3.3 Increased Production Smoothing?

Given the decline in output volatility, it is natural to ask whether firms have increased their use of inventories to smooth production in the postmoderation period. Although the impact coefficients in the UC model can be related to the different motives for holding inventories, a lack of significance for their changes across subsamples provides no indication that production smoothing has increased. Meanwhile, as noted in Section 2.3, the persistence of inventory investment should depend on the relative costs motivating production smoothing and stockout avoidance. Thus, we can look at changes in the persistence of transitory inventories for our UC model to infer changes in the relative importance of these motives.[20] The estimate of ϕ_i^* is 0.86 in the premoderation period, suggesting that the costs motivating production smoothing were relatively high. However, the estimate of ϕ_i^* is 0.68 in the postmoderation period, suggesting somewhat less of a desire to smooth production in recent years.[21] Furthermore, this change is significant. Thus, the UC model

19. The exact numbers in Table 1 and the corresponding estimates in Table 4 are different because Table 4 reports posterior means, which tend to be different than posterior modes for variances. Indeed, the posterior modes (not reported) are almost identical to the statistics in Table 1, confirming that our priors are largely uninformative.

20. The persistence of inventory investment based on the levels of output and sales has a monotonic relationship with the persistence of our residual measure of inventory investment based on log output and log sales. Meanwhile, the UC model implies an ARMA(2,2) process for the residual measure of inventory investment with the same AR coefficients as transitory inventories. Thus, changes in the relative importance of motives should produce changes in the persistence of transitory inventories.

21. The coefficient ϕ_i^* is the sum of the two autoregressive coefficients for an AR(2) specification of transitory inventories. Thus, we are implicitly using the sum of the AR coefficients as our measure of persistence. However, the estimated reduction in persistence is also evident if we consider the largest

TABLE 5
COUNTERFACTUAL EXPERIMENTS

	Change in $SD(\Delta y_t)$
Actual	−0.53 (0.14)
Sales process	−0.31 (0.15)
Inventory process	−0.05 (0.27)
Shocks only	−0.12 (0.06)
Transitory shocks only	−0.11 (0.05)
Propagation only	0.15 (0.32)

NOTES: Posterior means of implied changes in volatility, measured in terms of the standard deviation of output growth, are reported, with posterior standard deviations reported in parentheses.

estimates provide more support for a decrease in production smoothing than an increase.

3.4 Counterfactuals

Next, we conduct counterfactual experiments to help disentangle the role of inventories from that of sales in explaining the overall decline in output volatility.[22] Our main objective is to determine the extent to which changes in the inventory process—(i) less volatile shocks and/or (ii) changes in the propagation mechanism (autoregressive and impact coefficients)—could have accounted for the Great Moderation. To do this, we hold the parameters of the sales process fixed at their premoderation values and let the parameters associated with inventories (σ_ω, σ_υ, ϕ_i^*, $\lambda_{\kappa\eta}$, $\lambda_{y\eta}$, $\lambda_{y\epsilon}$, and $\lambda_{i\omega}$) change to their postmoderation values. We also try to isolate the roles of different inventory shocks (σ_ω and σ_υ) and the propagation mechanism (ϕ_i^*, $\lambda_{\kappa\eta}$, $\lambda_{y\eta}$, $\lambda_{y\epsilon}$, and $\lambda_{i\omega}$) by changing only subsets of parameters at a time. For completeness, we consider an experiment in which the inventory process remains fixed and only the parameters of the sales process (σ_η, σ_ϵ, ϕ_s^*, and $\lambda_{s\eta}$) are allowed to change. Table 5 reports posterior means and standard deviations for the actual and counterfactual changes in output growth volatility.

According to the results in Table 5, a change in the sales process on its own could have generated about half of the overall actual decline in the standard deviation of output growth, with the change being significant. Given that the autoregressive dynamics for sales are quite similar in the pre- and postmoderation periods, this result conforms to the "good luck" hypothesis in the sense that smaller sales shocks

inverse root of the characteristic equation for the AR lag polynomial or the half-life based on an impulse response function.

22. See Stock and Watson (2003), Ahmed, Levin, and Wilson (2004), Sims and Zha (2006), and Kim, Morley, and Piger (2008), among many others, for counterfactual experiments with VAR models. Of particular relevance to the analysis here, Kim, Morley, and Piger (2008) discuss the benefits of Bayesian inference for counterfactual quantities. Specifically, Bayesian analysis produces posterior moments for the counterfactual quantities, thus providing a sense of estimation uncertainty that is not available in the classical context.

rather than a change in their propagation appears to be a key element of the Great Moderation.

In terms of inventories, the results in Table 5 suggest that they could not have generated as much of a reduction in output volatility on their own as for a change in the sales process. Furthermore, the counterfactuals clearly support the idea that the excess volatility reduction in output compared to sales was driven by smaller inventory shocks rather than a change in their propagation. Consistent with the lack of support for increased production smoothing discussed in the previous subsection, a change in inventory propagation alone would not have generated any reduction in volatility. Instead, almost the entire reduction in volatility that can be related to inventories appears to be due to a reduction in transitory inventory shocks, with this change being significant. Meanwhile, the sum of the counterfactual reductions in volatility is less than the overall reduction, suggesting that there was an important interaction between the changes in the sales and inventory processes in explaining the Great Moderation.

3.5 Implied Forecasting Role of Inventories

Even if increased production smoothing is not responsible for the reduction in output volatility, a question remains as to whether it is necessary to explain the changed forecasting role of inventories with the Great Moderation. Based on Table 4, a larger proportion of overall inventory investment is predictable from one period to the next, which is certainly consistent with increased production smoothing in anticipation of future sales. However, the analysis in Section 3.3 suggests that the forecasting role of inventories can also change with the composition of inventory forecast errors. Therefore, we consider whether smaller transitory inventory shocks that explain so much of the excess volatility reduction in output compared to sales can also explain the changed forecasting role of inventories.

We determine the implied forecasting role of inventories given a change in the composition of shocks by calculating the marginal effects presented in Table 3 based on our parameter estimates. Then, we weight these marginal effects by the contribution of each underlying shock to the overall forecast error, where the weights are given by the ratio of the standard deviation of a shock relative to the standard deviation of the overall inventory forecast error. This weighted average provides us with implied error-correction coefficients (in the absence of predictable inventory changes). Table 6 reports posterior means and standard deviations for the implied error-correction coefficients and their changes.

The estimates in Table 6 are qualitatively in line with the VECM results in Table 2. Specifically, there is a diminished negative forecasting relationship between inventories and future output growth and an increased positive forecasting relationship between inventories and future sales growth in the postmoderation period. The changes are not significant and the quantitative effects are somewhat different from the VECM results in Table 2, but this likely reflects the fact that the predictability of inventory investment has also changed along with the composition of shocks. The

TABLE 6

IMPLIED ERROR-CORRECTION COEFFICIENTS

	Premoderation (1960Q1–1984Q1)	Postmoderation (1984Q2–2014Q1)	Change across subsamples
$\frac{\partial \Delta y_{t+1}}{\partial \Delta i_t^{fe}}$	−1.11 (0.22)	−0.81 (0.39)	0.30 (0.43)
$\frac{\partial \Delta s_{t+1}}{\partial \Delta i_t^{fe}}$	−0.10 (0.18)	0.19 (0.31)	0.29 (0.35)

NOTES: Posterior means of error-correction coefficients implied by the UC model are reported, with posterior standard deviations in parentheses. The marginal impacts of the underlying shocks are weighted by their relative standard deviations.

main point is that the results in Table 6 are consistent with the changing composition of shocks (specifically smaller transitory inventory shocks) explaining the changed forecasting role of inventories with the Great Moderation, without needing to rely on increased production smoothing.

3.6 Robustness

When analyzing inventory behavior, there is always a question of which data to consider. Because our focus is on explaining the Great Moderation as manifested in U.S. real GDP, we consider the residual measure of inventory investment for the aggregate data. A reasonable question, though, is whether our main findings are robust to consideration of inventory stock data instead of the residual-based measure of inventories or durable goods data instead of the aggregate data.

Using the inventory stock data (NIPA Table 5.8.6A for the sample period of 1969Q1–2014Q1 and extrapolating back to 1960Q1 using actual inventory investment), we found the following results: first, the sample statistics for inventory investment are similar to those for our residual-based measure in Table 1. Second, sales and inventories are not cointegrated, supporting the inclusion of an additional trend in our UC model. Third, the estimate of $\lambda_{\kappa\eta}$ in our UC model is not significant when using the inventory stock measure, suggesting that the permanent inventory–sales shock, ω_t, is indeed independent of the permanent sales shock, as is assumed in our model. Fourth, the forecasting relationships as captured by error-correction coefficients using the first differences of the log stock of inventories, which is approximately the change in inventories as a percentage of the lagged stock of inventories, are not as strong as for our residual measure. Therefore, focusing on the residual-based measure of inventories is not only important because it allows us to directly relate changes in the sales and inventory processes from our UC model to the change in output volatility, but also because it is more relevant for understanding the changed forecasting role of inventories for output and sales.

When we estimated our UC model using quarterly output and sales data for durable goods from the BEA (NIPA Tables 1.2.3 and 1.2.5), the results were also robust and some our key findings were even more pronounced than for the aggregate data. Again, we find that sales and inventories are not cointegrated.[23] Meanwhile, the residual measure of inventory investment is responsible for a larger portion of the overall decline in output volatility than for the aggregate data. Consistent with this finding, the counterfactual analysis for the durable goods data suggests that inventories played a larger role than sales in the overall decline in durable goods output volatility. As with the aggregate data, inventory shocks played the primary role in the excess volatility reduction of output, with smaller transitory shocks accounting for most of this excess reduction. Meanwhile, the VECM results and forecasting implications from the UC model were quite similar to those for the aggregate data.

4. CONCLUSION

In this paper, we have investigated the role of inventories in the Great Moderation. Based on a UC model that identifies inventory and sales shocks and their propagation in the aggregate data, we find no evidence for increased production smoothing in recent years. Instead, smaller transitory inventory shocks are responsible for the excess volatility reduction in output compared to sales, with the rest of the overall decline in output volatility accounted for by smaller sales shocks. The smaller transitory inventory shocks are notable because they also appear to explain the changed forecasting role of inventories with the Great Moderation.[24]

In contemplating whether or not the Great Moderation is now over, it is important to consider what might have caused a reduction in transitory inventory shocks in the first place. To the extent that these shocks reflect informational errors about future sales and arise due to the fact that some production must be set in advance, their reduction could correspond to improved informational flows about future sales or to greater flexibility in terms of setting production closer to sales. Distinguishing between these two hypotheses is difficult. However, we might expect improved informational flows to reflect a change in the predictability of the sales process. Thus, our finding that the dynamics of transitory sales remain unchanged with the Great Moderation does not lend itself to an "improved forecast" hypothesis. Also, somewhat contrary to improved forecasts, which presumably occur gradually due to learning, is the fact that the volatility reduction appears to have been sudden (see Kim and Nelson 1999, McConnell and Perez-Quiros 2000, Eo and Morley 2015). Therefore, the rise of "just-in-time" production (see McConnell, Mosser, and Perez-Quiros 1999) is the more compelling explanation for smaller transitory inventory shocks, as it is more

23. Notably, this finding suggests that the lack of cointegration for the aggregate data is not simply due to a compositional effect, noted by Ramey and Vine (2004), whereby the inventory–sales ratio changes as services become a larger component of sales.

24. Despite a very different approach and data, our findings are in line with Herrera and Pesavento (2005). Specifically, they consider sectoral data and find that the decline in the volatility of inventories with the Great Moderation was larger and more prevalent among input goods than for finished goods. Given that production smoothing primarily relates to finished goods, their finding also argues against increased production smoothing explaining the Great Moderation, whereas it is entirely consistent with smaller inventory shocks.

plausible that new production processes were implemented quickly, especially after the deep recessions of the early 1980s. Also, our finding that the implied costs motivating production smoothing have declined relative to the costs motivating stockout avoidance is consistent with the idea that less production needs to be set in advance.

Although transitory inventory shocks might have become smaller for structural and technological reasons, it is unlikely that they will disappear altogether. In particular, the extra volatility in U.S. real GDP compared to final sales during the 2007–9 recession strongly supports the idea that some production must be set in advance and inventory mistakes will continue to be made. At the same time, given their links to technology and despite some large changes in inventories during the recent recession, a smaller variance for transitory inventory shocks provides a much more optimistic prognosis for the continuation of the Great Moderation than the "good luck" hypothesis (or, for that matter, the "better policy" hypothesis).

On a related note, it has long been understood that the role of inventories in output fluctuations is asymmetric in terms of business cycle phases, with a much larger role being played in recessions than in expansions (see, e.g., Blinder and Maccini 1991, and Golob 2000).[25] However, our analysis is based on a linear model and therefore does not capture this asymmetry. Given the predominance of expansions in the sample period covered in this paper, our results should reflect the behavior of output, sales, and inventories during expansions more than during recessions. This could, in part, explain some of the differences between our conclusions and those in a recent study by Maccini and Pagan (2013). They explicitly measure movements in output related to business cycle phases and find little role for inventories in the changed behavior of output with the Great Moderation.[26] It also means that we cannot draw strong conclusions about possible changes in recession and recovery dynamics due to inventories (see Camacho, Perez-Quiros, and Rodriguez-Mendizabal 2011). Modeling business cycle asymmetries associated with inventories presents its own challenges and opportunities, which we leave for future research.

APPENDIX A: COST MINIMIZATION SOLUTION

In this appendix, we solve the cost minimization problem given in Section 2.3 and derive the key theoretical results reported there.

For simplicity, we generate our key theoretical results by focusing on the long-run motives. Specifically, we set $a_1 = a_3 = 0$ in the cost function, though we consider

25. Granger and Lee (1989) find evidence of asymmetric error-correction effects depending on the sign of inventory investment relative to its mean and the sign of the cointegrating error for inventories and sales.

26. More consistent with our findings, Maccini and Pagan (2013) find that increased production smoothing does not play a role in the Great Moderation. Instead, they show that an estimated structural model based on premoderation data could only have generated the observed reduction in output volatility if the volatilities of the sales process and technology shocks declined by about half. In this sense, their results are strongly supportive of the "good luck" hypothesis. However, their model does not allow for inventory mistakes.

the short-run motives when interpreting some of the parameters for our empirical model in Section 3.1.

Defining $W_t \equiv I_t - a_5 S_t^*$ and $Z_t \equiv -a_5 e_{p,t} - e_{s,t} - (1/a_2)u_{c,t}$, the first-order condition for the simplified cost minimization problem is

$$E_t[(1+\beta+\theta)W_t - W_{t-1} - \beta W_{t+1}] = Z_t,$$

where $\theta \equiv \beta a_4/a_2$. Following Hansen and Sargent (1980), we can solve the above polynomial to get

$$W_t = \varphi W_{t-1} + \varphi Z_t, \tag{A1}$$

where $\varphi = \delta - (\delta^2 - 4\beta^{-1})^{1/2}/2$ is the stable root of the polynomial, with $\delta \equiv \beta^{-1}(1+\beta+\theta)$.

Because φ is the stable root, W_t will be stationary. Furthermore, it can be shown that $0 < \varphi < 1$ given the assumptions on the cost coefficients and the discount factor. Based on the stationarity of W_t and the elements of Z_t, we get the first result reported in Section 2.3.

It should be noted that in standard linear-quadratic models, such as Ramey and West (1999), the production cost using our notation is $0.5a_2Y_t^2 + u_{c,t}Y_t$, assuming $a_1 = 0$. Thus, if output is nonstationary, the marginal cost of production will go to infinity. By contrast, abstracting from the short-run cost to inventory adjustment and the incorporation of transitory sales shocks, our model is identical to Hamilton's (2002) model. Thus, the marginal cost of production remains finite, with the cointegrating relationship as given in the first result.

Based on equation (A1) and the assumptions for the sales process in Section 2.3, it is possible to then show that inventory investment follows an ARMA(1,1) process:

$$(1-\varphi L)\Delta I_t = a_5(1-\varphi)e_{p,t} - \varphi(1-L)e_{s,t} - \frac{1}{a_2}\varphi(1-L)u_{c,t}, \tag{A2}$$

where L is the lag operator. Because $0 < \varphi < 1$, we get the second result reported in Section 2.3, with the comparative statics for the effects of production smoothing and stockout avoidance on persistence given as follows:

$$\frac{\partial\varphi}{\partial a_2} = \frac{\partial\varphi}{\partial\delta}\frac{\partial\delta}{\partial\theta}\frac{\partial\theta}{\partial a_2} = -\frac{a_4}{2a_2^2}(1-\delta(\delta^2-4\beta^{-1})^{-1/2}),$$

$$\frac{\partial\varphi}{\partial a_4} = \frac{\partial\varphi}{\partial\delta}\frac{\partial\delta}{\partial\theta}\frac{\partial\theta}{\partial a_4} = \frac{1}{2a_2}(1-\delta(\delta^2-4\beta^{-1})^{-1/2}),$$

where, because $(\delta^2 - 4\beta^{-1})^{1/2} > 0$ for a real root, $0 < \varphi < 1$, and assuming positive cost coefficients, it is straightforward to show that $\partial\varphi/\partial a_2 > 0$ and $\partial\varphi/\partial a_4 < 0$.

Based on the sales process, the inventory identity, and the optimal inventory investment process in equation (A2), we can then solve for the relative variances and covariances under different assumptions about the prevailing shocks. First, consider only permanent sales shocks, setting the variances of the other shocks to zero. This implies the following variance ratio and covariance expressions:

$$\frac{\text{var}(\Delta Y_t)}{\text{var}(\Delta S_t)} = 1 + 2a_5(1-\varphi) + 2a_5^2\frac{(1-\varphi)^2}{1+\varphi},$$

$$\frac{\text{cov}(\Delta S_t, \Delta^2 I_t)}{\text{var}(\Delta S_t)} = a_5(1-\varphi).$$

It is straightforward to show that the variance ratio is strictly greater than one. Meanwhile, given that $a_5 > 0$ and $0 < \varphi < 1$, the covariance will be positive. Second, consider only transitory sales shocks, which implies the following expressions:

$$\frac{\text{var}(\Delta Y_t)}{\text{var}(\Delta S_t)} = \frac{(1-\varphi)^2}{1+\varphi},$$

$$\frac{\text{cov}(\Delta S_t, \Delta^2 I_t)}{\text{var}(\Delta S_t)} = \frac{\varphi(\varphi-3)}{2}.$$

Again, given $0 < \varphi < 1$, the variance ratio will be strictly less than one and the covariance will be negative. Finally, consider only cost shocks, which implies the following expressions:

$$\frac{\text{var}(\Delta Y_t)}{\text{var}(u_{c,t})} = \frac{2\varphi^2(3-\varphi)}{a_2^2(1+\varphi)},$$

$$\text{cov}(\Delta S_t, \Delta^2 I_t) = 0.$$

In this case, we cannot standardize by the variance of sales because it is zero. But it is trivial to see that output will be more volatile than sales and the covariance of inventories and sales will be zero. Taking all of these findings together, we get the third result reported in Section 2.3.

Then, for the fourth and last result reported in Section 2.3, we can calculate the partial effects of an unpredictable change in inventories on future output growth and future sales growth given a prevailing shock. To do this, we compute the ratio of the marginal effects of each shock on future output and sales growth and on inventories. First, consider a permanent sales shock:

$$\frac{\partial \Delta Y_{t+1}}{\partial e_{p,t}} \bigg/ \frac{\partial \Delta I_t}{\partial e_{p,t}} = \varphi,$$

$$\frac{\partial \Delta S_{t+1}}{\partial e_{p,t}} \bigg/ \frac{\partial \Delta I_t}{\partial e_{p,t}} = 0.$$

Because $0 < \varphi < 1$, inventory investment will have a positive relationship with future output growth, whereas there is no relationship with future sales growth. Second, consider a transitory sales shock:

$$\frac{\partial \Delta Y_{t+1}}{\partial e_{s,t}} \Big/ \frac{\partial \Delta I_t}{\partial e_{s,t}} = \frac{1 - \varphi + \varphi^2}{\varphi},$$

$$\frac{\partial \Delta S_{t+1}}{\partial e_{s,t}} \Big/ \frac{\partial \Delta I_t}{\partial e_{s,t}} = \frac{1}{\varphi}.$$

Again because $0 < \varphi < 1$, it is straightforward to show that inventory investment will have a positive relationship with future output growth and future sales growth, with second expression greater than one and greater than the first expression—that is, the effect on future sales growth is greater than the effect on future output growth. Finally, consider a cost shock:

$$\frac{\partial \Delta Y_{t+1}}{\partial u_{c,t}} \Big/ \frac{\partial \Delta I_t}{\partial u_{c,t}} = \varphi - 1,$$

$$\frac{\partial \Delta S_{t+1}}{\partial u_{c,t}} \Big/ \frac{\partial \Delta I_t}{\partial u_{c,t}} = 0.$$

In this case, because $0 < \varphi < 1$, inventory investment will have a negative relationship with future output growth, whereas there is no relationship with future sales growth.

APPENDIX B: STATE-SPACE REPRESENTATION

In this appendix, we present the state-space representation of the UC model in Section 3.1.

The observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\boldsymbol{\beta}_t,$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} s_t \\ i_t \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1\ 0\ 0\ 0\ 1\ 0 \\ 0\ 0\ 1\ 0\ 1\ 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} s_t - \tau_t^* \\ s_{t-1} - \tau_{t-1}^* \\ i_t - i_t^* \\ i_{t-1} - i_{t-1}^* \\ \tau_t^* \\ \kappa_t \end{bmatrix}.$$

The state equation is

$$\boldsymbol{\beta}_t = \tilde{\boldsymbol{\mu}} + \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\mathbf{v}}_t,$$

where

$$\tilde{\boldsymbol{\mu}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu_\tau \\ \mu_\kappa \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \phi_{s,1} & \phi_{s,2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{i,1} & \phi_{i,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \tilde{\boldsymbol{\upsilon}}_t = \begin{bmatrix} \lambda_{s\eta}\eta_t + \epsilon_t \\ 0 \\ \lambda_{i\eta}\eta_t + \lambda_{i\omega}\omega_t + \lambda_{i\epsilon}\epsilon_t + \upsilon_t \\ 0 \\ \eta_t \\ \omega_t + \lambda_{\kappa\eta}\eta_t \end{bmatrix},$$

and the covariance matrix of $\tilde{\boldsymbol{\upsilon}}_t$, $\mathbf{Q}$, is given by

$$\mathbf{Q} = \begin{pmatrix} \lambda_{s\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2 & 0 & \lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{s\eta}\sigma_\eta^2 & \lambda_{s\eta}\lambda_{\kappa\eta}\sigma_\eta^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{i\eta}^2\sigma_\eta^2 + \lambda_{i\omega}^2\sigma_\omega^2 + \lambda_{i\epsilon}^2\sigma_\epsilon^2 + \sigma_\upsilon^2 & 0 & \lambda_{i\eta}\sigma_\eta^2 & \lambda_{i\eta}\lambda_{\kappa\eta}\sigma_\eta^2 + \lambda_{i\omega}\sigma_\omega^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s\eta}\sigma_\eta^2 & 0 & \lambda_{i\eta}\sigma_\eta^2 & 0 & \sigma_\eta^2 & \lambda_{\kappa\eta}\sigma_\eta^2 \\ \lambda_{s\eta}\lambda_{\kappa\eta}\sigma_\eta^2 & 0 & \lambda_{i\eta}\lambda_{\kappa\eta}\sigma_\eta^2 + \lambda_{i\omega}\sigma_\omega^2 & 0 & \lambda_{\kappa\eta}\sigma_\eta^2 & \sigma_\omega^2 \end{pmatrix}.$$

APPENDIX C: IMPLIED VARIANCES

In this appendix, we solve the UC model in Section 3.1 for the residual-based measure of inventory investment, sales growth, and output growth. We then show how to calculate the implied unconditional variances of these variables, as well as for an inventory forecast error and expected inventory investment. In addition, we calculate two unconditional covariance terms, $\text{cov}(\Delta s_t, \Delta^2 i_t)$ and $\text{cov}(\Delta s_t, \Delta i_t^{fe})$.

In terms of the UC model, the residual-based measure of inventory investment is given by

$$\Delta i_t = \Delta i_t^* + (1 - L)(i_t - i_t^*) = \mu_\tau + \mu_\kappa + (1 + \lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i,$$

where $(1 - \phi_{i,1}L - \phi_{i,2}L^2)z_t^i = (1 - L)x_t^i$ and $x_t^i = \lambda_{i\eta}\eta_t + \lambda_{i\omega}\omega_t + \lambda_{i\epsilon}\epsilon_t + \upsilon_t$. Continuously compounded sales growth is given by

$$\Delta s_t = \eta_t + z_t^s,$$

where $(1 - \phi_{s,1}L - \phi_{s,2}L^2)z_t^s = (1 - L)x_t^s$ and $x_t^s = \lambda_{s\eta}\eta_t + \epsilon_t$. Then, using the inventory identity in equation (1), continuously compounded output growth is given by

$$\begin{aligned} \Delta y_t &= \Delta s_t + (1 - L)\Delta i_t \\ &= (\eta_t + z_t^s) + (1 + \lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i - (1 + \lambda_{\kappa\eta})\eta_{t-1} - \omega_{t-1} - z_{t-1}^i. \end{aligned}$$

We can then write a vector representation for z_t^s and z_t^i as

$$\mathbf{z}_t = \mathbf{K}\mathbf{z}_{t-1} + \mathbf{w}_t,$$

where

$$\mathbf{z}_t = \begin{bmatrix} z_t^s \\ z_{t-1}^s \\ z_t^i \\ z_{t-1}^i \\ x_t^s \\ x_t^i \end{bmatrix}, \ \mathbf{K} = \begin{bmatrix} \phi_{s,1} & \phi_{s,2} & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{i,1} & \phi_{i,2} & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{w}_t = \begin{bmatrix} x_t^s \\ 0 \\ x_t^i \\ 0 \\ x_t^s \\ x_t^i \end{bmatrix}.$$

Let $\mathbf{W}$ be the covariance matrix of $\mathbf{w}_t$, with the following nonzero entries: $\mathbf{W}[1, 1] = \mathbf{W}[1, 5] = \mathbf{W}[5, 1] = \mathbf{W}[5, 5] = \lambda_{s\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2$, $\mathbf{W}[1, 3] = \mathbf{W}[3, 1] = \mathbf{W}[1, 6] = \mathbf{W}[6, 1] = \mathbf{W}[3, 5] = \mathbf{W}[5, 3] = \mathbf{W}[5, 6] = \mathbf{W}[6, 5] = \lambda_{s\eta}\lambda_{i\eta}\sigma_\eta^2 + \lambda_{i\epsilon}\sigma_\epsilon^2$, and $\mathbf{W}[3, 3] = \mathbf{W}[3, 6] = \mathbf{W}[6, 3] = \mathbf{W}[6, 6] = \lambda_{i\eta}^2\sigma_\eta^2 + \lambda_{i\omega}^2\sigma_\omega^2 + \lambda_{i\epsilon}^2\sigma_\epsilon^2 + \sigma_\upsilon^2$. Then, noting that $vec(\text{var}(\mathbf{z}_t)) = (I - \mathbf{K} \otimes \mathbf{K})^{-1} vec(\mathbf{W})$, the unconditional variance of inventory investment is given by

$$\begin{aligned} \text{var}(\Delta i_t) &= \text{var}((1 + \lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i) \\ &= (1 + \lambda_{\kappa\eta})^2\sigma_\eta^2 + \sigma_\omega^2 + \text{var}(z_t^i) + 2\text{cov}((1 + \lambda_{\kappa\eta})\eta_t, z_t^i) + 2\text{cov}(\omega_t, z_t^i) \\ &= (1 + \lambda_{\kappa\eta})^2\sigma_\eta^2 + \lambda_{\kappa\epsilon}^2\sigma_\epsilon^2 + \sigma_\omega^2 + \text{var}(z_t^i) + 2(1 + \lambda_{\kappa\eta})\lambda_{i\eta}\sigma_\eta^2 + 2\lambda_{i\omega}\sigma_\omega^2, \end{aligned}$$

where $\text{var}(z_t^i)$ is the [3, 3] element of $\text{var}(\mathbf{z}_t)$. The unconditional variances of the two expectational components of inventory investment are given by

$$\text{var}(\Delta i_t^{fe}) = (\lambda_{y\eta} - \lambda_{s\eta})^2\sigma_\eta^2 + (\lambda_{y\epsilon} - 1)^2\sigma_\epsilon^2 + \lambda_{y\omega}^2\sigma_\omega^2 + \sigma_\upsilon^2,$$

and

$$\text{var}(\Delta i_t^e) = \text{var}(\Delta i_t) - \text{var}(\Delta i_t^{fe}).$$

The unconditional variance of sales growth is given by

$$\text{var}(\Delta s_t) = \text{var}(\eta_t + z_t^s) = \sigma_\eta^2 + \text{var}(z_t^s) + 2\lambda_{s\eta}\sigma_\eta^2,$$

where $\text{var}(z_t^s)$ is the [1, 1] element of $\text{var}(\mathbf{z}_t)$, whereas the unconditional variance of output growth is given by

$$\begin{aligned} \text{var}(\Delta y_t) = \text{var}(\Delta s_t + \Delta i_t - \Delta i_{t-1}) &= \text{var}(\Delta s_t) + 2\text{var}(\Delta i_t) \\ &+ 2\text{cov}(\Delta s_t, \Delta i_t) - 2\text{cov}(\Delta s_t, \Delta i_{t-1}) - 2\text{cov}(\Delta i_t, \Delta i_{t-1}), \end{aligned}$$

with

$$\begin{aligned}\mathrm{cov}(\Delta s_t, \Delta i_t) &= \mathrm{cov}\big(\eta_t + z_t^s, (1+\lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i\big)\\ &= (1+\lambda_{\kappa\eta})\sigma_\eta^2 + (1+\lambda_{\kappa\eta})\lambda_{s\eta}\sigma_\eta^2 + \mathrm{cov}\big(z_t^s, z_t^i\big) + \lambda_{i\eta}\sigma_\eta^2,\end{aligned}$$

$$\begin{aligned}\mathrm{cov}(\Delta s_t, \Delta i_{t-1}) &= \mathrm{cov}\big(\eta_t + z_t^s, (1+\lambda_{\kappa\eta})\eta_{t-1} + \omega_{t-1} + z_{t-1}^i\big)\\ &= \mathrm{cov}\big(z_t^s, z_{t-1}^i\big) + \mathrm{cov}\big(z_t^s, (1+\lambda_{\kappa\eta})\eta_{t-1} + \omega_{t-1}\big)\\ &= \mathrm{cov}\big(z_t^s, z_{t-1}^i\big) + (\phi_{s,1}-1)\lambda_{s\eta}(1+\lambda_{\kappa\eta})\sigma_\eta^2,\end{aligned}$$

and

$$\begin{aligned}\mathrm{cov}(\Delta i_t, \Delta i_{t-1}) &= \mathrm{cov}\big((1+\lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i, (1+\lambda_{\kappa\eta})\eta_{t-1} + \omega_{t-1} + z_{t-1}^i\big)\\ &= \mathrm{cov}\big(z_t^i, (1+\lambda_{\kappa\eta})\eta_{t-1} + \omega_{t-1} + z_{t-1}^i\big)\\ &= (\phi_{i,1}-1)\big(\lambda_{i\eta}(1+\lambda_{\kappa\eta})\sigma_\eta^2 + \lambda_{i\omega}\sigma_\omega^2\big) + cov\big(z_t^i, z_{t-1}^i\big),\end{aligned}$$

where $\mathrm{cov}(z_t^s, z_t^i)$, $\mathrm{cov}(z_t^s, z_{t-1}^i)$, and $\mathrm{cov}(z_t^i, z_{t-1}^i)$ are the $[1, 3]$, $[1, 4]$, and $[3, 4]$ elements of $\mathrm{var}(\mathbf{z}_t)$, respectively.

Finally, the two unconditional covariance terms are given by

$$\begin{aligned}\mathrm{cov}\big(\Delta s_t, \Delta i_t^2\big) &= \mathrm{cov}\big(\eta_t + z_t^s, (1+\lambda_{\kappa\eta})\eta_t + \omega_t + z_t^i - (1+\lambda_{\kappa\eta})\eta_{t-1} - \omega_{t-1} - z_{t-1}^i)\big)\\ &= \mathrm{cov}\big(z_t^s, z_{t-1}^i\big) + \mathrm{cov}\big(z_t^s, (1+\lambda_{\kappa\eta})\eta_{t-1} + \omega_{t-1}\big)\\ &= (1+\lambda_{\kappa\eta})\sigma_\eta^2 + \lambda_{i\eta}\sigma_\eta^2 + \lambda_{s\eta}(1+\lambda_{\kappa\eta})\sigma_\eta^2 - (\phi_{s,1}-1)\lambda_{s\eta}(1+\lambda_{\kappa\eta})\sigma_\eta^2\\ &\quad + \mathrm{cov}\big(z_t^s, z_t^i\big) - \mathrm{cov}\big(z_t^s, z_{t-1}^i\big),\end{aligned}$$

and

$$\begin{aligned}\mathrm{cov}(\Delta s_t, \Delta i_t^{fe}) &= \mathrm{cov}(\eta_t + z_t^s, (\lambda_{y\eta} - \lambda_{s\eta})\eta_t + (\lambda_{y\epsilon} - 1)\epsilon_t + \lambda_{y\omega}\omega_t + \upsilon_t)\\ &= (\lambda_{y\eta} - \lambda_{s\eta})\sigma_\eta^2 + (\lambda_{y\epsilon} - 1)\sigma_\epsilon^2 + \lambda_{s\eta}(\lambda_{y\eta} - \lambda_{s\eta})\sigma_\eta^2.\end{aligned}$$

LITERATURE CITED

Ahmed, Shaghil, Andrew Levin, and Beth A. Wilson. (2004) "Recent U.S. Macroeconomic Stability: Good Luck, Good Policies, or Good Practices?" *Review of Economics and Statistics*, 86, 824–32.

Blanchard, Olivier J. (1983) "The Production and Inventory Behavior of the American Automobile Industry." *Journal of Political Economy*, 91, 585–614.

Blinder, Alan S., and Louis J. Maccini. (1991) "Taking Stock: A Critical Assessment of the Recent Research on Inventories." *Journal of Economic Perspectives*, 5, 73–96.

Camacho, Maximo, Gabriel Perez-Quiros, and Hugo R. Mendizabal. (2011) “High-Growth Recoveries, Inventories and the Great Moderation.” *Journal of Economic Dynamics and Control*, 35, 1322–39.

Chib, Siddhartha, and Edward Greenberg. (1995) “Understanding the Metropolis-Hastings Algorithm.” *American Statistician*, 49, 327–35.

Clarida, Richard H., Jordi Gali, and Mark L. Gertler. (2000) “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *Quarterly Journal of Economics*, 115, 147–80.

Clark, Todd E. (2009) “Is the Great Moderation Over? An Empirical Analysis.” *Federal Reserve Bank of Kansas City Economic Review*, 4, 5–42.

Davis, Steven J., and James A. Kahn. (2008) “Interpreting the Great Moderation: Changes in the Volatility of Economic Activity at the Macro and Micro Levels.” *Journal of Economic Perspectives*, 22, 155–80.

Eo, Yunjong, and James C. Morley. (2015) “Likelihood-Ratio-Based Confidence Sets for the Timing of Structural Breaks.” *Quantitative Economics*, 6, 463–97.

Golob, John E. (2000) “Post-1984 Inventories Revitalize the Production Smoothing Model.” Mimeo.

Granger, Clive W. J., and Tae H. Lee. (1989) “Investigation of Production, Sales and Inventory Relationships Using Multicointegration and Non-Symmetric Error Correction Models.” *Journal of Applied Econometrics*, 4, S145–S159.

Hamilton, James D. (2002) “On the Interpretation of Cointegration in the Linear-Quadratic Inventory Model.” *Journal of Economic Dynamics and Control*, 26, 2037–49.

Hansen, Lars P., and Thomas J. Sargent. (1980) “Formulating and Estimating Dynamic Linear Rational Expectations Model.” *Journal of Economic Dynamics and Control*, 2, 7–46.

Herrera, Ana M., Irina Murtazashvili, and Elena Pesavento. (2008) “The Comovement in Inventories and in Sales: Higher and Higher.” *Economics Letters*, 99, 155–8.

Herrera, Ana M., and Elena Pesavento. (2005) “The Decline in U.S. Output Volatility: Structural Changes and Inventory Investment.” *Journal of Business & Economic Statistics*, 23, 462–72.

Holt, Charles C., Franco Modigliani, John F. Muth, and Herbert A. Simon. (1960) *Planning Production, Inventories, and Work Force*. Englewood Cliffs, NJ: Prentice Hall.

Irvine, Owen F., and Scott Schuh. (2005) “The Roles of Comovement and Inventory Investment in the Reduction of Output Volatility.” Federal Reserve Bank of Boston, Research Paper Series, Working Paper No. 05–9.

Kahn, James A., Margaret M. McConnell, and Gabriel Perez-Quiros. (2002) “On the Causes of the Increased Stability of the U.S. Economy.” *FRBNY Economic Policy Review*, 8, 183–202.

Kim, Chang-Jin, James C. Morley, and Jeremy Piger. (2008) “Bayesian Counterfactual Analysis of the Sources of the Great Moderation.” *Journal of Applied Econometrics*, 23, 173–91.

Kim, Chang-Jin, and Charles R. Nelson. (1999) “Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle.” *Review of Economics and Statistics*, 81, 608–16.

Maccini, Louis J., Bartholomew Moore, and Huntley Schaller. (2015) “Inventory Behavior with Permanent Sales Shocks.” *Journal of Economic Dynamics and Control*, 53, 290–313.

Maccini, Louis J., and Adrian Pagan. (2013) "Inventories, Fluctuations, and Goods Sector Cycles." *Macroeconomic Dynamics*, 17, 89–122.

McCarthy, Jonathan, and Egon Zakrajsek. (2007) "Inventory Dynamics and Business Cycles: What Has Changed?" *Journal of Money, Credit and Banking*, 39, 615–38.

McConnell, Margaret M., Patricia C. Mosser, and Gabriel Perez-Quiros. (1999) "A Decomposition of the Increased Stability of GDP Growth." *FRBNY Current Issues in Economics and Finance*, 5, 1–6.

McConnell, Margaret M., and Gabriel Perez-Quiros. (2000) "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?" *American Economic Review*, 90, 1464–76.

Miron, Jeffrey A., and Stephen P. Zeldes. (1988) "Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories." *Econometrica*, 56, 877–908.

Morley, James C. (2007) "The Slow Adjustment of Aggregate Consumption to Permanent Income." *Journal of Money, Credit and Banking*, 39, 615–38.

Morley, James C., Charles R. Nelson, and Eric Zivot. (2003) "Why Are the Beveridge-Nelson and Unobserved-Component Decompositions of GDP so Different?" *Review of Economics and Statistics*, 85, 235–43.

Ramey, Valerie A., and Daniel J. Vine. (2004) "Why Do Real and Nominal Inventory-Sales Ratios Have Different Trends?" *Journal of Money, Credit and Banking*, 36, 959–63.

Ramey, Valerie A., and Kenneth D. West. (1999) "Inventories." In *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, pp. 863–923. New York: North Holland, Elsevier.

Sims, Christopher A., and Tao Zha. (2006) "Were There Regime Switches in US Monetary Policy?" *American Economic Review*, 96, 54–81.

Sinclair, Tara. (2009) "The Relationship between Permanent and Transitory Movements in U.S. Output and the Unemployment Rate." *Journal of Money Credit and Banking*, 41, 529–42.

Stock, James, and Mark Watson. (2003) "Has the Business Cycle Changed and Why?" *NBER Macroeconomics Annual 2002*, 17, 159–218.

Wen, Yi. (2005) "Understanding the Inventory Cycle." *Journal of Monetary Economics*, 52, 1533–55.

West, Kenneth D. (1986) "A Variance Bounds Test of the Linear Quadratic Inventory Model." *Journal of Political Economy*, 94, 374–401.

West, Kenneth D. (1990) "The Sources of Fluctuations in Aggregate Inventories and GNP." *Quarterly Journal of Economics*, 105, 939–72.