The Adjustment of Prices and the Adjustment of the Exchange Rate

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Abstract

The purchasing power parity puzzle relates to the adjustment of real exchange rates. Real exchange rates are extremely volatile, suggesting that temporary shocks emanate from the monetary sector. But the half-life of real exchange rate deviations is extremely large – 2.5 to 5 years. This half-life seems too large to be explained by the slow adjustment of nominal prices. We offer a different interpretation. We maintain that nominal exchange rates and prices need not converge at the same rate, as is implicit in rational-expectations sticky-price models of the exchange rate. Evidence from an unobserved components model for nominal prices and nominal exchange rates that imposes relative purchasing power parity in the long run indicates that nominal exchange rates converge much more slowly than nominal prices. The real puzzle is why nominal exchange rates converge so slowly.

JEL codes: F3, F4

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We thank David DeJong, Linda Goldberg, Bruce Hansen, Shang-Jin Wei, Ken West, and Holger Wolf for useful comments and discussion. Engel acknowledges research support from a National Science Foundation grant to the National Bureau of Economic Research.

Since the advent of floating exchange rates in 1973, real exchange rates among advanced countries have been persistent and volatile. There are two explanations for this outcome, but neither is entirely satisfactory. The first is that real productivity shocks and real demand shocks to economies have been very persistent. But it is difficult to identify shocks that would lead to such great volatility of real exchange rates.

A second view builds on rational-expectations sticky-price (RESP) models of open economy in the tradition of Dornbusch (1976). Those models demonstrate that monetary shocks could lead to a high degree of real exchange rate volatility through the overshooting effect. Real exchange rates can be persistent because they adjust at the same rate as nominal prices adjust.

However, empirical studies of real exchange rate adjustment have found very long half-lives for transitory shocks to real exchange rates. Typically, the half-life of real exchange rates is estimated to be from 2.5 to 5 years.[1] That adjustment seems to be too slow to be explained by stickiness of nominal prices. Hence, we have the "purchasing power parity puzzle", as defined by Rogoff (1996):

> How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out? Most explanations of short-term exchange rate volatility point to financial factors such as changes in portfolio preferences, short-term asset price bubbles, and monetary shocks. Such shocks can have substantial effects on the real economy in the presence of sticky nominal wages and prices. Consensus estimates for the rate at which PPP deviations damp, however, suggest a half-life of three to five years, seemingly far too long to be explained by nominal rigidities. It is not difficult to rationalize slow adjustment if real shocks – shocks to tastes and technology – are predominant. But existing models based on real shocks cannot account for short-term exchange-rate volatility. (pp. 647-648.)[2]

[1] See for example Frankel (1986), Lothian and Taylor (1996), Wu (1996), Papell (1997), Cheung and Lai (2000) and Murray and Papell (2000).

[2] Earlier, Stockman (1987) also questions whether the slow convergence of real exchange rates can be explained by slow adjustment of nominal prices.

Here we do not provide a full resolution to the purchasing power parity puzzle, but we do offer a refinement: it is nominal exchange rates, not prices, that adjust slowly toward purchasing power parity. In fact, when we allow nominal prices and exchange rates to adjust at different speeds, we find that nominal exchange rates usually take years to converge, while prices often converge within months. Why then do Rogoff (1996), Stockman (1987), and others mate the convergence speed of the real exchange rate with the convergence speed of prices? Probably it is because that is the sort of dynamics that arise from RESP models. In those models, prices, nominal exchange rates, and real exchange rates converge to the long run at the same rate.

Our finding raises a new puzzle: why does the nominal exchange rate converge so slowly? We do not present an alternative theory that answers this question. The model we present is purely empirical. Perhaps this new puzzle is related to the empirical failure of uncovered interest parity (UIP). In terms of the RESP model, the forward-looking behavior implicit in rational expectations modeling of the UIP condition is the key to the solution that puts exchange rates and prices on a saddle path, and reduces the dimensionality of the system. However, we do not attempt any theoretical modeling of an alternative to UIP. The UIP puzzle has been very resistant to theoretical explanations, so we leave it and this new puzzle for future research.

Our model is one in which nominal prices converge toward equilibrium price levels that are unobserved. The exchange rate between any two countries converges toward an equilibrium exchange rate that is linked to prices in the long run by purchasing power parity. The model has a state-space representation that can be estimated with the help of the Kalman filter.

Superficially, our empirical model appears similar to models in the macroeconomics literature in which variables (such as GDP) are decomposed into a transitory and a random walk component. But our formulation of the state-space model allows more flexibility than many other applications in macroeconomics. To emphasize the difference, we refer to "equilibrium" and "disequilibrium" components, rather than "permanent" and "transitory" components of our time series. There are some important distinctions between our model and the permanent-transitory decompositions. For one, our unobserved equilibrium price levels and exchange rates are not simply posited to be pure random walks. We allow transitory dynamics both in the equilibrium prices and exchange rates, as well as in the disequilibrium components. Also, identification of our model does not require arbitrary independence restrictions on the covariance matrix of innovations to equilibrium and disequilibrium variables. Indeed, RESP models could not be nested in our formulation if we required equilibrium and disequilibrium innovations to be independent. An underlying structural monetary shock, for example, must be allowed to influence both equilibrium prices and exchange rates and deviations from the equilibrium.

There are three reasons why we are able to build a state-space model with these attractive features. First, we make use of a reformulation of the standard state-space model due to Morley, Nelson and Zivot (2001). Second, our model is multivariate, which in some cases allows identification with fewer covariance restrictions than in univariate models when there are cross-equation restrictions on the behavior of the variables. Third, and most importantly, we make use of structural identifying restrictions. In particular, we use the long-run PPP restriction and also rely on the economic structure of RESP models to guide our formulation of the decomposition between equilibrium and disequilibrium components of exchange rates and prices.

In section 1, we lay out the empirical model. Section 2 relates the model to RESP models directly, as a way to develop some restrictions that are helpful in estimation. (We build a model that nests a RESP model as a special case.) In section 3, we discuss intuitively where identification of the model comes from. Section 4 reports results, and the outcome of some specification tests. Section 5 compares our approach to other recent studies that have allowed different speeds of adjustment for exchange rates and prices. In section 6, we conclude and speculate on what type of economic behavior might produce the results we find. There are two appendices. The first rigorously relates our model to RESP models, and the second gives the detail of our set-up of the Kalman filter.

1. Model

We propose an unobserved components (UC) model to examine price level and exchange rate adjustment. The log price levels and the log nominal exchange rate for a given pair of countries gravitate over time toward an unobserved equilibrium based on purchasing power parity (PPP), but have transitory deviations from the equilibrium.

In its most general form, our model has the observed log price levels, p_{it}, $i = 1,\ldots,n$, and the log exchange rates, s_{jt}, $j = 2,\ldots,n$, (where the exchange rates are expressed as the price of country *j*'s currency in terms of the country 1's currency) adjust toward unobserved equilibrium values according to stationary autoregressive processes:

$$\phi_p^i(L)(p_{it} - \bar{p}_{it}) = v_{it}, \tag{1}$$

$$\phi_s^j(L)(s_{jt} - \bar{s}_{jt}) = v_{jt}^s . \tag{2}$$

The lag operators, $\phi_p^i(L)$ and $\phi_s^j(L)$, are all k-th order, and the roots lie outside the unit circle; $\bar{p}_{it}$ is the equilibrium price level in country i, and $\bar{s}_{jt}$ represents the equilibrium value of s_{jt}; v_{it} represents a disequilibrium innovation to country i's price level, and v_{jt}^s is a disequilibrium innovation to j's exchange rates. Meanwhile, the first differences of the unobserved equilibrium log price levels adjust according to autoregressive processes:

$$\phi_{\bar{p}}^i(L)(\Delta\bar{p}_{it} - \mu_i) = \bar{v}_{it} . \tag{3}$$

Again, $\phi_{\bar{p}}^i(L)$ is a k-th order lag operator whose roots lie outside the unit circle; μ_i represents a deterministic drift in country i's equilibrium price level; and $\bar{v}_{it}$ is an innovation to the equilibrium price level. The equilibrium exchange rate for country j relative to country 1 (the base country) relates to equilibrium price levels according to PPP:

$$\bar{s}_{jt} = \bar{p}_{1t} - \bar{p}_{jt} . \tag{4}$$

Finally, the equilibrium and disequilibrium innovations have mean zero and a joint Normal distribution.

Equation (1) takes the form of price-adjustment equations in open-economy models presented by Mussa (1982) and Obstfeld and Rogoff (1984). The equilibrium prices, $\bar{p}_{it}$, are interpreted in those models as the price level that would prevail in each country if prices were perfectly flexible, given the current values and history of the exogenous variables. Under this interpretation, equation (3) describes what the evolution of p_{it} would be if prices were perfectly flexible. Our model incorporates a unit root in these equilibrium prices, but does not require that they follow a random walk. For example, with fixed money demand, nominal prices could follow such a process if money supplies were exogenously generated as unit root processes.

Equation (4) imposes long-run purchasing power parity. Rogoff (1997) claims there is a growing consensus on this empirical regularity.[3] Equation (2) indicates there are transitory deviations from purchasing power parity.

It is easy to relate this model to stochastic versions of the RESP model. In section 2 we discuss the relationship in detail. It is useful now to point out the main contrast between this model and the RESP models: in RESP models, $\phi_p^i(L)$ and $\phi_s^j(L)$ are restricted to be the same as each other.

2. Estimation

To keep the dimensionality of our model reasonable, we impose three specification assumptions prior to estimation. First, for simplicity and transparency in terms of convergence properties, we assume first-order autoregressive adjustment processes (i.e., $k = 1$). Second, we impose some restrictions, discussed below, on the covariance matrix of the equilibrium and disequilibrium innovations. Third, since our main focus is on the difference between the speeds of adjustment for nominal prices and for nominal exchange rates, we assume that nominal prices adjust at the same speed for

[3] However, see Engel (2000). The permanent deviations from PPP that Engel argues may exist have very small effects on real exchange rates over the horizons we are investigating.

each country (i.e., $\phi_p^i = \phi_p$ and $\phi_{\bar{p}}^i = \phi_{\bar{p}}$ for all *i*) and nominal exchange rates adjust at the same speed for each country pair ($\phi_s^j = \phi_s$ for all *j*).

We do not assume that all of the innovations to equilibrium and disequilibrium prices and exchange rates are independent. Such a strong assumption is not necessary to identify the model. Furthermore, independence would have the drawback of not nesting RESP-style dynamics. Appendix 1 presents a RESP model for a two-country case, and discusses the restrictions implied by that model. In this section, we discuss those restrictions more informally and describe how they are accommodated in our estimation.

Consider equations (1) and (3), the price-adjustment equation and the equation determining the dynamics of equilibrium prices. In the RESP model, the innovation $\bar{v}_{it}$ embodies structural monetary and aggregate demand shocks that move the equilibrium price level. If we were to assume independent innovations, the error term in the price-adjustment equation (1), v_{it}, would not be correlated with $\bar{v}_{it}$. The implication from equation (1) is that any shock that pushes up $\bar{p}_{it}$ would push p_{it} up immediately by exactly the same amount. But this kind of immediate proportional response of prices, p_{it}, to shocks that affect equilibrium prices, $\bar{p}_{it}$, is completely inconsistent with the price-stickiness assumptions of RESP models. RESP models assume negative correlation between v_{it} and $\bar{v}_{it}$. Indeed, a literal representation of predetermined nominal prices has these terms perfectly negatively correlated: $v_{it} = -\bar{v}_{it}$. Under this assumption, the price adjustment equation (1) can be written as:

$$p_{it} = (1-\phi_p^i(L))(p_{it} - \bar{p}_{it}) + E_{t-1}\bar{p}_{it}.$$

In practice, we assume that while v_{it} and $\bar{v}_{it}$ might be correlated, there is not perfect negative correlation. The assumption of perfect negative correlation means that prices do not respond at all in the current period to shocks that affect $\bar{p}_{it}$. That is an impractical assumption in our empirical model. Our data are sampled quarterly, so the assumption means that, even after one full quarter, prices show no response to $\bar{v}_{it}$ innovations. We find in our empirical work that prices actually adjust fairly quickly – generally more than half of the adjustment occurs within six months. Even if prices do not respond on impact to $\bar{v}_{it}$ innovations to equilibrium prices, we should allow for the possibility that some of the adjustment occurs within the first quarter. So, we allow $Cov(v_{it}, \bar{v}_{it})$ to be non-zero, but we do not impose perfect negative correlation.

Another instance in which it is important not to assume independence is between the innovations to s_{jt} and to $\bar{p}_{1t}$ and $\bar{p}_{jt}$. A key feature of the RESP model is that exchange rates instantaneously reflect shocks that ultimately are reflected in goods prices. To accommodate this behaviour, we also allow for non-zero values of $Cov(v^s_{jt}, \bar{v}_{jt})$ and $Cov(v^s_{jt}, \bar{v}_{1t})$.

Then, since the innovations to the exchange rate equation, v^s_{jt}, and the innovations to prices, v_{jt} and v_{1t}, are correlated with the innovations to equilibrium prices, $\bar{v}_{jt}$ and $\bar{v}_{1t}$, it is logical to allow v^s_{jt} to be correlated with $\bar{v}_{jt}$ and $\bar{v}_{1t}$. So, we also allow $Cov(v_{jt}, v^s_{jt})$ and $Cov(v_{1t}, v^s_{jt})$ to be non-zero.

Meanwhile, we assume $Cov(v_{it}, v_{jt}) = 0$, $Cov(v_{it}, \bar{v}_{jt}) = 0$, and $Cov(\bar{v}_{it}, \bar{v}_{jt}) = 0$,

$i \neq j$. These are typical assumptions in RESP models. They correspond to an assumption that domestic monetary and aggregate demand shocks are uncorrelated with the corresponding foreign shocks.

Our model generalizes the models of Mussa (1982) and Obstfeld and Rogoff (1984) in two ways. The first is relatively trivial. As we discussed above, we do not impose the restriction that innovations to current and equilibrium prices in each country are perfectly negatively correlated. The second is crucial. The two-country model yields saddle-path dynamics in which prices and the exchange rate converge at the same speed. It has a linear restriction of the form:

$$s_{jt} - \bar{s}_{jt} = -\eta_j (p_{jt} - \bar{p}_{jt}) + \eta_1 (p_{1t} - \bar{p}_{1t}),$$

where η_j and η_1 are constants, with a symmetric model $(\eta_1 = \eta_j)$ implying that $\phi_p^1(L)$, $\phi_p^j(L)$, and $\phi_s^j(L)$ are all the same. We do not impose this restriction. Instead, we allow prices to have one speed of convergence and the exchange rate to have another. Indeed, it is by jettisoning the restriction that $\phi_p^1(L)$, $\phi_p^j(L)$, and $\phi_s^j(L)$ are the same that we move from a model in which we can speak meaningfully about the speed of adjustment of the real exchange rate to a model that focuses on the speed of adjustment of nominal prices and nominal exchange rates.

As we have mentioned, we do impose that $\phi_p^1(L)$ and $\phi_p^j(L)$ are identical. The literature that links the slow adjustment of the real exchange rate to the speed of adjustment of nominal prices has made this assumption. Without that assumption, RESP models would not imply that real exchange rates could be represented by low-order autoregressive processes. We maintain the assumption of identical speeds of adjustment of prices, but break the link to exchange rate adjustment imposed by RESP models since it is this link that we are interested in testing.

In section 4, we estimate the model for the G7 countries. We first estimate the model pairwise for the U.S. as the base country and each of the other six countries separately. Then we estimate the model jointly for all seven countries. In the two-country models, we impose further restrictions that arise in the RESP model. These proportionality restrictions hold for the symmetric RESP model, discussed in the appendix, and might well be expected to hold for our model given the assumption that nominal prices adjust at the same speed for each country.

The first proportionality restriction we impose is that, while the direction is opposite, the degree of exchange overshooting or undershooting should be the same in response to equal shocks to $\overline{p}_t$ and $\overline{p}_t^*$:

$$\frac{Cov(v_{jt}^s, \overline{v}_{jt})}{Var(\overline{v}_{jt})} = \frac{-Cov(v_{jt}^s, \overline{v}_{1t})}{Var(\overline{v}_{1t})}, \tag{5}$$

Intuitively, the model implies that $\overline{p}_t$ and $\overline{p}_t^*$ respond one for one to shocks to domestic and foreign money supplies respectively. The exchange rate may overshoot (or undershoot) in its initial response to money shocks, so that $v_{jt}^s = k(\overline{v}_{jt} - \overline{v}_{1t})$. Equation (5) follows, given our assumption of the independence of domestic and foreign monetary shocks.

Model symmetry yields the second restriction we impose. It is that the relationship between equilibrium and disequilibrium price shocks is proportional in each country:

$$\frac{Cov(v_{jt},\bar{v}_{jt})}{Var(\bar{v}_{jt})}=\frac{Cov(v_{1t},\bar{v}_{1t})}{Var(\bar{v}_{1t})}, \tag{6}$$

The third restriction is natural in light of the previous two. It is that the relationship between disequilibrium price shocks and disequilibrium exchange shocks is proportional with opposite signs in each country:

$$\frac{Cov(v^{s}_{jt},v_{jt})}{Var(v_{jt})}=\frac{-Cov(v^{s}_{jt},v_{1t})}{Var(v_{1t})}. \tag{7}$$

We do not impose these restrictions in the model in which all seven countries are handled simultaneously. This seven-country model is more stable (more strongly identified) than the two-country models, so we need fewer restrictions. In addition, it is considerably less tractable to impose these restrictions in the seven-country model.

3. Interpretation

The unobserved components model that we use resembles the permanent-transitory decompositions of GDP by Harvey (1985) and Clark (1987). Those models decompose a single GDP time series into a random walk component and a transitory component modeled as an AR(2) process, which are assumed to be independent. Superficially we seem to be doing something similar to prices and exchange rates. But our "equilibrium" prices and exchange rates are not constrained to be pure random walks. They can have transitory dynamics. Moreover, we do not need to impose restrictions that the innovations to the equilibrium and disequilibrium components are independent. However, it is intuitive to compare our approach with the GDP decompositions of Harvey (1985) and Clark (1987) to get a sense of where our results come from.

First, the assumption of independence between the permanent and transitory components used by Harvey (1985) and Clark (1987) is not needed even in their models. Morley, Nelson and Zivot (2001) show how the same model can be estimated without imposing any assumption about the correlation of the permanent and transitory components. They speculate that part of the reason previous studies have imposed independence is that they write down the state-space representation in such a way that the transitory component is in the observation equation of the Kalman filter, and the permanent component is in the state equation. The usual implementation of the Kalman filter assumes independence of the errors in the state equation and the measurement equation.[4] But, Morley, Nelson and Zivot (2000) show that if the model is written such that both the transitory and permanent components are in the state equation, it is easy to use the Kalman filter allowing the two components to be correlated. We make use of that insight in setting up the Kalman filter for our model. Both the equilibrium and disequilibrium variables are in the state equation.

The cross-equation restriction that we have imposed – that purchasing power parity holds for the equilibrium exchange rate – also helps identify our equilibrium and disequilibrium prices and exchange rates in practice. That is, our model does not separately decompose nominal prices for each country and each nominal exchange rate into equilibrium and disequilibrium components. The equilibrium component of the exchange rate between countries *i* and *j* is constrained to equal $\bar{p}_{it} - \bar{p}_{jt}$.

We rely on the structure of the RESP model, as well, to distinguish between equilibrium and disequilibrium components. The notion of the equilibrium price level

[4] Versions of the Kalman filter exist where the errors in the state and measurement equations are correlated.

arises in the context of a nominal price adjustment equation. Our model implies a univariate ARMA(2,2) model for $p_{it} - p_{it-1}$. We determine the "equilibrium" and "disequilibrium" dynamics in the context of price adjustment in RESP models, which have prices gradually returning to the equilibrium value. So our equation (1), which is based on the price adjustment behavior modeled by Mussa (1982) and Obstfeld and Rogoff (1984), and others, puts structure on the data generating processes of prices.

We have also imposed restrictions on the covariance matrix. These are not zero restrictions on the covariances between the equilibrium and disequilibrium components. Instead they are assumptions implying uncorrelated monetary shocks across countries. (Also, in the two-country models, we impose further proportionality restrictions that arise in symmetric RESP models.) While not all of the restrictions are necessary for strict identification, they are all reasonable and help us derive stable estimates in practice without altering our main conclusions.

Appendix 2 discusses the Kalman filter and maximum likelihood estimation of the model.

4. Results

We consider six country pairs based on the G7 countries, with the US always serving as the home country. The other countries are Canada, France, Germany, Italy, Japan, and the UK. The prices are consumer price indexes (not seasonally adjusted) in the third month of each quarter. The exchange rates are end-of-quarter prices of foreign

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Such versions of the filter are more complicated and not frequently used.

currency expressed in US dollars. All data are from Datastream. The data are converted into logarithms and multiplied by 100. The sample period is 1974Q1 to 1998Q2.

We employ the OPTMUM procedure for the GAUSS programming language to obtain maximum likelihood estimates. Numerical derivatives are used for estimation and the calculation of asymptotic standard errors. Estimates appear robust to a variety of starting values.

4a. Two-Country Models

Table 1 presents the maximum likelihood estimates for our model and the country pairs a) US and Canada, b) US and France, c) US and Germany, d) US and Italy, e) US and Japan, and f) US and UK, respectively. The table reports the autoregressive parameters for prices, ϕ_p; equilibrium prices, $\phi_{\bar{p}}$; and exchange rates, ϕ_s; the innovation standard deviations for disequilibrium prices in the U.S., $\sigma_{p,1}$, and the other country, $\sigma_{p,2}$; the innovation standard deviations for equilibrium prices in the U.S., $\sigma_{\bar{p},1}$, and the other country, $\sigma_{\bar{p},2}$; and the innovation standard deviations for the disequilibrium exchange rates, σ_s.[5]

The main result we highlight is that, for every country pair, the adjustment of prices to the PPP equilibrium is much faster than the adjustment of the exchange rate. The half-lives of price deviations from equilibrium are less than a quarter in the first three cases and less than two quarters in the remaining three cases. Meanwhile, the half-lives

[5] To conserve space, we do not report estimates of the initial values of the equilibrium prices and exchange rates, the unconditional means of the equilibrium inflation rates, or the off diagonal elements of the covariance matrix. These estimates generally have large standard errors, so we do not draw any strong conclusions from them.

of exchange rate deviations from equilibrium range from two years for the US/UK case, to as many as thirteen years for the US/Canada case.

The half-life estimates for prices do not provide much fodder either for advocates of models where slow nominal price adjustment is an important element in business-cycle behavior, or for supporters of models with rapidly adjusting nominal prices. Our point estimates are consistent with the degree of price stickiness estimated in recent empirical studies of sticky-price models, but the standard errors on the coefficient estimates are large enough to encompass both alternatives.[6] What is remarkable, of course, is the very slow adjustment of nominal exchange rates.

Equilibrium inflation is very persistent for every country pair. It seems unlikely that we would be able to reject a unit root in equilibrium inflation in any of the cases. However, if a unit root really were present, accounting for it should only serve to strengthen evidence for the fast adjustment of prices. In particular, an omitted nonstationary component from equilibrium prices would show up in the estimated deviations of prices from equilibrium, thus putting an upward bias on our estimates of the persistence of those deviations.

Innovations to exchange rate deviations from equilibrium have standard deviations an order of magnitude larger than innovations to equilibrium prices and price deviations. This is not too surprising given the relative volatility of observed prices and exchange rates, which is the main stylized fact RESP overshooting models try to account for. But, it is notable since it potentially explains why other studies have found that

[6] For example, Galí, Gertler, and López-Salido's (2000) estimates imply a half-life of six months for nominal prices in Europe.

nominal exchange rates do most of the adjustment towards PPP, even if prices adjust more quickly. We discuss this point in further detail in the next section.

Figure 1 presents plots of the equilibrium price levels for each two-country model, the deviations of prices from equilibrium, and the actual and equilibrium exchange rates. One reassuring aspect of these estimates is that the estimated equilibrium and disequilibrium price levels for the U.S. appear to be quite similar across all six models, though there is no constraint imposed here that they be the same. (The seven-country model reported in the next section, of course, imposes that constraint.) The extreme persistence of the exchange rate deviations from the equilibrium level is apparent in these graphs. It does not appear that the persistence arises as the result of a single episode, such as the large swing in the value of the dollar in the 1980s.

The first row of Table 2 presents formal likelihood ratio tests of the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment. Except for the US/Italy and US/Japan cases, the likelihood ratio statistics are quite large, suggesting that the overall evidence for different speeds of adjustment is strong. Thus, the results for the likelihood ratio test generally support what the point estimates seem to suggest: prices adjust toward PPP more quickly than exchange rates.

The second row of Table 2 reports the results for a likelihood ratio test of the various symmetry restrictions (same speed of adjustment for nominal prices across countries and proportionality restrictions on the covariances) against the alternative of no symmetry restrictions. The $\chi^2(5)$ likelihood ratio statistics are generally not significant.

Only the US/Japan case is significant at the 10% level. Both the same speed of adjustment restriction on prices and the proportionality restrictions are insignificant when tested for separately. Thus, the symmetry restrictions in our model appear to be justified, with estimates changing little when the restrictions are relaxed. Again, we impose the restrictions to keep the dimensionality of our model reasonable and to focus on the difference between price adjustment and exchange rate adjustment.

The third row of Table 2 reports the results for a likelihood ratio test of the null hypothesis that all of the innovations are independent. The $\chi^2(3)$ likelihood ratio statistics are not significant at conventional levels. Thus, while it is important to relax a strict independence restriction on the innovations in order to nest RESP-style dynamics, this result reflects the fact that our main findings are not merely a product of the more general covariance specification.[7]

The fourth row of Table 2 reports the results for a likelihood ratio test of no break in the unconditional mean of equilibrium inflation for each country against the alternative of a structural break in 1980 from equation (3). As an empirical fact, the G7 countries uniformly had higher inflation in the 1970s than they did afterwards. A reasonable question, then, is whether our modeling assumption of a constant unconditional mean throughout the sample period is strongly at odds with the data and is, in any way, responsible for our main findings. The $\chi^2(2)$ likelihood ratio statistics for a structural break are generally insignificant, reflecting that, even though point estimates for the unconditional means are greatly reduced after the 1970s, they are not estimated with any

[7] For a model with independent innovations, the likelihood ratio test results for the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment are essentially the same as the results reported in the first row of Table 2.

great precision. Meanwhile, the final row of Table 2 reports the results given a structural break in 1980 for a likelihood ratio test of the hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment. The results are similar to the model with no structural break, except that we can no longer reject the null for the U.S./Canada case.

4b. Seven-Country Model

Here we report the estimation results for a model of price levels and exchange rates for all of the G7 countries. The only real ambiguity in extending the model to all seven countries is whether there should be additional restrictions related to the exchange rate components. Our approach is to allow all of the exchange rate components to be correlated with each other and with the other unobserved components. That is, we do not impose any additional zero covariance restrictions.

Table 3 presents the maximum likelihood estimates for the key parameters from a model of the G7 price levels and exchange rates. Encouragingly, the estimates for the autoregressive coefficients and volatility parameters are quite similar to the estimates in Table 1.[8] As before, the results suggest that prices adjust more quickly than exchange rates. The half-life of a deviation of prices from equilibrium is less than two quarters, while the half-life of a deviation of exchange rates from equilibrium is more than two years.

The $\chi^2(1)$ likelihood ratio test statistic for the null hypothesis that prices and the exchange rate adjust at the same speed against the alternative of different speeds of adjustment is highly significant. Its value is 16.336, which has a *p*-value smaller than 0.001.

Figure 2 presents plots of the equilibrium and disequilibrium price components, and the actual and equilibrium exchange rates. The plots are strikingly similar to those derived from the two-country models.

We do not undertake further specification tests of the seven-country model because of the enormous computational burden associated with estimation, and because of the uniformly positive results from the specification tests of the two-country models.

5. Discussion

Our main finding that prices adjust more quickly than exchange rates appears at first glance to contradict the results of other related studies. For example, Wei and Parsley (1995), and Goldfajn and Valdes (1999), contend that the exchange rate is responsible for most of the adjustment toward purchasing power parity, rather than nominal prices. The simple point we make here is that there is a distinction between the "size" of the adjustment and the "speed" of adjustment. Since the nominal exchange rate has a much larger innovation variance than prices, it deviates from its equilibrium more than prices do when there is a shock. So the exchange rate must adjust more – but that does not contradict our finding that it adjusts more slowly than prices.

It is useful to frame the discussion by drawing a contrast between our approach, and vector error correction models (VECM) (e.g., Cheung, Lai, and Bergman (1999).)

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[8] Again, we do not report the initial values, mean inflation rates, or covariance parameter estimates to conserve space, although it should be noted that the transitory exchange rate shocks are highly correlated with each other.

Consider the following VECM for relative prices $(p_t \equiv p_{1t} - p_{2t})$ and the exchange rate s_t for a pair of countries:

$$p_{t+1} - p_t = \alpha_p (s_t - p_t) + u_{t+1}^p, \tag{8}$$

$$s_{t+1} - s_t = \alpha_s (s_t - p_t) + u_{t+1}^s, \tag{9}$$

where α_p and α_s are error correction coefficients and u_t^p and u_t^s are stationary residuals.[9] One might expect to find (as Cheung, Lai, and Bergman do) that α_s is always much larger in magnitude than α_p. That is, exchange rates adjust much more than relative prices in response to a deviation from PPP.

The speed of adjustment is a measure of how fast a variable returns to some equilibrium. Thus, in the traditional PPP literature, the real exchange rate is assumed to converge to some constant level, $\overline{q}$, in the long run. We can measure the speed of adjustment by determining how much of the gap $q_t - \overline{q}$ is carried through to the next period in $q_{t+1} - \overline{q}$. In our model, we look at speeds of adjustment for p_t and s_t individually. For example, the speed of adjustment for the nominal exchange rate is measured by the degree to which $s_{t+1} - \overline{s}_{t+1}$ has adjusted to the gap $s_t - \overline{s}_t$.

However, the VECM parameters do not measure speeds of adjustment. For example, the parameter α_p is a measure of how relative inflation, $p_{t+1} - p_t$, responds to the real exchange rate gap, $q_t - \overline{q}$. (We can rewrite equations (8) and (9) so that the error correction term can be written as $q_t - \overline{q}$.) The error correction term in (8) and (9) is not the same as the exchange rate gap $(s_t - \overline{s}_t)$ or the relative price gap $(\overline{s}_t - p_t)$ implicit in our UC representation of prices and the exchange rate, but is, instead, equal to their difference. So, our UC representation has prices adjusting only to the relative price gap, while the VECM representation imposes that prices adjust equally to both gaps. α_p will not be large compared to our ϕ_p because α_p measures the response of prices to a very large gap, $q_t - \overline{q}$, while ϕ_p measures the response of prices to the smaller gap, $p_t - \overline{p}_t$. ϕ_p captures how quickly prices are adjusting to their deviation from equilibrium, while the error correction parameter α_p measures how much prices are responding to the price gap *and* the exchange-rate gap.

An example makes this clear. If p_t follows a random walk, then by construction they would adjust to equilibrium instantaneously (i.e., very quickly indeed!). There would be no relative price gap, only an exchange rate gap. However, since relative prices follow a random walk, they would not adjust toward the exchange rate gap at all, implying that α_p would actually be zero.

It appears from our findings that the main reason exchange rates adjust more than relative prices is that the exchange rate gap is much larger than the relative price gap. Specifically, we find disequilibrium exchange rate innovations are always an order of magnitude more volatile than disequilibrium price innovations.

Another way to think about the distinction between our UC modeling approach and the VECM approach concerns the left-hand-side variable. Consider, for example, the

[9] Note that a finite-order VECM can only approximate the dynamics of the infinite-order vector MA

nominal exchange rate. In our UC model, we examine changes in the exchange rate relative to its equilibrium value: $s_{t+1} - \bar{s}_{t+1} - (s_t - \bar{s}_t)$. The left-hand-side variable in the VECM approach is simply $s_{t+1} - s_t$. It is, of course, an empirical question as to which modeling approach fits the data the best.[10] Our approach is easier to understand as a generalization of the RESP model, and it is easier to infer the "speed of adjustment" from our parameter estimates.

Thus when one carefully distinguishes between the "size" and the "speed" of adjustment, it becomes clear that our main findings do not contradict the conclusion that exchange rates are responsible for most of the adjustment toward PPP.

7. Conclusions

Our results suggest a new way of describing the purchasing power parity puzzle. Nominal prices converge relatively rapidly to their equilibrium value, but exchange rates converge slowly. To be clear ours is not an economic model, and we have not undertaken tests of any economic model. We have merely presented a new statistical model of exchange rates and prices, but one that might be provocative to exchange-rate modelers.

We reject the label that our model is one with "sticky" exchange rates. All of the RESP models have exchange rates converging slowly – at the same speed as nominal prices. Stickiness refers to the innovation variance of relative prices or exchange rates. A model with purely sticky nominal prices, for example, would have $Var(v_{it} + \bar{v}_{it}) = 0$.

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representation that corresponds to our UC model of prices and the exchange rate.

Our model does not imply "sticky" nominal exchange rates, because the variance of innovations to the exchange rate is very large, and much larger than the innovation variance of prices, $v_{it} + \bar{v}_{it}$. What we find is that exchange rates are very volatile, but converge to the PPP equilibrium much more slowly than nominal prices.

What could explain the result that prices converge fairly quickly in each country to their equilibrium levels, but the exchange rate moves only very slowly to the PPP value? One possible explanation is that persistent real shocks are important. Our PPP model does not incorporate real shocks, but it is easy to see how a model with real shocks could produce persistent nominal exchange rate deviations. If nominal prices adjust quickly, but there are real shocks that imply a slowly-adjusting real exchange rate, then the nominal exchange rate necessarily will adjust slowly to the PPP equilibrium. We are skeptical that real shocks can explain our findings. As we have noted in our introduction, the extreme volatility of real exchange rates suggests that the underlying source of shocks is monetary or financial. Most theories of how real shocks affect real exchange rates is through their influence on the relative price of nontraded goods. Engel (1999) documents that virtually none of the short-run variation in real exchange rates for these advanced countries is attributable to movements in the relative price of nontraded goods.

Rogoff's (1997) speculation is apropos:

> One is left with a conclusion that would certainly make the godfather of purchasing power parity, Gustav Cassel, roll over in his grave. It is simply this: International goods markets, though becoming more integrated all the time, remain quite segmented, with large trading frictions across a broad range of goods. These frictions may be due to transportation costs, threatened or actual tariffs, nontariff barriers, information costs or lack of labor mobility. As a consequence of various adjustment costs, there is a large buffer within which nominal exchange rates can move without

[10] However, the two models are not easily nested in a more general model. Model comparison based, for example, on out-of-sample forecasting ability would be one approach to compare the models, but is beyond the scope of this paper.

> producing an immediate proportional response in relative domestic prices. International goods markets are highly integrated, but not yet nearly as integrated as domestic goods markets. This is not an entirely comfortable conclusion, but for now there is no really satisfactory alternative explanation to the purchasing power parity puzzle. (p. 667-668.)

Perhaps, in addition, when these frictions are present, there is more scope for herding behavior and bubbles. Bubbles or herding might temporarily send the exchange rate off on disequilibrium paths that result in the appearance of slow convergence to the equilibrium. It is also suggestive to note that our empirical model of exchange rates is consistent with the RESP model except in one respect: it implies uncovered interest parity will not hold. (See Appendix 1.)

There is still a purchasing power parity puzzle, but this paper refines the puzzle. The new stylized fact that we document is that it is not unbelievably slow nominal price convergence that accounts for the persistence of real exchange rates. The challenge is to produce a theory that is consistent with the findings that nominal prices adjust relatively quickly (though our findings do not contradict either flexible-price or sticky-price models), that nominal and real exchange rates are highly volatile, and that nominal exchange rates converge very slowly to the PPP equilibrium.

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Appendix 1

The purpose of this appendix is to derive the behavior of real exchange-rate adjustment from a RESP model. The derivation helps understand the implicit restrictions that are usually put on price and exchange-rate changes, and where we differ. We present a two-country version of the RESP model.

Start with money demand equations in the home and foreign country, and interest parity (all constant terms will be suppressed for simplicity):

$$u_{1t} - p_{1t} = -\lambda_1 i_{1t} \tag{A1.1}$$

$$u_{2t} - p_{2t} = -\lambda_2 i_{2t} \tag{A1.2}$$

$$i_{1t} - i_{2t} = E_t(s_{t+1}) - s_t\,. \tag{A1.3}$$

Here, u_{1t} (u_{2t}) is the log of the money supply less money demand shifters in the home (foreign) country, and i_{1t} (i_{2t}) is the home (foreign interest rate.)

We define the equilibrium price, $\overline{p}_{1t}$ $(\overline{p}_{2t})$ as the level that p_{1t} (p_{2t}) would equal given current value of u_{1t} (u_{2t}). Under flexible prices, real interest rates are assumed constant, so nominal interest rates are assumed to equal the expected rate of inflation (plus a constant).

$$u_{1t} - \overline{p}_{2t} = -\lambda_1 (E_t(\overline{p}_{1,t+1}) - \overline{p}_{1t}) \tag{A1.4}$$

$$u_{2t} - \overline{p}_{2t} = -\lambda_2 (E_t(\overline{p}_{2,t+1}) - \overline{p}_{2t}) \tag{A1.5}$$

Each of (A1.4) and (A1.5) are univariate rational expectations difference equations. They have solutions of the form:

$$\bar{p}_{1t} = A_1(L)u_{1t} \tag{A1.6}$$

$$\bar{p}_{2t} = A_2(L)u_{2t} \tag{A1.7}$$

Here $A_1(L)$ ($A_2(L)$) is the lag-operator on money supply and money demand shocks in the home (foreign) country that solves equation (A1.4) (equation (A1.5)).

We posit that nominal prices in each country adjust slowly toward their equilibrium levels. But, we make two adjustments. First, only a fraction δ_1 of prices are sticky. A fraction $1-\delta_1$ adjust instantaneously. (In the foreign country, a fraction δ_2 of prices are sticky.) Second, we allow an i.i.d. shock to hit prices, so that even when $\delta_1 = 1$ or $\delta_2 = 1$ there can be some deviation of the actual price level from its expected level:

$$p_{1,t+1} - p_{1t} = -\theta_1(p_{1t} - \bar{p}_{1t}) + \delta_1 E_t(\bar{p}_{1,t+1}) + (1-\delta_1)\bar{p}_{1,t+1} - \bar{p}_{1t} + \varepsilon_{1,t+1} \tag{A1.8}$$

$$p_{2,t+1} - p_{2t} = -\theta_2(p_{2t} - \bar{p}_{2t}) + \delta_2 E_t(\bar{p}_{2,t+1}) + (1-\delta_2)\bar{p}_{2,t+1} - \bar{p}_{2t} + \varepsilon_{2,t+1} \tag{A1.9}$$

Prices each period adjust part of the way toward their equilibrium value, under the assumptions: $0 < \theta_1 < 1$ and $0 < \theta_2 < 1$. There are also terms that account for drift in the equilibrium prices.

Equations (A1.1), (A1.2) and (A1.3) imply

$$E_t(s_{t+1}) = s_t + \frac{1}{\lambda_1}(p_{1t} - u_{1t}) - \frac{1}{\lambda_2}(p_{2t} - u_{2t}) \tag{A1.10}$$

If long-run PPP holds, so $\bar{s}_t = \bar{p}_{1t} - \bar{p}_{2t}$, equations (A1.4) and (A1.5) yield:

$$E_t(\bar{s}_{t+1}) = \bar{s}_t + \frac{1}{\lambda_1}(\bar{p}_{1t} - u_{1t}) - \frac{1}{\lambda_2}(\bar{p}_{2t} - u_{2t}) \qquad \text{(A1.11)}$$

Subtracting (A1.11) from (A1.10),

$$E_t(s_{t+1}) - E_t(\bar{s}_{t+1}) = s_t - \bar{s}_t + \frac{1}{\lambda_1}(p_{1t} - \bar{p}_{1t}) - \frac{1}{\lambda_2}(p_{2t} - \bar{p}_{2t}) \qquad \text{(A1.12)}$$

Equations (A1.8), (A1.9) and (A1.12) can be written in matrix form as a three-equation homogenous system of difference equations:

$$\begin{bmatrix} E_t(p_{1,t+1} - \bar{p}_{1,t+1}) \\ E_t(p_{2,t+1} - \bar{p}_{2,t+1}) \\ E_t(s_{t+1} - \bar{s}_{t+1}) \end{bmatrix} = \begin{bmatrix} 1-\theta_1 & 0 & 0 \\ 0 & 1-\theta_2 & 0 \\ 1/\lambda_1 & 1/\lambda_2 & 1 \end{bmatrix} \begin{bmatrix} p_{1t} - \bar{p}_{1t} \\ p_{2t} - \bar{p}_{2t} \\ s_t - \bar{s}_t \end{bmatrix} \qquad \text{(A1.13)}$$

Diagonalizing equation (A1.13) yields

$$\begin{bmatrix} E_t(p_{1,t+1} - \bar{p}_{1,t+1}) \\ E_t(p_{2,t+1} - \bar{p}_{2,t+1}) \\ E_t(z_{t+1}) \end{bmatrix} = \begin{bmatrix} 1-\theta_1 & 0 & 0 \\ 0 & 1-\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{1t} - \bar{p}_{1t} \\ p_{2t} - \bar{p}_{2t} \\ z_t \end{bmatrix} \qquad \text{(A1.14)}$$

where

$$z_t = p_{1t} - \bar{p}_{1t} - \frac{\lambda_1\theta_1}{\lambda_2\theta_2}(p_{2t} - \bar{p}_{2t}) + \lambda_1\theta_1(s_t - \bar{s}_t). \qquad \text{(A1.15)}$$

Inspection of equation (A1.14) shows that imposing the condition that the system be expected to converge to the steady state requires $z_t = 0$. This is an important property of the RESP model, and the key difference between our model and the RESP model: that model makes $s_t - \bar{s}_t$ be a linear combination of $p_{1t} - \bar{p}_{1t}$ and $p_{2t} - \bar{p}_{2t}$. This is the requirement that the economy be on a stable saddle path. Our model does not impose that. As we discuss further below, our model is fundamentally different than the RESP model, even the version of the RESP model in which $\theta_1 \neq \theta_2$.

If $\theta_1 \neq \theta_2$, we will be unable to represent the dynamics of the real exchange rate only in terms of lagged values of the real exchange rate, because domestic and foreign prices converge at different speeds. But, if $\theta_1 = \theta_2 = \theta$ and $\lambda_1 = \lambda_2 = \lambda$, we can use equations (A1.12), (A1.15) and the condition that $z_t = 0$ to get:

$$E_t(s_{t+1} - \bar{s}_{t+1}) = (1-\theta)(s_t - \bar{s}_t) \tag{A1.16}$$

Equations (A1.8), (A1.9) and (A1.16) show that domestic prices, foreign prices and the exchange rate all converge at the same speed (in expectations) when $\theta_1 = \theta_2$ and $\lambda_1 = \lambda_2$. Defining the real exchange rate as $q_t \equiv s_t - p_{1t} + p_{2t}$, we have:

$$E_t(q_{t+1} - \bar{q}_{t+1}) = (1-\theta)(q_t - \bar{q}_t)\,.$$

It may seem that merely relaxing the assumptions of $\theta_1 = \theta_2$ and $\lambda_1 = \lambda_2$ yields a model in which domestic prices, foreign prices and exchange rates converge at different speeds. Clearly in this case, domestic prices converge at a rate of θ_1 and foreign prices converge at the rate θ_2. The exchange rate equation could be written, for example, as:

$$E_t(s_{t+1} - \bar{s}_{t+1}) = (1-\theta_1)(s_t - \bar{s}_t) - \frac{1}{\lambda_2}(1 - \frac{\theta_1}{\theta_2})(p_{2t} - \bar{p}_{2t})$$

However, there is no unique way to write the exchange rate equation, because the condition that $z_t = 0$ implies that $s_t - \bar{s}_t$ is a linear combination of $p_{1t} - \bar{p}_{1t}$ and

$p_{2t} - \bar{p}_{2t}$. That is, there are only two independent equations in the dynamic system (whether or not $\theta_1 = \theta_2$) in the RESP model. The reduced dimension of the system is a result of the requirement that is imposed that the system converges to steady state. The exchange rate must jump in response to shocks so it is on the path that leads to the steady state.

So, our model can be thought of as generalizing the RESP model in two ways: we do not require that prices in both countries and the exchange rate converge at the same speed, and we allow for three independent equations for $s_t - \bar{s}_t$, $p_{1t} - \bar{p}_{1t}$, and $p_{2t} - \bar{p}_{2t}$.

To write the system of stochastic equations implied by the RESP model, note

$$\bar{p}_{1,t+1} - E_t(\bar{p}_{1,t+1}) = A_{1,0} u_{1,t+1}, \tag{A1.17}$$

where $A_{1,0}$ is the first term in $A_1(L)$. Similarly:

$$\bar{p}_{2,t+1} - E_t(\bar{p}_{2,t+1}) = A_{2,0} u_{2,t+1}. \tag{A1.18}$$

We can use this to write equations for $p_{1,t+1} - \bar{p}_{1,t+1}$ and $p_{2,t+1} - \bar{p}_{2,t+1}$:

$$p_{1,t+1} - \bar{p}_{1,t+1} = (1-\theta_1)(p_{1t} - \bar{p}_{1t}) - \delta_1 A_{1,0} u_{1,t+1} + \varepsilon_{1,t+1},$$

$$p_{2,t+1} - \bar{p}_{2,t+1} = (1-\theta_2)(p_{2t} - \bar{p}_{2t}) - \delta_2 A_{2,0} u_{2,t+1} + \varepsilon_{2,t+1}.$$

Then define $v_{1t} \equiv -\delta_1 A_{1,0} u_{1t} + \varepsilon_{1t}$, $v_{2t} \equiv -\delta_2 A_{2,0} u_{2t} + \varepsilon_{2t}$, $\bar{v}_{1t} \equiv A_{1,0} u_{1t}$, and $\bar{v}_{2t} \equiv A_{2,0} u_{2t}$. These random variables correspond to the error terms in the price-adjustment equations (1), and the equilibrium price equations (3) in the text.

Then, because of the saddle path property that tells us $z_{t+1}=0$, we have:

$$s_{t+1}-\bar{s}_{t+1}=\frac{-1}{\lambda_1\theta_1}(p_{1,t+1}-\bar{p}_{1,t+1})+\frac{1}{\lambda_2\theta_2}(p_{2,t+1}-\bar{p}_{2,t+1})$$

Therefore, $$v_{t+1}^s=\frac{-1}{\lambda_1\theta_1}v_{1,t+1}+\frac{1}{\lambda_2\theta_2}v_{2,t+1}. \tag{A1.19}$$

Define $\kappa_1\equiv 1/\lambda_1\theta_1$ and $\kappa_2\equiv 1/\lambda_2\theta_2$. Then we can write the covariance matrix as:

$$Var\begin{bmatrix} v_{1t} \\ v_{2t} \\ v_t^s \\ \bar{v}_{1t} \\ \bar{v}_{2t} \end{bmatrix}=\begin{bmatrix} \sigma_{p,1}^2 & 0 & -\kappa_1\sigma_{p,1}^2 & -\delta_1\sigma_{\bar{p},1}^2 & 0 \\ 0 & \sigma_{p,2}^2 & \kappa_2\sigma_{p,2}^2 & 0 & -\delta_2\sigma_{\bar{p},2}^2 \\ -\kappa_1\sigma_{p,1}^2 & \kappa_2\sigma_{p,2}^2 & \kappa_1{}^2\sigma_{p,1}^2+\kappa_2{}^2\sigma_{p,2}^2 & \delta_1\kappa_1\sigma_{\bar{p},1}^2 & -\delta_2\kappa_2\sigma_{\bar{p},2}^2 \\ -\delta_1\sigma_{\bar{p},1}^2 & 0 & \delta_1\kappa_1\sigma_{\bar{p},1}^2 & \sigma_{\bar{p},1}^2 & 0 \\ 0 & -\delta_2\sigma_{\bar{p},2}^2 & -\delta_2\kappa_2\sigma_{\bar{p},2}^2 & 0 & \sigma_{\bar{p},2}^2 \end{bmatrix} \tag{A1.20}$$

In equation (A1.20), there are only eight independent elements to estimate: δ_1, δ_2, κ_1, κ_2, $\sigma_{p,1}^2$, $\sigma_{p,2}^2$, $\sigma_{\bar{p},1}^2$, and $\sigma_{\bar{p},2}^2$. Of course, the usual restriction that the lower and upper triangles be identical reduces the dimension of the matrix to fifteen. There are four additional zero restrictions that reduce the dimension to eleven. The other three restrictions come about because of the saddle-path restriction in equation (A1.19). Without that saddle-path restriction, there would be eleven elements to estimate:

$$Var\begin{bmatrix} v_{1t} \\ v_{2t} \\ v_t^s \\ \bar{v}_{1t} \\ \bar{v}_{2t} \end{bmatrix}=\begin{bmatrix} \sigma_{p,1}^2 & 0 & \sigma_{sp,1} & \sigma_{p\bar{p},1} & 0 \\ 0 & \sigma_{p,2}^2 & \sigma_{sp,2} & 0 & \sigma_{p\bar{p},2} \\ \sigma_{sp,1} & \sigma_{sp,2} & \sigma_s^2 & \sigma_{s\bar{p},1} & \sigma_{s\bar{p},2} \\ \sigma_{p\bar{p},1} & 0 & \sigma_{s\bar{p},1} & \sigma_{\bar{p}}^2 & 0 \\ 0 & \sigma_{p\bar{p},2} & \sigma_{s\bar{p},2} & 0 & \sigma_{\bar{p},2}^2 \end{bmatrix}. \tag{A1.21}$$

In our estimates of the bivariate models, we impose $\delta_1 = \delta_2$, and $\kappa_1 = \kappa_2$. Those restrictions imply the proportionality restrictions of equations (5), (6) and (7).

Finally, as noted in the conclusions section, if we retain all of the equations of the RESP model (equations (A1.1), (A1.2), (A1.4)-(A1.9)), but do not assume uncovered interest parity (A1.3) and instead assume that exchange rates adjust to equilibrium at some rate $1-\zeta$:

$$s_{t+1} - \overline{s}_{t+1} = (1-\zeta)(s_t - \overline{s}_t) + v_t^s ,$$

we can solve to find that the uncovered interest parity condition does not hold:

$$E_t(s_{t+1}) - s_t = i_{1t} - i_{2t} - \frac{1}{\lambda_1}(p_{1t} - \overline{p}_{1t}) + \frac{1}{\lambda_2}(p_{2t} - \overline{p}_{2t}) - \zeta(s_t - \overline{s}_t) . \qquad \text{(A1.22)}$$

Appendix 2

This Appendix details estimation of the two-country models. The estimation of the seven-country model generalizes in the obvious ways.

For estimation given the restrictions, we cast the model in state-space form and apply the Kalman filter and maximum likelihood based upon the prediction error decomposition as discussed in Harvey (1993). The state equation, which represents the evolution of the unobserved components, is

$$\beta_t = \tilde{\mu} + F\beta_{t-1} + \tilde{v}_t, \tag{A2.1}$$

where

$$F = \begin{bmatrix} \phi_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\phi_{\bar{p}} & -\phi_{\bar{p}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\phi_{\bar{p}} & -\phi_{\bar{p}} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\beta_t = \begin{bmatrix} p_{1t} - \bar{p}_{1t} \\ p_{2t} - \bar{p}_{2t} \\ s_t - \bar{s}_t \\ \bar{p}_{1t} \\ \bar{p}_{1,t-1} \\ \bar{p}_{2t} \\ \bar{p}_{2,t-1} \end{bmatrix}, \quad \tilde{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mu_1(1-\phi_{\bar{p}}) \\ 0 \\ \mu_2(1-\phi_{\bar{p}}) \\ 0 \end{bmatrix}, \text{ and } \tilde{v}_t = \begin{bmatrix} v_{1t} \\ v_{2t} \\ v_t^s \\ \bar{v}_{1t} \\ 0 \\ \bar{v}_{2t} \\ 0 \end{bmatrix}.$$

Note that the covariance matrix for $\tilde{v}_t$, denoted $Q \equiv E[\tilde{v}_t \tilde{v}_t']$, is a simple linear transformation of (A1.21). Meanwhile, the observation equation, which relates the price levels and exchange rate to their unobserved components, is

$$y_t = A + H\beta_t, \tag{A2.2}$$

where

$$y_t = \begin{bmatrix} p_{1t} \\ p_{2t} \\ s_t \end{bmatrix},\ A = \begin{bmatrix} \bar{p}_{1,-1} \\ \bar{p}_{2,-1} \\ \bar{s}_{-1} \end{bmatrix},\ H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 \end{bmatrix}.$$

The inclusion of a separate initial value for the equilibrium exchange rate corresponds to relative, rather than absolute, PPP.[11] Meanwhile, we include initial values for the equilibrium price levels in A to address the lack of appropriate startup values for the Kalman filter. In particular, equilibrium prices follow unit root processes that have no unconditional expected values. By including initial values in estimation here, we are able to normalize the corresponding initial state variables to zero. Then, we estimate equilibrium prices by adding the estimated initial values to the filter output.[12]

The Kalman filter for this state-space model is given by the following six equations:

$$\beta_{t|t-1} = \tilde{\mu} + F\beta_{t-1|t-1} \tag{A2.3}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \tag{A2.4}$$

$$\eta_{t|t-1} = y_t - H\beta_{t|t-1} \tag{A2.5}$$

$$f_{t|t-1} = HP_{t|t-1}H' \tag{A2.6}$$

$$\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1} \tag{A2.7}$$

[11] Since price data is in index form, only relative PPP is tenable.
[12] An alternative approach would be to make an arbitrary guess about the corresponding Kalman filter startup values and assign our guess an extremely large variance.

$$P_{t|t} = P_{t|t-1} - K_t H P_{t|t-1} \tag{A2.8}$$

where $\beta_{t|t-1} \equiv E_{t-1}[\beta_t]$, for example, denotes the expectation of β_t conditional on information up to time $t-1$; $P_{t|t-1}$ is the variance-covariance of $\beta_{t|t-1}$; $\eta_{t|t-1}$ is a vector of the conditional forecast errors of the observed series; $f_{t|t-1}$ is the variance-covariance of $\eta_{t|t-1}$; and $K_t \equiv P_{t|t-1} H' f_{t|t-1}^{-1}$ is the Kalman gain.

Given arbitrary initial parameter estimates and initial values $\beta_{0|0}$ and $P_{0|0}$ based on unconditional expected values and the normalizations discussed above, we solve equations (A2.3)-(A2.8) recursively for $t = 1,\ldots,T$ to obtain filtered inferences about β_t conditional on information up to time *t*.

Then, as a by-product of the Kalman filter, we obtain $\eta_{t|t-1}$ and $f_{t|t-1}$, which allow us to calculate maximum likelihood estimates of the various parameters based on the prediction error decomposition (Harvey, 1993):

$$\max_{\theta}\left\{ l(\theta) = -\frac{1}{2}\sum_{t=1}^{T} \ln((2\pi)^3 |f_{t|t-1}|) - \frac{1}{2}\sum_{t=1}^{T} \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1} \right\}, \tag{A2.9}$$

where θ is the vector of parameters.

Table 1
Maximum Likelihood Estimates for the Two-Country Models

Parameter	US/Canada	US/France	US/Germany	US/Italy	US/Japan	US/UK
ϕ_p	0.273 (0.201)	0.478 (0.128)	0.480 (0.114)	0.681 (0.244)	0.641 (0.163)	0.569 (0.120)
ϕ_s	0.987 (0.015)	0.942 (0.033)	0.928 (0.032)	0.927 (0.034)	0.958 (0.022)	0.919 (0.038)
$\phi_{\bar{p}}$	0.955 (0.026)	0.965 (0.020)	0.926 (0.029)	0.938 (0.028)	0.962 (0.019)	0.935 (0.030)
$\sigma_{p,1}$	0.430 (0.059)	0.397 (0.050)	0.358 (0.059)	0.421 (0.069)	0.421 (0.078)	0.327 (0.039)
$\sigma_{p,2}$	0.365 (0.058)	0.235 (0.037)	0.396 (0.049)	0.359 (0.100)	0.497 (0.068)	0.783 (0.032)
σ_s	2.193 (0.158)	5.612 (0.423)	5.900 (0.426)	5.435 (0.398)	6.191 (0.489)	5.265 (0.389)
$\sigma_{\bar{p},1}$	0.230 (0.039)	0.263 (0.042)	0.295 (0.050)	0.276 (0.050)	0.252 (0.050)	0.324 (0.036)
$\sigma_{\bar{p},2}$	0.267 (0.060)	0.268 (0.042)	0.212 (0.046)	0.527 (0.103)	0.299 (0.055)	0.535 (0.021)

Note: Standard errors in parentheses

Table2
Likelihood Ratio Specification Tests

Test	US/Canada	US/France	US/Germany	US/Italy	US/Japan	US/UK
Speed of Adjustment $H_0: \phi_p = \phi_s$ $H_1: \phi_p \neq \phi_s$	5.585 (0.018) 1 d.f.	11.633 (0.000) 1 d.f.	7.555 (0.224) 1 d.f.	1.477 (0.224) 1 d.f.	1.772 (0.183) 1 d.f.	3.665 (0.056) 1 d.f.
Symmetry Restrictions $H_0: \phi_p^1 = \phi_p^2; \phi_{\bar{p}}^1 = \phi_{\bar{p}}^2$ and equations (5),(6),(7) H_1: no symmetry restrictions	6.778 (0.238) 5 d.f.	1.946 (0.857) 5 d.f.	8.144 (0.148) 5 d.f.	4.102 (0.535) 5 d.f.	9.784 (0.082) 5 d.f.	3.838 (0.573) 5 d.f.
Independent Innovations H_0: diagonal covariance H_1: reported model	1.344 (0.719) 3 d.f.	4.406 (0.221) 3 d.f.	3.235 (0.357) 3 d.f.	2.658 (0.447) 3 d.f.	5.961 (0.114) 3 d.f.	1.542 (0.673) 3 d.f.

Table2 (continued) Likelihood Ratio Specification Tests						
Test	US/Canada	US/France	US/Germany	US/Italy	US/Japan	US/UK
Structural Break in 1980 H_0 : No break in mean of inflation process H_1 : Structural break	3.301 (0.192) 2 d.f.	4.257 (0.119) 2 d.f.	2.941 (0.230) 2 d.f.	5.570 (0.062) 2 d.f.	2.562 (0.278) 2 d.f.	3.979 (0.137) 2 d.f.
Speed of Adjustment Conditional on Structural Break in 1980 $H_0 : \phi_p = \phi_s$ $H_1 : \phi_p \neq \phi_s$	0.604 (0.437) 1 d.f.	7.590 (0.006) 1 d.f.	5.913 (0.015) 1 d.f.	1.580 (0.209) 1 d.f.	0.021 (0.885) 1 d.f.	3.799 (0.051) 1 d.f.

Notes: Chi-square statistics are reported. P-values are in parentheses. "d.f." are degrees of freedom.

Table 3
Maximum Likelihood for the Seven-Country Model

Parameter	US	Canada	France	Germany	Italy	Japan	UK
ϕ_p	0.607 (0.035)						
ϕ_s	0.965 (0.010)						
$\phi_{\bar{p}}$	0.965 (0.010)						
$\sigma_{p,i}$	0.538 (0.034)	0.449 (0.030)	0.303 (0.025)	0.457 (0.031)	0.524 (0.082)	0.525 (0.033)	1.064 (0.084)
$\sigma_{s,j}$		2.320 (0.042)	6.093 (0.001)	6.167 (0.002)	5.778 (0.037)	6.755 (0.079)	5.921 (0.111)
$\sigma_{\bar{p},i}$	0.191 (0.004)	0.210 (0.028)	0.225 (0.005)	0.176 (0.003)	0.453 (0.015)	0.317 (0.007)	0.274 (0.003)

Note: Standard errors are reported in parentheses. For computational reasons, we calculate the standard errors for the seven-country model using the outer-product-gradient method and holding the off-diagonal elements of the variance-covariance matrix fixed at their estimated values. Thus, these standard errors will tend to be biased downwards and are reported for illustrative purposes only.

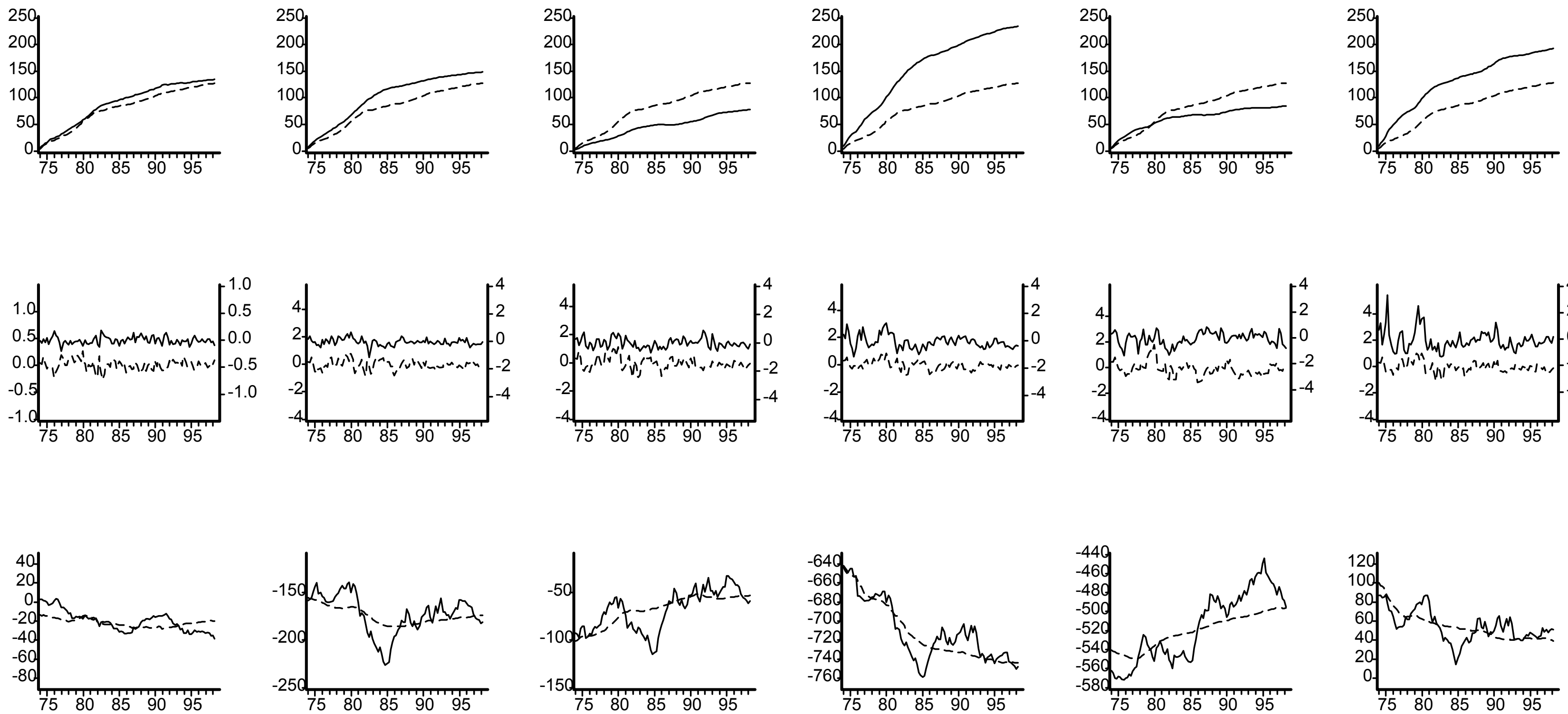

Fig. 1 – Price Components and Exchange Rates for the Two-Country Models
The first row displays estimated equilibrium price levels for country pairs US/Canada, US/France, US/Germany, US/Italy, US/Japan, and US/UK, respectively. In order to depict cumulative inflation over the sample period, the equilibrium prices are normalized to begin at zero. The second row displays the estimated movements away from equilibrium for the same six country pairs. In both the first and second rows, the dashed lines correspond to the US components. The third row displays the observed nominal exchange rates for the country pairs, with the dashed lines representing estimated equilibrium exchange rates implied by PPP. Estimates of the unobserved components come from the Kalman filter for the two-country models. The *y*-axis units are logarithms multiplied by 100.

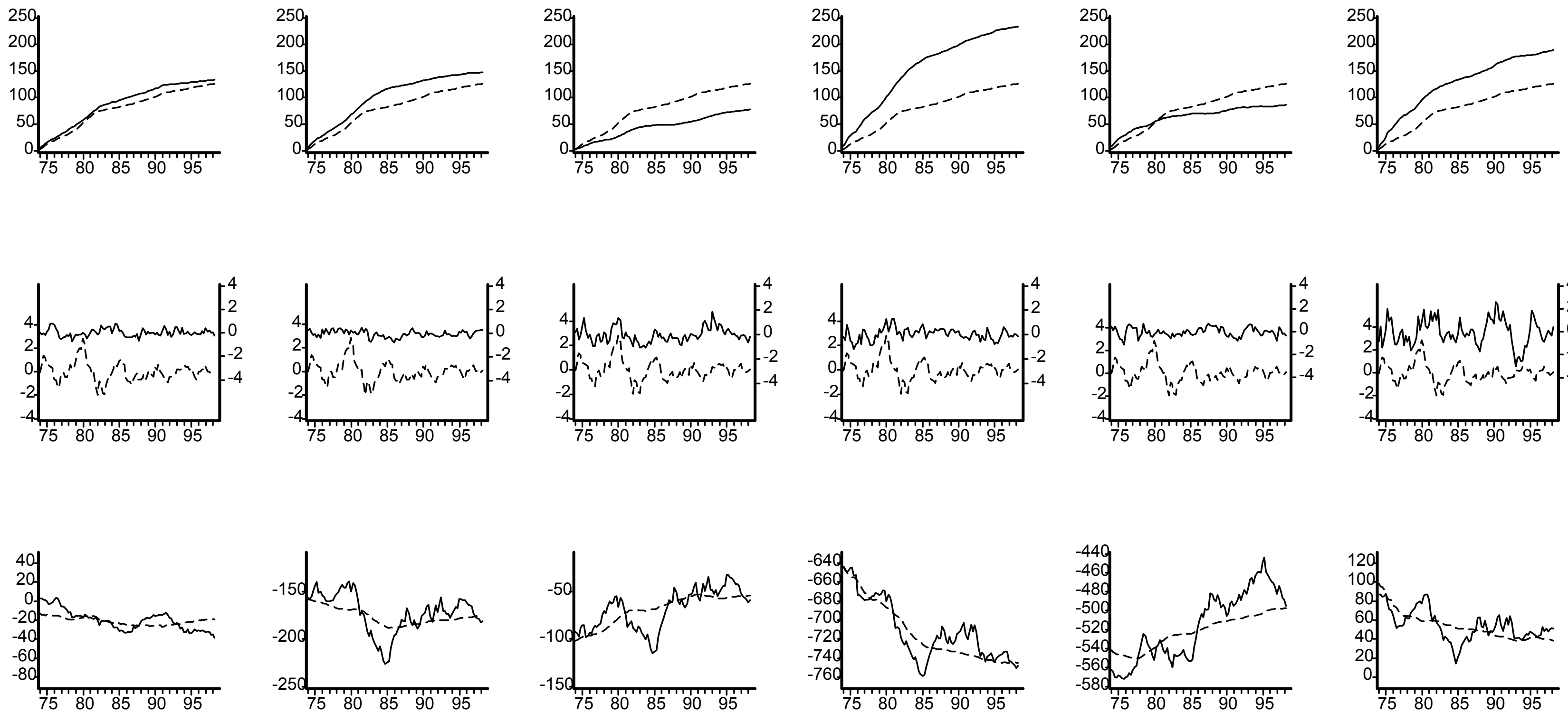

Fig. 2 – Price Components and Exchange Rates for the Seven-Country Model
The first row displays estimated equilibrium price levels for country pairs US/Canada, US/France, US/Germany, US/Italy, US/Japan, and US/UK, respectively. In order to depict cumulative inflation over the sample period, the equilibrium prices are normalized to begin at zero. The second row displays the estimated movements away from equilibrium for the same six country pairs. In both the first and second rows, the dashed lines correspond to the US components. The third row displays the observed nominal exchange rates for the country pairs, with the dashed lines representing estimated equilibrium exchange rates implied by PPP. Estimates of the unobserved components come from the Kalman filter for the seven-country model. The *y*-axis units are logarithms multiplied by 100.